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Bellman-Ford Algorithm for Shortest Path

Problem: Given a graph and a source vertex *src* in graph, find shortest paths from *src* to all vertices in the given graph. The graph may contain negative weight edges.

We have discussed Dijkstra's algorithm for this problem. Dijkstra's algorithm is a Greedy algorithm and time complexity is O(VLogV) (with the use of Fibonacci heap). *Dijkstra doesn't work for Graphs with negative weight edges, Bellman-Ford works for such graphs. Bellman-Ford is also simpler than Dijkstra and suites well for distributed systems. But time complexity of Bellman-Ford is O(VE), which is more than Dijkstra.*

Algorithm: Following are the detailed steps.

- *Input:* Graph and a source vertex *src*.
- *Output:* Shortest distance to all vertices from *src*. If there is a negative weight cycle, then shortest distances are not calculated, negative weight cycle is reported.

1. This step initializes distances from source to all vertices as infinite and distance to source itself as 0. Create an array dist[] of size |V| with all values as infinite except dist[src] where src is source vertex.
2. This step calculates shortest distances. Do following |V|-1 times where |V| is the number of vertices in given graph. Do following for each edge u-v:
   - If dist[v] > dist[u] + weight of edge uv, then update dist[v] as: *dist[v] = dist[u] + weight of edge uv*.
3. This step reports if there is a negative weight cycle in graph. Do following for each edge u-v. If dist[v] > dist[u] + weight of edge **uv**, then "Graph contains negative weight cycle".

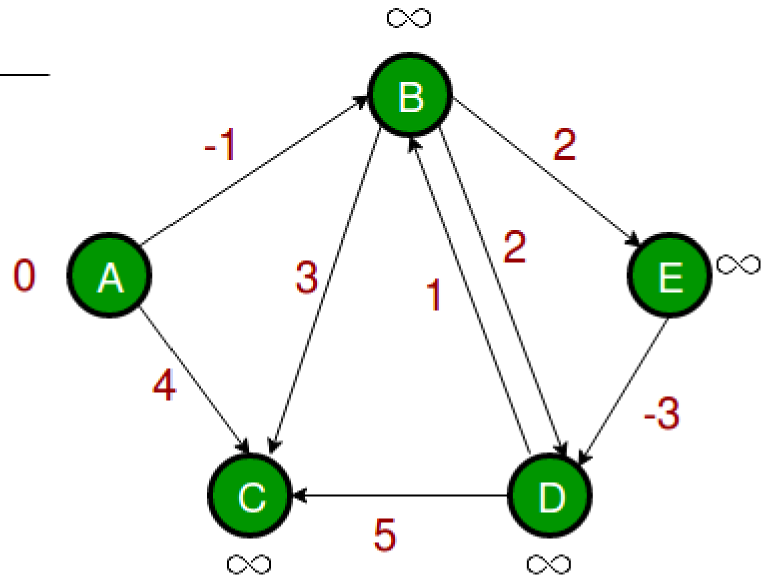
The idea of step 3 is, step 2 guarantees the shortest distances if the graph doesn't contain a negative weight cycle. If we iterate through all edges one more time and get a shorter path for any vertex, then there is a negative weight cycle.

How does this work? Like other Dynamic Programming Problems, the algorithm calculates shortest paths in a bottom-up manner. It first calculates the shortest distances which have at most one edge in the path. Then, it calculates the shortest paths with at-most 2 edges, and so on. After the i-th iteration of the outer loop, the shortest paths with at most **i** edges are calculated. There can be maximum **|V| - 1** edge in any simple path, that is why the outer loop runs |v| - 1 time. The idea is, assuming that there is no negative weight cycle, if we have calculated shortest paths with at most i edges, then an iteration over all edges guarantees to give shortest path with at-most (i+1) edge.

Example:
Let us understand the algorithm with following example graph. The images are taken from this source.

Let the given source vertex be 0. Initialize all distances as infinite, except the distance to the source itself. Total number of vertices in the graph is 5, so *all edges must be processed 4 times.*

A	B	C	D	E
0	∞	∞	∞	∞



Let all edges are processed in following order: (B,E), (D,B), (B,D), (A,B), (A,C), (D,C), (B,C), (E,D). We get following distances when all edges are processed first time. The first row in shows initial distances. The second row shows distances when edges (B,E), (D,B), (B,D) and (A,B) are processed. The third row shows distances when (A,C) is processed. The fourth row shows when (D,C), (B,C) and (E,D) are processed.

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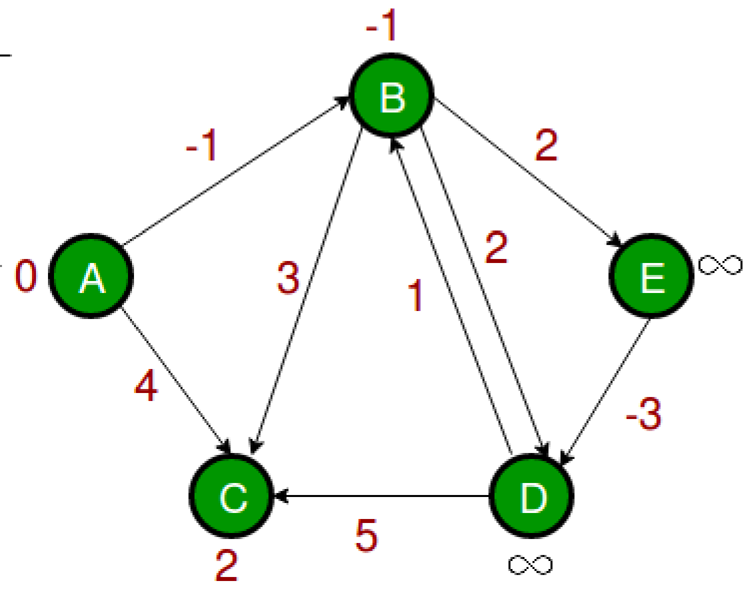
A	B	C	D	E
0	∞	∞	∞	∞
0	-1	∞	∞	∞
0	-1	4	∞	∞
0	-1	2	∞	∞



The first iteration guarantees to give all shortest paths which are at most 1 edge long. We get the following distances when all edges are processed a second time (The last row shows final values).

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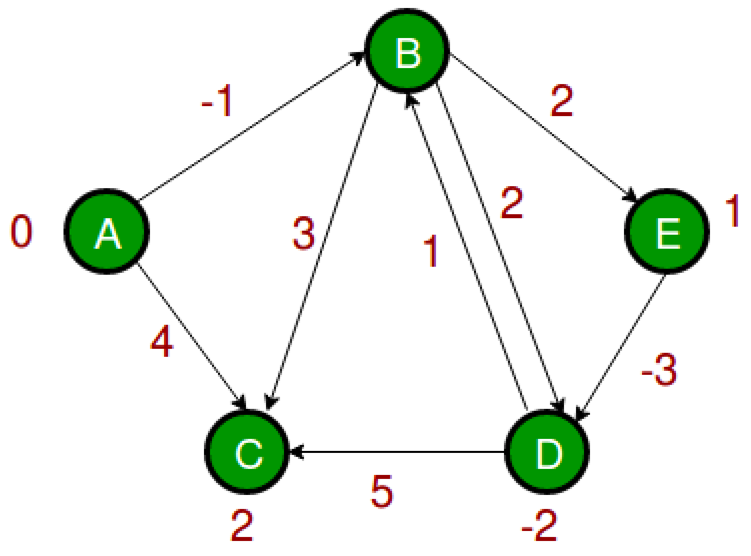
A	B	C	D	E
0	∞	∞	∞	∞
0	-1	∞	∞	∞
0	-1	4	∞	∞
0	-1	2	∞	∞
0	-1	2	∞	1
0	-1	2	1	1
0	-1	2	-2	1



The second iteration guarantees to give all shortest paths which are at most 2 edges long. The algorithm processes all edges 2 more times. The distances are minimized after the second iteration, so third and fourth iterations don't update the distances.

Implementation:

C++ Java

```
// A Java program for Bellman-Ford's single source shortest path
// algorithm.
import java.util.*;
import java.lang.*;
import java.io.*;

// A class to represent a connected, directed and weighted graph
```

```
class Edge {
    int src, dest, weight;
    Edge() {
        src = dest = weight = 0;
    }
};

int V, E;
Edge edge[];

// Creates a graph with V vertices and E edges
Graph(int v, int e)
{
    V = v;
    E = e;
    edge = new Edge[e];
    for (int i=0; i<e; ++i)
        edge[i] = new Edge();
}

// The main function that finds shortest distances from src
// to all other vertices using Bellman-Ford algorithm.  The
// function also detects negative weight cycle
void BellmanFord(Graph graph,int src)
{
    int V = graph.V, E = graph.E;
    int dist[] = new int[V];

    // Step 1: Initialize distances from src to all other
    // vertices as INFINITE
    for (int i=0; i<V; ++i)
        dist[i] = Integer.MAX_VALUE;
    dist[src] = 0;

    // Step 2: Relax all edges |V| - 1 times. A simple
    // shortest path from src to any other vertex can
    // have at-most |V| - 1 edges
    for (int i=1; i<V; ++i)
```

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```
        {
            for (int j=0; j<E; ++j)
            {
                int u = graph.edge[j].src;
                int v = graph.edge[j].dest;
                int weight = graph.edge[j].weight;
                if (dist[u]!=Integer.MAX_VALUE &&
                    dist[u]+weight<dist[v])
                    dist[v]=dist[u]+weight;
            }
        }

        // Step 3: check for negative-weight cycles.  The above
        // step guarantees shortest distances if graph doesn't
        // contain negative weight cycle. If we get a shorter
        //  path, then there is a cycle.
        for (int j=0; j<E; ++j)
        {
            int u = graph.edge[j].src;
            int v = graph.edge[j].dest;
            int weight = graph.edge[j].weight;
            if (dist[u] != Integer.MAX_VALUE &&
                dist[u]+weight < dist[v])
              System.out.println("Graph contains negative weight cycle");
        }
        printArr(dist, V);
    }

    // A utility function used to print the solution
    void printArr(int dist[], int V)
    {
        System.out.println("Vertex   Distance from Source");
        for (int i=0; i<V; ++i)
            System.out.println(i+"\t\t"+dist[i]);
    }

    // Driver method to test above function
    public static void main(String[] args)
    {
        int V = 5;  // Number of vertices in graph
        int E = 8;  // Number of edges in graph
```

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```
Graph graph = new Graph(V, E);

// add edge 0-1 (or A-B in above figure)
graph.edge[0].src = 0;
graph.edge[0].dest = 1;
graph.edge[0].weight = -1;

// add edge 0-2 (or A-C in above figure)
graph.edge[1].src = 0;
graph.edge[1].dest = 2;
graph.edge[1].weight = 4;

// add edge 1-2 (or B-C in above figure)
graph.edge[2].src = 1;
graph.edge[2].dest = 2;
graph.edge[2].weight = 3;

// add edge 1-3 (or B-D in above figure)
graph.edge[3].src = 1;
graph.edge[3].dest = 3;
graph.edge[3].weight = 2;

// add edge 1-4 (or A-E in above figure)
graph.edge[4].src = 1;
graph.edge[4].dest = 4;
graph.edge[4].weight = 2;

// add edge 3-2 (or D-C in above figure)
graph.edge[5].src = 3;
graph.edge[5].dest = 2;
graph.edge[5].weight = 5;

// add edge 3-1 (or D-B in above figure)
graph.edge[6].src = 3;
graph.edge[6].dest = 1;
graph.edge[6].weight = 1;

// add edge 4-3 (or E-D in above figure)
graph.edge[7].src = 4;
graph.edge[7].dest = 3;
```

```
        graph.edge[7].weight = -3;

        graph.BellmanFord(graph, 0);
    }
}
```

Output:

```
Vertex   Distance from Source
0                0
1                -1
2                2
3                -2
4                1
```

Important Notes:

1. Negative weights are found in various applications of graphs. For example, instead of paying the cost for a path, we may get some advantage if we follow the path.
2. Bellman-Ford works better (better than Dijkstra's) for distributed systems. Unlike Dijksra's where we need to find the minimum value of all vertices, in Bellman-Ford, edges are considered one by one.