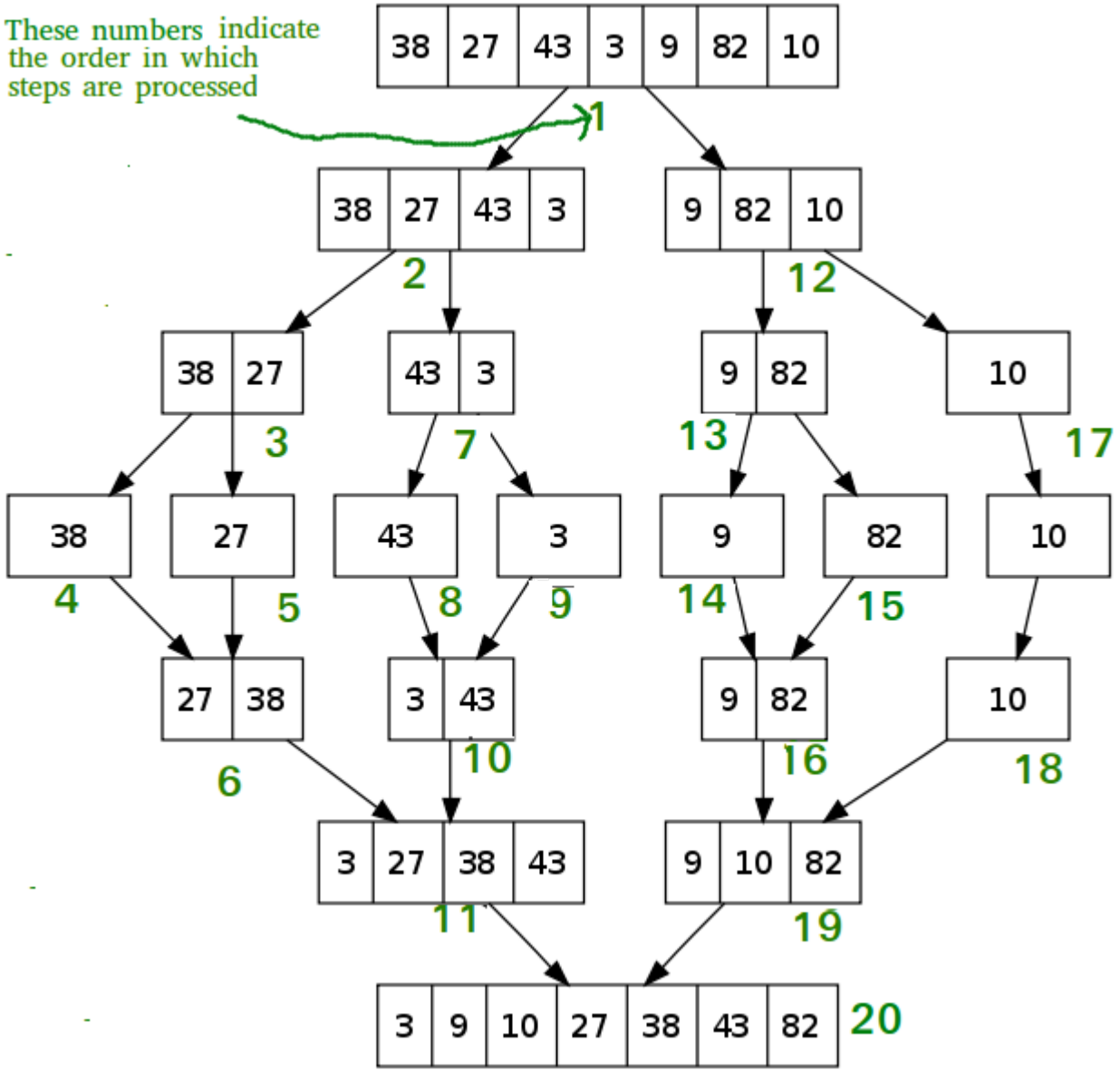
Merge Sort

Merge Sort is a Divide and Conquer algorithm. It divides the input array in two halves, calls itself for the two halves and then merges the two sorted halves. **The merge() function** is used for merging two halves. The merge(arr, l, m, r) is key process that assumes that arr[l..m] and arr[m+1..r] are sorted and merges the two sorted sub-arrays into one in a sorted manner. See following implementation for details:

```
MergeSort(arr[], l,  r)
If r > l
     1. Find the middle point to divide the array into two halves:
             middle m = (l+r)/2
     2. Call mergeSort for first half:
             Call mergeSort(arr, l, m)
     3. Call mergeSort for second half:
             Call mergeSort(arr, m+1, r)
     4. Merge the two halves sorted in step 2 and 3:
             Call merge(arr, l, m, r)
```

The following diagram from wikipedia shows the complete merge sort process for an example array {38, 27, 43, 3, 9, 82, 10}. If we take a closer look at the diagram, we can see that the array is recursively divided in two halves till the size becomes 1. Once the size becomes 1, the merge processes comes into action and starts merging arrays back till

the complete array is merged.



Implementation:

Java

```
/* Java program for Merge Sort */
class MergeSort {
    // Merges two subarrays of arr[].
    // First subarray is arr[l..m]
    // Second subarray is arr[m+1..r]
    void merge(int arr[], int l, int m, int r)
    {
        // Find sizes of two subarrays to be merged
        int n1 = m - l + 1;
        int n2 = r - m;

        /* Create temp arrays */
        int L[] = new int[n1];
        int R[] = new int[n2];

        /*Copy data to temp arrays*/
        for (int i = 0; i < n1; ++i)
            L[i] = arr[l + i];
        for (int j = 0; j < n2; ++j)
            R[j] = arr[m + 1 + j];

        /* Merge the temp arrays */
```

```
// Initial indexes of first and second subarrays
int i = 0, j = 0;

// Initial index of merged subarray array
int k = l;
while (i < n1 && j < n2) {
    if (L[i] <= R[j]) {
        arr[k] = L[i];
        i++;
    }
    else {
        arr[k] = R[j];
        j++;
    }
    k++;
}

/* Copy remaining elements of L[] if any */
while (i < n1) {
    arr[k] = L[i];
    i++;
    k++;
```



```
/* Copy remaining elements of R[] if any */
while (j < n2) {
    arr[k] = R[j];
    j++;
    k++;
```

```
        }
    }

    // Main function that sorts arr[l..r] using
    // merge()
    void sort(int arr[], int l, int r)
    {
        if (l < r) {
            // Find the middle point
            int m = l + (r - l) / 2;

            // Sort first and second halves
            sort(arr, l, m);
            sort(arr, m + 1, r);

            // Merge the sorted halves
            merge(arr, l, m, r);
        }
    }

    /* A utility function to print array of size n */
    static void printArray(int arr[])
    {
        int n = arr.length;
        for (int i = 0; i < n; ++i)
            System.out.print(arr[i] + " ");
        System.out.println();
    }
```

C++

```
// C++ program for Merge Sort
#include <iostream>
using namespace std;

// Merges two subarrays of array[].
// First subarray is arr[begin..mid]
// Second subarray is arr[mid+1..end]
void merge(int array[], int const left, int const mid,
           int const right)
{
    auto const subArrayOne = mid - left + 1;
    auto const subArrayTwo = right - mid;

    // Create temp arrays
    auto *leftArray = new int[subArrayOne],
         *rightArray = new int[subArrayTwo];

    // Copy data to temp arrays leftArray[] and rightArray[]
    for (auto i = 0; i < subArrayOne; i++)
        leftArray[i] = array[left + i];
    for (auto j = 0; j < subArrayTwo; j++)
        rightArray[j] = array[mid + 1 + j];
```

```
auto indexOfSubArrayOne
    = 0, // Initial index of first sub-array
    indexOfSubArrayTwo
    = 0; // Initial index of second sub-array
int indexOfMergedArray
    = left; // Initial index of merged array

// Merge the temp arrays back into array[left..right]
while (indexOfSubArrayOne < subArrayOne
       && indexOfSubArrayTwo < subArrayTwo) {
    if (leftArray[indexOfSubArrayOne]
        <= rightArray[indexOfSubArrayTwo]) {
        array[indexOfMergedArray]
            = leftArray[indexOfSubArrayOne];
        indexOfSubArrayOne++;
    }
    else {
        array[indexOfMergedArray]
            = rightArray[indexOfSubArrayTwo];
        indexOfSubArrayTwo++;
    }
    indexOfMergedArray++;
}
// Copy the remaining elements of
// left[], if there are any
while (indexOfSubArrayOne < subArrayOne) {
    array[indexOfMergedArray]
        = leftArray[indexOfSubArrayOne];
    indexOfSubArrayOne++;
```

```
        indexOfMergedArray++;
    }
    // Copy the remaining elements of
    // right[], if there are any
    while (indexOfSubArrayTwo < subArrayTwo) {
        array[indexOfMergedArray]
            = rightArray[indexOfSubArrayTwo];
        indexOfSubArrayTwo++;
        indexOfMergedArray++;
    }
    delete[] leftArray;
    delete[] rightArray;
}

// begin is for left index and end is
// right index of the sub-array
// of arr to be sorted */
void mergeSort(int array[], int const begin, int const end)
{
    if (begin >= end)
        return; // Returns recursively

    auto mid = begin + (end - begin) / 2;
    mergeSort(array, begin, mid);
    mergeSort(array, mid + 1, end);
    merge(array, begin, mid, end);
}

// UTILITY FUNCTIONS
```

```
// Function to print an array
void printArray(int A[], int size)
{
    for (auto i = 0; i < size; i++)
        cout << A[i] << " ";
}
```

C

```
// Merges two subarrays of arr[].
// First subarray is arr[l..m]
// Second subarray is arr[m+1..r]
void merge(int arr[], int l, int m, int r)
{
    int i, j, k;
    int n1 = m - l + 1;
    int n2 =  r - m;

    /* create temp arrays */
    int L[n1], R[n2];

    /* Copy data to temp arrays L[] and R[] */
    for (i = 0; i < n1; i++)
        L[i] = arr[l + i];
    for (j = 0; j < n2; j++)
```

```
    R[j] = arr[m + 1+ j];

/* Merge the temp arrays back into arr[l..r]*/
i = 0; // Initial index of first subarray
j = 0; // Initial index of second subarray
k = l; // Initial index of merged subarray
while (i < n1 && j < n2)
{
    if (L[i] <= R[j])
    {
        arr[k] = L[i];
        i++;
    }
    else
    {
        arr[k] = R[j];
        j++;
    }
    k++;
}

/* Copy the remaining elements of L[], if there
   are any */
while (i < n1)
{
    arr[k] = L[i];
    i++;
    k++;
}
```

```
    /* Copy the remaining elements of R[], if there
       are any */
    while (j < n2)
    {
        arr[k] = R[j];
        j++;
        k++;
    }
}

/* l is for left index and r is right index of the
   sub-array of arr to be sorted */
void mergeSort(int arr[], int l, int r)
{
    if (l < r)
    {
        // Same as (l+r)/2, but avoids overflow for
        // large l and h
        int m = l+(r-l)/2;

        // Sort first and second halves
        mergeSort(arr, l, m);
        mergeSort(arr, m+1, r);

        merge(arr, l, m, r);
    }
}
```

```java
/* Java program for Merge Sort */
class MergeSort {
    // Merges two subarrays of arr[].
    // First subarray is arr[l..m]
    // Second subarray is arr[m+1..r]
    void merge(int arr[], int l, int m, int r)
    {
        // Find sizes of two subarrays to be merged
        int n1 = m - l + 1;
        int n2 = r - m;

        /* Create temp arrays */
        int L[] = new int[n1];
        int R[] = new int[n2];

        /*Copy data to temp arrays*/
        for (int i = 0; i < n1; ++i)
            L[i] = arr[l + i];
        for (int j = 0; j < n2; ++j)
            R[j] = arr[m + 1 + j];

        /* Merge the temp arrays */

        // Initial indexes of first and second subarrays
        int i = 0, j = 0;

        // Initial index of merged subarray array
```

```
    int k = l;
    while (i < n1 && j < n2) {
        if (L[i] <= R[j]) {
            arr[k] = L[i];
            i++;
        }
        else {
            arr[k] = R[j];
            j++;
        }
        k++;
    }

    /* Copy remaining elements of L[] if any */
    while (i < n1) {
        arr[k] = L[i];
        i++;
        k++;
    }

    /* Copy remaining elements of R[] if any */
    while (j < n2) {
        arr[k] = R[j];
        j++;
        k++;
    }
}

// Main function that sorts arr[l..r] using
```

```
    // merge()
    void sort(int arr[], int l, int r)
    {
        if (l < r) {
            // Find the middle point
            int m = l + (r - l) / 2;

            // Sort first and second halves
            sort(arr, l, m);
            sort(arr, m + 1, r);

            // Merge the sorted halves
            merge(arr, l, m, r);
        }
    }

    /* A utility function to print array of size n */
    static void printArray(int arr[])
    {
        int n = arr.length;
        for (int i = 0; i < n; ++i)
            System.out.print(arr[i] + " ");
        System.out.println();
    }
```

Java

```
/* Java program for Merge Sort */
class MergeSort {
```

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```
// Merges two subarrays of arr[].
// First subarray is arr[l..m]
// Second subarray is arr[m+1..r]
void merge(int arr[], int l, int m, int r)
{
    // Find sizes of two subarrays to be merged
    int n1 = m - l + 1;
    int n2 = r - m;

    /* Create temp arrays */
    int L[] = new int[n1];
    int R[] = new int[n2];

    /*Copy data to temp arrays*/
    for (int i = 0; i < n1; ++i)
        L[i] = arr[l + i];
    for (int j = 0; j < n2; ++j)
        R[j] = arr[m + 1 + j];

    /* Merge the temp arrays */

    // Initial indexes of first and second subarrays
    int i = 0, j = 0;

    // Initial index of merged subarray array
    int k = l;
    while (i < n1 && j < n2) {
        if (L[i] <= R[j]) {
            arr[k] = L[i];
```

```
            i++;
        }
        else {
            arr[k] = R[j];
            j++;
        }
        k++;
    }

    /* Copy remaining elements of L[] if any */
    while (i < n1) {
        arr[k] = L[i];
        i++;
        k++;
    }

    /* Copy remaining elements of R[] if any */
    while (j < n2) {
        arr[k] = R[j];
        j++;
        k++;
    }
}

// Main function that sorts arr[l..r] using
// merge()
void sort(int arr[], int l, int r)
{
    if (l < r) {
```

```
            // Find the middle point
            int m = l + (r - l) / 2;

            // Sort first and second halves
            sort(arr, l, m);
            sort(arr, m + 1, r);

            // Merge the sorted halves
            merge(arr, l, m, r);
        }
    }

    /* A utility function to print array of size n */
    static void printArray(int arr[])
    {
        int n = arr.length;
        for (int i = 0; i < n; ++i)
            System.out.print(arr[i] + " ");
        System.out.println();
    }
```

Time Complexity: Sorting arrays on different machines. Merge Sort is a recursive algorithm and time complexity can be expressed as following recurrence relation.
$T(n) = 2T(n/2) + \Theta(n)$
The above recurrence can be solved either using Recurrence Tree method or Master method. It falls in case II of Master Method and solution of the recurrence is $\Theta(nLogn)$.

Time complexity of Merge Sort is **$\Theta(nLogn)$** in all 3 cases (worst, average and best) as merge sort always divides the array in two halves and take linear time to merge two halves.

Auxiliary Space: O(n)