

Tutorial-2, CSN-212

Paras Chetal, Enrollment No. 15114049

February 6, 2017

4.1

4.1 (b) $T(n) = T(7n/10) + n$

Let no. of terms = k

$$\left(\frac{7}{10}\right)^k n = 1 \implies k = \log_{\frac{10}{7}} n$$

$$T(n) = n + \left(\frac{7}{10}\right)n + \left(\frac{7}{10}\right)^2 n + \left(\frac{7}{10}\right)^3 n \dots \quad (k \log_{\frac{10}{7}} n \text{ times})$$

$$\text{We know, } \sum_{r=0}^k \left(\frac{7}{10}\right)^r = \frac{\left(1 - \left(\frac{7}{10}\right)^{\log_{\frac{10}{7}} n}\right)}{\left(1 - \frac{7}{10}\right)}$$

$$\implies T(n) = \frac{10}{3}n(1 - n^{-1}) = \frac{10}{3}(n - 1)$$

$$\implies O(n)$$

4.1 (c) $T(n) = 16T(n/4) + n^2$

Let no. of terms = k

$$k = \log_4 n$$

$$T(n) = n^2 + 16(n^2/16 + T(n/4^2))$$

$$T(n) = n^2 + n^2 + 16^2(n^2/16^2 + 16(T/4^3))$$

$$T(n) = kn^2 + 16^k T(1)$$

$$\implies T(n) = n^2 \log_4 n + 16^{\log_4 n} T(1)$$

$$\implies O(n^2 \log n)$$

4.1 (f) $T(n) = 2T(n/4) + \sqrt{n}$

Let no. of terms = k

$$k = \log_4 n$$

$$T(n) = \sqrt{n} + \sqrt{n} + 2^2(\sqrt{n}^2 + 2(T/4^3))$$

$$\begin{aligned}
T(n) &= k\sqrt{n} + 2^k T(1) \\
\implies T(n) &= \sqrt{n} \log_4 n + 2^{\log_4 n} T(1) \\
\implies O(\sqrt{n} \log n)
\end{aligned}$$

4.1 (g) $T(n) = T(n-2) + n^2$

Let no. of terms = k

$$n - 2k = 1$$

$$k = (n-1)/2$$

$$T(n) = n^2 + (n-2)^2 + (n-3)^2 \dots (k \text{ times})$$

$$T(n) = \sum_{r=0}^k (n^2 - 4rn + 4r^2)$$

$$T(n) = n^2 k - 4n \frac{k(k-1)}{2} + 4 \frac{k(k+1)(2k+1)}{6}$$

Since $k = \Theta(n)$

$$\implies O(n^3)$$

4.2

4.2 (a) $T(n) = 4T(n/3) + n \log n$

Let no. of terms = k

$$k = \log_3 n$$

$$T(n) = n \log n + 4(n/3 \log(n/3) + 4T(n/3^2))$$

$$T(n) = n \log n (1 + 4/3 + (4/3)^2 \dots (k \text{ times})) - n \log 3 (4/3 + 2(4/3)^2 + 3(4/3)^3 \dots (k \text{ times}))$$

$$T(n) = 4n \log n - 24n + 25n^l \log_4 3$$

$$\implies O(n \log n)$$