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1 Literature Review

Several studies have employed Bayesian methods to model and update the knowledge of space objects and their orbits. These studies focus on various aspects of space situational awareness, such as orbit determination, orbit prediction, and collision probability assessment.

There has been research in understanding the limitations of current debris characterization techniques and models, M.J. Matney[1] from NASA Johnson Space Center found that uncertainties in orbital debris measurement arise from bias in observational data, statistical fluctuations, and model errors. These uncertainties can significantly affect the reliability of orbital debris models. Particle filters[2] have been proposed to represent the full probability density function (PDF). By using Monte Carlo runs to generate particles approximating the full PDF, the PF demonstrates proficiency in comparison to the Extended Kalman Filter and Splitting Gaussian Mixture algorithms, providing a more accurate representation of orbit states and a potential method for modeling uncertainty in debris characteristics. The paper contributes to the field of orbit determination by presenting a method that can effectively handle non-Gaussian uncertainties using a particle filter. There have been efforts to use machine-learning[3] and ontology-based Bayesian networks to characterize and understand the behavior of Orbital Debris to predict and mitigate potential collisions. Furfaro and team proposed a deep meta-learning approach for the identification and characterization of space debris, showcasing the potential of machine learning techniques in this domain. There have been studies in propagating uncertainties in satellites states[4] for orbit injection errors which could be used to validate some findings from this Thesis. The proposed methodology demonstrates an advanced alternative to traditional methods, potentially improving the accuracy of debris tracking and collision prediction or debris capture.

There are several challenges and future directions for applying Bayesian methods to orbital debris modeling. These include improving the efficiency and accuracy of the Bayesian updating process, incorporating non-Gaussian uncertainties, and fusing data from multiple sensors and databases. Another potential direction is to combine the Bayesian framework with machine learning methods to enhance the accuracy and efficiency of debris characterization.

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[1] M. J. Matney, 'Uncertainty in Orbital Debris Measurements and Models', 587 (2005), 45.
[2] 'Mashiku et al. - 2012 - Statistical Orbit Determination Using the Particle.Pdf' <https://ntrs.nasa.gov/api/citations/20120016941/downloads/20120016941.pdf> [accessed 5 May 2023].
[3] Roberto Furfaro, Richard Linares, and Vishnu Reddy, 'Space Debris Identification and Characterization via Deep Meta-Learning', 2019.
[4] Nikolas Bascue, Atri Dutta, and Pradipto Ghosh, 'Impact of Launch Injection Errors on Orbit-Raising of All-Electric Satellites', in *AIAA Scitech 2020 Forum* (presented at the AIAA Scitech 2020 Forum, Orlando, FL: American Institute of Aeronautics and Astronautics, 2020) <https://doi.org/10.2514/6.2020-0959>.

Conclusion

This literature review highlights the use of Bayesian principles to model uncertainty in orbital debris flux using tools such as the ORDEM database and the LEGEND tool from NASA. The study revealed that the LEGEND tool was able to predict the debris characteristics but uses a Monte Carlo approach without using any prior knowledge of the debris characteristics, while the ORDEM tool/database uses Bayesian Principles to predict the future state of orbit characteristic, it is more tailored towards calculating the debris flux with an intention to create space situational awareness as part of future mission planning. This thesis will focus on providing a framework on quantifying the uncertainty of **individual** debris states using Bayesian principles and debris data from NORAD (North American Aerospace Defense Command) and the SSN (United States Space Surveillance Network) via CelesTrak.

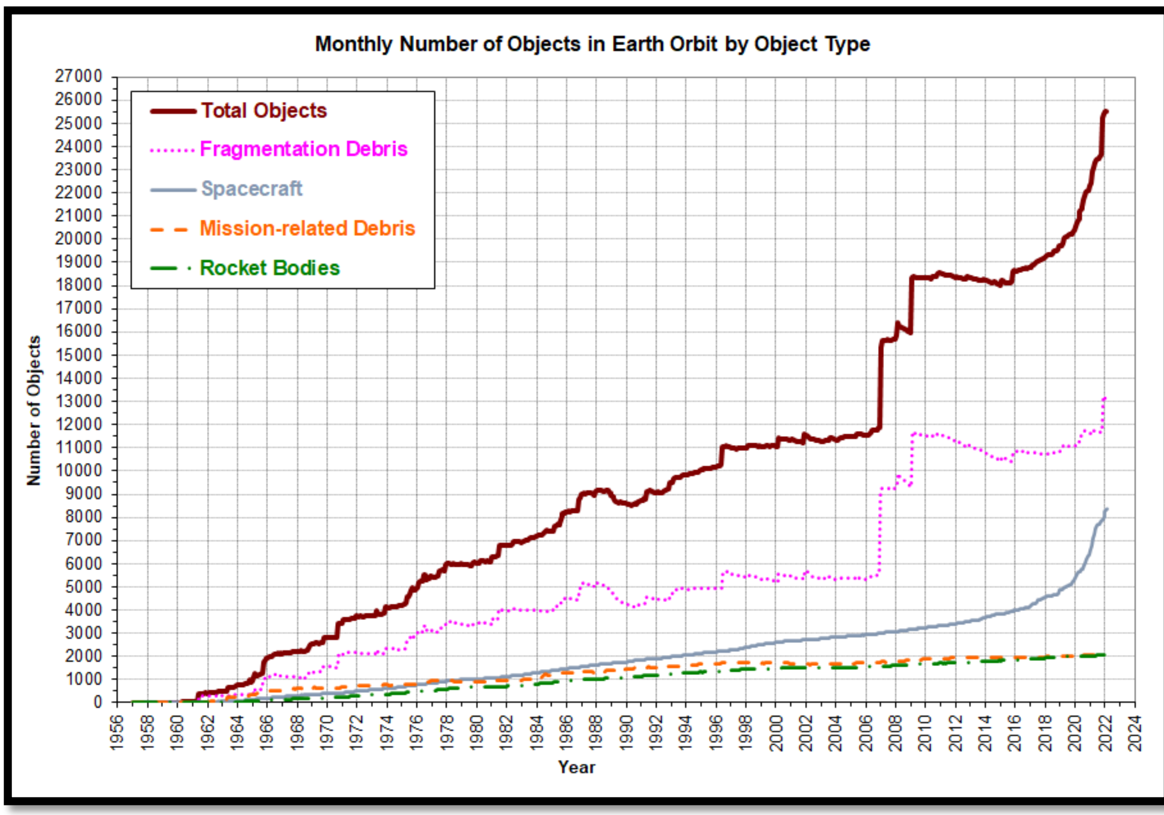

Figure 1.1. Chart showing number of objects >10 cm in LEO. Credit: NASA ODPO.

Orbital debris historical Data review.

1.1 ORDEM (Orbital Debris Engineering Model)[5]

NASA’s Orbital Debris Program Office develops ORDEM 3.1. It is a tool for spacecraft designers to understand the long-term risk of collisions with orbital debris. The model covers low Earth orbit (LEO) to geosynchronous orbit (GEO) altitudes for the years 2016-2050.It models fluxes for object sizes > 10 µm within LEO and > 10 cm in GEO. The deterministic portion is based on the U.S. Space Surveillance Network (SSN) catalog. The tool uses observational datasets from radar, in situ, and optical sources to extrapolate to smaller sizes.

Provides fluxes (number per m^2 per year) of debris for a given year. Includes "spacecraft mode" and "telescope/radar mode" settings. It models fluxes for objects greater than 10 µm in LEO and greater than 10 cm in GEO. Breakdown of debris into material density categories. It models five populations, including intact, low-density fragments, medium-density fragments, high-density fragments, and sodium-potassium (NaK) coolant droplets.

1.2 LEGEND (LEO-to-GEO Environment Debris model)[6]

The LEGEND model consists of a three-dimensional simulation program that uses daily solar flux F10.7 values combined with the J77 atmospheric model for the drag calculation. The model adopts a deterministic approach to mimic the known historical populations of debris and a Monte Carlo approach to simulate potential future on-orbit explosions and collisions. The model generates fragments down to 1 mm in size based on the NASA Standard Breakup Model, which describes the size, area-to-mass, and velocity distributions of the breakup fragments. Each breakup fragment is assigned a unique identification character to track its origin: explosion, intact-intact collision, intact-fragment collision, or fragment-fragment collision. The model also considers perturbations such as Earth's J2, J3, solar-lunar gravitational perturbations, atmospheric drag, solar radiation pressure, and Earth's shadow effects.

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[5] ‘Orbital Debris Engineering Model (ORDEM), Version 3.2.0(MSC-25457-1) | NASA Software Catalog’ <https://software.nasa.gov/software/MSC-25457-1> [accessed 4 May 2023].

[6] J.-C. Liou and N.L. Johnson, ‘Instability of the Present LEO Satellite Populations’, *Advances in Space Research*, 41.7 (2008), 1046–53 <https://doi.org/10.1016/j.asr.2007.04.081>.

1.3 CELESTRAK[7]

CelesTrak provides Two-Line Element Sets (TLEs) and other related data, which is crucial for the tracking of artificial satellites in Earth orbit. The TLEs are sets of numerical values that describe the orbit of a satellite at a particular point in time. Researchers, engineers, and amateur satellite observers for various applications, including satellite tracking and collision avoidance, often use these TLEs.

CelesTrak's data is frequently updated and is sourced from authoritative bodies such as the North American Aerospace Defense Command (NORAD) and the United States Space Surveillance Network. The website also provides educational and analytical resources, such as software tools and articles, to help users understand and interpret the data.

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[7] 'CelesTrak: Current GP Element Sets' <https://celestrak.org/NORAD/elements/> [accessed 16 September 2023].

2 Data and Methodology

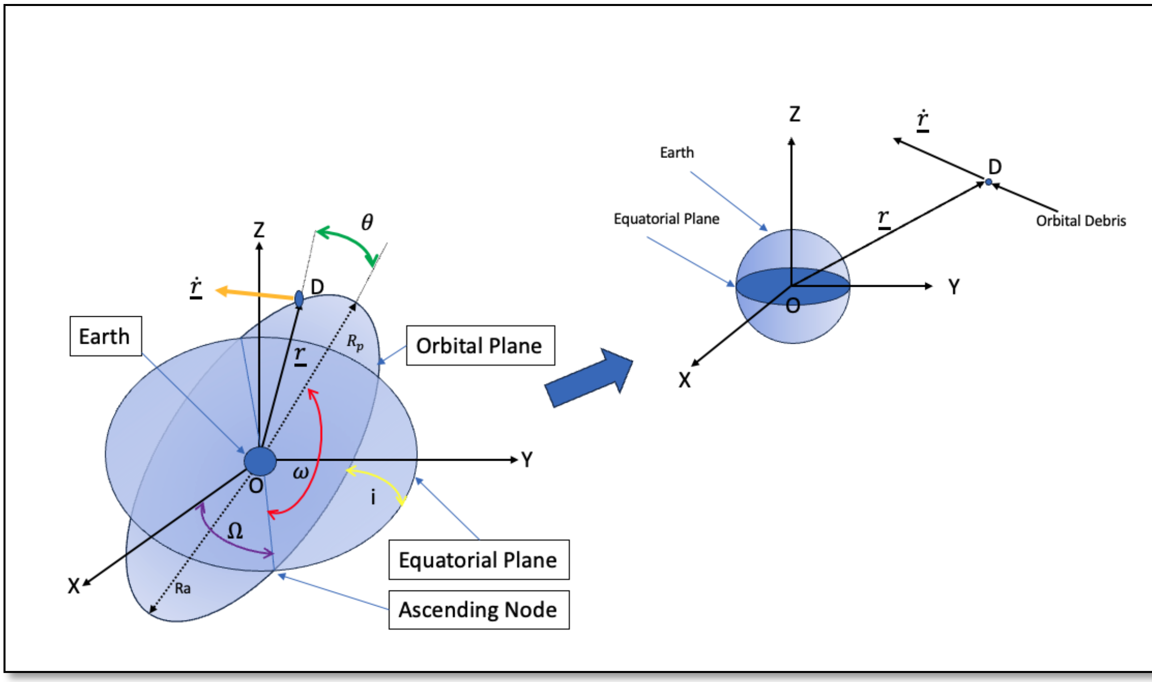
In this section we discuss the different models that will be used in this thesis. We start with the deterministic model where we use the orbital parameters of the debris to calculate the state vectors in addition to capturing different perturbations on the debris. The equations of motions and then use numerical methods to calculate the future state; whereas in the uncertainty model, we use the data from NORAD[8] to create a prior to create a Bayesian box to better estimate the certainty on our deterministic model's prediction. The Uncertainty model also provides the posterior distribution of the state parameters which can be used to derive samples using algorithms like Markov Chain Monte Carlo (MCMC).The objective of this thesis is to compare these two results and provide insights into using Bayesian Framework for orbital parameter estimations.

2.1 Deterministic Model

The equations of motion for a debris object can be expressed as a system of ordinary differential equations (ODEs) using the debris' position and velocity as state variables. Let's denote the position vector of the debris as $\underline{r} = (x, y, z)$ and the velocity vector as $\underline{\dot{r}} = (v_x, v_y, v_z)$ with respect to the earth.

r_p	This is the radial distance of the perigee, the closest point in the orbit.
r_a	This is the radial distance of the apogee, the farthest point in the orbit.
i	The state i denotes the angle of inclination, the angle between the orbital plane and equatorial plane.
Ω	The angle Ω denotes the right ascension of the ascending node, the angle between the x axis and the line joining the ascending node.
ω	The angle ω denotes the argument of periapsis, the angle made between the perigee and the ascending node line.
θ	The angle θ denotes the true anomaly angle which is the angle between perigee and the line connecting the debris to origin in earth.

[8] 'CelesTrak: IRIDIUM 33 Debris' <https://celestrak.org/NORAD/elements/table.php?GROUP=iridium-33-debris&FORMAT=2le> [accessed 6 September 2023].



Picture 1 : Orbital Elements in the ECI Reference Frame

Let’s consider a vector D(t) that represents the orbital parameters of a debris orbiting somewhere in the LEO around the planet earth at a given time t.

$$D(t) = [\ R_p(t)\ \ R_a(t)\ \ I(t)\ \ \ \Omega(t)\ \ \ \omega(t)\ \ \ \theta(t)\ ]$$

The orbital parameter vectors can be converted into Cartesian coordinates containing the position and velocity components as detailed below.

The different relationships between semi-major axis(a), eccentricity(e), radial distance (r) and specific angular momentum (h) are shown below :

$$e = 1 - \left(\frac{R_p}{a}\right) \qquad a = \frac{R_p + R_a}{2} \tag{1,2}$$

$$h = \sqrt{a(1 - e^2)\mu} \qquad r = \frac{a(1-e^2)}{1+ecos(\theta)} \tag{3,4}$$

with the above equations in place, we can go ahead and calculate the position vector $\underline{r}$

$$\underline{r} = r \begin{bmatrix} cos\,\Omega\, cos(\omega+\theta) - sin\,\Omega\, sin(\omega+\theta)\, cos\, i \\ sin\,\Omega\, cos(\omega+\theta) + cos\,\Omega\, sin(\omega+\theta)\, cos\, i \\ sin(\omega+\theta)\, sin\, i \end{bmatrix} \tag{5}$$

$$\underline{v} = \left(-\frac{\mu}{h}\right) \begin{bmatrix} \cos(\sin(\omega+\theta) + e\sin\Omega) + \sin\Omega\,(\cos(\omega+\theta) + e\cos\Omega)\cos i \\ \sin\Omega(\sin(\omega+\theta) + e\sin\Omega) - \cos\Omega(\cos\omega+\theta) + e\cos\Omega)\cos i \\ -(\cos(\omega+\theta) + e\sin\omega)\sin i \end{bmatrix} \tag{6}$$

2.1.1 Orbital Debris Dynamics[9]

Considering Keplerian motion and the forces acting on the debris comprising of gravitational perturbations, aerodynamics drag and solar radiation pressure and no external forces , we can calculate the debris state in the future using its equations of motion as follows.

$$\boxed{\ddot{\underline{r}} = -\mu\frac{r}{r^3} + \underline{p}} \tag{7}$$

μ is the gravitational parameter for Earth due to the debris mass being significantly smaller compared to Earth. $\underline{p}$ is the resultant acceleration vector produced by perturbations listed below.

Atmospheric drag. ($\underline{\boldsymbol{a_D}}$)

Atmosphere is a spherically symmetric 1000 km thick gas layer around the planet Earth. Atmospheric density decreases with the altitude hence the effect of atmosphere is a function of the altitude of the debris. The perturbing force is given by the following equations.

[9] D. A. Vallado, *Fundamentals of Astrodynamics and Applications*, 2nd edition (Dordrecht ; Boston: Springer, 2001).

$$\underline{F_D} = \frac{1}{2}\rho S C_D v_r^2 \left(-\frac{\underline{v_r}}{v_r}\right)$$

Here $\underline{v_r} = \underline{v} - \underline{v_{atm}}$

$$\underline{v_{atm}} = \underline{\omega_e}\, X\, \underline{r}$$

$$\boxed{\underline{a_D} = -\frac{1}{2}\rho \frac{SC_D}{m} v_r \underline{v_r}} \tag{8}$$

*Perturbing acceleration due to drag is computed using a correction factor f to account for the effect of latitude which is usually less than 10%

$$a_D = \frac{1}{2}\rho v^2 f^2 \frac{SC_D}{m} \qquad f = 1 - \left(\frac{\omega_e r P0}{vP0}\cos i\right)$$

where C_D is the drag coefficient, ρ is the atmospheric density, and v_r is the relative velocity between the debris object and the atmosphere. The drag force $\underline{F_D}$ acts in the direction opposite to the relative velocity vector $\underline{v_r}$.

The semi-major axis decreases over time due to the loss of the energy , also known as contraction. The debris' eccentricity decreases over time because of the debris experiencing a higher deceleration at the periapsis compared to the apoapsis , also known as circularization.

Gravitational perturbations (J2).($\underline{a_{J2}}$)

The J2 effect arises due to the Earth's oblate shape. The gravitational potential can be expanded in a series of spherical harmonics, and the J2 term is the first non-zero zonal harmonic. The J2 perturbation accelerations ($\underline{a_{J2}}$)can be expressed as:

$$\boxed{\underline{a_{J2}} = \left(\frac{3}{2}\right) J2 \left(\frac{\mu}{R_{Earth}^2}\right) \underline{r}\, \frac{\left(5 * \left(\frac{z^2}{r^2}\right) - 1\right)}{r^5}} \tag{9}$$

Where J2 is the Earth's second zonal harmonic, R_{Earth} is the Earth's mean equatorial radius, and r is the distance between the debris object and the center of the Earth.

There are other perturbations that add to the uncertainty of the deterministic model but for the sake of simplicity these will not be incorporated in our current model.

Solar radiation pressure (a_{rad}) ,Earth albedo radiation pressure (a_{albe}),Solid Earth Tides (a_{ET}),Ocean Tides (a_{OT}) are other perturbations that have been discussed in literature[10].

Now, we can combine all the accelerations to form the complete system of ODEs: (10)

$$\boxed{\begin{aligned} \dot{x} &= v_x \\ \dot{y} &= v_y \\ \dot{z} &= v_z \\ \ddot{x} &= -\frac{\mu x}{r^3} + a_{x_{J2}} + a_{x_{drag}} \\ \ddot{y} &= -\frac{\mu y}{r^3} + a_{y_{J2}} + a_{y_{drag}} \\ \ddot{z} &= -\frac{\mu z}{r^3} + a_{z_{J2}} + a_{z_{drag}} \end{aligned}}$$

These ODEs describe the motion of a debris object in Earth's orbit, considering the Earth's gravitational force, J2 perturbations, solar radiation pressure, and atmospheric drag. The ODEs can be solved numerically using various methods, such as the Runge-Kutta or Verlet algorithms, to obtain the position and velocity of the debris object over time.

[10] Smriti Nandan Paul, Richard J. Licata, and Piyush M. Mehta, 'Advanced Ensemble Modeling Method for Space Object State Prediction Accounting for Uncertainty in Atmospheric Density', *Advances in Space Research*, 71.6 (2023), 2535–49 <https://doi.org/10.1016/j.asr.2022.12.056>.

2.2 Measurement Model

The measurement model relates the true state of the debris T (position and velocity) to the observations (e.g., radar, telescopes, and satellite-based measurements). In our case the NORAD tool is going to act a source of measurement. Due to the inherent uncertainties in measurement devices, there will be errors between the true state and the observed state. This measurement model should account for these errors, which are often assumed to follow a Gaussian distribution with a mean of zero and a known covariance matrix.

Let's consider an example of a measurement model for debris tracking using a radar system. The radar system provides range (ρ), azimuth angle (α), and elevation angle (ε) measurements for the observed debris.

The true state vector T of the debris may include position (r_x, r_y, r_z) and velocity (v_x, v_y, v_z) in an Earth-centered inertial (ECI) coordinate system:

$$T = \left[r_x, r_y, r_z, v_x, v_y, v_z\right]^T$$

The non-linear function h(x) maps the true state vector x to the expected measurements (ρ, α, ε). In this case, we can define h(x) using radar geometry equations:

$$\rho = \sqrt{(r_x - r_{rx})^2 + \left(r_y - r_{ry}\right)^2 + (r_z - r_{rz})^2}$$

$$\alpha = atan2\left(\left(r_y - r_{ry}\right), (r_x - r_{rx})\right) = \text{asin}\ ((r\_z - r\_rz)\ /\ \rho)$$

$$\in = \text{arctan}\ \left(\frac{r_z - r_{rz}}{\sqrt{(r_x - r_{rx})^2 + \left(r_y - r_{ry}\right)^2}}\right)$$

Where (r_{rx}, r_{ry}, r_{rz}) represents the position of the radar system in the ECI coordinate system.

The measurement noise vector $v = \left[v_\rho, v_\alpha, v_\varepsilon\right]^T$ represents the uncertainties in the radar measurements, which can be assumed to be a zero-mean Gaussian noise with a known covariance matrix R. The elements of R can be obtained from the specifications of the radar system or by analyzing the residuals of the measurements.

The measurement model can be written as:

y = h(x) + v

Where $y = [\rho, \alpha, \varepsilon]^T$ is the vector of observed radar measurements.

In this example, the NORAD tool provides the measurement data on the debris characteristic. This information can be incorporated into the Bayesian framework to refine calculate the likelihood function.

The NORAD tool leverages the SGP4 Orbit Determination[11] to provide the measurement data.

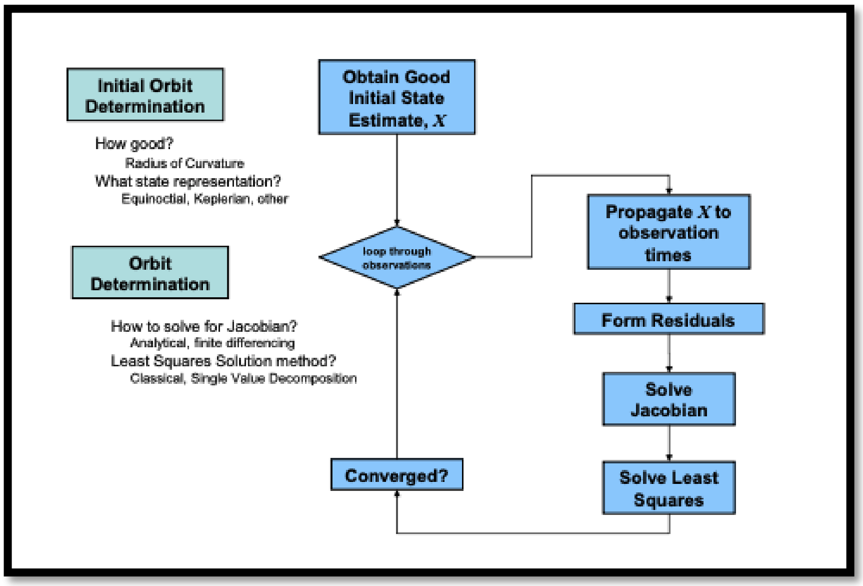


Figure 2 : Orbit determination algorithm

[11] David Vallado and Paul Crawford, 'SGP4 Orbit Determination', in *AIAA/AAS Astrodynamics Specialist Conference and Exhibit* (presented at the AIAA/AAS Astrodynamics Specialist Conference and Exhibit, Honolulu, Hawaii: American Institute of Aeronautics and Astronautics, 2008) <https://doi.org/10.2514/6.2008-6770>.

2.3 Uncertainty Model

Orbital debris tracking is a challenging task due to various uncertainties, such as initial condition errors, environmental factors, and the limitations of mathematical models. This thesis proposes a comprehensive Uncertainty Quantification (UQ) model based on Bayesian principles to address these issues. The goal is to develop a probabilistic framework that can effectively quantify the uncertainties associated with orbital debris parameters. By providing a measure of the confidence levels associated with debris trajectory predictions, this UQ model aims to improve the reliability of collision risk assessments as well as lay ground for future debris capture missions.

The main relation that captures the essence of the Bayesian statistics is the one of posterior probability[12].

$$P\ (H|D) = \frac{P(H)P(D|H)}{P(D)} \tag{11}$$

2.3.1 Hypothesis

H is the hypothesis that we want to be certain or confident of given that we have observed data D. In our case, the deterministic model predicts $\hat{X}$, the state vector of the debris in the future state at time t. This is our hypothesis; we want to find the certainty on our hypothesis $\hat{X}$.

$$H: X\ =\ \hat{X}$$

Let's introduce $\hat{X}(t) = [\ \widehat{r_p}(t)\ \ \widehat{r_a}(t)\ \ \hat{\imath}(t)\ \ \widehat{\Omega}(t)\ \ \widehat{\omega}(t)\ \ \widehat{\theta}(t)\ ]$

2.3.2 Prior

P(H) is the prior probability; it provides the confidence information on our Hypothesis $\hat{X}$ prior to observing the data D. It is usually characterized by its mean and variance.

In our case, the mean of each orbital parameter is the prior of that parameter. We have accumulated 1 year worth of prior data on 10 IRIDIUM 33[13] debris objects that will be used for this purpose.

$$\text{Prior } P(\hat{X}) = (\mu, \sigma^2) \tag{12}$$

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[12] Brendon J Brewer, '1 Bayesian Inference and Computation: A Beginner's Guide', *Cambridge University Press* <https://assets.cambridge.org/97811071/02132/excerpt/9781107102132_excerpt.pdf>.
[13] 'CelesTrak: IRIDIUM 33 Debris'.

$$\mu = \frac{1}{N}\sum_{i=1}^{N}\hat{x}_i$$

$$\sigma_i^2 = \frac{1}{N}\sum_{i=1}^{N}(\hat{x}_i - \mu)^2$$

2.3.3 Likelihood Function

P(D|$\hat{X}$) is the likelihood of observing the data D if the Hypothesis $\hat{X}$ were true.

A Gaussian distribution can be used to predict the observed data D if the Hypothesis $\hat{X}$ were true. Beta Distribution and Poisson Distribution were also analyzed , for a small sample the Gaussian distribution seemed more adequate.

$$p(D|\hat{X}) = \prod_{i=1}^{N}\frac{1}{\delta_i\sqrt{2\pi}}\exp\left[-\frac{1}{2\sigma^2}(\mathrm{D}-\mu)^2\right] \tag{13}$$

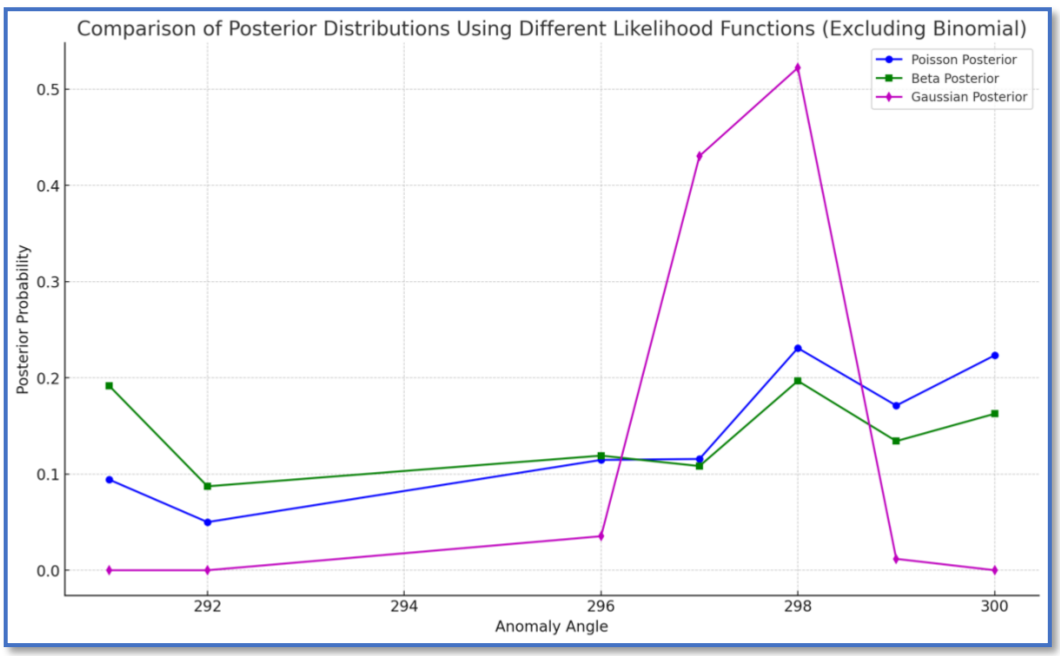

Figure 3 : Comparison of Posterior Distributions Using Different Likelihood Functions.

2.3.4 Posterior Distribution

Now that we have calculated the intermediate sub-components of the Bayesian Framework , it's time to calculate the posterior distribution which is given by the equation below,

$$P\left(\hat{X}\middle|D\right) = \frac{P(\hat{X})P\left(D\middle|\hat{X}\right)}{P(D)} \tag{14}$$

P(D) is the marginal likelihood which describes the probability that the data D would have been observed whether $\hat{X}$ is true or not.This likelihood is calculated by adding the two mutually exclusive probabilities.

i) If $\hat{X}$ is true, the probability is P($\hat{X}$)P(D|$\hat{X}$)

ii) If $\hat{X}$ is false, the probability is P($\overline{\hat{X}}$)P(D|$\overline{\hat{X}}$)

P(D) = P($\hat{X}$)P(D|H)+ P($\overline{\hat{X}}$)P(D|$\overline{\hat{X}}$) which is summation of the Prior*likelihood term.

$$P(D) = \sum P\left(\hat{X}\right)P\left(D\middle|\hat{X}\right) \tag{15}$$

Here is an example of a Bayes' Box that holds the different elements that makeup up the posterior distribution relations.

For this example, we are working on Bayes' box for the true anomaly angle of the debris' orbit. The Hypothesis is the different hypothesis of what the angles could be. From the years of data, we can come up with the prior information using the relationship explained above. Let us assume that the measurement model predicted the true anomaly angle to be 297,297 and 298 degrees in three different measurements.

Anomaly Angle	**Prior**	**Gaussian Likelihood**	**Prior X Gaussian Likelihood**	**Gaussian Posterior**
291	0.111	3.40E-28	3.77E-29	6.28E-27
292	0.056	1.30E-20	7.28E-22	1.21E-19
296	0.111	3.20E-03	3.55E-04	5.91E-02
297	0.111	3.90E-03	4.33E-04	7.20E-02
298	0.222	2.30E-02	5.11E-03	8.49E-01
299	0.167	7.10E-04	1.19E-04	1.97E-02
300	0.222	1.10E-06	2.44E-07	4.06E-05
Total	1			1.00E+00

Table 1: Bayes' box for true Anomaly angle Example.

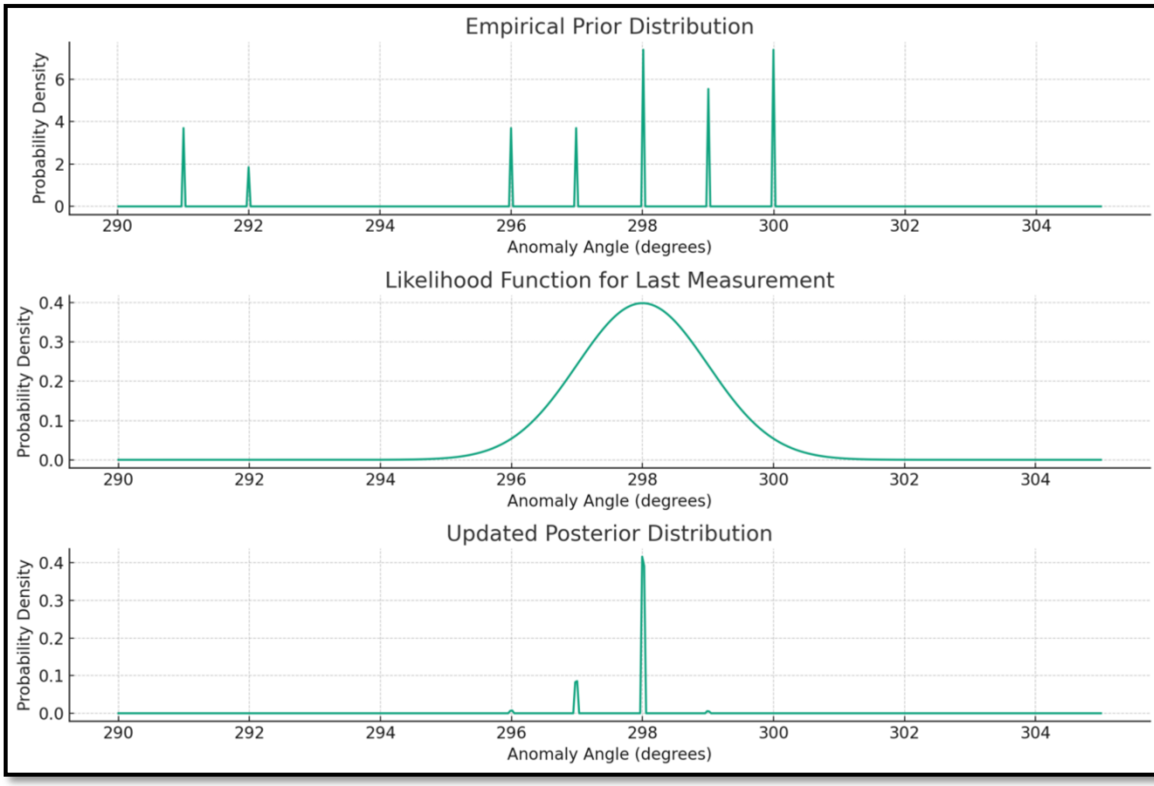

Figure 4 : Illustration of the different distributions used in Bayesian Framework.

Bayes' rule can be applied to a set of hypotheses simultaneously as shown below:

$$P\ (H_i|D) = \frac{P(H_i)P(D|H_i)}{P(D)}$$

Hypotheses	prior	likelihood	Prior x Likelihood	Posterior
H_1 H_2 …	$P(H_1)$ $P(H_2)$ …	$P(D\|H_1)$ $P(D\|H_2)$ …	$P(H_1)$ x $P(D\|H_1)$ $P(H_2)$ x $P(D\|H_2)$ …	$P(H_1\|D)$ $P(H_2\|D)$ …
Totals	1		P(D)	1

Table 2: A Bayes' Box for multiple hypothesis.

Epoch Date	Inclination	Right Ascension of Ascending Node	Eccentricity	Argument of Perigee	Mean Anomaly	Revolution Number
1/4/23 13:20	86.4245	299.2326	0.0175605	287.04	71.161	44075
1/25/23 3:58	86.4234	290.972	0.017045	221.4898	137.3245	44366
1/2/23 13:44	86.4242	300.0286	0.0176132	293.2553	65.0172	44047
1/7/23 12:44	86.4247	298.0405	0.0174759	277.6978	80.4392	44117
1/7/23 5:56	86.4249	298.154	0.0174873	278.5782	79.5623	44113

Sample Data for an IRIDIUM -33 Debris from NORAD

2.3.5 Parameter Estimation

Once we have the posterior distribution, we need to leverage that to do parameter estimation of the Debris' Orbital parameters. MCMC allows us to compute probabilities, expectations etc. from the posterior distribution.

Metropolis-Hasting's algorithm will be used to solve the MCMC problem. A MATLAB/Python code will be provided to perform the Markov Chain Monte Carlo along with simulations.

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“Atri Dutta” – “Nano Satellites, spring 2023, Wichita State University”