CS 201c, Tutorial 1: Asymptotic Analysis

Assumptions. n varies over natural numbers. $f(n), f_1(n), f_2(n)$ and $g(n), g_1(n), g_2(n)$ are strictly greater than 0 for every natural number n. $\mathbb{N}$ denotes the set of natural numbers.

1. Show that if $f_1(n) = \Theta(g_1(n))$ and $f_2(n) = \Theta(g_2(n))$, then $f_1(n) \cdot f_2(n) = \Theta(g_1(n) \cdot g_2(n))$.

   Can we also say that $\frac{f_1(n)}{f_2(n)} = \Theta\left(\frac{g_1(n)}{g_2(n)}\right)$?

2. (a) Show that if $f(n) \sim g(n)$, then $f(n) = \Theta(g(n))$.

   (b) Show that the converse of the above statement is not always true.

3. Show that:

$$\log_2(1) + \log_2(2) + \cdots + \log_2(n) = \Theta(n(\log_2(n))^2)$$

4. Show that $2^n = o(3^n)$.

5. Is $2^n \sim 2^{n+\log_2(n)}$? Is $2^n \sim 2^{n+\frac{1}{\log_2(n)}}$? Is $n^{(\log_2(n))^2} = o(2^n)$?

6. Show that:

$$\sqrt{1} + \sqrt{2} + \cdots + \sqrt{n} = \Theta(n^{\frac{3}{2}})$$

   In general, for any real number $\alpha > 0$, prove that:

$$1^\alpha + 2^\alpha + \cdots + n^\alpha = \Theta(n^{1+\alpha})$$

7. For a positive real number T, let

$$f(T) = max\{n \mid n \in \mathbb{N} \; and \; n \log_2(n) \leq T\}$$

   Show that $f(T) \sim \frac{T}{\log_2(T)}$.