

OBJECTIVES

1. Determine the optimum turbofan configuration for a passenger jet for minimum TSFC (Thrust Specific Fuel Consumption) during cruise by varying design parameters such as β (Bypass Ratio), P_{rc} (Compressor Pressure Ratio) and maximum cycle temperature (T_{04})
 - a) Determine the engine sizing for a take-off thrust of 106,760N.
 - b) Determine the TSFC (Thrust Specific fuel consumption) at cruise by selecting the maximum cycle temperature such that the engine thrust is 16,000N
 - c) Determine the β (Bypass Fan Ratio) and P_{rc} (Compressor Pressure Ratio) that minimizes TSFC for the following variations.
 - β from 2 to 20 in steps of 2 (10 cases)
 - P_{rc} from 5 to 100 in steps of 5 (20 cases)
 - d) Analyze the data and provide a detailed summary of the results.
2. Determine the total sound intensity level (in db) for the variations mentioned in 1(c).

Technique and Procedure - Equations

- Calculate Temperature(s) and Pressure(s) at different stations during takeoff to get the engine sizing for that combination of β and P_{rc} .

$$u_{\infty} = M_a \sqrt{\gamma_D R T_a}$$

$$T_{02} = T_a \left[1 + \frac{1}{2} (\gamma_D - 1) M_a^2 \right]$$

$$P_{02} = P_a \{ 1 + \eta_D [(T_{02}/T_a) - 1] \}^{\frac{\gamma_D}{\gamma_D - 1}}$$

$$\rho_{02} = P_{02} / (R T_{02})$$

$$U_{02} = M_{02} \sqrt{\gamma_D R T_{02}} = 0.3 \sqrt{\gamma_D R T_{02}}$$

$$T_{03} = T_{02} \left[1 + \frac{1}{\eta_C} (P_{rc} (\gamma_C - 1) / \gamma_C - 1) \right]$$

$$f = \frac{T_{04} - T_{03}}{(Q_R / c_{p,B}) - T_{04}}$$

$$T_{08} = T_{02} \left[1 + \frac{1}{\eta_F} (P_{rF}^{(\gamma_F - 1) / \gamma_F} - 1) \right]$$

$$T_{05} = T_{04} - \left(\frac{1}{1 + f} \right) \left(\frac{h_{Comp}}{c_{p,T}} \right) - \left(\frac{\beta}{1 + f} \right) \left(\frac{h_{Fan}}{c_{p,T}} \right)$$

$$P_{05} = P_{04} \left[1 - \frac{1}{\eta_T} \left(1 - \frac{T_{05}}{T_{04}} \right) \right]^{\frac{\gamma_T}{\gamma_T - 1}}$$

$$u_{e,C} = \sqrt{2 \eta_N \frac{\gamma_N R}{\gamma_N - 1} T_{05} \left[1 - \left(\frac{P_a}{P_{05}} \right)^{\frac{\gamma_N - 1}{\gamma_N}} \right]}$$

$$u_{e,F} = \sqrt{2 \eta_{N,F} \frac{\gamma_{N,F} R}{\gamma_{N,F} - 1} T_{08} \left[1 - \left(\frac{P_a}{P_{08}} \right)^{\frac{\gamma_{N,F} - 1}{\gamma_{N,F}}} \right]}$$

$$T / \dot{m}_a = (1 + f) u_{e,C} + \beta u_{e,F} - (\beta + 1) u_{\infty}$$

$$\text{Core } \dot{m}_a = 106760 / \left(\frac{T}{\dot{m}_a} \right)$$

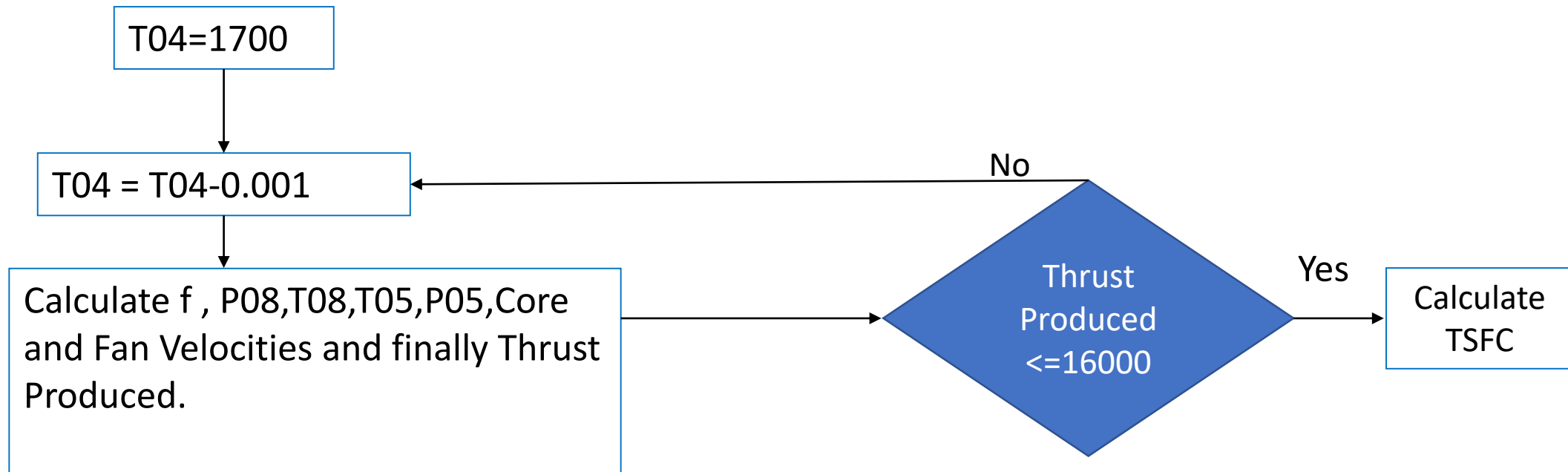
$$\text{Core area } A_{02} = \frac{\dot{m}_a}{(\rho_{02} U_{02})}$$

$$\text{TSFC} = f / (T / \dot{m}_a)$$

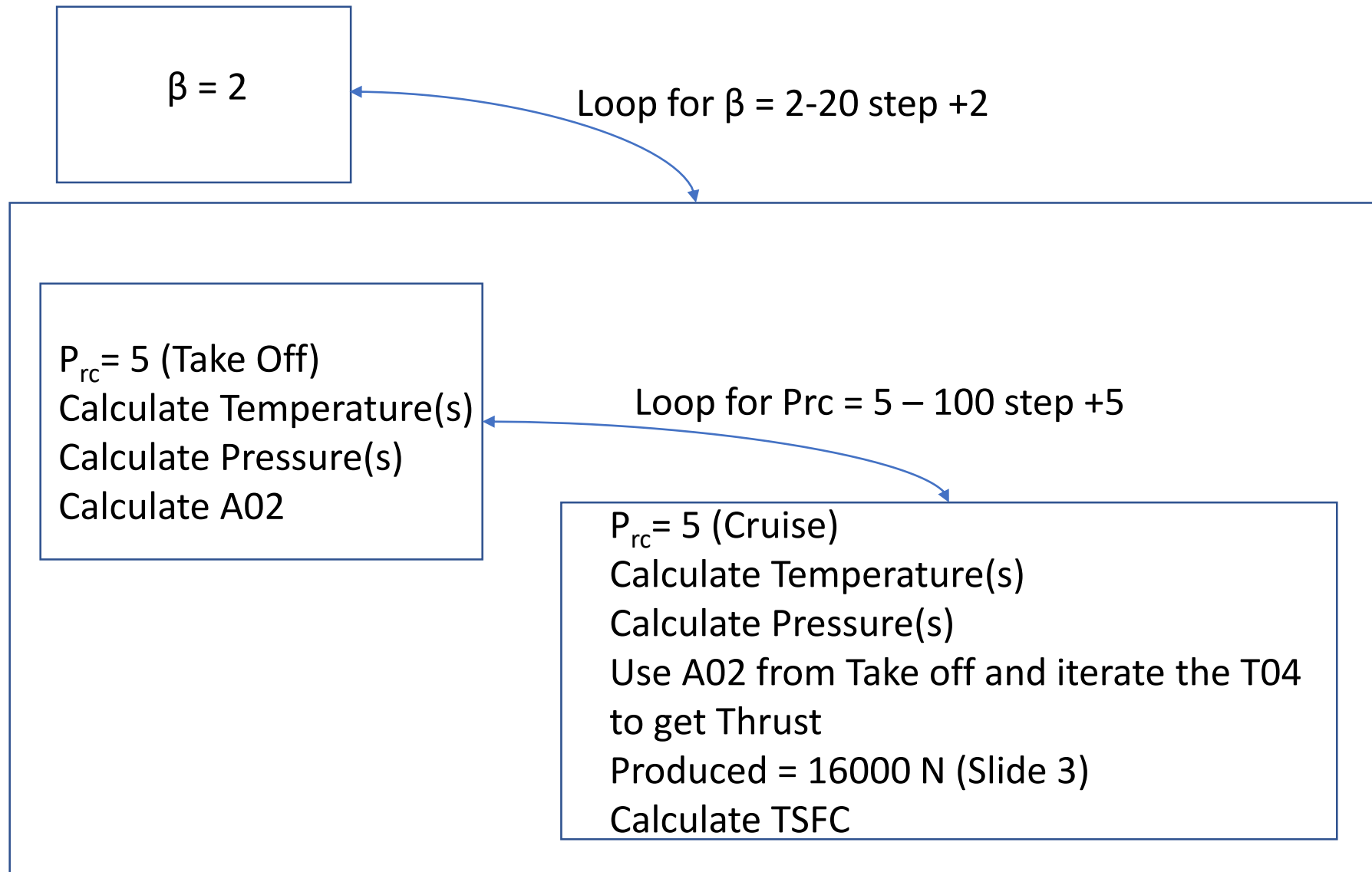
$$\left(2 \eta_N \frac{\gamma_N}{\gamma_N - 1} - 1 \right) T_{05} \left[1 - \left(\frac{P_a}{P_{05}} \right)^{\frac{\gamma_N - 1}{\gamma_N}} \right]$$

Technique and Procedure – T04 Calculation for Cruise

- Use the Engine Sizing from take-off to then calculate the maximum cycle Temperature (T04) during cruise such that the thrust produced is equal to 16,000N.
- We start with T04 = 1700 K and iterate the calculations till the Thrust produced = 16000N.



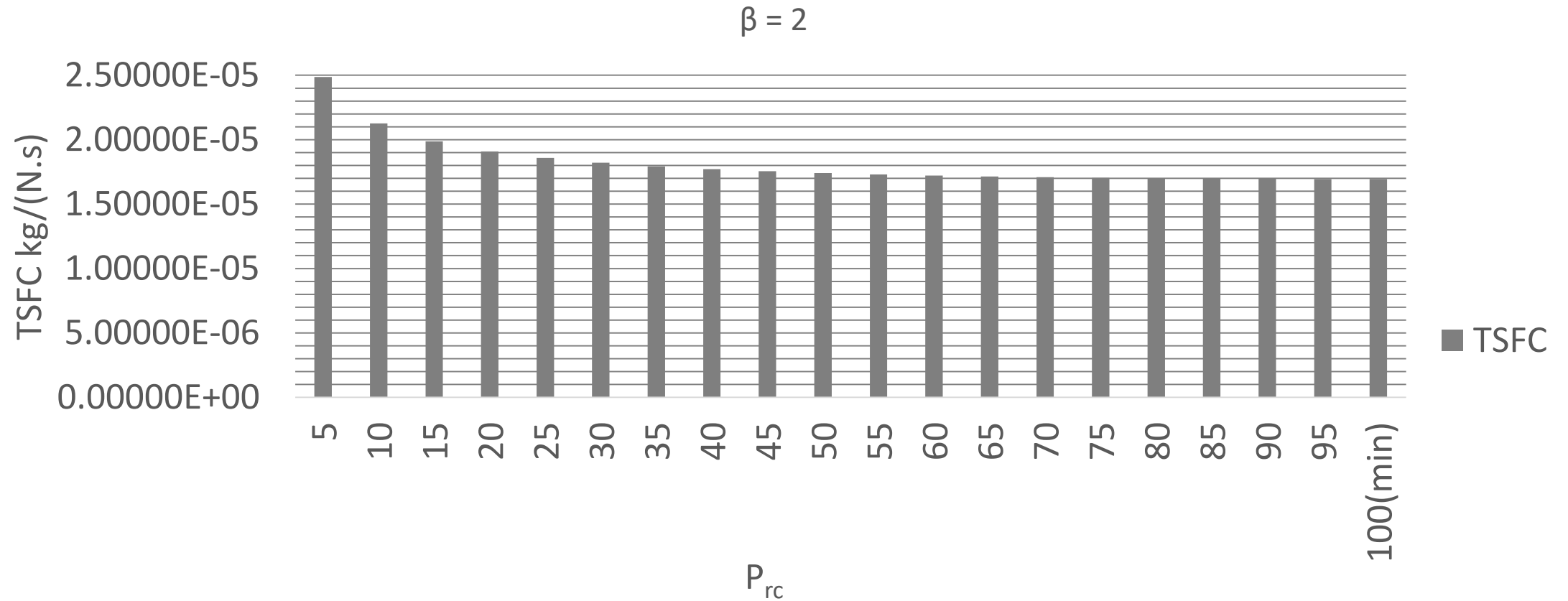
Technique and Procedure – Calculating Data for the Variants



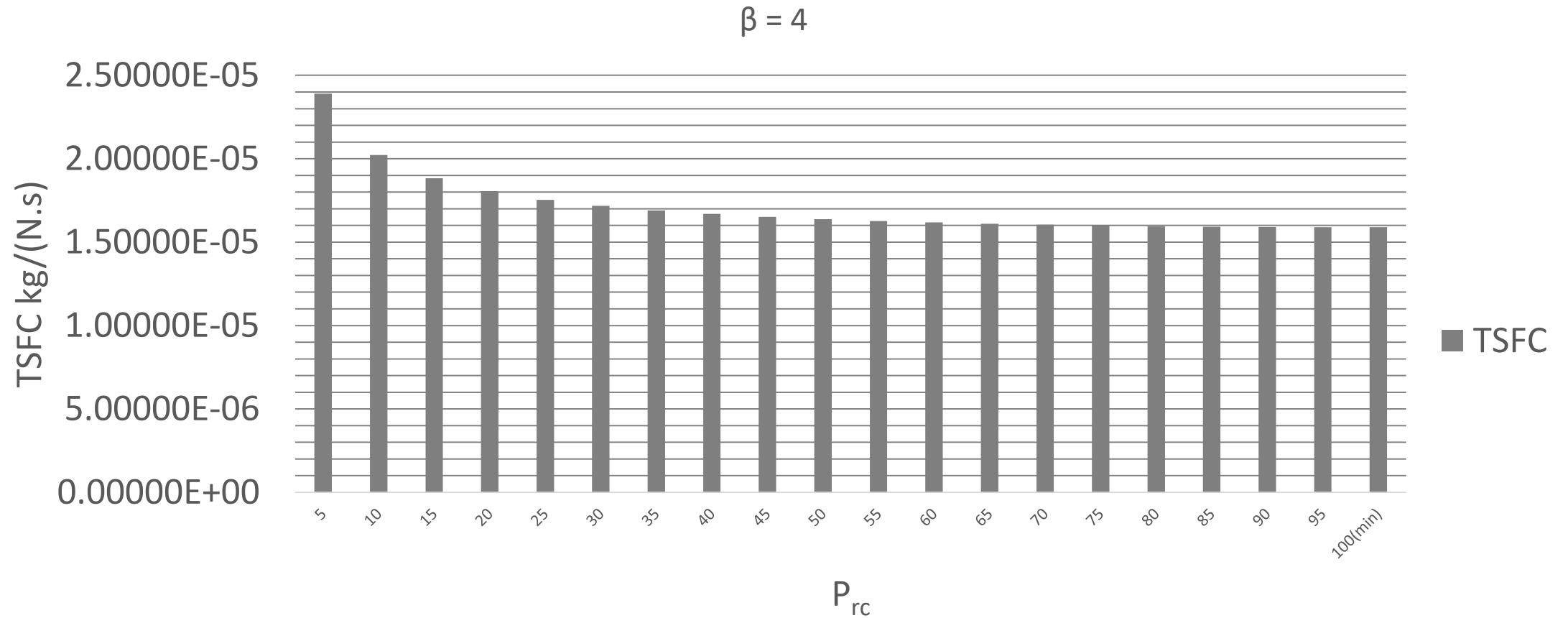
Technique and Procedure – Calculating Total sound intensity for Take-off

- Sound Intensity Level (db) = $10 \cdot \log_{10}(I/I_0)$
- where threshold of hearing $I_0 = 10^{-12} \text{ W/m}^2 = 10\text{db}$
- Sound Intensity $I = \rho / \left[4\pi \left(r_{\text{sphere}} \right)^2 \right]$
- Low Speed Acoustic Power for $\frac{u_e}{a_0} < 2$:
 - $\rho = 10^{-4} \rho_0 u_e^8 A_e / a_0^5$ where $A_e = \pi D^2 / 4$ & $a_0 = \sqrt{\gamma R T_a}$
- High Supersonic Acoustic Power $\frac{u_e}{a_0} > 2$
 - $\rho = 0.003 \rho_0 M_e^3 A_e a_0^3$ where $M_e = u_e / a_e$
- Multiple Noise Sources: $I = 10 \log_{10} \left\{ \Sigma [10^{I_i / I_0}] \right\} = 10 \log_{10} \left\{ 10^{I_1 / I_0} + \dots + 10^{I_n / I_0} \right\}$
- A_e for core = A_{02} , A_e for fan = $\beta * A_{02}$
- a_e is calculated by calculating T_7 for the Core for high speed. $T_7 = T_{05} \left\{ 1 - \eta_N \left[1 - \left(\frac{P_a}{P_{05}} \right)^{(\gamma_N - 1) / \gamma_N} \right] \right\}$

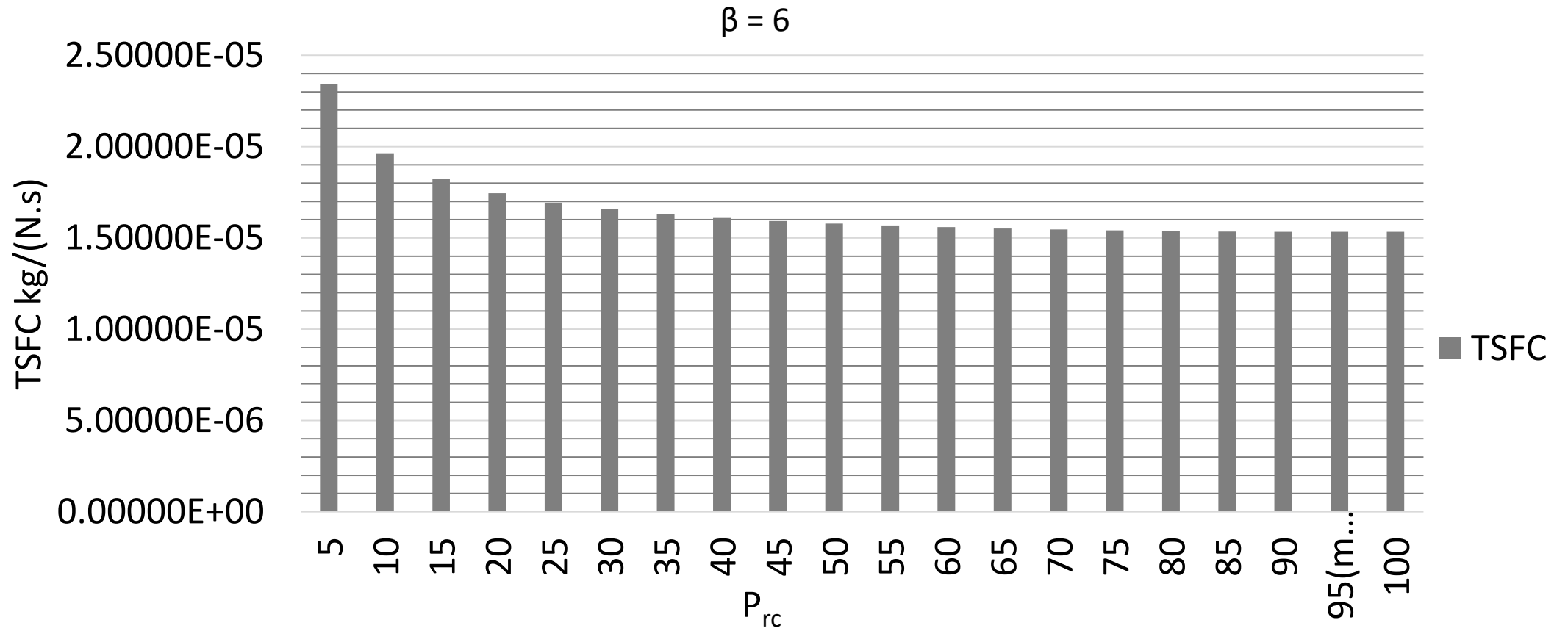
TSFC Vs PRC Variants for a $\beta = 2$ (Results -1)



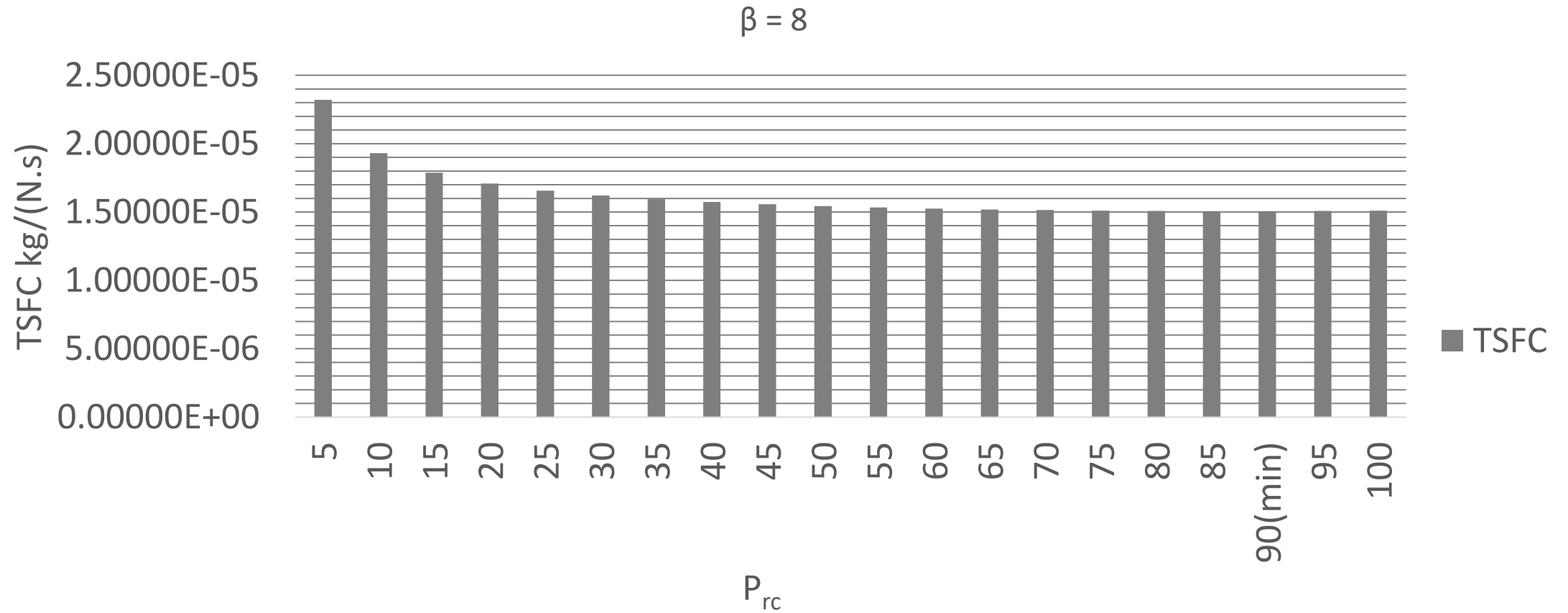
TSFC Vs PRC Variants for a $\beta = 4$ (Results -2)



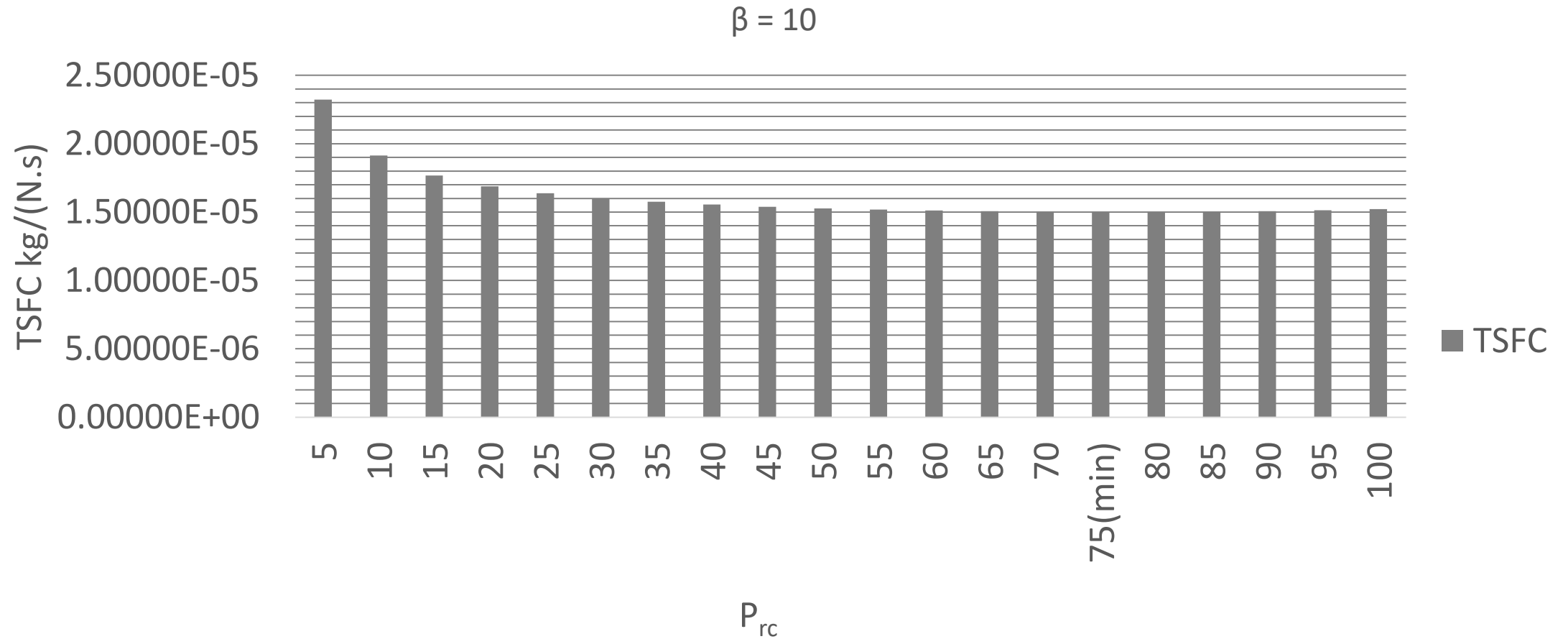
TSFC Vs PRC Variants for a $\beta = 6$ (Results – 3)



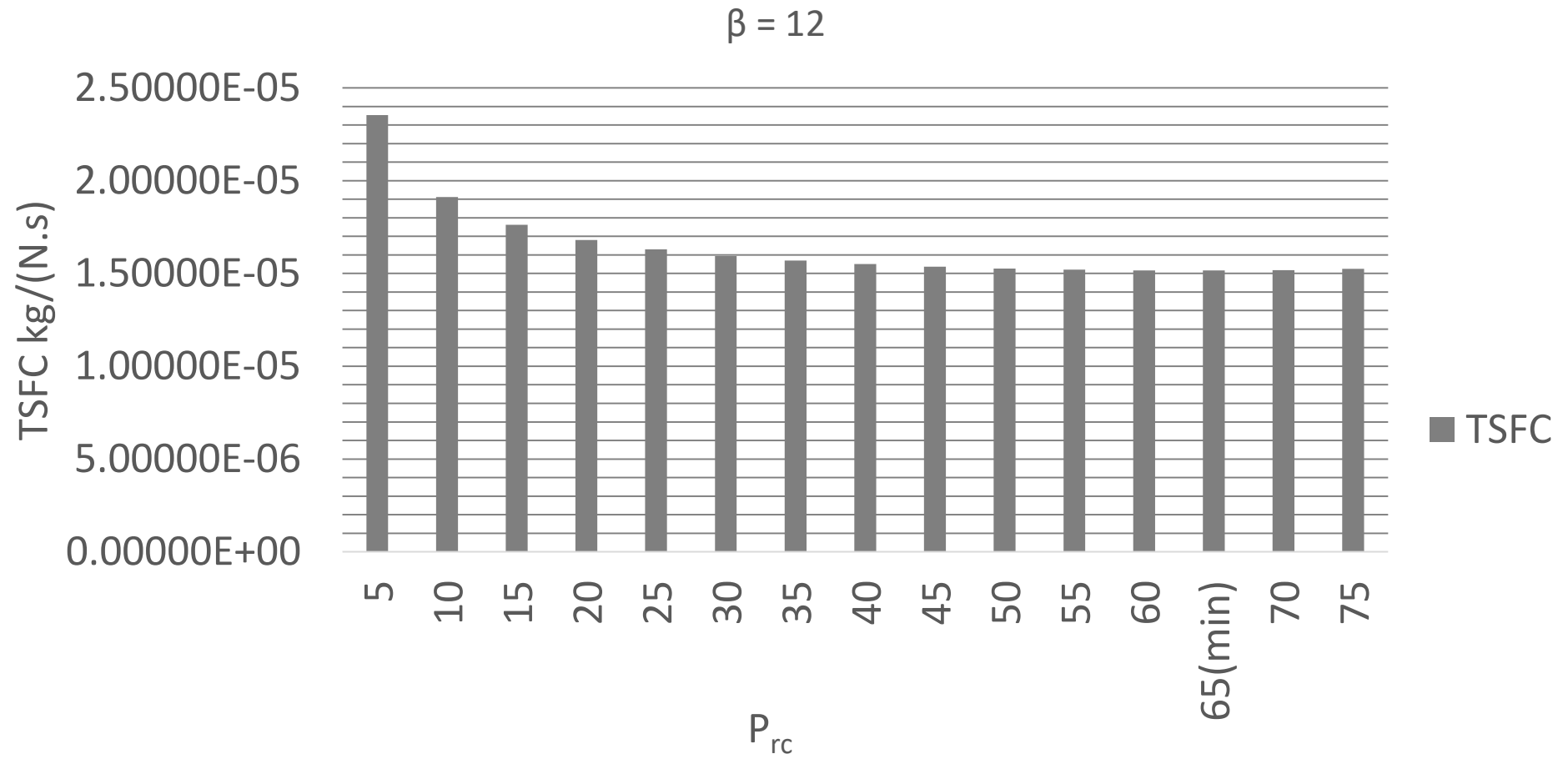
TSFC Vs PRC Variants for a $\beta = 8$ (Results – 4)



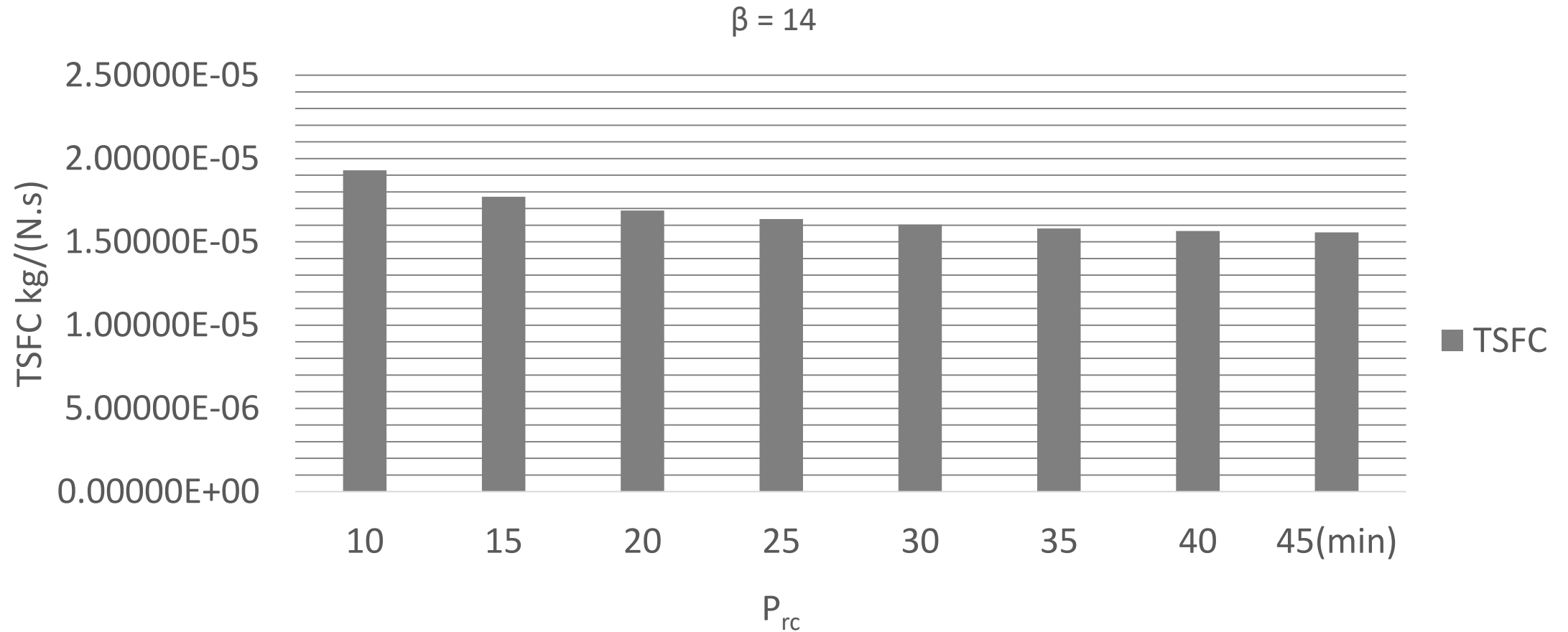
TSFC Vs PRC Variants for a $\beta = 10$ (Results -5)



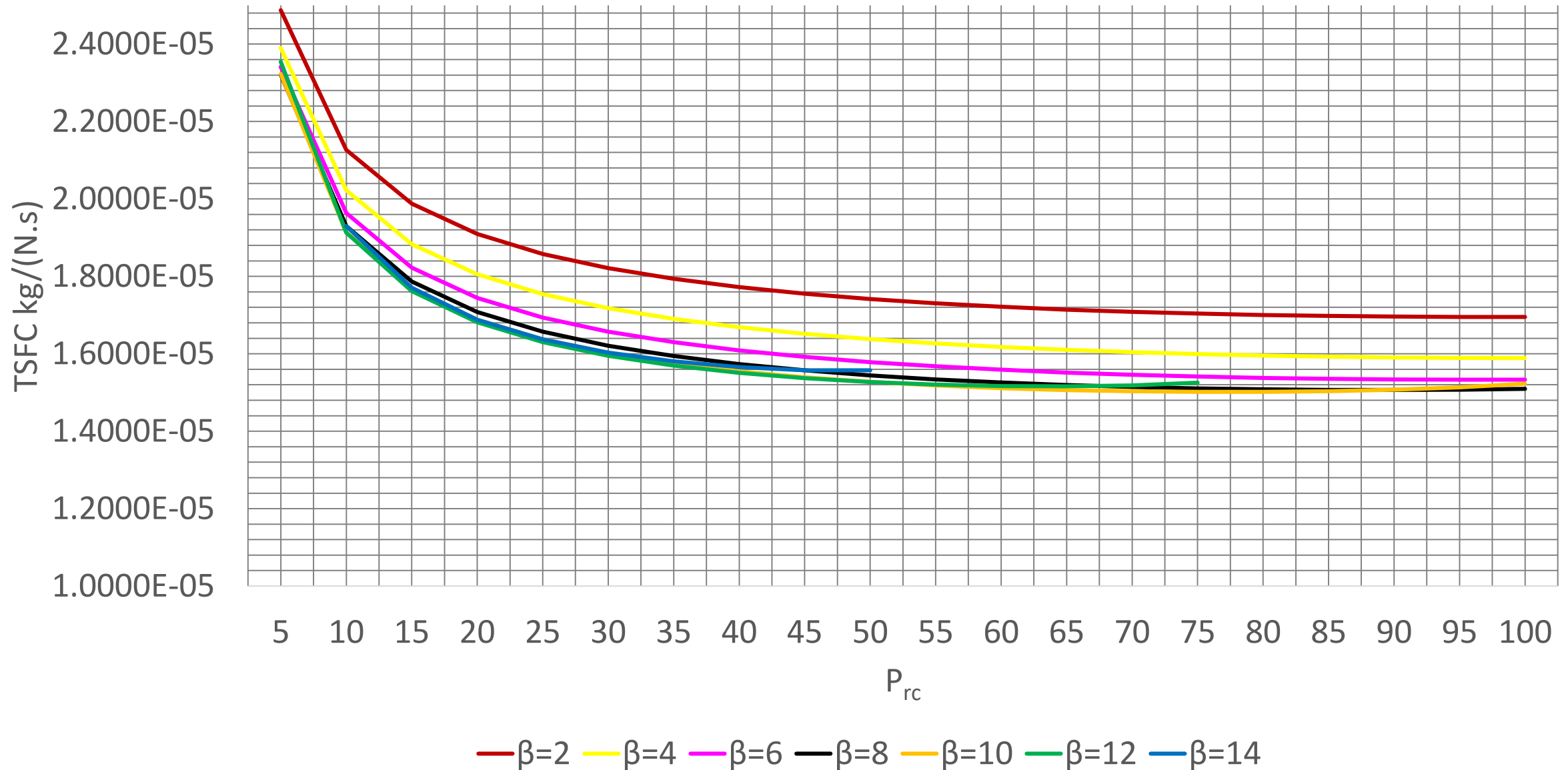
TSFC Vs PRC Variants for a $\beta = 12$ (Results -6)



TSFC Vs PRC Variants for a $\beta = 14$ (Results -7)



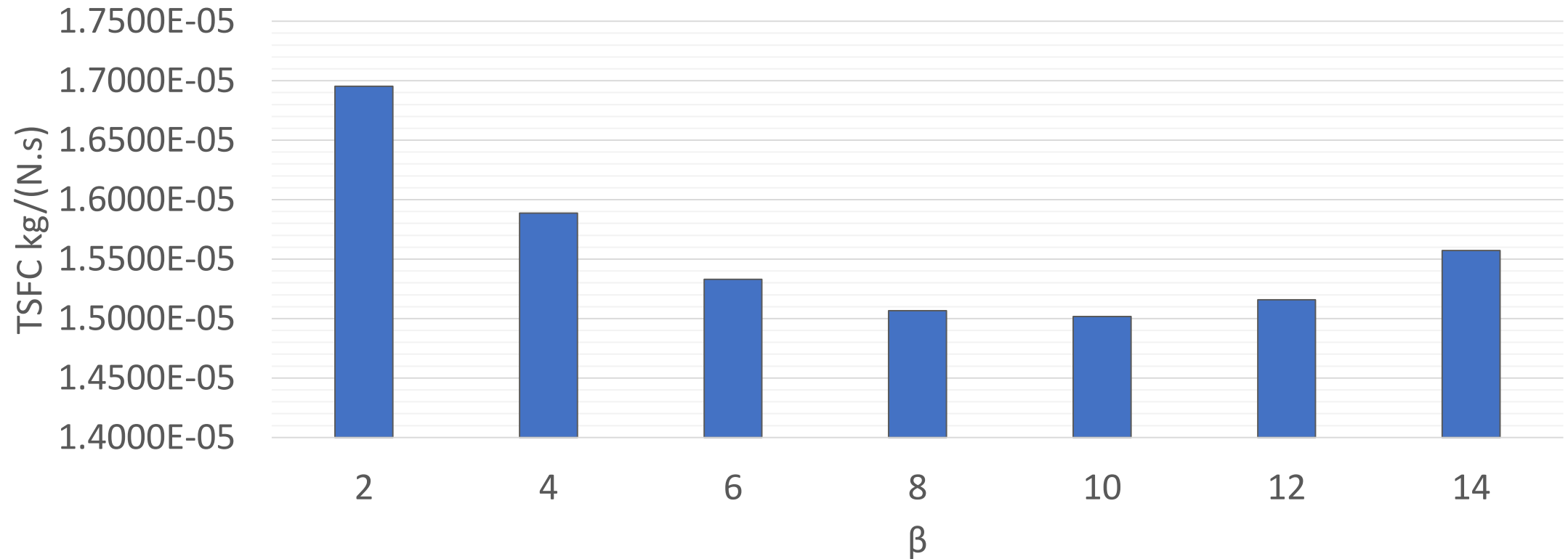
Comparison of TSFC for Different β (Results – 8)



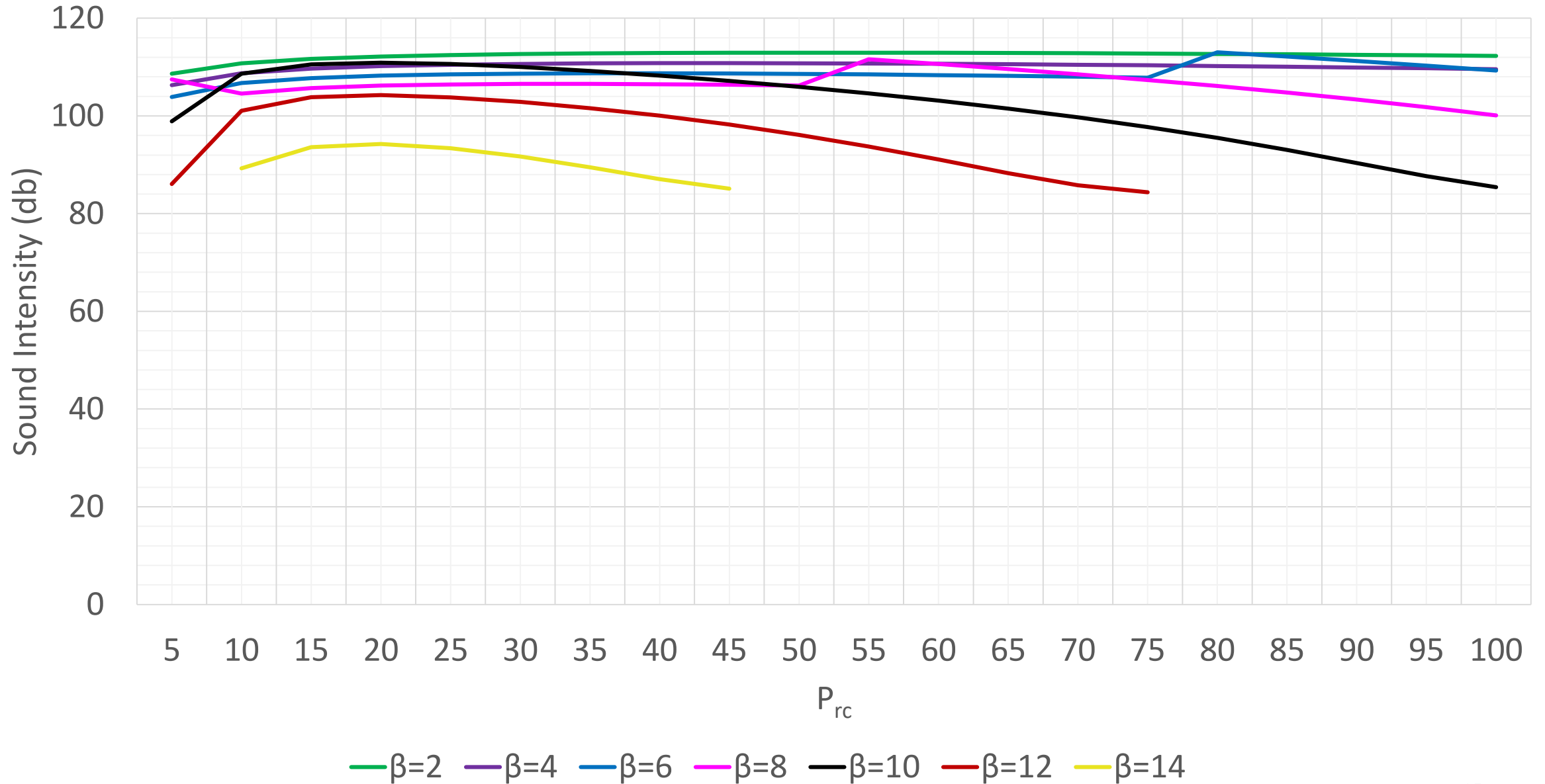
Minimum TSFC data – (Results -9)

β	2	4	6	8	10	12	14
P_{rc}	100	100	95	90	75	65	45
Min-TSFC	1.6953E-05	1.5887E-05	1.5329E-05	1.5067E-05	1.5017E-05	1.5159E-05	1.5573E-05

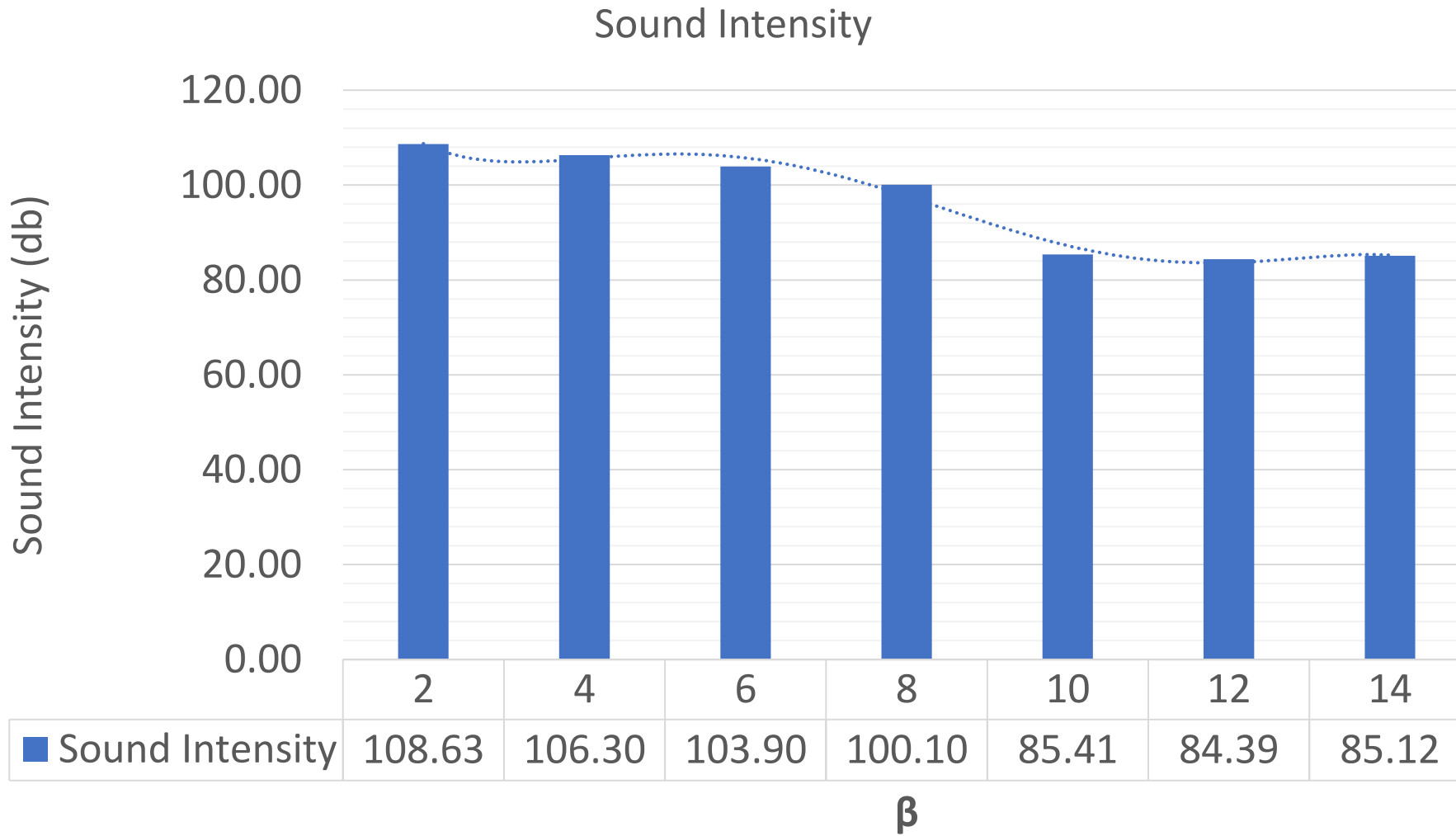
Min-TSFC for the Different β



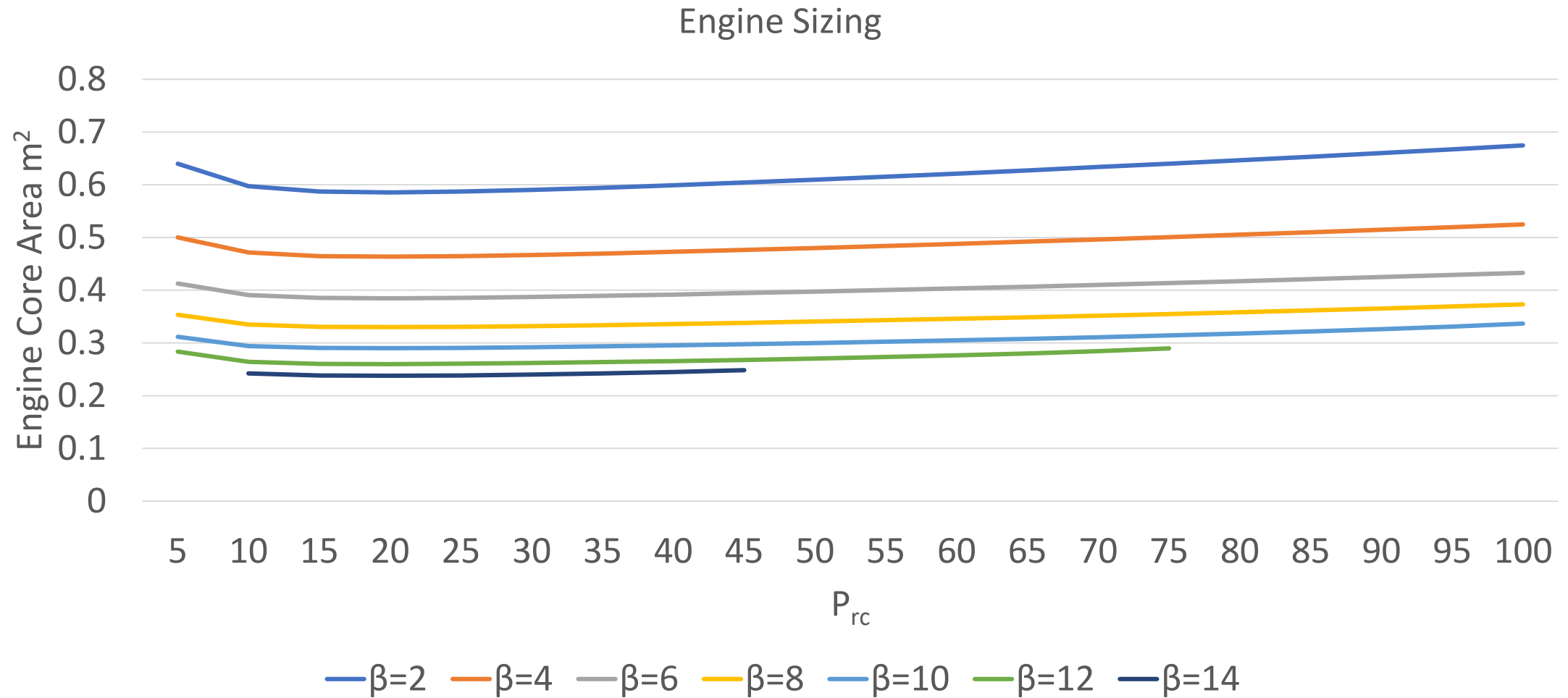
Sound Intensity Vs Prc for Different β (Results -10)



Minimum Sound Intensity Vs β for Different β (Results -11)



Engine Sizing for engine variants (Results -12)



Discussion – Bypass Ratio and Prc

- From the charts (Slide 12) above we can clearly see that the TSFC decreases as the β increases. The reason for this behavior is because TSFC is inversely proportional to (T/\dot{m}_a) (Slide 2) and hence directly proportional to the mass flow rate \dot{m}_a . The Larger the Fan Size (Bypass Ratio), the overall engine diameter/area increases leading to increase in mass flow and a lower TSFC.
- As the Bypass ratio was increased the turbofan was able to achieve the cruise thrust of 16000N at lower Prc(s) compared to smaller β , increasing the Prc beyond nominal values showed an increase in TSFC primarily because the compressor had to work more to rotate the larger fan.
- Higher Bypass Ratio engines ($\beta > 14$) are not viable because the turbine outlet pressure (P_{05}) is lesser than P_a (Ambient atmospheric pressure). Some cases in $\beta = 12$ and $\beta = 14$ are not viable because of the same reason at higher PrC and at Prc = 5 for $\beta = 14$.
- In the equation below if the P_a/P_{05} ratio is greater than 1 the exit velocity becomes complex. The engine would be sucking air from the exhaust at this point.

$$u_{e,C} = \sqrt{2\eta_N \frac{\gamma_N R}{\gamma_N - 1} T_{05} \left[1 - \left(\frac{P_a}{P_{05}} \right)^{\frac{\gamma_N - 1}{\gamma_N}} \right]}$$

Discussion – Noise Intensity

- We can see from slide 15 that the bypass ratio is inversely proportional to the noise intensity. This is primarily because of the following reasons.
 - Increasing the bypass ratio reduces the core exhaust velocity . More flow is bypassed through the fan.
 - This reduces the overall jet noise.
- We can see that there are sudden drops and rise in the sound intensity this is because of the acoustic power changing from Low speed to high speed supersonic for the core exhaust velocity as described by the equations on slide 5.

Summary 1 – TSFC at Cruise

- Slide 14 shows the comparison of the different configurations and establishes that at $\beta = 10$ and $P_{rc} = 75$ have the minimum TSFC of $1.5017E-05 \text{ kg}/(\text{N.s})$.
 - Objective 1C slide 1 has been achieved.

Summary 2 – Noise Intensity at Take-Off

- Slide 15 shows the overall comparison on Sound intensity for the takeoff condition. The results show that higher Bypass ratio Turbofan engines can help reduce sound. The lowest sound is produced by the Engine with the P_{rc} of 75 and Bypass ratio of 12.
 - Objective 2 of slide 1
- Slide 15 also shows that the Sound Intensity is effected by the P_{rc}
- Slide 16 analyses the minimum Sound intensity for each bypass ratio and predicts that a β of 12 has the lowest sound level for our project calculations.

Summary 3 – Engine Sizing

- Slide 17 shows us the core engine size decreases with the increase in the bypass fan ratio.
 - Objective 1 (a) , Slide 1

Appendix

- Tools used.
 - Programming – python 3
 - Report – Microsoft PowerPoint and Adobe reader.
- Reference
 - AE 742 Homework Materials and handouts.