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# Search, Part 4

## Lecture 7

Chapter 3, Sections 3.5-3.6

Jim Rehg

College of Computing  
Georgia Tech

January 29, 2016

# Where Was Prof. Rehg last Wed?

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NSF I-Corps Kick-Off Meeting (Houston, TX)



Program to explore commercialization opportunities for research technology

Participating with Agata Rozga (Res. Scientist), Yin Li (PhD student), and Ernesto Escobar (Mentor)

\$50K to conduct interviews and pursue customer discovery

# Where Was Prof. Rehg on Mon and Wed?

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Dagstuhl Seminar 16042

Eyewear Computing – Augmenting the Human with Head-Mounted Wearable Assistants



Well-known international forum for exchange of research ideas

I was co-organizer with researchers from Germany, Japan, and Google (CA)

Focus on “wearable cameras meet HCI”

# A\* Search – Additional Details

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Identify the relationship between

A\* Search

Greedy Best-First Search

Uniform Cost Search

# A\* Search – Additional Details

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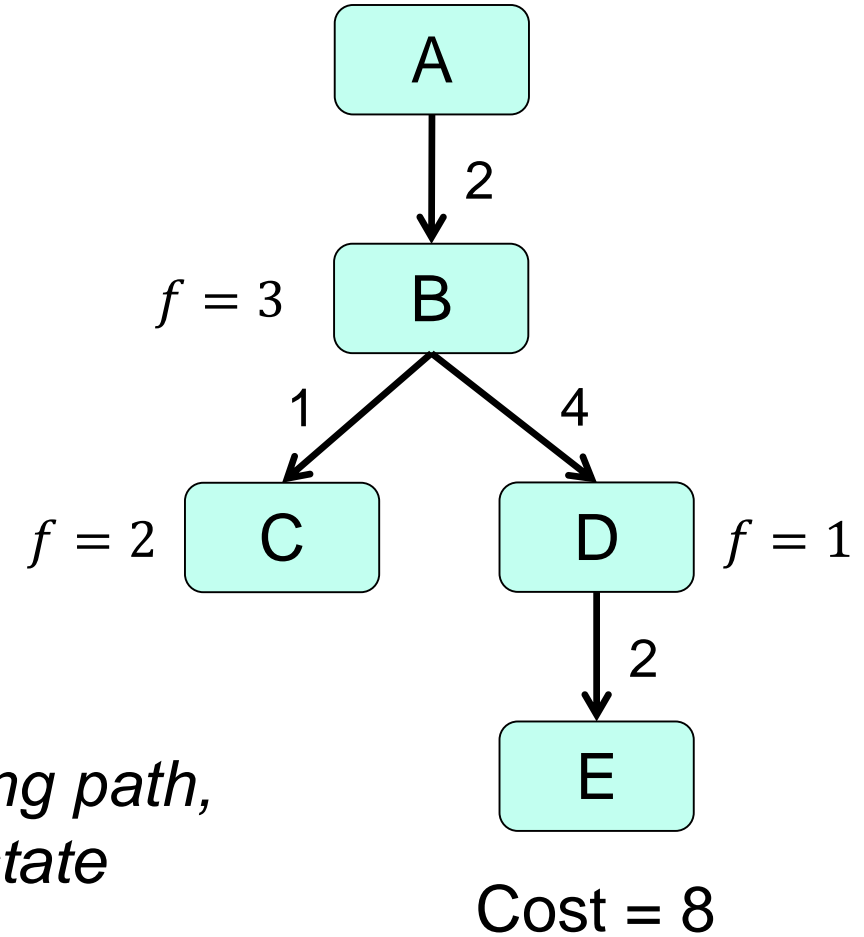
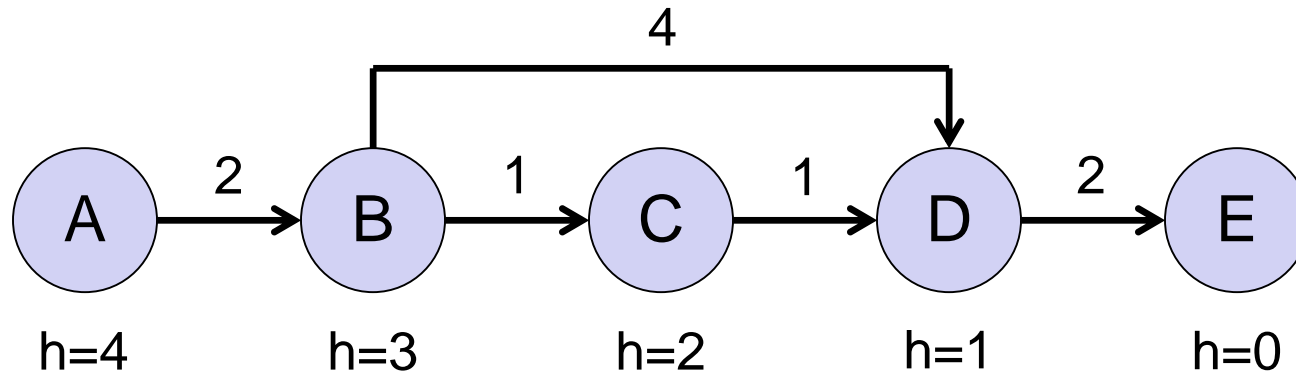
Identify the relationship between

**A\* Search** – Optimal heuristic search

**Greedy Best-First Search** – Greedy search with “wrong” cost

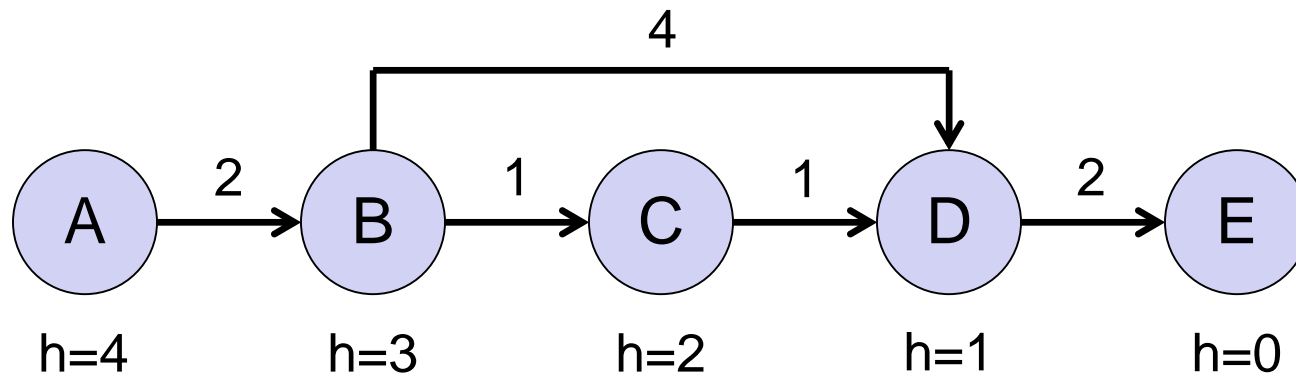
**Uniform Cost Search** – Basic cost-based search method, basis for A\* when using optimal heuristic

# Problem with Best-First Greedy Search

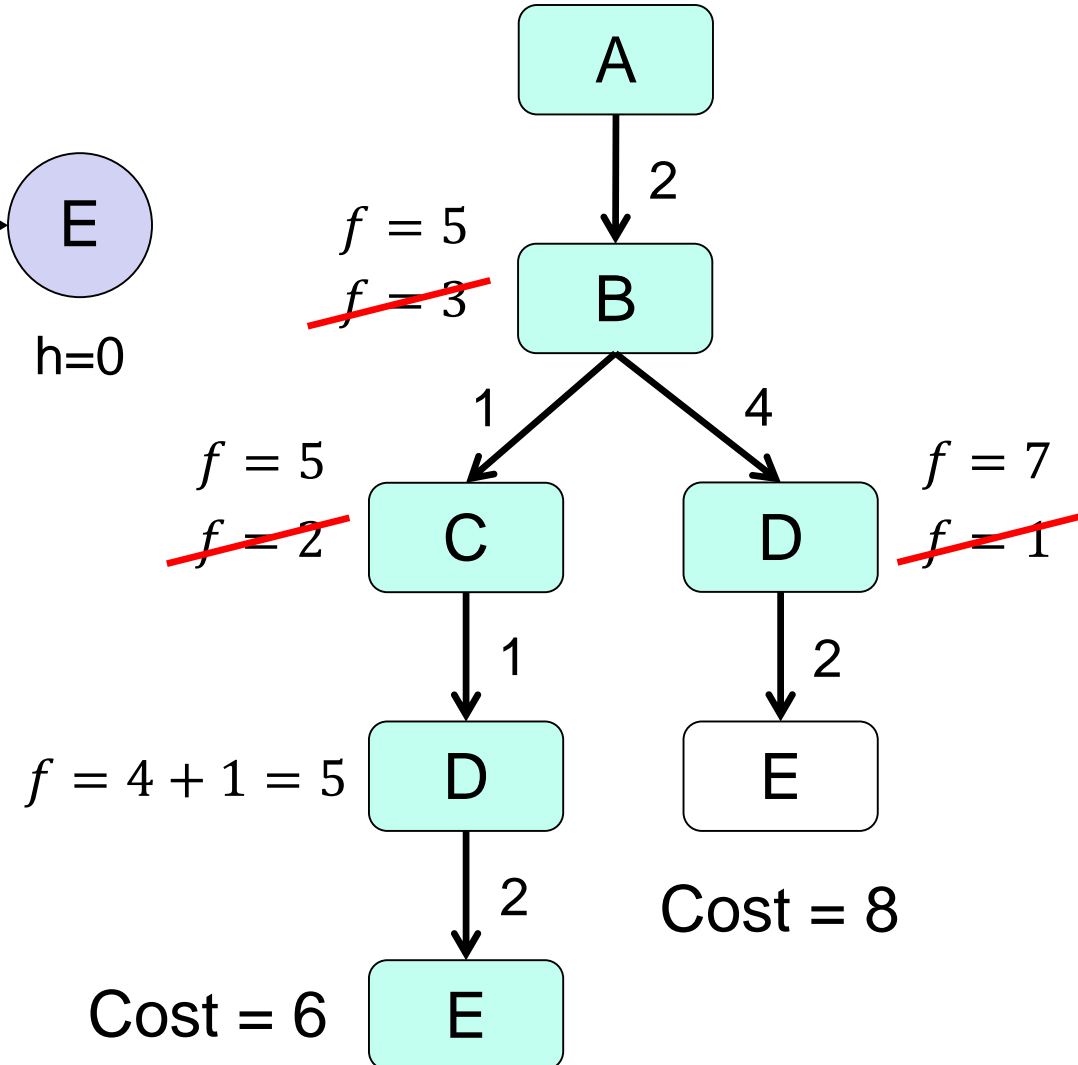


*Best-First doesn't keep track of cost along path,  
and takes a high cost arc to a low cost state*

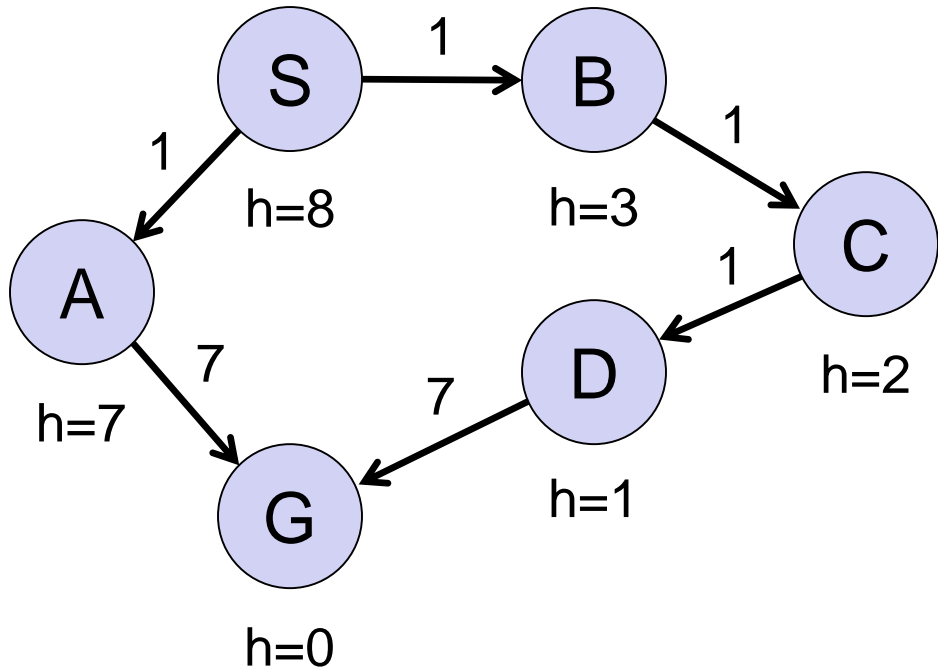
# Problem with Best-First Greedy Search



*A\* uses path cost + heuristic,  
correctly chooses the lower  
cost path*



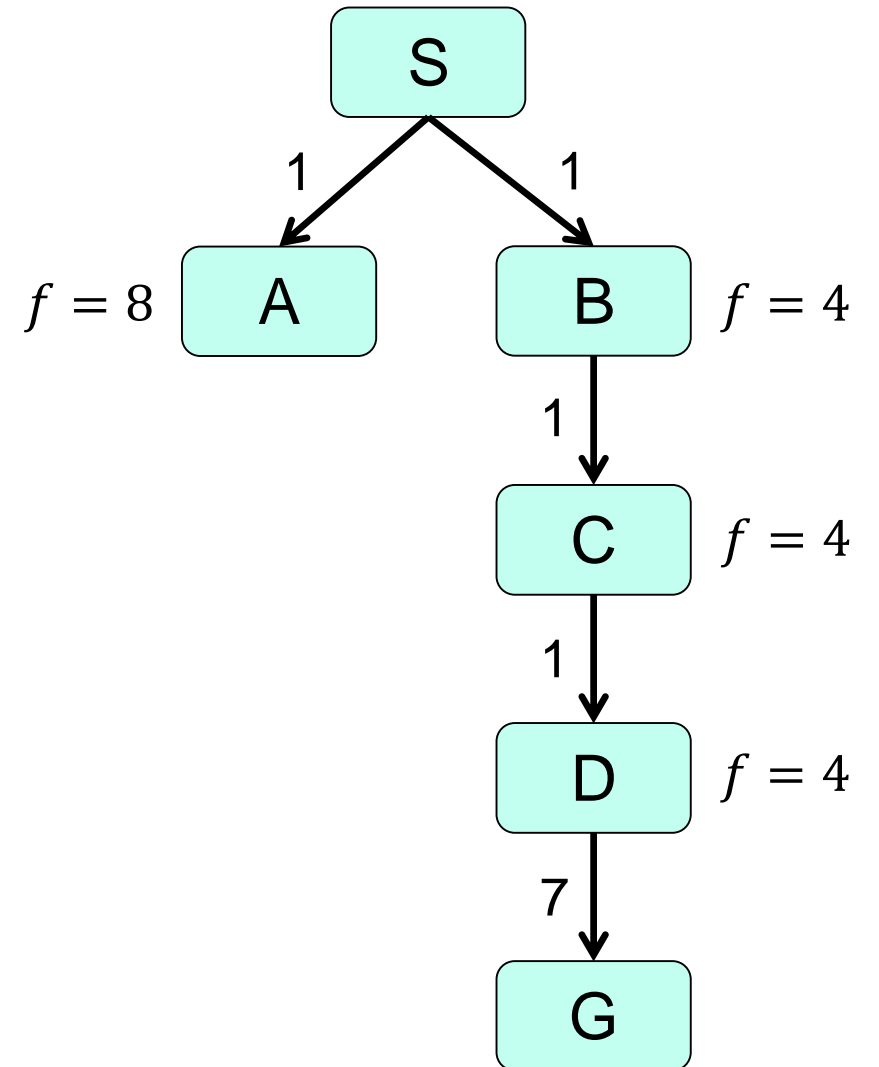
# When Should A\* Terminate?



## Frontier (Queue)

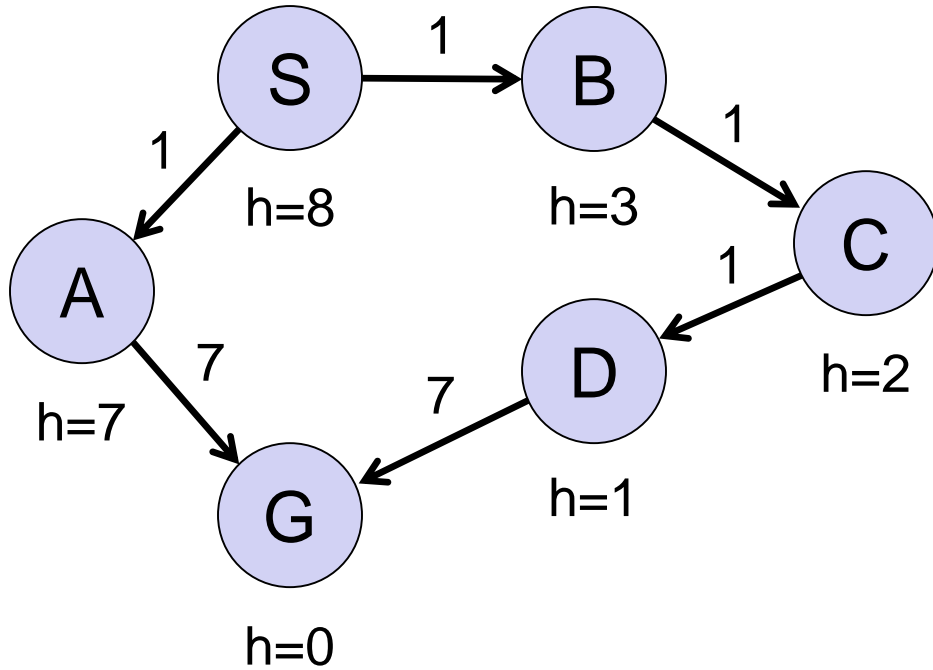
- 1) S(8)
- 2) B(4) A(8)
- 3) C(4) A(8)
- 4) D(4) A(8)

Expanding D gives goal node,  
but terminating at this point  
gives suboptimal solution





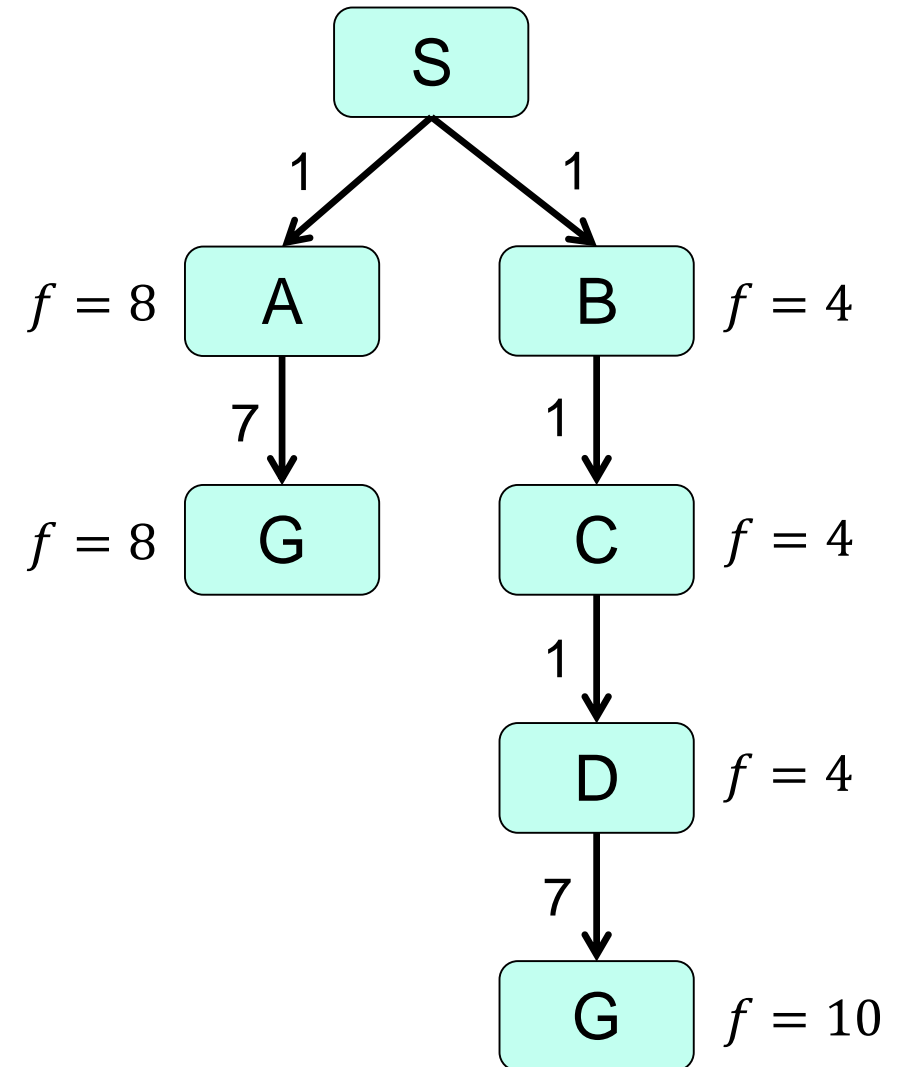
# When Should A\* Terminate?



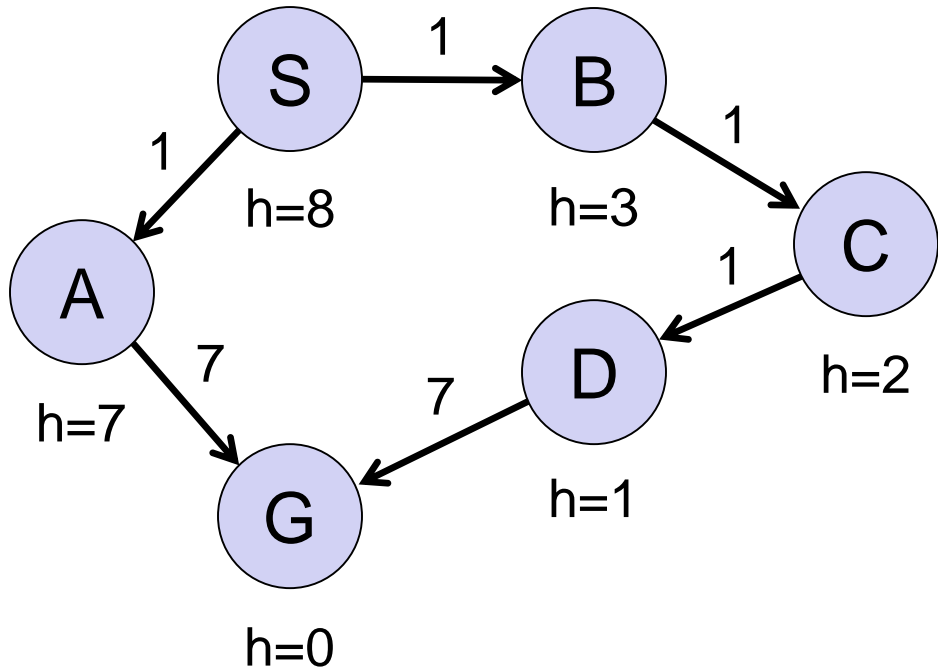
Push goal node on queue,  
expanding A leads to  
optimal path to goal

## Frontier (Queue)

- 1) S(8)
- 2) B(4) A(8)
- 3) C(4) A(8)
- 4) D(4) A(8)
- 5) A(8) G(10)
- 6) G(8) ~~G(10)~~



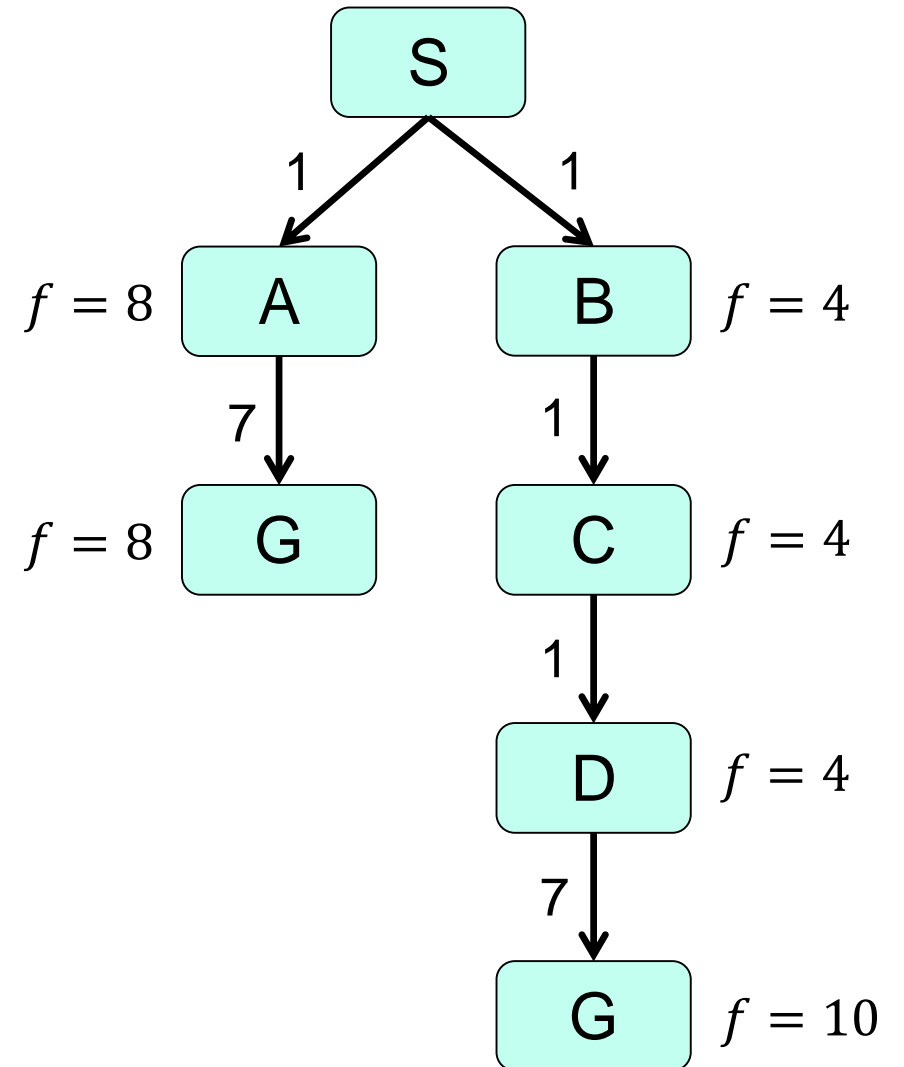
# When Should A\* Terminate?



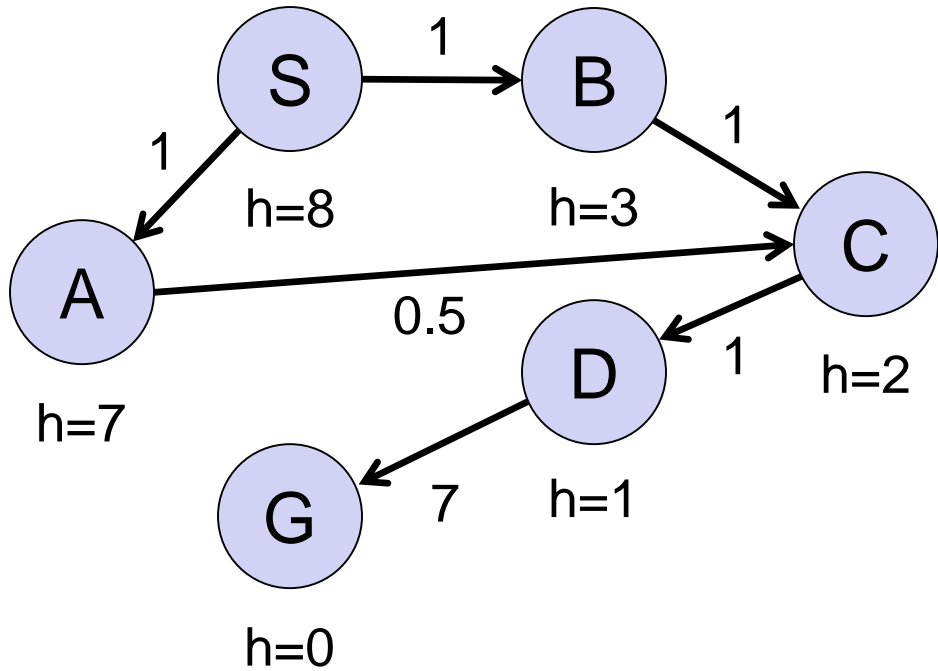
*Answer:* Terminate  
when goal node is  
**popped** from queue

## Frontier (Queue)

- 1) S(8)
- 2) B(4) A(8)
- 3) C(4) A(8)
- 4) D(4) A(8)
- 5) A(8) G(10)
- 6) **G(8)**

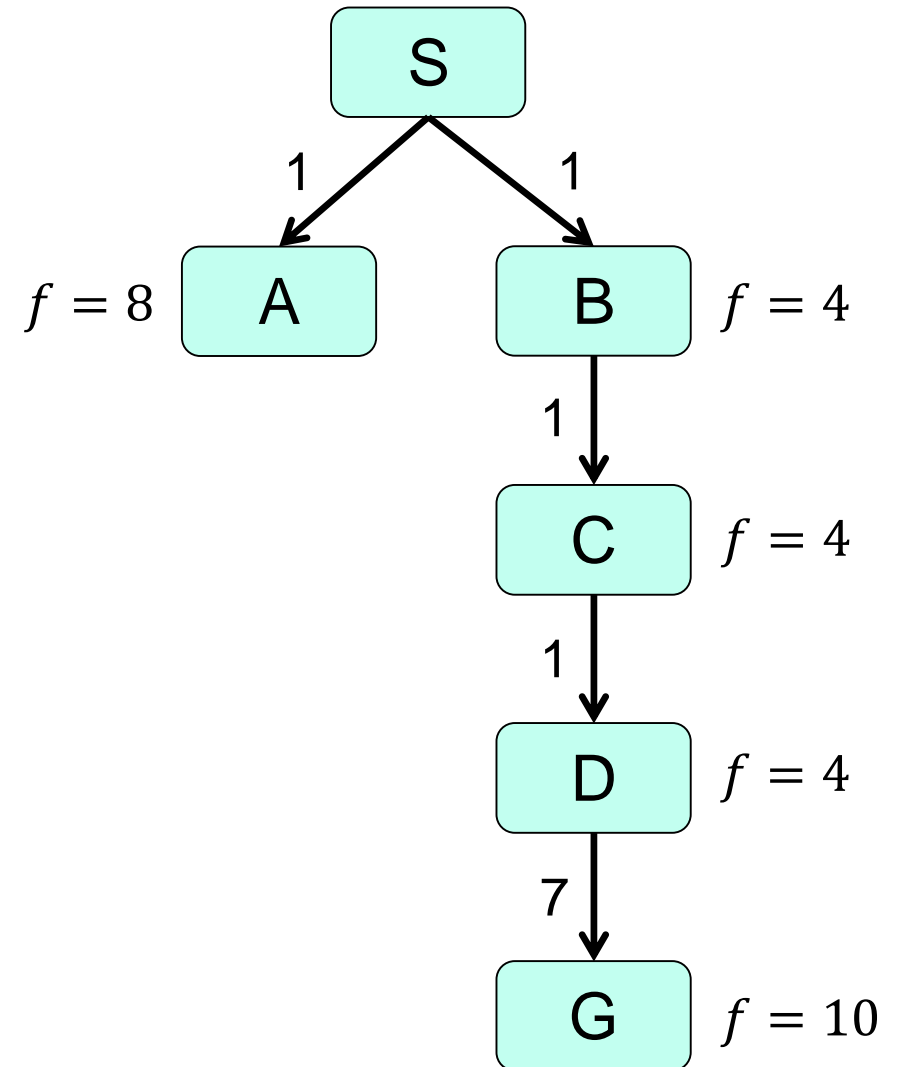


# When Does A\* Revisit States?

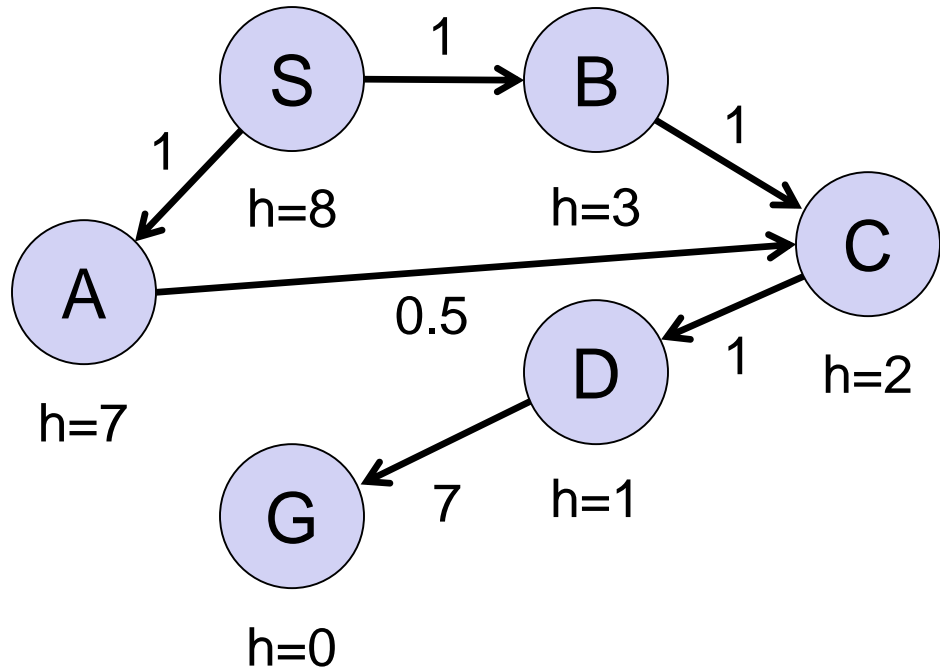


## Frontier (Queue)

- 1) S(8)
- 2) B(4) A(8)
- 3) C(4) A(8)
- 4) D(4) A(8)
- 5) A(8) G(10)



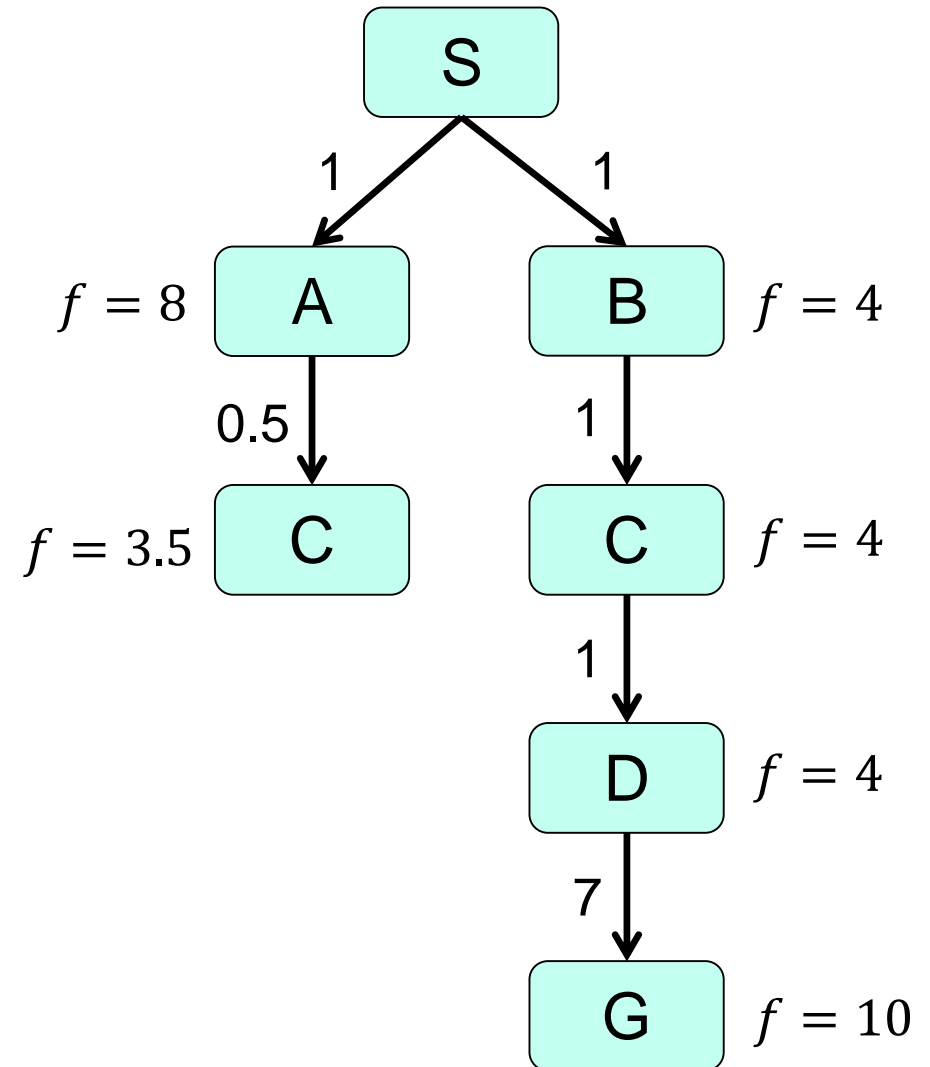
# When Does A\* Revisit States?



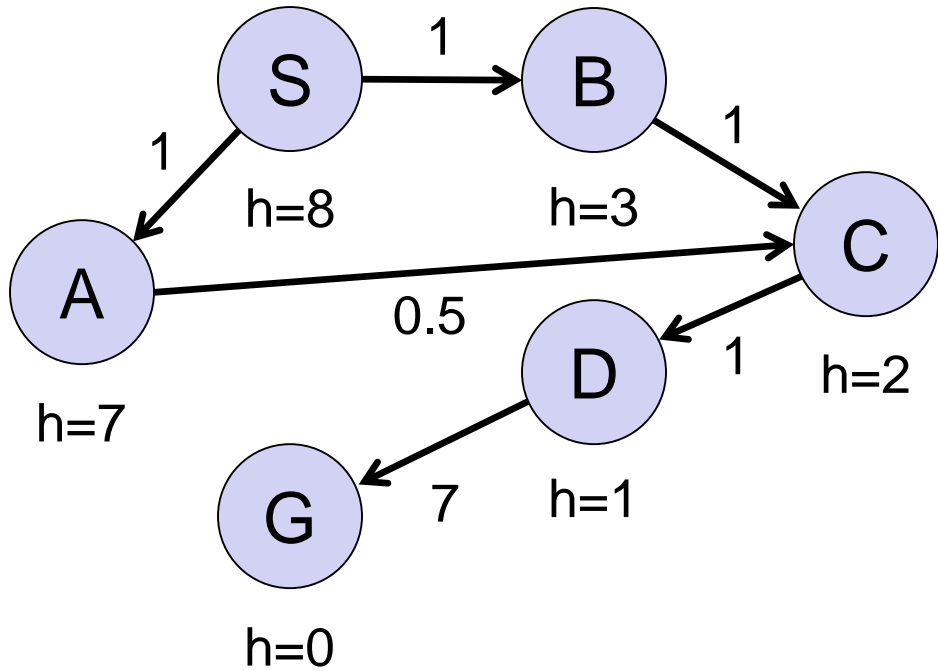
C visited on path S-B-C,  
added to visited list, and  
then re-visited on S-A-C  
at lower cost

## Frontier (Queue)

- 1) S(8)
- 2) B(4) A(8)
- 3) C(4) A(8)
- 4) D(4) A(8)
- 5) A(8) G(10)
- 6) C(3.5) G(10)



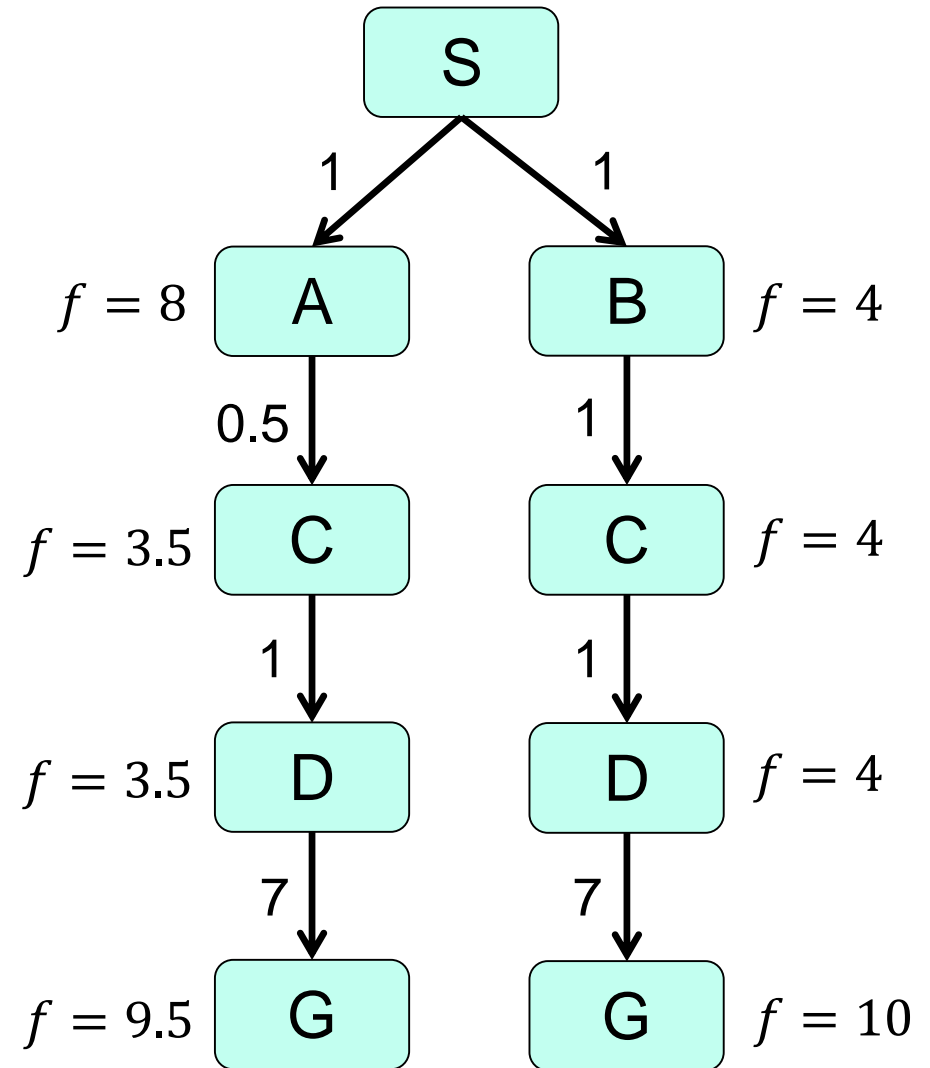
# When Does A\* Revisit States?



States on closed list can be revisited at a lower cost

## Frontier (Queue)

- 1) S(8)
- 2) B(4) A(8)
- 3) C(4) A(8)
- 4) D(4) A(8)
- 5) A(8) G(10)
- 6) C(3.5) G(10)
- 7) D(3.5) G(10)
- 8) G(9.5)



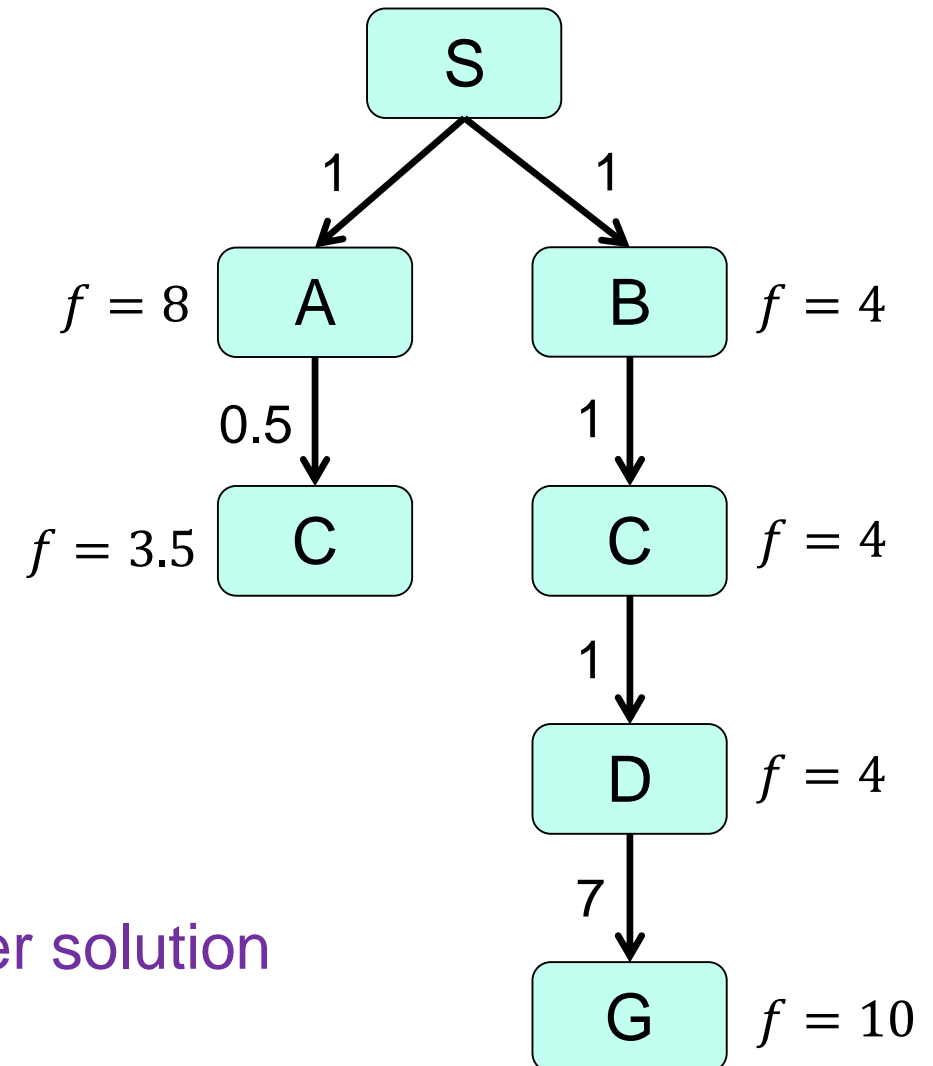
# Visited List

## Visited (Closed List)

- 1) (S, 8, null)
- 2) (B, 4, S)
- 3) (C, 4, B)
- 4) (D, 4, C)
- 5) (A, 8, S)

## Frontier (Queue)

- 1) S(8)
- 2) B(4) A(8)
- 3) C(4) A(8)
- 4) D(4) A(8)
- 5) A(8) G(10)
- 6) C(3.5) G(10)



Maintain visited list for expanded nodes:  
(state, cost, *backpointer*)

Will allow us to recover solution  
once we reach goal

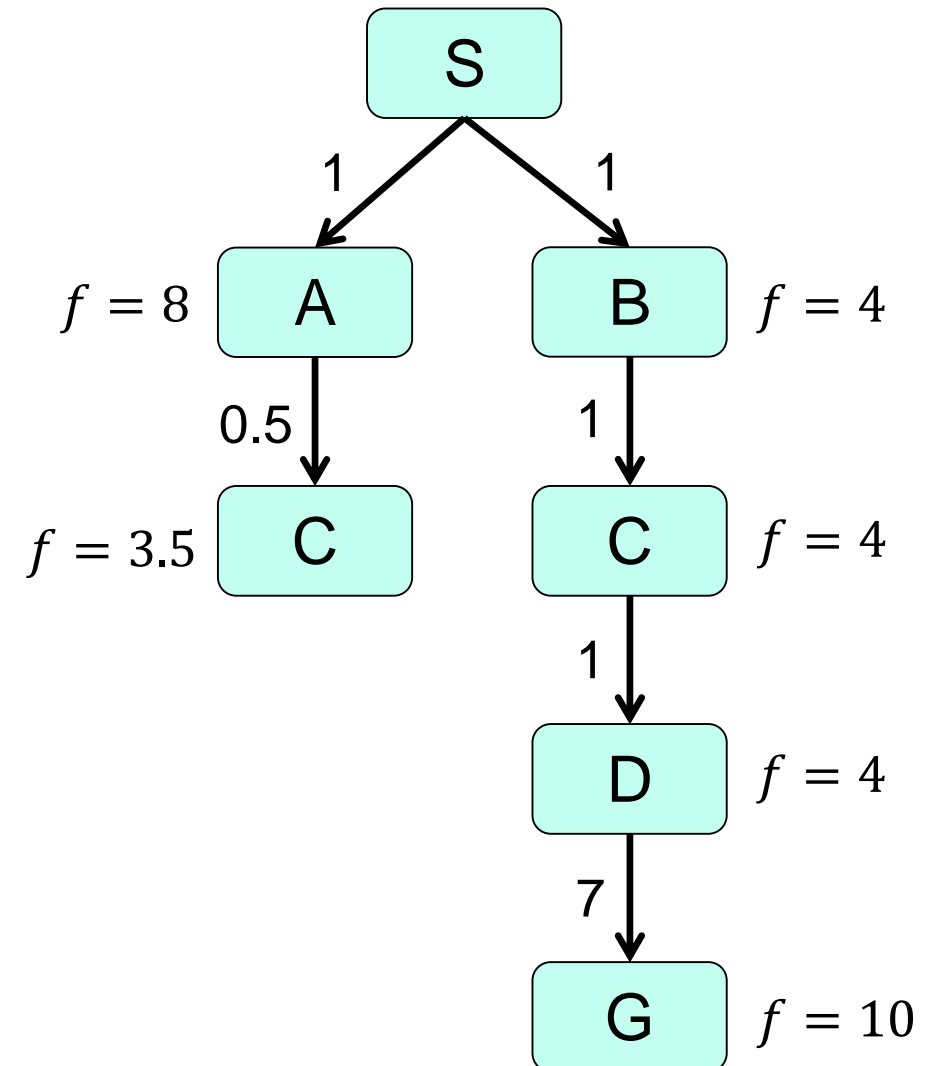
# Visited List

## Visited (Closed List)

- 1) (S, 8, null)
- 2) (B, 4, S)
- 3) ~~(C, 4, B)~~
- 4) (D, 4, C)
- 5) (A, 8, S)
- 6) (C, 3.5, A)

## Frontier (Queue)

- 1) S(8)
- 2) B(4) A(8)
- 3) C(4) A(8)
- 4) D(4) A(8)
- 5) A(8) G(10)
- 6) C(3.5) G(10)



States that are revisited with lower cost  
replace previously-added nodes

Maintain visited list, add expanded nodes:  
(state, cost, *backpointer*)

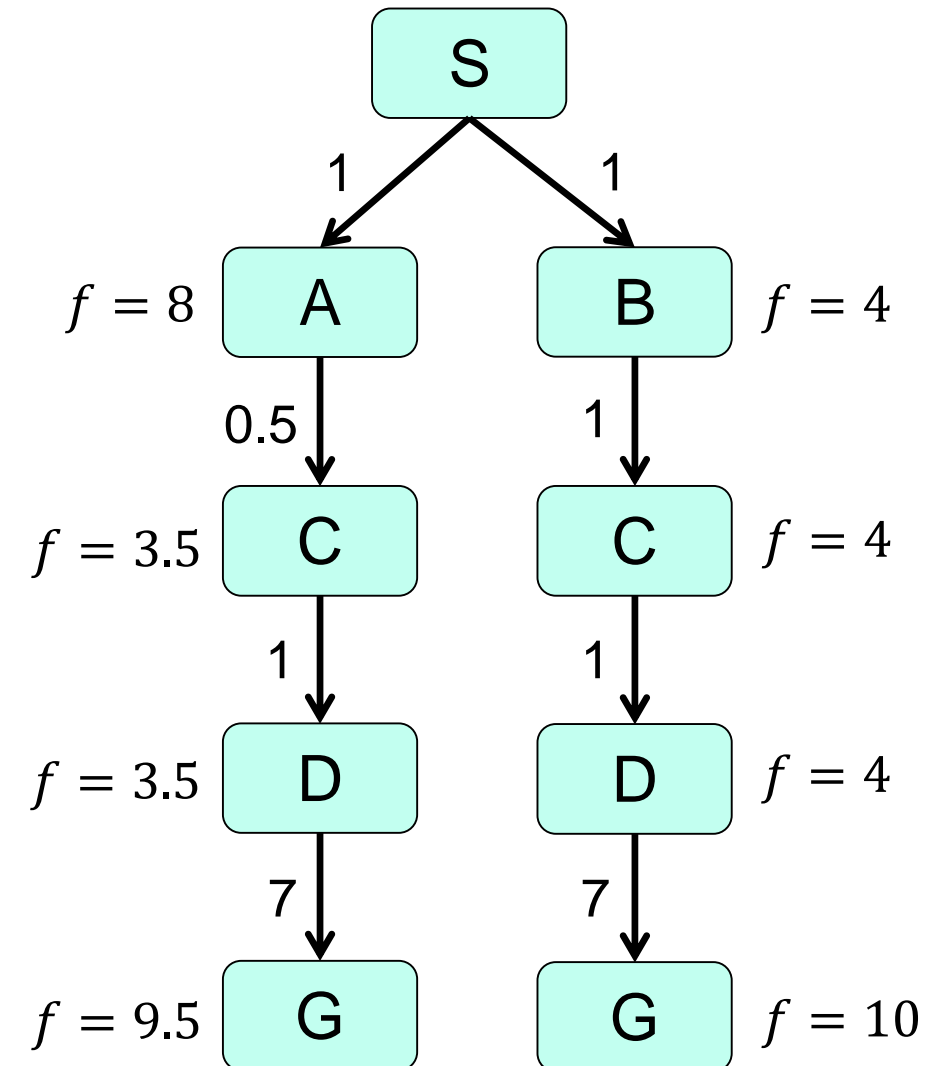
# Visited List

## Visited (Closed List)

- 1) (S, 8, null)
- 2) (B, 4, S)
- 3) ~~(C, 4, B)~~
- 4) ~~(D, 4, C)~~
- 5) (A, 8, S)
- 6) (C, 3.5, A)
- 7) (D, 3.5, C)
- 8) (G, 9.5, D)

## Frontier (Queue)

- 1) S(8)
- 2) B(4) A(8)
- 3) C(4) A(8)
- 4) D(4) A(8)
- 5) A(8) G(10)
- 6) C(3.5) G(10)
- 7) D(3.5) G(10)
- 8) G(9.5)



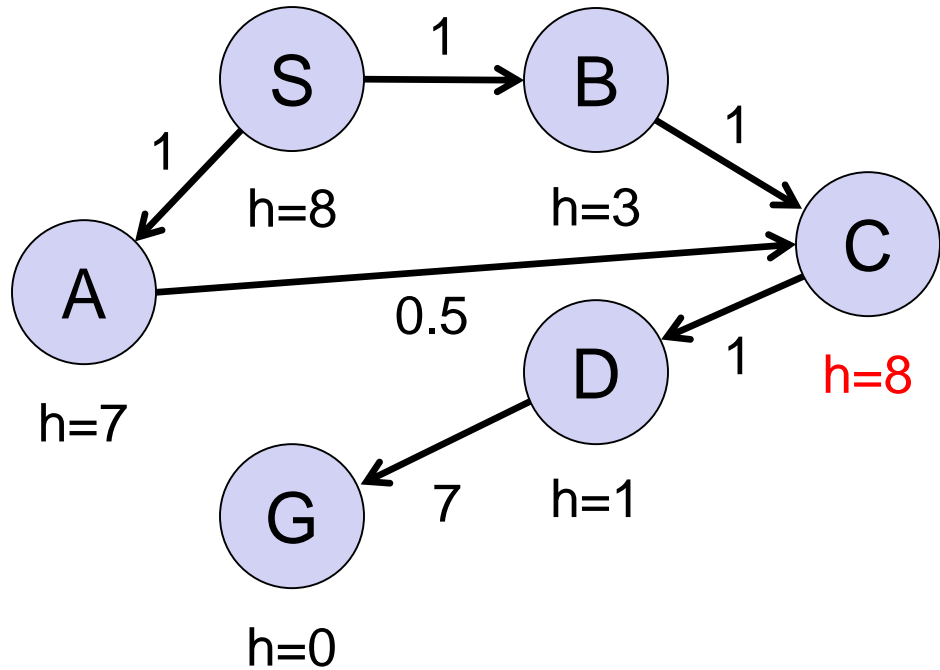
Backchain to retrieve state sequence:

G, D, C, A, S

and reverse it to obtain action plan



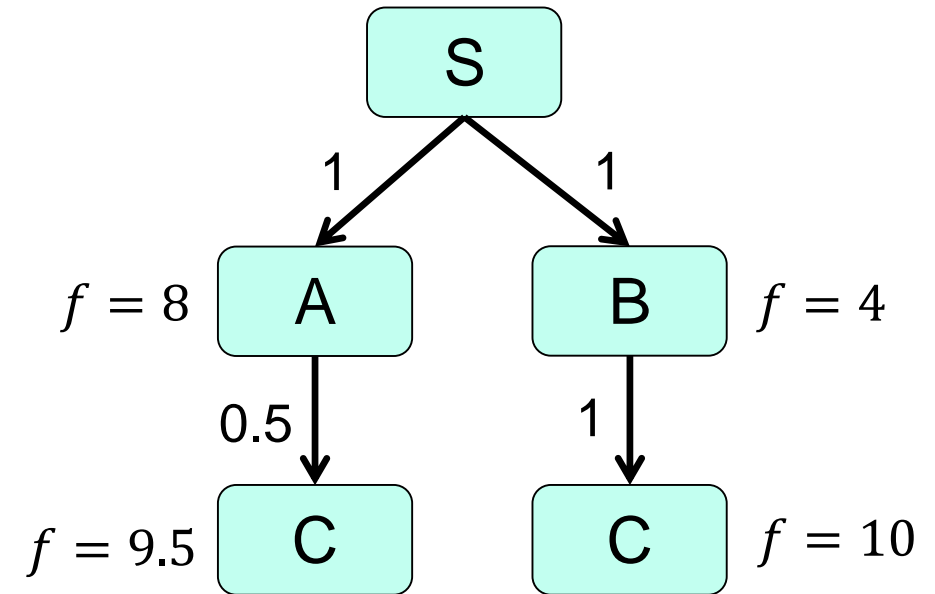
# When Does A\* Revisit States?



## Frontier (Queue)

- 1) S(8)
- 2) B(4) A(8)
- 3) A(8) C(10)
- 4) C(9.5) ~~C(10)~~

S-A-C (9.5) vs. S-B-C(10)



With change in heuristic, state C is promoted with a lower cost after being added to queue initially

*To avoid confusion, you can always write the path explicitly when adding node to queue*

# Pseudocode for A\* Search

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**function** ASTAR-SEARCH(*problem*) **returns** *solution*

Initialize priority queue (*Q*) and visited list (*V*) with starting state

**while** not empty *Q* **do**

pop node *n* with lowest  $f(n)$  from *Q*

**if** *n* is goal node **then return** BACKCHAIN(*V*)

**foreach** node *s* **in** *problem*.SUCCESSORS(*n*) **do**

$f' = g(n) + \text{cost}(n, s) + h(s)$

**if** (*s* not already in *V*) OR  $f(s) > f'$  **then**

Add *s* to *Q* with cost  $f'$

Update *V* with (*s*,  $f'$ , *n*)

**return** *failure*

# 8-Puzzle Example

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Which of the following are admissible heuristics?

- 1)  $h(n)$  = Number of tiles in wrong position in state  $n$
- 2)  $h(n) = 0$
- 3)  $h(n)$  = Sum of Manhattan distances between each tile and its goal location
- 4)  $h(n) = 1$
- 5)  $h(n) = \min(2, h^*(n))$
- 6)  $h(n) = h^*(n)$
- 7)  $h(n) = \max(2, h^*(n))$

*Example State*

4	2	7
6	5	
1	8	3

*Goal State*

1	2	3
4	5	6
7	8	

# 8-Puzzle Example

---

Which of the following are admissible heuristics?

1)  $h(n)$  = Number of tiles in wrong position in state  $n$  YES

2)  $h(n) = 0$  YES

3)  $h(n)$  = Sum of Manhattan distances between each tile and its goal location YES

4)  $h(n) = 1$  NO

5)  $h(n) = \min(2, h^*(n))$  YES

6)  $h(n) = h^*(n)$  YES

7)  $h(n) = \max(2, h^*(n))$  NO

*Example State*

4	2	7
6	5	
1	8	3

*Goal State*

1	2	3
4	5	6
7	8	

# Dominance

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Reminder: Heuristic  $h(n)$  is *admissible* if  $h(n) \leq h^*(n)$

If  $h_1$  and  $h_2$  *admissible* heuristics, and  
 $h_2(n) \geq h_1(n)$  for all  $n$ ,

Then  $h_2$  *dominates*  $h_1$ , and  
 $h_2$  is better for search

Why?

Note:  $h'(n) = \max\{h_1(n), h_2(n)\}$  is admissible and  
dominates  $h_1$  and  $h_2$

# Dominant Heuristic is Better

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In  $A^*$  every node  $n$  with

$f(n) < C^*$  will be expanded

$h(n) < C^* - g(n)$  will be expanded

$h_2(n) > h_1(n)$  for all  $n$ , then

Set of  $n$  for which  $h_1(n) < C^* - g(n)$

Will be *larger* than set of  $n$  for which  $h_2(n) < C^* - g(n)$

*Thus  $h_1$  will expand more nodes*

# Questions?

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