CS201c Tutorial
Topic: Hashing

1. Suppose the universe U consists of the 2^k bit strings of length k each. Suppose you use hashing with chaining, in a hash table H with 2^l slots where l < k.

Let M be a random 0-1 matrix with l rows and k columns. (Each entry of M is obtained by tossing an unbiased coin.)

Consider the hash function:

$$h_{M}(x) = M \cdot x$$ where x is a bit string in universe U.

(M . x denotes the result of multiplying matrix M with column vector x, where you use the XOR operation in place of addition.)

(i) Let y and z be two *distinct* bit strings in universe U. Show that:

$$Pr_{M} (h_{M}(x) == h_{M}(y)) = 1/(2^l)$$

(ii) Use part (i) to analyze the expected time for both successful and unsuccessful search, if you hash n keys into an empty hash table H using a *random* hash function h_{M}.

2. Suppose you use hashing with linear probing with a *perfectly random (i.e., ideal)* hash function.

Assume that the hash table H has m slots and is initially empty. Further, suppose you insert n keys 1, 2, …, n in hash table H in this order. (We assume that n=2k, where k is a positive integer and n<=m.)

(a) What is the probability that the first n entries H[0…(n-1)] of hash table are [1 2 3 … (2k-1) 2k]?
(b) What is the probability that the first n entries H[0…(n-1)] of hash table are [2k, (2k-1) (2k-2) … 2 1]?
(c) Let t be any permutation of {1, 2, …, 2k-1, 2k}. Give an expression for the probability that the first n entries H[0…(n-1)] of hash table are t(1), t(2), t(3), …, t(2k) in this order?

3. Let p be an odd prime. Suppose you use hashing with quadratic probing in a hash table H with p slots. For integers c and d, the probe sequence for an element x of the universe is defined as:

$$(h(x)+c*i+d*(i^2)) \bmod p \quad : \quad \text{for } i=0,1,2,...,p-1$$

where h is a *perfectly random* hash function.

What is the maximum number of distinct locations in hash table H that can be visited by a probe sequence of the above form?