# Logical Agents

## CH 9

Inference in First Order Logic

# First-Order Logic

* Constants, variables, functions, predicates
  E.g.: Anna, x, MotherOf(x), Friends(x, y)
* Literal: Predicate or its negation
* Clause: Disjunction of literals
* Grounding: Replace all variables by constants
  E.g.: Friends (Anna, Bob)
* World (model, interpretation):
  Assignment of truth values to all ground predicates

# Unification

* Basic step in inference for FOL
* Find a unifier (substitution) t which makes two sentences p,q identical
    * UNIFY(p,q) = t    where SUBST(t,p)=SUBST(t,q)
* Example unifications:
    * Knows(John,x) & Knows(John,Jane) : {x/Jane}
    * Knows(John,x) & Knows(y, Bill) : {x/Bill, y/John}
    * Knows(John,x) & Knows(y, Mother(y)) :
      {x/Mother(John), y/John}
    * Knows(John,x) & Knows(x,Elsie) : Fail

# Unification Issues

* Knows(John,x) & Knows(x,Elsie) : Fail
  * Solve by standardizing apart
    * Knows(John,x) & Knows(y,Elsie) : {x/Elsie, y/John}
* Most general unifier
  * Knows(John,x) & Knows(y,z) :
    {y/John, x/z} OR {y/John, x/John, z/John}
    Knows(John,z)   Knows(John,John)
  * Choose most general unifier
  * Always exists and is unique up to renaming/subst

# Inference Approaches

* Forward chaining
* Backward chaining
* Resolution

* (See book for details, *these methods will not be on the final*)

# Wumpus FOL

# Interacting with FOL KBs

* Suppose Wumpus agent perceives smell breeze and glitter at time t=5

Tell(KB, Percept([Smell, Breeze, Glitter], 5)

* Does the KB entail any particular action?

Ask(KB, ∃a BestAction(a, 5))

* The query returns a substitution

Answer: {a/Grab}

* **Perception sentences:**

$\forall bgt$ Percept([Smell, b, g], t) $\Rightarrow$ Smell(t)

$\forall sbt$ Percept([s, b, Glitter], t) $\Rightarrow$ AtGold(t)

* **Reflex action: grab gold:**

$\forall t$ AtGold(t) $\Rightarrow$ Action(Grab, t)

* **Something is at square s at time t:**

At(a, s, t)

* **Wumpus is in a fixed location:**

$\forall t$ At(Wumpus, [3, 1], t)

* bg **There is a pit in square s:**

Pit(s)

* Definition of two squares being **adjacent**:

$$\forall x,y,a,b\ Adjacent([x,y],[a,b]) \Leftrightarrow$$
$$(x = a \wedge (y=b-1 \vee y=b+1)) \vee (y=b \wedge (x=a-1 \vee x=a+1))$$

* If an agent perceives breeze in a square then that square is breezy:

$$\forall s,t\ \ At(Agent,s,t) \wedge Breeze(t) \Rightarrow Breezy(s)$$

* Breeze means pit in adjacent square:

$$\forall s\ \mathrm{Breezy}(s) \Leftrightarrow \exists r\ \mathrm{Adjacent}(s, r) \wedge \mathrm{Pit}(r)$$

# Knowledge Engineering

* Deciding how to best write down the concepts of a domain
* Decide on the vocabulary of symbols, predicates and functions
* Put all the sentences of the domain in your KB (and debug) so that some expected inferences can be made

**The Cyc knowledge base** (KB) is a formalized representation of a vast quantity of fundamental human knowledge: facts, rules of thumb, and heuristics for reasoning about the objects and events of everyday life.

**OpenCyc** is the open source version of the Cyc technology, the world's largest and most complete general knowledge base and commonsense reasoning engine.

# Inference in FOL

Chapter 9

# Inference & Quantifiers

* One idea: Apply simple rules to remove quantifiers, then FOL is converted to PL
* Better idea: Inference methods that operate on FOL sentences directly

# Universal Instantiation

Every instantiation of a universally quantified sentence is entailed by it:

$$\frac{\forall v \quad \alpha}{\textsc{Subst}(\{v/g\}, \alpha)}$$

for any variable $v$ and ground term $g$

E.g., $\forall x \;\; King(x) \land Greedy(x) \;\Rightarrow\; Evil(x)$ yields

$$\begin{array}{l}
King(John) \land Greedy(John) \;\Rightarrow\; Evil(John) \\
King(Richard) \land Greedy(Richard) \;\Rightarrow\; Evil(Richard) \\
King(Father(John)) \land Greedy(Father(John)) \;\Rightarrow\; Evil(Father(John)) \\
\quad \vdots
\end{array}$$

# Existential Instantiation

For any sentence $\alpha$, variable $v$, and constant symbol $k$
**that does not appear elsewhere in the knowledge base:**

$$\frac{\exists v \quad \alpha}{\textsc{Subst}(\{v/k\}, \alpha)}$$

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**that does not appear elsewhere in the knowledge base:**

$$\frac{\exists v \;\; \alpha}{\textsc{Subst}(\{v/k\}, \alpha)}$$

E.g., $\exists x \;\; Crown(x) \land OnHead(x, John)$ yields

$$Crown(C_1) \land OnHead(C_1, John)$$

provided $C_1$ is a new constant symbol, called a Skolem constant

# Inference & Quantifiers

* **Universal Instantiation:** can be applied several times to **add** new sentences; new KB logically equivalent to the old
* **Existential Instantiation:** apply once to **replace** the existential sentence; new KB is not equivalent, but if a sentence is satisfiable in the new KB also in the old

# Reduce to Prop. Logic

Suppose the KB contains just the following:

$$\forall x \;\; King(x) \land Greedy(x) \Rightarrow Evil(x)$$
$$King(John)$$
$$Greedy(John)$$
$$Brother(Richard, John)$$

# Reduce to Prop. Logic

Suppose the KB contains just the following:

$$\forall x \;\; King(x) \land Greedy(x) \;\Rightarrow\; Evil(x)$$
$$King(John)$$
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Instantiating the universal sentence in **all possible** ways, we have

$$King(John) \land Greedy(John) \;\Rightarrow\; Evil(John)$$
$$King(Richard) \land Greedy(Richard) \;\Rightarrow\; Evil(Richard)$$
$$King(John)$$
$$Greedy(John)$$
$$Brother(Richard, John)$$

The new KB is propositionalized: proposition symbols are

$King(John)$, $Greedy(John)$, $Evil(John)$, $King(Richard)$ etc.

# Problem with Propositionalization

Propositionalization seems to generate lots of irrelevant sentences.
E.g., from

$$\forall x \;\; King(x) \land Greedy(x) \;\Rightarrow\; Evil(x)$$
$$King(John)$$
$$\forall y \;\; Greedy(y)$$
$$Brother(Richard, John)$$

it seems obvious that $Evil(John)$, but propositionalization produces lots of facts such as $Greedy(Richard)$ that are irrelevant

With $p$ $k$-ary predicates and $n$ constants, there are $p \cdot n^k$ instantiations

With function symbols, it gets nuch much worse!

**Let's just instantiate what is relevant to our query!**

# Unification

We can get the inference immediately if we can find a substitution $\theta$ such that $King(x)$ and $Greedy(x)$ match $King(John)$ and $Greedy(y)$

$\theta = \{x/John, y/John\}$ works

$\textsc{Unify}(\alpha, \beta) = \theta$ if $\alpha\theta = \beta\theta$

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$\textsc{Unify}(\alpha, \beta) = \theta$ if $\alpha\theta = \beta\theta$

| $p$ | $q$ | $\theta$ |
|---|---|---|
| $Knows(John, x)$ | $Knows(John, Jane)$ | |
| $Knows(John, x)$ | $Knows(y, OJ)$ | |
| $Knows(John, x)$ | $Knows(y, Mother(y))$ | |
| $Knows(John, x)$ | $Knows(x, OJ)$ | |

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| $p$ | $q$ | $\theta$ |
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| $Knows(John, x)$ | $Knows(John, Jane)$ | $\{x/Jane\}$ |
| $Knows(John, x)$ | $Knows(y, OJ)$ | $\{x/OJ, y/John\}$ |
| $Knows(John, x)$ | $Knows(y, Mother(y))$ | $\{y/John, x/Mother(John)\}$ |
| $Knows(John, x)$ | $Knows(x, OJ)$ | $fail$ |

Standardizing apart eliminates overlap of variables, e.g., $Knows(z_{17}, OJ)$

# Generalized Modus Ponens

$$\frac{p_1',\ p_2',\ \ldots,\ p_n',\ (p_1 \wedge p_2 \wedge \ldots \wedge p_n \Rightarrow q)}{q\theta}$$

where $p_i'\theta = p_i\theta$ for all $i$

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$King(John)$ $\quad \forall y \; Greedy(y)$ $\quad \forall x \; King(x) \wedge Greedy(x) \Rightarrow Evil(x)$

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$King(John)$ $\quad \forall y \; Greedy(y)$ $\quad \forall x \; King(x) \wedge Greedy(x) \Rightarrow Evil(x)$

$p_1'$ is $King(John)$
$p_2'$ is $Greedy(y)$

$p_1$ is $King(x)$
$p_2$ is $Greedy(x)$

$q$ is $Evil(x)$

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$p_1{}'$ is $King(John)$
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$p_1$ is $King(x)$
$p_2$ is $Greedy(x)$

$q$ is $Evil(x)$

$\theta$ is $\{x/John, y/John\}$
$q\theta$ is $Evil(John)$

# Forward Chaining in FOL

# FOL Forward Chaining

```
function FOL-FC-ASK(KB, α) returns a substitution or false

   repeat until new is empty
      new ← { }
      for each sentence r in KB do
         ( p1 ∧ . . . ∧ pn ⇒ q) ← STANDARDIZE-APART(r)
         for each θ such that (p1 ∧ . . . ∧ pn)θ = (p'1 ∧ . . . ∧ p'n)θ
                         for some p'1, . . . , p'n in KB
            q' ← SUBST(θ, q)
            if q' is not a renaming of a sentence already in KB or new then do
                 add q' to new
                 φ ← UNIFY(q', α)
                 if φ is not fail then return φ
      add new to KB
   return false
```

# Summary Ch 8+9

* First Order Logic lets us talk about objects, properties, and relations
* Much more expressive than ProLog
* Variables and Quantifiers require new inference rules
* Unification finds a substitution of variables consistent with the desired inference
* Generalized Modes Ponens for Forward/ Backward chaining in FOL

# Example Knowledge Base

The law says that it is a crime for an American to sell weapons to hostile nations. The country Nono, an enemy of America, has some missiles, and all of its missiles were sold to it by Colonel West, who is American.

Prove that Col. West is a criminal

# Example Knowledge Base

. . . it is a crime for an American to sell weapons to hostile nations:

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$$American(x) \wedge Weapon(y) \wedge Sells(x, y, z) \wedge Hostile(z) \Rightarrow Criminal(x)$$

Nono . . . has some missiles

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Nono . . . has some missiles, i.e., $\exists\, x\ Owns(Nono, x) \land Missile(x)$:

$Owns(Nono, M_1)$ and $Missile(M_1)$

. . . all of its missiles were sold to it by Colonel West

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$\forall\, x \;\; Missile(x) \land Owns(Nono, x) \;\Rightarrow\; Sells(West, x, Nono)$

Missiles are weapons:

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$Missile(x) \Rightarrow Weapon(x)$

An enemy of America counts as "hostile":

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Missiles are weapons:

$Missile(x) \Rightarrow Weapon(x)$

An enemy of America counts as "hostile":

$Enemy(x, America)\ \Rightarrow\ Hostile(x)$

West, who is American ...

$American(West)$

The country Nono, an enemy of America ...

$Enemy(Nono, America)$

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$\boxed{American(West)}$ $\boxed{Missile(M1)}$ $\boxed{Owns(Nono,M1)}$ $\boxed{Enemy(Nono,America)}$

... it is a crime for an American to sell weapons to hostile nations:

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... all of its missiles were sold to it by Colonel West

$\forall x\ \ Missile(x) \wedge Owns(Nono, x)\ \Rightarrow\ Sells(West, x, Nono)$ x = M1

Missiles are weapons:

$Missile(x) \Rightarrow Weapon(x)$ x = M1

An enemy of America counts as "hostile":

$Enemy(x, America)\ \Rightarrow\ Hostile(x)$ x = Nono

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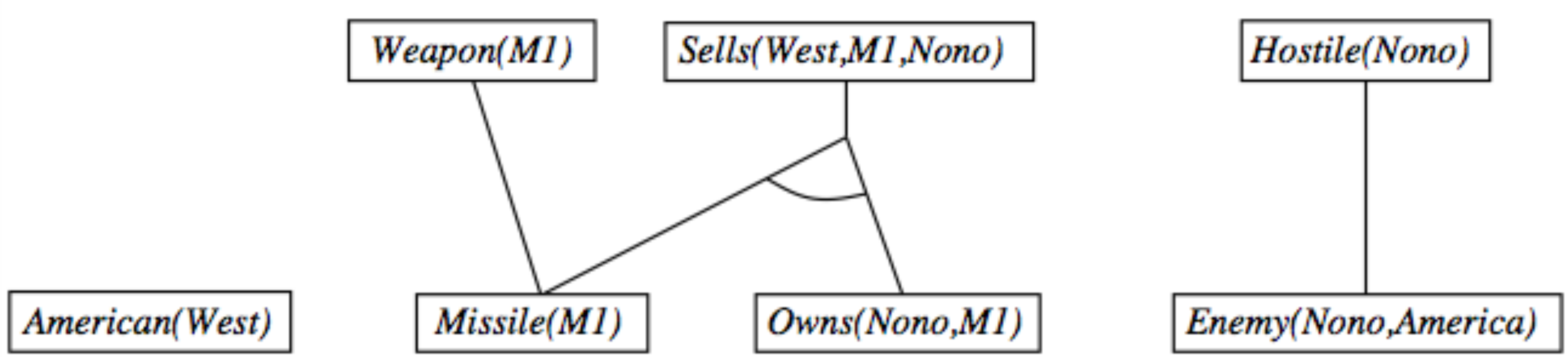

. . . it is a crime for an American to sell weapons to hostile nations:

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x = West

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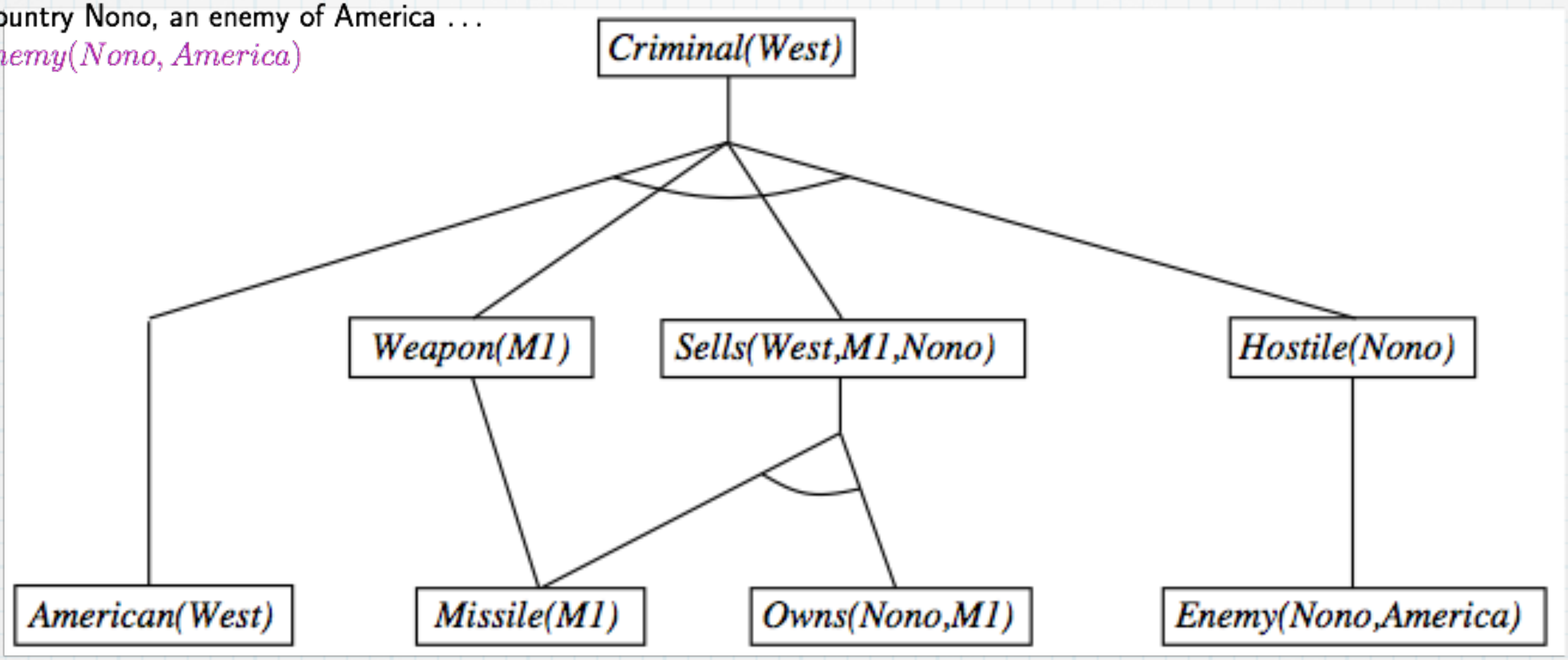


Sound and complete for first-order definite clauses
(proof similar to propositional proof)

Datalog = first-order definite clauses + **no functions** (e.g., crime KB)
FC terminates for Datalog in poly iterations: at most $p \cdot n^k$ literals

May not terminate in general if $\alpha$ is not entailed

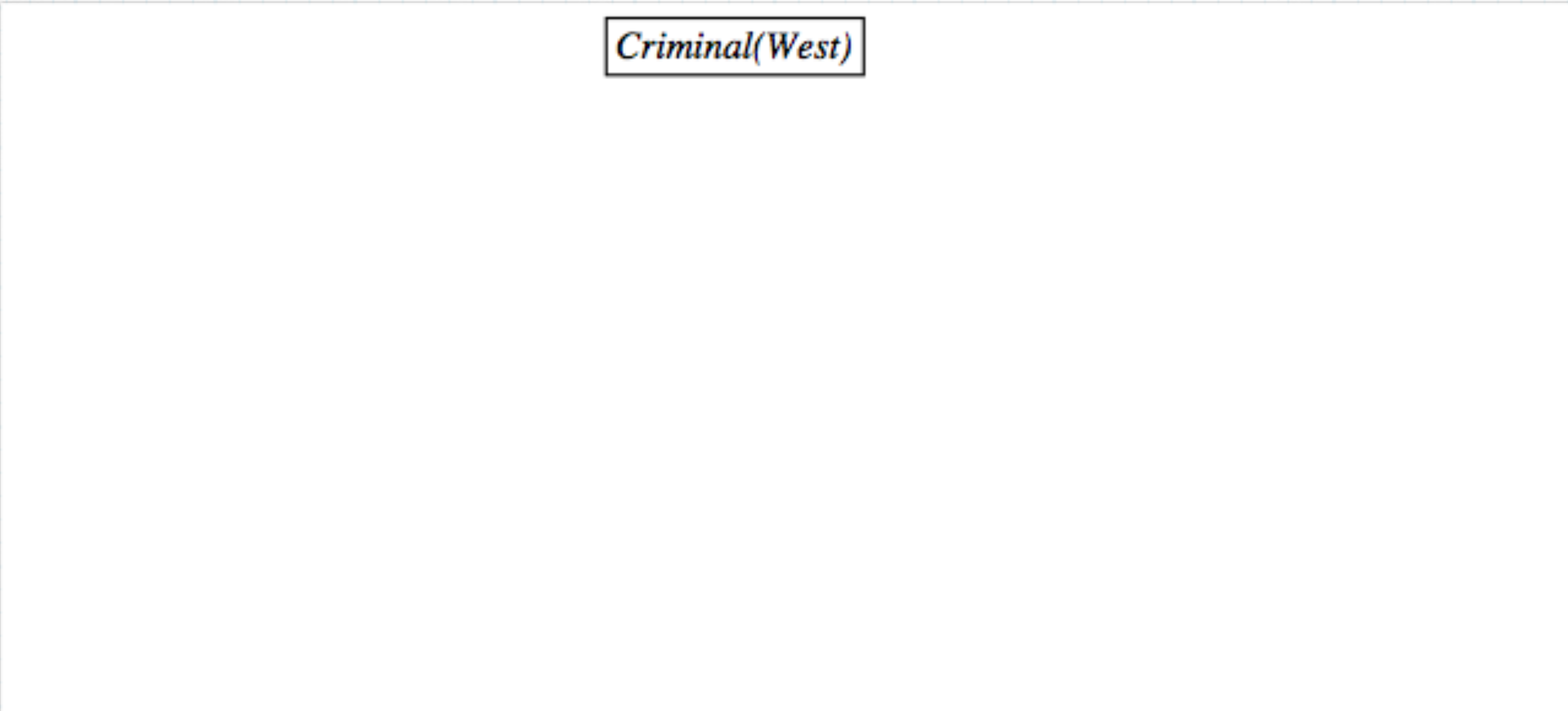
This is unavoidable: entailment with definite clauses is semidecidable

# Backward chaining algorithm

**function** FOL-BC-ASK(*KB*, *goals*, $\theta$) **returns** a set of substitutions
    **inputs**: *KB*, a knowledge base
        *goals*, a list of conjuncts forming a query ($\theta$ already applied)
        $\theta$, the current substitution, initially the empty substitution { }
    **local variables**: *answers*, a set of substitutions, initially empty

    **if** *goals* is empty **then return** $\{\theta\}$
    $q' \leftarrow$ SUBST($\theta$, FIRST(*goals*))
    **for each** sentence $r$ **in** *KB*
        where STANDARDIZE-APART($r$) = ( $p_1 \wedge \ldots \wedge p_n \Rightarrow q$)
        and $\theta' \leftarrow$ UNIFY($q$, $q'$) succeeds
      *new_goals* $\leftarrow [p_1, \ldots, p_n |$ REST(*goals*)$]$
      *answers* $\leftarrow$ FOL-BC-ASK(*KB*, *new_goals*, COMPOSE($\theta'$, $\theta$)) $\cup$ *answers*
    **return** *answers*

# Backward Chaining



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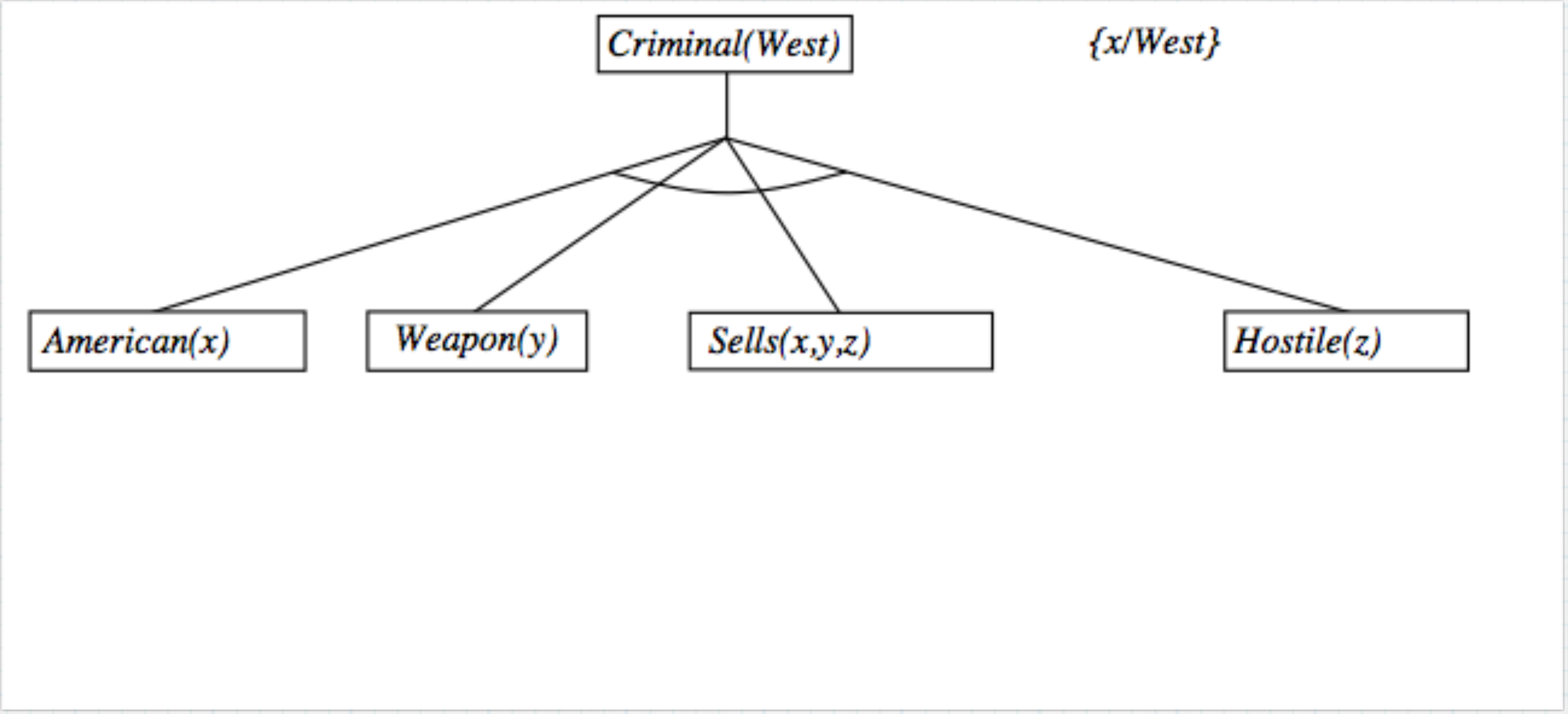

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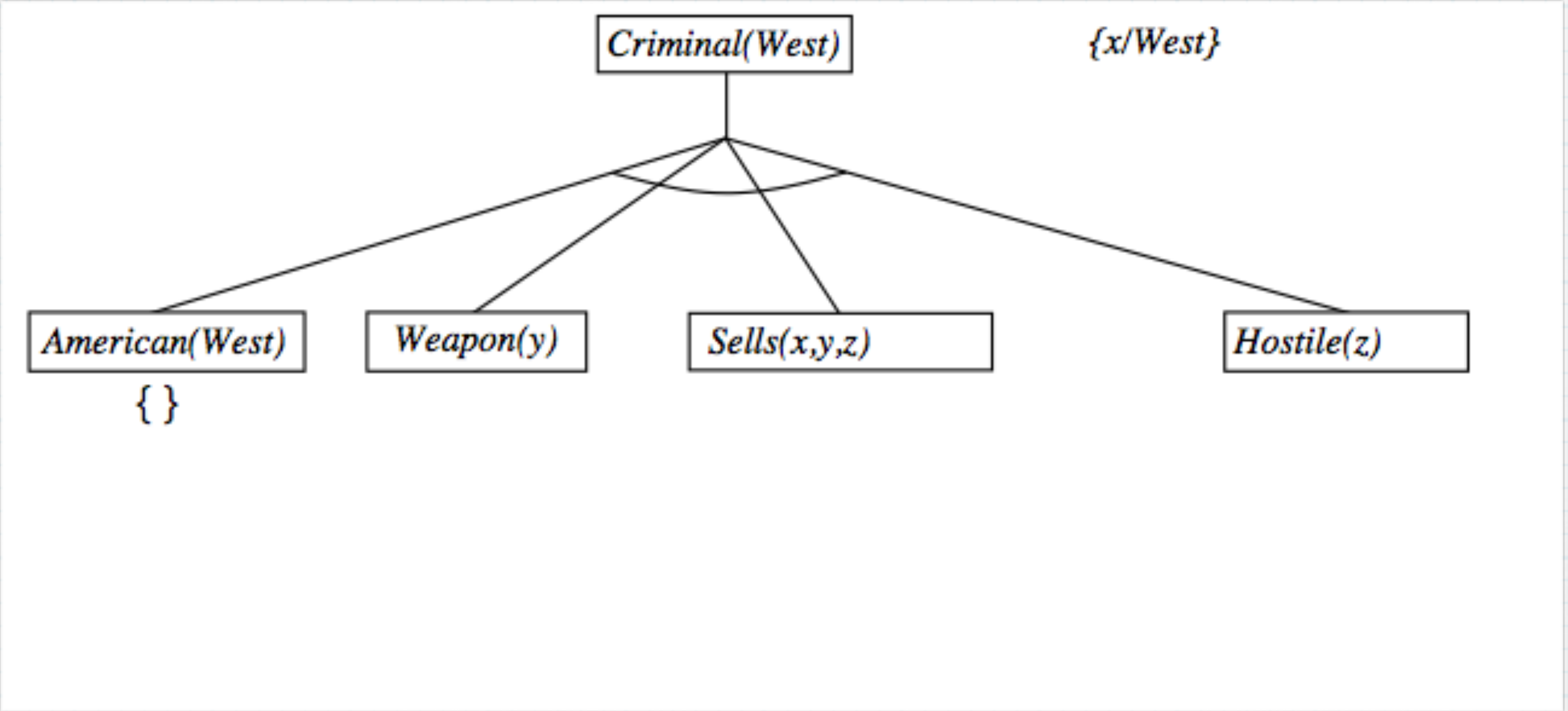

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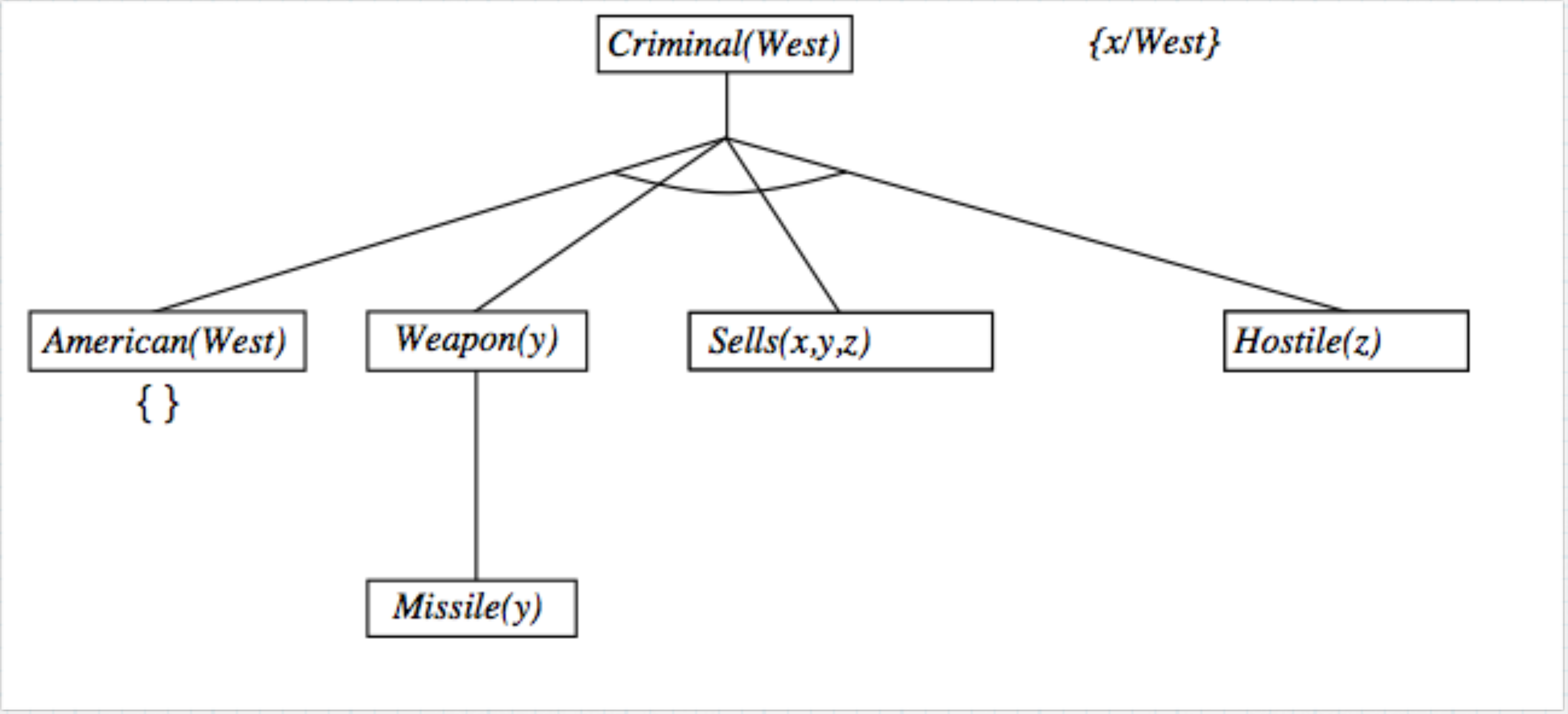

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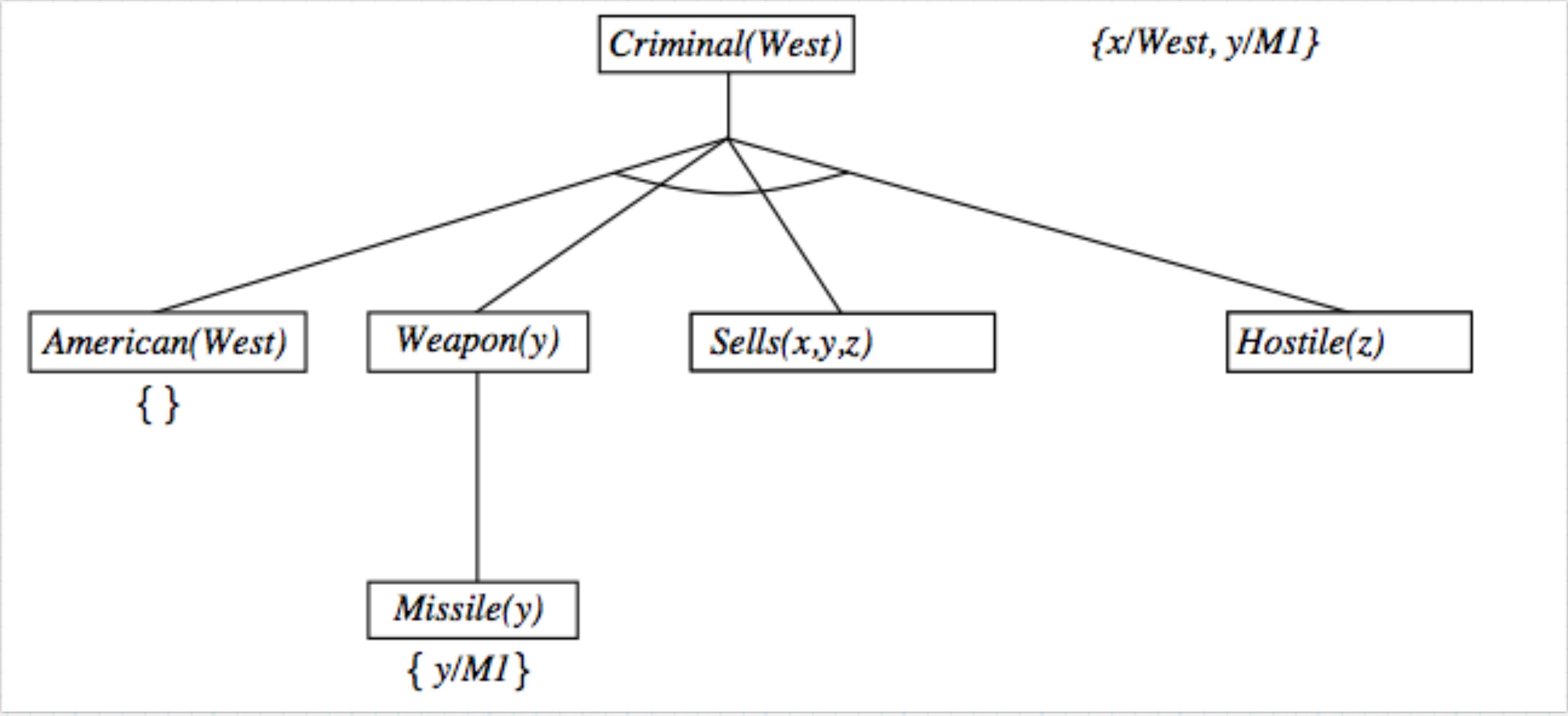

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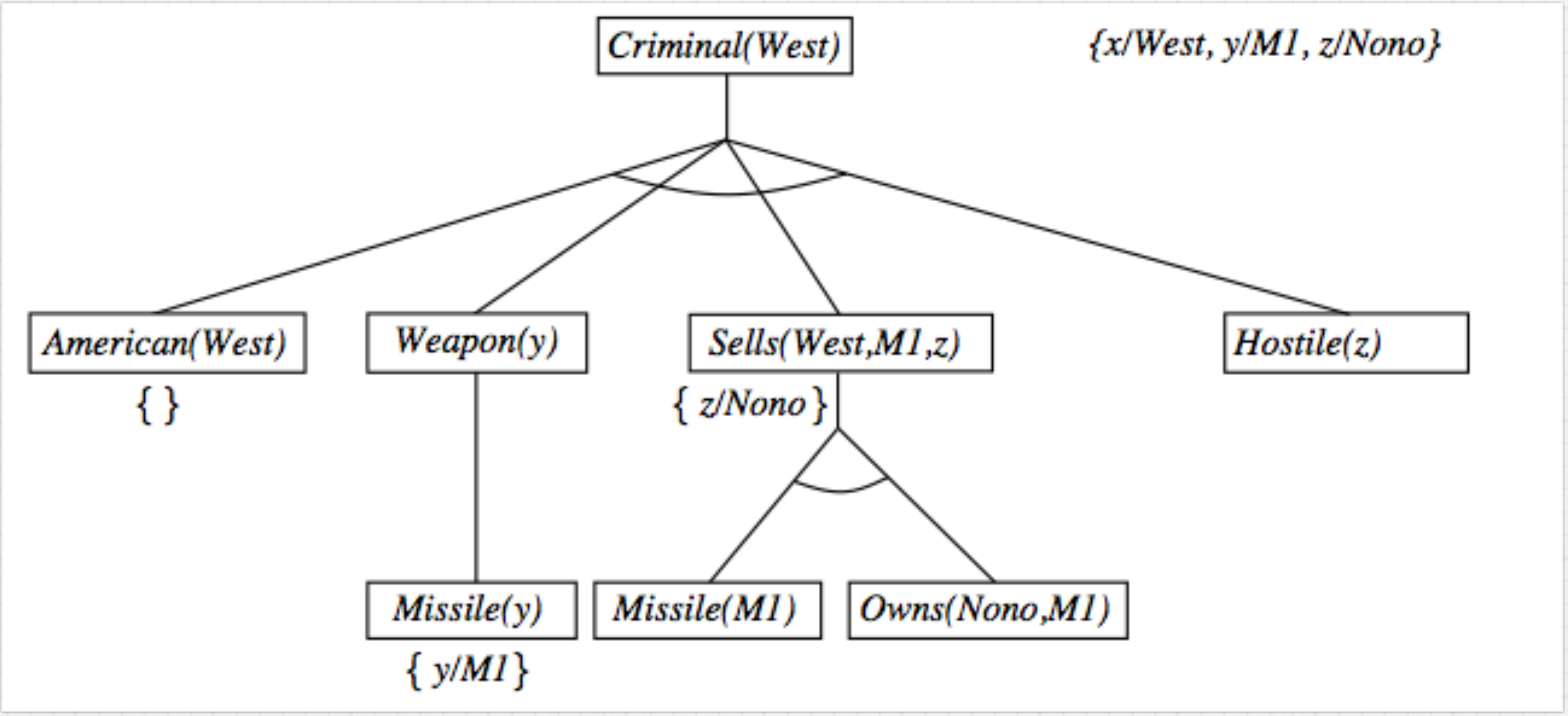

# Properties of Backward Chaining

Depth-first recursive proof search: space is linear in size of proof

Incomplete due to infinite loops
$\Rightarrow$ fix by checking current goal against every goal on stack

Inefficient due to repeated subgoals (both success and failure)
$\Rightarrow$ fix using caching of previous results (extra space!)

Widely used (without improvements!) for logic programming

# Resolution

Full first-order version:

$$\frac{\ell_1 \vee \cdots \vee \ell_k, \qquad m_1 \vee \cdots \vee m_n}{(\ell_1 \vee \cdots \vee \ell_{i-1} \vee \ell_{i+1} \vee \cdots \vee \ell_k \vee m_1 \vee \cdots \vee m_{j-1} \vee m_{j+1} \vee \cdots \vee m_n)\theta}$$

where $\textsc{Unify}(\ell_i, \neg m_j) = \theta$.

For example,

$$\frac{\begin{array}{l}\neg Rich(x) \vee Unhappy(x) \\ Rich(Ken)\end{array}}{Unhappy(Ken)}$$

with $\theta = \{x/Ken\}$

Apply resolution steps to $CNF(KB \wedge \neg\alpha)$; complete for FOL

# Converting to CNF

Everyone who loves all animals is loved by someone:

$$\forall x \ [\forall y \ \ Animal(y) \ \Rightarrow \ Loves(x, y)] \ \Rightarrow \ [\exists y \ \ Loves(y, x)]$$

1. Eliminate biconditionals and implications

$$\forall x \ \ [\neg \forall y \ \ \neg Animal(y) \lor Loves(x, y)] \lor [\exists y \ \ Loves(y, x)]$$

2. Move $\neg$ inwards: $\neg \forall x, p \ \ \equiv \exists x \ \ \neg p$, $\ \ \neg \exists x, p \ \ \equiv \forall x \ \ \neg p$:

$$\forall x \ \ [\exists y \ \ \neg(\neg Animal(y) \lor Loves(x, y))] \lor [\exists y \ \ Loves(y, x)]$$
$$\forall x \ \ [\exists y \ \ \neg\neg Animal(y) \land \neg Loves(x, y)] \lor [\exists y \ \ Loves(y, x)]$$
$$\forall x \ \ [\exists y \ \ Animal(y) \land \neg Loves(x, y)] \lor [\exists y \ \ Loves(y, x)]$$

# Converting to CNF

3. Standardize variables: each quantifier should use a different one

$$\forall x \;\; [\exists y \;\; Animal(y) \wedge \neg Loves(x,y)] \vee [\exists z \;\; Loves(z,x)]$$

4. Skolemize: a more general form of existential instantiation.
   Each existential variable is replaced by a Skolem function
   of the enclosing universally quantified variables:

$$\forall x \;\; [Animal(F(x)) \wedge \neg Loves(x, F(x))] \vee Loves(G(x), x)$$

5. Drop universal quantifiers:

$$[Animal(F(x)) \wedge \neg Loves(x, F(x))] \vee Loves(G(x), x)$$

6. Distribute $\wedge$ over $\vee$:

$$[Animal(F(x)) \vee Loves(G(x), x)] \wedge [\neg Loves(x, F(x)) \vee Loves(G(x), x)]$$

$\neg$ *American(x)* $\vee$ $\neg$ *Weapon(y)* $\vee$ $\neg$ *Sells(x,y,z)* $\vee$ $\neg$ *Hostile(z)* $\vee$ *Criminal(x)*

$\neg$ *Criminal(West)*

... it is a crime for an American to sell weapons to hostile nations:

$American(x) \wedge Weapon(y) \wedge Sells(x,y,z) \wedge Hostile(z) \Rightarrow Criminal(x)$

Nono ... has some missiles, i.e., $\exists\, x\; Owns(Nono, x) \wedge Missile(x)$:

$Owns(Nono, M_1)$ and $Missile(M_1)$

... all of its missiles were sold to it by Colonel West

$\forall\, x\;\; Missile(x) \wedge Owns(Nono, x) \;\Rightarrow\; Sells(West, x, Nono)$

Missiles are weapons:

$Missile(x) \Rightarrow Weapon(x)$

An enemy of America counts as "hostile":

$Enemy(x, America) \;\Rightarrow\; Hostile(x)$

West, who is American ...

$American(West)$

The country Nono, an enemy of America ...

$Enemy(Nono, America)$

$\neg American(x) \lor \neg Weapon(y) \lor \neg Sells(x,y,z) \lor \neg Hostile(z) \lor Criminal(x)$

$\neg Criminal(West)$

$American(West)$

$\neg American(West) \lor \neg Weapon(y) \lor \neg Sells(West,y,z) \lor \neg Hostile(z)$

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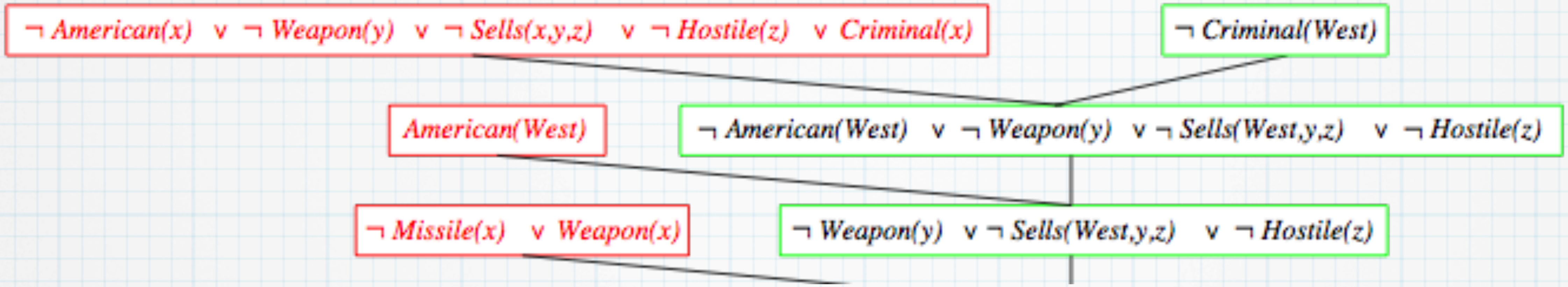


... it is a crime for an American to sell weapons to hostile nations:

$American(x) \wedge Weapon(y) \wedge Sells(x, y, z) \wedge Hostile(z) \Rightarrow Criminal(x)$

Nono ... has some missiles, i.e., $\exists x \; Owns(Nono, x) \wedge Missile(x)$:

$Owns(Nono, M_1)$ and $Missile(M_1)$

... all of its missiles were sold to it by Colonel West

$\forall x \;\; Missile(x) \wedge Owns(Nono, x) \;\Rightarrow\; Sells(West, x, Nono)$

Missiles are weapons:

$Missile(x) \Rightarrow Weapon(x)$

An enemy of America counts as "hostile":

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West, who is American ...

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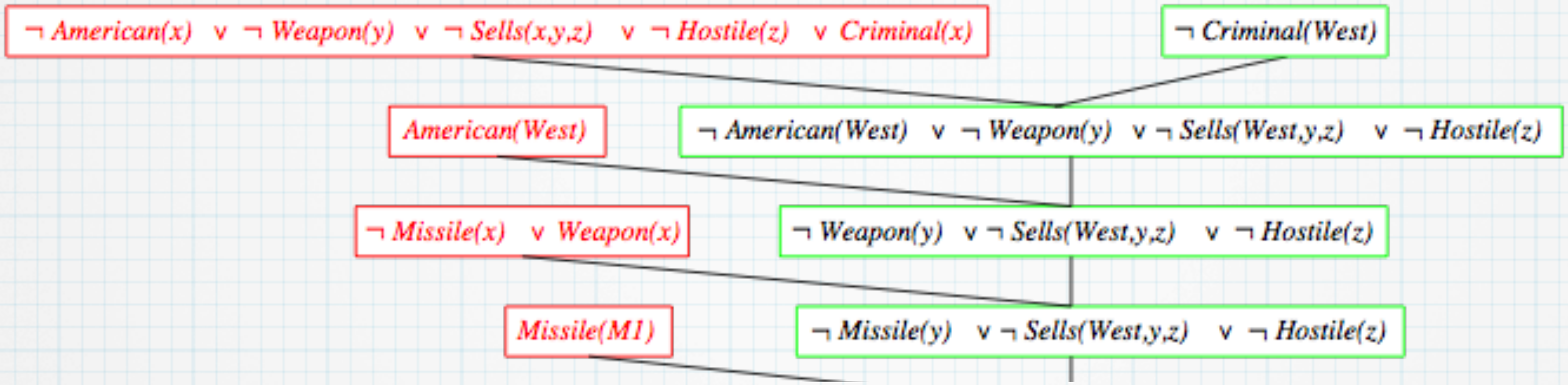


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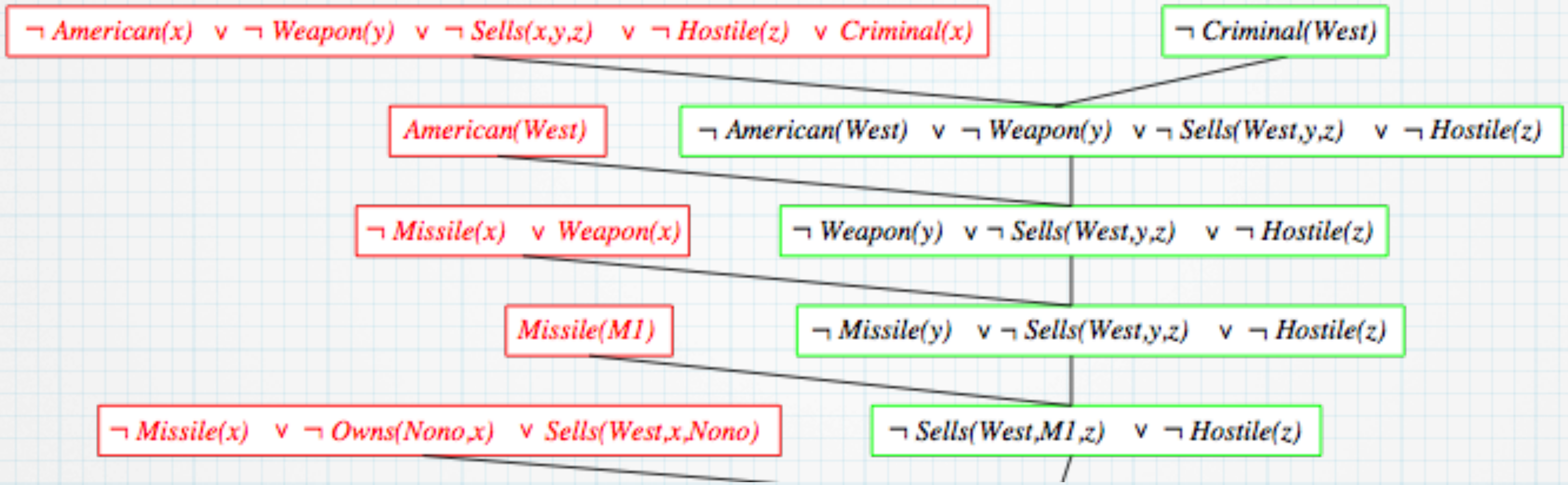


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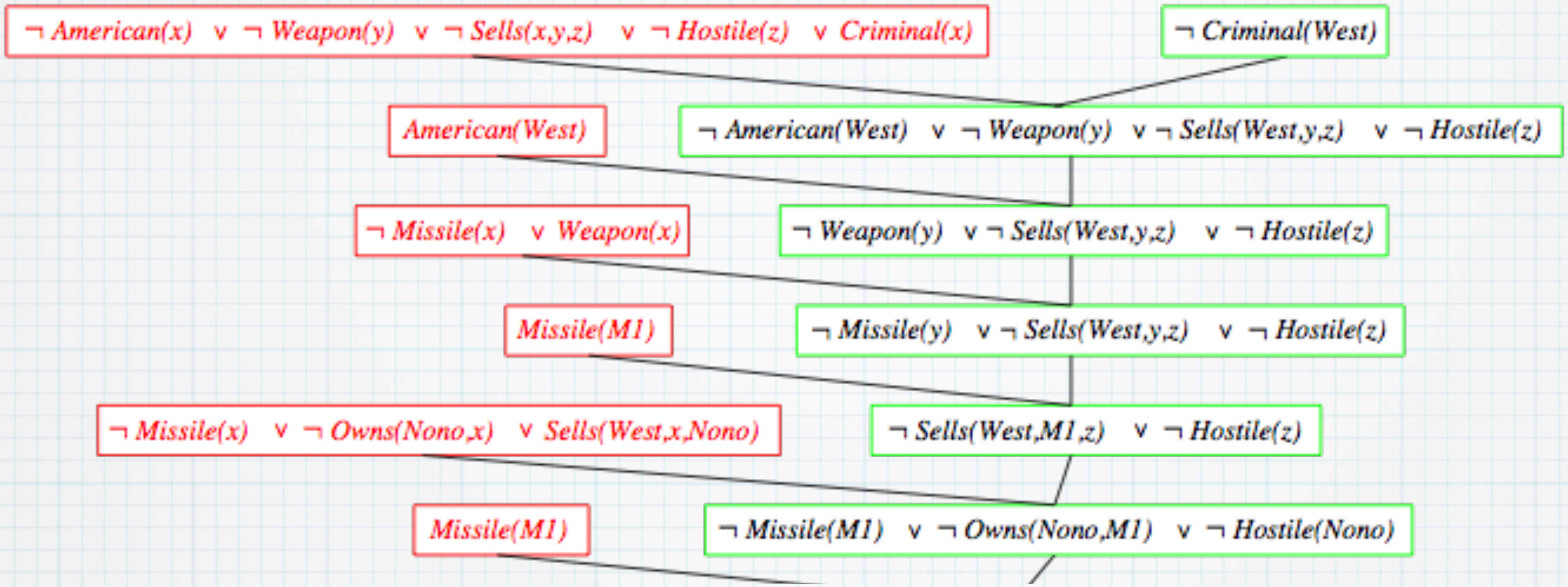

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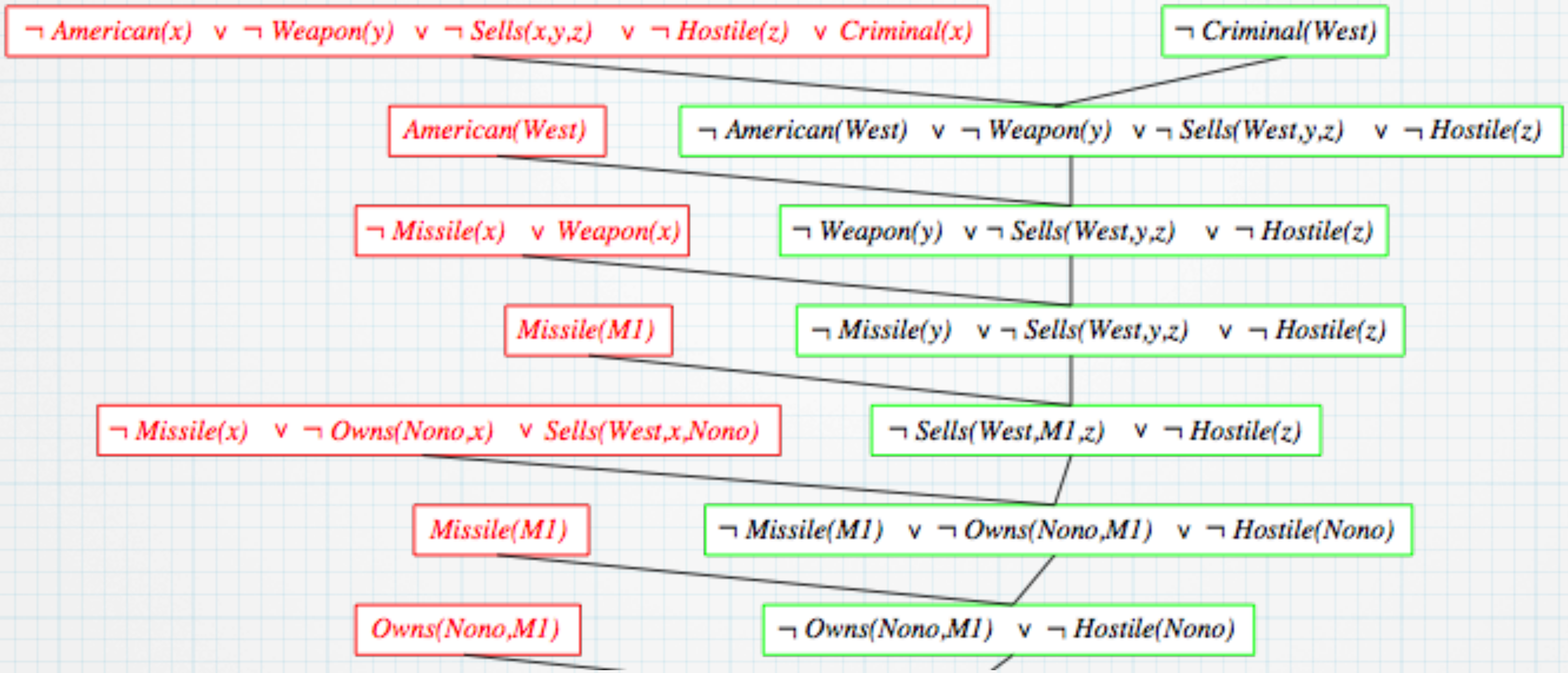


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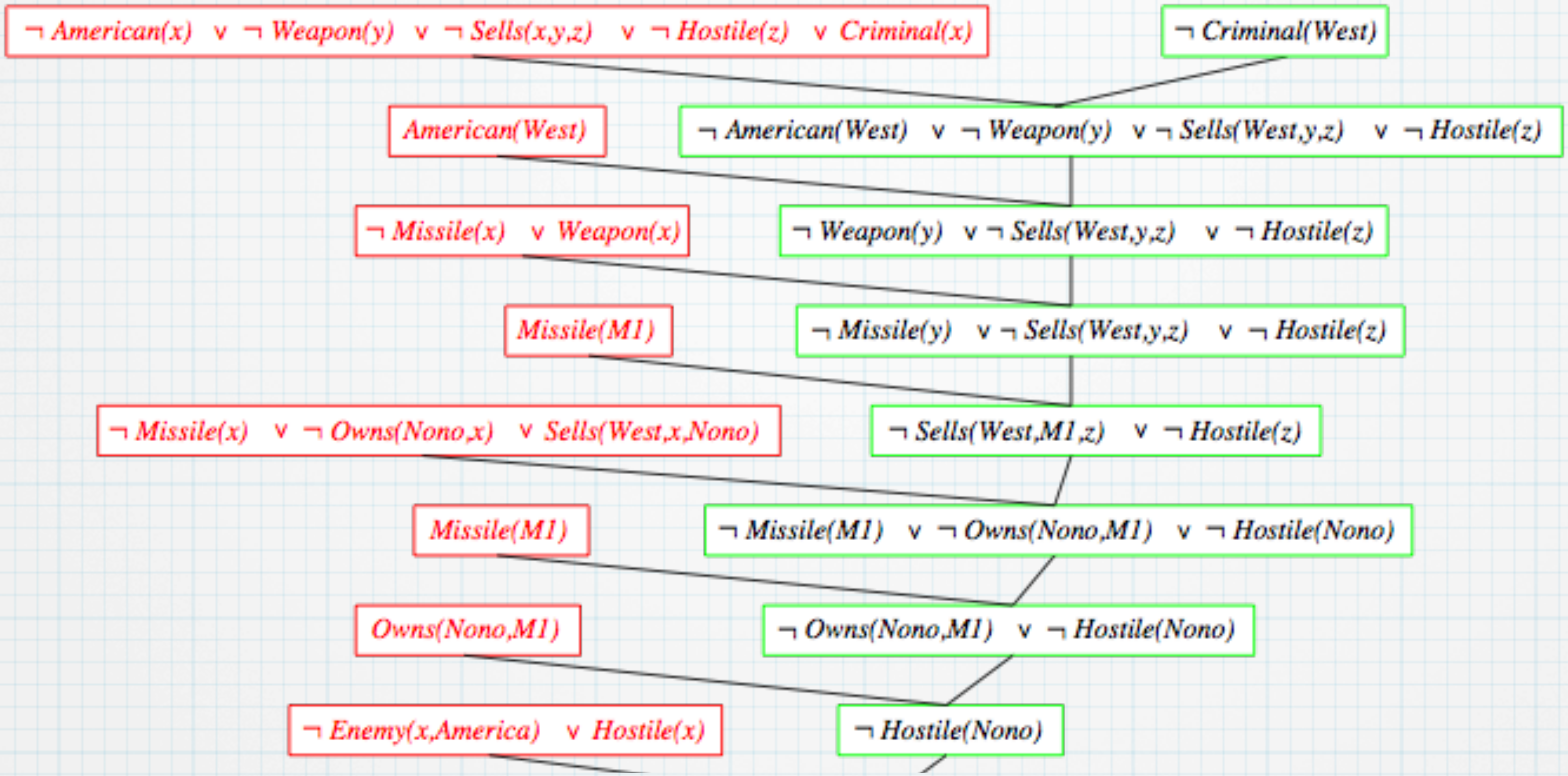


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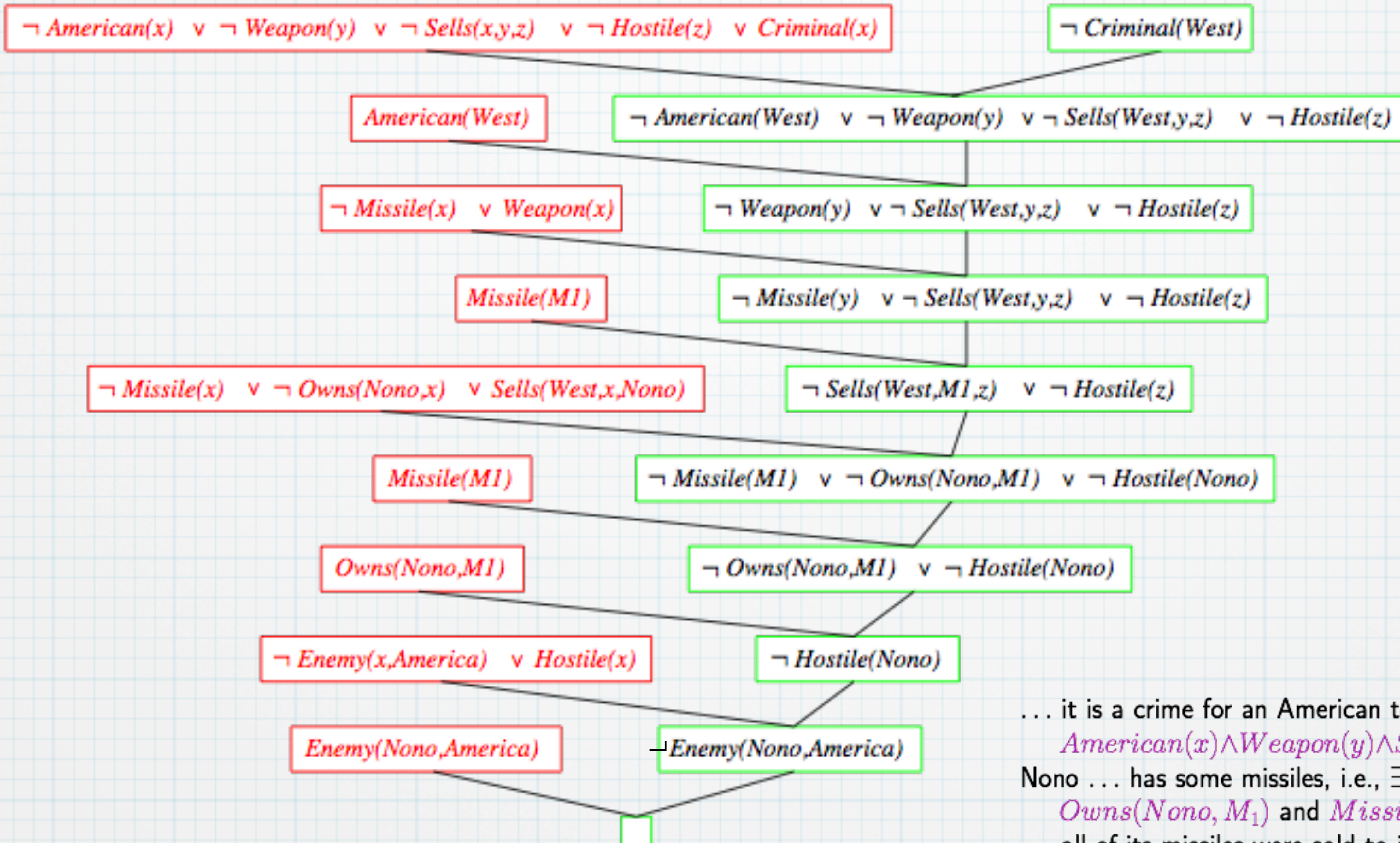


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