Making Complex Decisions

Markov Decision Processes
CH 17

Section Overview

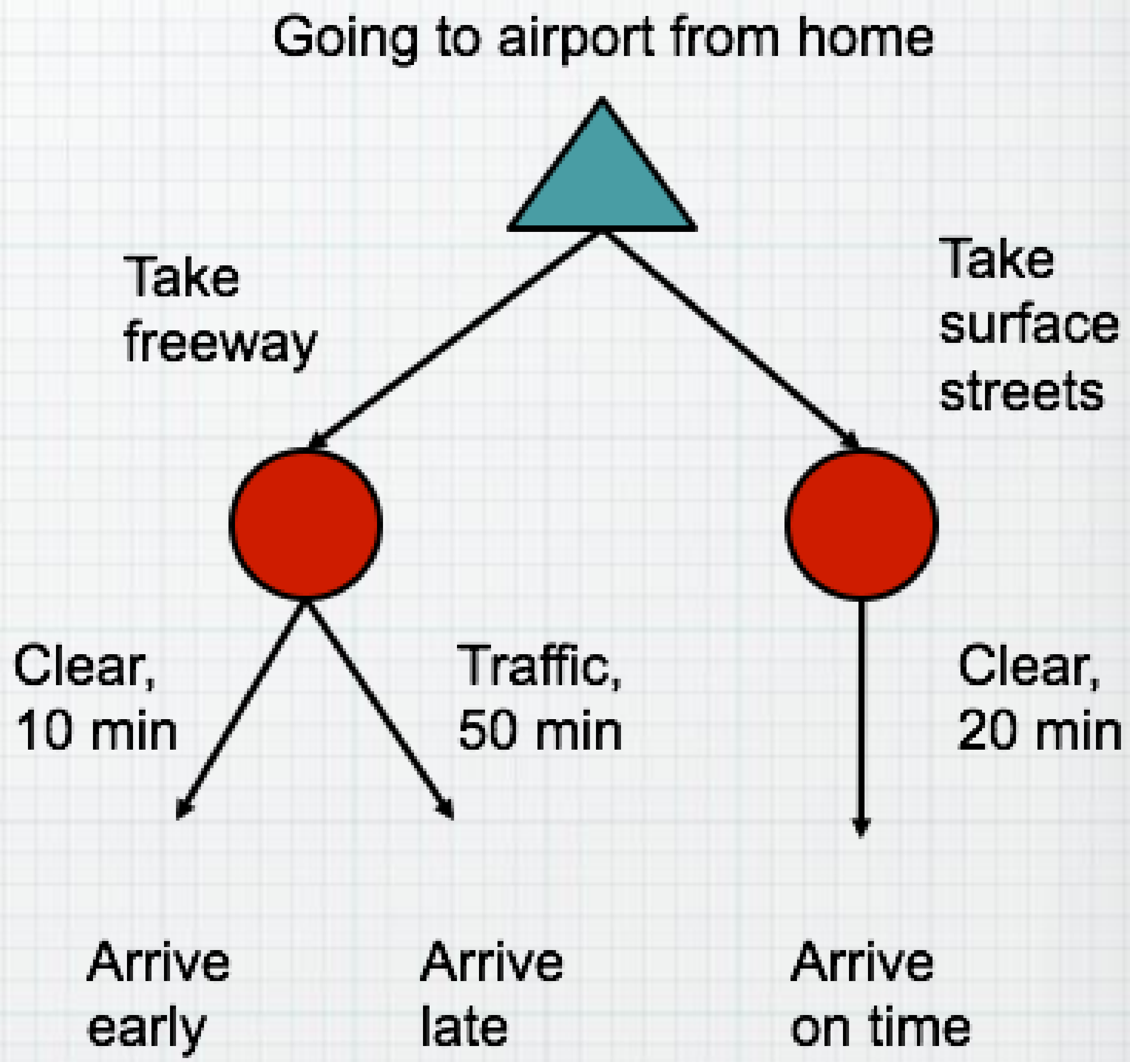
* Representing preferences
* Markov Decision Processes
* Solving MDPs
  * Value Iteration
  * Policy Iteration
* Partially Observable MDPs

Maximum Expected Utility

- Rational Agent = Maximizes its expected utility given its knowledge
- Questions:
  - Where do utilities come from?
  - Why expected utility?

Example

* One way has a chance to be better or worse
* How to decide?
* Which would you pick if you are catching a flight?
* Which if you are picking up a friend?



Assigning relative value to outcomes = Utilities

Agent Rational Decisions

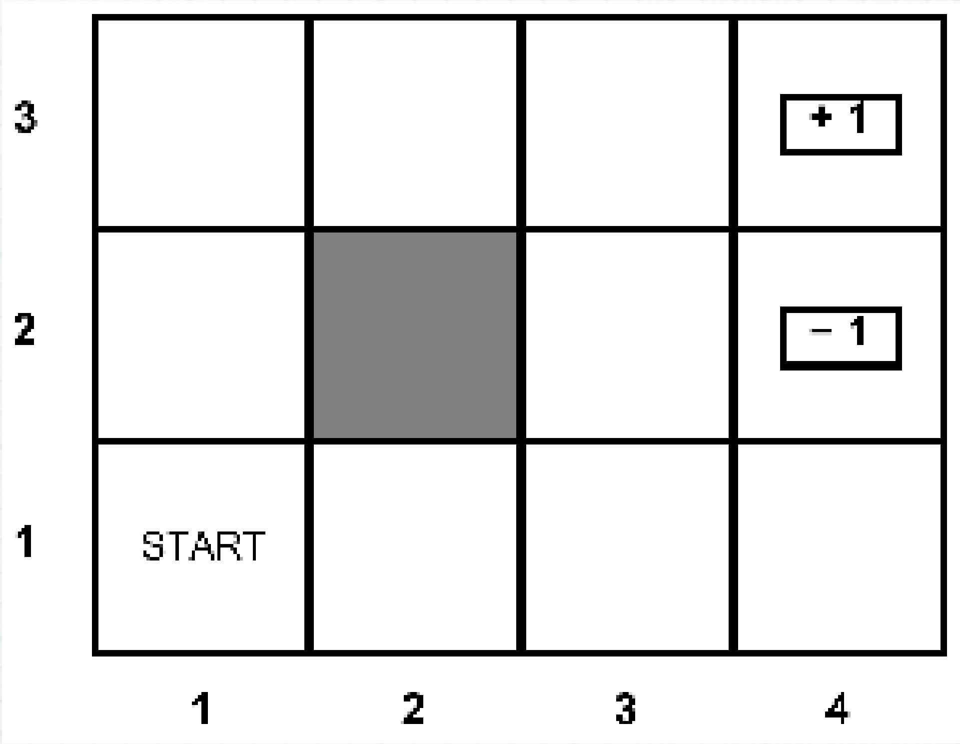
- Representing Decisions, Maximize Utility
  - Receive feedback = rewards
  - Utility defined by the reward function
  - Act to maximize expected rewards
  - Can learn to maximize rewards via Reinforcement Learning

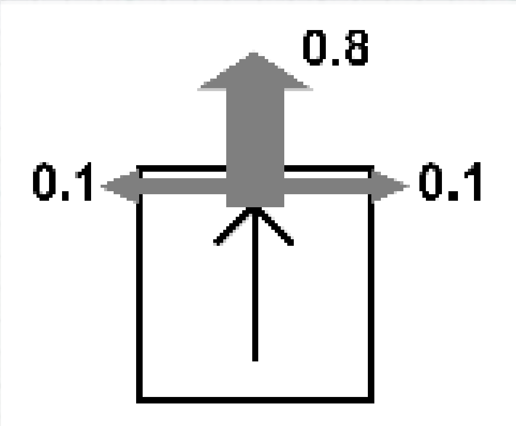
Reward Functions

* For example:
    * Playing a game, reward defined at the end for winning or losing
    * Vacuuming agent, reward for each piece of dirt
    * Autonomous taxi, reward for each passenger delivered

MDP Grid World

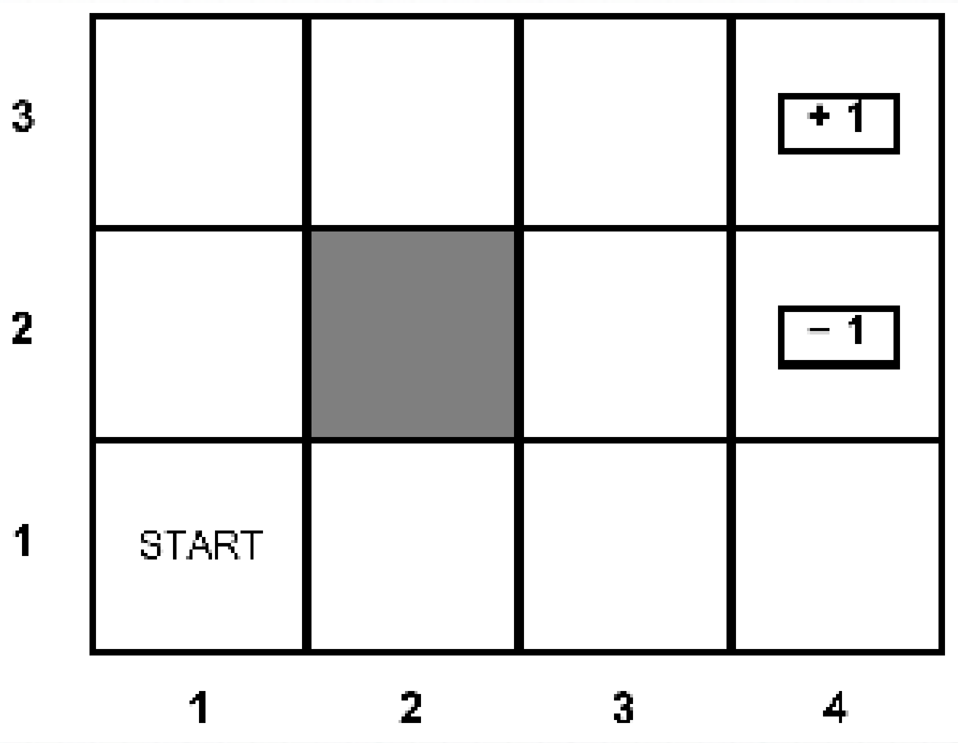
- Example agent for our MDP discussion
- Walls block the agent
- Actions only work 80% of the time
- Big rewards at the end

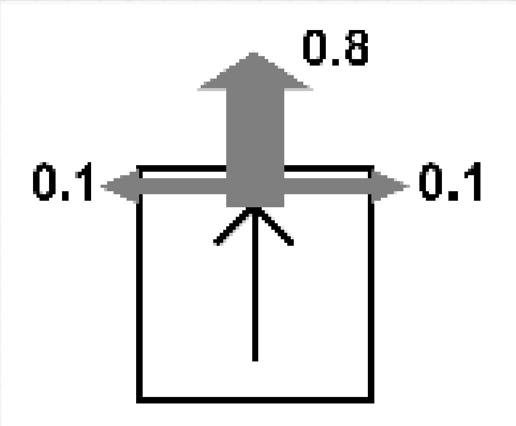




Markov Decision Processes

- MDP defined by
  - States $s \in S$
  - Actions $a \in A$
  - Transition function $T(s,a,s')$
  - Reward function $R(s,a,s')$





Just like our old search formulation but with non-det actions and rewards

What makes it Markov?

- Markov means: given the present state future and past are independent
- Specifically for MDPs Markov means

$$P(S_{t+1} = s' | S_t = s_t, A_t = a_t, S_{t-1} = s_{t-1}, A_{t-1}, \ldots S_0 = s_0)$$

$$=$$

$$P(S_{t+1} = s' | S_t = s_t, A_t = a_t)$$

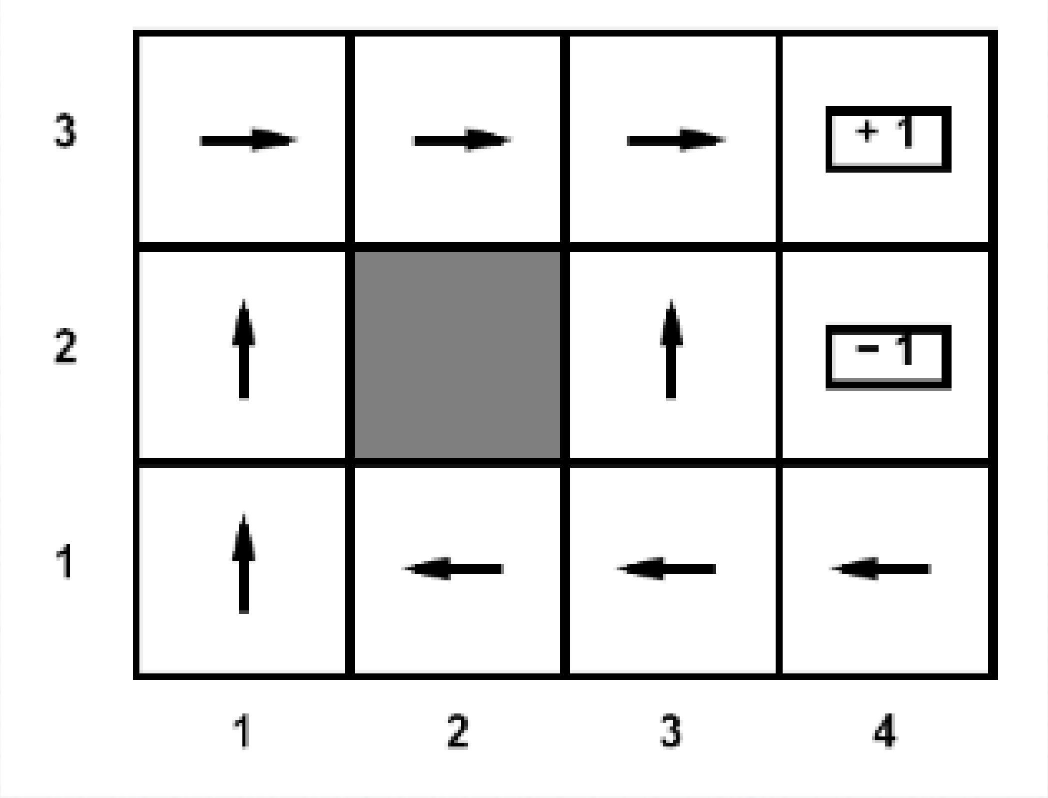
Solving MDPs

- In deterministic single-agent search problem we solved for an optimal plan, sequence of actions
- Now an optimal policy $\Pi^* = S \rightarrow A$
  - policy, Π, gives an action for every state
  - optimal policy, Π^*, maximizes expected utility
  - defines a reflex agent

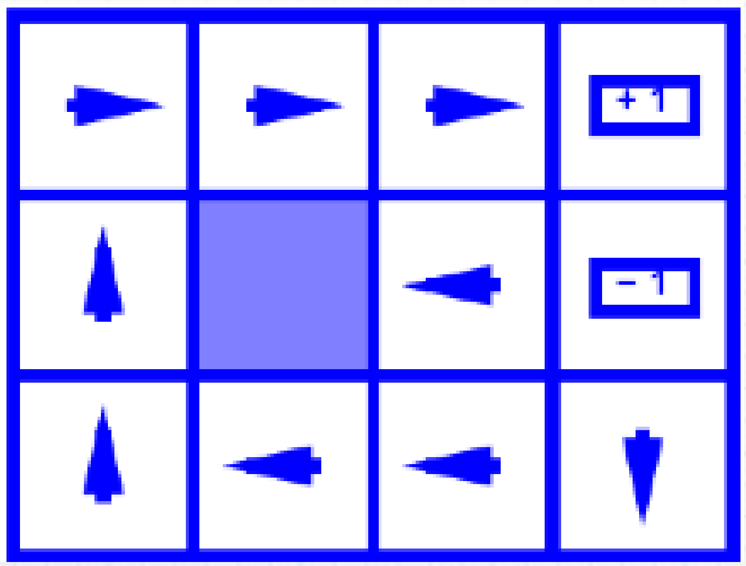
Solving MDPs

* In deterministic single-agent search problem we solved for an optimal plan, sequence of actions
* Now an optimal policy $\Pi^* = S \rightarrow A$

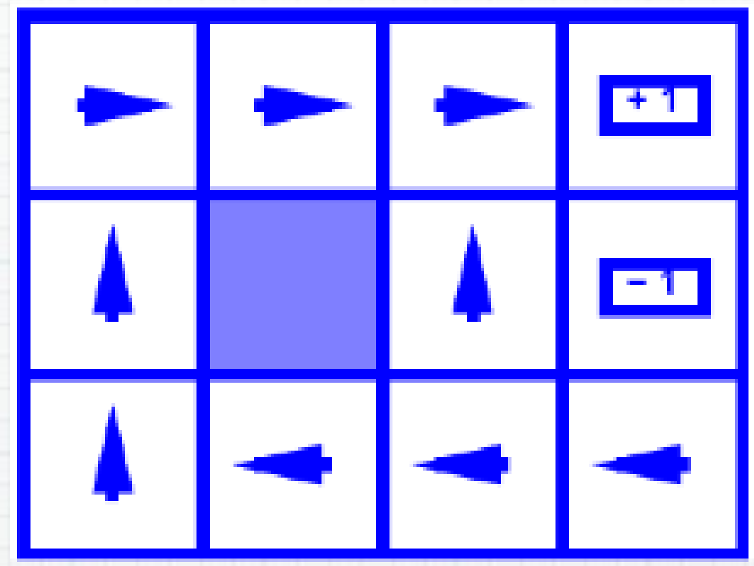
Optimal policy when R(s, a, s') = -0.03 for all non-terminals s



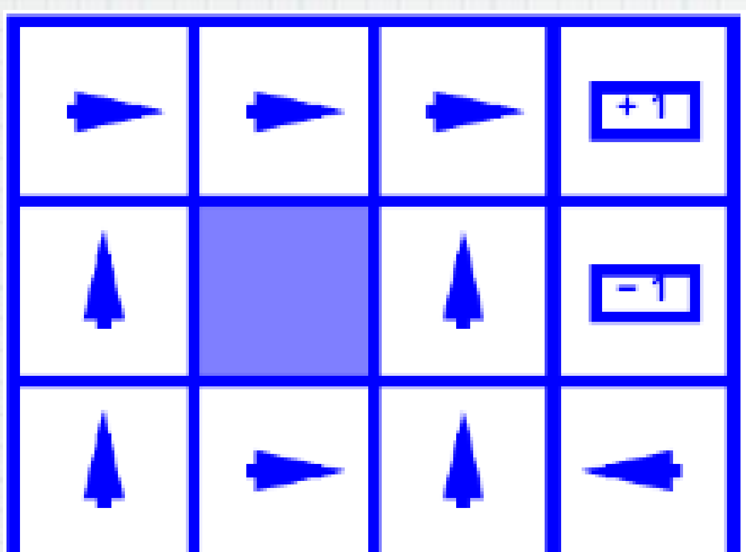
Example Optimal Policies



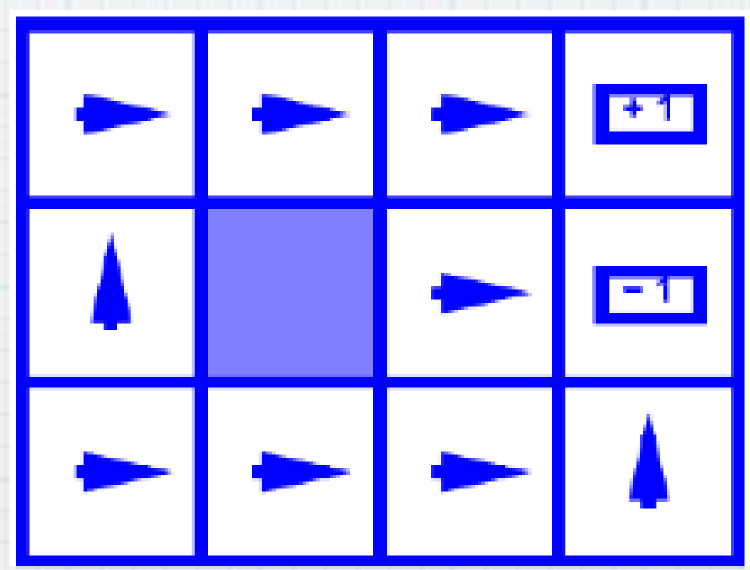
R(s) = -0.01



R(s) = -0.03



R(s) = -0.4



R(s) = -2.0

Summary + Preview

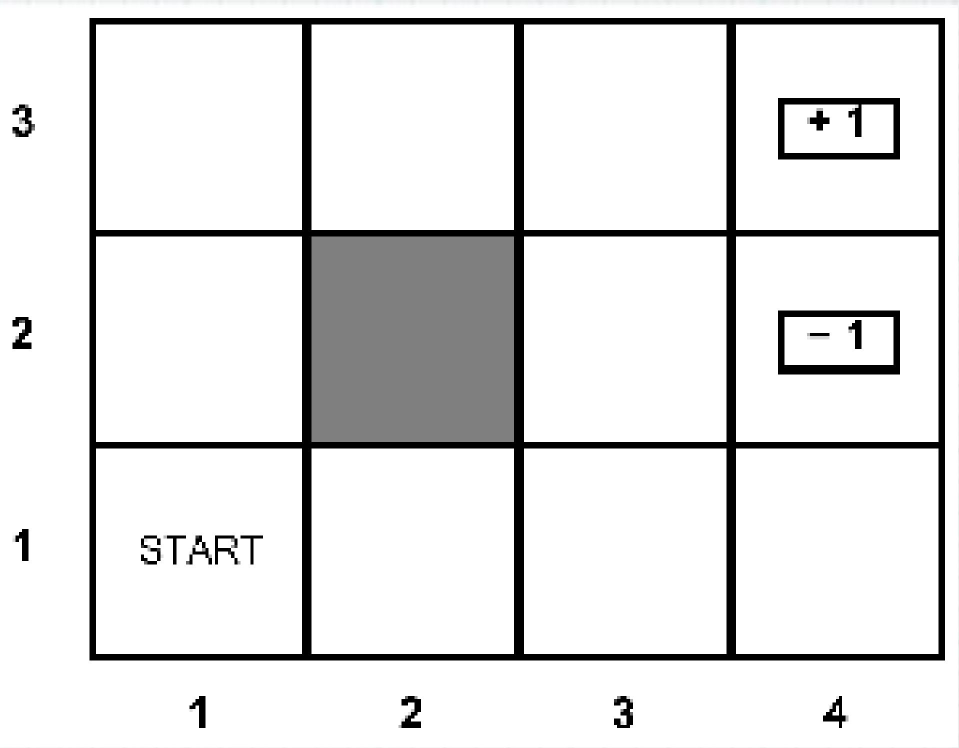
* Preferences & Utilities & Rewards
* Markov Decision Processes
* Next...
  * Two algorithms for solving an MDP
  * Partially observable states

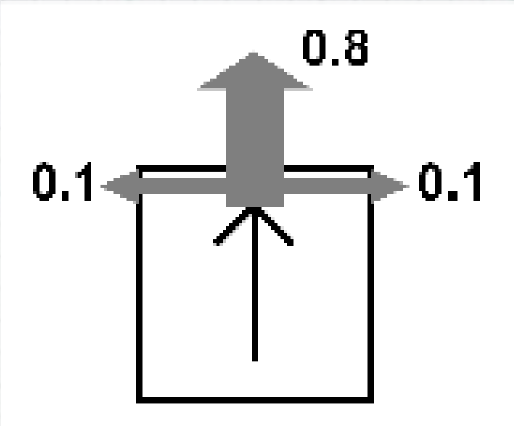
Making Decisions

* When agent will need to act in the same environment over and over, to achieve the same goal, with nondeterministic actions
* “Policy” of action instead of “Plans”

Markov Decision Processes

- MDP defined by
  - States s∈S
  - Actions a∈A
  - Transition function T(s,a,s’)
  - Reward function R(s,a,s’)





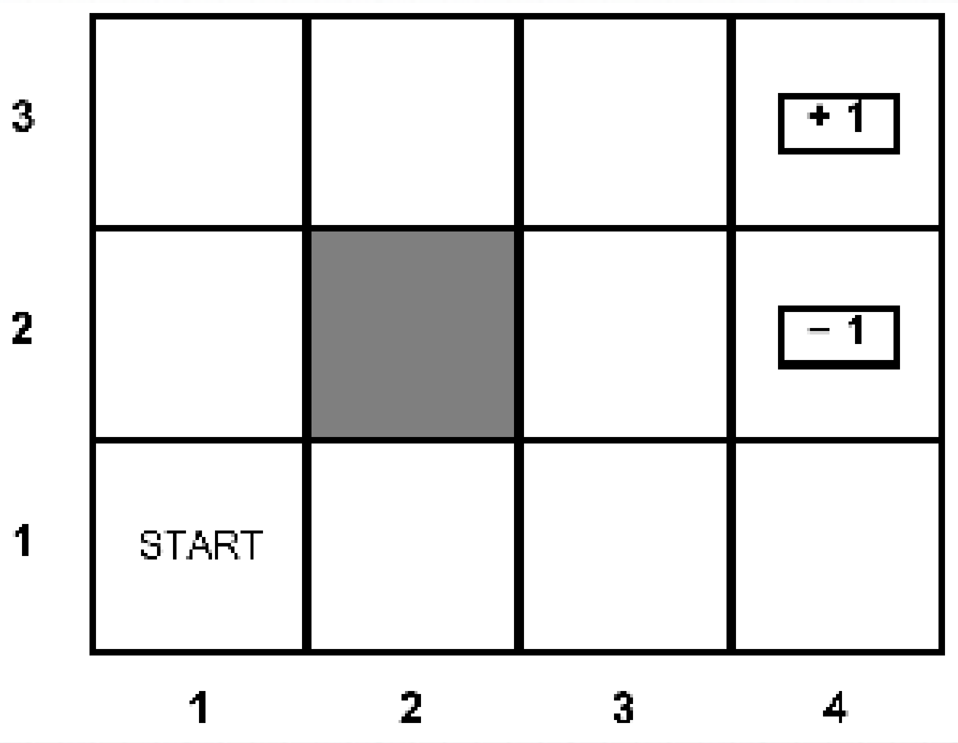
Trashcan Robot MDP

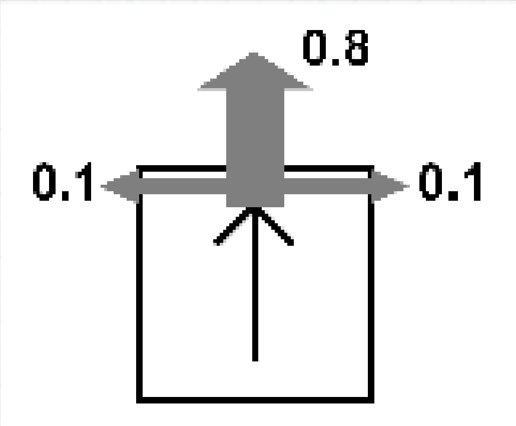
You are building a robot to move around an office building collecting trash (from tables, floors, people, etc.). Define how you would set up this task as a Markov Decision Process

- What is the state space?
- What are the actions needed?
- What should the rewards be?

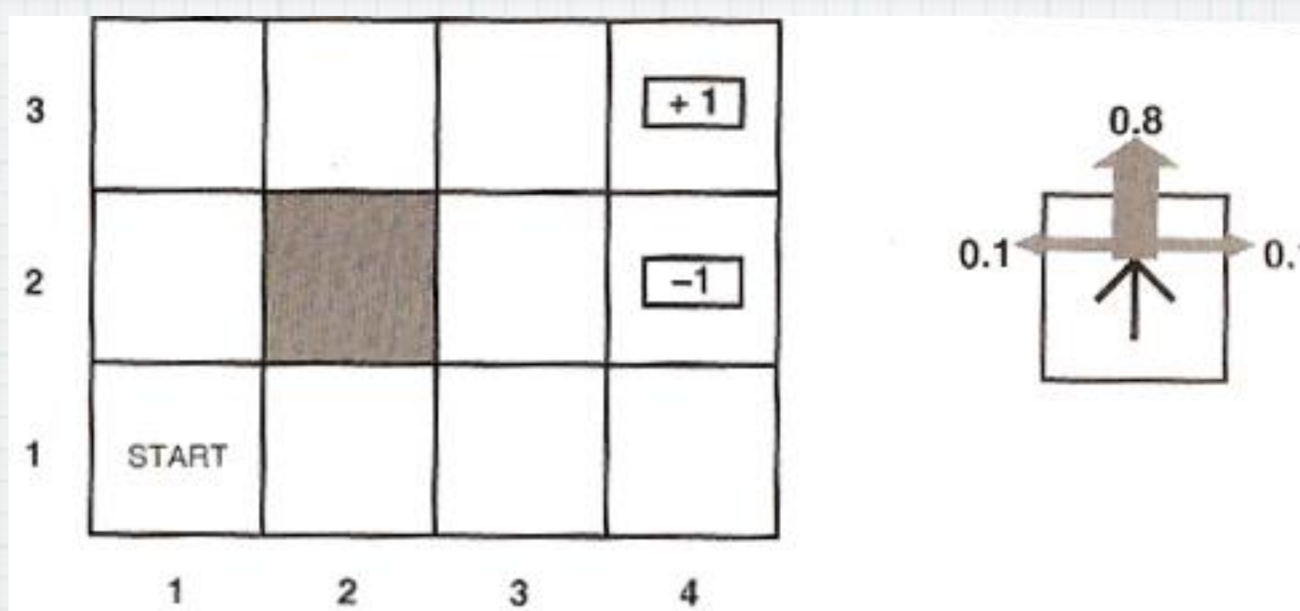
MDP Practice Exercise

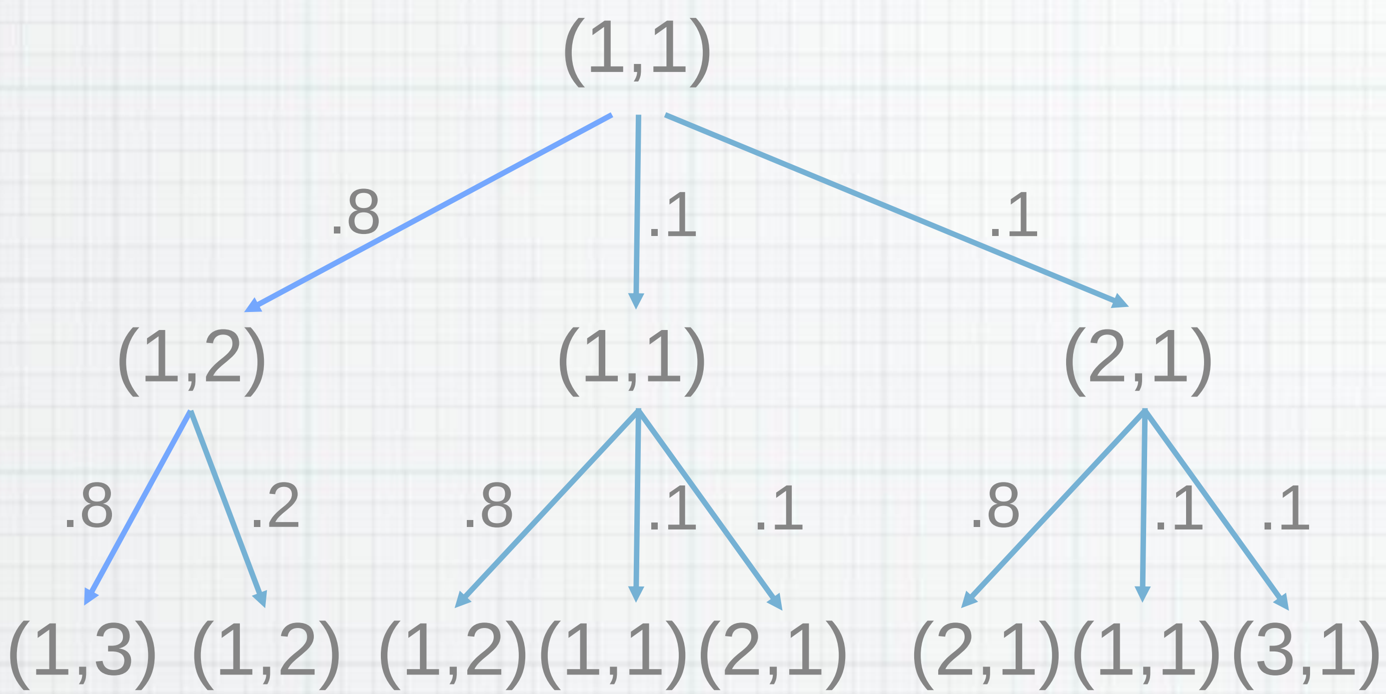
* What are the possible states the agent could be in after moving: a_1=North, a_2=North
* With what probabilities?





$$\begin{tikzcd} & (1,1) \arrow[dl, ".8"'] \arrow[d, ".1"] \arrow[dr, ".1"] & \\ (1,2) & (1,1) & (2,1) \end{tikzcd}$$




(1,1)
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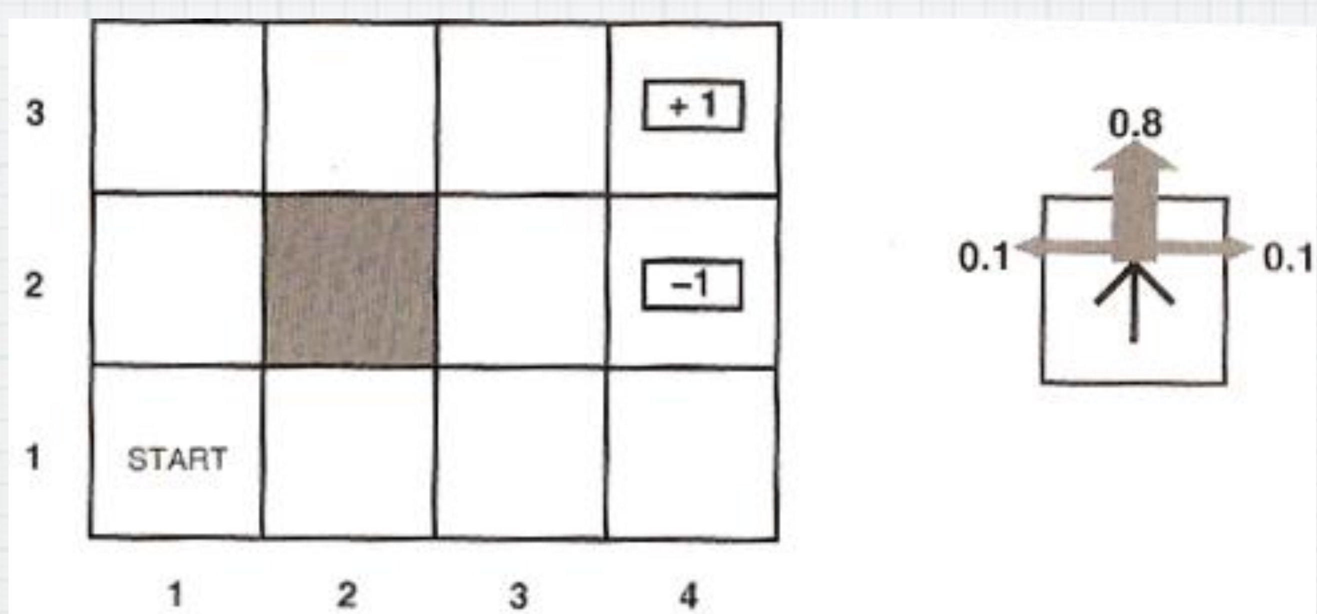
3
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START
1
2
3
4
0.8
0.1
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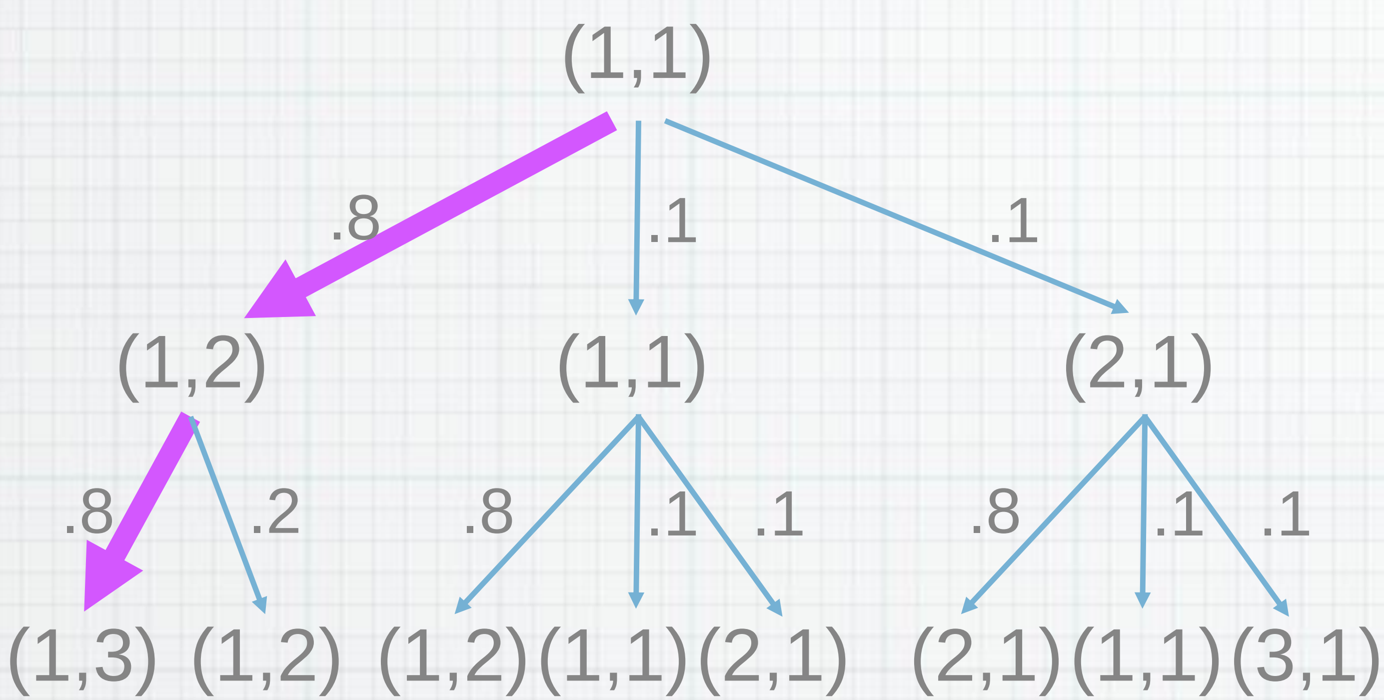
(1,1)
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(1,3)
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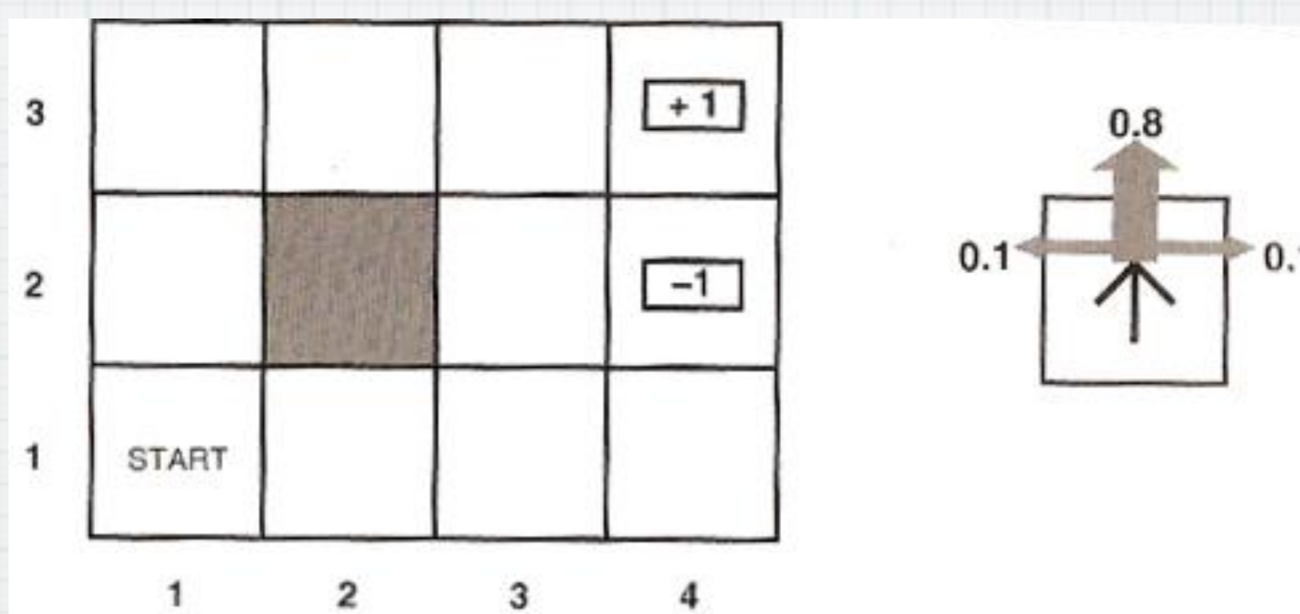
(2,1)
(2,1)
(1,1)
(3,1)

(1,3) = .8*.8 = .64


3
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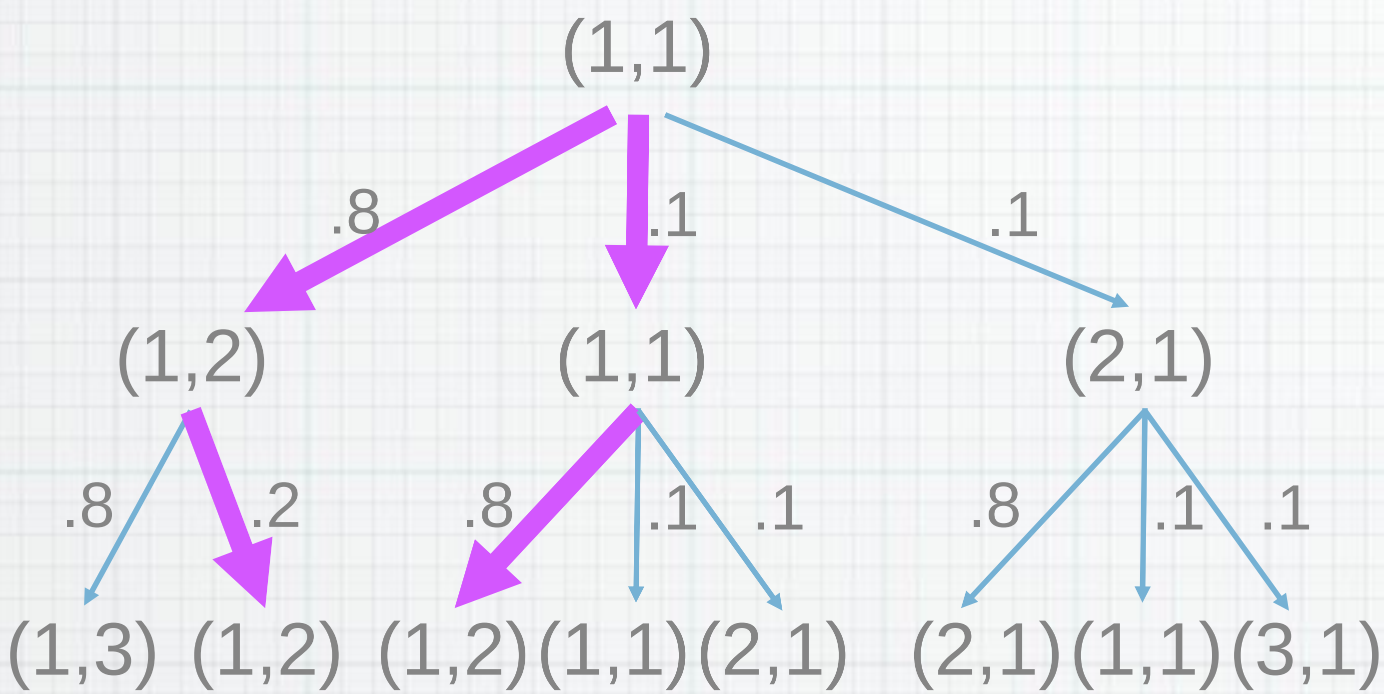
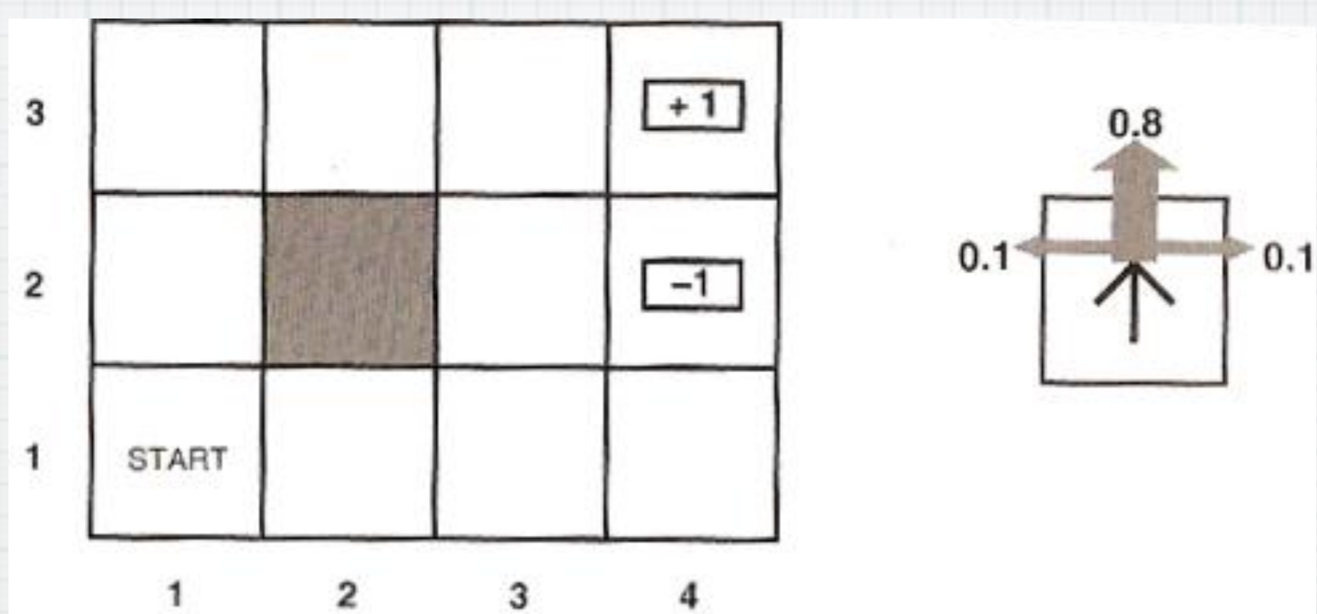
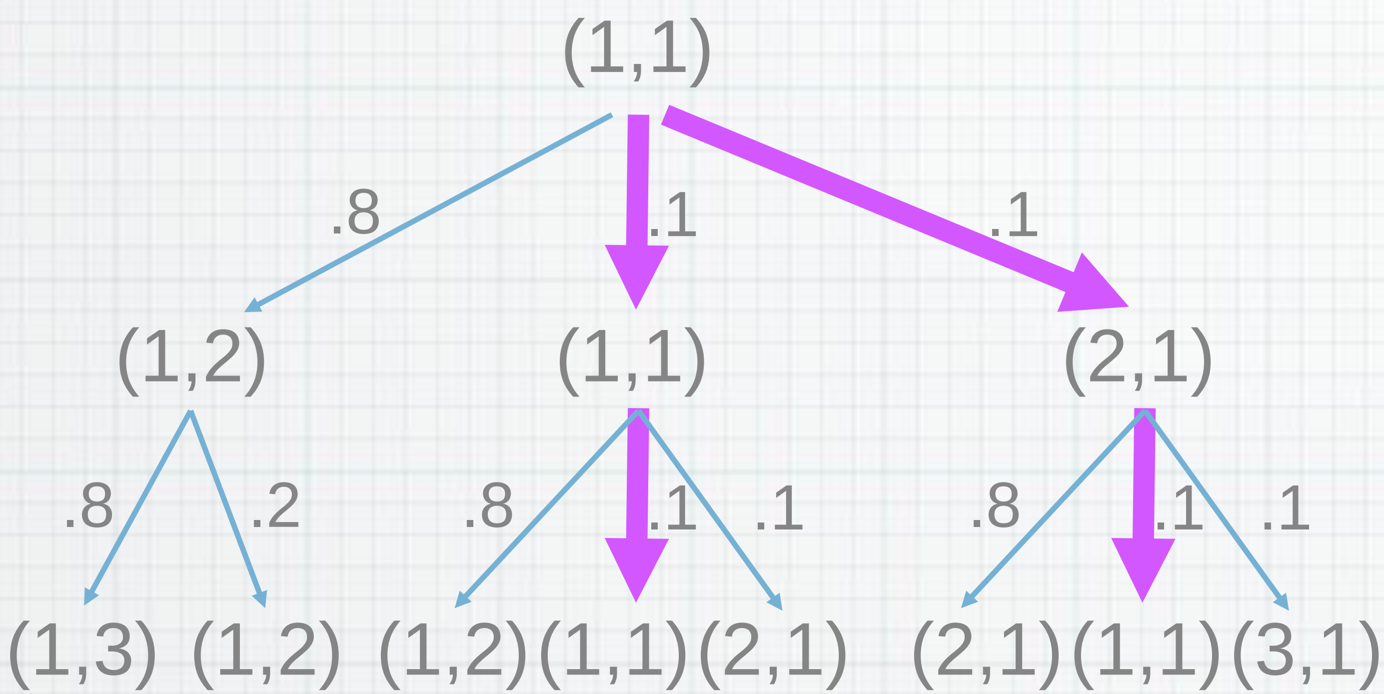
0.1



$(1,3) = .8*.8 = .64$

$(1,2) = .8*.2 + .1*.8 = .24$

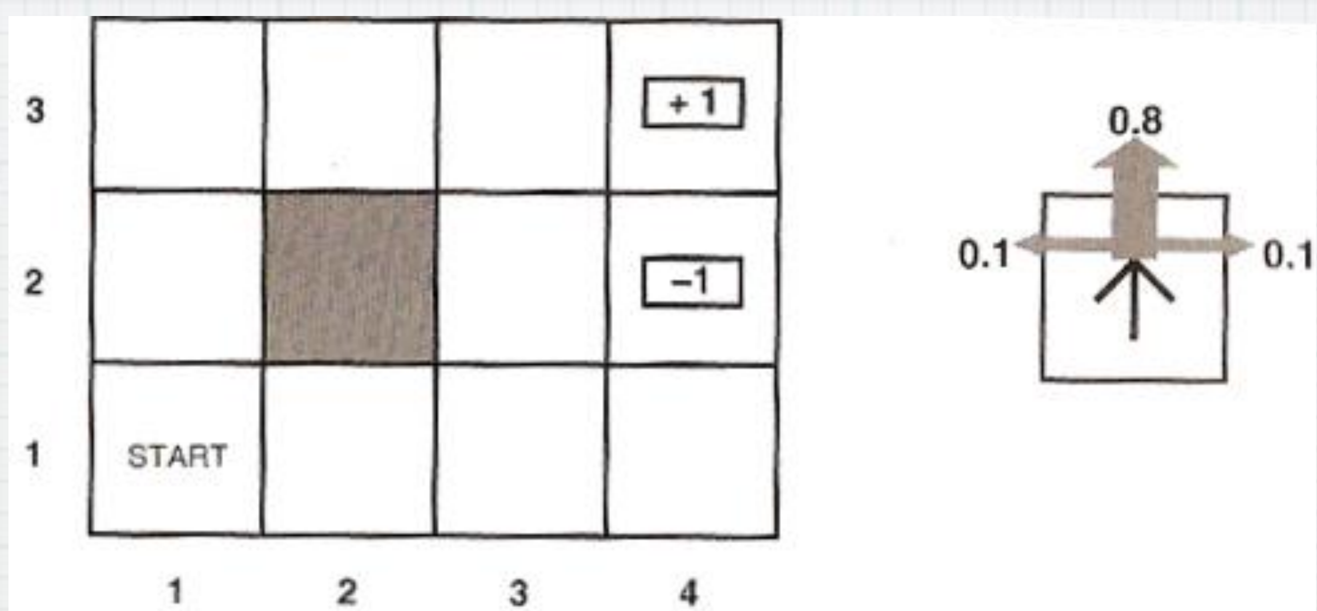


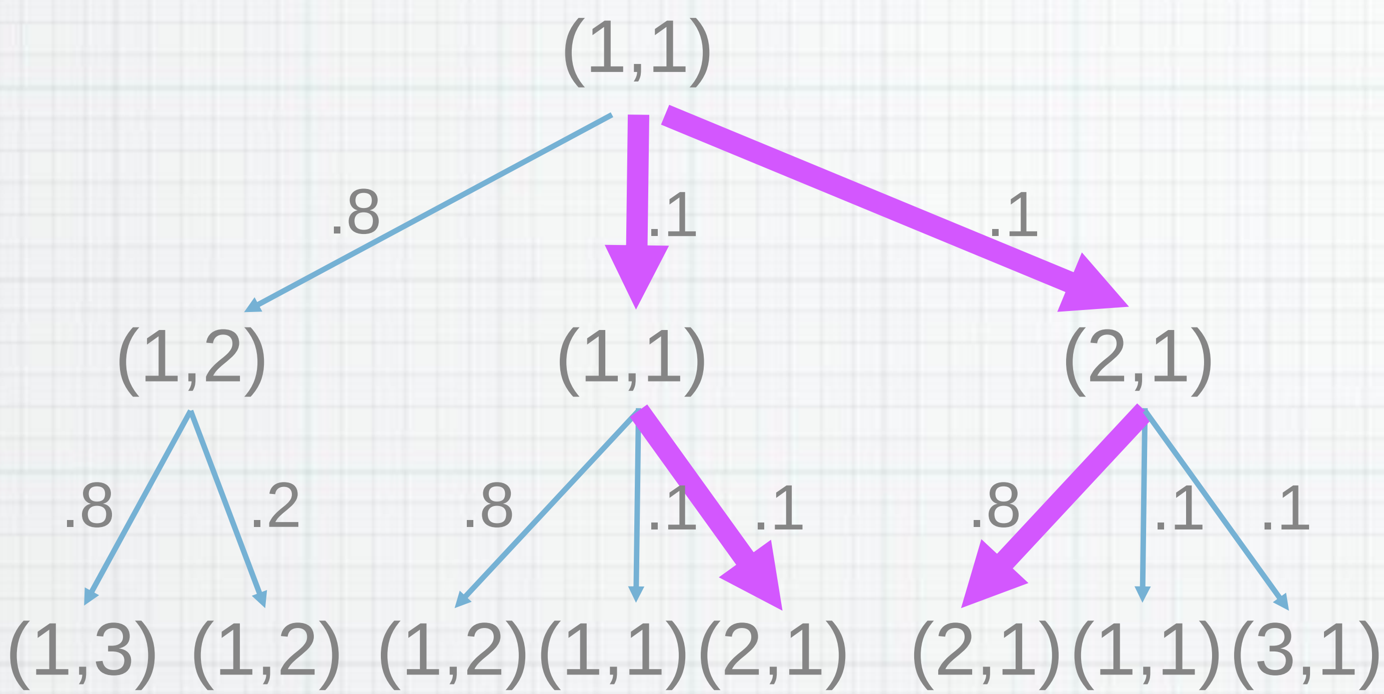


(1,3) = .8*.8 = .64

(1,2) = .8*.2 + .1*.8 = .24

(1,1) = .1*.1 + .1*.1 = .02



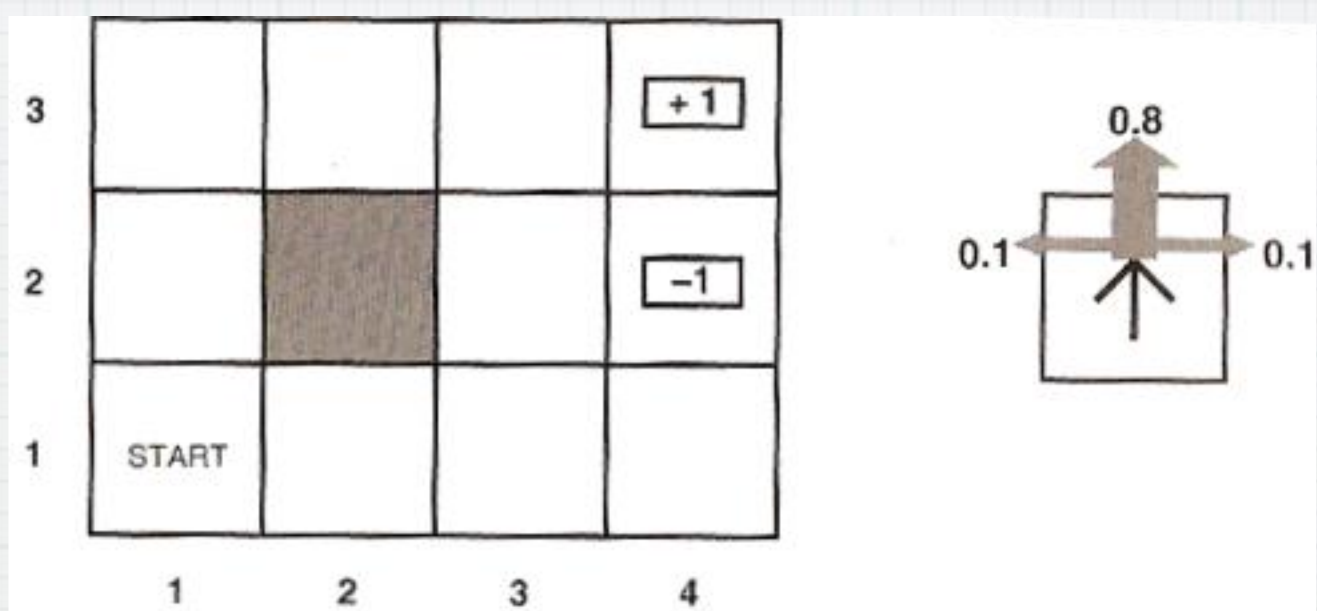


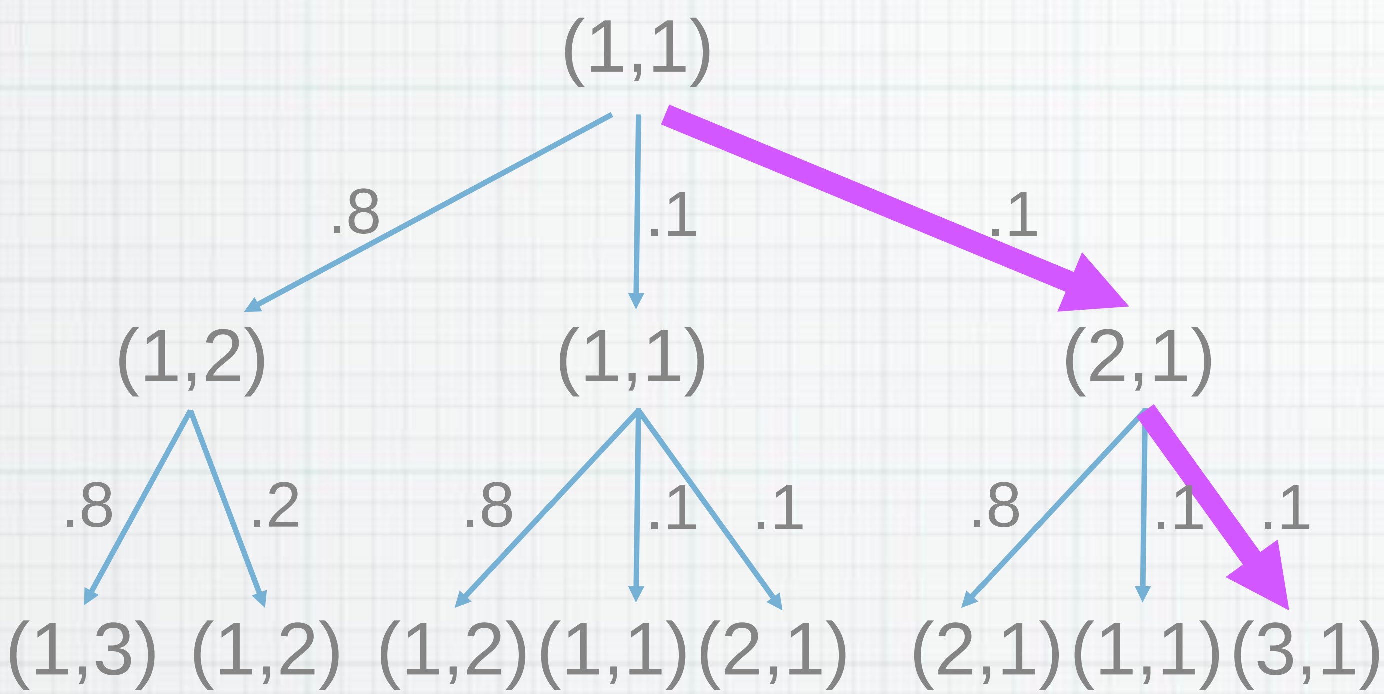
(1,3) = .8*.8 = .64

(1,2) = .8*.2 + .1*.8 = .24

(1,1) = .1*.1 + .1*.1 = .02

(2,1) = .1*.1 + .1*.8 = .09





P(s_2)

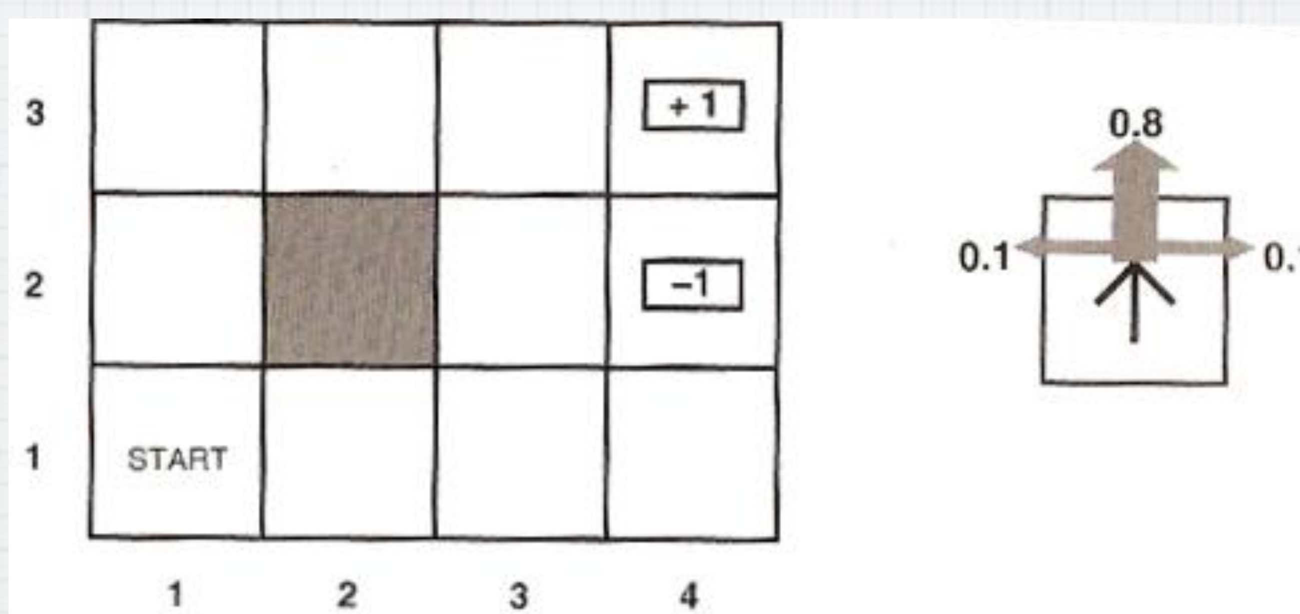
(1,3) = .8*.8 = .64

(1,2) = .8*.2 + .1*.8 = .24

(1,1) = .1*.1 + .1*.1 = .02

(2,1) = .1*.1 + .1*.8 = .09

(3,1) = .1*.1 = .01



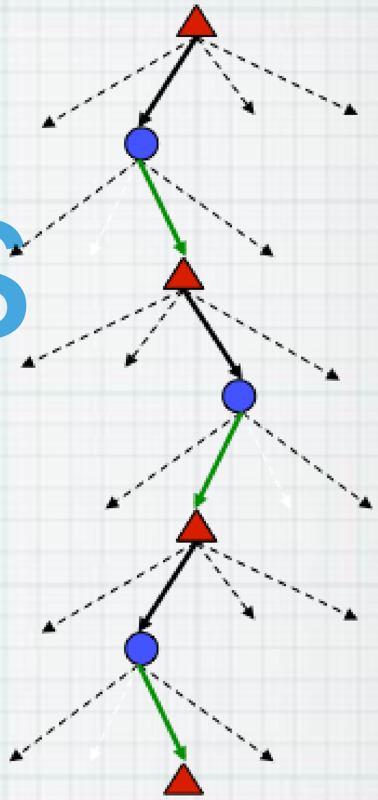
Markov Decision Process (MDP)

MDP Solution = policy: what action to do in every state

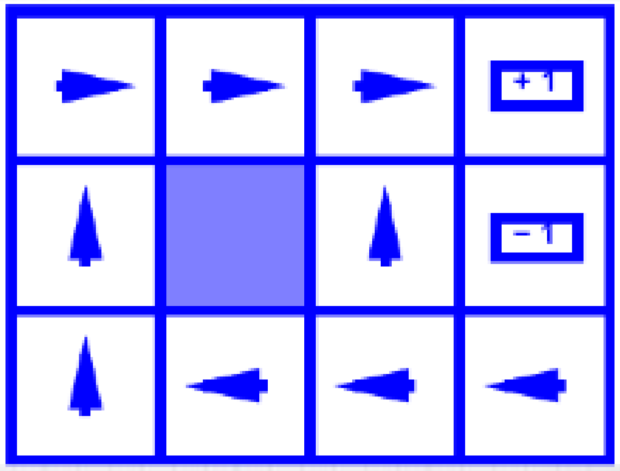
Expected Utility: the expected reward from executing a particular policy

Optimal Policy: has the highest expected utility

Utility of Sequences

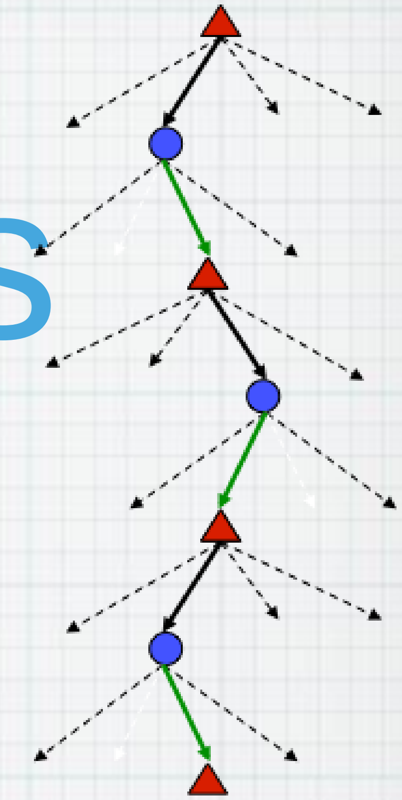


- Finite
  - fixed time the agent is around
  - the right action in a state depends on how much time left
- Infinite
  - no time limits, optimal action only depends on the state



R(s) = -0.03

Utility of Sequences



- Finite
  - fixed time the agent is around
  - the right action in a state depends on how much time left

Non-stationary

- Infinite
  - no time limits, optimal action only depends on the state

Stationary

Utilities of Sequences

- Problem: infinite sequences have infinite rewards

$$U([r_0, r_1, r_2, \ldots]) = r_0 + r_1 + r_2 + \cdots$$

- Solution?
    - cut-off sequences, to make finite
    - ...non-stationary policies

Utilities of Sequences

- Problem: infinite sequences have infinite rewards

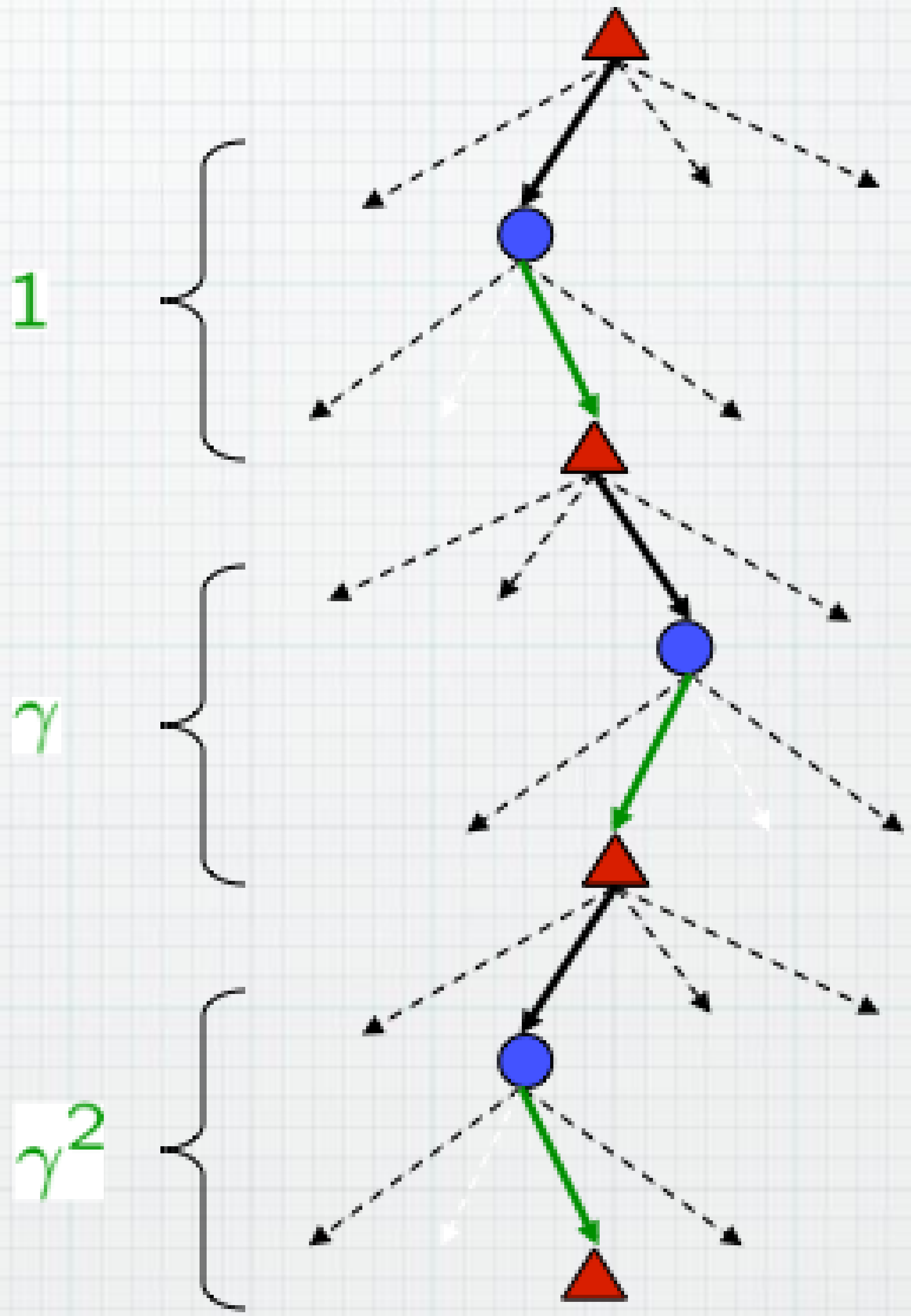
$$U([r_0, r_1, r_2, \ldots]) = r_0 + r_1 + r_2 + \cdots$$

- Solution: discount future rewards

$$U([r_0, r_1, r_2, \ldots]) = r_0 + \gamma r_1 + \gamma^2 r_2 \cdots$$

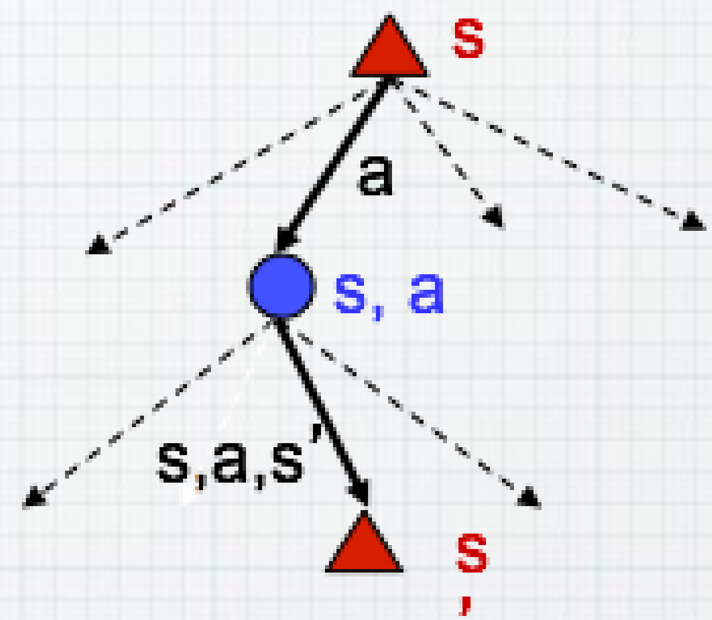
Discount Future

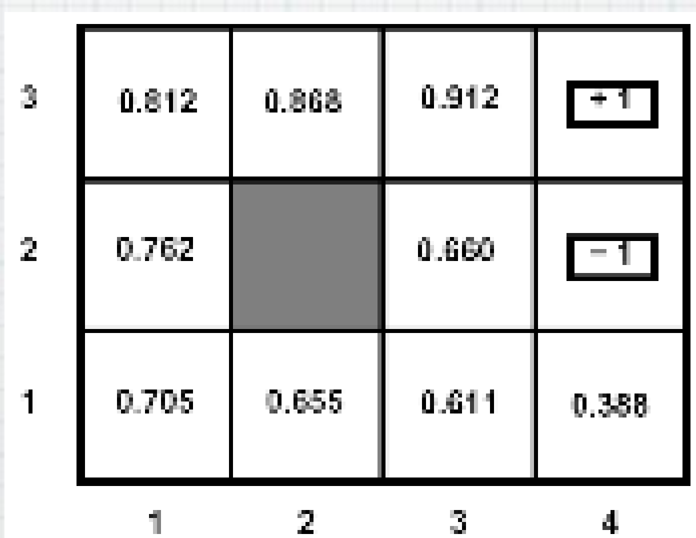
- Discount factor for each time step ɣ<1
- Earlier rewards have higher utility than later rewards
- With discounted rewards, the utility of an infinite sequence is finite

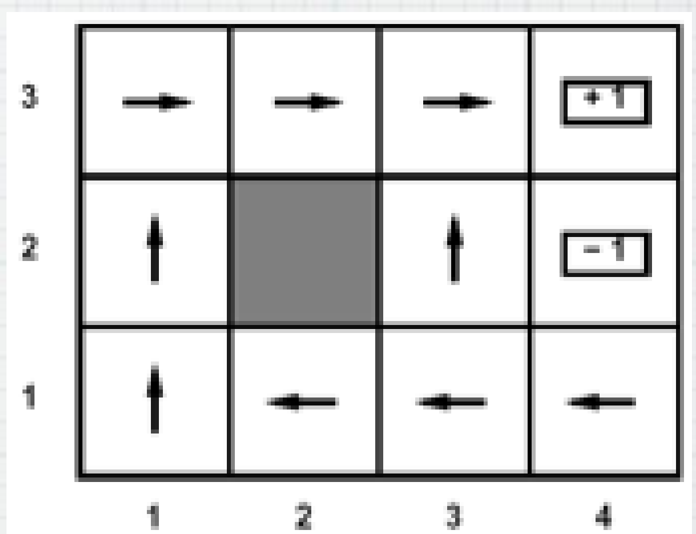


Optimal Values for States

- Optimal values define optimal policies
- U*(s) = expected utility starting in s and acting optimally
- Q*(s,a) = expected utility taking a in s, and then acting optimally







Calculating Policies

MDPs end up being a good representation for many real world problems.

Given an MDP description, several algorithms for solving for the optimal policy

Two general classes of algorithms:

Value Iteration and Policy Iteration

Optimal Utility

- Utility or Value of a state
  - Expected utility of the sequence to follow that state, with particular policy of action
  - U*(s) is the expected utility of following the optimal policy from s
- U(s) vs. R(s)

Optimal Policy

$$U^{\pi}(S) = E\left[\sum_{t=0}^{\infty} \gamma^t R(s_t)\right]$$

$$\pi_S^* = \arg\max_{\pi} U^{\pi}(S)$$

$$\pi_{s_t}^* = \arg\max_{a \in A(s_t)} \sum_{s_{t+1}} P(s_{t+1} \mid s_t, a) U(s_{t+1})$$

Expected utility