Propositional Logic

Logical Agents continued

CH 7

Slides courtesy of Andrea Thomaz

Overview

* Intro basic Logical Agent Design
* Wumpus World -- example environment
* Concepts of Logic in General
* **Propositional Logic**
* Inferences with ProLog
* First-order Logic
* Inferences with FOL

On the Midterm

Propositional Logic

Syntax, Semantics, Entailment

Propositional Logic: Syntax

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The proposition symbols P_1, P_2 etc are sentences

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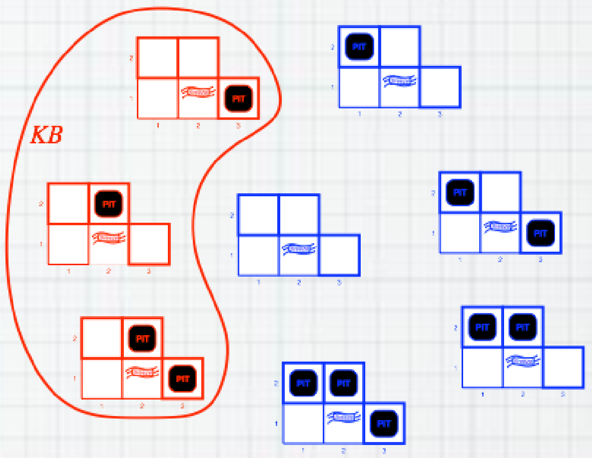
If S_1 and S_2 are sentences, $S_1 \Leftrightarrow S_2$ is a sentence (biconditional)

Propositional Logic: Semantics



Each model specifies true/false for each proposition symbol

E.g. $P_{1,2}$ $P_{2,2}$ $P_{3,1}$

$true$ $true$ $false$

(With these symbols, 8 possible models, can be enumerated automatically.)

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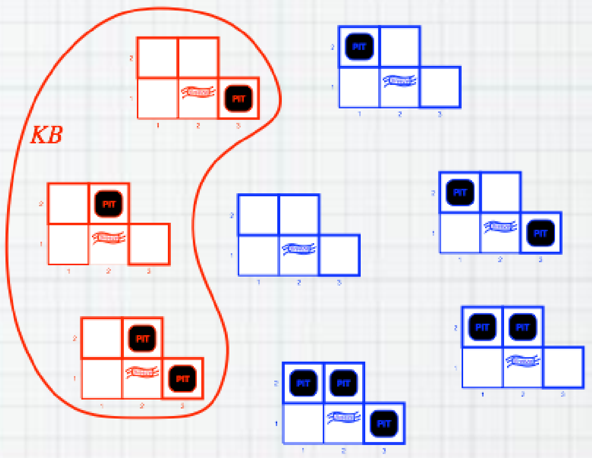

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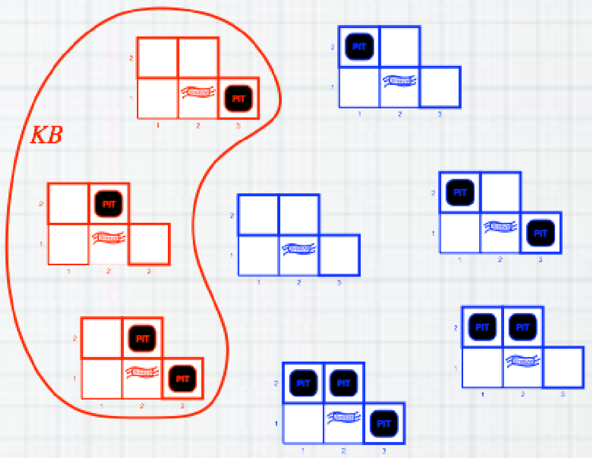

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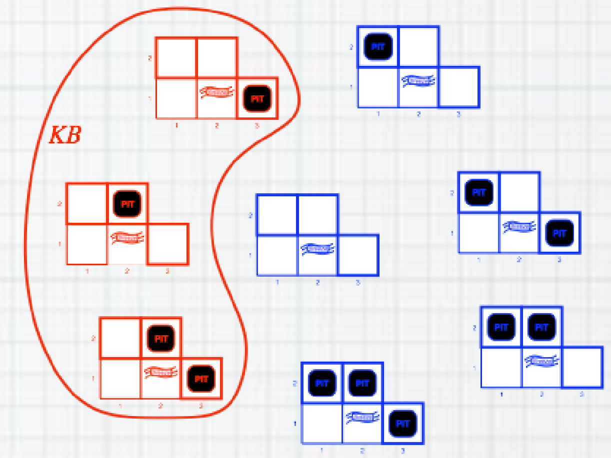

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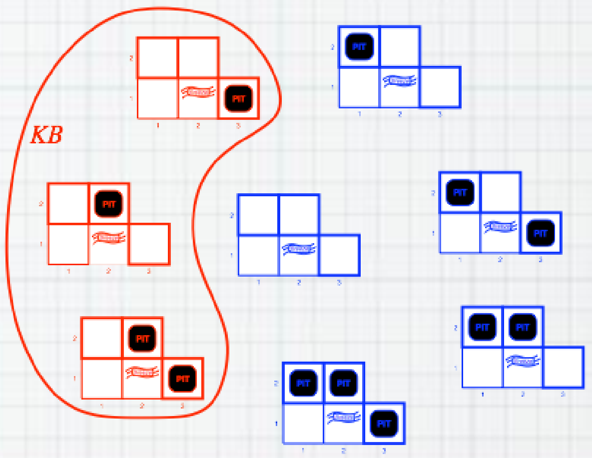

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E.g. $\begin{array}{ccc} P_{1,2} & P_{2,2} & P_{3,1} \\ true & true & false \end{array}$

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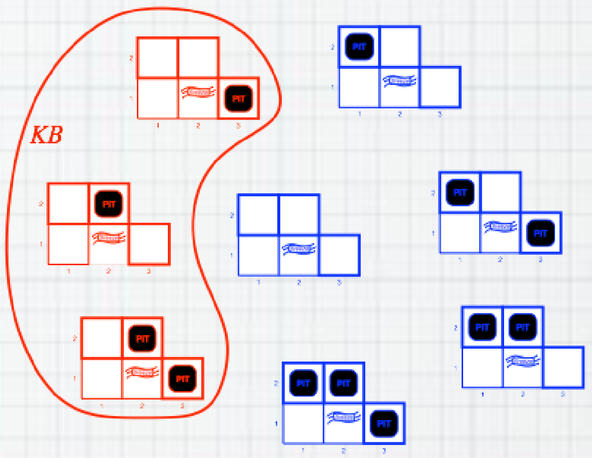

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$S_1 \Leftrightarrow S_2$	is true iff	$S_1 \Rightarrow S_2$	is true **and**	$S_2 \Rightarrow S_1$	is true

Truth Table for Connectives

Model Truth value w.r.t. given Model

P	Q	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \Rightarrow Q$	$P \Leftrightarrow Q$
false	*false*					
false	*true*					
true	*false*					
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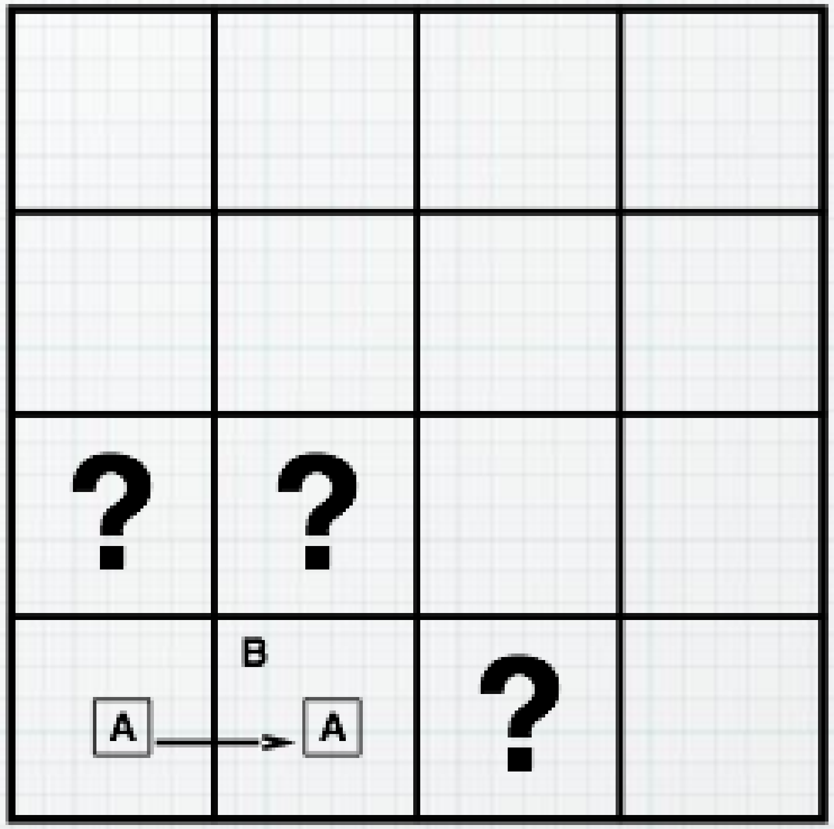
Wumpus Sentences

Let $P_{i,j}$ be true if there is a pit in $[i,j]$.
Let $B_{i,j}$ be true if there is a breeze in $[i,j]$.



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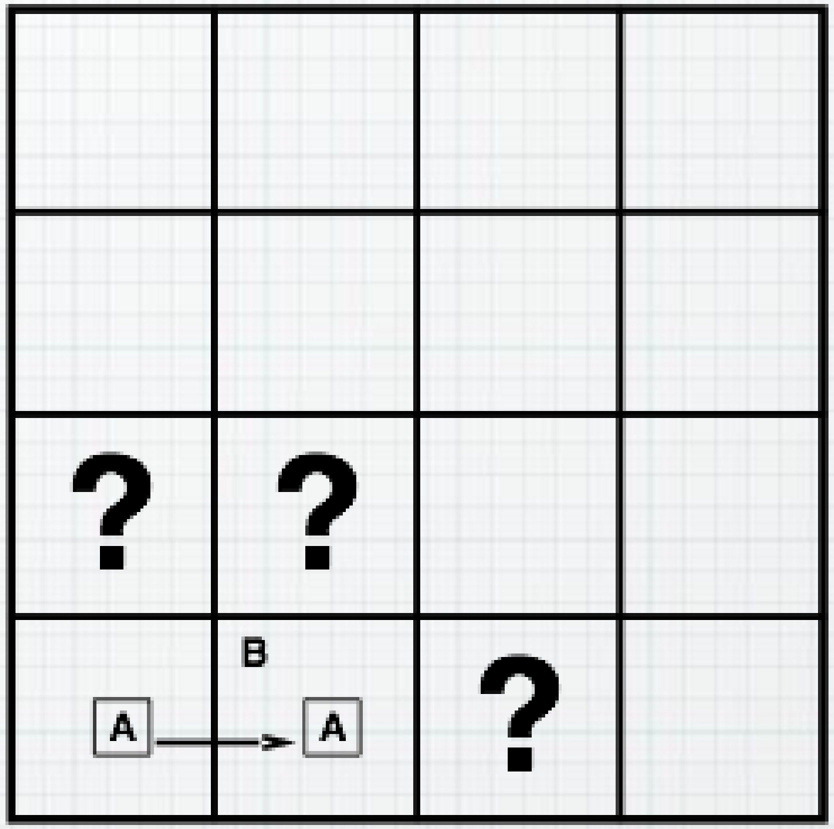
$$\neg P_{1,1}$$
$$\neg B_{1,1}$$
$$B_{2,1}$$



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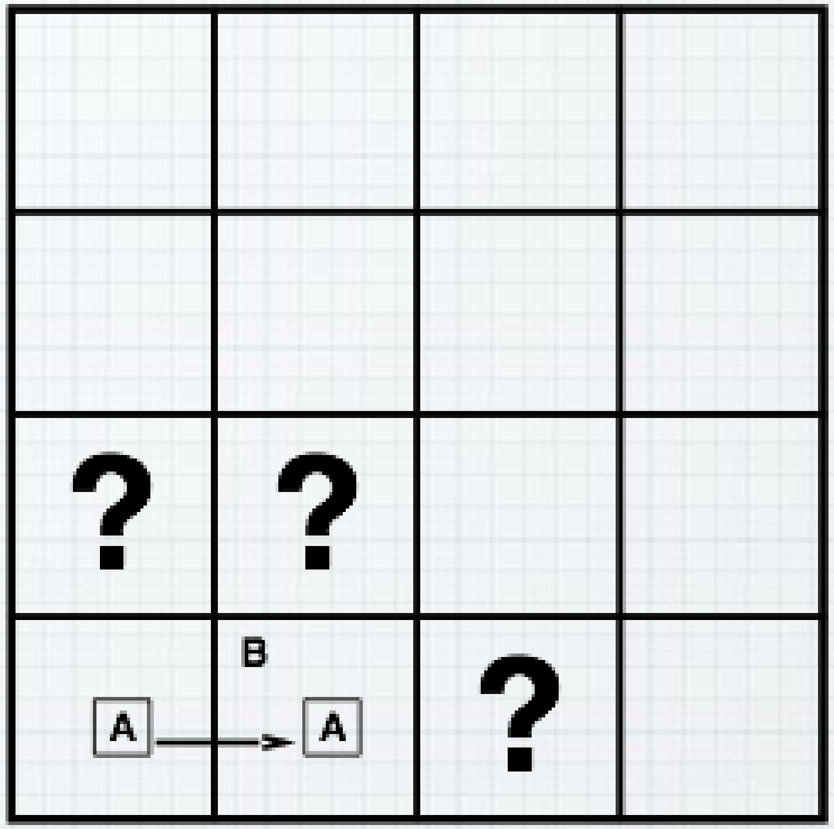
"Pits cause breezes in adjacent squares"

$$B_{1,1} \quad \Leftrightarrow \quad (P_{1,2} \vee P_{2,1})$$
$$B_{2,1} \quad \Leftrightarrow \quad (P_{1,1} \vee P_{2,2} \vee P_{3,1})$$

"A square is breezy **if and only if** there is an adjacent pit"



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$$B_{1,1} \Leftrightarrow (P_{1,2} \lor P_{2,1})$$
$$B_{2,1} \Leftrightarrow (P_{1,1} \lor P_{2,2} \lor P_{3,1})$$

"A square is breezy **if and only if** there is an adjacent pit"

We want to infer hidden state past the info in our KB

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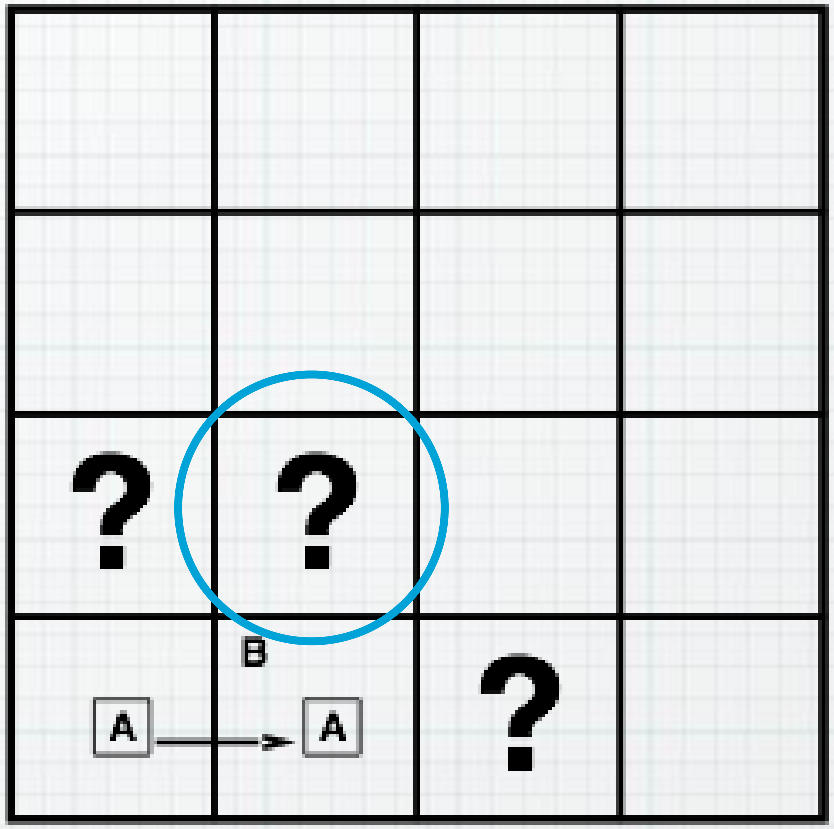

EX: Does our KB imply that there is a Pit in [2,2]?

Truth Tables for Inference

Model KB sentences

$B_{1,1}$	$B_{2,1}$	$P_{1,1}$	$P_{1,2}$	$P_{2,1}$	$P_{2,2}$	$P_{3,1}$	R_1	R_2	R_3	R_4	R_5	KB
false	*false*	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*false*	*false*
false	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*false*	*true*	*false*	*false*
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
false	*true*	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*false*	*true*	*true*	*false*
false	*true*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*false*	*true*	*false*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*true*	*false*	*false*	*true*	*false*	*false*	*true*	*true*	*false*
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
true	*true*	*true*	*true*	*true*	*true*	*true*	*false*	*true*	*true*	*false*	*true*	*false*

Inference by Enumeration:

For all models where KB is true, check if query is T

¬$P_{1,2}$: is entailed by our KB ¬$P_{2,2}$: is not...

¬$P_{2,1}$: is entailed by our KB

Inference by Enumeration

```
function TT-ENTAILS?(KB, α) returns true or false
    inputs: KB, the knowledge base, a sentence in propositional logic
            α, the query, a sentence in propositional logic

    symbols ← a list of the proposition symbols in KB and α
    return TT-CHECK-ALL(KB, α, symbols, [])
```

```
function TT-CHECK-ALL(KB, α, symbols, model) returns true or false
    if EMPTY?(symbols) then
        if PL-TRUE?(KB, model) then return PL-TRUE?(α, model)
        else return true
    else do
        P ← FIRST(symbols); rest ← REST(symbols)
        return TT-CHECK-ALL(KB, α, rest, EXTEND(P, true, model)) and
                    TT-CHECK-ALL(KB, α, rest, EXTEND(P, false, model))
```

- DFS enumeration of all symbols
- Checking if query T everywhere KB is T

Inference by Enumeration

* Sound: Yes, if says entailed it is
* Complete: Yes, enumerating everything so will find all entailed sentences
* Complexity: $O(2^n)$, n=#symbols.... exponential in size of input!