

CS201c: Quiz 2 solutions

Ans 1.

H[0]	H[1]	H[2]	H[3]	H[4]	H[5]	H[6]
16	3	-1	2	9	1	8

Key	Partial probe sequence till successful insert operation
1	5
2	3
3	1
8	5, 6
9	3, 4
16	3, 4, 5, 6, 0

Ans 2.

Probability that pivot rank is equal to i is exactly $1/(2z+1)$ for every $1 \leq i \leq (2z+1)$. (Pivot rank i means that pivot is the i -th smallest element in the array A .)

Pivot rank	Size of B
1	$2z$
2	$2z-1$
.	.
.	.
.	.
z	$z+1$
$z+1$	0
$z+2$	$z+1$
$z+3$	$z+2$
$z+4$	$z+3$
.	.
.	.
.	.
$2z+1$	$2z$

Thus, expected value of $|B|$

$$\begin{aligned} &= (2 [(z+1) + (z+2) + \dots + (2z)]) / (2z+1) \\ &= [(2^*z*z) + (z^*(z+1))] / (2z+1) \\ &= [z * (3z+1)] / (2z+1) \end{aligned}$$

Ans 3.

Remark. We can get a representative node of rank r with $\{1, 2, \dots, 2^r\}$ in its subtree as follows:

- (i) Recursively get a representative node v_1 of rank $r-1$ with $\{1, 2, \dots, 2^{r-1}\}$ in its subtree.
- (ii) Recursively get a representative node v_2 of rank $r-1$ with $\{2^{r-1}+1, 2^{r-1}+2, \dots, 2^r\}$ in its subtree.
- (iii) Do Union(v_1, v_2).

Sequence of union operations achieving a rank r ($1 \leq r \leq k$) node with all $n=2^k$ nodes in its subtree:

- Make union operations on first 2^r keys to get a set with 2^r nodes such that its representative node v has rank r (see Remark above).
- Do union(v, j) where $(2^{r+1}) \leq j \leq n$.
[Since rank(v)= r and rank(j)= $0 < r$, j becomes a child of v and v 's rank does not change.]
- Finally, v will be the root node of a single set, and rank of v will be r .