

Logical Agents

CH 9

Inference in First Order Logic

First-Order Logic

- * Constants, variables, functions, predicates
E.g.: Anna, x, MotherOf(x), Friends(x, y)
- * Literal: Predicate or its negation
- * Clause: Disjunction of literals
- * Grounding: Replace all variables by constants
E.g.: Friends (Anna, Bob)
- * World (model, interpretation):
Assignment of truth values to all ground predicates

Unification

- * Basic step in inference for FOL
- * Find a unifier (substitution) θ which makes two sentences p, q identical
- * $\text{UNIFY}(p, q) = \theta$ where $\text{SUBST}(\theta, p) = \text{SUBST}(\theta, q)$
- * Example unifications:
 - * $\text{Knows}(\text{John}, x) \ \& \ \text{Knows}(\text{John}, \text{Jane}) : \{x/\text{Jane}\}$
 - * $\text{Knows}(\text{John}, x) \ \& \ \text{Knows}(y, \text{Bill}) : \{x/\text{Bill}, y/\text{John}\}$
 - * $\text{Knows}(\text{John}, x) \ \& \ \text{Knows}(y, \text{Mother}(y)) :$
 $\{x/\text{Mother}(\text{John}), y/\text{John}\}$
 - * $\text{Knows}(\text{John}, x) \ \& \ \text{Knows}(x, \text{Elsie}) : \text{Fail}$

Unification Issues

- * $\text{Knows}(\text{John}, x) \ \& \ \text{Knows}(x, \text{Elsie})$: Fail
 - * Solve by standardizing apart
 - * $\text{Knows}(\text{John}, x) \ \& \ \text{Knows}(y, \text{Elsie})$: $\{x/\text{Elsie}, y/\text{John}\}$
- * Most general unifier
 - * $\text{Knows}(\text{John}, x) \ \& \ \text{Knows}(y, z)$:
 $\{y/\text{John}, x/z\}$ OR $\{y/\text{John}, x/\text{John}, z/\text{John}\}$
 $\text{Knows}(\text{John}, z) \ \text{Knows}(\text{John}, \text{John})$
 - * Choose most general unifier
 - * Always exists and is unique up to renaming/subst

Inference Approaches

- * Forward chaining
 - * Backward chaining
 - * Resolution
-
- * (See book for details, *these methods will not be on the final*)

Wumpus FOL

Interacting with FOL KBs

- * Suppose Wumpus agent perceives smell breeze and glitter at time $t=5$

Tell(KB, Percept([Smell, Breeze, Glitter], 5))

- * Does the KB entail any particular action?

Ask(KB, $\exists a$ BestAction(a, 5))

- * The query returns a substitution

Answer: {a/Grab}

*** Perception sentences:**

$\forall bgt \text{ Percept}([\text{Smell}, b, g], t) \Rightarrow \text{Smell}(t)$

$\forall sbt \text{ Percept}([s, b, \text{Glitter}], t) \Rightarrow \text{AtGold}(t)$

*** Reflex action: grab gold:**

$\forall t \text{ AtGold}(t) \Rightarrow \text{Action}(\text{Grab}, t)$

*** Something is at square s at time t:**

$\text{At}(a, s, t)$

*** Wumpus is in a fixed location:**

$\forall t \text{ At}(\text{Wumpus}, [3, 1], t)$

*** There is a pit in square s:**

$\text{Pit}(s)$

- * Definition of two squares being **adjacent**:

$$\forall x,y,a,b \text{ Adjacent}([x,y],[a,b]) \Leftrightarrow \\ (x = a \wedge (y=b-1 \vee y=b+1)) \vee (y=b \wedge (x=a-1 \vee x=a+1))$$

- * If an agent perceives breeze in a square then that square is breezy:

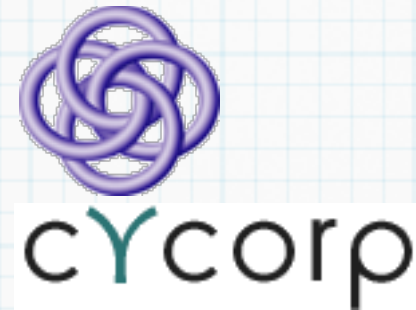
$$\forall s,t \text{ At}(\text{Agent},s,t) \wedge \text{Breeze}(t) \Rightarrow \text{Breezy}(s)$$

- * Breeze means pit in adjacent square:

$$\forall s \text{ Breezy}(s) \Leftrightarrow \exists r \text{ Adjacent}(s, r) \wedge \text{Pit}(r)$$

Knowledge Engineering

- * Deciding how to best write down the concepts of a domain
- * Decide on the vocabulary of symbols, predicates and functions
- * Put all the sentences of the domain in your KB (and debug) so that some expected inferences can be made



The Cyc knowledge base (KB) is a formalized representation of a vast quantity of fundamental human knowledge: facts, rules of thumb, and heuristics for reasoning about the objects and events of everyday life.

OpenCyc is the open source version of the Cyc technology, the world's largest and most complete general knowledge base and commonsense reasoning engine.

Inference in FOL

Chapter 9

Inference & Quantifiers

- * One idea: Apply simple rules to remove quantifiers, then FOL is converted to PL
- * Better idea: Inference methods that operate on FOL sentences directly

Universal Instantiation

Every instantiation of a universally quantified sentence is entailed by it:

$$\frac{\forall v \alpha}{\text{SUBST}(\{v/g\}, \alpha)}$$

for any variable v and ground term g

E.g., $\forall x \text{ King}(x) \wedge \text{Greedy}(x) \Rightarrow \text{Evil}(x)$ yields

$$\text{King}(\text{John}) \wedge \text{Greedy}(\text{John}) \Rightarrow \text{Evil}(\text{John})$$

$$\text{King}(\text{Richard}) \wedge \text{Greedy}(\text{Richard}) \Rightarrow \text{Evil}(\text{Richard})$$

$$\text{King}(\text{Father}(\text{John})) \wedge \text{Greedy}(\text{Father}(\text{John})) \Rightarrow \text{Evil}(\text{Father}(\text{John}))$$

⋮

Existential Instantiation

For any sentence α , variable v , and constant symbol k
that does not appear elsewhere in the knowledge base:

$$\frac{\exists v \alpha}{\text{SUBST}(\{v/k\}, \alpha)}$$

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E.g., $\exists x \text{Crown}(x) \wedge \text{OnHead}(x, \text{John})$ yields

$$\text{Crown}(C_1) \wedge \text{OnHead}(C_1, \text{John})$$

provided C_1 is a new constant symbol, called a **Skolem constant**

Inference & Quantifiers

- * **Universal Instantiation:** can be applied several times to **add** new sentences; new KB logically equivalent to the old
- * **Existential Instantiation:** apply once to **replace** the existential sentence; new KB is not equivalent, but if a sentence is satisfiable in the new KB also in the old

Reduce to Prop. Logic

Suppose the KB contains just the following:

$\forall x \text{ King}(x) \wedge \text{Greedy}(x) \Rightarrow \text{Evil}(x)$

$\text{King}(\text{John})$

$\text{Greedy}(\text{John})$

$\text{Brother}(\text{Richard}, \text{John})$

Reduce to Prop. Logic

Suppose the KB contains just the following:

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 $\text{King}(\text{John})$
 $\text{Greedy}(\text{John})$
 $\text{Brother}(\text{Richard}, \text{John})$

Instantiating the universal sentence in **all possible** ways, we have

$\text{King}(\text{John}) \wedge \text{Greedy}(\text{John}) \Rightarrow \text{Evil}(\text{John})$
 $\text{King}(\text{Richard}) \wedge \text{Greedy}(\text{Richard}) \Rightarrow \text{Evil}(\text{Richard})$
 $\text{King}(\text{John})$
 $\text{Greedy}(\text{John})$
 $\text{Brother}(\text{Richard}, \text{John})$

The new KB is **propositionalized**: proposition symbols are

$\text{King}(\text{John}), \text{Greedy}(\text{John}), \text{Evil}(\text{John}), \text{King}(\text{Richard})$ etc.

Problem with Propositionalization

Propositionalization seems to generate lots of irrelevant sentences.

E.g., from

$\forall x \text{ King}(x) \wedge \text{Greedy}(x) \Rightarrow \text{Evil}(x)$

$\text{King}(\text{John})$

$\forall y \text{ Greedy}(y)$

$\text{Brother}(\text{Richard}, \text{John})$

it seems obvious that $\text{Evil}(\text{John})$, but propositionalization produces lots of facts such as $\text{Greedy}(\text{Richard})$ that are irrelevant

With p k -ary predicates and n constants, there are $p \cdot n^k$ instantiations

With function symbols, it gets much much worse!

Let's just instantiate what is relevant to our query!

Unification

We can get the inference immediately if we can find a substitution θ such that $King(x)$ and $Greedy(x)$ match $King(John)$ and $Greedy(y)$

$\theta = \{x/John, y/John\}$ works

$UNIFY(\alpha, \beta) = \theta$ if $\alpha\theta = \beta\theta$

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p	q	θ
$Knows(John, x)$	$Knows(John, Jane)$	
$Knows(John, x)$	$Knows(y, OJ)$	
$Knows(John, x)$	$Knows(y, Mother(y))$	
$Knows(John, x)$	$Knows(x, OJ)$	

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p	q	θ
$Knows(John, x)$	$Knows(John, Jane)$	$\{x/Jane\}$
$Knows(John, x)$	$Knows(y, OJ)$	$\{x/OJ, y/John\}$
$Knows(John, x)$	$Knows(y, Mother(y))$	$\{y/John, x/Mother(John)\}$
$Knows(John, x)$	$Knows(x, OJ)$	fail

Standardizing apart eliminates overlap of variables, e.g., $Knows(z_{17}, OJ)$

Generalized Modus Ponens

$$p_1', p_2', \dots, p_n', \frac{(p_1 \wedge p_2 \wedge \dots \wedge p_n \Rightarrow q)}{q\theta}$$

where $p_i'\theta = p_i\theta$ for all i

Generalized Modus Ponens

$$\frac{p_1', p_2', \dots, p_n', (p_1 \wedge p_2 \wedge \dots \wedge p_n \Rightarrow q)}{q\theta} \quad \text{where } p_i'\theta = p_i\theta \text{ for all } i$$

$$King(John) \quad \forall y \, Greedy(y) \quad \forall x \, King(x) \wedge Greedy(x) \Rightarrow Evil(x)$$

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$$King(John) \quad \forall y Greedy(y) \quad \forall x King(x) \wedge Greedy(x) \Rightarrow Evil(x)$$

p_1' is $King(John)$

p_2' is $Greedy(y)$

p_1 is $King(x)$

p_2 is $Greedy(x)$

q is $Evil(x)$

Generalized Modus Ponens

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$$King(John) \quad \forall y \text{ Greedy}(y) \quad \forall x \text{ King}(x) \wedge \text{Greedy}(x) \Rightarrow \text{Evil}(x)$$

$$\begin{array}{l} p_1' \text{ is } King(John) \\ p_2' \text{ is } Greedy(y) \end{array}$$

$$\begin{array}{l} p_1 \text{ is } King(x) \\ p_2 \text{ is } Greedy(x) \end{array}$$

$$q \text{ is } Evil(x)$$

$$\begin{array}{l} \theta \text{ is } \{x/John, y/John\} \\ q\theta \text{ is } Evil(John) \end{array}$$

Forward Chaining in FOL

FOL Forward Chaining

```
function FOL-FC-Ask( $KB, \alpha$ ) returns a substitution or false
  repeat until new is empty
     $new \leftarrow \{ \}$ 
    for each sentence  $r$  in  $KB$  do
       $(p_1 \wedge \dots \wedge p_n \Rightarrow q) \leftarrow \text{STANDARDIZE-APART}(r)$ 
      for each  $\theta$  such that  $(p_1 \wedge \dots \wedge p_n)\theta = (p'_1 \wedge \dots \wedge p'_n)\theta$ 
        for some  $p'_1, \dots, p'_n$  in  $KB$ 
           $q' \leftarrow \text{SUBST}(\theta, q)$ 
          if  $q'$  is not a renaming of a sentence already in  $KB$  or new then do
            add  $q'$  to new
             $\phi \leftarrow \text{UNIFY}(q', \alpha)$ 
            if  $\phi$  is not fail then return  $\phi$ 
    add new to  $KB$ 
  return false
```

Summary Ch 8+9

- * First Order Logic lets us talk about objects, properties, and relations
- * Much more expressive than ProLog
- * Variables and Quantifiers require new inference rules
- * Unification finds a substitution of variables consistent with the desired inference
- * Generalized Modus Ponens for Forward/Backward chaining in FOL

Example Knowledge Base

The law says that it is a crime for an American to sell weapons to hostile nations. The country Nono, an enemy of America, has some missiles, and all of its missiles were sold to it by Colonel West, who is American.

Prove that Col. West is a criminal

Example Knowledge Base

... it is a crime for an American to sell weapons to hostile nations:

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$American(x) \wedge Weapon(y) \wedge Sells(x, y, z) \wedge Hostile(z) \Rightarrow Criminal(x)$

Nono ... has some missiles

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Nono ... has some missiles, i.e., $\exists x Owns(Nono, x) \wedge Missile(x)$:

$Owns(Nono, M_1)$ and $Missile(M_1)$

... all of its missiles were sold to it by Colonel West

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$\forall x Missile(x) \wedge Owns(Nono, x) \Rightarrow Sells(West, x, Nono)$

Missiles are weapons:

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Missiles are weapons:

$Missile(x) \Rightarrow Weapon(x)$

An enemy of America counts as "hostile":

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Missiles are weapons:

$Missile(x) \Rightarrow Weapon(x)$

An enemy of America counts as "hostile":

$Enemy(x, America) \Rightarrow Hostile(x)$

West, who is American ...

$American(West)$

The country Nono, an enemy of America ...

$Enemy(Nono, America)$

... it is a crime for an American to sell weapons to hostile nations:

$$\text{American}(x) \wedge \text{Weapon}(y) \wedge \text{Sells}(x, y, z) \wedge \text{Hostile}(z) \Rightarrow \text{Criminal}(x)$$

Nono ... has some missiles, i.e., $\exists x \text{ Owns}(\text{Nono}, x) \wedge \text{Missile}(x)$:

$$\text{Owns}(\text{Nono}, M_1) \text{ and } \text{Missile}(M_1)$$

... all of its missiles were sold to it by Colonel West

$$\forall x \text{ Missile}(x) \wedge \text{Owns}(\text{Nono}, x) \Rightarrow \text{Sells}(\text{West}, x, \text{Nono})$$

Missiles are weapons:

$$\text{Missile}(x) \Rightarrow \text{Weapon}(x)$$

An enemy of America counts as "hostile":

$$\text{Enemy}(x, \text{America}) \Rightarrow \text{Hostile}(x)$$

West, who is American ...

$$\text{American}(\text{West})$$

The country Nono, an enemy of America ...

$$\text{Enemy}(\text{Nono}, \text{America})$$

American(West)

Missile(M1)

Owns(Nono, M1)

Enemy(Nono, America)

... it is a crime for an American to sell weapons to hostile nations:

$$\text{American}(x) \wedge \text{Weapon}(y) \wedge \text{Sells}(x, y, z) \wedge \text{Hostile}(z) \Rightarrow \text{Criminal}(x)$$

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$$\text{Owns}(\text{Nono}, M_1) \text{ and } \text{Missile}(M_1)$$

... all of its missiles were sold to it by Colonel West

$$\forall x \text{ Missile}(x) \wedge \text{Owns}(\text{Nono}, x) \Rightarrow \text{Sells}(\text{West}, x, \text{Nono}) \quad \mathbf{x = M1}$$

Missiles are weapons:

$$\text{Missile}(x) \Rightarrow \text{Weapon}(x) \quad \mathbf{x = M1}$$

An enemy of America counts as "hostile":

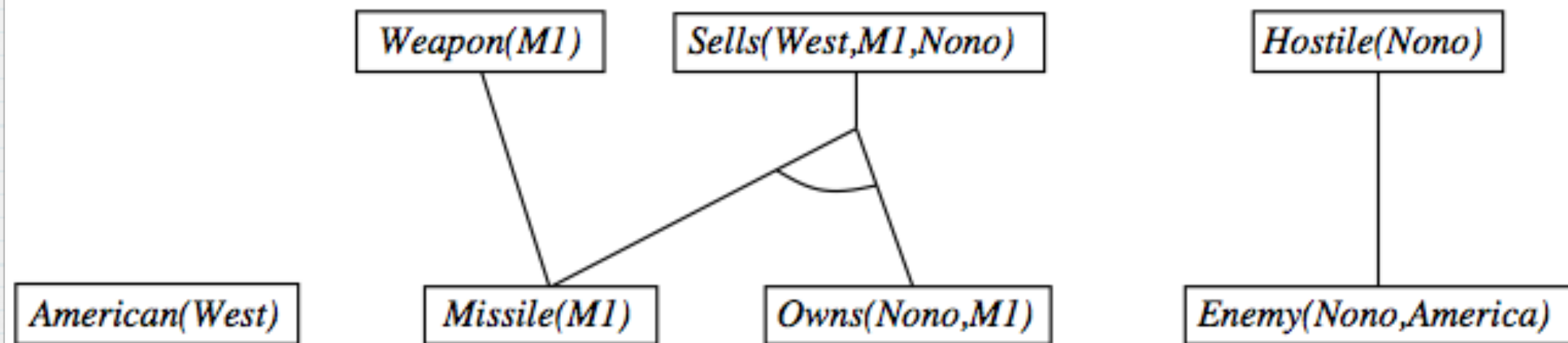
$$\text{Enemy}(x, \text{America}) \Rightarrow \text{Hostile}(x) \quad \mathbf{x = \text{Nono}}$$

West, who is American ...

$$\text{American}(\text{West})$$

The country Nono, an enemy of America ...

$$\text{Enemy}(\text{Nono}, \text{America})$$



... it is a crime for an American to sell weapons to hostile nations:

$American(x) \wedge Weapon(y) \wedge Sells(x, y, z) \wedge Hostile(z) \Rightarrow Criminal(x)$

$x = West$

Nono ... has some missiles, i.e., $\exists x Owns(Nono, x) \wedge Missile(x)$:

$Owns(Nono, M_1)$ and $Missile(M_1)$

... all of its missiles were sold to it by Colonel West

$\forall x Missile(x) \wedge Owns(Nono, x) \Rightarrow Sells(West, x, Nono)$

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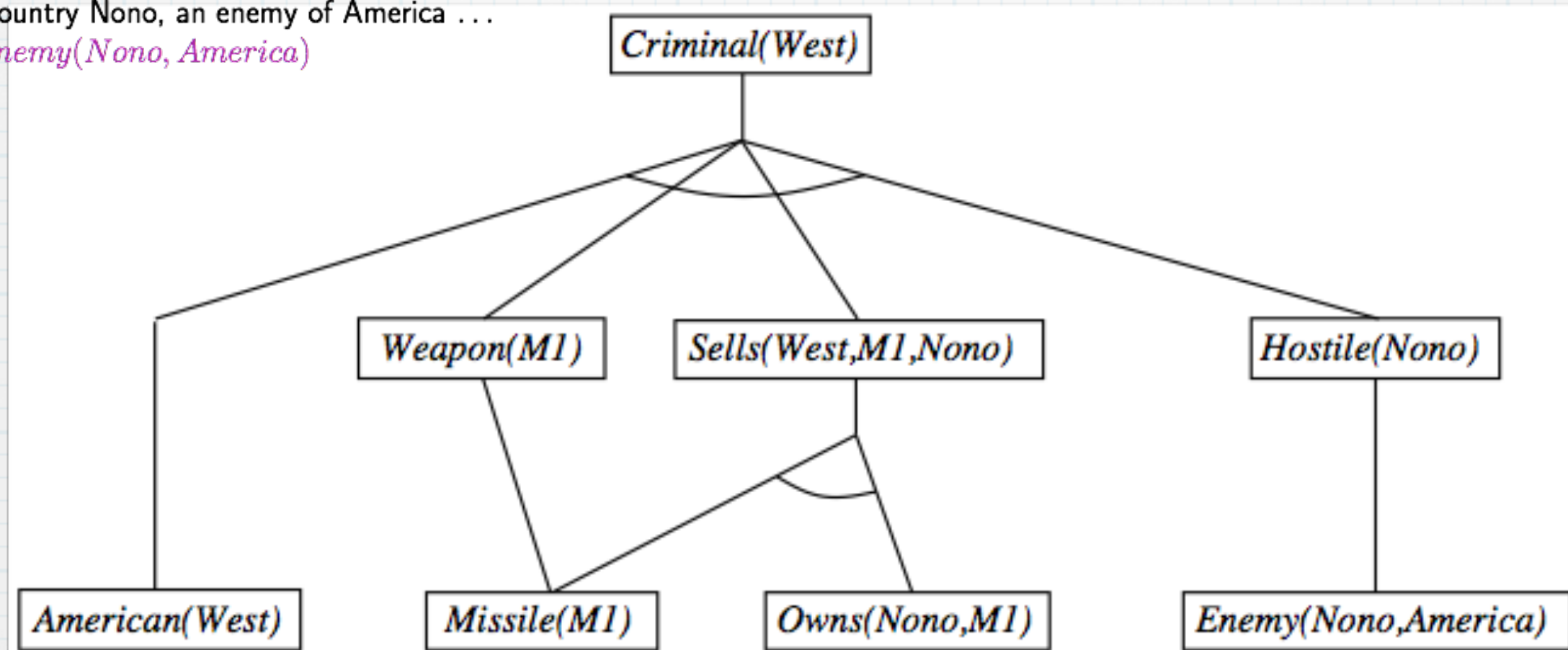
$Enemy(x, America) \Rightarrow Hostile(x)$

West, who is American ...

$American(West)$

The country Nono, an enemy of America ...

$Enemy(Nono, America)$



Sound and complete for first-order definite clauses
(proof similar to propositional proof)

Datalog = first-order definite clauses + **no functions** (e.g., crime KB)

FC terminates for Datalog in poly iterations: at most $p \cdot n^k$ literals

May not terminate in general if α is not entailed

This is unavoidable: entailment with definite clauses is semidecidable

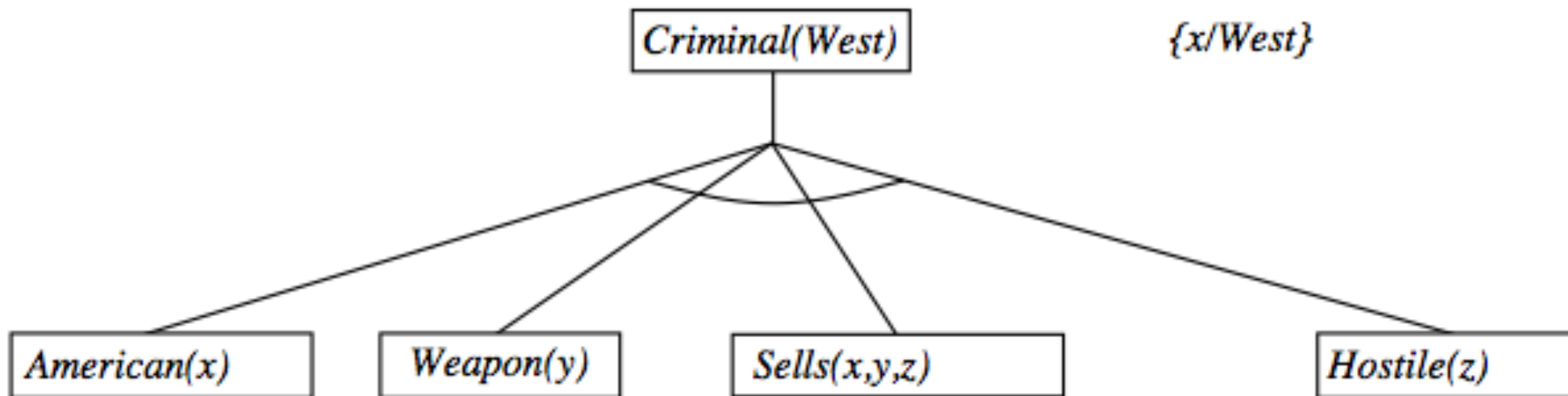
Backward chaining algorithm

```
function FOL-BC-Ask(KB, goals,  $\theta$ ) returns a set of substitutions
  inputs: KB, a knowledge base
           goals, a list of conjuncts forming a query ( $\theta$  already applied)
            $\theta$ , the current substitution, initially the empty substitution  $\{ \}$ 
  local variables: answers, a set of substitutions, initially empty
  if goals is empty then return  $\{ \theta \}$ 
   $q' \leftarrow \text{SUBST}(\theta, \text{FIRST}(\textit{goals}))$ 
  for each sentence r in KB
    where  $\text{STANDARDIZE-APART}(r) = (p_1 \wedge \dots \wedge p_n \Rightarrow q)$ 
    and  $\theta' \leftarrow \text{UNIFY}(q, q')$  succeeds
     $\textit{new goals} \leftarrow [p_1, \dots, p_n | \text{REST}(\textit{goals})]$ 
     $\textit{answers} \leftarrow \text{FOL-BC-ASK}(\textit{KB}, \textit{new goals}, \text{COMPOSE}(\theta', \theta)) \cup \textit{answers}$ 
  return answers
```

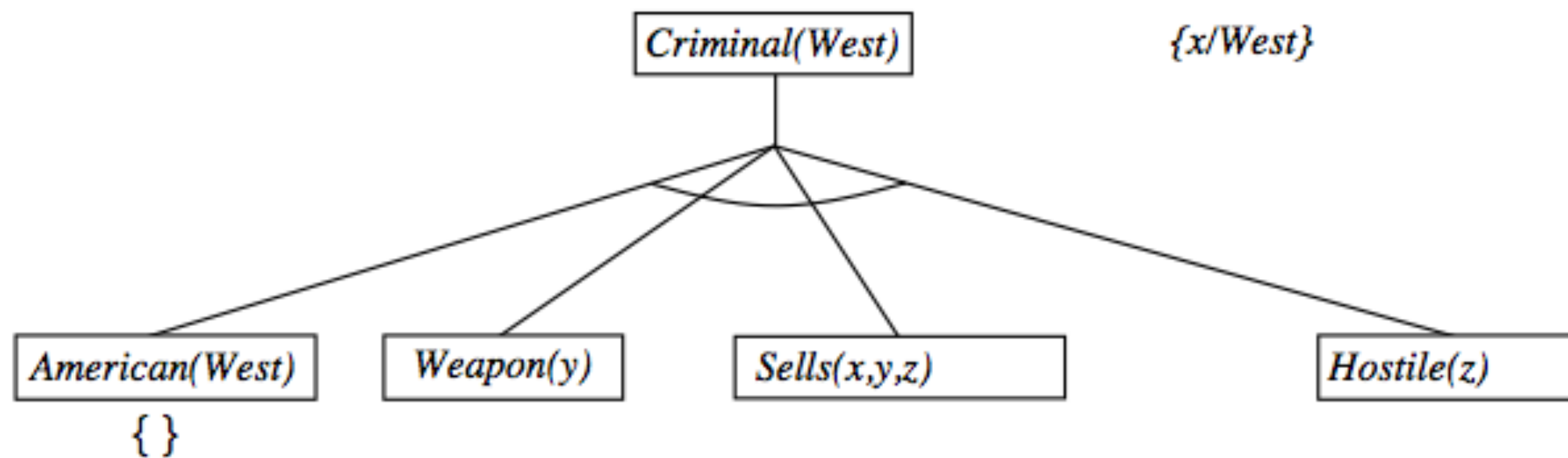
Backward Chaining

Criminal(West)

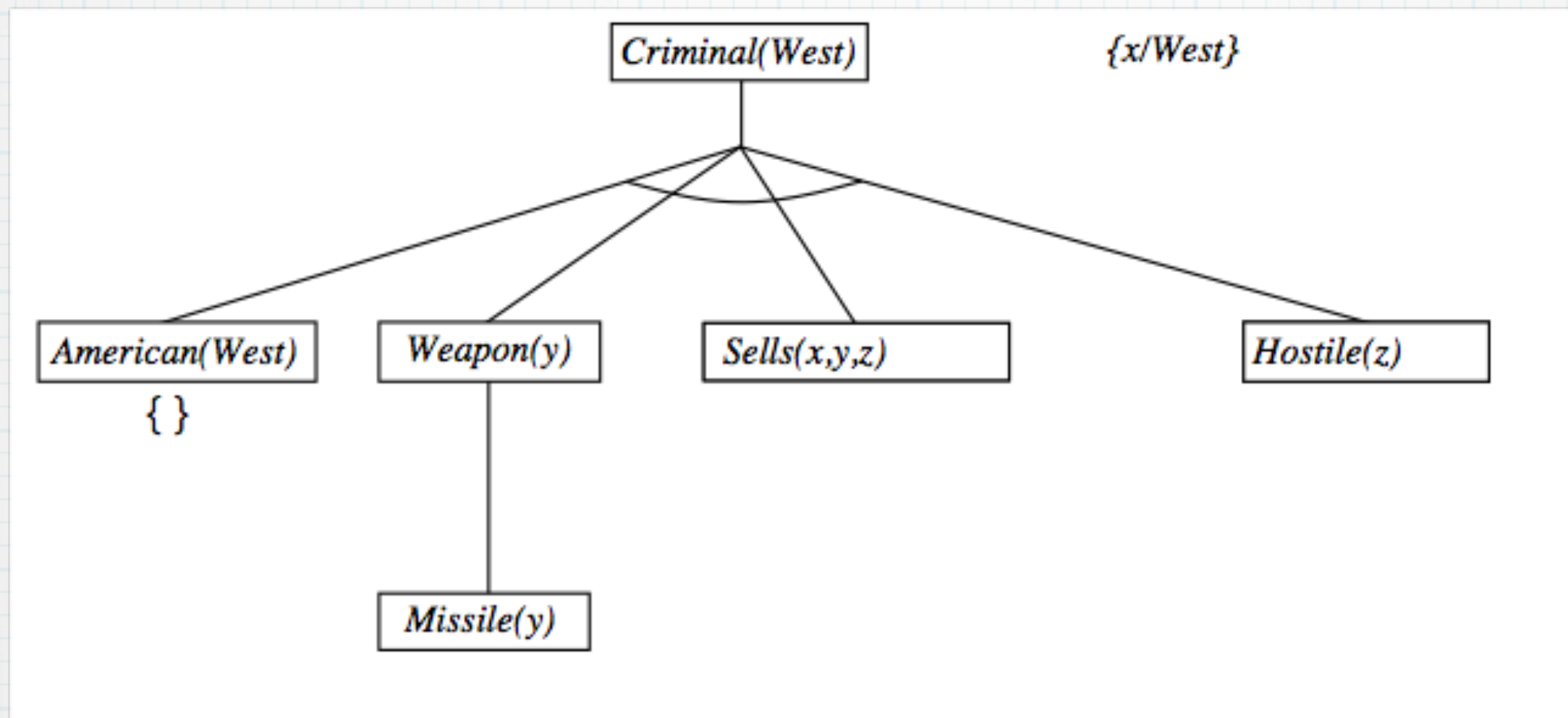
Backward Chaining



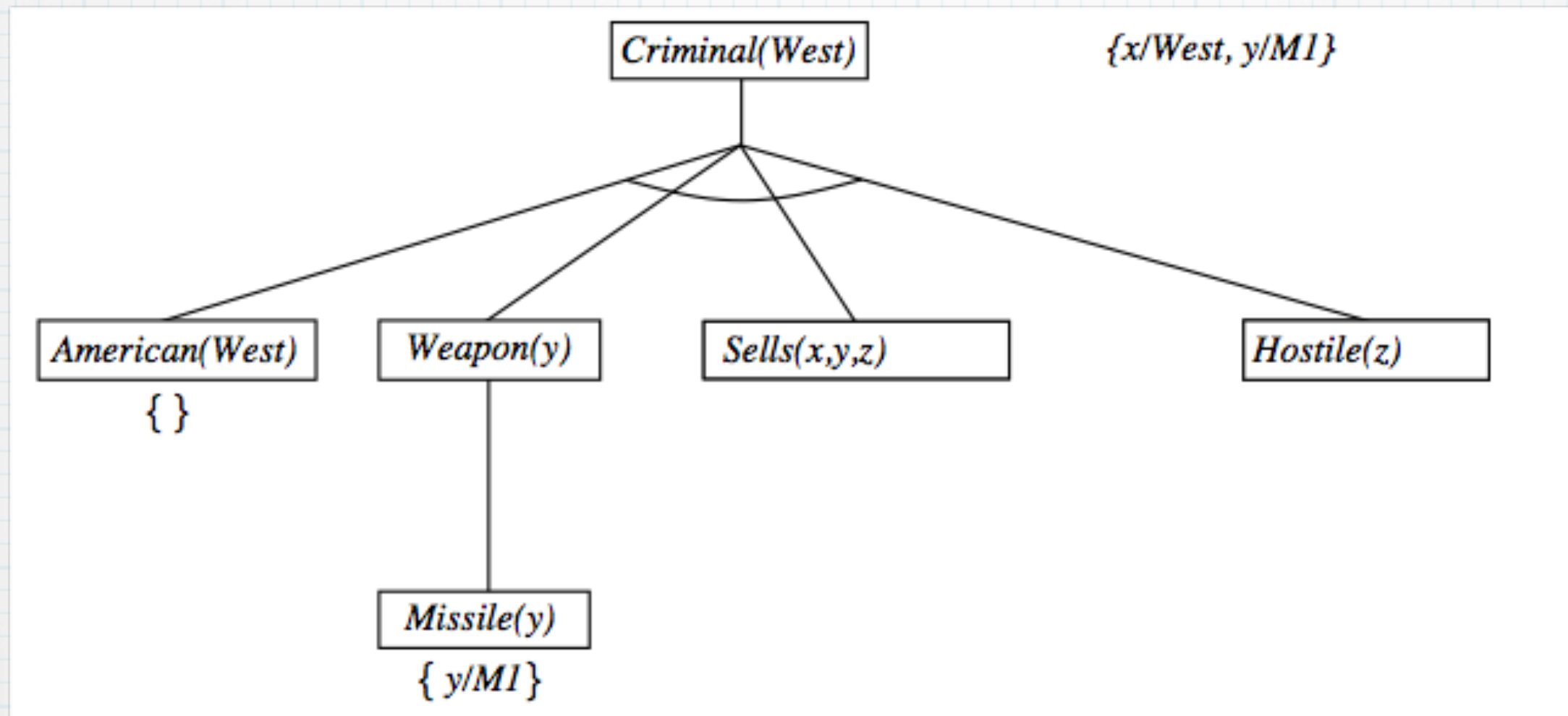
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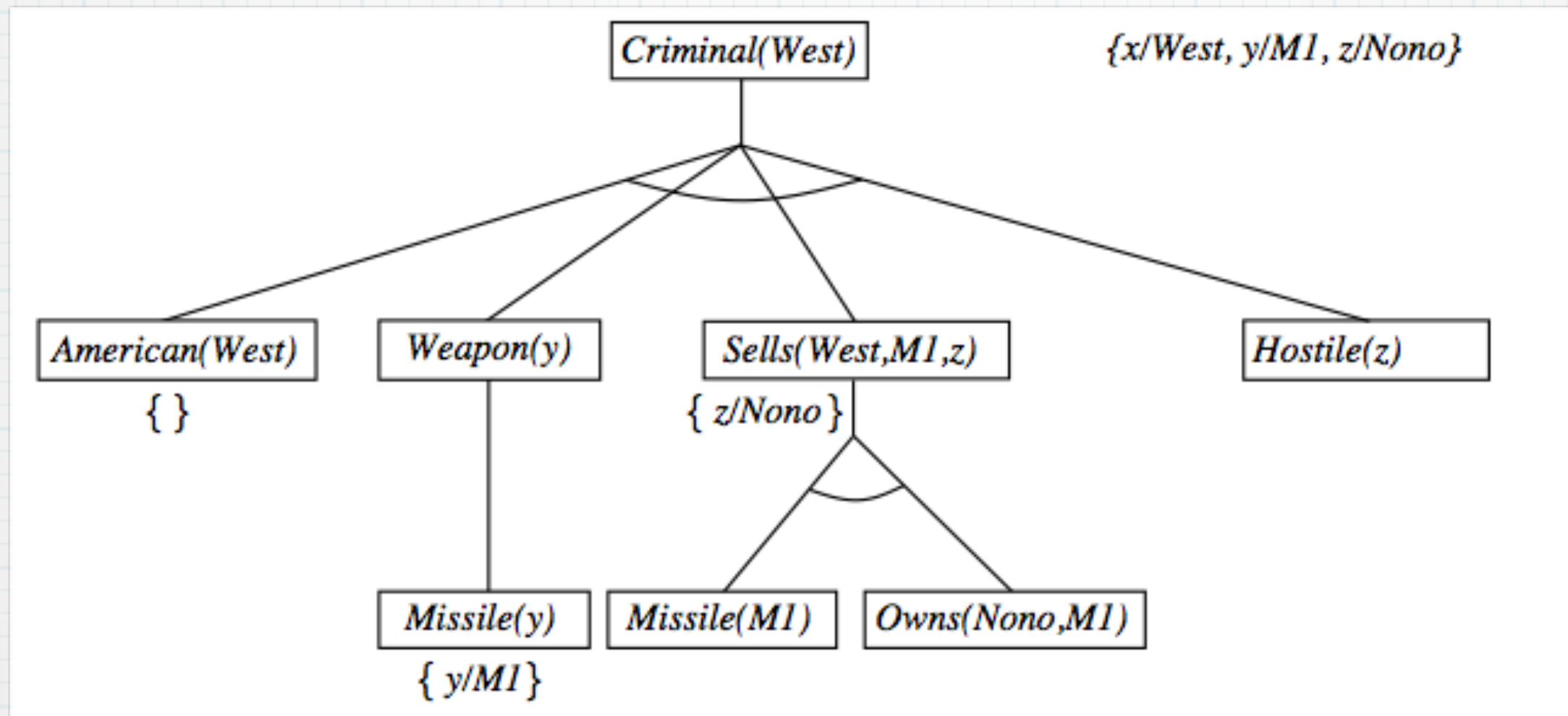
Backward Chaining



Backward Chaining



Backward Chaining



Properties of Backward Chaining

Depth-first recursive proof search: space is linear in size of proof

Incomplete due to infinite loops

⇒ fix by checking current goal against every goal on stack

Inefficient due to repeated subgoals (both success and failure)

⇒ fix using caching of previous results (extra space!)

Widely used (without improvements!) for logic programming

Resolution

Full first-order version:

$$\frac{\ell_1 \vee \dots \vee \ell_k, \quad m_1 \vee \dots \vee m_n}{(\ell_1 \vee \dots \vee \ell_{i-1} \vee \ell_{i+1} \vee \dots \vee \ell_k \vee m_1 \vee \dots \vee m_{j-1} \vee m_{j+1} \vee \dots \vee m_n)\theta}$$

where $\text{UNIFY}(\ell_i, \neg m_j) = \theta$.

For example,

$$\frac{\neg \text{Rich}(x) \vee \text{Unhappy}(x) \\ \text{Rich}(\text{Ken})}{\text{Unhappy}(\text{Ken})}$$

with $\theta = \{x/\text{Ken}\}$

Apply resolution steps to $\text{CNF}(KB \wedge \neg \alpha)$; complete for FOL

Converting to CNF

Everyone who loves all animals is loved by someone:

$$\forall x [\forall y \text{ Animal}(y) \Rightarrow \text{Loves}(x, y)] \Rightarrow [\exists y \text{ Loves}(y, x)]$$

1. Eliminate biconditionals and implications

$$\forall x [\neg \forall y \neg \text{Animal}(y) \vee \text{Loves}(x, y)] \vee [\exists y \text{ Loves}(y, x)]$$

2. Move \neg inwards: $\neg \forall x, p \equiv \exists x \neg p$, $\neg \exists x, p \equiv \forall x \neg p$:

$$\forall x [\exists y \neg(\neg \text{Animal}(y) \vee \text{Loves}(x, y))] \vee [\exists y \text{ Loves}(y, x)]$$

$$\forall x [\exists y \neg \neg \text{Animal}(y) \wedge \neg \text{Loves}(x, y)] \vee [\exists y \text{ Loves}(y, x)]$$

$$\forall x [\exists y \text{ Animal}(y) \wedge \neg \text{Loves}(x, y)] \vee [\exists y \text{ Loves}(y, x)]$$

Converting to CNF

3. Standardize variables: each quantifier should use a different one

$$\forall x [\exists y \text{ Animal}(y) \wedge \neg \text{Loves}(x, y)] \vee [\exists z \text{ Loves}(z, x)]$$

4. Skolemize: a more general form of existential instantiation.
Each existential variable is replaced by a **Skolem function** of the enclosing universally quantified variables:

$$\forall x [\text{Animal}(F(x)) \wedge \neg \text{Loves}(x, F(x))] \vee \text{Loves}(G(x), x)$$

5. Drop universal quantifiers:

$$[\text{Animal}(F(x)) \wedge \neg \text{Loves}(x, F(x))] \vee \text{Loves}(G(x), x)$$

6. Distribute \wedge over \vee :

$$[\text{Animal}(F(x)) \vee \text{Loves}(G(x), x)] \wedge [\neg \text{Loves}(x, F(x)) \vee \text{Loves}(G(x), x)]$$

$\neg \text{American}(x) \vee \neg \text{Weapon}(y) \vee \neg \text{Sells}(x,y,z) \vee \neg \text{Hostile}(z) \vee \text{Criminal}(x)$

$\neg \text{Criminal}(\text{West})$

... it is a crime for an American to sell weapons to hostile nations:

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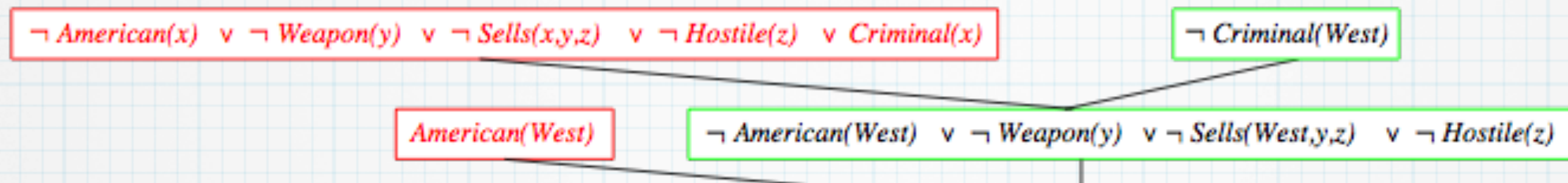
$\text{Enemy}(x, \text{America}) \Rightarrow \text{Hostile}(x)$

West, who is American ...

$\text{American}(\text{West})$

The country Nono, an enemy of America ...

$\text{Enemy}(\text{Nono}, \text{America})$



... it is a crime for an American to sell weapons to hostile nations:

$$American(x) \wedge Weapon(y) \wedge Sells(x,y,z) \wedge Hostile(z) \Rightarrow Criminal(x)$$

Nono ... has some missiles, i.e., $\exists x Owns(Nono, x) \wedge Missile(x)$:

$$Owns(Nono, M_1) \text{ and } Missile(M_1)$$

... all of its missiles were sold to it by Colonel West

$$\forall x Missile(x) \wedge Owns(Nono, x) \Rightarrow Sells(West, x, Nono)$$

Missiles are weapons:

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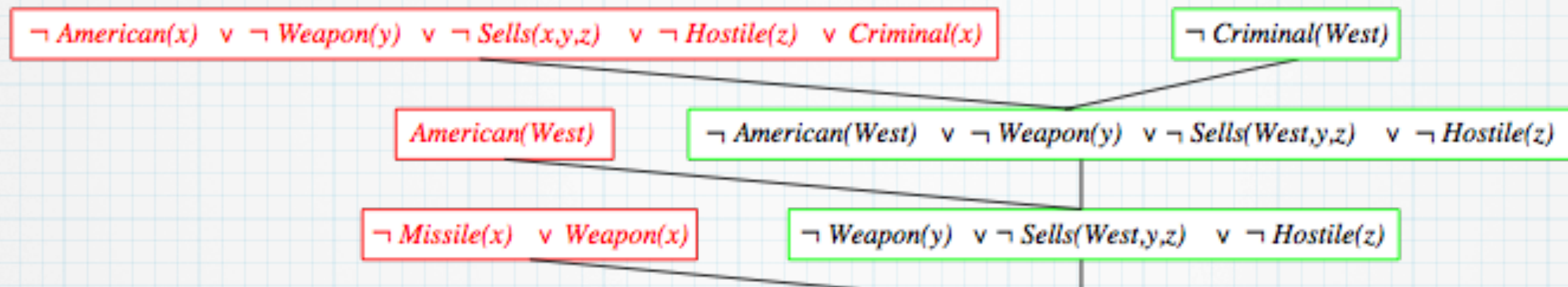
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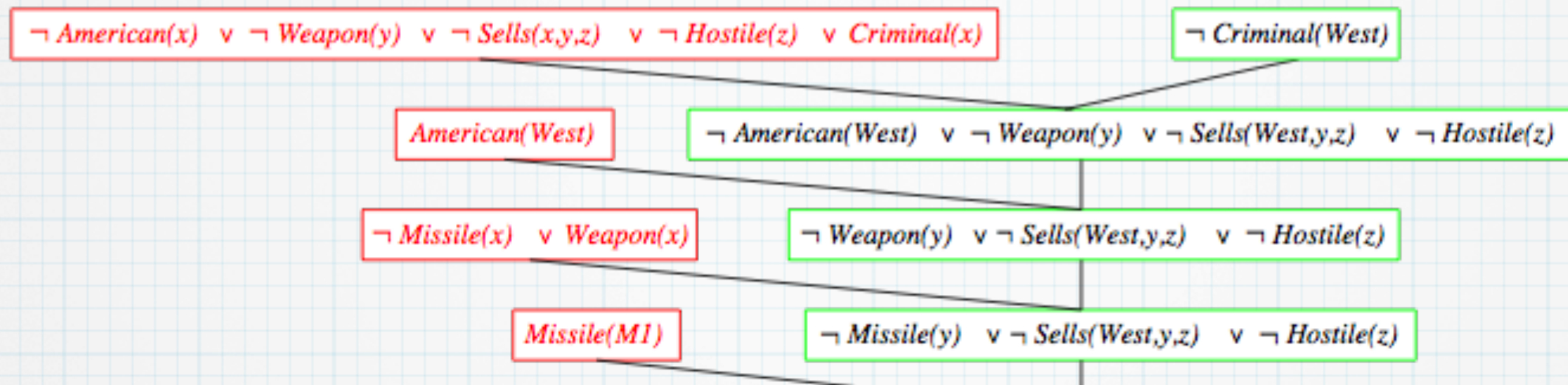
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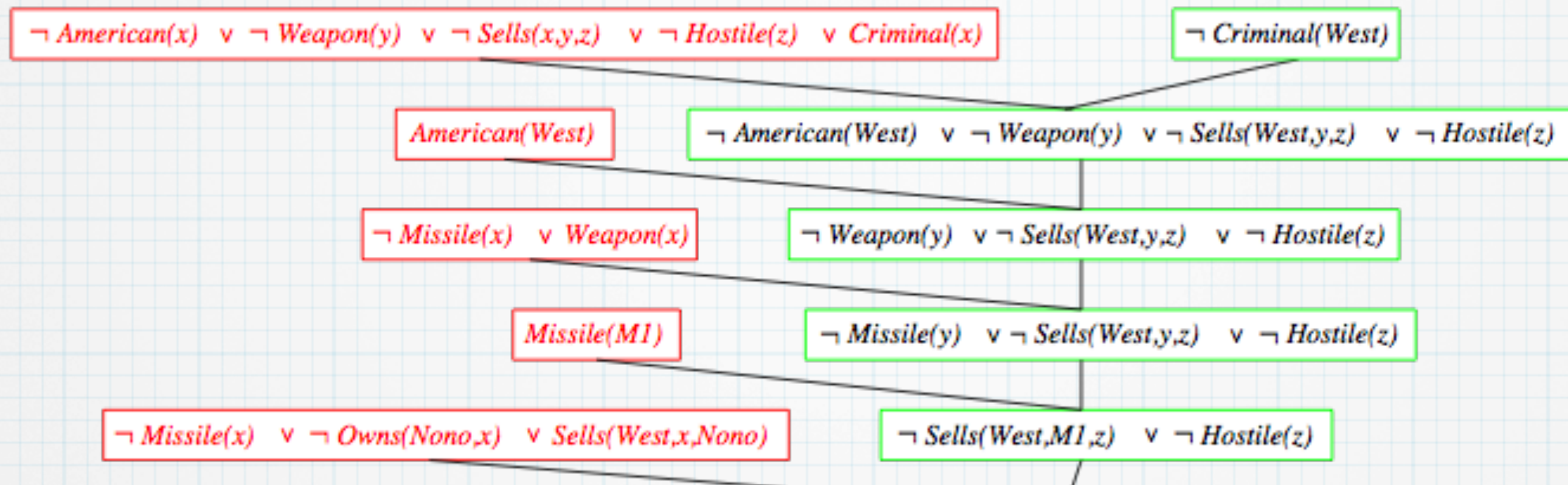
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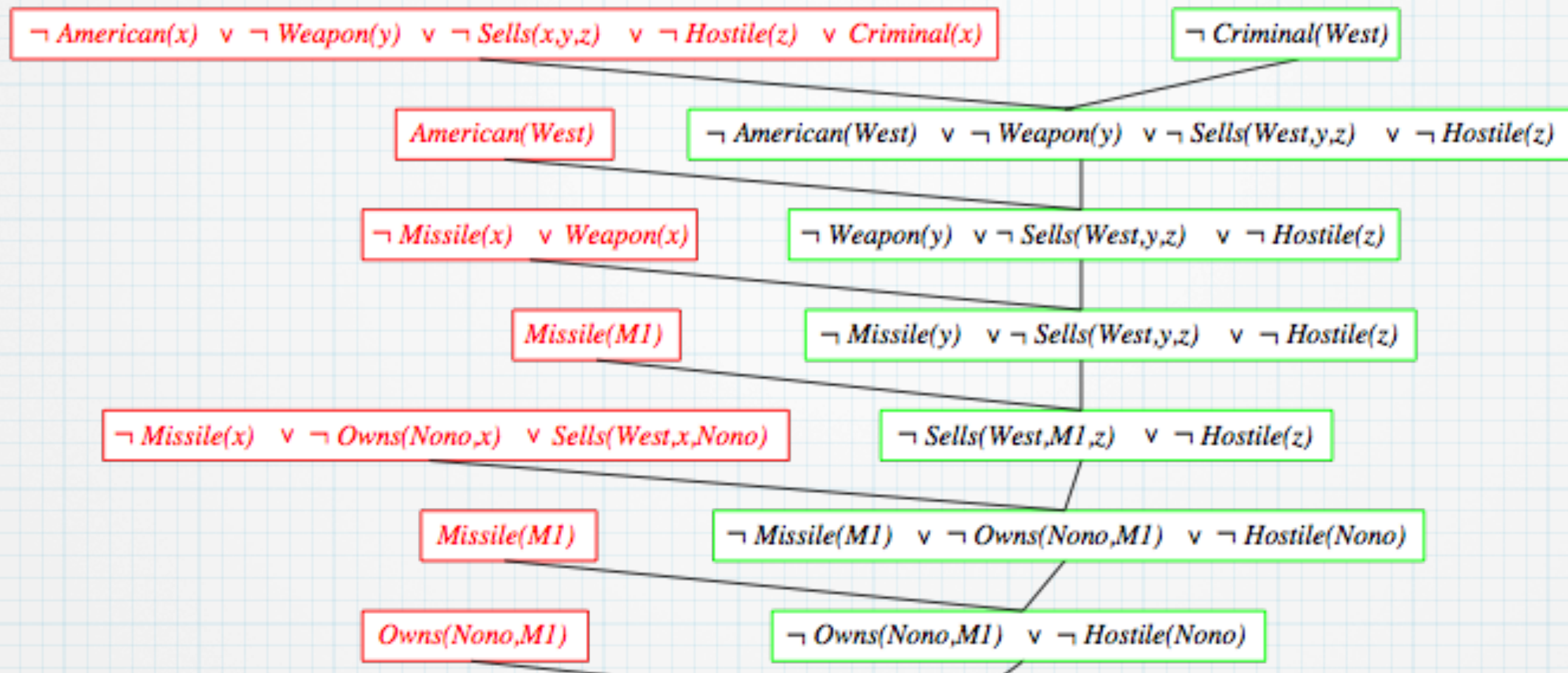
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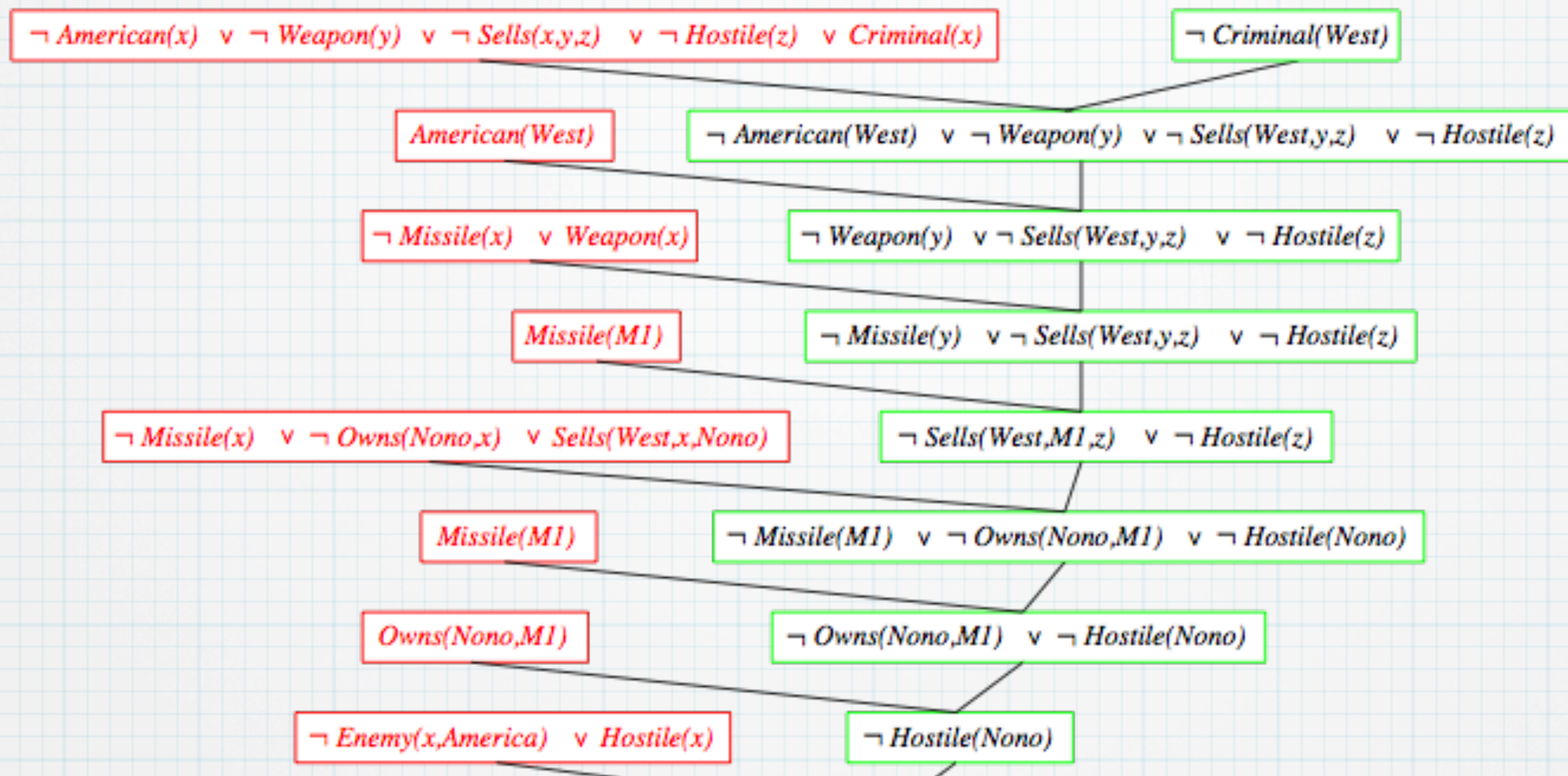
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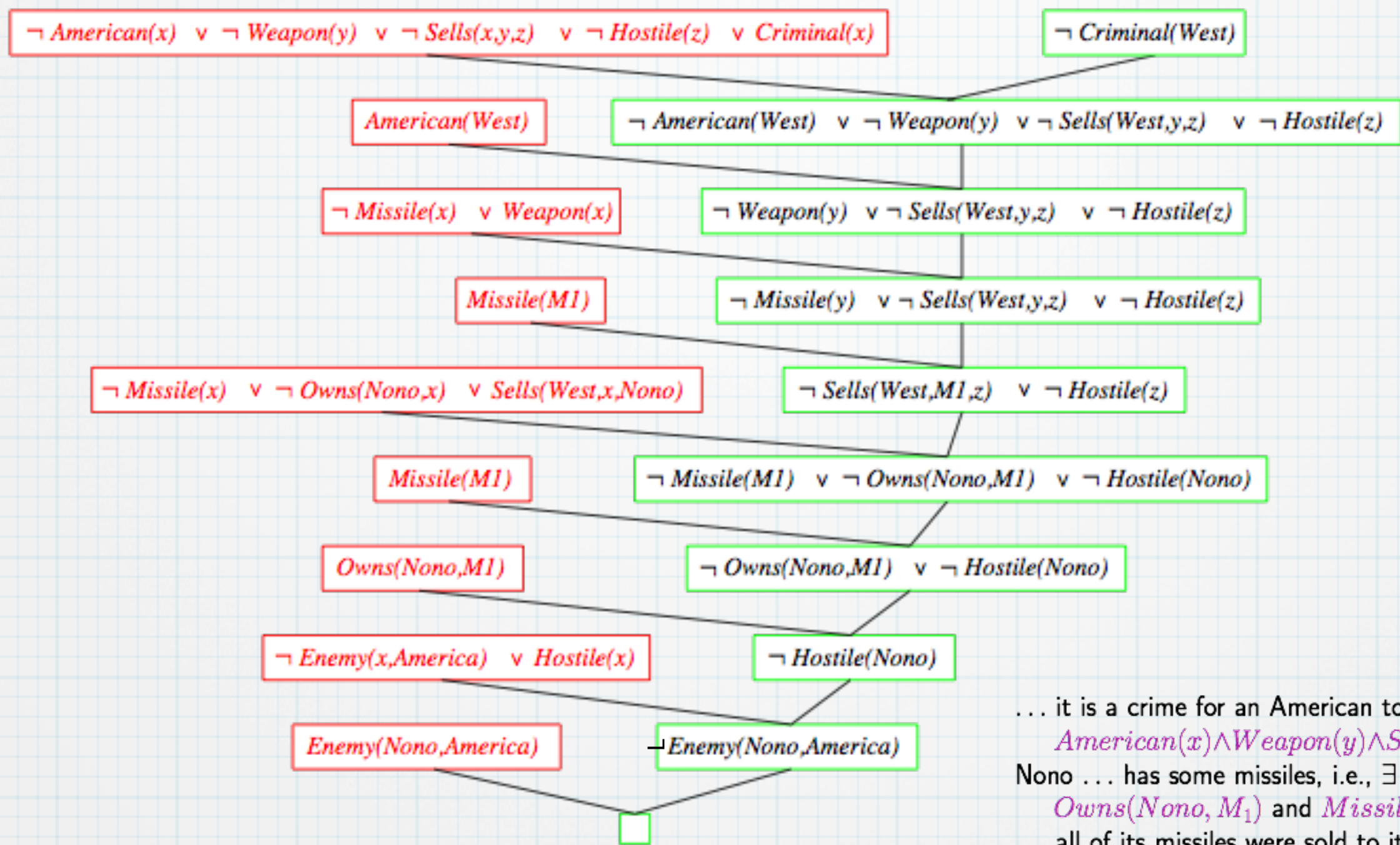
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