Assignment - DS

①

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Q1) $O(x^2)$

$t = 5$ sec size of input, $n = 10$

since time complexity $\approx O(x^2)$

$t \propto x^2$

$$\frac{t_1}{t_2} = \frac{(x_1)^2}{(x_2)^2}$$

$$\frac{5}{t_2} = \left(\frac{10}{50}\right)^2$$

$$\boxed{t_2 = 125 \text{ sec}}$$

Q2) T $A(n) = n^3$
$B(n) = 2n^2$

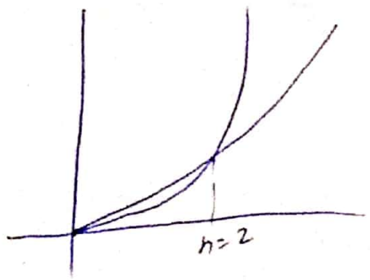
for break point

$n^3 = 2n^2$

$n^2(n-2) = 0$

$\boxed{n = 2}$

Break point at $n=2$



Q3) let $f(n) = n2^n$

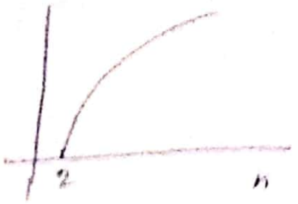
for $f(n)$ to be in $O(4^n)$

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \lim_{n\to\infty} \frac{n2^n}{4^n} = c, \quad c \geq 0$$

$$= \lim_{n\to\infty} \frac{n2^n}{2^{2n}}$$

$$= \lim_{n\to\infty} \frac{n}{2^n} = 0 = c$$ which lies in $[0, \infty)$

Q4) logarithmic functions

log n 

let n = 1 million

$\log(10^6) = 6$

$\log(10^9) = 9$

So, we see that time Pes by small amount. Hence, it grows very slowly.

Q5) (a) θ + O

if $\lim_{n\to\infty} \frac{f(n)}{g(n)} = c$, $c \geq 0$, then we say f(n) grows with O(g(n)) i.e we have upper bound f(n) by g(n)

if $\lim_{n\to\infty} \frac{f(n)}{g(n)} = c$, $c > 0$ then we say f(n) grows with almost same speed as g(n) we say $f(n) \approx g(n)$

(b) O and Ω

In O(n) we upper bond the f(n) with another fn g(n) i.e, $f(n) \approx O(g(n))$

In Ω (n) we lower bond the f(n) with g(x)

i.e $\lim_{n\to\infty} \frac{f(n)}{g(n)} \neq 0$.

Q-6) P.T $n^4 + \log n + 17 \approx O(n^4)$

$\log n < n^4$

$17 \leq 17n^4$

$n^4 + \log n + 17 \leq 17n^4$

$\lim_{n\to\infty} \frac{n^4 + \log n + 17}{\log n} \approx 1$

$= c$, $c > 0$

$n^4 + \log n + 17 \approx O(n^4)$

Q.7-A (a) Applying step count ②

R=1 is a declarative statement with initialisation hence step count is 1 -①

'while' has 0 step count = 0

k = k+1 is executed n times hence step count = n

End while has 0 step = 0

Total steps = n+1 + n + 1 = 2(n+1)

(b) for i=1 to n-1 → n

for j = i+1 to n → n-i+1 step

Steps = n, n-1, n-2, ---- 2

Total steps = $\frac{n(n+1)-2}{2} = \frac{n^2+n-2}{2}$

Swap = (n-1) + (n-2) + --- 1

$= \frac{(n-1)n}{2}$ times

end for = 0

End for = 0

Total steps = $n + \frac{n^2+n-2}{2} + \frac{n^2-n}{2}$

$= n^2+n-1$

Q-8 a) let $T = x^2+x+1$ be quad. eqⁿ

at n=20, $T_1 = 400 + 21 = 421$

$T_2 = 400+31 = 931$

$\boxed{T_2 \simeq 2T_1}$

Hence time nearly doubles for larger inputs

Q-9) $T_A = (100)^n$, $T_B = n^4$

but $n = 10^{10}$

$T_{A_1} = (100)^{10^{10}}$ $\quad$ $T_{B_1} = 10^{40}$

$= (10)^{2\times 10^{10}}$

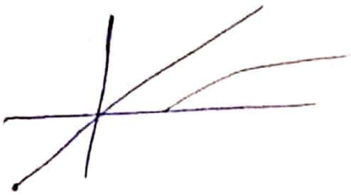
$T_{A_1} >> T_{B_1}$

also

$\log n^4 = 4\log n$

& $\log 100^n = 2n$

& growth of $n > \log n$



$(100)^n$ grows faster than n^4

Q-10) P.T $n\log n \in \Theta(\log(n!))$

for $f(n) \in \Theta(g(n))$, $f(n) \geq g(n)$

also $\therefore$

$\log(n) + \log(n-1) + \text{---} + \log(n-(n-1))$

$\leq \log(n) + \text{---} + \log n$

$\Rightarrow \log n! \leq n\log n$

$\lim_{n\to\infty} \frac{n\log n}{\log n!} \geq 0$

hence $n\log n \in \Theta(\log(n!))$

Q11) (a) $2^{n-1} + 4^{n+1}$

let $f(n) = 2^{n-1} + 4^{n+1}$

for $f(n) = \theta(g(n))$

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = C, \; C > 0$$

let $g(n) = 4^n$

$$\lim_{n\to\infty} \frac{2^{n-1} + 4^{n+1}}{4^n} = 4 > 0$$

hence $(2^{n-1} + 4^{n+1}) \approx \theta(4^n)$

(b) $(n^2+6)^8$

let $g(n) = n^{16}$

$$\lim_{n\to\infty} \frac{(n^2+6)^8}{(n^2)^8} = 1 > 0$$

hence $(2^{n-1} + 4^{n+1}) \approx \theta(n^{16})$

Q-12)
$TA = n^2$

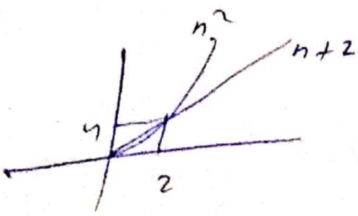
$TA = n+2$

for breaking Point

$n^2 = n+2$

$n = 2, -1$

$\boxed{n=2}$ as $n > 0$



Q13 a) for ($i=n$; $i=1$; $i=i-4$)
{
$\boxed{}$ — A
}

let $n=16$

i	A (no of times)
16	1
12	2
8	3
4	4
0	

In order of $n/4$

$O(n)$

b) for ($i=1$; $i \le n$; $i=i+5$)
{
A
}

let $n=20$

i	A
1	1
6	2
11	3
16	4
21	

order of $n/5$

$O(n)$

(c) for ($i=n-1$; $i \ge 1$; $i=i/2$)
{
A
}

i	A
$n-1$	1
$\frac{n-1}{2}$	2
$\frac{n-1}{2^2}$	3
$\frac{n-1}{2^3}$	4

$\frac{n-1}{2^k}$ $\quad k$

$2^k \ge n-1$ or

$2^k \ge n$

$k \ge \log_2 n$

$O(\log_2 n)$

d) for (i=1; i<n; i=i*2)
{
A
}

⇒ i A
1 1
1×2 2
2^2 3
2^3 4
⋮ ⋮
2^k

for n = 16
i A
1 1
2 2
2^2 3
2^3 4
2^4 5

$2^k < n$

$k < \log n$

$O(\log n)$

(e) for (i=0; i<n; i++)
□ → $c_1 \times n$
for (j=0; j<n; j++)
□ → $c_2 \times n \times n$

⇒ i j
0 n
1 n
⋮ |
n n

$= n(1+2+ \cdots +n)$

$= n^2$

inner loop executes n^2 times

total time = $c_1 \times n + c_2 \times n^2$

$= O(n^2)$

Q) for (i=0; i<n; i++) → n.
for (i=n-1; i>=1; i= i/2) → $\log_2 n$
$\square$ — $C_1 \times n \log_2 n$
for (j=0 ; j<n ; j++) n.
⇒ $\square$ → $C_2 \times n \times n$

~~total time~~

i	i (inner)	j
0	log n	n
1	log n	n
⋮	⋮	⋮
n	log n	n
total	n log n	n^2

total time = $C_1 n \log n + C_2 n^2$
= $O(n^2)$

Q) for (i=0; i<n ; i++)
for (j=0; j<n; j++)
for (k=0; k<n; k++)
$\square$ — C_1
⇒ for (i=0; i<n ; i++)
for (j=10 ; j>=1; j--)
$\square$ — C_2

1st nested loops

i	j	k
0	n	n
1	n	n^2
⋮	⋮	⋮
n	n^2	n^3

no of iteration = n^3

Second nested loop

i	j
0	10
1	10
2	10
⋮	⋮
n	10n

no. of iteration = 10n

total time = $C_1 n^3 + C_2 * 10n$
= $O(n^3)$

(5)

for (i=0 ; i<n ; i++)
  for (j=0 ; j≤i ; j++)
    $\square \rightarrow A$

Outer loop

	i	j
1	0	1
2	1	2
3	2	3
⋮	⋮	⋮
n	n-1	n

no. of times j execute $= 1+2+3+ \ldots n$

$= \frac{n(n+1)}{2} = \frac{n^2}{2} + \frac{n}{2}$

$= O(n^2)$

(i) for (i=0 ; i<n-1 ; i++)
  for (j=i+1 ; j<n ; j++)
    $\square \rightarrow A$

Ans.

outer loop

	i	j
1	0	n-1
2	1	n-2
3	2	⋮
⋮	⋮	⋮
n-1	n-2	1

total no of Execution of j $= 1+2+3+ \ldots n-1$

$= \frac{n(n-1)}{2} = O(n^2)$

(j) for (x=n-2 ; x≥0 ; x--)
  for (j=0 ; j≤x ; j++)

1) outer loop $\quad n \quad$ inner loop (j)

outer loop	n	inner loop (j)
1	$n-2$	$n-1$
2	$n-3$	$n-2$
3	$n-4$	$n-3$
⋮	⋮	⋮
$n-1$	0	1

no. of execution $= 1 + 2 + \cdots n-1$

$$= \frac{n(n-1)}{2} = O(n^2)$$

(k) for $(i=0; i<n; i++)$

for $(j=n-1; j\geq 1; j=i/2)$

$\square - c_1$

if (c)

for $(i=0; i<n; i++)$

for $(j=0; j<n; j++)$

$\square - c_2$

else

for $(i=0; i<n; i++)$

$\square - c_3$

2) In nested loops

i	j
0	$\log n$
1	$\log n$
⋮	⋮
n	$n\log n$

$\Rightarrow O(n\log n)$

In if-else

i	j
0	n
1	n
⋮	⋮
n	n^2

$\Rightarrow c_2 n^2 = O(n^2)$

else $c_3(n) = O(n)$

total time $= O(n^2)$

(l) $i = 1$, $S = 1$ ⑥

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while (S ≤ n)
{
    i++;        → c1
    S = S + i;  → c2
}
```

Ans)

i	S	
1	1	
2	3	$= 1+2$
3	6	$= 1+2+3$
4	10	
5	15	$\vdots$

$S = \frac{k(k+1)}{2}$

$\frac{k(k+1)}{2} > n$

$k \approx \sqrt{n}$ $\sim O(\sqrt{n})$

(m) for($i = 1$; $i^2 \le n$; $i++$)

$\square - c_1$

i	i^2
1	$(1)^2$
2	$(2)^2$
3	$(3)^2$
$\vdots$	$\vert$
n	$(k)^2$ no. of iterations

$k^2 < n$

$k \approx \sqrt{n}$ $\sim O(\sqrt{n})$

(m) for (i=1, i ≤ n ; i++)
for (j=1; j ≤ i ; j++)
for (k=1 ; k ≤ 100 ; k++)

→

i	j	k
1	1	1×100
2	2	2×100
3	3	3×100
⋮	⋮	⋮
n	n	n×100

Total times k execute = $n(n \times 100)$
$= n^2 100 = O(n^2)$

(o) for (i = 1; i ≤ n ; i++)
for (j = 1 ; j ≤ i² ; j++)
for (k = 1 ; k ≤ $\frac{n}{2}$; k++)

→

i	j	k
1	$(1)^2$	$\frac{n}{2} \times (1)^2$
2	$(2)^2$	$\frac{n}{2} \times (2)^2$
3	$(3)^2$	
⋮	⋮	⋮
n	$(n)^2$	$\frac{n}{2} \times (n^2) \times n$

total times k execute $= \frac{n}{2}\left[1^2 + (2)^2 + \ldots n^2\right]$

$= \frac{n}{2}\left[\frac{n(n+1)(2n+1)}{6}\right]$

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$O(n^4)$

for ($i = \frac{n}{2}$; $i \le n$; i++)

for (j=1 ; $j \le \frac{n}{2}$; j++)

for (k=1 ; $k \le n$; k=k*2)

→

i	j	k
$\frac{n}{2}$	$n/2$	$\log n$
$\frac{n}{2}+1$	$n/2$	$\log n$
|	|	|
n	$\frac{n}{2}(\frac{n}{2})$	$\frac{n}{2} \log n \times \frac{n}{2}$

total time k executes $= \frac{n}{2} \times \frac{n}{2} \times \log n$

$= O(n^2 \log n)$

(9) for ($i = \frac{n}{2}$; $i \le n$; i++)

for (j=1; j<n ; j=2*j)

for (k=1; $k \le n$; k=2*k)

→

i	j	k
$\frac{n}{2}$	$\log n$	$\log n$
$\frac{n}{2}+1$	$\log n$	$\log n$
|	|	|
n	$\frac{n}{2} \log n$	$\frac{n}{2} \log n \times \log n$

total time k execute $= \frac{n}{2} \log n \times \log n$

$= O(n \log^2 n)$