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ML – H/W – 1

Ans-1 :→ (i) Let us suppose that A and B are similar. Then there exists a invertible '$n \times n$' matrix P such that $P^{-1}AP = B$.

Now, taking determinant on both sides, we get,

$$\det|P^{-1}AP| = |B|$$

$$\det(P^{-1}AP) = \det(B)$$
$$\det(P^{-1}) \det(A) \det(P) = \det(B) \quad \text{--- (1)}$$

Now, as we know that $\det(M^{-1}) = \det(M)^{-1}$
$$\Rightarrow \det(M^{-1}) = \frac{1}{\det(M)} \quad \text{--- (2)}$$

⇒ Replacing (2) in (1), we get,

$$\det(A) \det(P) \boxed{\tfrac{1}{}} \det(P) = \det(B)$$

$$\Rightarrow \det(A) \cancel{\det(P)} \left(\frac{1}{\cancel{\det(P)}}\right) = \det(B)$$

$$\Rightarrow \det(A) = \det(B)$$

(ii) Now, it is given that $A \sim B$ and from the similarity of A and B, we know that

$$\det(A) = \det(B),$$

Using these properties, we can say that,

The characteristic polynomials prove the equality of eigen values of similar matrices

Mathematically speaking, we have,

Let $p_B(\lambda)$ and $p_A(\lambda)$ denote the characteristic polynomial of A and B.

$$p_B(\lambda) = \det(B - \lambda I)$$
$$p_A(\lambda) = \det(A - \lambda I)$$

Starting from $p_B(\lambda)$, we have,

$$p_B(\lambda) = \det(B - \lambda I)$$

Replacing B by $P^{-1}AP$, we have,

$$p_B(\lambda) = \det(P^{-1}AP - \lambda I)$$

$$p_B(\lambda) = \det(P^{-1}\lambda A P - P^{-1}AP)$$

$$p_B(\lambda) = \det(P^{-1}(A - \lambda I)P)$$

$$p_B(\lambda) = \det(P^{-1}) \det(A - \lambda I) \det(P)$$

Now, since we know that $\det(P^{-1}) = \frac{1}{\det(P)}$, we have

$$p_B(\lambda) = \cancel{(\det(P))^{-1}} \det(A - \lambda I) \cancel{\det(P)}$$

$$p_B(\lambda) = \det(A - \lambda I)$$

$\Rightarrow p_B(\lambda) = p_A(\lambda)$ $\Rightarrow$ Hence proved.

Ans) (2) (i) let $u = (u_1, u_2, u_3, \ldots, u_n)$

Now, we have to calculate the value of $u \cdot u$

Now, by definition of dot product, we have,

~~[illegible]~~

$$u \cdot v = u_1 v_1 + u_2 v_2 + u_3 v_3 + \ldots + u_n v_n$$

Now, here, v is u itself, therefore, we have,

$$\Rightarrow u \cdot u = u_1 u_1 + u_2 u_2 + u_3 u_3 + \ldots + u_n u_n$$

$$\Rightarrow u \cdot u = (u_1)^2 + (u_2)^2 + (u_3)^2 + \ldots + (u_n)^2$$

$$\Rightarrow u \cdot u = \sum_{i=1}^{n} (u_i)^2$$

Now, taking the "square root" of "square" on the Right side, we have,

$$\Rightarrow u \cdot u = \left(\sqrt{\sum_{i=1}^{n} (u_i)^2} \right)^2$$

$$\Rightarrow u \cdot u = \|\vec{u}\|^2$$

where $\|u\|$ represents the 2-D norm.

Hence Proved.

Ans) (2) ii) $Q \rightarrow n \times n$ orthogonal matrix
$\vec{x} \longrightarrow n \times n$ vector
$I_n \rightarrow n \times n$ Identity matrix

Now, to prove
$$\|Q\vec{x}\| = \|\vec{x}\|$$

Taking the left hand side and squaring it, we have

$$\|Q\vec{x}\|^2 = Q\vec{x} \cdot Q\vec{x}$$

Now, as we know that for a vector $\vec{y}$,
$$\vec{y} \cdot \vec{y} = y^T y,$$

Applying this property to the above equation, we have,

$$\|Q\vec{x}\|^2 = (Q\vec{x})^T Q\vec{x}$$

$$\Rightarrow \|Q\vec{x}\|^2 = Q^T \vec{x}^T Q \vec{x}$$

$$\Rightarrow \|Q\vec{x}\|^2 = Q^T Q \vec{x}^T \vec{x}$$

Now, we know that $Q^T Q = I_n$

$$\Rightarrow \|Q\vec{x}\|^2 = Q^T Q \vec{x}^T \vec{x}$$

$$\Rightarrow \|Q\vec{x}\|^2 = I_n \vec{x}^T \vec{x}$$

$$\Rightarrow \|Q\vec{x}\|^2 = (\vec{x}^T \vec{x})$$ (from inverse Transpose property)

$$\|Q\vec{x}\|^2 = \|\vec{x}\|^2$$ $\Rightarrow$ Hence Proved

Ans 3) The Gaussian function can be defined as given :-

$$N(x, \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\left[\frac{-1}{2\sigma^2}(x-\mu)^2\right]}$$

This can be written as :-

$$N(x, \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\left[\frac{(x-\mu)^2}{(-2\sigma^2)}\right]}$$

Now, in order to find out the maximum value for this function, we have to find out the double derivative which will give the critical points of the function and hence we can find out the maximum value.

$$\Rightarrow \frac{d}{dx}\left[(N(x, \mu, \sigma^2))\right] = \frac{d}{dx}\left[\left(\frac{1}{\sqrt{2\pi\sigma^2}}\right) e^{\left[-\frac{(x-\mu)^2}{2\sigma^2}\right]}\right]$$

Now, since we have to find derivative w.r.t 'x', we can treat other variables as constants.

$$\Rightarrow \frac{d}{dx}\left[(N(x, \mu, \sigma^2))\right] = \frac{1}{\sqrt{2\pi\sigma^2}} \frac{d}{dx}\left[e^{\left[-\frac{(x-\mu)^2}{2\sigma^2}\right]}\right]$$

By applying the derivative rule of e^x, we have,

$$\Rightarrow \frac{d}{dx}\left[N(x, \mu, \sigma^2)\right] = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\left[-\frac{(x-\mu)^2}{2\sigma^2}\right]} \cdot \frac{d}{dx}\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

$$\Rightarrow \frac{d}{dx}\left[N(x,\mu,\sigma^2)\right] = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\left[\frac{-(x-\mu)^2}{2\sigma^2}\right]} \frac{(-\cancel{2})(x-\mu)}{\cancel{2}\sigma^2}$$

$$\Rightarrow \frac{d}{dx}\left[N(x,\mu,\sigma^2)\right] = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\left[-\frac{(x-\mu)^2}{2\sigma^2}\right]} \frac{(-x+\mu)}{\sigma^2}$$

Now, taking the derivative again, we have,

$$\Rightarrow \frac{d}{dx}\left[\frac{d}{dx}\left[N(x,\mu,\sigma^2)\right]\right] = \frac{d}{dx}\left(\frac{1}{\sqrt{2\pi\sigma^2}} e^{\left[-\frac{(x-\mu)^2}{2\sigma^2}\right]} \left[\frac{-x+\mu}{\sigma^2}\right]\right)$$

Taking out the constants, we have,

$$\Rightarrow \frac{d}{dx}\left[\frac{d}{dx}\left[N(x,\mu,\sigma^2)\right]\right] = \frac{1}{(\sqrt{2\pi\sigma^2})\sigma^2} \frac{d}{dx}\left[\left(e^{-\frac{(x-\mu)^2}{2\sigma^2}}\right)\left[-x+\mu\right]\right]$$

By applying the multiplication rule for derivatives, we have,

$$\frac{d}{dx}\left[\frac{d}{dx}\left[N(x,\mu,\sigma^2)\right]\right] = \frac{1}{(\sqrt{2\pi\sigma^2})\sigma^2}\left[\left(e^{-\frac{(x-\mu)^2}{2\sigma^2}}\right)\frac{(-x+\mu) + \text{Constant}}{(x-\mu)}\right]$$

Now, putting this '= 0', we get the critical point as below:

$$\frac{1}{(\sqrt{2\pi\sigma^2})\sigma^2}\left[\left(e^{-\frac{(x-\mu)^2}{2\sigma^2}}\right)\frac{(-x+\mu)^4 + \text{Constant}}{(-x+\mu)}\right] = 0$$

Now, as we can see that the only factors that can be zero here are:

$(x-\mu)$ or $(x+\mu)$ as $\frac{1}{(\sqrt{2\pi\sigma^2})\sigma^2}$ = constant $\neq 0$

$e^{-f(x)} \neq 0$

Constant $\neq 0$

$\Rightarrow (x-\mu)=0$ or $(x+\mu)=0$

$\Rightarrow x=\mu$ or $x=(-\mu)$

$\Rightarrow$ From here we can say that the maxima occurs at (μ)

Hence Proved.

Ans 4) (i) When a fair coin is flipped 3 times, the sample space is {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

H → denotes Head

T → denotes Tail

~~[illegible]~~

Now, we can see that, for finding the Expected Number of heads, we can take probability of success as getting heads.

X → Random variable indicating number of heads,

$P(H) \to \frac{1}{2} \Rightarrow$ Probability of head

$P(T) = \frac{1}{2} \Rightarrow$ Probability of Tail

Now, seeing the sample space, we can say that

$P(x=0)$ = all tails $= \frac{1}{8}$

$P(x=1)$ = 1 head and 2 tails = HTT, THT, TTH $= \frac{3}{8}$

↳ This can also be written as $\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2} + \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2} + \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}$

$$\Rightarrow \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{3}{8}$$

$P(X=2) = \frac{3}{8}$

$P(X=3) = \frac{1}{8}$

So, now, we can see that the probability of getting a head becomes as

$$P(X) = \frac{7}{8}$$

⇒ Expected value becomes as $\Rightarrow 3 \times \frac{1}{2} \Rightarrow \frac{3}{2} = 1.5$

This can also be done by Binomial $E(x)$.

where $E(x) = np$ where $n =$ ~~success~~ trials of success (head)

$p =$ probability of success

$E(x) = 3 \times 1/2$

$E(x) = 3/2 = 1.5$

(ii) for the first toss of coin, we have,

$P(H) =$ Probability of $H = \frac{1}{2}$

$P(T) =$ Probability of $T = \frac{1}{2}$

Now, for the second toss, we have the following outcomes in conjunction with the first toss, :→

$\Rightarrow \{HH, HT, TH, TT\}$

Now, $P(H) = TH = \frac{1}{4}$, $P(T) = TT = \frac{1}{4}$

$P(H) = TH =$ can be rewritten as $\left(\frac{1}{2} \cdot \frac{1}{2}\right)$

Now, for the third toss, we have the following outcomes in conjunction with the previous two tosses :→

$\Rightarrow \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

$\Rightarrow$ P(H) at third outcome = TTH = $\frac{1}{8}$

This can be rewritten as $\left(\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{4}\right)$

Now, by following this pattern, we have,

$\underset{\text{at } n^{th} \text{ outcome}}{P(H)} = \left(\frac{1}{2}\right)^n$; $P(T) = \left(1 - \frac{1}{2}\right)^{n-1}$

This is the probability of success

This is the probability of failure

Now, as we know that the above mentioned pattern is of Geometric series, therefore,

$$E(H) = \sum_{i=1}^{n} n\, P(X=n)$$

$$E(H) = \sum_{i=1}^{n} n\, (\text{G.P. sum})$$

$$E(H) = \sum_{i=1}^{n} n \cdot (\text{sum of G.P})$$ (Here sum of G.P = $\frac{1}{1-r}$ where r = interval)

$$\Rightarrow E(H) = \sum_{i=1}^{n} \left(n \cdot \frac{1}{1-p}\right)$$

$$\Rightarrow E(H) = (1-p)^{n-1} (p)$$ where p ≡ probability of success

$$\Rightarrow E(H) = \left(1-\frac{1}{2}\right)^{n-1} \left(\frac{1}{2}\right)$$

Now, since we know that the first head terminates the experiment, therefore

$E(X = \text{Head}) = \frac{r}{1-p}$ where $r = 1$ → steps of heads moving, $p = \frac{1}{2}$

$E(X = \text{Head}) = \frac{1}{1-\frac{1}{2}} = \frac{1}{\frac{1}{2}}$ ⇒ (2)

⇒ first Head will occur at second toss.

(iii) Let the expected Number of coin flips be 'x'.

Now there are 3 possible cases as listed below;

(a) If a tail appears on the first flip of a coin →
$p(X) = \frac{1}{2}$ and no. of steps required $= (x+1)$

(b) If a head appears on the first flip and a tail appears on the second flip.

$p(X) = \frac{1}{4}$ and no. of flips $= (x+2)$

(c) Two consecutive heads on two flips;

$p(X) = \frac{1}{4}$ and no. of flips '2'.

Now, by using the general law of probability and equations, we have,

$E(x) = \sum x p(x)$.

$$\Rightarrow X = \left(\frac{1}{2}\right)(x+1) + \left(\frac{1}{4}\right)(x+2) + \left(\frac{1}{4}\right)(2)$$

$$\Rightarrow x = \frac{1}{2}x + \frac{1}{2} + \frac{1}{4}x + \frac{1}{2} + \frac{1}{2}$$

$$\Rightarrow x = \left(\frac{1}{2}x + \frac{1}{4}x\right) + \frac{1}{2} + \frac{1}{2} + \frac{1}{2}$$

$$\Rightarrow x = \left(\frac{2x+x}{4}\right) + \frac{3}{2}$$

$$\Rightarrow x = \frac{3x}{4} + \frac{3}{2}$$

$$x - \frac{3x}{4} = \frac{3}{2} \Rightarrow \frac{x}{\cancel{4}_2} = \frac{3}{\cancel{2}} \Rightarrow \boxed{x = 6}$$

$\boxed{\text{Required Answer}}$ by $E(x)$ recursive

Ans5) Now, from definition, we know that

$$Cov[X,Y] = E[[(X - E(X)][Y - E(Y)]]$$

$$Cov[X,Y] = E[XY - X(EY) - (EX)Y + E(X)E(Y)]$$

$$Cov[X,Y] = E[XY] - E(X)E(Y) - \cancel{E(X)E(Y)} + \cancel{E(X)E(Y)}$$

$$Cov[X,Y] = E[XY] - (EX)(EY) \quad -①$$

Also, $Var(X) = E[XY] - (E(X)E(Y)) \quad -②$

From ① and ② we can say that,

$$Var(Z) = Cov(Z, Z)$$

Now Replacing $Z = X+Y$, we have,

$$Var(X+Y) = Cov(X+Y, X+Y)$$

$$Var(X+Y) = Cov(X, X) + Cov(X, Y) + Cov(Y, X) + Cov(Y, Y)$$

$$Var(X+Y) = Var(X) + Var(Y) + 2\,Cov(X, Y)$$

Hence Proved

Ans 6) $A \to$ Event A is going to same school ⟹ whom you recognize $P(A)$
$B \to$ Event B is vaguely recognising those people at the party $P(B)$

$$P\left(\frac{B}{A}\right) = \frac{1}{2}$$

$$P(A) = \frac{1}{10}$$

$$P(B) = \frac{1}{5}$$

$$P\left(\frac{A}{B}\right) = \frac{P\left(\frac{B}{A}\right) P(A)}{P(B)}$$

$$P\left(\frac{A}{B}\right) = \frac{\left(\frac{1}{2}\right)\left(\frac{1}{10}\right)}{\frac{1}{5}} = \frac{1}{20} \times 5$$

$$= 0.25$$

Ans 7) LOGICAL 'NOT' :-

The logical Not function is as below's

A	NOT A
False	True
True	False

⇒

A	Not A
0	1
1	0

The prediction algorithm states that

$$\text{Prediction } (y') = \begin{cases} 1 & \text{if } wx+b \geq 0 \\ 0 & \text{if } Wx+b < 0 \end{cases}$$

To train the perceptron for Logical Not gate, we have,

① Initialize weight values and bias.

$w_1x_1 + b$ where $w_1 = 1$, $b = (-1)$

$x1(1) - 1$

② Passing NOT logic table, we have,

$\Rightarrow x1 = 0 \Rightarrow x1 - 1 \Rightarrow 0 - 1 = (-1)$

$\Rightarrow$ Not correct as $Wx+b < 0$,

$y' = 0$

③ $\Rightarrow$ let $x1 = 0$

$\Rightarrow 0+1 = 0 \Rightarrow 1$

$y' = 1$ $\Rightarrow$ Rule $wx+b>0 \Rightarrow$ Rule works

(4) let $x_1 = 1$

$\Rightarrow wx+b \Rightarrow 1+1=2$

Now, $wx+b>0 \Rightarrow y'=1$ but here $y'=2$

$\Rightarrow$ Does not work

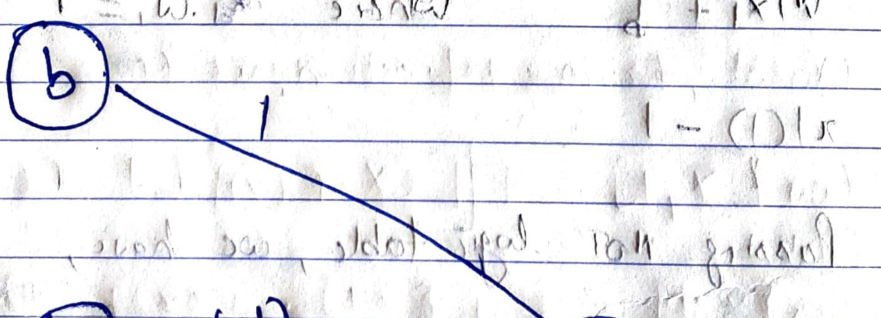
(5) Now, if we change the $w_1=(-1)$, we have,

$wx+b \Rightarrow (-1)+1=0 \Rightarrow y'=0$

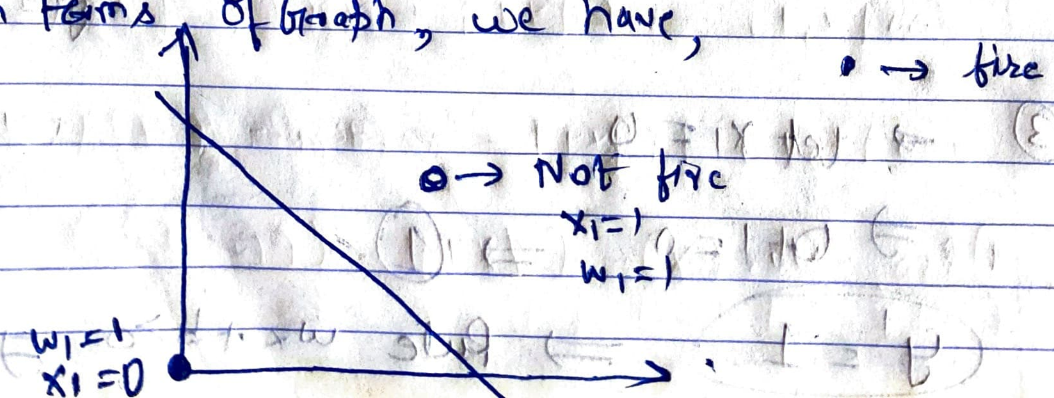
This follows as $wx+b \le 0 = 0$

$\Rightarrow$ The NOT Gate can be traced as below:

$-x_1 + 1$



In terms of Graph, we have,



LOGICAL NOR Gate :-

The logical NOR Gate is below:-

A	B	Output
0	0	1
0	1	0
1	0	0
1	1	0

from the diagram, we can say that

Now, the perceptron algorithm gives us the following eq^n → $w_1x_1 + w_2x_2 + b$

① Initializing w_1 and w_2 as 1 and $b = (-1)$, we get,

$x1(1) + x_2(1) - 1$

$\Rightarrow x1 + x_2 - 1$

② Putting the values from the first row of the truth table, we have,

$x_1 = 0, x_2 = 0$ in $x_1 + x_2 - 1$

$\Rightarrow 0 + 0 - 1 = (-1)$

from the perceptron rule, we have

$wx + b \leq 0$, then $y' = 0$ But above, $y' = (-1)$

$\Rightarrow$ incorrect

⇒ So, we change $b = 1$, we have,

$0 + 0 + 1 = 1$ ⇒ rule works

⇒ Now, for $\boxed{\text{Row-2}}$ of the truth table, we have,
$x_1 = 0$, $x_2 = 1$, we get,

$0 + 1 + 1 = 2$

From the perceptron rule, if $wx + b > 0$, then $y' = 1$.
⇒ Incorrect row.

⇒ So, we will have to make changes to the input as follows:- $x_1 = 0$, $x_2 = 1$ ⇒ $w_1 = (-1)$
using the perceptron rule again, we have,
~~⇒~~ ~~$wx+b$~~. $0 - 1 + 1 = 0$

⇒ correct

⇒ Now, for $\boxed{\text{Row-3}}$, we have, $x_1 = 1$, $x_2 = 0$, we get,
$1 + 0 + 1 = 2$
From perceptron rule, we have,
$wx + b > 0$ ⇒ $y' = 1$ ⇒ In correct

Making changes to the values, $w_1 = (-1)$, $x_1 = 0$, $x_2 = 1$.

⇒ $(-1) + 0 + 1 = 0$ ⇒ Satisfied

Now, for $\boxed{\text{Row-4}}$, we have, $x_1=1$, $x_2=1$,

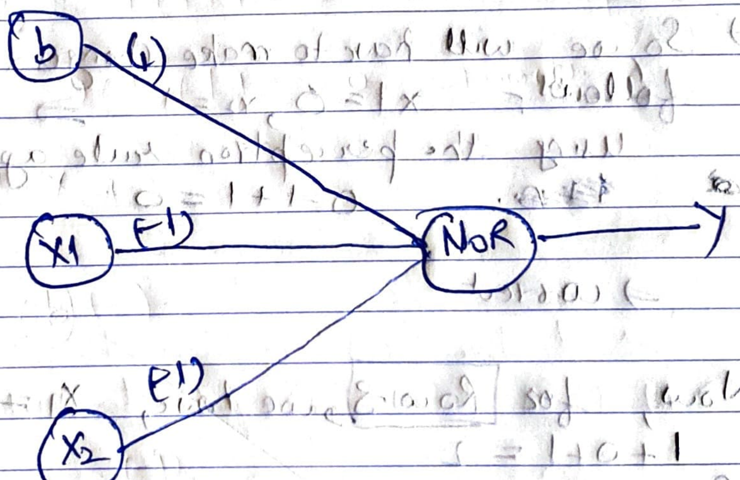
$(1)+(-1)+1 = (-1)$

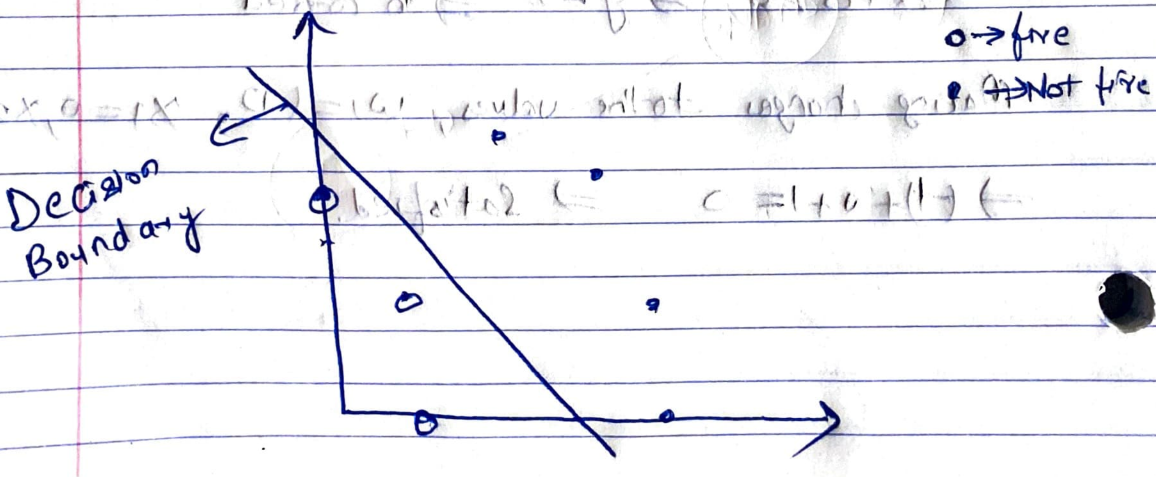
⇒ Satisfies the perceptron rule.

⇒ The final perceptron equation for NOR gate becomes as:→

$-x_1 - x_2 + 1$

Depicting it graphically, we have,





LOGICAL NAND GATE :-

From Boolean Algebra, we know that a NAND gate can be written as :-

A	B	output
0	0	1
0	1	1
1	0	1
1	1	0

From the truth table above, we can say that the NAND gate is 0 only if both inputs are 1.

Now, for $\boxed{\text{Row-1}}$ of the truth table, we have,

① $W_1X_1 + W_2X_2 + b \rightarrow$ perceptron rule
Initializing W_1 and $W_2 = 1$ and $b = (-1)$, we get,

$(X_1)(1) + X_2(1) - 1$
Putting $X_1 = 0$ and $X_2 = 0$, we have

$0 + 0 - 1 = (-1) = y'$

Now, from the perceptron rule, if $wx + b \leq 0$, then $y' = 0$
$\Rightarrow$ This row's incorrect.

Now, updating the weights, we have,
$X_1 = 0$, $X_2 = 0$, $b = 1$, we have,

$0 + 0 + 1 = 1$ $\Rightarrow$ works for the perceptron rule.

Now, for $\boxed{\text{Row-2}}$, we have,

$x_1 = 0$, $x_2 = 1$, we have,

$0+1+1=2 \Rightarrow 0+1+1=2$

works

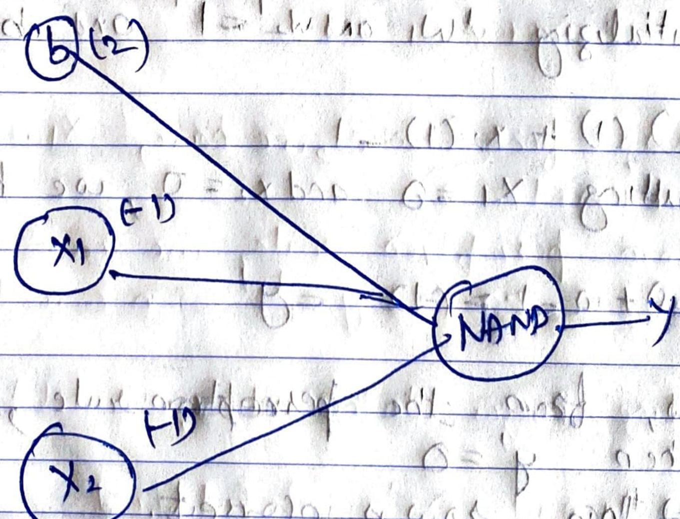
$\boxed{\text{Row-3}}$, we have,

$\hookrightarrow 0+1+1=2 \Rightarrow$ works.

$\boxed{\text{Row-4}}$ $1+1+1=3 \Rightarrow$ Not expected.

$\Rightarrow (-1)+1+2=0 \Rightarrow$ works.

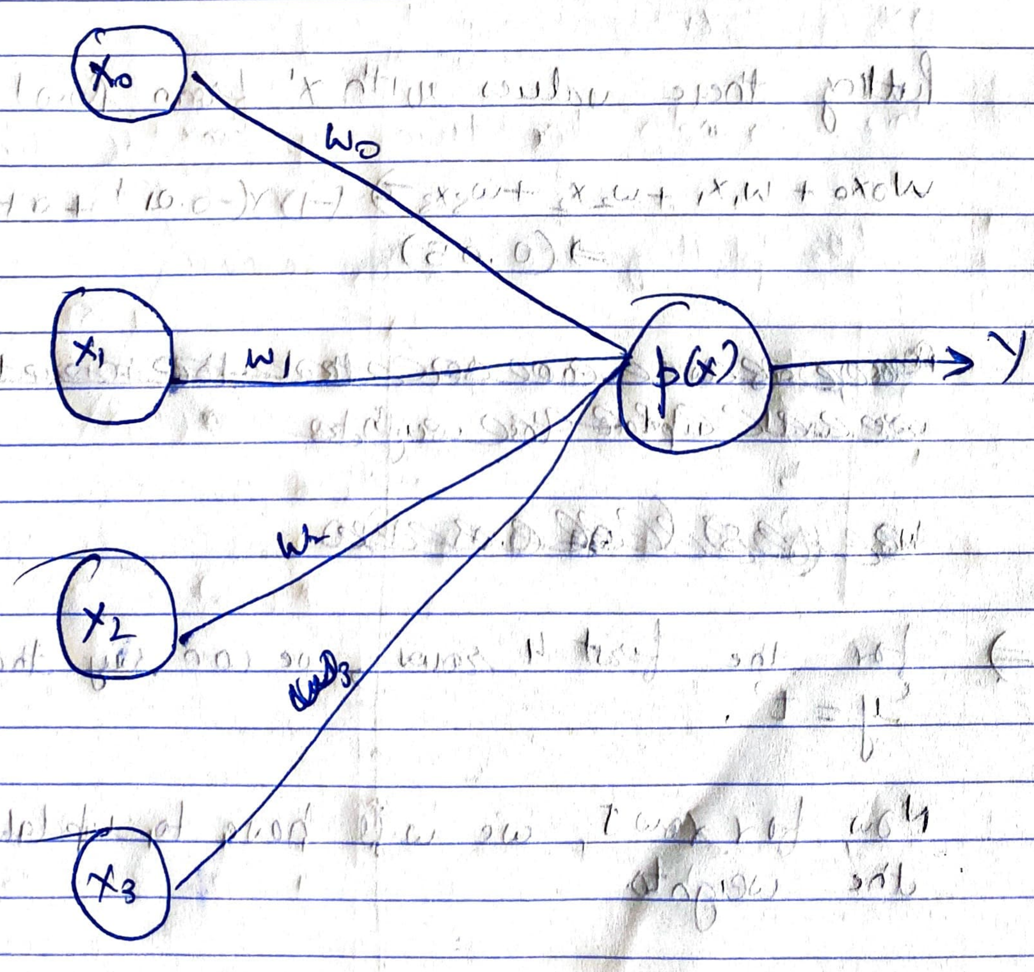
$\Rightarrow -x_1 - x_2 + 2$



Ans 8) Now, as we all know that there are 3 inputs, the table of inputs and outputs is given below:-

A	B	C	output
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

The Graphical Representation for this is given below:-



Now, the Rules of perceptron learning algorithm states that

$y = 1$ if $\left\{ \sum_{i=1}^{n} w_i x_i + b > 0 \right.$ or $y = 0$ if $\sum_{i=1}^{n} w_i x_i + b \leq 0$

and

$\Delta w_i = \eta (y^k - t^k) x_i^k$

let the initial weights be

$w_0 = (-0.05)$
$w_1 = (-0.2)$
$w_2 = (0.2)$
$w_3 = (0.3)$

Putting these values with x' from Row 1, we get,

$w_0 x_0 + w_1 x_1 + w_2 x_2 + w_3 x_3 \Rightarrow (-1) \times (-0.05) + 0 + 0 + 0$
$\Rightarrow (0.05)$

~~[illegible] we will update the weights~~

~~[illegible]~~

$\Rightarrow$ for the first 4 rows, we can say that it is '$y = t$'.

Now, for row 5, we will have to update the weights

$w_0 = 0.2 - 0.25\,(0-1)\,x' = (-0.05)$
$w_1 = -0.02 - 0.25\,(0-1)\,x' = 0.25$
$w_2 = 0.02 - 0.25\,(0-1)\,x_1 = 0.27$
$w_3 = 0.3 - 0.25\,(0-1)\,x_1 = 0.05$

⇒ for the next 3 rows, we can say that

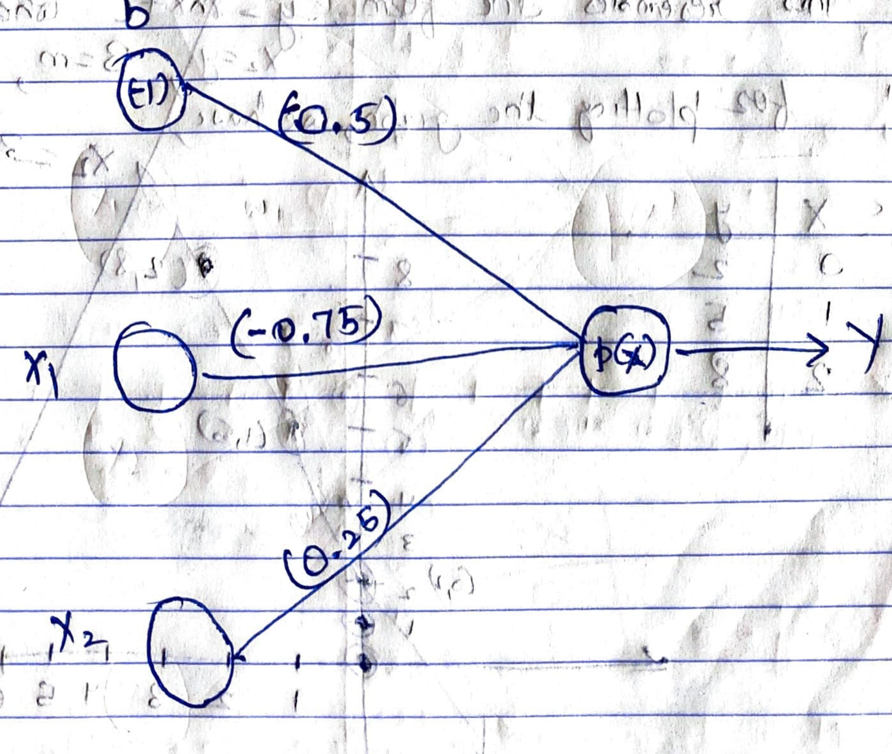
$y \neq t$

⇒ perceptron cannot learn.

Ans 10)

x_1	x_2	Class
0	0	1
0	1	1
1	0	0
1	1	0

(a)



(b) The equation for the perceptron from the above question becomes as :-

$$-0.75x_1 + 0.25x_2 + 0.5 = 0$$

Now, writing the above equation in the slope intercept form, we have,

$$-0.75x_1 + 0.25x_2 + 0.5 = 0$$

$$-0.75x_1 + 0.25x_2 = (-0.5)$$

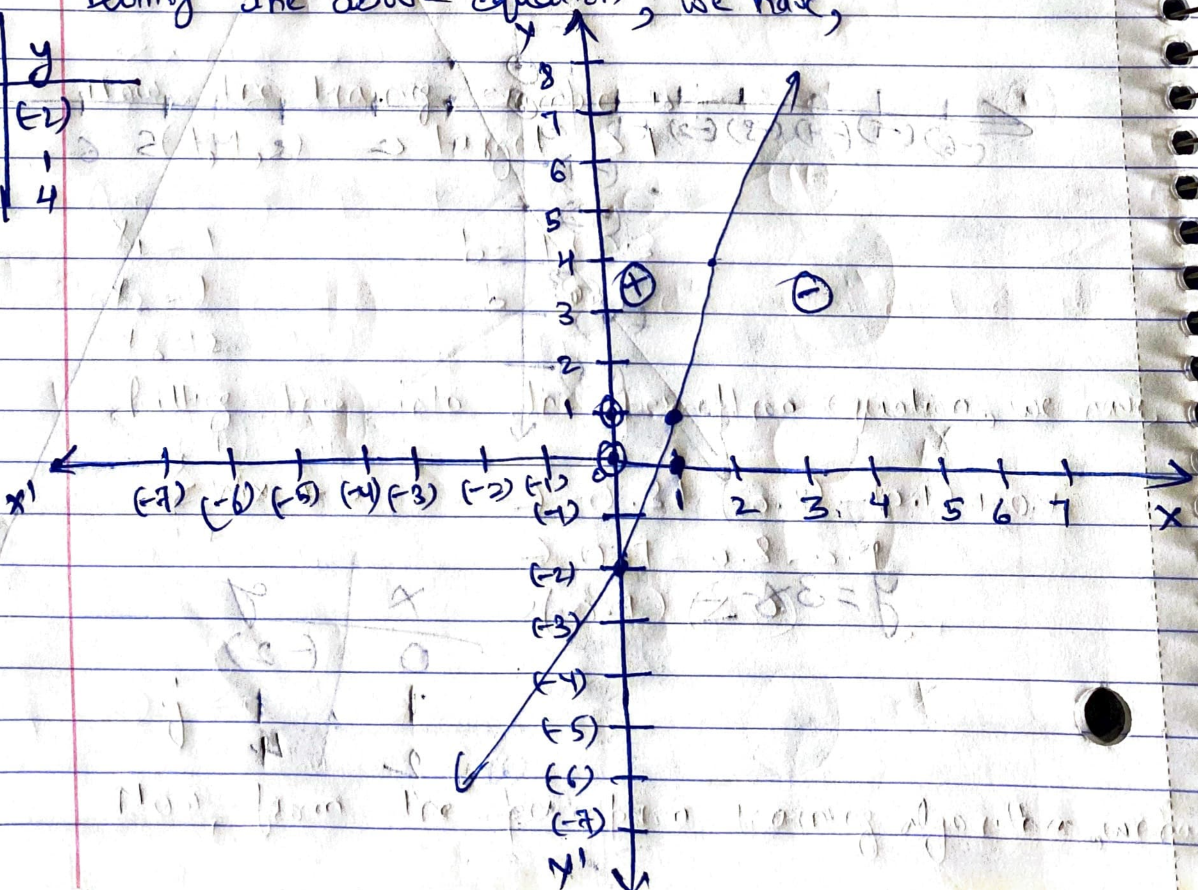
$$0.25x_2 = 0.75x_1 - 0.5$$

$$x_2 = \frac{\overset{3}{\cancel{0.75}}x_1}{\cancel{0.25}} - \frac{\cancel{0.50}^{\,2}}{\cancel{0.25}}$$

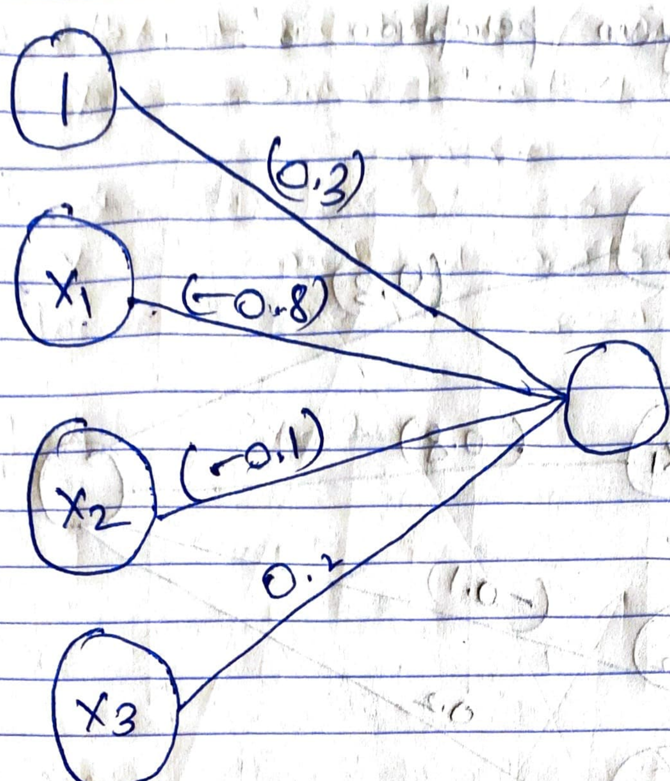
$$x_2 = 3x_1 - 2$$

Plotting the above equation, we have,

x	y
0	(-2)
1	1
2	4



(c)



$x_1 = (-1)$

$x_2 = 1$

$x_3 = (-3)$

Using the perceptron algorithm, we have,

$\sum_{i=1}^{n} w_i x_i + b \Rightarrow$

$\Rightarrow w_1x_1 + w_2x_2 + w_3x_3 + w_0x_0$

$\Rightarrow 0.3 + 0.8 + (-0.1) + (-3) \times 0.2$

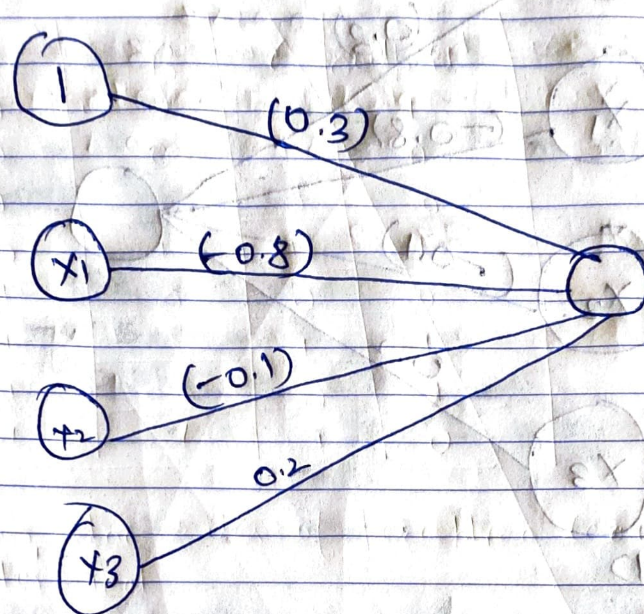
$\Rightarrow 0.3 + 0.8 - 0.1 - 0.6$

$\Rightarrow 1.1 - 0.7$

$\Rightarrow 0.4 \Rightarrow y > 0$

$\Rightarrow$ class 1 (by) $y = \sum_{i=0}^{n} x_i w_i + b \begin{cases} = 1, & \text{if } > 0 \\ = 0, & \text{if } \le 0 \end{cases}$

(d) The given perceptron is :-



Now, The training example is :-

(1, 1, 2) $\rightarrow$ target $\rightarrow$ 1

$x_1 = 1$ $\quad$ $b = 1 \times 0.3$

$x_2 = 1$ $\quad$ $b = 0.3$

$x_3 = 2$

Putting this into the perceptron equation, we have,

$$\sum_{i=1}^{n} w_i x_i + b \Rightarrow 0.3 + (-0.8) - (0.1) + 0.4$$

$$\Rightarrow 0.7 - 0.8 - 0.9$$

$$\Rightarrow (-0.9) \quad (<0)$$

$\Rightarrow \hat{y} < 0$

Now, from the perceptron learning algorithm, we have

$$\sum_{i=1}^{n} w_i x_i + b = 1 \begin{cases} \text{if } wx+b > 0 \\ = 0 \quad \text{if } wx+b \leq 0 \end{cases}$$

Now the target was 1 but we got 0.
⇒ weights must be corrected & another iteration will be required.

→ Now, The training example second is :-
$(1, -2, -2) \longrightarrow$ target $\longrightarrow 0$

⇒ $x_1 = 1$ $\quad b = 1 \times 0.3$
$x_2 = (-2)$ $\quad b = 0.3$
$x_3 = (-2)$

Putting these into the perceptron eqn, we have,

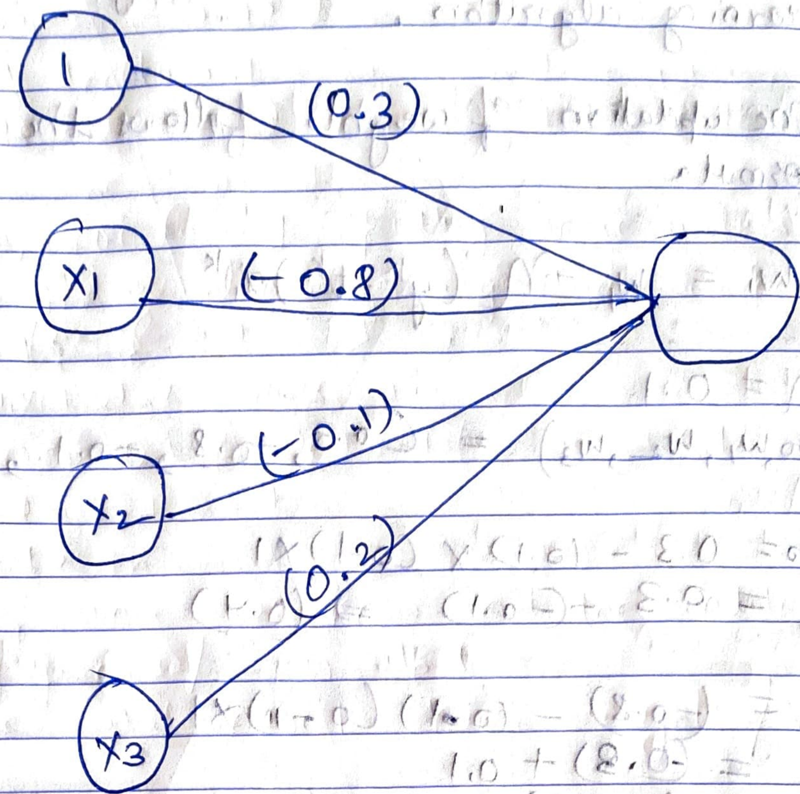
$$\sum_{i=1} w_i x_i + b \Rightarrow 0.3 + (-0.8) + 0.12 - 0.4$$
$$\Rightarrow 0.3 + 0.2 - 0.8 - 0.4$$
$$\Rightarrow 0.5 - 0.12 \Rightarrow (0.7) \Rightarrow y' < 0$$

$$\Rightarrow \sum_{i=1}^{n} w_i x_i + b = 0 \quad \text{for target } 0.$$

(e) Now, as we can see from the above question, we can see that for first training set, the perceptrons wrongly predicts and for the second training set, the perceptron correctly predicts the target.
Therefore, we can say that the accuracy is 50% as

$$\text{Correct \%} = \frac{1}{2} \times 100 \Rightarrow 50\%$$

(f) The perceptron given is :-



→ FIRST TRAINING EXAMPLE :→

$X_1 = 1$ $\quad b = 1 \times 0.3$ $\quad$ target = 1

$X_2 = 1$ $\quad b = 0.3$

$X_3 = 2$

Now, applying this in the perceptron equation, we have,

$$\sum_{i=1}^{n} w_i x_i + b \Rightarrow w_1 x_1 + w_2 x_2 + b$$

$$\Rightarrow (-0.8) - (0.1) + 0.4 + 0.3$$

$$\Rightarrow (-0.9) + (0.7)$$

$$\Rightarrow (-0.2) < 0$$

$\Rightarrow y' < 0$ $\quad \Rightarrow$ incorrect.

Now since the target is wrongly predicted, we will have to update the weights using Perceptron Learning algorithm.

The updation of weights follows the following formula

$$w_i = w_i - \eta (y^k - t^k) x_i^k$$

$\eta = 0.1$

$(w_0, w_1, w_2, w_3) = (0.3, -0.8, -0.1, 0.3)$

$w_0 = 0.3 - (0.1) \times (0-1) \times 1$
$= 0.3 - (-0.1) \Rightarrow (0.4)$

$w_1 = (-0.8) - (0.1)(0-1) \times 1$
$= (-0.8) + 0.1$
$= (-0.7)$

$w_2 = (-0.1) - (0.1)(0-1) \times 1$
$= 0$

$w_3 = 0.2 - (0.1)(0-1) \times 2$
$= 0.4$

new weights $\Rightarrow (0.4, -0.7, 0, 0.4)$

SECOND TRAINING EXAMPLE

(ii) Now, for the second case, we will take the new weights as below:-

$x_1 = 1$
$x_2 = (-2)$
$x_3 = (-2)$

$(w_0, w_1, w_2, w_3) = (0.4, (-0.7), 0, 0.4)$

Again, using the formula used in (i) part,

$y = (1 \times 0.4) + (1 \times 0.7) + (-2 \times 0) + (-2 \times 0.4)$

$y = 0.4 - 0.7 + 0 - 0.8$
$y = (-1) + (-0.1)$
$y = (-1.1) < 0$

$\Rightarrow y = 0$
$y^x = t^x$

weights after epoch are $\Rightarrow \{0.4, (-0.7), 0, 0.4\}$

(g) Since the new weights after one epoch of training worked for the second training data

$\Rightarrow$ Accuracy $= 100\%$.

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PSU ID:- 923928157

Ans 11) In the perceptron learning algorithm, we have the variables as weights and the perceptron should learn to improve the accuracy by updating the weights in each iteration (epochs).

This updating of the weights will mean that the perceptron is learning with each passing input set in each iteration.

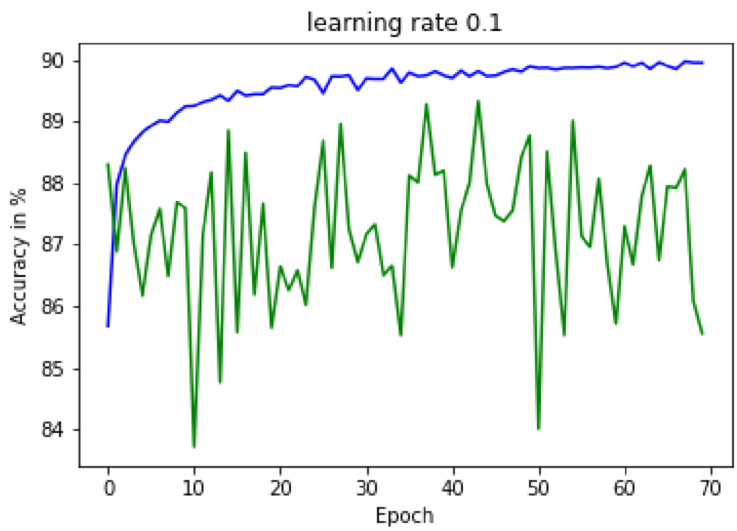
Now, in order to achieve good accuracy, we have in this experiment, used three learning rates as 0.1,0.01,0.001.

Experiment-1: -

In this experiment, I have used learning rate as 0.1. Here, as the initial weights are used with the input data sets, the perceptron starts learning. With each passing epoch, the accuracy starts increasing indicating that the perceptron is making decisions as per the target more accurately.

The more correctly the perceptron is predicting the outcomes, the more the accuracy increases.

In this particular experiment, the accuracy percentage plateaus at 90% for 70 epochs which can be seen as per the graph below.



This Experiment also gives us a confusion matrix which is plotted below

```
CONFUSION MATRIX OF TRAIN SET : learning rate 0.1

[[5689    3   31   24   11   60   41   10   32   22]
 [   1 6458   55   23   23   23   14   19  100   26]
 [  36   59 5304  168   35   52   88   67  134   11]
 [  16   36  148 5321    9  242   11   57  216   85]
 [  12    5   57   10 5311   50   50   59   38  253]
 [  71   36   41  261   20 4588   77   11  247   64]
 [  34    7   76   22   55   92 5580    3   44    3]
 [   9   26   62   63   48   18    3 5714   26  298]
 [  43  105  161  177   60  236   51   22 4905   87]
 [  12    7   23   62  270   60    3  303  109 5100]]
CONFUSION MATRIX OF TEST SET : learning rate 0.1

[[ 961    0   15    4    4   12   18    3   14   10]
 [   0 1122   38    1    3    3    4   11   26    3]
 [   1    2  736    8    3    0    7   19    7    1]
 [   2    3  162  933    7   56    2   29  194   18]
 [   0    0    6    2  778    5    4   10   11    4]
 [   8    3   19   36    2  777   53    2  105    7]
 [   2    3   12    1   16    8  864    0    8    1]
 [   3    0    6   10    1    3    0  859    4    8]
 [   1    1   27    5   12   17    5    4  570    1]
 [   2    1   11   10  156   11    1   91   35  956]]
```

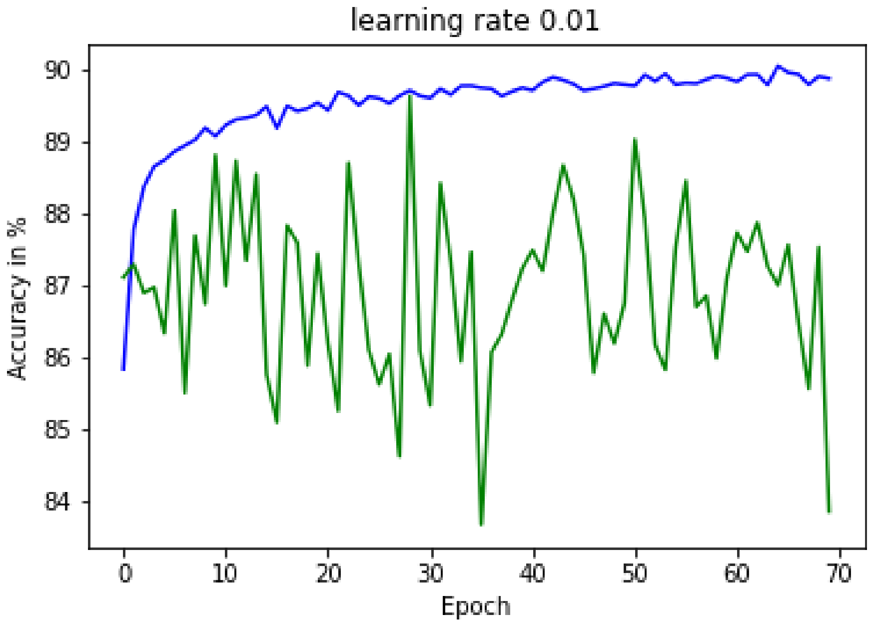
As we know that the accuracy is above 90% but is not near to 100%, the confusion matrix is diagonally dominant but not completely diagonally dependent.

2) Experiment-2

In this experiment, I have used learning rate as 0.01. Here, as the initial weights are used with the input data sets, the perceptron starts learning. With each passing epoch, the accuracy starts increasing indicating that the perceptron is making decisions as per the target more accurately.

The more correctly the perceptron is predicting the outcomes, the more the accuracy increases.

In this particular experiment, the accuracy percentage plateaus at 90% for 70 epochs which can be seen as per the graph below.



The confusion matrix for this experiment is given below: -

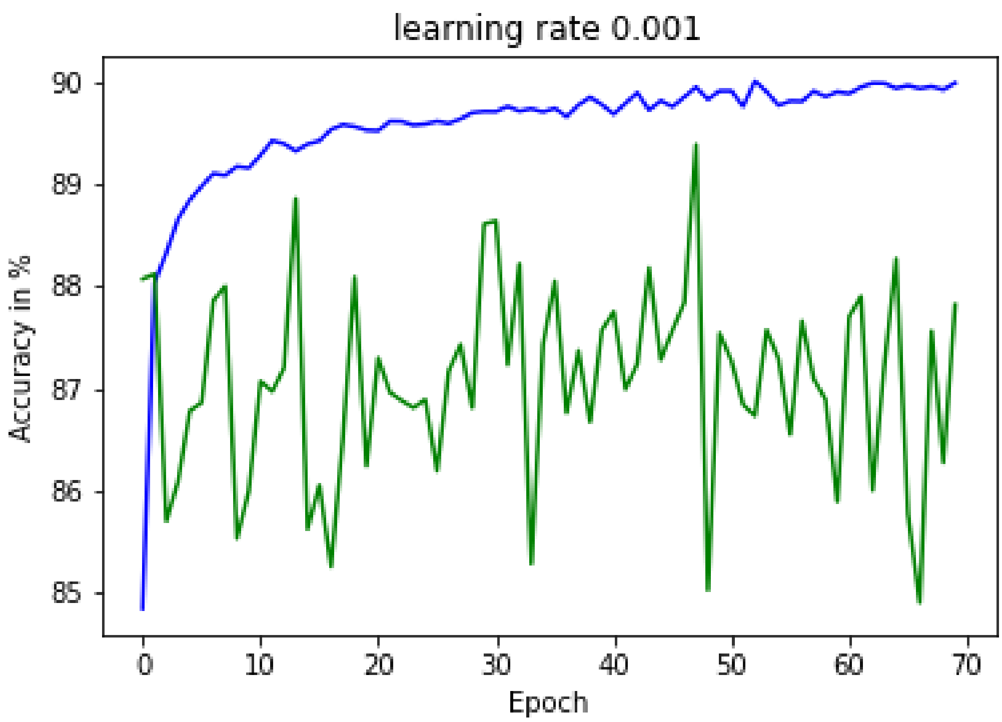
```
CONFUSION MATRIX OF TRAIN SET : learning rate 0.01

[[5689    1   34   26   15   56   35   11   35   22]
 [   1 6446   64   22   20   21   16   20  108   25]
 [  38   74 5284  177   40   53   90   62  133   12]
 [  16   39  132 5315   10  231    6   58  235   86]
 [  12    6   57    9 5332   55   47   53   32  241]
 [  62   28   39  251   16 4585   94   17  258   69]
 [  38    5   82   17   49   96 5579    6   43    2]
 [   8   25   63   58   38   18    3 5723   23  302]
 [  42  110  178  184   57  240   45   21 4882   94]
 [  17    8   25   72  265   66    3  294  102 5096]]
CONFUSION MATRIX OF TEST SET : learning rate 0.01

[[ 963    0   11    3    3   15   24    3    8    9]
 [   0 1116   23    1    1    6    4   12   20    6]
 [   3    1  734    8    5    0   19   18    4    1]
 [   5    9  214  975   11  173    7   65  273   30]
 [   0    1    9    2  789   12    5   10   10    5]
 [   4    2    3    5    0  623   41    2   19    3]
 [   1    3   10    0    8    6  854    0    6    1]
 [   2    1    5    7    2    4    0  780    2    3]
 [   2    2   16    4   15   35    4    6  603    2]
 [   0    0    7    5  148   18    0  132   29  949]]
```

2) Experiment-2

In this experiment, I have used learning rate as 0.001. Here, as the initial weights are used with the input data sets, the perceptron starts learning. With each passing epoch, the accuracy starts increasing indicating that the perceptron is making decisions as per the target more accurately.

The more correctly the perceptron is predicting the outcomes, the more the accuracy increases.

In this particular experiment, the accuracy percentage plateaus at 90% for 70 epochs which can be seen as per the graph below.



The confusion matrix for this experiment is given below:-

```
CONFUSION MATRIX OF TRAIN SET : learning rate 0.001

[[5678    1   34   25   18   53   40    9   35   26]
 [   2 6444   68   25   27   21   12   19  105   22]
 [  33   64 5305  169   35   56   82   60  136   19]
 [  15   36  134 5334   13  234    4   52  220   86]
 [  13    7   59   10 5303   53   54   47   40  257]
 [  75   35   34  264   17 4589   96   14  237   61]
 [  37    5   76   21   54   95 5584    4   44    2]
 [  10   25   64   49   41   16    3 5736   21  299]
 [  43  114  157  163   58  241   39   31 4910   90]
 [  17   11   27   71  276   63    4  293  103 5087]]
CONFUSION MATRIX OF TEST SET : learning rate 0.001

[[ 967    0   23   15    4   28   20    3   20   10]
 [   0 1123   39    1    3    7    5   16   42    8]
 [   2    3  850   27    3    4    8   25   21    1]
 [   1    1   60  889    4   32    1   15   95   10]
 [   1    0   10    4  883   14    6   16   15   18]
 [   2    1    5   36    0  730    9    2   26    5]
 [   2    4   19    5   21   30  907    0   22    1]
 [   3    1    5   11    1    5    1  871    8    9]
 [   2    2   14    9    5   25    1    4  690    5]
 [   0    0    7   13   58   17    0   76   35  942]]
```