Table 1: For instructor's use

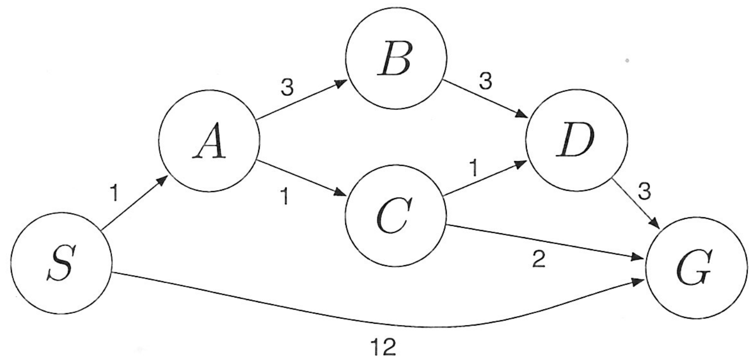
Question	Points Scored	Possible Points
1		12
2		12
3		12
4		12
5		6

MIDTERM SOLUTIONS

This exam is closed book. You are allowed 4 sheets of notes (8 pages front and back). You may use any format for your notes that you like. Please explain all of your answers fully to receive any extra credit.

Question 1. (12 points) Search

Answer the following graph search questions using the graph below, where S is the start node and G is the goal node. Break any ties alphabetically.



(a) (2 pt) What *solution path* would breadth-first graph search return for this problem?

S - G

(b) (2 pt) What *visited list and solution path* would uniform cost graph search return for this problem?

S
A(1) G(12)
C(2) B(4) G(12)
D(3) B(4) G(4) G(12)
B(4) G(4) G(6) G(12)
G(4) G(6) G(12)

V: S A C D B G

SOLN: S - A - C - G

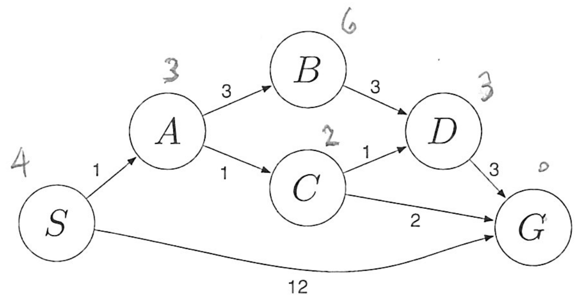
(c) (2 pt) What *visited list and solution path* would depth-first graph search return for this problem?

V: S A B D G

SOLN: S - A - B - D - G

(d) (2 pt) What *solution path* would A* graph search, using a consistent heuristic, return for this problem?

S - A - C - G



(e) (4 pt) (Graph repeated above for your convenience) Consider the two heuristics for this search problem shown in the tables below. Answer the following **YES/NO questions**. *If your answer is NO, give specific evidence to support your claim.*

State	h_1
S	5 X
A	3
B	6
C	2
D	3
G	0

State	~~h_1~~ h_2
S	4
A	2
B	6
C	1
D	3
G	0

ADMISSIBLE

$h(n) \le h^*(n)$

CONSISTENT

$h(n) \le c(n,n') + h(n')$

$\Rightarrow h(n) - h(n') \le c(n, n')$

i. (1 pt) Is h_1 admissible? **Yes** **No**

$h(s) = 5 > h^*(s) = 4$

ii. (1 pt) Is h_1 consistent? **Yes** **No**

$h(s) \le c(s, A) + h(A)$

$5 \le 1 + 3$ X

iii. (1 pt) Is h_2 admissible? **Yes** **No**

iv. (1 pt) Is h_2 consistent? **Yes** **No**

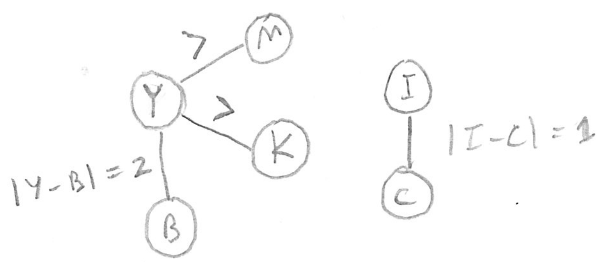
$h(s) \le c(s, A) + h(A)$

$4 \le 1 + 2$ X

Question 2. (12 points) Constraint Satisfaction Problem
You promise your daughter Barbara that her friends can stay at your house over Spring Break. Your house has six bedrooms and and you decide to figure out the room assignments using a CSP in which the variables are You (**Y**), Barbara (**B**), Charlotte (**C**), Kristin (**K**), Indigo (**I**), and Matilda (**M**). The value of each variable is the room assignment, from $1-6$. The assignments must satisfy the following constraints:

i) Each person must stay in a room by themselves
ii) **Y** > 3
iii) **K** is less than **Y**
iv) **M** is either 5 or 6
v) **Y** $>$ **M**
vi) **B** is even
vii) **I** is not 1 or 6
viii) $|$**I-C**$| = 1$
ix) $|$**Y-B**$| = 2$

(a) (2 pt) Constraint graph Draw the constraint graph for all of the binary constraints *except* for the constraints due to (i) (as otherwise it will be too cluttered).



(b) (2 pt) Unary constraints On the grid below cross out the values from each domain that are eliminated by enforcing *unary constraints*.

Y	~~1~~	~~2~~	~~3~~	4	5	6
B	~~1~~	2	~~3~~	4	~~5~~	6
C	1	2	3	4	5	6
K	1	2	3	4	5	6
I	~~1~~	2	3	4	5	~~6~~
M	~~1~~	~~2~~	~~3~~	~~4~~	5	6

(b) (1 pt) MRV According to the Minimum Remaining Value (MRV) heuristic, which variable should be assigned a value first?

Y B C K I M

2

Constraints Repeated for Convenience:

i) Each person must stay in a room by themselves
ii) **Y** > 3
iii) **K** is less than **Y**
iv) **M** is either 5 or 6
v) **Y** > **M**
vi) **B** is even
vii) **I** is not 1 or 6
viii) |**I**-**C**| = 1
ix) |**Y**-**B**| = 2

(c) (2 pt) Forward Checking For the purposes of decoupling this problem from your solution to the previous problem, assume we choose to assign Y first, and assign it the value 6. What are the resulting domains after enforcing unary constraints (from part b) and running forward checking for this assignment?

Y						6
B	~~1~~	~~2~~	~~3~~	4	~~5~~	~~6~~
C	1	2	3	4	5	~~6~~
K	1	2	3	4	5	~~6~~
I	~~1~~	2	3	4	5	~~6~~
M	~~1~~	~~2~~	~~3~~	~~4~~	5	~~6~~

(d) (3 pt) Constraint Propagation Now suppose that you assign $C = 3$ in addition to the solution from part (c) and run *constraint propagation*. Update the domains below to indicate the remaining values for all variables.

Y						6
B	~~1~~	~~2~~	~~3~~	4	~~5~~	~~6~~
C	~~1~~	~~2~~	3	~~4~~	~~5~~	~~6~~
K	1	~~2~~	~~3~~	~~4~~	~~5~~	~~6~~
I	~~1~~	2	~~3~~	~~4~~	~~5~~	~~6~~
M	~~1~~	~~2~~	~~3~~	~~4~~	5	~~6~~

(e) (2 pt) Solution Is there an assignment of rooms which satisfies all of the constraints? Why or why not?

Y = 6
B = 4
C = 3
K = 1
I = 2
M = 5

Question 3.(12 points) Mark each of the following statements as *TRUE or FALSE*. You do not have to correct the false sentences. Each question is worth one point.

(a) Graph Search

Consider a graph search problem where every edge has a cost which is greater than zero, *with varying costs for different edges.* Assume that we use a consistent heuristic $h_1(s)$ for A* search unless stated otherwise.

True **False** Depth-first graph search is guaranteed to return an optimal solution. DFS not optimal

True **False** Breadth-first graph search is guaranteed to return an optimal solution. ONLY OPTIMAL IF ALL COSTS EQUAL

True **False** Uniform-cost graph search is guaranteed to return an optimal solution.

True **False** Greedy best-first graph search is guaranteed to return an optimal solution. Not optimal

True **False** A* graph search is guaranteed to return an optimal solution.

True **False** A* graph search is guaranteed to expand no more nodes than depth-first graph search. DFS could go directly to suboptimal soln.

True **False** A* graph search is guaranteed to expand no more nodes than uniform-cost graph search.

True **False** Now define the heuristic $h_2(s) = 2h_1(s)$: A* graph search with $h_2(s)$ is guaranteed to return an optimal solution.

$h_1(s) \leq h^*(s) \not\Rightarrow 2h_1(s) \leq h^*(s)$

(b) CSP

Consider a CSP with n variables and d possible values for each variables (in other words, d is the size of the domain for each variable).

True **False** When enforcing arc consistency through constraint propagation (AC-3), the set of values which remain when the algorithm terminates does not depend upon the order in which arcs are processed from the queue.

True **False** The motivation for using the Minimum Remaining Values (MRV) heuristic is to increase the likelihood that subsequent variable assignments will succeed, thereby extending the current search path as far as possible. FAST FAIL

True **False** $O(d^n)$ is the maximum number of times a backtracking search algorithm might have to backtrack (i.e. the number of times it generates an assignment that violates the constraints) before finding a solution or concluding that none exists. WORST CASE TRY ALL ASSIGNMENTS

True **False** After assigning a variable and running forward checking, we are guaranteed that all remaining values in the domains of all variables are consistent with the constraints.

FORWARD CHECKING ONLY MODIFIES NEIGHBORS
NEED AC-3 FOR FULL PROPAGATION

Question 4. (12 points) Propositional Logic
Answer the following questions. **Show your work in order to receive full credit.**

(a) (8 pt) Determine, for each of the following propositional logic formulas, whether it is:

- *valid* - true for all possible assignments of literals
- *satisfiable* - true for at least one assignment of literals
- *unsatisfiable* - not true for any assignments of literals

For example, $p \wedge \neg p$ is unsatifiable because it is false for both true and false assignments to p. Note that each of the rows below is considered separately.

1. $p \vee \neg p$ VALID $p=0$ $p=1$ TRUE

2. $p \vee q \vee \neg r$ SAT $p=1$ $q=1$ $r=0$

3. $(\overset{1}{p} \wedge \overset{1}{q}) \wedge \overset{0}{\neg}(p \vee q)$ UNSAT
   MUST BE T $p=1, q=1$ WILL BE F $p=1, q=1$

4. $(p \wedge q) \Rightarrow r$ SAT $p=1$ $q=1$ $r=1$

5. $(p \Rightarrow q) \Rightarrow \underbrace{((p \Rightarrow r) \Rightarrow (p \Rightarrow r))}_{\text{T ALWAYS}}$ VALID OR $\neg(p \Rightarrow q) \vee \neg(p \Rightarrow r) \vee \underbrace{(p \Rightarrow r)}_{\text{ALWAYS T}}$
   CAN BE T V F

6. $p \Rightarrow (q \Rightarrow p)$ $\underbrace{\neg p \vee (\neg q \vee p)}_{\text{ALWAYS TRUE}}$ VALID

7. $p \Rightarrow (p \Rightarrow q)$ $\neg p \vee (\neg p \vee q)$ SAT $q=1$

8. $(\overset{0}{p} \overset{1}{\Leftrightarrow} q) \Rightarrow (\overset{0}{p} \overset{1}{\Rightarrow} q)$ VALID $p \Rightarrow q$ FALSE FOR $p=0, q=1$ $p \Leftrightarrow q$ FALSE ALSO $0 \Rightarrow 0$ ✓
   $\neg((\neg p \vee q) \wedge (\neg q \vee p)) \vee (\neg p \vee q)$
   $\neg(\neg p \vee q) \vee \underbrace{\neg(\neg q \vee p) \vee (\neg p \vee q)}_{\text{ALWAYS T}}$

(b) (4pt) Translate the following English sentences into propositional logic and use resolution theorem proving (using the method of proof by contradiction) to prove that Bob is going to be fired.

A $\neg I$ W

Bob attended the meeting or he was not invited. If the boss wanted Bob at the meeting, then he was invited. Bob did not attend the meeting. If the boss did not want Bob there, and the boss did not invite him there, then he is going to be fired.

F

i. (1 pt) List your propositional symbols and what they mean

A I W F

ii. (1 pt) Write the list of propositional sentences representing the passage above.

$A \vee \neg I$ A

$W \Rightarrow I \rightarrow \neg W \vee I$

$\neg A$ □

$(\neg W \wedge \neg I) \Rightarrow F$

$\hookrightarrow (W \vee I \vee F)$

$W \vee I \rightarrow I$

$\neg F$

ii. (2 pt) Use resolution theorem proving to show that Bob will be fired via a proof by contradiction. *Hint:* Look for resolution steps that will produce the literal corresponding to "Bob attended the meeting."

Question 5. (6 points) Probability Models

You are worried about the safety of your home, and constantly monitor whether your front door is open (**D**) using an iphone app. When your door is open there are two possible causes: your spouse is at home (**S**), and may have left the door open by accident, or your house was robbed (**R**). You are given the prior probabilities $P(S = 1) = 0.5$ and $P(R = 1) = 0.1$. The following conditional probability table gives the probability of the door being open:

S	R	$P(D = 1 \mid S, R)$	$P(D = 0 \mid S, R)$
0	0	0.01	0.99
0	1	0.25	0.75
1	0	0.05	0.95
1	1	0.75	0.25

(a) (1 pt) Why don't each of the columns in the probability table above sum to 1?

NOT PDF

(b) (1 pt) What is the probability that the door is open conditioned on being told that your spouse is at home and your house was not robbed?

$P(D=1 \mid S=1, R=0) = 0.05$

(c) (1 pt) Calculate the joint probability $P(D = 1, S = 1, R = 0)$. *Hint:* Use the definition of conditional probability from part (b). How does this probability differ from your answer in part (b)? Is the probability lower or higher? Why?

$$P(D=1, S=1, R=0) = P(D=1 \mid S=1, R=0)\, P(S=1)\, P(R=0)$$
$$= (0.05)(0.5)(0.9) = 0.0225$$

Lower, because just joint event, not cond. on evidence

(d) (1 pt) Suppose that you only know that your spouse is at home. Compute the conditional probability that the door is open (e.g. $P(D = 1 \mid S = 1)$).

$$P(D=1 \mid S=1) = \frac{\sum_{R=0}^{1} P(D=1, S=1, R)}{P(S=1)} = \frac{0.0225 + 0.375}{0.5}$$

$$P(D=1, S=1, R=1) = P(D=1 \mid S=1, R=1)\, P(S=1)\, P(R=1)$$
$$= (0.75)(0.5)(0.1) = 0.375$$

0.12

(e) (1 pt) Give the formula for Bayes Rule. Give an example of a probability distribution in the context of the current problem that we could compute using Bayes Rule.

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)} \qquad P(R|D=1)$$

(f) (1 pt) Give one advantage of probabilitic agents over logical agents in solving challenging AI problems. Conversely, can you think of any scenarios where a logical agent approach would be a better choice?

PROBABILISTIC AGENTS CAN REASON ABOUT UNCERTAINTY
IMPORTANT FOR NON DETERMINISTIC ENVIRONMENTS.

IF REPRESENTATION IS DETERMINISTIC, AS IN SAT PROBS,
LOGIC AGENT ALL YOU NEED.