

# Is The Given Year a Leap Year?

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## 1 Logic

A leap year occurs on any year evenly divisible by 4, but not on a century unless it is divisible by 400.<sup>1</sup> Let  $p(x)$  be “ $x$  is evenly divisible by 4.”,  $q(x)$  be “ $x$  is evenly divisible by 100.”, and  $r(x)$  be “ $x$  is evenly divisible by 400.”.

So a *year* is a leap year if and only if  $p(\text{year}) \wedge (\neg r(\text{year}) \rightarrow \neg q(\text{year}))$ . This evaluates to  $p(\text{year}) \wedge (r(\text{year}) \vee \neg q(\text{year}))$ . By applying the distributive law for propositions we obtain  $(p(\text{year}) \wedge r(\text{year})) \vee (p(\text{year}) \wedge \neg q(\text{year}))$ .

Although it is obvious, but here we show  $p(x) \wedge r(x) \leftrightarrow r(x)$  is a tautology, or in other words,  $p(x) \wedge r(x)$  is equivalent to  $r(x)$ . We rewrite  $p(x) \wedge r(x) \leftrightarrow r(x)$  as  $(p(x) \wedge r(x) \rightarrow r(x)) \wedge (r(x) \rightarrow p(x) \wedge r(x))$ .  $p(x) \wedge r(x) \rightarrow r(x)$  is a tautology, so by identity law the statement will be equivalent to  $r(x) \rightarrow p(x) \wedge r(x)$ .  $p(x) \wedge r(x) \rightarrow p(x)$  is also a tautology, and so by identity law,  $r(x) \rightarrow p(x) \wedge r(x)$  is equivalent to  $(r(x) \rightarrow p(x) \wedge r(x)) \wedge (p(x) \wedge r(x) \rightarrow p(x))$ , and by applying the transitive law, the statement evaluates to  $r(x) \rightarrow p(x)$  which is a tautology, because if a number is evenly divisible by 400, it is also evenly divisible by 4. So we conclude that  $p(x) \wedge r(x)$  is equivalent to  $r(x)$ . Therefore a *year* is a leap year if and only if  $r(\text{year}) \vee (p(\text{year}) \wedge \neg q(\text{year}))$ .

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<sup>1</sup><https://projecteuler.net/problem=19>