A Study of Opinion Dynamics Models

Team Members

Nishok Kumar Sundaressan

Sumeet Shirgure

Nilay Pochhi

Parantap Dansana

Introduction.

Opinion dynamics is the study of the evolution of opinions in a community over a specific interval of time.

The study of opinion dynamics can be useful in understanding decision-making behaviour, tracking the spread of ideas in society, creating effective bias combating techniques, and following the emergence and popularity of new ideas and technologies.

Scope.

We have considered a number of models to experiment with.

Some of them are based on a specific network of connections between the members, and hence are modelled as graphs. Others just take into account the opinions as a condition for interaction.

The opinions themselves can be either discrete or continuous, scalar or a vector.

Multidimensional FJ Model - Basic Description

- Promotes and extends the DeGroot iterative pooling scheme, taking its origins in French's "Theory of Social Power".
- Actors can also factor their initial opinions, or prejudices, into every iteration of opinion.
- The multi-dimensional model explored accounts for interdependent issues.
- Each agent allocates weights to the displayed opinions of others under the constraint of an ongoing allocation of weight to the agent's initial opinion.
- Opinion values are continuous in nature.
- The network is represented as a row-stochastic matrix.

Multidimensional FJ Model - Mathematical formulation

Update step at each stage k = 0,1,2,3,..

$$X_i(k+1) = l_{ii} * C * \sum_{j=1}^{n} (w_{ij} * X_j(k)) + (1 - l_{ii}) * u_i$$

Convergence criteria

$$x_i = \lim_{k \to \infty} x_i(k)$$ Or $$X = ((\Lambda W) \otimes C) * X + (I_n - \Lambda) * X(0)$$

and C should be row-stochastic in order for model to be stable and C should be regular for model to converge.

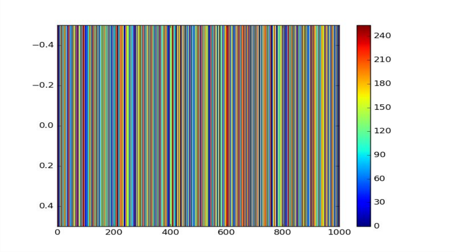
- The elements of each vector xi(k)=(x1 i (k),...,xm i (k)) stand for the opinions of the ith agent on m different issues.
- W ∈ Rn×n is a row-stochastic matrix of interpersonal influences.
- L is a diagonal matrix of actors' susceptibilities to the social influence $0 \le l_{ii} \le l$ for all i
- $x_i(0) = u_i$ for all i.
- C ∈ Rm×m is a constant "coupling matrix" , henceforth called the matrix of multi-issues dependence structure (MiDS).

Multidimensional FJ Model - Experiment Parameters

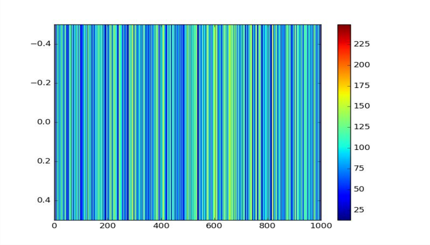
- Number of social actors = 1000
- Row-stochastic matrix W is made in such a way that each agent has four neighbours.
- Number of different issues = 10
- Initialization of matrix is done in two ways - with random initialization and with initialization from Gaussian distribution.
- The various time steps (10000 interactions per user) at which each agent interacts are obtained with help of a poisson distribution. Parameter for distribution is kept same for all users in one experiment, and is randomly fixed for each user in another experiment.
- Value of delta matrix is sampled from uniform distribution. For the MiDS matrix, the diagonal values are initialized with a value between 0.7 and 0.9 (to ensure convergence) and rest of values are initialized in such a manner that MiDS is row-stochastic.

Multidimensional FJ Model - Plots

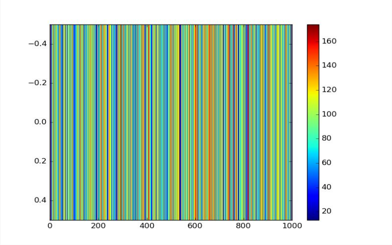
Experiment 1: W initialized using Uniform Distribution



Initial opinions at stage 0

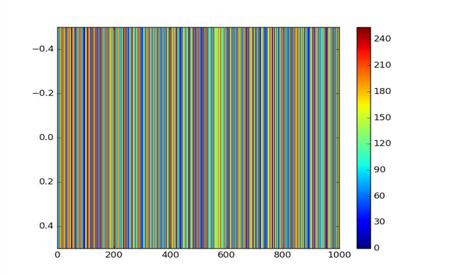


Opinions at stage 12000

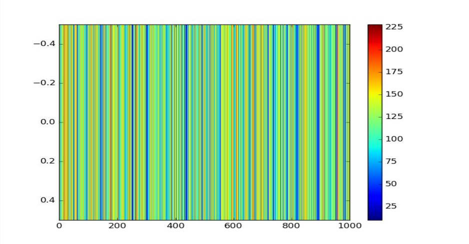


Final converged opinions

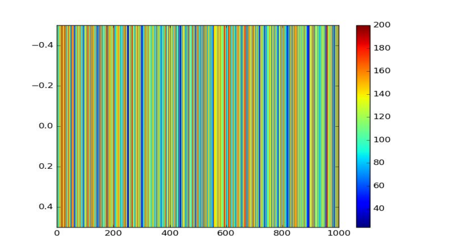
Experiment 2: W initialized using Gaussian Distribution



Initial opinions at stage 0



Opinions at stage 10000



Final converged opinions

Voter Model - Basic Description

- Proposed by Yildiz et. al
- This model has potential applications in engineering arena for coordinating mobile agents interconnected through a communication networks, to modeling collective behavior in human societies
- Opinion of an individual is represented by a discrete value often referred to as a colour.
- The interactions are modeled as an undirected graph.
- Opinion evolution may be of two types:
  - Random
  - Majority (called as Label Propagation Algorithm)

Voter Model - Mathematical Formulation

- Each node schedules its next update with a poisson process of rate 1.

$$c_i[k+1] = X \qquad X \sim \frac{1}{d_i} \sum_{c \epsilon \{1,2,\ldots,N\}} Q_{i,c}[k] \delta(x-c)$$

  - Q is number of neighbours of node i having colour c
  - d_i is degree of node i

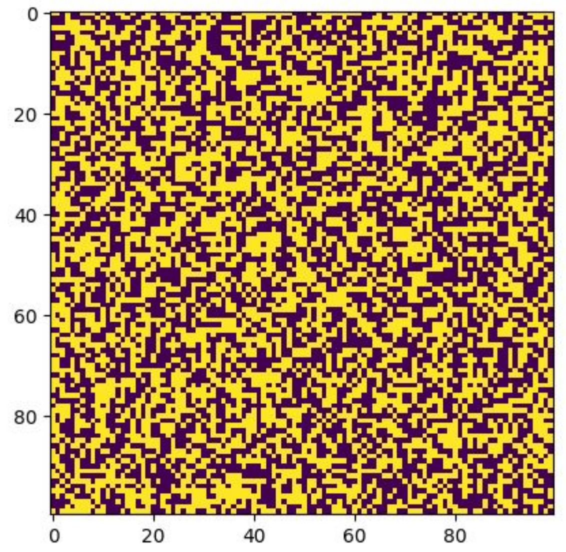
- Can be simply defined as randomly choosing a neighbour’s value.

Voter Model - Experiment Parameters

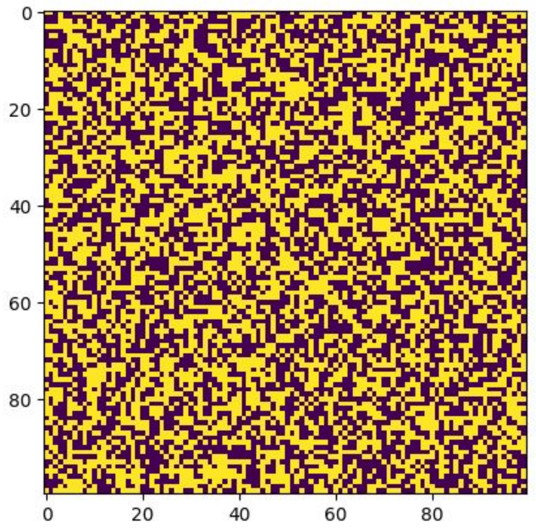
- Number of nodes - 10000
- Experiments are performed on opinions modeled as a ***Lattice Graph.***
- Opinion Initializations experimented upon:
  - Random
  - Gaussian Kernel
- Poisson process rate used - 1
- Ways in which opinions are changed:
  - Random
  - Majority
- Number of timesteps: 100000
- Opinions used - 0/1

Voter Model - Illustration (Random Voter)

- Random Initialization - Initial step

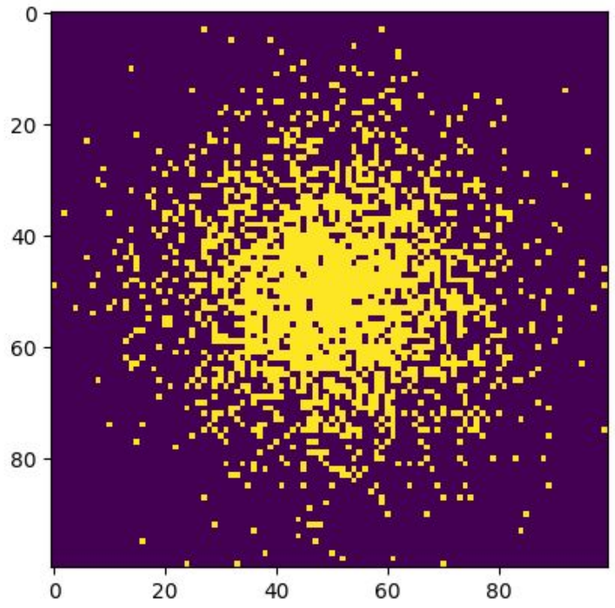


- Random Initialization - Final step

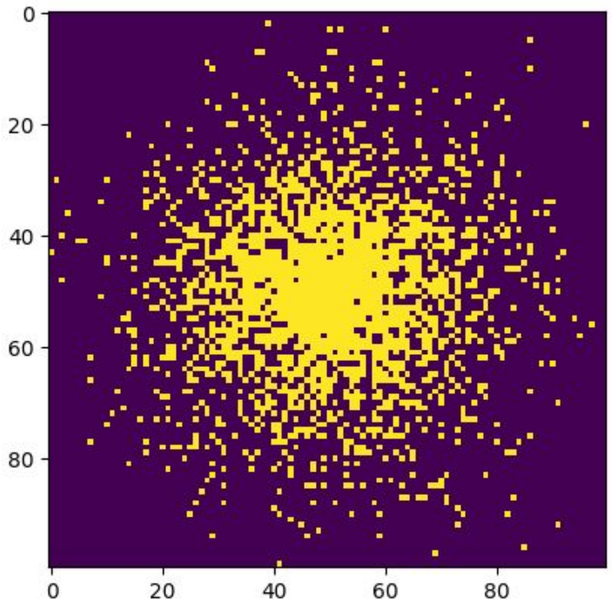


Voter Model - Illustration (Random Voter)

- Gaussian Initialization - Initial step

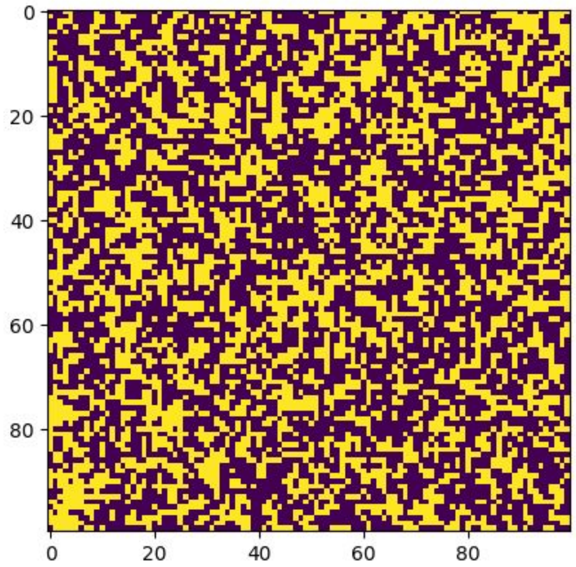


- Gaussian Initialization - Final step

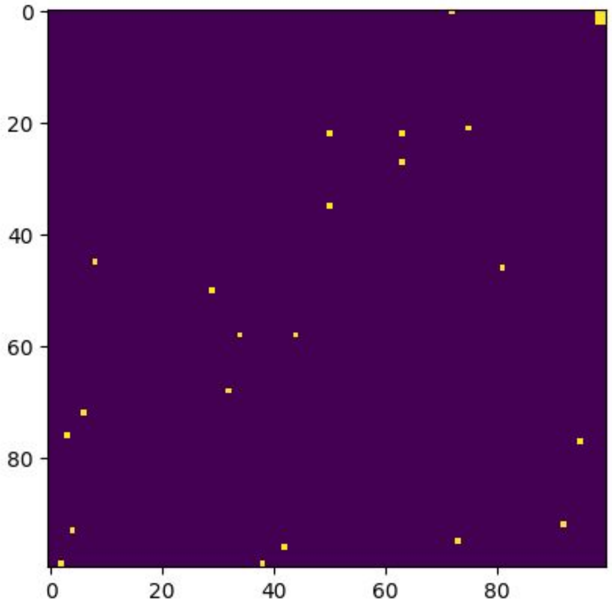


Voter Model - Illustration (Majority Voter)

- Random Initialization - Initial step

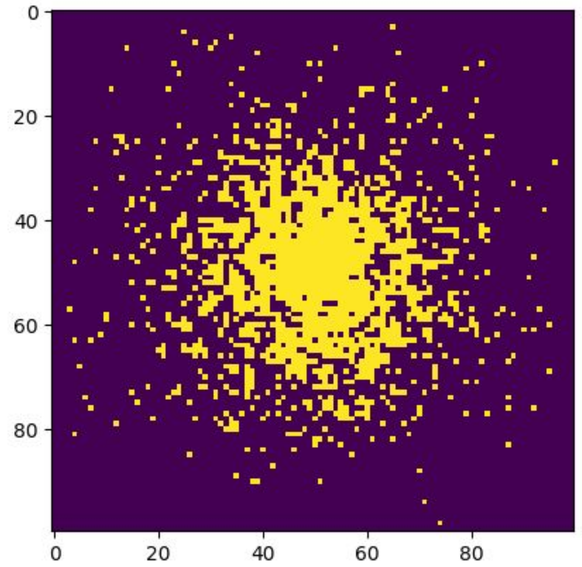


- Random Initialization - Final step

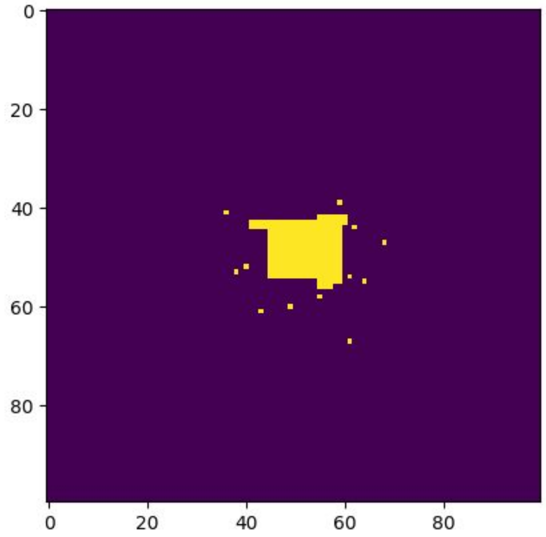


Voter Model - Illustration (Majority Voter)

- Gaussian Initialization - Initial step



- Gaussian Initialization - Final step



AxelRod's Model - Basic Description

- This model here deals with how people do indeed become more similar as they interact, but also provides an explanation of why the tendency to converge stops before it reaches completion.

- In this model there is a L×L square lattice of cells. Each cell represents a stationary individual who is endowed with a certain culture.

- An individual's culture is characterised by a list of f features, or dimensions of culture (e.g. language, religion, style of dress...); for each feature there is a set of q traits, which are the alternative values the feature may have.

AxelRod’s Model - Basic Description

- All agents share the same value for f, and all features have the same value q. Thus, individual i's culture is represented by a vector xi of f variables, where each variable takes an integer value in the range [0, q - 1]. Initially, individuals are assigned a random culture.

- When a agent i interacts with agent j, agent i inherits one of the feature chosen randomly of agent j .

- In this way, agent i approaches agent j's cultural interests.

AxelRod’s Model - Mathematical Formulation

- the state of the system can be characterised by a L×L array where each element corresponds to one cell of the grid and stores its agent's cultural vector.

- The number of possible states is q^{f*L^2}.

- In particular, the transition probability between any two states that differ in more than one agent's cultural feature is necessarily zero.

AxelRod's Model - Mathematical Formulation

- The transition probability from state i to state j in general:
  - If in state i every pair of neighbouring agents have cultures that are either identical or completely different, then state i is absorbing, i.e. $p_{i,i}$= 1 and $p_{i,j}$ = 0
  - If, on the other hand, in state i there are at least two neighbouring agents who share at least one cultural attribute and their cultures are not identical, then $p_{i,j}$ < 1.
  - $$p_{i,j} = \frac{1}{L^2 \cdot h_{ij}} \sum_{r \in H_{ij}} \frac{n_{kr}}{f} \cdot \frac{1}{f - n_{kr}}$$

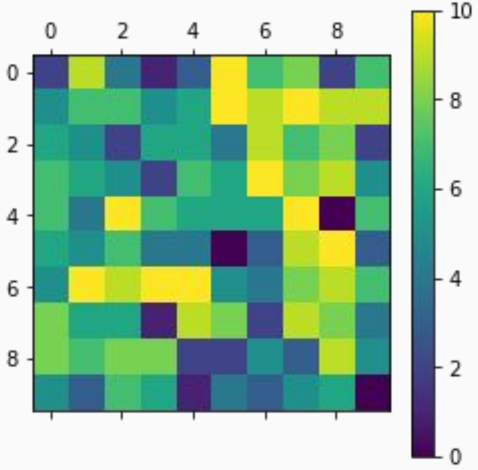
AxelRod's Model - Experiment Parameters

- Number of nodes - 100
- Experiments are performed on opinions modeled as a ***Lattice Graph.***
- Opinion Initializations experimented upon:
  - Uniform Random Distribution
  - Gaussian Random Distribution
- Ways in which opinions are changed:
  - Uniform Random
- Number of timesteps: 105259
- Number of features: 20
- Opinions used - [0, 10]

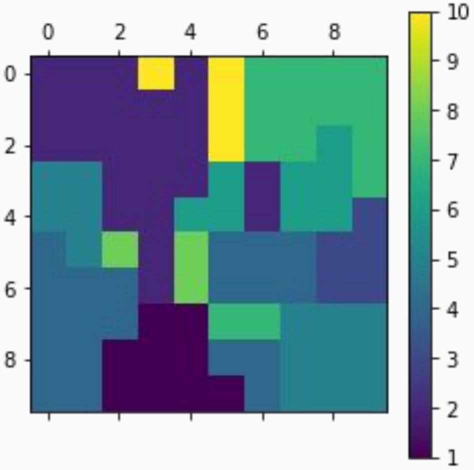
AxelRod’s Model - Plots

Features initialized using Gaussian Distribution:

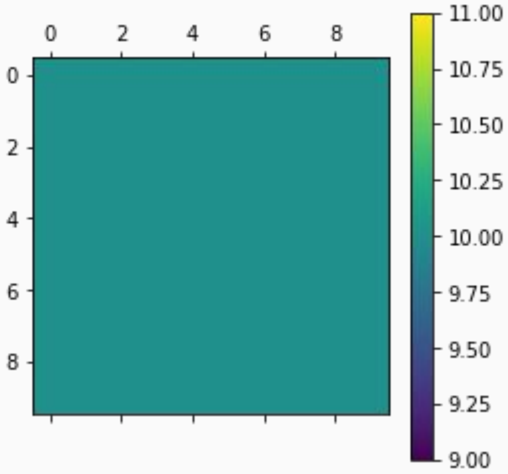
Feature1



Initial opinions at stage 0



opinions after timestamp 10525

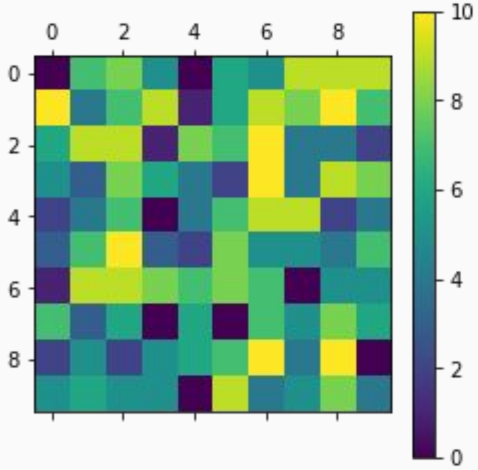


opinions after convergence

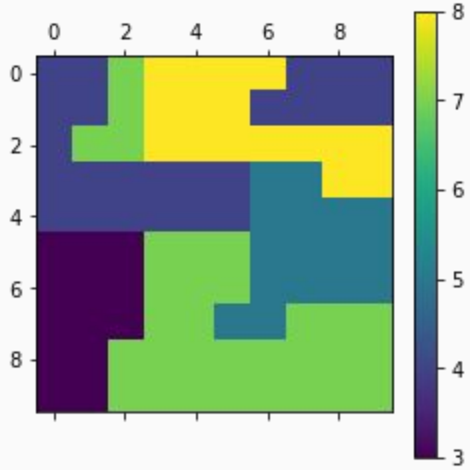
AxelRod’s Model - Plots

Features initialized using Gaussian Distribution:

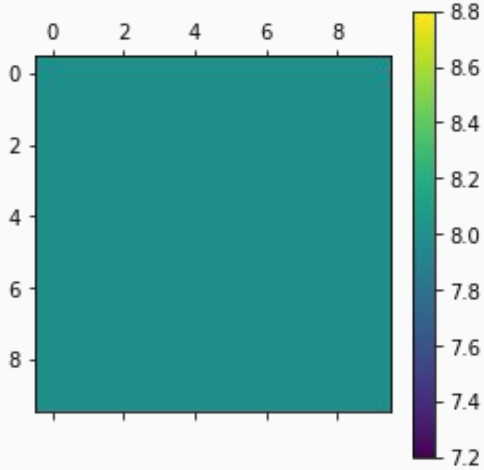
Feature2



Initial opinions at stage 0



opinions after timestamp 10525

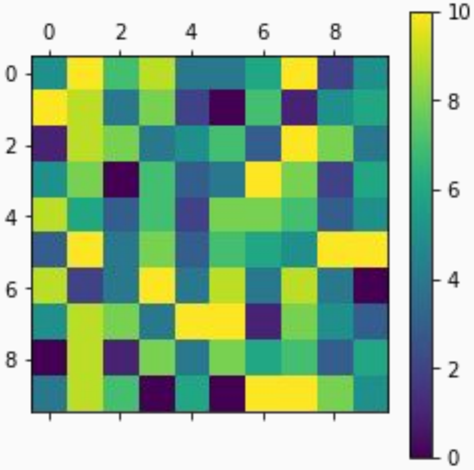


opinions after convergence

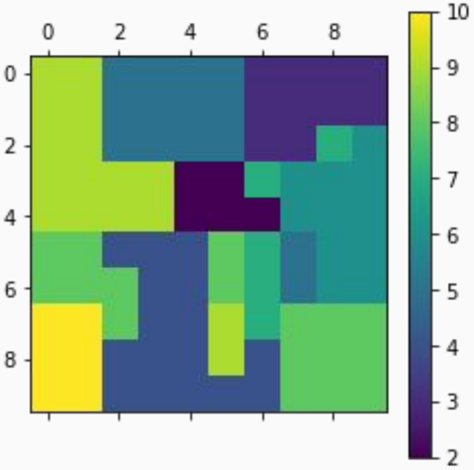
AxelRod's Model - Plots

Features initialized using Gaussian Distribution:

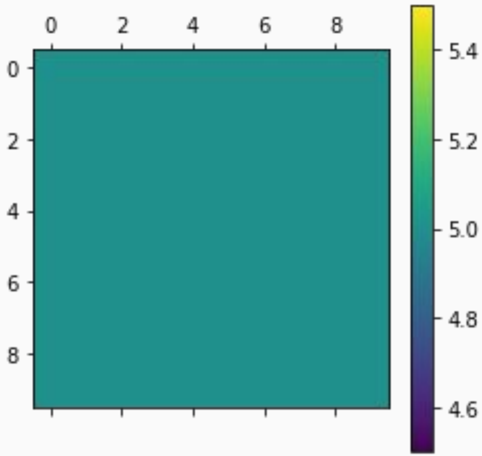
Feature3



Initial opinions at stage 0



opinions after timestamp 10525



opinions after convergence

The HK Model (Bounded Confidence Model)

- Introduced by Hegselmann and Krause.
- Non linear model, that assumes no social structure.
- Opinions are modelled as a real valued scalar function.
- A member's opinion is influenced only by the opinions that do not differ much.
- This threshold represents the confidence a member has in their own opinion.
- Simulations of such a dynamic allows us to observe the situations where the members reach consensus.

The HK Model (Bounded Confidence Model)

- Mathematically, the opinions of all the members are given by a vector $\mathbf{x}$.
- The values of x_i and x_j changes according to the equation :

$$x_i^{(t+1)} = x_i^{(t)} + \mu(x_j^{(t)} - x_i^{(t)})$$

$$x_j^{(t+1)} = x_j^{(t)} + \mu(x_i^{(t)} - x_j^{(t)})$$

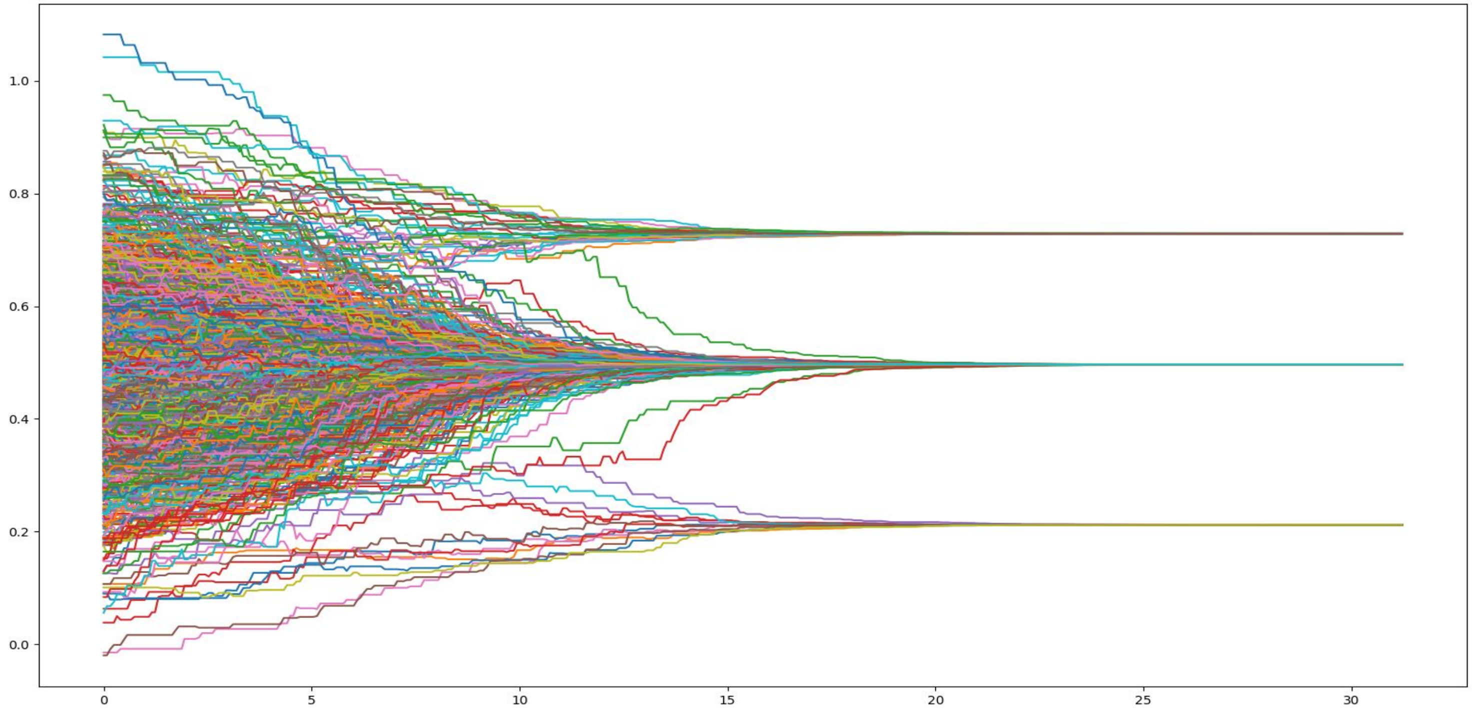
- The distribution of these interactions (between member i and j) is again modelled as a Poisson process.

The HK Model (Bounded Confidence Model)

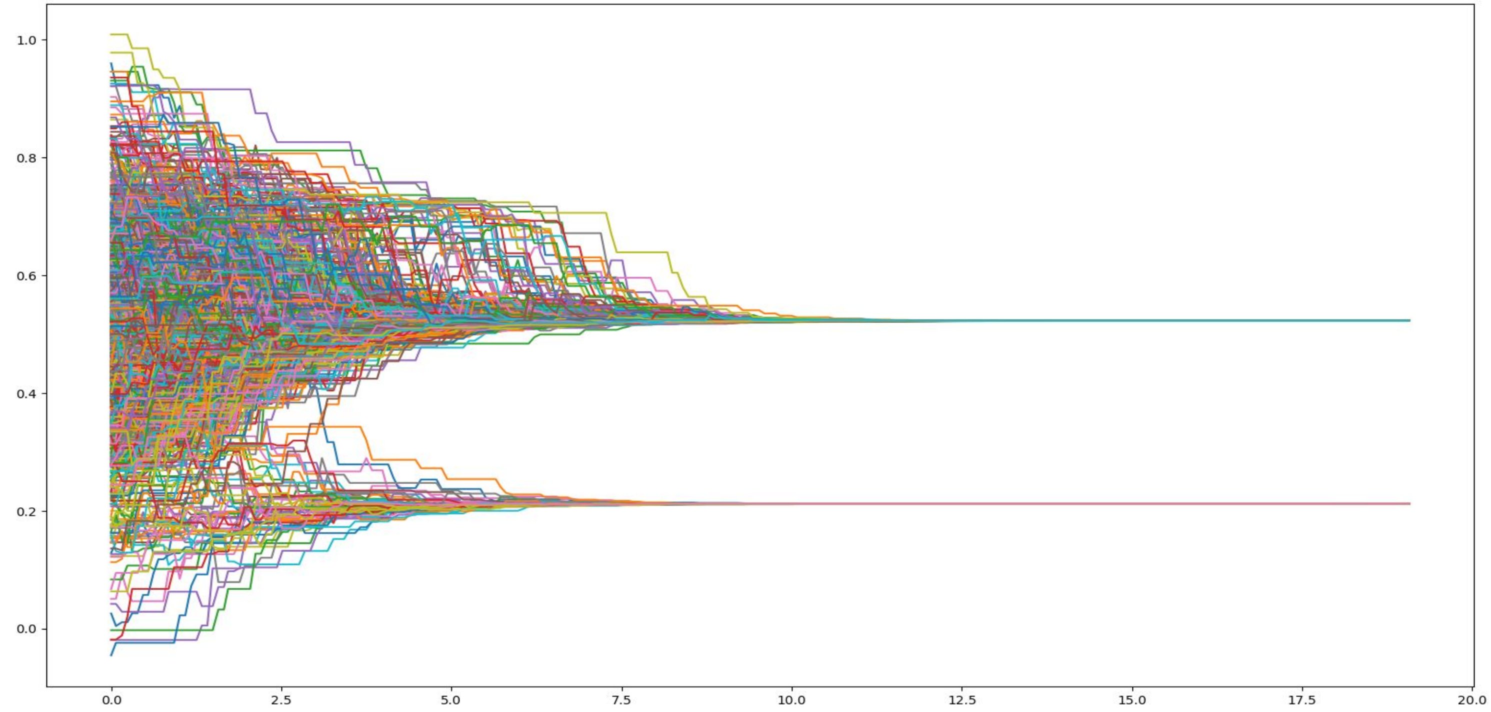
Parameters used in the model :

- 1000 members, each with a normally distributed initial opinion.
- Each member has a confidence interval that varies uniformly between .2 and .3
- The values of mu are varied over steps of 0.1.

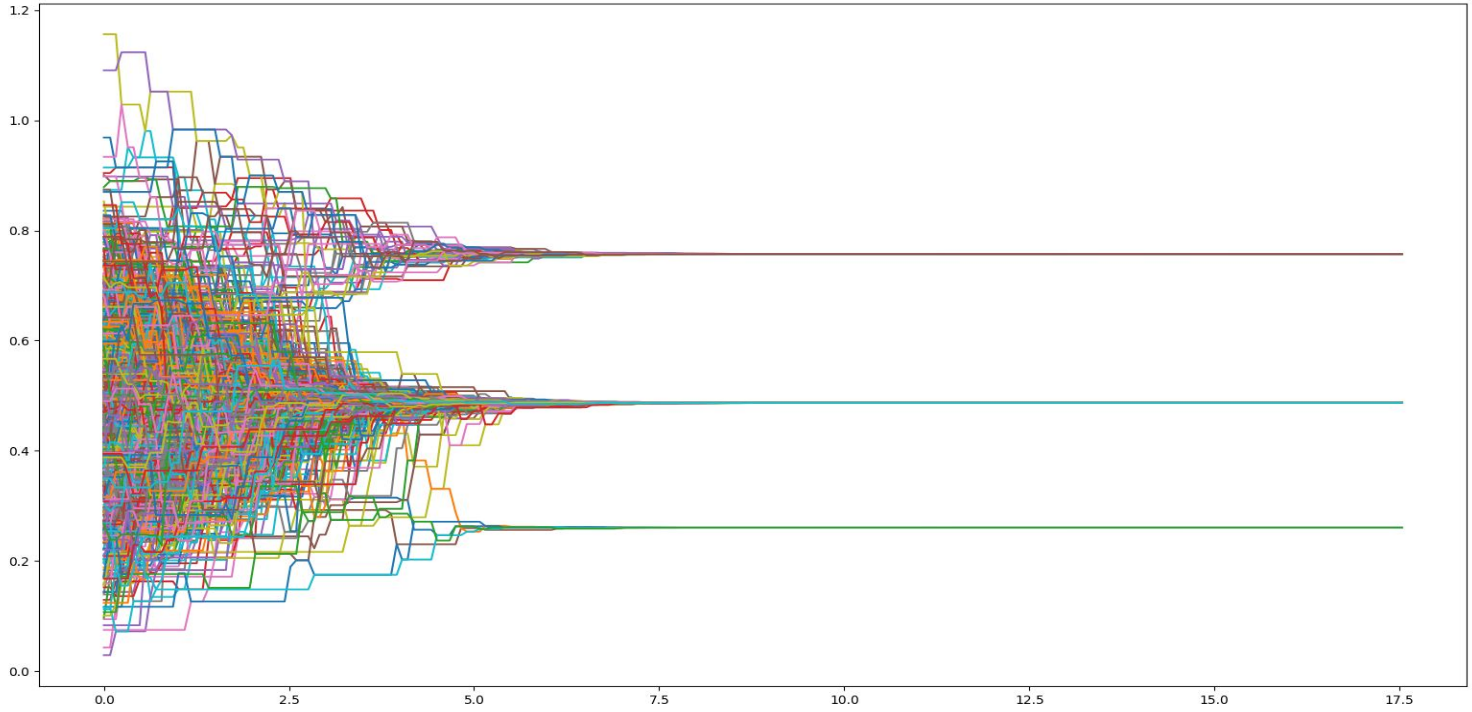
Plot for mu = 0.1



Plot for mu = 0.3



Plot for mu = 0.5



Bias and Segregation. - Future work.

In real world systems, the interactions are not always ideal, and can have inherent bias towards a particular opinion.

To model this bias itself, we plan on incorporating it into our simulations. To do so, we allow two members i and j to interact with a probability that varies as :

$$P \alpha f((op_i - op_j)^{-\gamma})$$

This makes interactions between like minded people more frequent.

Bias and Segregation. - Future work.

If the opinions are polarised and the interactions are biased, then there is a possibility of formation of clusters of opinions in a much faster rate.

(**Segregation analysis on time axis**)

We also think that this might lead to formation of a larger number of clusters.

(**Segregation analysis on effective clustering**)