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Yaman Parasher

Photonics Integrated Circuits, Sensors & Netoworks
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Assignment No. 2
Comparative study of 3 methods used for the analysis of mode structures in a rectangular waveguide

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1 Introduction

In todays's world rectangular dielectric waveguide is considered as the most commonly used structure in integrated optics, especially in semiconductor diode lasers for a number of applications. I learn from this chapter that the the infinite slab and circular waveguides that we have already studied are not practical for use on a substrate because the slab waveguide has no lateral confinement, and the circular fiber is not compatible with the planar processing technology being used to make planar structures for data applications such as high-speed data backplanes in integrated electronics, waveguide filters, optical multiplexors, and optical switches.

In this assignement, I studied several methods for analyzing the mode structure of rectangular structures.These methods gives an idea about the coupling in different cases,like while design filter how one can modeled evanescent coupling in the adjacent waveguides, is a one such issue that needs to be figure out through proper mode structure analysis.Starting from Maractalli, I inspect Perbutative approach and then all the way to the effective index method in the end. The results obtained from the three methods will then be used to compare their effectiveness.

2 Theorectical Aspects

Before starting to review the three prime analysis methods for the mode structure,here in this section, I would like to provide a glimpse of the whole process on how I get down to these methods.My primary goal here is to determine the mode structure of these kind of different types of rectangular waveguides as shown in Fig. 1. They illustrate the surface waveguide, the buried waveguide, and the ridge waveguide.

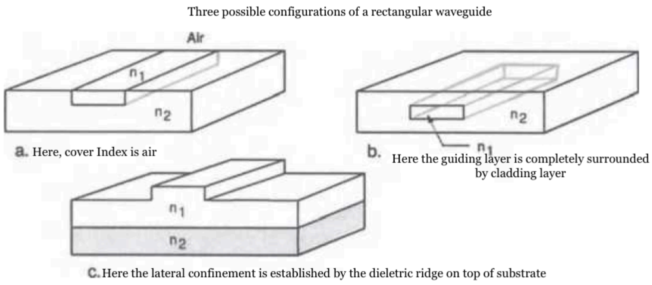

Figura 1: Three possible configurations for rectangular waveguides

Similarly,a cross-section of a generalized embedded waveguide is shown in Fig. 2. Though there are nine distinct dielectric regions in this structure to analyze but the real difficulty lies in analyzing the structure that originates in the four shaded regions.These regions act as the coupling zones for the x and y solutions of the field.

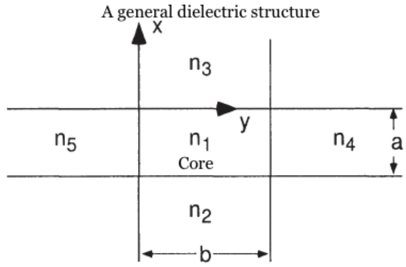

Figura 2: A general dielectric structure of waveguide

Well above cut-off, the electromagnetic mode is tightly confined within the core, and the amount of energy in the comer regions is negligible, so the wave equation can be solved using standard separation of variable While Near 'Cutoff,the mode will have a significant amount of power in the corner regions.The x and y dependent solutions will be then strongly coupled through the boundary conditions in the comer regions, making them mathematically inseparable. Therefore, neglecting the field in the comer regions is the only sensible decision.

In rectangular dielectric waveguides the field is neither purely TE or TM.In the limit of small index differences, the guided optical fields are essentially transverse, and the transverse component of the electric field will be aligned either with the x or y axis of the structure. While going through the analysis part in the chapter, I observed that the rectangular waveguide mode, to first order, is simply the product of two orthogonal spatial modes, one which acts like a TE wave, and the other a TM wave. This structure can therefore be well described as a product of two orthogonal spatial

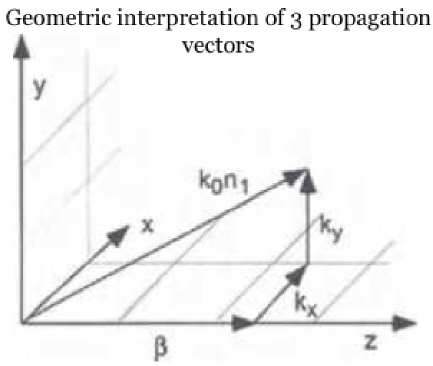

Figura 3: Geometric interpretation of the three propagation vectors in a rectangular waveguide

modes such that the x dependence is found by solving a slab waveguide problem considering no structure in y direction. Similarly, the y dependence is found by solving a waveguide problem considering no structure in x direction. The two solutions are coupled through the selection of the propagation coefficientβ, where both transverse propagation coefficients, κ_x and κ_y are subtracted quadratically from $\kappa_o n1$. The critical cut-off condition will be determined by the smaller of the two dimensions (a or b) of the waveguide. The normalized frequency for a rectangular waveguide is defined as

$$V = k_o \frac{a}{2} \sqrt{n_1^2 - n_2^2} \tag{1}$$

3 Marcatili Analysis

3.1 Introduction

As explained in the previous section, above the cut-off, the electromagnetic mode is tightly confined in the core so the energy in the corner regions is negligible. Keeping this point into consideration, in Marcatili Method the corner regions of the waveguide can be ignored, and the wave equation can be solved using standard separation of variables.

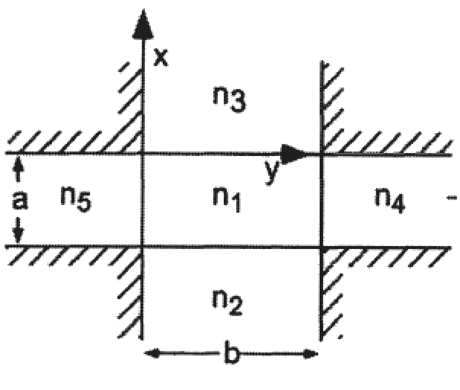

Figura 4: A buried dielectric waveguide structure

Wavelength νm	V, Normalized Frequency	b, Normalized Propagation constant
0.4	2.1505	0.9127
0.5	1.7204	0.8760
0.6	1.4337	0.4470
0.7	1.2289	0.3421
0.8	1.0753	0.2440
0.9	0.9558	0.1531
1.0	0.8602	0.0695
1.1	0.7820	-0.0072
1.2	0.7168	-0.0775
1.3	0.6617	-0.1417
1.4	0.6144	-0.2005

Tabela 1: V & b calculated using Marcatili Method for wavelength range from 0.4-1.4νm

It can be seen in Fig. 4 that ignoring the four corner regions leaves behind five regions. In region 1 i.e. the core, the guided mode solutions should vary sinusoidally along both x and y direction. As the boundary conditions require that the transverse component of the fields be continuous across each interface so the y dependence in region 2 and 3 should have the same sinusoidal structure as in the core but the x component should decay exponentially and similarly for regions 4 and 5 the x dependence should have the same sinusoidal structure as core whereas the y component should decay exponentially. Solving the boundary conditions, we get the following equation of β,b & V.

$$\beta = \kappa_o^2 n_1^2 - \kappa_x^2 - \kappa_y^2 \tag{2}$$

$$b = \frac{\beta^2 - k_o^2 n_2^2}{\kappa_o^2 n_1^2 - \kappa_o^2 n_2^2} \tag{3}$$

$$V = k_o \frac{a}{2}\sqrt{n_1^2 - n_2^2} \tag{4}$$

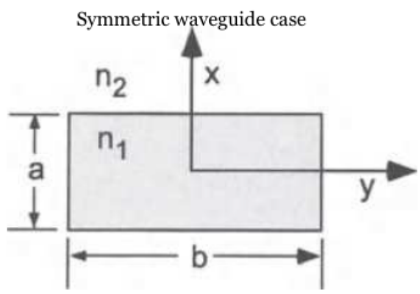

Figura 5: A symmetric waveguide structure

For the analysis, I have taken value of n_1, n_2, a and b as 1.5, 1.499, 5νm and 10νm respectively. The wavelength was varied in the range of 0.5-1.4 νm and the corresponding values of V and b was noted down in table 1.

4 Perturbation Approach

4.1 Introduction

Using Marcatili method we get negative values for b which is not physically possible. The major reason for inaccuracies in Marcatili Method are the four corner regions that we ignored. Perturbation Method give us a better approximation of b for corresponding wavelengths. To find improved solution we can apply perturbative correction to the solution found by Marcatili Method. In this method we take into consideration the four corner regions too and calculate the correction in b

Wavelength νm	V, Normalized Frequency	b, Normalized Propagation constant	Δb	b Corrected
0.4	2.1505	0.9127	-0.0052	0.9075
0.5	1.7204	0.8760	-0.0120	0.8640
0.6	1.4337	0.4470	0.0083	0.4552
0.7	1.2289	0.3421	0.0151	0.3572
0.8	1.0753	0.2440	0.0247	0.2687
0.9	0.9558	0.1531	0.0372	0.1903
1.0	0.8602	0.0695	0.0524	0.1219
1.1	0.7820	-0.0072	0.0702	0.0630
1.2	0.7168	-0.0775	0.0903	0.0128
1.3	0.6617	-0.1417	0.1127	-0.0295
1.4	0.6144	-0.2005	0.1357	-0.0647

Tabela 2: V & corrected b using Perturbation approach for wavelength range from 0.4-1.4νm

due to these regions using equation 5. This is then used to calculate a better approximation of normalized propagation constant.

$$\Delta b = [1+(\frac{\kappa_o^2(n_1^2-n_2^2}{\kappa_x^2}-1)^{1/2}(\frac{\kappa_x a \pm sin(\kappa_x a}{1 \pm cos(\kappa_x a)})]^{-1}[1+(\frac{\kappa_o^2(n_1^2-n_2^2}{\kappa_y^2}-1)^{1/2}(\frac{\kappa_y b \pm sin(\kappa_y b}{1 \pm cos(\kappa_y b)})]^{-1} \quad (5)$$

It can be seen in table 2 that this correction gives us a better approximation of b values.

5 Effective Index Method

5.1 Introduction

Perturbation method gives a little better approximation of normalized propagation constant, but we still get negative values, which as mentioned earlier are not practically possible. In the Effective Index Method, we convert a single 2-D problem into two 1-D problems as shown in Fig 6.

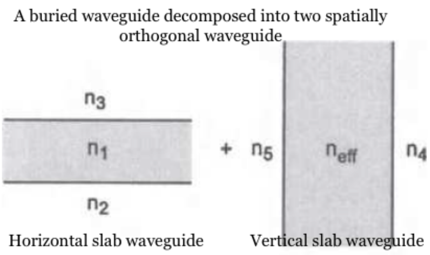

Figura 6: Two spatially orthogonal waveguides from a buried waveguide structure

For this method, we first stretch the waveguide along its thin axis, in our case y axis converting it into a planar slab waveguide. The wave analysis is then done on this slab waveguide and the value of β is found on the wavelengths of interest. Using this value of β we can find the effective refractive index of the slab waveguide with equation 6. The original rectangular waveguide is then stretched along the wider axis, in our case x axis again forming a slab waveguide. The modes for this waveguide are then found using the effective refractive index we found in the previous slab waveguide. To get accurate results, the aspect ratio of the width/height of the waveguide must exceed a factor of three. Thus, the effective index method is not applicable to square waveguides.

$$n_{eff} = \frac{\beta}{\kappa_o} \quad (6)$$

The values of V and b were calculated for the similar wavelengths as those taken in the previous methods, and as we can see in table 3 the effective method gives us the most accurate results.

Wavelength νm	$n_effective$	V, Normalized Frequency	b, Normalized Propagation constant
0.4	1.5	2.1505	1
0.5	1.5	1.7204	1
0.7	1.4995	0.8689	0.6852
0.8	1.4995	0.7603	0.6335
1.0	1.4994	0.544	0.4909
1.1	1.4993	0.4283	0.3865
1.2	1.4993	0.3926	0.3500
1.4	1.4993	0.3365	0.2889

Tabela 3: V, neff & b using Effective Index Method

6 Results & Discussions

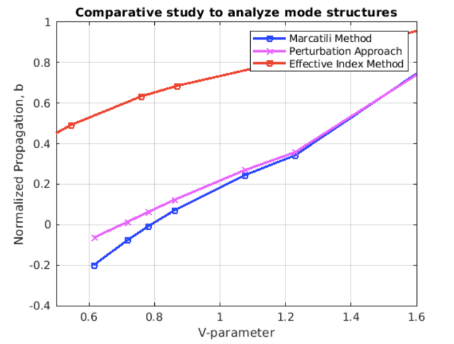

Figura 7: Comparative Ananlysis of the three methods

For a comparative study of the three methods a graph has been plotted which can be shown in Fig. 7 .It is clear from this figure that Effective Index Method gives us the best approximation for lower values of V. I observed that Ignoring the four corner regions surrounding the rectangular waveguide in Marcatili Method did not affect the accuracy of the results as long as we were well above the cut-off. Because in this case the mode is tightly confined in the core and the amount of energy in the comer regions is negligible.However when we get closer to the cut-off , the mode will have a significant amount of power in the corner regions. Therefore,because of this the x and y dependent solutions will be strongly coupled through the boundary conditions in the corner regions, thus making them mathematically inseparable. So,neglecting the corner regions surrounding the waveguide gives incorrect values of b. While on the other hand the Perturbation approach, decrease the amount of inaccuracy in the b values. The curves of Marcatili and Perturbation Method are superimposed on each other which illustrates the difference that the perturbation adds to the solution of Marcatili Method.
The perturbation correction does not bring the cut-off point to V= 0 however it does improve the overall curve. The effective index method is the only technique which predicts that there will be at least one mode. However, the Effective Index Method is only accurate when the aspect ratio of width/height is greater than 3.

7 Conclusion

In this assignment,the analysis of the three methods led me to conlcude that the perturbative correction is good to about V=0.7 for the fundamental mode whereas Marcatili Method gave increasingly inaccurate solution in the region V<2. However,on the other hand, Effective Index Method gives an accurate values for the normalized propagation constant in the region V<0.7. Hence Perturbation gives most accurate solution but it's very difficult to compute and also for V<0.7, where only the Effective Index method gives the accuracy. Unlike the other two methods, the Effective Index Method is capable to predict at least one mode in the symmetric waveguide structure. However, it has the limitation that it is accurate only if the aspect ratio of width/height is greater than 3.