original

Chapter- 4.

(1)

CIRCUIT THEOREMS

~~4.1~~ 4.0 INTRODUCTION

In chapter 3, we have used Kirchhoff's laws and main advantage of using ~~these laws~~ KCL and KVL is that ~~two laws~~ we can analyze a circuit without tampering with its original configuration. A major drawback of this approach is that, for a large and complex circuit, tedious computation is involved. To handle the complexity of the circuits, over the years engineers have developed some circuit theorems to simplyfy circuits analysis. Such theorems include Thevenin's theorem and Norton's theorem. These theorems are applicable to linear circuits and in this chapter, we ~~will~~ first discuss the concept of circuit linearity. In this Chapter we will also discuss the concepts of superposition, source transformation and maximum power transfer.

4.1: LINEARITY PROPERTY

Linearity: It is the property of an element describing a linear relationship between cause and effect.

Although linearity property applies to many circuit elements, but in this chapter we shall limit its applicability to resistors only. The linearity property is a combination of both the homogeneity (scaling) property and the additivity property.

Homogeneity property: It requires that if the input (called excitation) is multiplied by constant, then the output (called response) is multiplied by the same constant.

For example, for a resistor, Ohm's law relates the input current i to the output voltage v,

$$v = iR \quad \text{.....} \quad (4.1)$$

~~If the current is increased (decreased)~~

If the current is increased (or decreased) by a constant K, then the voltage increases (or decreases) correspondingly by K, that is

$$(Ki)R = Kv \quad \text{....} \quad (4.2)$$

Additivity property: It requires that the response to a sum of inputs is the sum

Fig. 3.1: Different symbols for indicating a reference node
(a) chassis ground (b) common ground
(c) ground.

all circuits. When the potential of the earth is used as reference, we use the earth ground as shown in Fig. 3.1(b) or 3.1(c). In this book, we shall always use the symbol of Fig. 3.1(a).

After selecting a reference node, assign voltage designations to nonreference nodes. For example, consider the circuit of Fig. 3.2(a). Node 0 is the

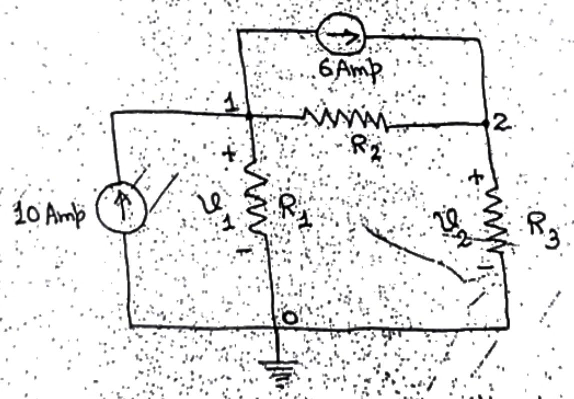


Fig. 3.2(a): circuit with two independent current sources.

reference node ($v_0 = 0$). Nodes 1 and 2 are assigned voltages v_1 and v_2, respectively. A node voltage is defined as the voltage rise from the reference node to a nonreference node.

The Second step in the nodal analysis is to apply KCL to each nonreference node in the circuit. For further explanation, circuit of Fig. 3.2(a) is redrawn in Fig 3.2(b) to avoid putting too much information on the same circuit.

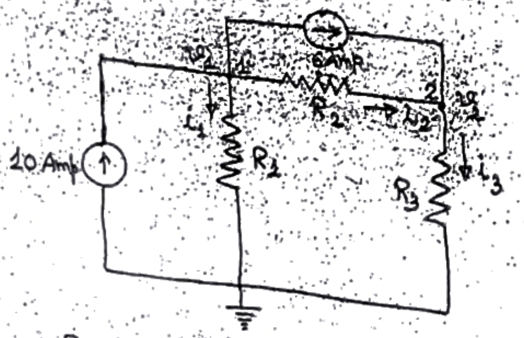


Fig. 3.2(b): Circuit of Fig. 3.2(a) is redrawn with useful information.

Applying KCL at node 1, we have,

$$10 = 6 + i_1 + i_2 \quad \text{----} \quad (3.1)$$

At node 2

$$6 + i_2 = i_3 \quad \text{----} \quad (3.2)$$

To obtain the unknown currents i_1, i_2 and i_3 in terms of node voltages, we apply Ohm's law.

Soln.

Applying KVL, we obtain,

$$12 i_1 - 4 i_2 + v = 0 \quad \text{....(i)}$$

$$-4 i_1 + 16 i_2 - v - v_x = 0 \quad \text{....(ii)}$$

But $v_x = 2 i_1$, equation (ii) becomes

$$-4 i_1 + 16 i_2 - 2 i_1 = v$$

$$\therefore -6 i_1 + 16 i_2 = v \quad \text{....(iii)}$$

When $v = 3$ Volt, solving eqns. (i) and (iii), we obtain $i_0 = i_2 = \frac{3}{28}$ Amp and when $v = 6$ Volt, $i_0 = i_2 = \frac{6}{28}$ Amp.

This clearly shows that when source voltage (input) is doubled, i_0 also doubles. Hence, the circuit is linear.

EX-4.2: Determine v_o, when $i = 5$ Amp and $i = 10$ Amp of the circuit shown in Fig.4.3.

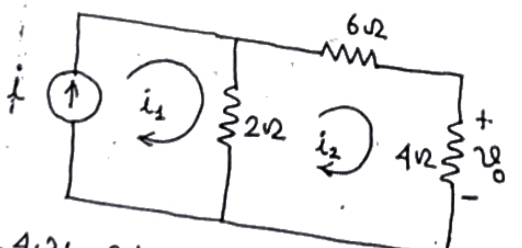


Fig. 4.3: Circuit for EX-4.2

Soln.

$$i_1 = i \quad \cdots\cdots (i)$$

$$12 i_2 - 2 i_1 = 0$$

$$\therefore \quad i_2 = \frac{i_1}{6} \quad \cdots\cdots (ii)$$

Also

$$v_0 = 4 i_2 \quad \cdots\cdots (iii)$$

when $i = 5$ Amp; $i_1 = 5$ Amp, $i_2 = \frac{i_1}{6} = \frac{5}{6}$ Amp;

$v_0 = 4 i_2 = 4 \times \frac{5}{6} = \frac{20}{6}$ Volt.

Similarly, when $i = 10$ Amp; $i_1 = 10$ Amp;

$i_2 = \frac{i_1}{6} = \frac{10}{6}$ Amp; $v_0 = 4 \times \frac{10}{6} = \frac{40}{6}$ Volt.

This shows that when source current (input) is doubled, v_0 also doubles. Hence, the circuit is linear.

4.2: SUPERPOSITION PRINCIPLE

If a circuit has two or more independent sources one can determine the contribution of each independent source to the variable and then add them up. This approach is known as the superposition. The idea of superposition rests on the linearity property.

The superposition principle states that the current through (or voltage across) an element in a linear circuit is the algebraic sum of the currents through (or voltages across) that element due to each independent source acting alone.

$$\therefore \Delta = a_{11}a_{22}a_{33} + a_{21}a_{32}a_{13} + a_{31}a_{12}a_{23} - a_{13}a_{22}a_{31} - a_{23}a_{32}a_{11} - a_{33}a_{12}a_{21} \quad \cdots (3.18)$$

Ex-3.1: Determine the node voltages in the circuit shown in Fig. 3.3(a).

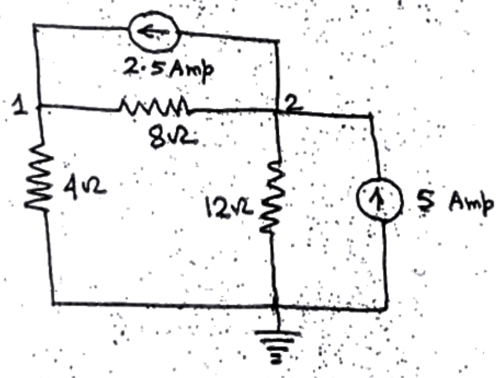


Fig. 3.3(a): Circuit for Ex-3.1

Soln.

Fig. 3.3(b) shows the circuit for analysis of Fig. 3.3(a).

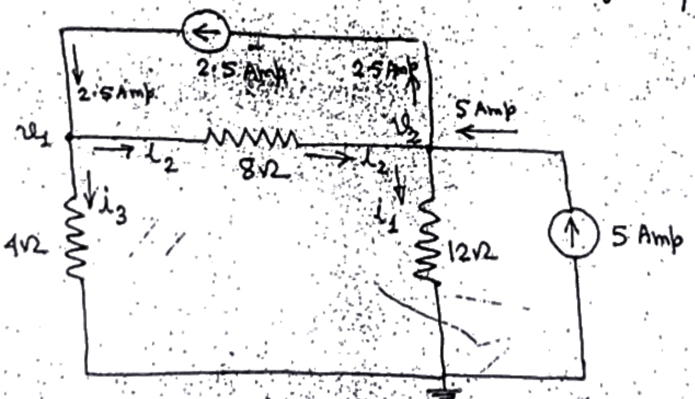


Fig. 3.3(b): Circuit for analysis of original circuit shown in Fig. 3.3(a).

At node 1, applying KCL and Ohm's law gives,

$$2.5 = i_2 + i_3 = \frac{v_1 - v_2}{8} + \frac{v_1 - 0}{4}$$

$$\therefore \quad 3v_1 - v_2 = 20 \quad \ldots (i)$$

At node 2,

$$5 + i_2 = 2.5 + i_1$$

$$\therefore \quad \frac{2.5}{1} + \frac{v_1 - v_2}{8} = \frac{v_2 - 0}{12}$$

$$\therefore \quad -3v_1 + 5v_2 = 60 \quad \ldots (ii)$$

By solving eqns. (i) and (ii), we get

$v_1 = 13.33$ Volt, $v_2 = 20$ Volt.

Ex. 3.2: Calculate the node voltages in the circuit shown in Fig. 3.4(a).

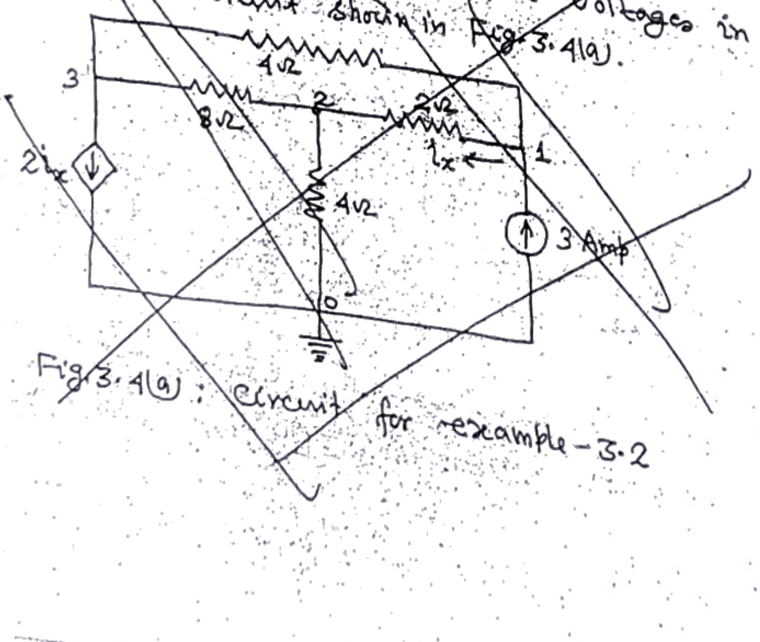


Fig. 3.4(a): Circuit for example-3.2

Solving eqns. (xi) and (xii), we get,

$$i_x'' = -\frac{60}{17} \text{ Amp} \quad \cdots \text{(xiii)}$$

Therefore,

$$i_x = i_x' + i_x'' = \frac{52}{17} - \frac{60}{17} = \frac{-8}{17} \text{ Amp}$$

EX- 4.5: Determine i using superposition theorem of the circuit shown in Fig. 4.8

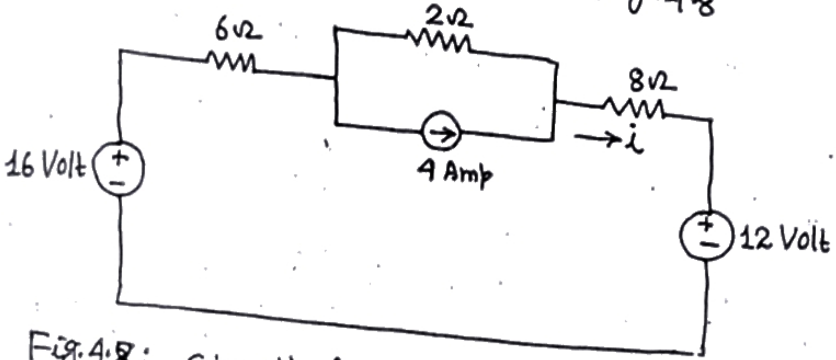
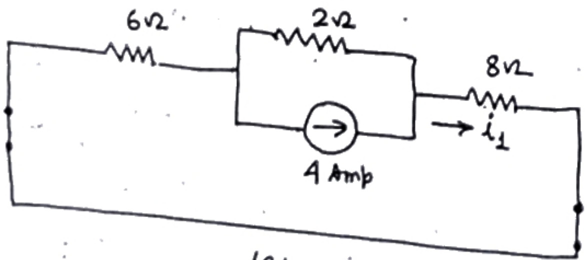
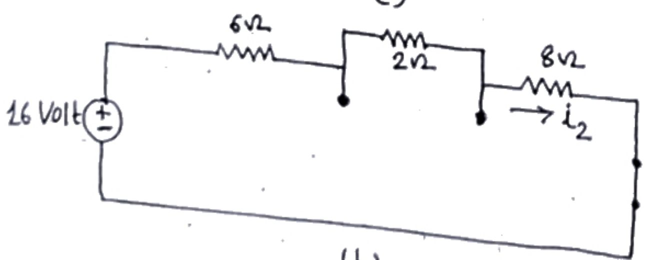


Fig. 4.8: Circuit for EX- 4.5

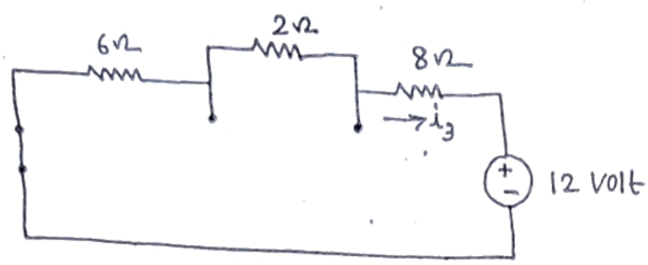
Soln.



(a)



(b)



(c)

Fig. 4.9: (a) 16 Volt and 12 Volt sources are turned-off

(b) 4 Amp current source and 12 Volt voltage source are turned off

(c) 4 Amp current source and 16 Volt voltage source are turned-off.

In Fig. 4.9 (a), we apply current division principle,

$$i_1 = \frac{2}{(6+2+8)} \times 4 = 0.5 \text{ Amp}$$

In Fig. 4.9(b), we apply KVL,

$$i_2 = \frac{16}{16} = 1 \text{ Amp}$$

Similarly from Fig. 4.9(c), we obtain

$$i_3 = -\frac{12}{16} = -\frac{3}{4} \text{ Amp} = -0.75 \text{ Amp}$$

Hence

$$i = i_1 + i_2 + i_3 = 0.5 + 1 - 0.75 = 0.75 \text{ Amp.}$$

4.3: SOURCE TRANSFORMATION

Source transformation is a tool to simplify circuit analysis. Basic idea behind this is concept of equivalance. An equivalent circuit is one whose v-i characteristics are identical with the original circuit.

A source transformation, shown in Fig.4.10, allows a voltage source in series with a resistor to be replaced by a current source in parallel with the same resistor or vice versa.

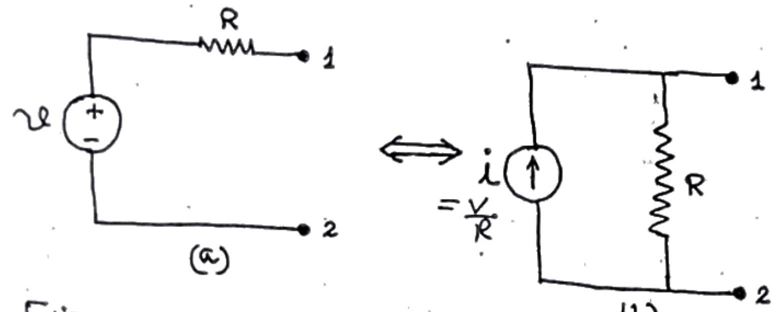


Fig. 4.10: Source transformations

Double headed arrow in Fig.4.10 emphasizes that a source transformation is bilateral, that is we can start with either configuration and derive the other.

The two circuits in Fig.4.10, are equivalent, provided they have same voltage-current relation at terminals 1-2. Equivalence is achieved if any resistor R_L experiences the same current flow, and thus the same voltage drop, whether connected between nodes 1, 2 in Fig. 4.10(a) or Fig. 4.10(b).

Suppose R_L is connected between nodes 1, 2 in Fig. 4.10(a). Using Ohm's law, the current in R_L is

$$i_L = \frac{v}{R + R_L} \quad \cdots \quad (4.6)$$

Now suppose the same Resistor R_L is connected between nodes 1, 2 in Fig. 4.10(b). We find the current in R_L is

$$i_L = \frac{R}{R + R_L} i \quad \cdots \quad (4.7)$$

If the two circuits in Fig 4.10(a) and Fig. 4.10(b) are equivalent, these resistor currents must be the same. Equating the right hand side of eqns (4.6) and (4.7) and simplifying, we obtain

$$i = \frac{v}{R} \quad \text{or} \quad v = iR \quad \cdots \quad (4.8)$$

When eqn. (4.8) is satisfied for the circuits in Fig. 4.10, the current i_L is the same for both circuits in the ~~fig.~~ Fig. 4.10 for all values of R_L. If the current through R_L is the same in both circuits, then the voltage drop across R_L is the same in both circuits, and the circuits are equivalent at nodes 1, 2. If the polarity of v is reversed, the orientation of i must be reversed to maintain equivalence.

Source transformation also applies to dependent sources. As shown in Fig. 4.11, a dependent voltage source in series with a resistor can be transformed to a dependent current source in parallel with the resistor or vice versa where we make sure that that eqn.(4.8) is satisfied.

Fig. 4.11: Transformation of dependent sources.

Ex-4.6: Using source transformation, determine v_o in the circuit shown in Fig. 4.12.

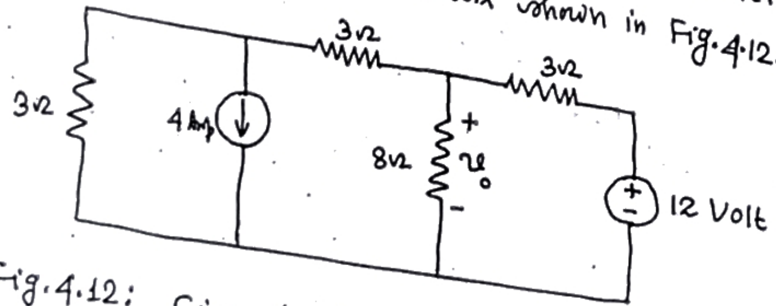


Fig. 4.12: Circuit for Ex-4.6

Soln.

First transform the current and voltage sources to obtain the circuit in Fig. 4.13(a)

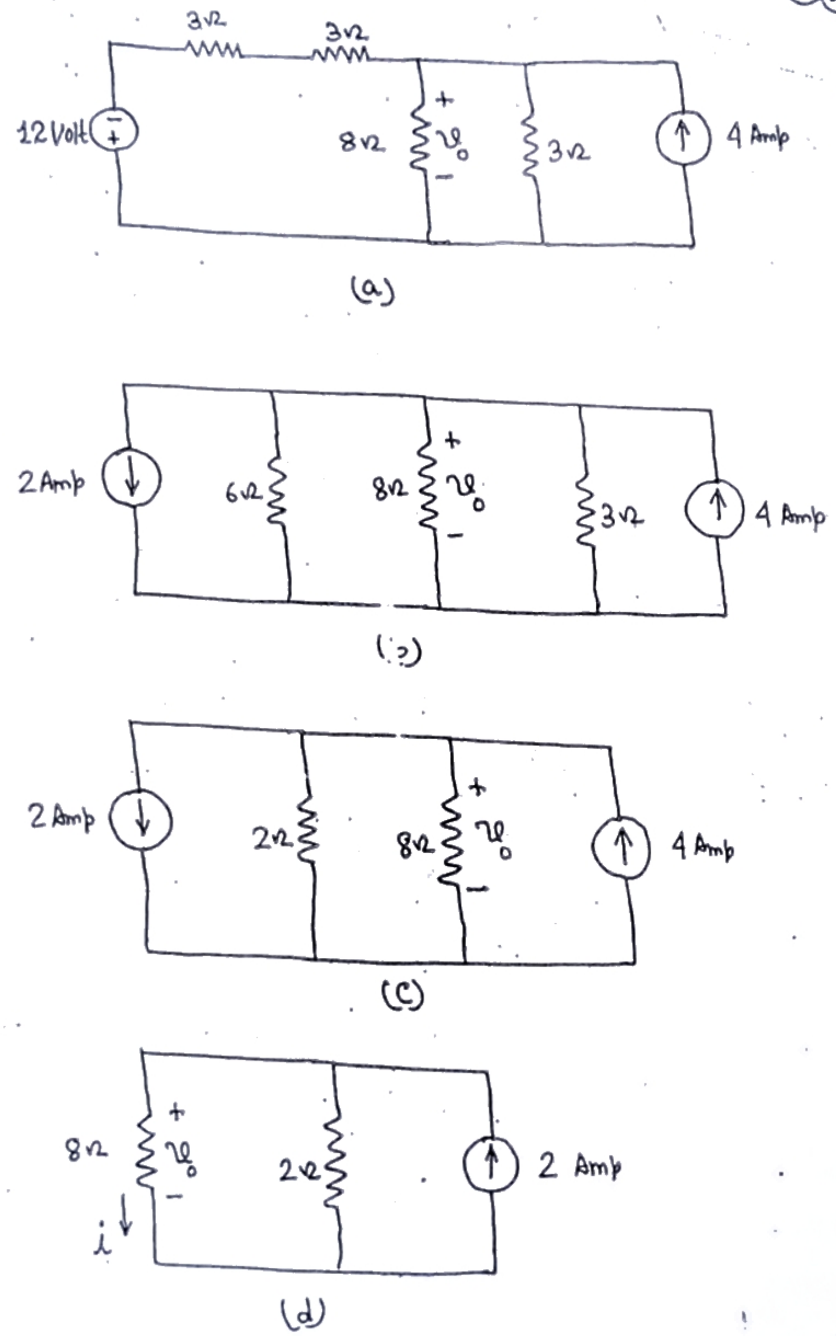


Fig. 4.13: For EX-4.6

Combine 3Ω and 3Ω resistors in series and transforming the 12 volt voltage source in Fig. 4.13(a) gives us Fig. 4.13(b). Now combine 6Ω and 3Ω resistors in parallel to get 2Ω and the equivalent circuit is shown in Fig. 4.13(c). Also combine the 2 Amp and 4 Amp current sources in Fig. 4.13(c) to get equivalent ~~circuit shown in Fig. 4.13(d).~~ 2 Amp current source and the circuit is shown in Fig. 4.13(d).

From Fig. 4.13(d),

$$i = \frac{2}{(2+8)} \times 2 = 0.4 \text{ Amp}$$

$$\therefore v_o = 8i = 8 \times 0.4 = 3.2 \text{ Volt}.$$

EX-4.7: Using source transformation, determine i_x in the circuit shown in Fig. 4.14

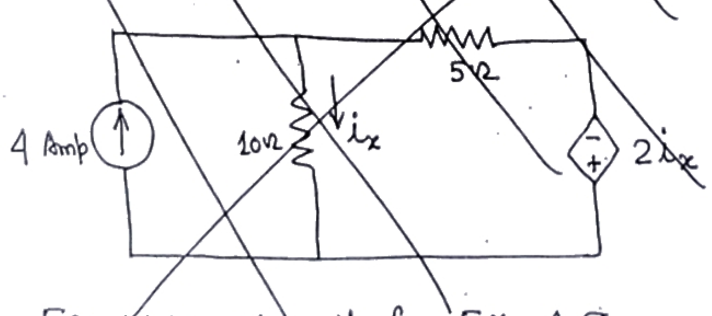


Fig. 4.14: Circuit for EX-4.7.

Soln.

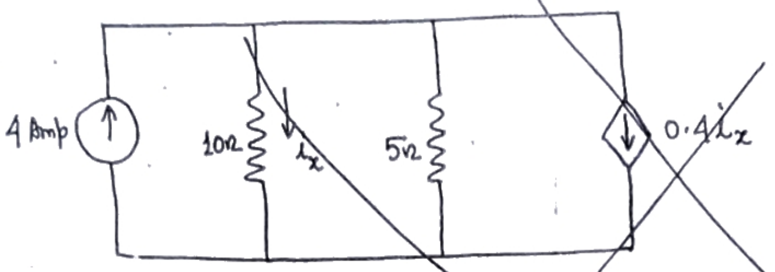


Fig. 4.15: For EX-4.7

We convert dependent voltage source to current source shown in Fig. 4.15. From Fig. 4.15, we can easily write by inspection,

$$i_x = \frac{5}{(5+10)}(4 - 0.4i_x)$$

$$\therefore 3.4 i_x = 4$$

$$\therefore i_x = \frac{4}{3.4} = 1.176 \text{ Amp.}$$

EX-4.8: Using source transformation technique, determine the current through load resistance $R_L = 4\Omega$ of Fig. 4.16.

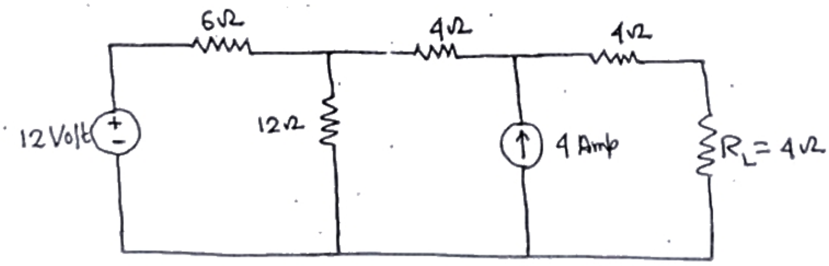


Fig. 4.16: Circuit for EX-4.8

Soln.

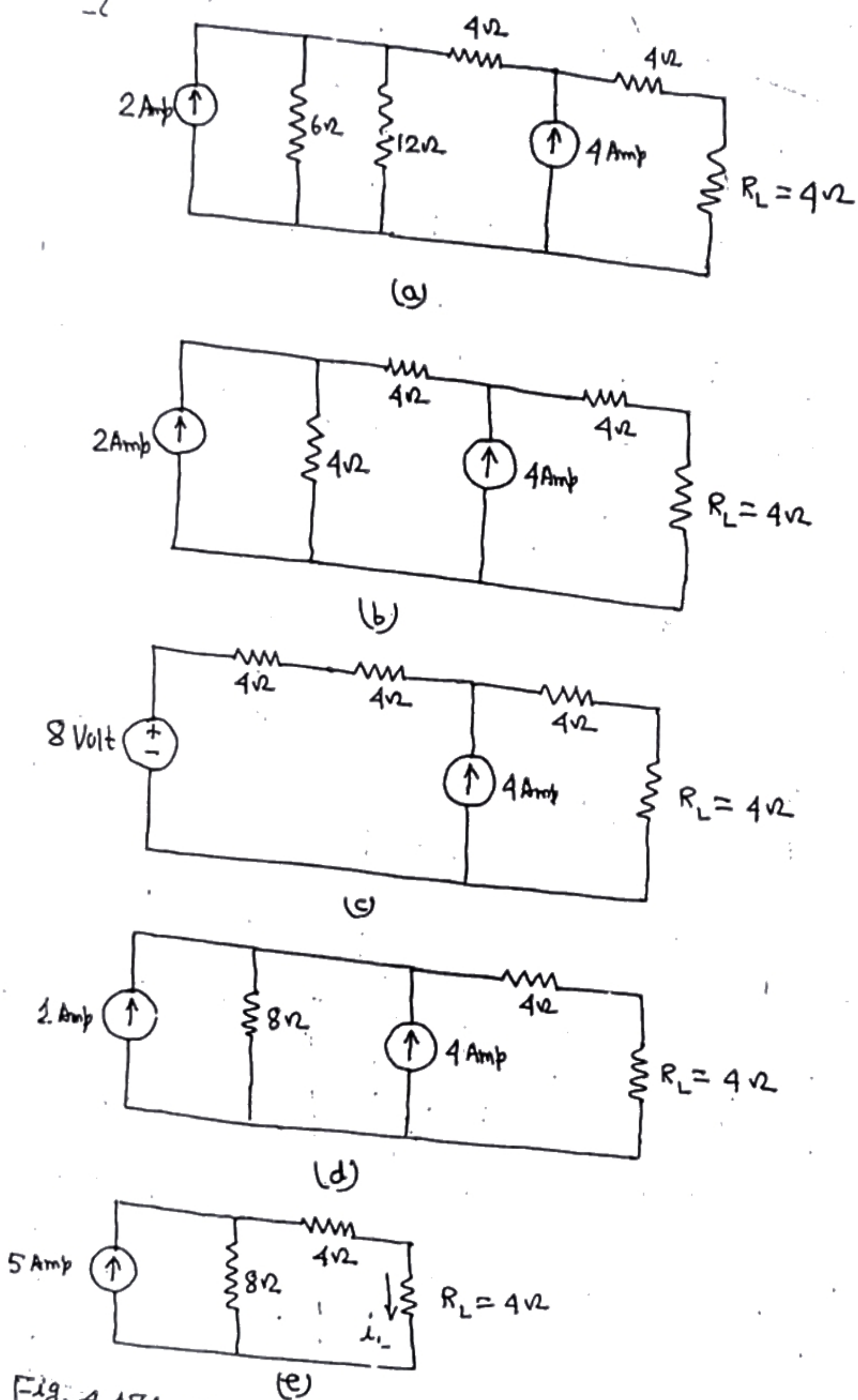


Fig. 4.17: For Ex- 4.8

The 12 Volt voltage source with 6Ω series resistor is converted to a current source and in parallel with 6Ω resistor as shown in Fig. 4.17(a)

6Ω and 12Ω resistors of Fig. 4.17(a) are in parallel and their equivalent is $6\times12/(6+12) = 4\Omega$ as shown in Fig. 4.17(b).

In Fig. 4.17(b), 2 Amp current source is in parallel with 4Ω resistor and is transformed into 8 Volt voltage source with 4Ω series resistor as shown in Fig. 4.17(c).

Next, in Fig. 4.17(c), 8 Volt voltage source with $(4+4) = 8\Omega$ series resistor is transformed into current source of 1 Amp with 8Ω parallel resistor as shown in ~~Fig. 4.17~~ Fig. 4.17(d).

Finally two current sources of 1 Amp and 4 Amp are combined to give a single current source of 5 Amp as shown in Fig. 4.17(e).

Therefore current through load resistance $R_L = 4\Omega$ is given by

$$i_4 = \frac{8}{(8+4+4)} \times 5 = 2.5 \text{ Amp.}$$

4.4: THEVENIN'S THEOREM

In this section, we learn how to replace two-terminal circuits containing resistances and sources by simple equivalent circuits. As a typical example, a household outlet terminal may be connected to different electrical appliances constituting a variable load. Each time the variable is changed, the entire circuit has to be analyzed again. To avoid this problem, Thevenin's theorem gives a good technique by which fixed part of the circuit can be replaced by an equivalent circuit. By a two-terminal circuit, we mean that the original circuit has only two points that can be connected to other circuits. However, a restriction is that the controlling variables for any controlled sources must appear inside the original circuit.

Fig. 4.18(a) shows a linear circuit. The circuit to the left of the terminals 1-2 in Fig. 4.18(b) is known as the Thevenin equivalent circuit. It was developed by M. Leon Thevenin (1857-1926) in 1883, a French telegraph engineer.

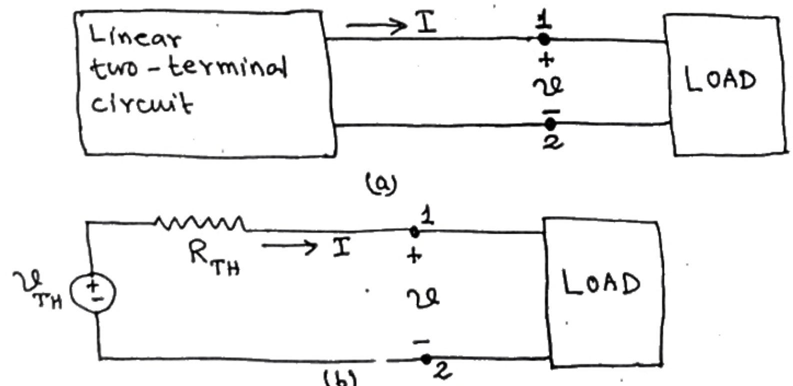


Fig. 4.18: (a) Original circuit (b) Thevenin equivalent circuit

In Fig. 4.18, LOAD may be a single resistor or another circuit.

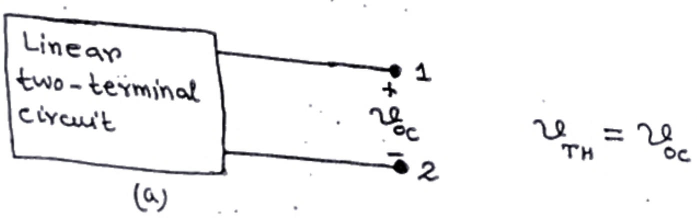
- Thevenin's theorem states a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source v_{TH} in series with a resistor R_{TH}.

Where

v_{TH} = Open circuit voltage at the terminals

R_{TH} = Input or equivalent resistance at the terminals when the independent sources are turned off.

Our major objective is now to find v_{TH} and R_{TH}. Suppose two circuits in Fig. 4.18 are equivalent.



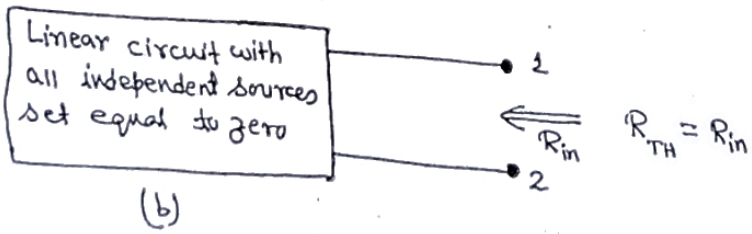


Fig. 4.19: (a) finding v_{TH} (b) finding R_{TH}.

For finding out the Thevenin resistance R_{TH}, we need to consider two cases.

Case-1: If the network has no dependent sources, turn off all the independent sources. Then determine R_{TH}, which is the input resistance of the network looking between terminals 1 and 2 as shown in Fig. 4.19(b).

Case-2: If the network has dependent sources, turn off all independent sources. Now apply a voltage source v_o at terminals 1 and 2 and obtain the resulting current i_o. Then $R_{TH} = v_o/i_o$ as shown in Fig. 4.20(a). Alternatively, a current source i_o can be inserted at terminals 1-2 as shown in Fig. 4.20(b). Then find terminal voltage v_o and $R_{TH} = v_o/i_o$. Both the approaches give identical results. We may assume any value of v_o and i_o. For example, we may use $v_o = 1$ Volt or $i_o = 1$ Amp or any unspecified values of v_o and i_o.

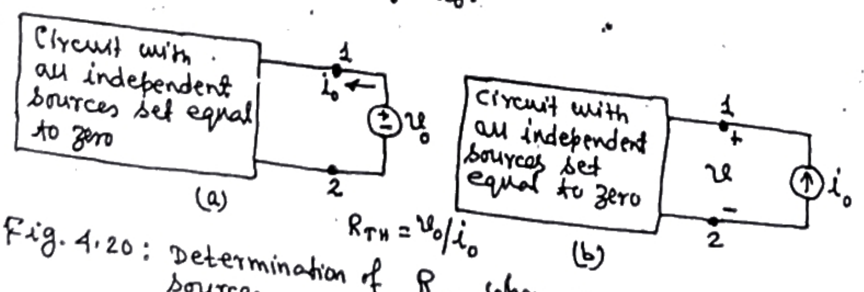


Fig. 4.20: Determination of R_{TH} when circuit has dependent sources.

Now let us examine the two cases.

1. If the terminals 1-2 are open circuited (by removing the LOAD), no current flows in Fig. 4.18(a), i.e $I = 0$, so that open circuit voltage across the terminals 1-2 in Fig. 4.18(a) must be equal to the voltage source v_{TH} in Fig. 4.18(b), since the two circuits are equivalent. Thus v_{TH} is the open-circuit voltage across the terminals as shown in Fig. 4.19(a), that is, $v_{TH} = v_{oc}$.

2. Again, terminals 1-2 are open circuited with the LOAD disconnected and turn off all independent sources. The input resistance or equivalent resistance of the dead circuit at the terminals ~~of the~~ 1-2 in Fig. 4.18(a) must be equal to R_{TH} in Fig. 4.18(b) since the two circuits are equivalent. Hence R_{TH} is the input resistance at the terminals 1-2, when the independent sources are turned off, as shown in Fig. 4.19(b), that is $R_{TH} = R_{in}$.

Thevenin's theorem helps to simplify a circuit and is very important in circuit analysis. A large circuit can be replaced by a single independent voltage source and a single resistor and this replacement technique is a powerful tool in circuit design.

It may happen that R_{TH} has a negative value. The negative value ($v = -iR$) implies that the circuit is supplying power and this is possible in a circuit with dependent sources.

Ex- 4.9: Find the Thevenin equivalent circuit of the circuit shown in Fig.4.21, to the left of the terminals 1-2. Then find current through $R_L = 8\Omega$.

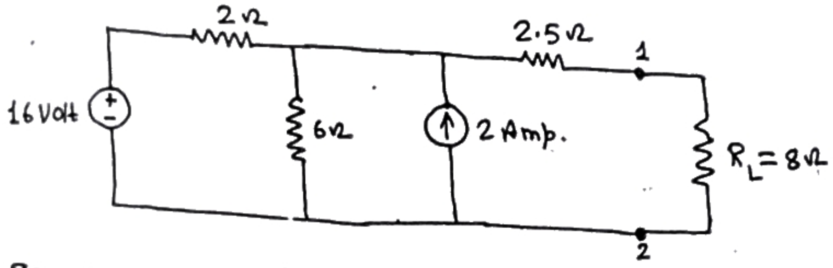


Fig. 4.21: Circuit for Ex-4.9

Soln.

To determine R_{TH}, we turn off the 16 volt independent voltage source (replacing it with a short circuit) and 2 Amp current source (replacing it with an open circuit). The circuit is shown in Fig. 4.22(a).

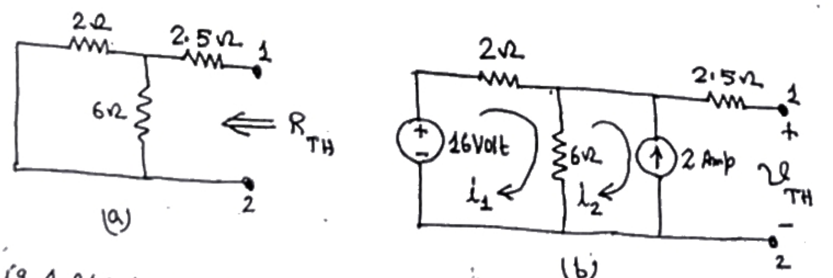


Fig. 4.21: (a) finding R_{TH} (b) finding v_{TH}.

From Fig. 4.21(a),

$$R_{TH} = \frac{2\times6}{2+6} + 2.5 = 4\Omega$$

In Fig. 4.21(b), applying mesh analysis,

$$-16 + 2i_1 + 6(i_1 - i_2) = 0 \quad \text{- - (i)}$$

and

$$i_2 = -2 \text{ Amp} \quad \text{- - - (ii)}$$

$\therefore \; i_1 = 0.5$ Amp

$$\therefore \; v_{TH} = 6(i_1 - i_2) = 6(0.5+2) = 15 \text{ Volt}$$

The Thevenin equivalent circuit is shown in Fig. 4.22.

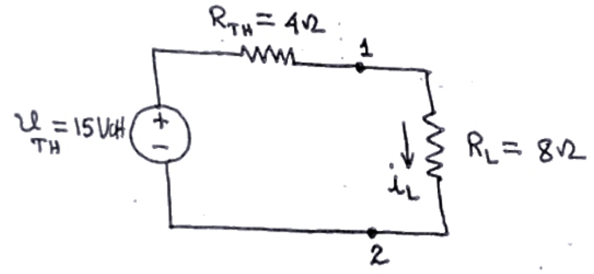


Fig. 4.22: Thevenin equivalent circuit for Ex-4.9

$$i_L = \frac{v_{TH}}{R_{TH}+R_L} = \frac{15}{4+8} = \frac{15}{12} = 1.25 \text{ Amp.}$$

EX-4.10: Determine v_{TH} of Ex-4.9 by using nodal analysis.

Soln.

We ignore the 2.5Ω resistor since no current flows through it. Fig. 4.23 shows the circuit of Ex-4.9 for determining v_{TH} using nodal analysis.

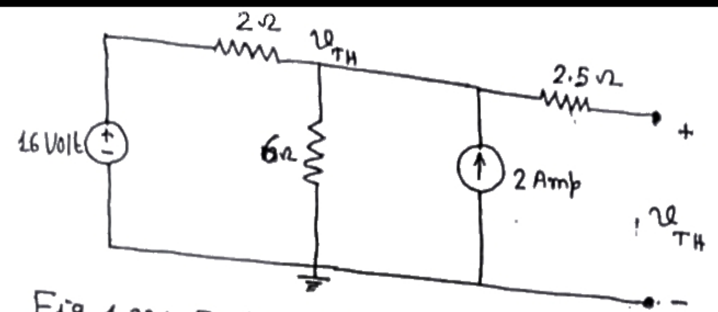


Fig. 4.23: Finding v_{TH} using nodal analysis

From Fig. 4.23, we can write,

$$\frac{16 - v_{TH}}{2} + 2 = \frac{v_{TH}}{6}$$

$$\therefore \; 8 - \frac{v_{TH}}{2} + 2 = \frac{v_{TH}}{6}$$

$$\therefore \; \frac{2}{3} v_{TH} = 10 \qquad \therefore v_{TH} = 15 \text{ Volt.}$$

Ex- 4.11: Determine Thevenin equivalent circuit of Fig. 4.24.

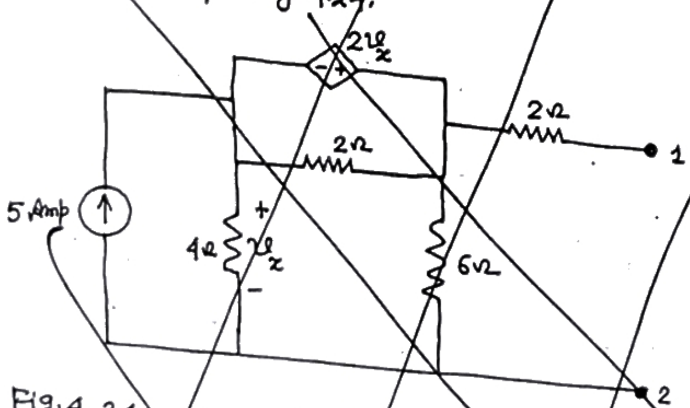


Fig. 4.24: Circuit for Ex- 4.11.

Soln.

To determine R_{TH}, we leave the dependent source alone and set the independent source equals to zero.

EX-4.13: Using Thevenin's theorem, determine voltage across load resistance $R_L = 4\Omega$ of the circuit shown in Fig. 4.30.

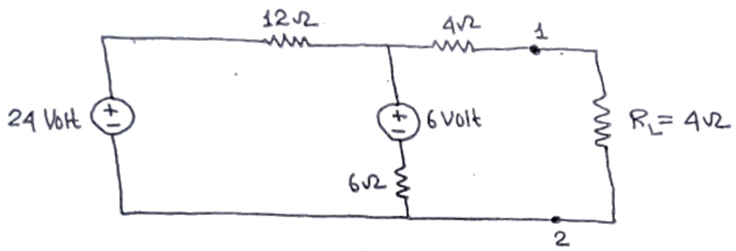


Fig. 4.30: Circuit for Ex-4.13

Soln.

First we remove load resistance $R_L = 4\Omega$ from terminals 1-2. Therefore terminals 1-2 are open and resulting circuit is shown in Fig. 4.31.

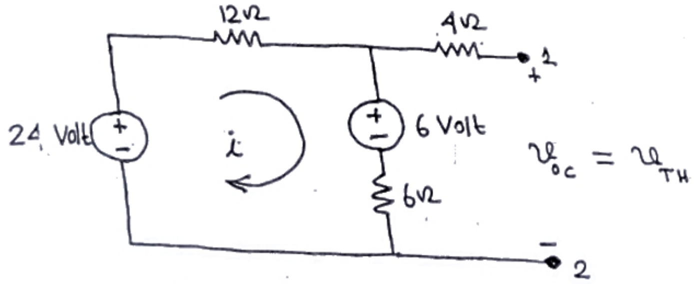


Fig. 4.31: Finding $v_{oc} = v_{TH}$ for Fig. 4.30 of Ex-4.13

Applying KVL, we have

$$18i + 6 - 24 = 0 \quad \therefore \; i = 1 \text{ Amp}$$

Thus

$$v_{oc} = v_{TH} = 6 + 6i = 6 + 6 \times 1 = 12 \text{ Volt.}$$

To determine R_{TH}, independent sources of Fig. 4.31 are turned off (short circuited) and the circuit is shown in Fig. 4.32.

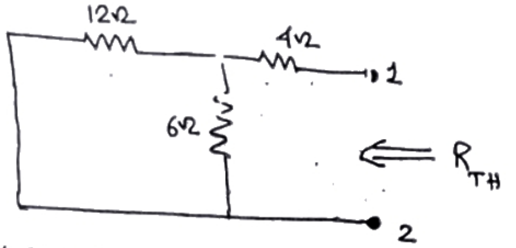


Fig. 4.32: Finding R_{TH} for Fig. 4.30 of Ex-4.13

$$\therefore R_{TH} = \frac{12 \times 6}{12+6} + 4 = 8\Omega.$$

Thevenin equivalent circuit is shown in Fig. 4.33.

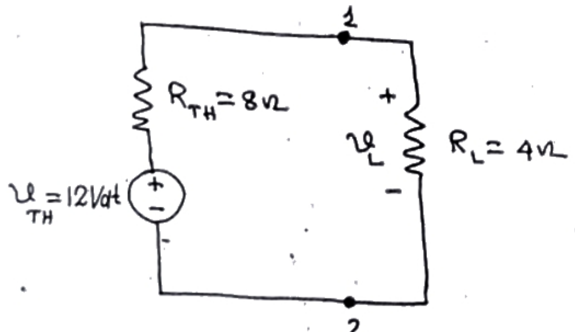


Fig. 4.33: Thevenin equivalent circuit for Ex-4.13.

Voltage across $R_L = 4\Omega$ resistance is

$$v_L = \frac{v_{TH}}{(R_{TH} + R_L)} \times R_L$$

$$\therefore v_L = \frac{12}{(8+4)} \times 4 = 4 \text{ Volt.}$$

EX- 4.14: By using Thevenin's theorem, determine current flowing through 3Ω resistor between points 1-2 of Fig. 4.32.

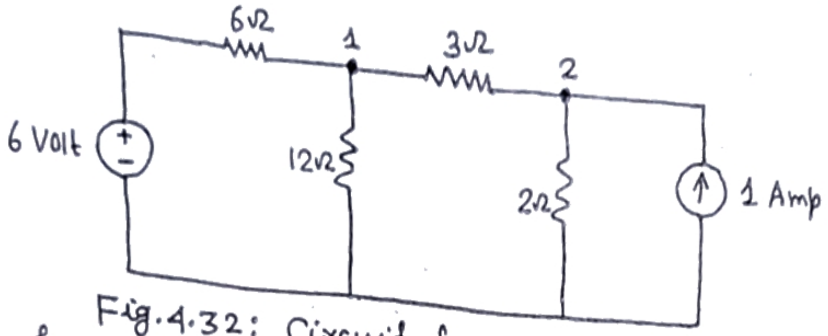


Fig. 4.32: Circuit for ~~Ex-4.32~~ EX-4.14

Soln.

Opening 3Ω resistor across terminals 1-2 to determine $v_{oc} = v_{TH}$ and the circuit is shown in Fig. 4.33.

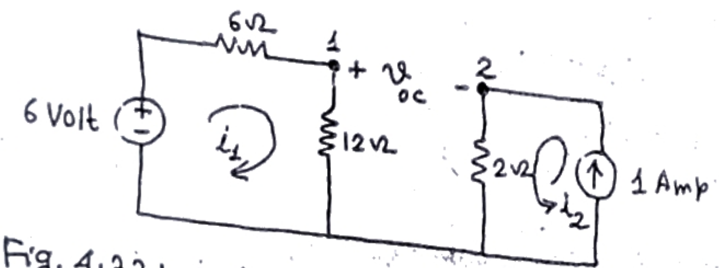


Fig. 4.33: Determining $v_{oc} = v_{TH}$ for EX-4.14

$i_1 = \frac{6}{6+12} = \frac{1}{3}$ Amp; $i_2 = 1$ Amp

$\therefore v_{oc} = v_{TH} = 12 i_1 - 2 i_2 = 12 \times \frac{1}{3} - 2 \times 1 = 2$ Volt

Circuit for determining R_{TH} is shown in Fig. 4.34

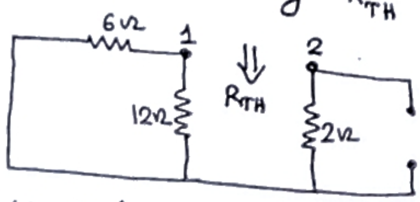


Fig. 4.34: Determining R_{TH} for EX-4.14

$$R_{TH} = \frac{6 \times 12}{(6+12)} + 2 = 4 + 2 = 6\Omega.$$

Thevenin equivalent circuit is shown in Fig. 4.35.

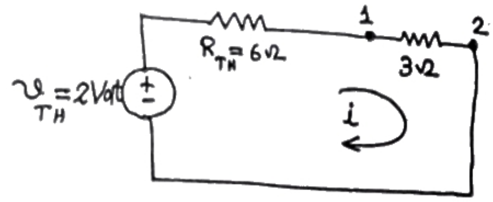


Fig. 4.35: Thevenin equivalent circuit for EX-4.14

Current through 3Ω resistor is

$$i = \frac{v_{TH}}{R_{TH} + 3} = \frac{2}{6+3} = \frac{2}{9} \text{ Amp.}$$

EX-4.15: Using Thevenin's theorem, determine current through 10Ω resistor of the circuit shown in Fig. 4.36.

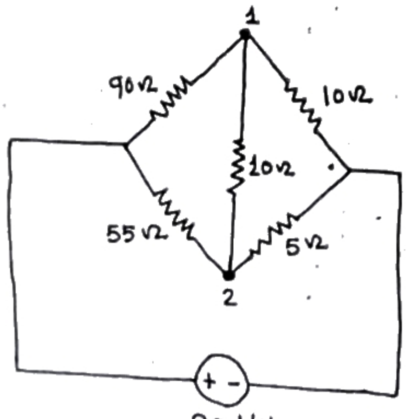


Fig. 4.36: Circuit for EX-4.15

Soln.

For determining $v_{oc} = v_{TH}$, removing $10\,\Omega$ resister across the terminals 1-2. and the resulting circuit is shown in Fig. 4.37.

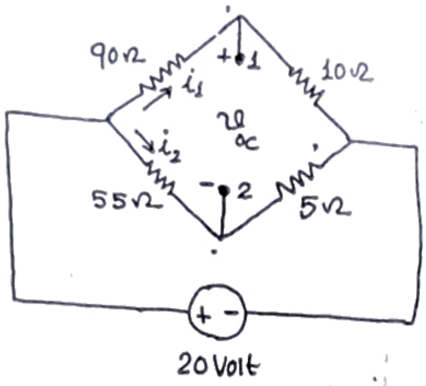


Fig. 4.37: Determing $v_{oc} = v_{TH}$ for Fig. 4.36 of EX-4.25.

From Fig. 4.37,

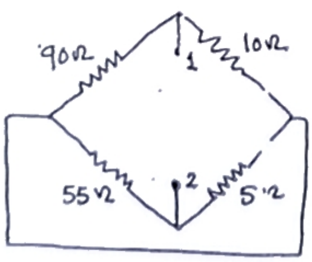
$$i_1 = \frac{20}{100} = 1/5 \text{ Amp} ; \quad i_2 = \frac{20}{(55+5)} = \frac{1}{3} \text{ Amp.}$$

$$\therefore \quad 10 i_1 - 5 i_2 - v_{oc} = 0$$

$$\therefore v_{oc} = 10 \times \frac{1}{5} - 5 \times \frac{1}{3} = 2 - \frac{5}{3} = \frac{1}{3} \text{ Volt}$$

$$\therefore v_{TH} = v_{oc} = \frac{1}{3} \text{ Volt.}$$

For determining R_{TH}, independent voltage source is short circuited. and the resulting circuit is shown in Fig. 4.38(a)



(a)

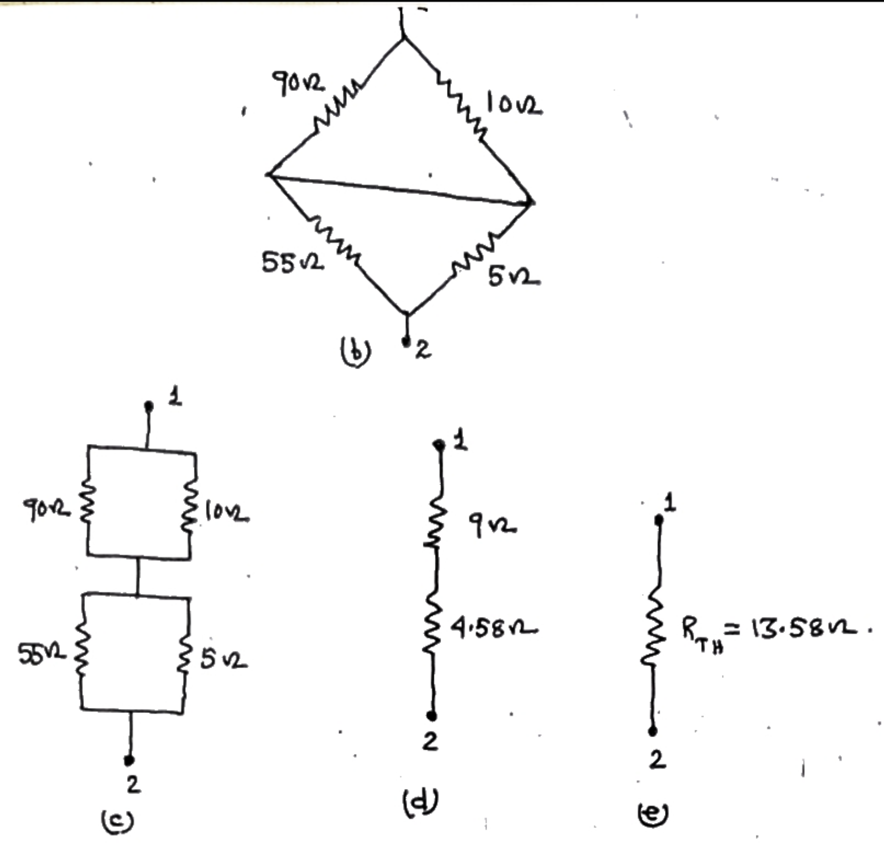


Fig. 4.38: Finding R_{TH} for Fig. 4.36 of Ex - 4.15

Thevenin equivalent circuit is shown in Fig. 4.39

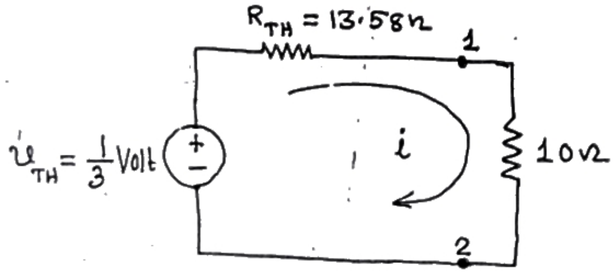


Fig. 4.39: Thevenin equivalent circuit for EX - 4.15

$$i = \frac{v_{TH}}{R_{TH}+10} = \frac{1/3}{(13.58+10)} = 0.0141 \text{ Amp}$$

EX-4.16: Determine current through 50Ω resistor of the circuit shown in Fig. 4.40 using Thevenin's theorem.

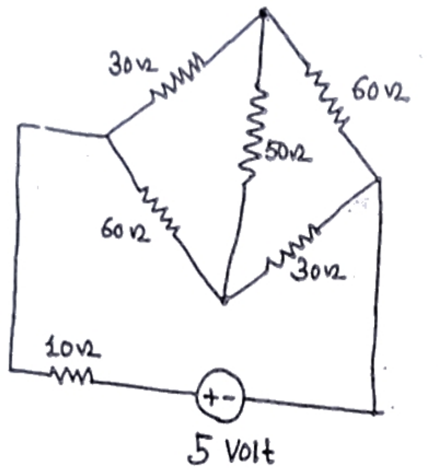


Fig. 4.40: Circuit for EX-4.16

Soln

Removing 50Ω resistor to determine $v_{oc} = v_{TH}$ and the resulting circuit is shown in Fig. 4.41

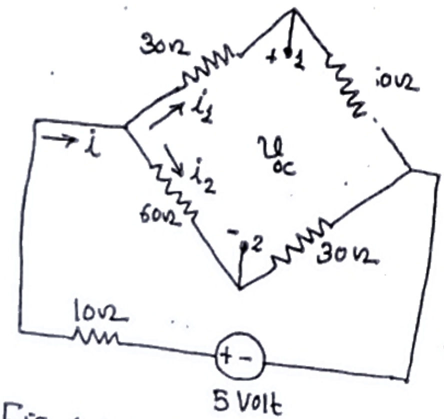


Fig. 4.41: Finding $v_{oc} = v_{TH}$

From Fig. 4.41,

$$i = i_1 + i_2 \quad \cdots (i)$$

Also $i_1 = i_2 \quad \cdots (ii)$

Applying KVL,

$$10(i_1+i_2) + i_1(30+60) - 5 = 0$$

But $i_1 = i_2$

$$\therefore \quad 20i_1 + 90i_1 = 5 \qquad \therefore i_1 = \frac{5}{110} = \frac{1}{22} \text{ Amp}$$

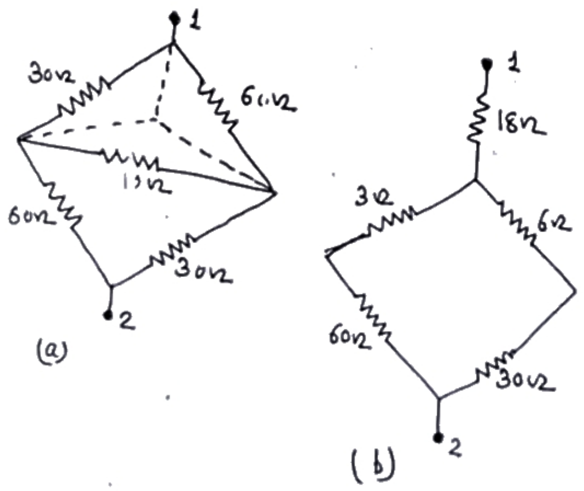
$$\therefore \quad i_2 = i_1 = \frac{1}{22} \text{ Amp}$$

Thus,

$$60i_1 - 30i_2 - v_{oc} = 0$$

$$\therefore \quad v_{oc} = \frac{30}{22} \text{ Volt} = 1.36 \text{ Volt}$$

To determine R_{TH}, independent voltage source is short circuited and the resulting circuits are shown in Fig. 4.42.



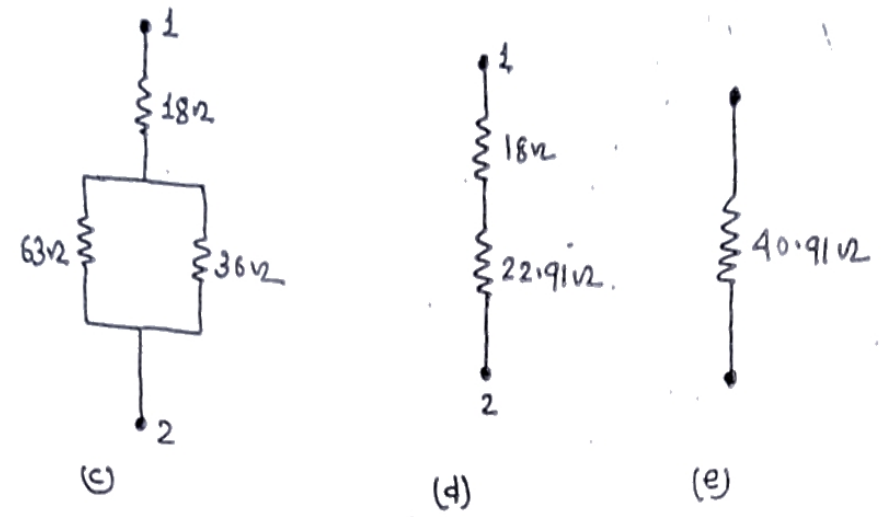


Fig. 4.42: Finding R_{TH}

$\therefore \; R_{TH} = 40.91\Omega$

Thevenin equivalent circuit is shown in Fig. 4.43

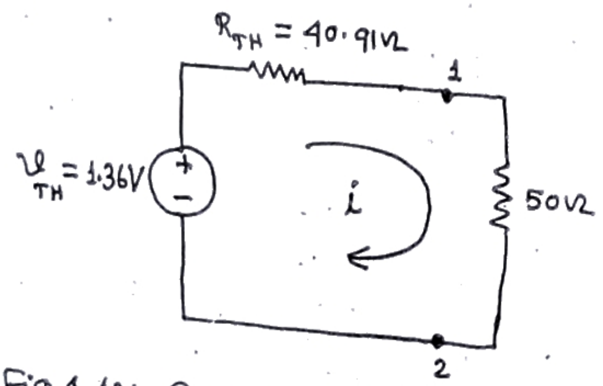


Fig. 4.43: Thevenin equivalent circuit for EX-4.16

$$\therefore i = \frac{1.36}{(40.91 + 50)} = 0.015 \text{ Amp.}$$

EX-4.17: ~~Determine the input resistance~~ R_{in} of the ~~circuit shown in Fig. 4.44.~~

~~Solving eqn.(ii') and (iii'), we get,~~

$$\cancel{v = 33.3 \text{ Volts.}}$$

$$\cancel{\therefore \; R_{in} = R_{TH} = \frac{v}{i} = \frac{33.3}{1} = 33.3\,\Omega.}$$

Ex-4.18: Find the Thevenin equivalent of the circuit shown in Fig. 4.46.

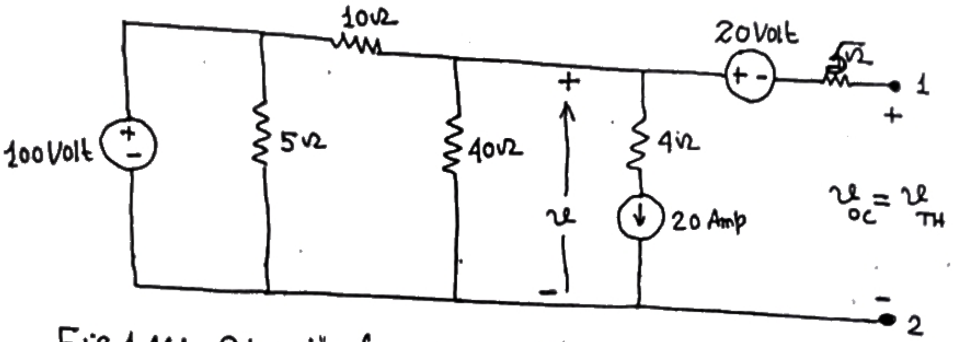


Fig.4.46: Circuit for Ex-4.18

Soln.

Applying nodal analysis, we have

$$\frac{v-100}{10} + \frac{v}{40} + 20 = 0$$

$$\therefore \quad v = -80 \text{ Volt.}$$

Thus,

$$-20 + v - v_{oc} = 0$$

$$\therefore \; v_{oc} = v_{TH} = -20 - 80 = -100 \text{ Volt.}$$

Fig. 4.47 shows the circuit with the voltage sources replaced by short circuits and the current source by an open circuit.

Note that $5\,\Omega$ resistor has no effect on R_{TH} because it is shorted and neither does the 4Ω resistor because it is in series with an open circuit.

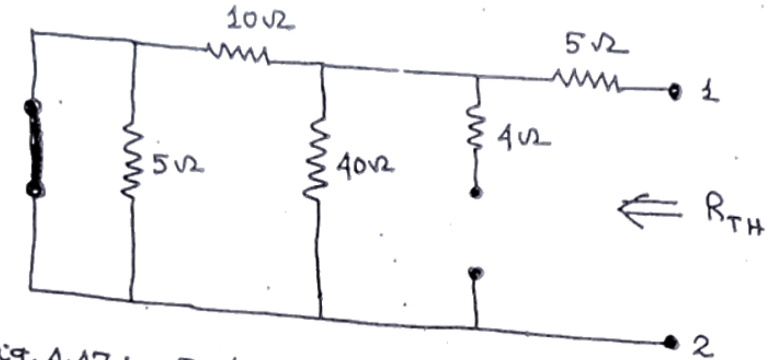


Fig. 4.47: Finding R_{TH}.

$$\therefore \quad R_{TH} = 5 + \left(\frac{40 \times 10}{40+10}\right) = 13\,\Omega$$

Thevenin equivalent circuit is shown in Fig. 4.48.

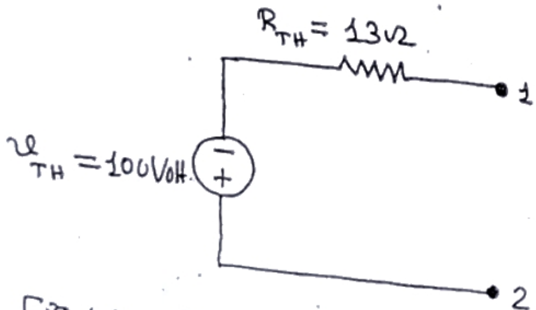


Fig. 4.48: Thevenin equivalent circuit for Ex-4.18

<u>Ex-4.19</u> Determine the current through 10Ω resistor of the circuit shown in Fig. 4.49. Use Thevenin's Theorem.

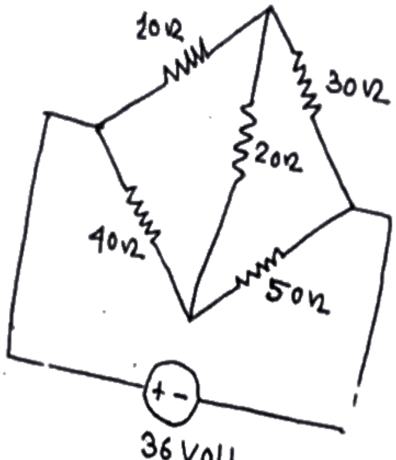
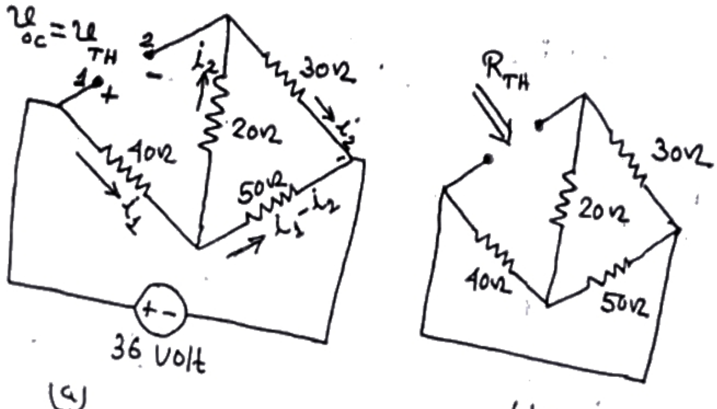


Fig. 4.49: Circuit for Ex- 4.19

Soln.



(a) (b)

Fig. 4.50: (a) finding v_{TH} (b) finding R_{TH}.

Fig. 4.50 shows the circuit for finding v_{TH} and R_{TH}.

From Fig. 4.50(a),

$$i_1 = 2i_2 \quad \text{--- (i)}$$

$$40i_1 + 50(i_1 - i_2) - 36 = 0 \quad \text{--- (ii)}$$

Solving eqn (i) & (ii), we get,

$i_1 = \frac{36}{65}$ Amp ; $i_2 = \frac{18}{65}$ Amp.

$\therefore v_{TH} = 20i_2 + 40i_1 = 100i_2 = 100 \times \frac{18}{65}$ Volt.

$\therefore v_{TH} = \frac{360}{13}$ Volt.

From Fig. 4.50(b),

$$R_{TH} = (40 \| 50 + 20) \| 30 = \frac{228}{13}\,\Omega$$

current through $10\,\Omega$ resistor,

$$i = \frac{v_{TH}}{R_{TH}+10} = \frac{360/13}{\left(\frac{228}{13}+10\right)} = \frac{360}{358}\text{ Amp.}$$

EX-4.20: Obtain the current in $2\,\Omega$ resistor of the circuit shown in Fig. 4.51. Use Thevenin's theorem.

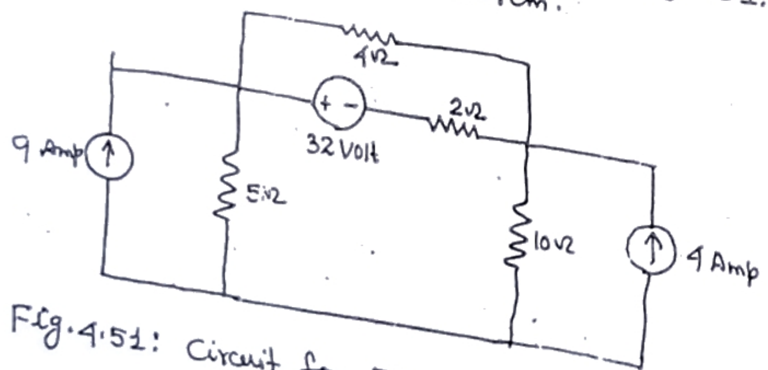


Fig. 4.51: Circuit for EX-4.20

Soln.

$2\,\Omega$ resistor is removed from Fig. 4.51 and the resulting circuit is shown in Fig. 4.52 to determine v_{TH}.

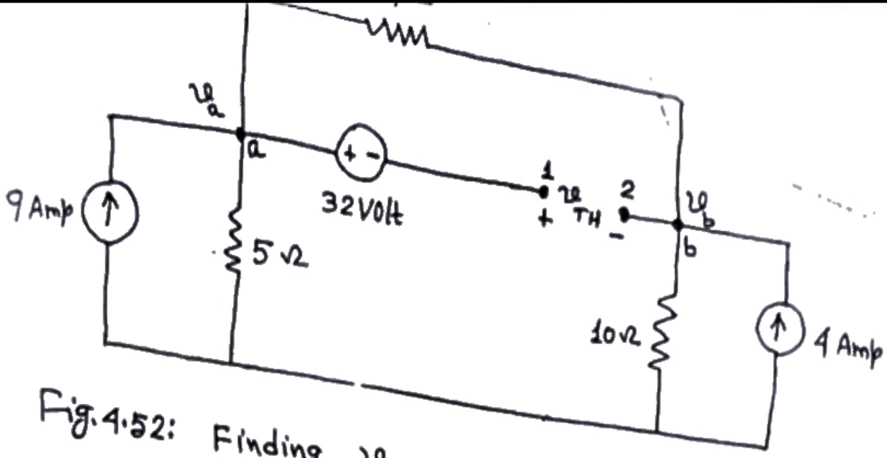


Fig. 4.52: Finding v_{TH} for Ex – 4.20

At ~~nd~~ node a,

$$\frac{v_a}{5} + \frac{v_a - v_b}{4} = 9 \quad \cdots (i)$$

At node b

$$\frac{v_a - v_b}{4} + 4 = \frac{v_b}{10} \quad \cdots (ii)$$

Solving eqns. (i) and (ii), we get,

$$v_a = \frac{830}{19} \text{ Volt} \; ; \; v_b = \frac{810}{19} \text{ Volt.}$$

Thus,

$$v_a - v_b - v_{TH} - 32 = 0$$

$$\therefore v_{TH} = \frac{830}{19} - \frac{810}{19} - 32 = -30.947 \text{ Volt.}$$

For determing R_{TH}, resulting circuit is shown in Fig. 4.53.

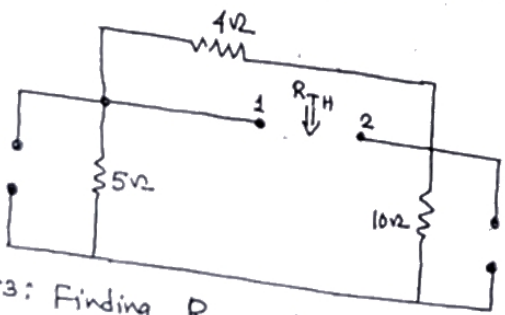


Fig. 4.53: Finding R_{TH} for Ex – 4.20

From Fig. 4.53, we have,

$$R_{TH} = \frac{(10+5)\times 4}{(10+5)+4} = \frac{60}{19}\ \Omega$$

Thevenin equivalent circuit is shown in Fig. 4.54

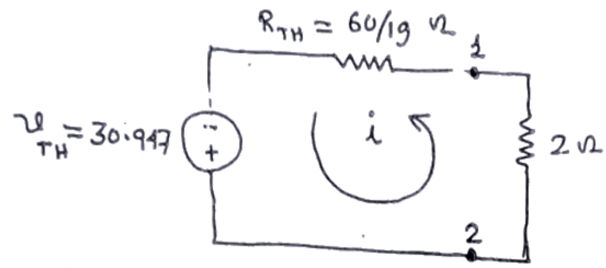
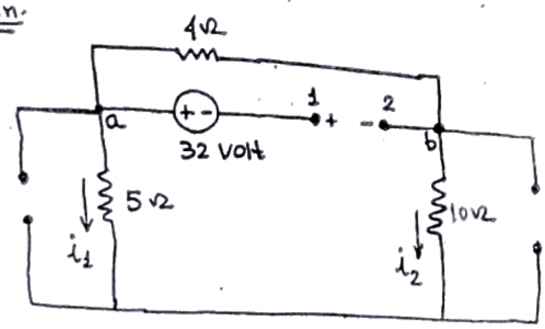


Fig. 4.54: Thevenin equivalent circuit for Ex-4.20

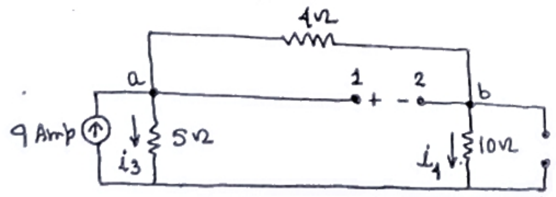
$$\therefore\ i = \frac{v_{TH}}{R_{TH}+2} = \frac{30.947}{\left(\frac{60}{19}+2\right)} = 6\ \text{Amp.}$$

EX-4.21 : Obtain the Thevenin voltage shown in the circuit of Fig. 4.51 for Ex-4.20

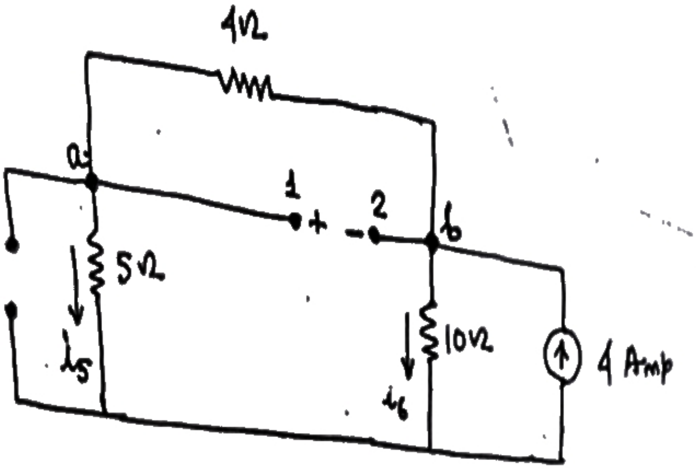
Soln.



(a)



(b)



(c)

Fig. 4.55: (a) Two independent current sources are turned off
(b) Voltage source and 4 Amp current sources are turned off
(c) Voltage source and 9 Amp current source are turned off.

~~Fig. 4.5~~ Fig. 4.55 gives three different circuits for obtaining v_{TH} using superposition theorem.

From Fig. 4.55(a),

$$i_1 = 0.0 \, ; \quad i_2 = 0.0$$

From Fig. 4.55(b),

$$i_3 = \left(\frac{10+4}{10+4+5}\right) \times 9 = 6.63 \text{ Amp}; \quad i_4 = 9 - i_3 = 9 - 6.63 = 2.37 \text{ Amp}$$

From Fig. 4.55(c),

$$i_5 = \frac{10}{(10+5+4)} \times 4 = 2.11 \text{ Amp}; \quad i_6 = 4 - 2.11 = 1.89 \text{ Amp}$$

$$\therefore \quad i_{5\Omega} = i_1 + i_3 + i_5 = 0 + 6.63 + 2.11 = 8.74 \text{ Amp}$$

$$i_{10\Omega} = i_2 + i_4 + i_6 = 0 + 2.37 + 1.89 = 4.26 \text{ Amp}$$

$$\therefore \quad v_a = 5 \times i_{5\Omega} = 5 \times 8.74 = 43.7 \text{ Volt}$$

$$v_b = 10 \times i_{10\Omega} = 10 \times 4.26 = 42.6 \text{ Volt}$$

Thus

$$v_{TH} = v_a - v_b - 32 = 43.7 - 42.6 - 32 = -30.9 \text{ Volt.}$$

4.5: NORTON'S THEOREM

Norton's theorem states that a linear two terminal circuit can be replaced by an equivalent circuit consisting of a current source i_N in parallel with a resistor R_N.

where,

i_N = short circuit current through the terminals

R_N = input or equivalent resistance at the terminals when the independent sources are turned off.

Consider the circuit given in Fig. 4.56(a). This circuit can be replaced by the one given in Fig. 4.56(b). We find R_N in the same way we find R_{TH}. In fact, Thevenin and Norton resistances are equal, that is

$$R_N = R_{TH} \quad \text{- - - -} \quad (4.9)$$

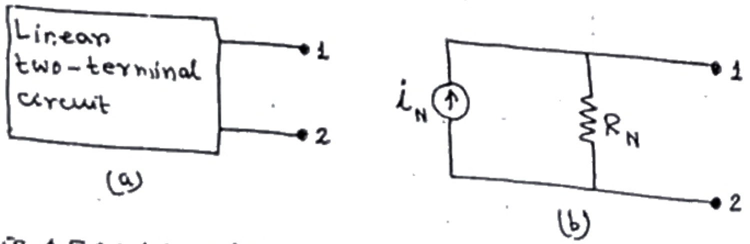


Fig. 4.56: (a) Original circuit (b) Norton equivalent circuit.

To determine the Norton current i_N, first compute the short circuit current flowing from terminal 1 to 2 in both circuits in Fig. 4.56

It is evident that the short circuit current in Fig. 4.56(b) is i_N and this must be the same short circuit current from terminal 1 to 2 in Fig. 4.56(a). Since the circuits of Fig. 4.56(a) and Fig. 4.56(b) are equivalent, thus,

$$i_N = i_{sc} \quad \cdots \quad (4.10)$$

Fig. 4.57 shows the circuit for finding Norton current i_N.

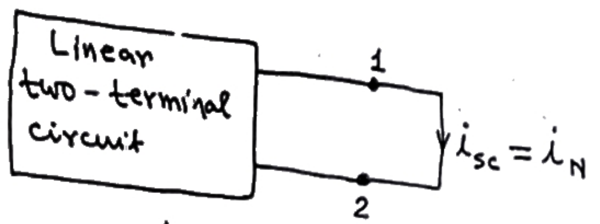


Fig.4.57: Finding Norton current i_N

Also

$$i_N = \frac{v_{TH}}{R_{TH}} \quad \cdots \quad (4.11)$$

Note that dependent and independent sources are treated the same way as in Thevenin's theorem.

The Thevenin and Norton equivalent circuits are related by a source transformation. For this reason, source transformation is often called Thevenin - Norton transformation.

To determine the Thevenin or Norton equivalent circuits require that we find:

1. The open circuit voltage v_{oc} across terminals 1 and 2.

2. The short circuit current i_{sc} at terminals 1 and 2.

3. The equivalent or input resistance R_{in} at terminals 1 and 2 when all independent sources are turned off.

We summarize the relationships:

$$v_{TH} = v_{oc} \quad \text{--- (4.12)}$$

$$i_N = i_{sc} \quad \text{--- (4.13)}$$

$$R_{TH} = \frac{v_{oc}}{i_{sc}} = R_N \quad \text{--- (4.14)}$$

Open circuit and short circuit tests are sufficient to find any Thevenin or Norton equivalent.

Ex-4.22: Determine Norton equivalent circuit of the circuit shown in Fig. 4.58

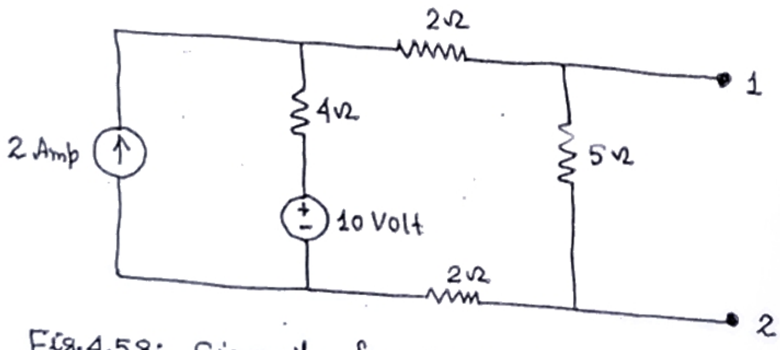


Fig. 4.58: Circuit for Ex-4.22.

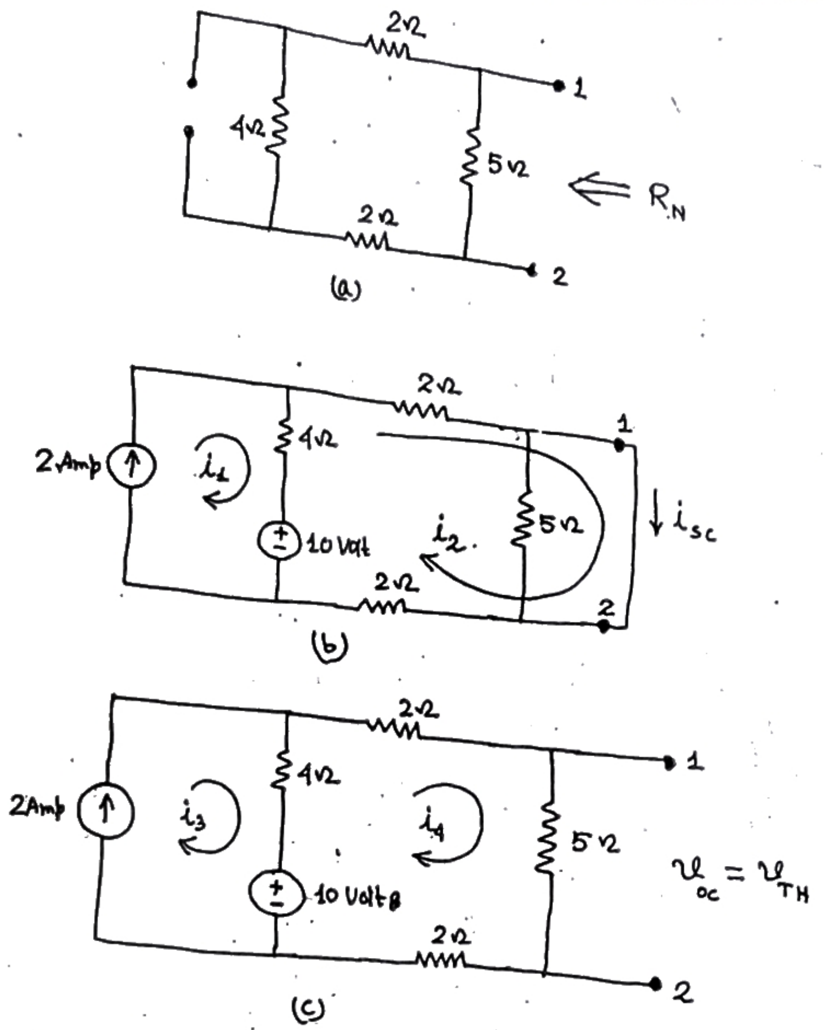


Fig. 4.59: (a) finding R_N (b) finding $i_N = i_{sc}$
(c) finding $v_{oc} = v_{TH}$

We determine R_N in the same way we find R_{TH} in the Thevenin equivalent circuit. All the independent sources are turned off and this leads to the circuit in Fig. 4.59(a). Thus

$$R_N = \frac{5 \times (2+4+2)}{5 + (2+4+2)} = 3.077\Omega = R_{TH}$$

To determine i_N, terminals 1 and 2 are short circuited as shown in Fig. 4.59(b). 5 Ω resistor is ignored because it has been short circuited.
Applying mesh analysis, we get,

$$i_1 = 2\text{ Amp} \quad \text{and} \quad 8i_2 - 4i_1 = 10$$

$$\therefore\ 8i_2 = 4\times 2 + 10 \qquad \therefore\ i_2 = 2.25\text{ Amp.}$$

$$\therefore\ i_{sc} = i_2 = 2.25\text{ Amp.}$$

Alternatively, we can determine $i_N = v_{TH}/R_{TH}$.
We obtain v_{TH} as the open circuit voltage across terminals 1 and 2 in Fig. 4.59(c). Using mesh analysis, we obtain,

$$i_3 = 2\text{ Amp};$$

$$13i_4 - 4i_3 = 10 \qquad \therefore\ 13i_4 = 4\times 2 + 10$$

$$\therefore\ i_4 = 1.384\text{ Amp.}$$

$$\therefore\ v_{TH} = v_{oc} = 5i_4 = 5\times 1.384 = 6.923\text{ Volt.}$$

Hence,

$$i_N = \frac{v_{TH}}{R_{TH}} = \frac{6.923}{3.077} = 2.25\text{ Amp}$$

as obtained previously.
This also serves to confirm eqn (4.14), that

$$R_{TH} = R_N = \frac{v_{oc}}{i_{sc}} = \frac{6.923}{2.25} = 3.077\,\Omega.$$

Thevenin and Norton equivalent circuits are shown in Fig. 4.60(a) and Fig. 4.60(b) respectively.

...nd R_N, we turned off the independent voltage source and connect a voltage source of $v_o = 1$ Volt to the terminals 1 and 2 and resulting circuit is shown in Fig. 4.62(a). We ignore the 8Ω resistance of Fig. 4.62(a) because it is short circuited. Also due to the short circuit, the dependent current source, 5Ω resistor and independent voltage source are in parallel.

Hence, $i_y = 0.0$. ~~At node 1~~, $i_o = \cancel{v_o}$

At node 1,

$$i_o = \frac{v_o}{5} = \frac{1}{5} \text{ Amp.}$$

and

$$R_N = \frac{v_o}{i_o} = \frac{1}{(1/5)} = 5\Omega.$$

To determine i_N, terminals 1 and 2 are short circuited and the resulting circuit is shown in Fig. 4.62(b). In Fig. 4.62(b), dependent current source, 5Ω resistor, 20 Volt and 8Ω resistor all are in parallel. Hence,

$$i_y = \frac{20}{8} = 2.5 \text{ Amp};$$

At node 1, KCL gives

$$i_{sc} = \frac{20}{5} + 2i_y = 4 + 2 \times 2.5 = 9 \text{ Amp.}$$

Thus,

$$i_N = i_{sc} = 9 \text{ Amp.}$$

EX-4.24: Determine the current in 10Ω resistor of the circuit shown in Fig. 4.63 by Norton's theorem.

48

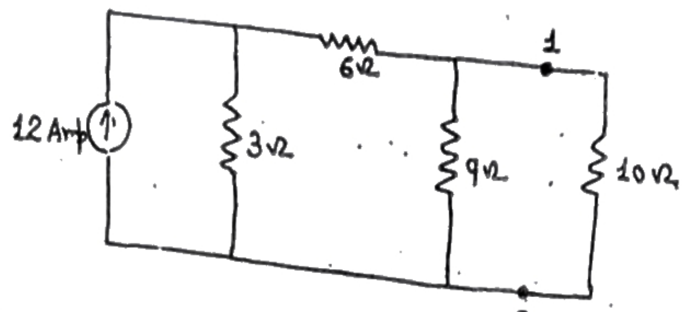
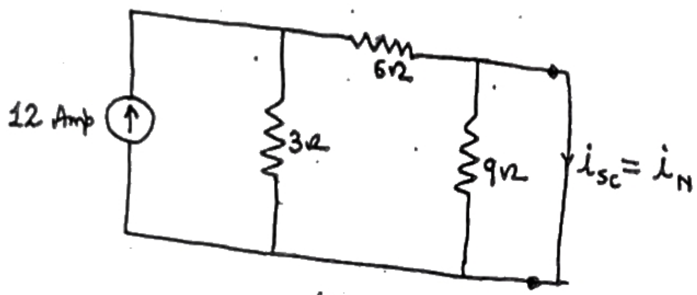
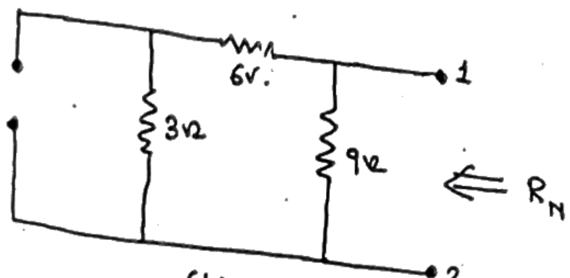


Fig. 4.63: Circuit for Ex-4.24

Soln.



(a)



(b)

Fig. 4.64: (a) finding i_N (b) finding R_N

In Fig. 4.64(a), 9 Ω resistor is short circuited.
Hence,

$$i_N = \frac{3}{(3+6)} \times 12 = 4 \text{ Amp}$$

From Fig. 4.64(b),

$$R_N = \frac{9 \times (6+3)}{9+(6+3)} = 4.5\Omega$$

Fig. 4.65 shows Norton equivalent circuit. (49)

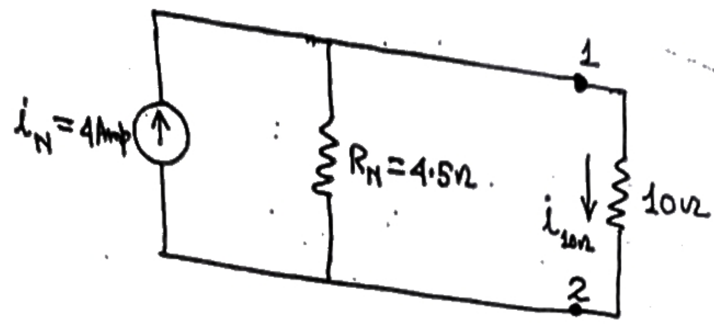


Fig. 4.65: Norton Equivalent circuit for EX-4.24.

Current through $10\ \Omega$ resistor is given by

$$i_{10\Omega} = \frac{4.5}{(4.5+10)} \times 4 = 1.241 \text{ Amp.}$$

EX-4.25: Obtain the Norton equivalent circuit for the circuit shown in Fig. 4.66 to determine the current in the 50Ω resistor.

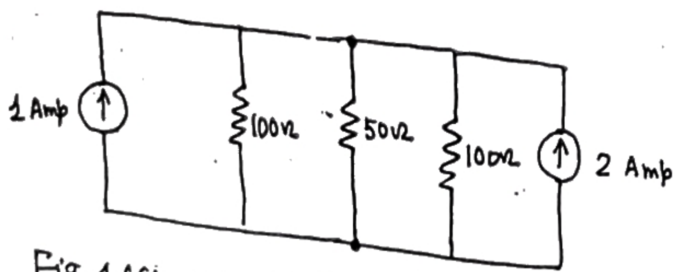
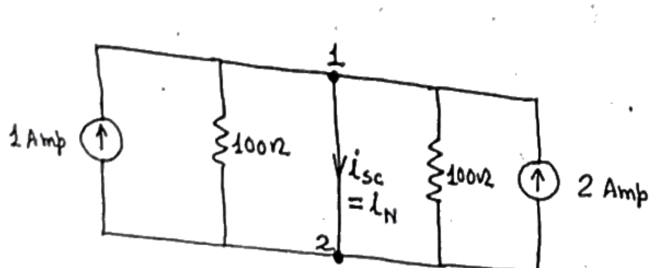


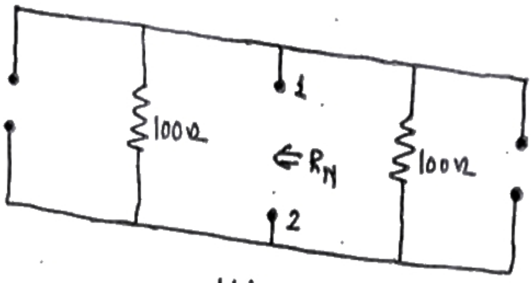
Fig. 4.66: Circuit for EX-4.25

Soln.



(a)

50



(b)

Fig. 4.67: (a) finding i_N (b) finding R_N

In Fig. 4.67(a), both 100Ω resistors are short circuited. Hence,

$$i_{sc} = i_N = 1 + 2 = 3 \text{ Amp}$$

In Fig. 4.67(b), Both 100Ω resistors are in parallel, hence,

$$R_N = \frac{100 \times 100}{100 + 100} = 50\Omega$$

Norton equivalent circuit is given in Fig. 4.68

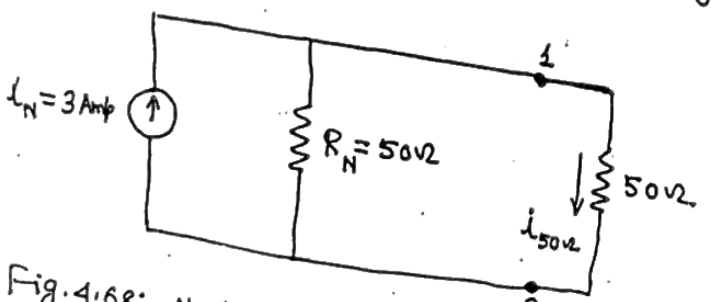


Fig. 4.68: Norton equivalent circuit for Ex-4.25

$$\therefore i_{50\Omega} = 3 \times \frac{50}{50+50} = 1.5 \text{ Amp.}$$

Ex-4.26: Obtain Norton equivalent circuit of the circuit shown in Fig. 4.69.

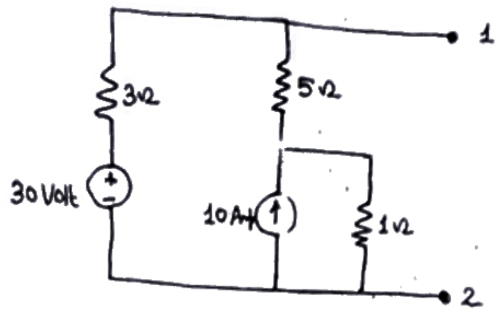


Fig. 4.69: Circuit for Ex-4.26

Soln.

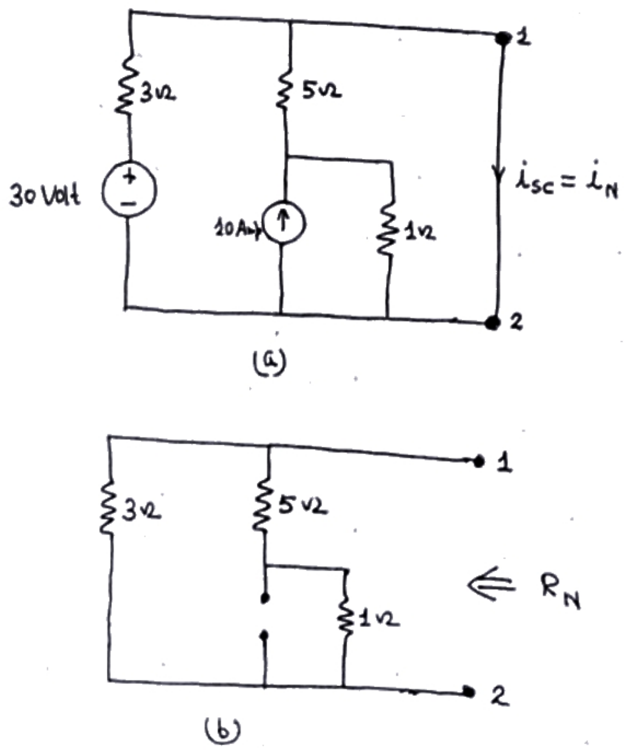


Fig. 4.70: (a) finding i_N (b) finding R_N

In Fig. 4.70(a), short circuit current is given by

$$i_{sc} = \left(\frac{30}{3}\right) + \frac{1}{(5+1)} \times 10$$

$$\therefore i_{sc} = 10 + \frac{5}{3} = \frac{35}{3} \text{ Amp}$$

$$\therefore i_N = i_{sc} = \frac{35}{3} \text{ Amp.}$$

From Fig. 4.70(b),

$$R_N = \frac{3 \times (5+1)}{3+(5+1)} = 2\Omega$$

Norton equivalent circuit is given in Fig. 4.71.

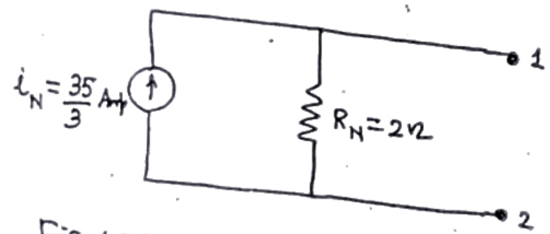


Fig. 4.71: Norton equivalent circuit for Ex-4.26

<u>Ex-4.27:</u> Determine R_N of the circuit shown in Fig. 4.72 using $R_N = v_{oc}/i_{sc}$.

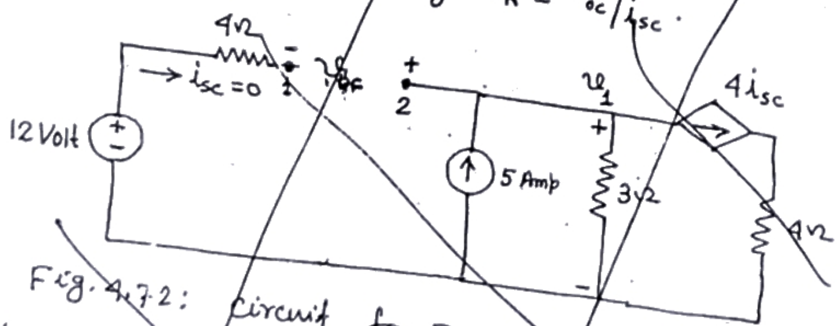


Fig. 4.72: Circuit for Ex.-4.27.

<u>Soln.</u>

In the circuit of Fig. 4.72, $i_{sc} = 0$, hence, $v_1 = 5 \times 3 = 15$ Volt

Note that there is no independent source in Fig. 4.74, hence, $v_{TH} = 0$. The Thevenin equivalent circuit is shown in Fig. 4.76.

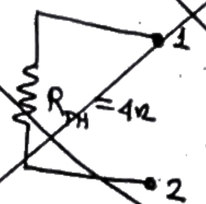


Fig. 4.76: Thevenin equivalent circuit for EX-4.28

4.6: MAXIMUM POWER TRANSFER

In many practical cases, a circuit is designed to provide power to a load. There are applications in many areas where it is desirable to maximize the power delivered to the load.

The Thevenin equivalent is useful in finding the maximum power, a linear circuit can deliver to a load. We assume that load resistance R_L is adjustable. If the entire circuit is replaced by its Thevenin equivalent except for the adjustable load resistance as shown in Fig. 4.77, the power delivered to the load is

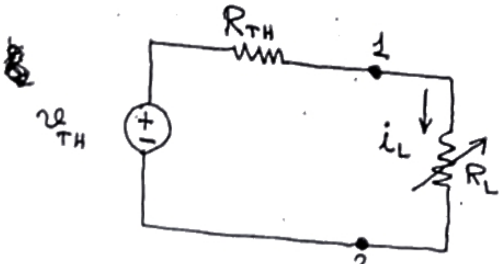


Fig. 4.77: The circuit used for maximum power transfer.

$$p_L = i_L^2 R_L = \left(\frac{v_{TH}}{R_{TH}+R_L}\right)^2 R_L \quad \text{---- (4.15)}$$

For a given circuit, v_{TH} and R_{TH} are fixed. By varying R_L, the power delivered to the load can be varied and it is sketched in Fig. 4.78

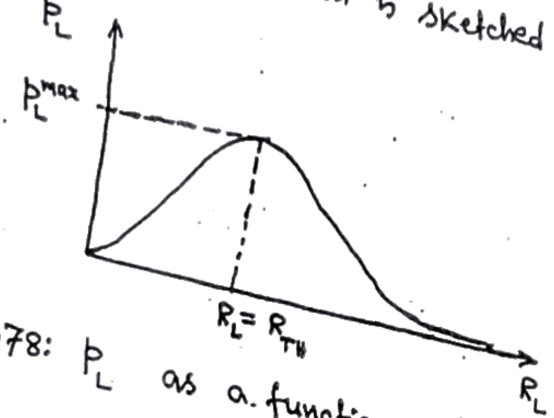


Fig. 4.78: p_L as a function of R_L

From Fig. 4.78, we notice that p_L is small for small or large value of R_L but maximum for some value R_L between 0 and ∞.

To determine the condition for maximum power transfer, we set,

$$\frac{dp_L}{dR_L} = 0 \quad \text{---- (4.16)}$$

$$\therefore \; v_{TH}^2 \left[\frac{(R_{TH}+R_L)^2 - 2R_L(R_{TH}+R_L)}{(R_{TH}+R_L)^4}\right] = 0$$

$$\therefore \; R_{TH} + R_L - 2R_L = 0$$

$$\therefore \; R_L = R_{TH} \quad \text{---- (4.17)}$$

refore, maximum power occurs when $R_L = R_{TH}$. is known as maximum power theorem.

Substituting $R_L = R_{TH}$, in eqn.(4.15), we get,

$$p_L^{max} = \frac{v_{TH}^2}{4R_{TH}} \quad \text{------- (4.18)}$$

Ex-4.29: Determine the value of R_L for maximum power transfer in the circuit shown in Fig. 4.79. Also find the maximum power.

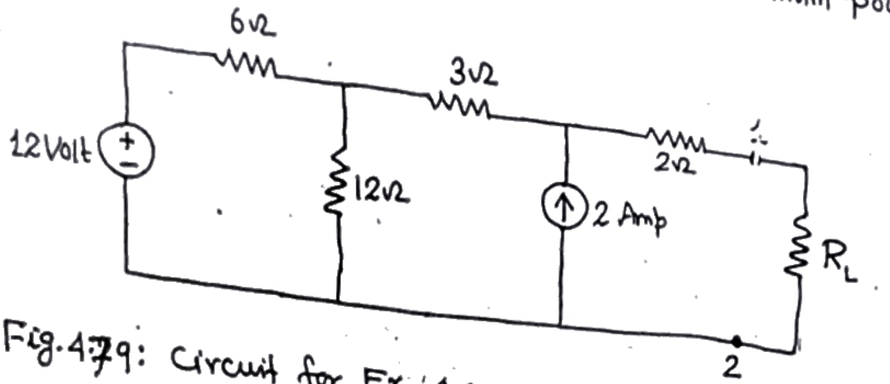


Fig. 4.79: Circuit for Ex-4.29

Soln.

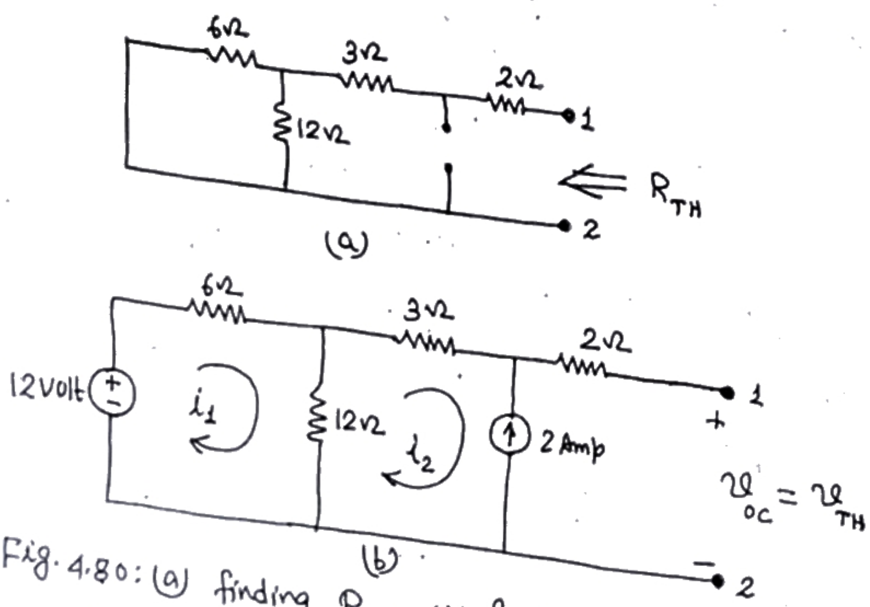


Fig. 4.80: (a) finding R_{TH} (b) finding v_{TH}.