Markov Decision Processes (part 2)

CS 3600 Intro to AI

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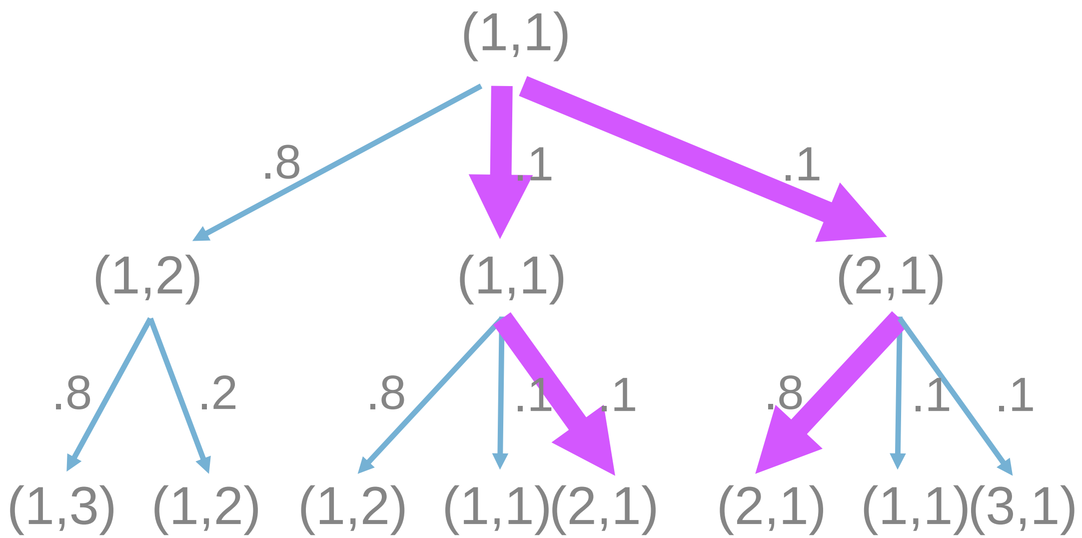
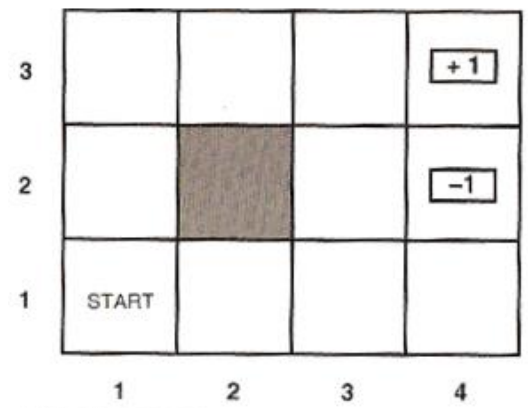
Computing Utilities for States

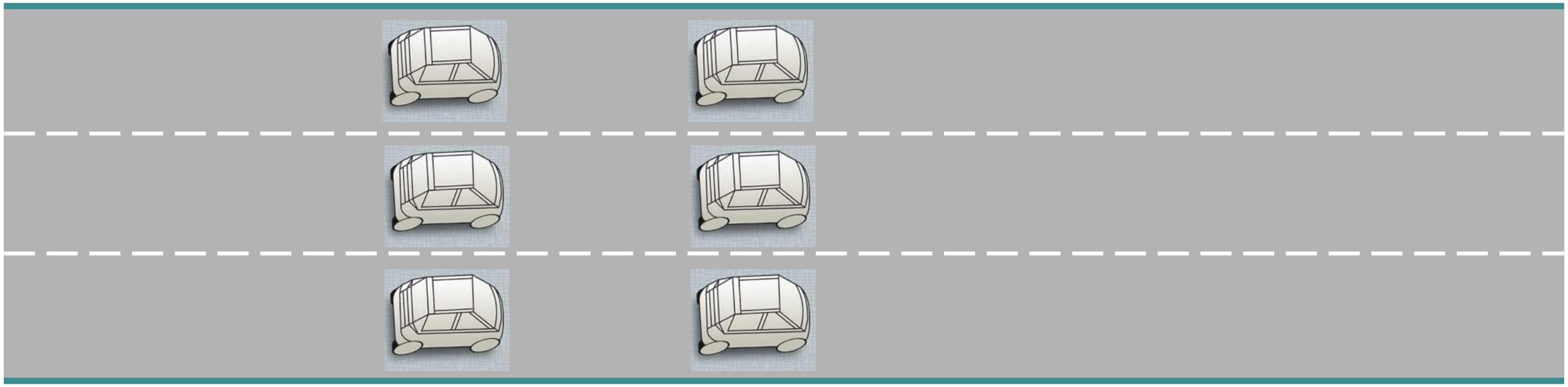


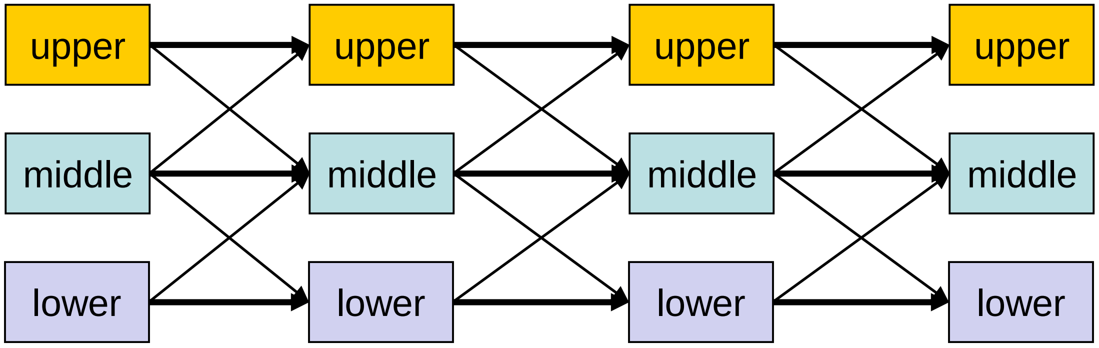


Need to do something smart to avoid exponential blowup

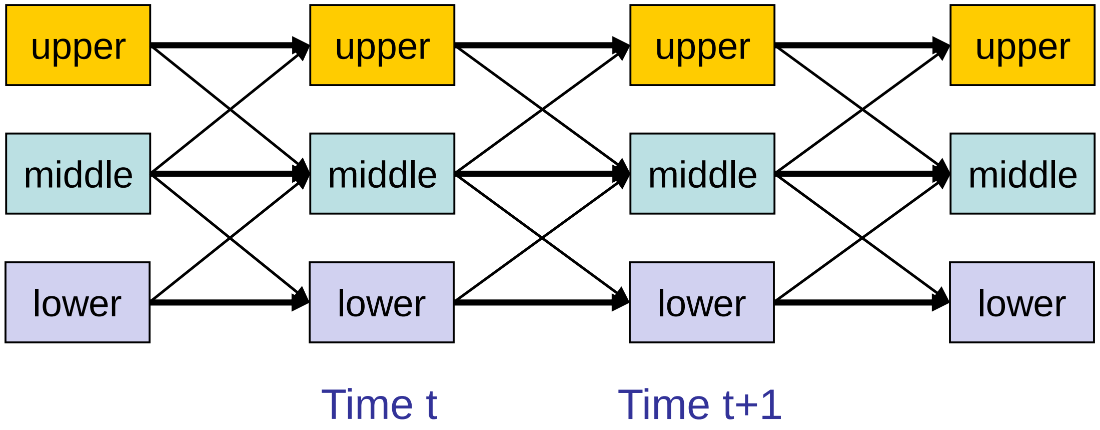
Key Idea: Decompose into subproblems (subtrees)

1-D Example




upper
middle
lower
upper
middle
lower
upper
middle
lower
upper
middle
lower
Time t
Time t+1

1-D Example

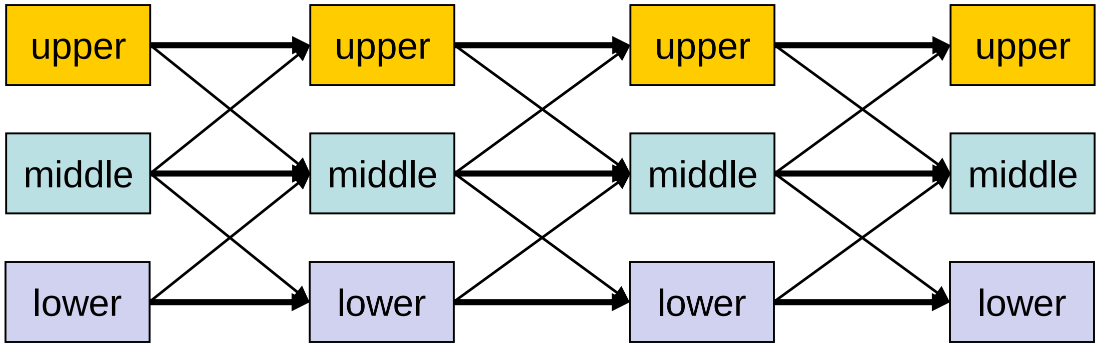


$$U^*(s_t) = R(s_t) + \gamma \max_{a \in A(s_t)} \sum_{s_{t+1}} P(s_{t+1} \mid s_t, a) U^*(s_{t+1})$$

Optimal Utility at t

Expected Optimal Utility at t+1

1-D Example



Time t

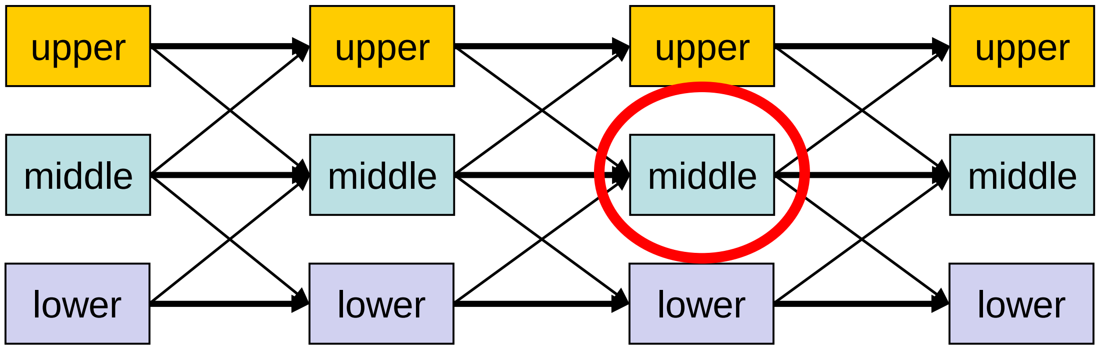
Time t+1

$$U^*(s_T) = R(s_T)$$

Work Backwards!

Optimal Utility at the terminal T

Bellman Equations for Utility



Time t   Time t+1

$$U(s_t^u) = -0.04 + \gamma \max_{a_t} \{ [0.8U(s_{t+1}^u) + 0.2U(s_{t+1}^m)] I(a_t^u)$$

upper -> upper

$$+ [0.2U(s_{t+1}^u) + 0.8U(s_{t+1}^m)] I(a_t^m) \}$$

upper -> middle

$$U(s_t^m) = -0.04 + \gamma \max_{a_t} \{ [0.8U(s_{t+1}^u) + 0.2U(s_{t+1}^m)] I(a_t^u)$$

middle -> upper

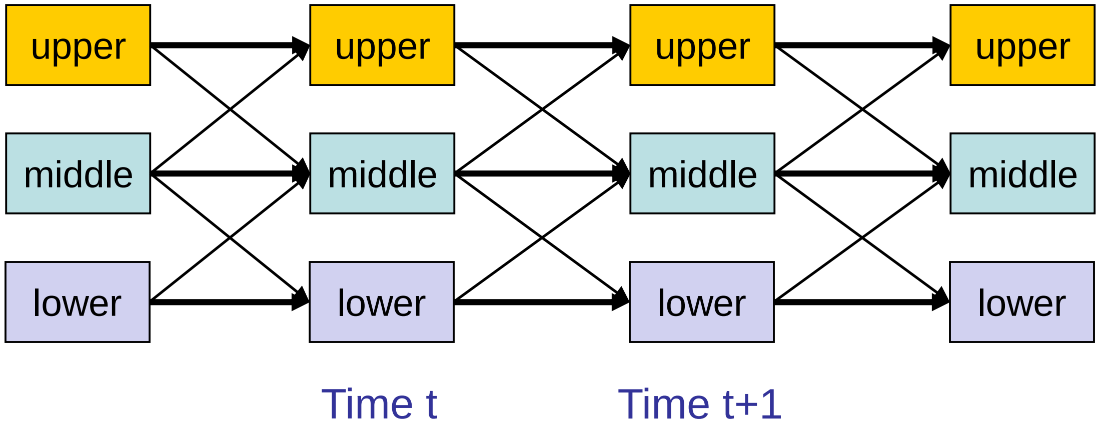
$$+ [0.1U(s_{t+1}^u) + 0.8U(s_{t+1}^m) + 0.1U(s_{t+1}^l)] I(a_t^m)$$

$$+ [0.2U(s_{t+1}^m) + 0.8U(s_{t+1}^l)] I(a_t^l) \}$$

middle -> middle

middle -> lower

Value Iteration



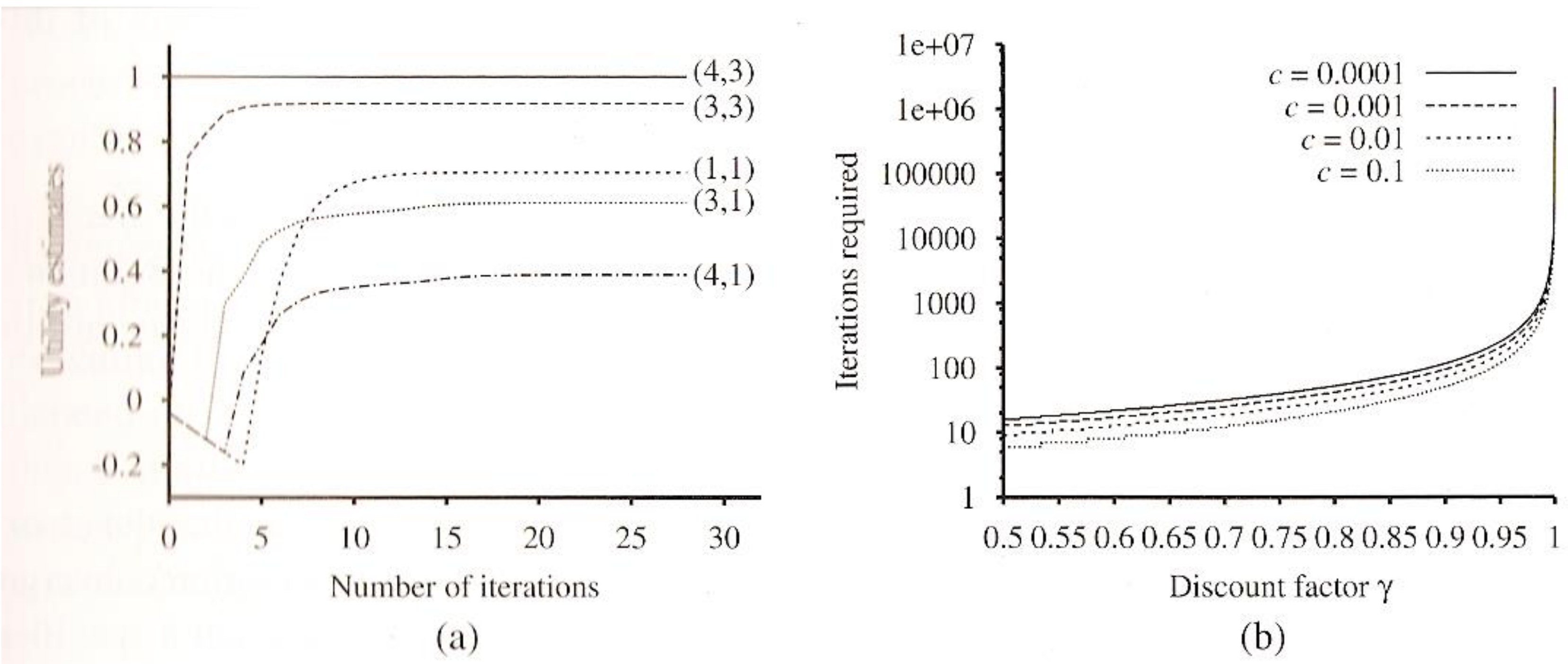
Iteratively solve the system of coupled nonlinear equations

Bellman update:

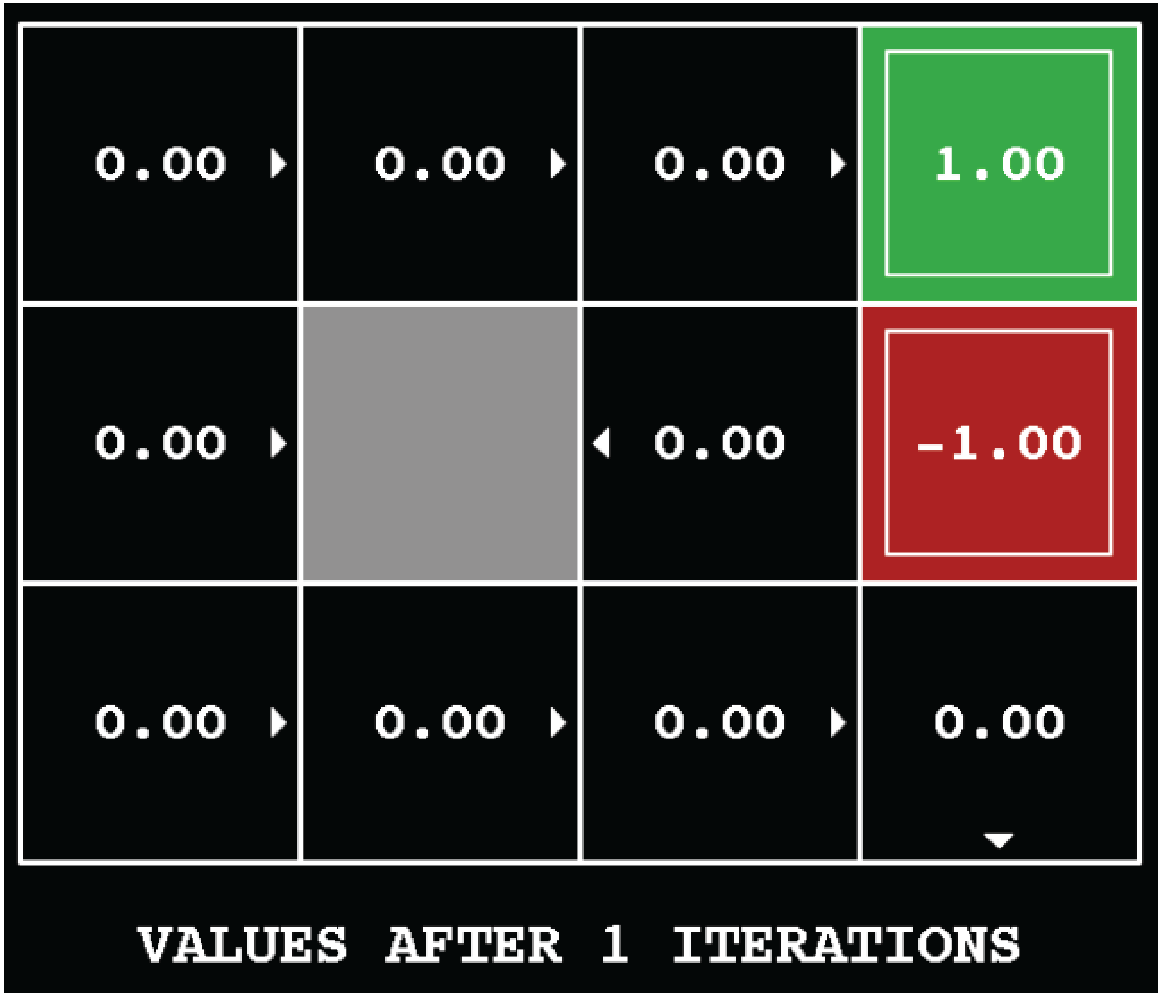
$$U_{i+1}(s_t) \leftarrow R(s_t) + \gamma \max_{a \in A(s_t)} \sum_{s_{t+1}} P(s_{t+1} \mid s_t, a) U_i(s_{t+1})$$

Convergence of Value Iteration

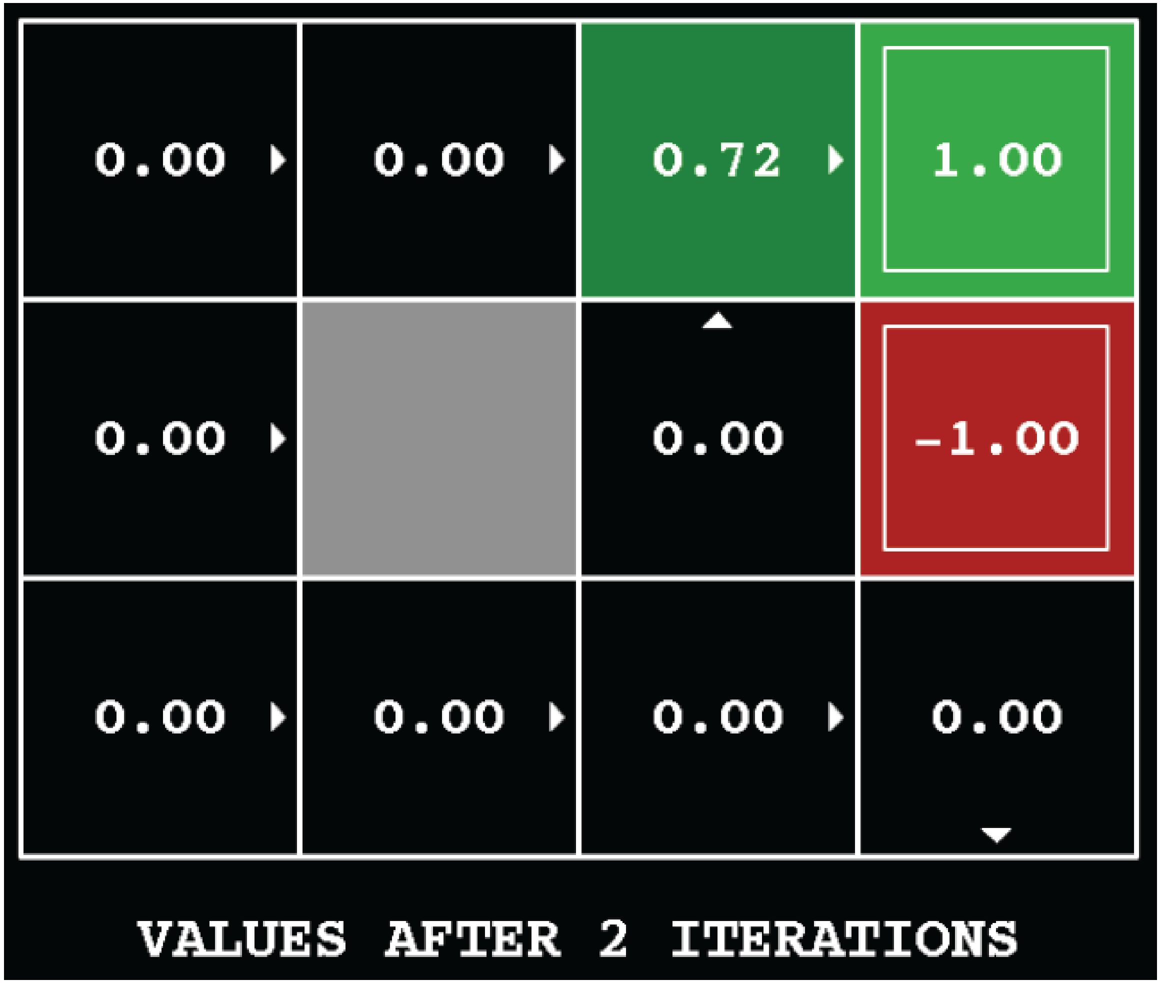
- Guaranteed to converge to a unique solution that gives an optimal policy
- Intuition: Utility values only need to be relatively correct to select for the best action



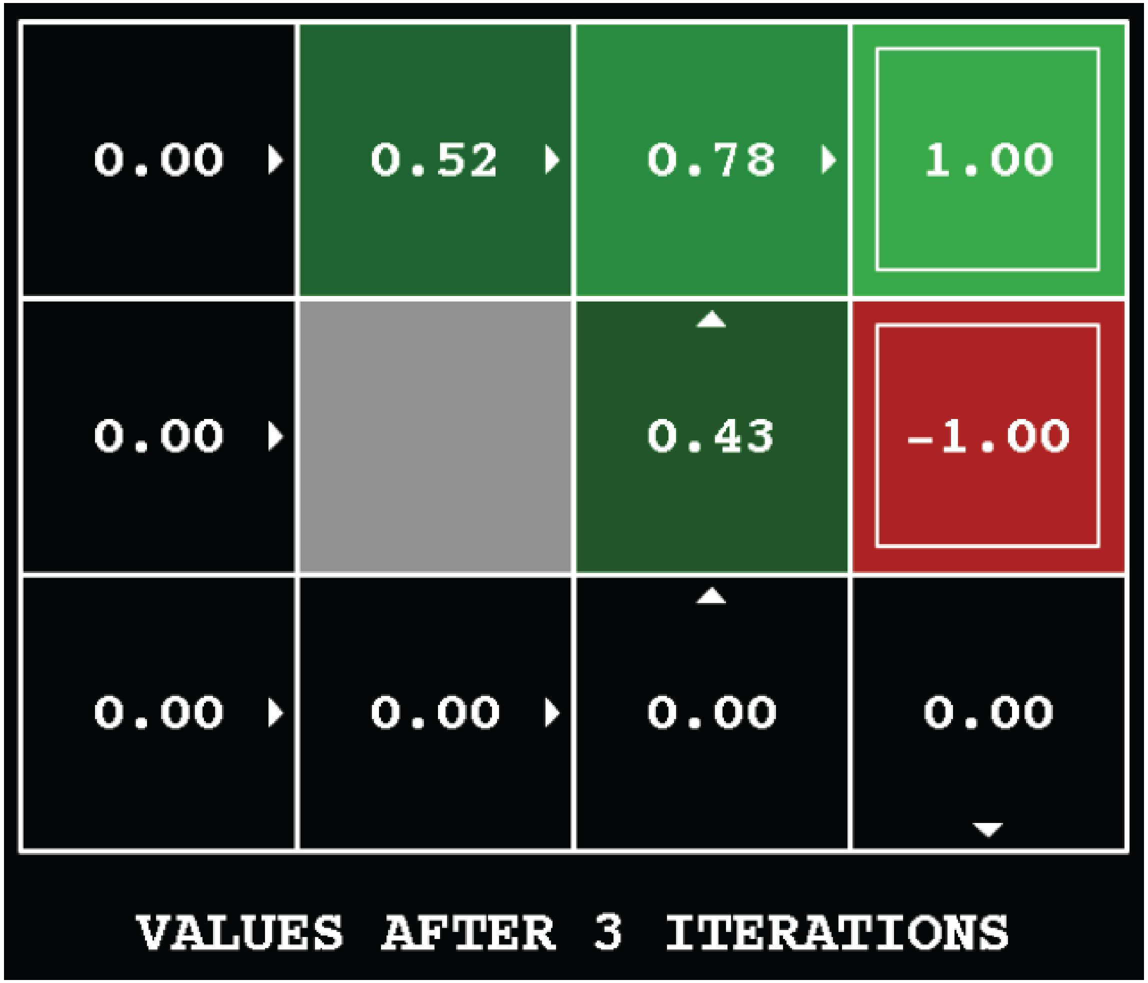
Example of Value Iteration



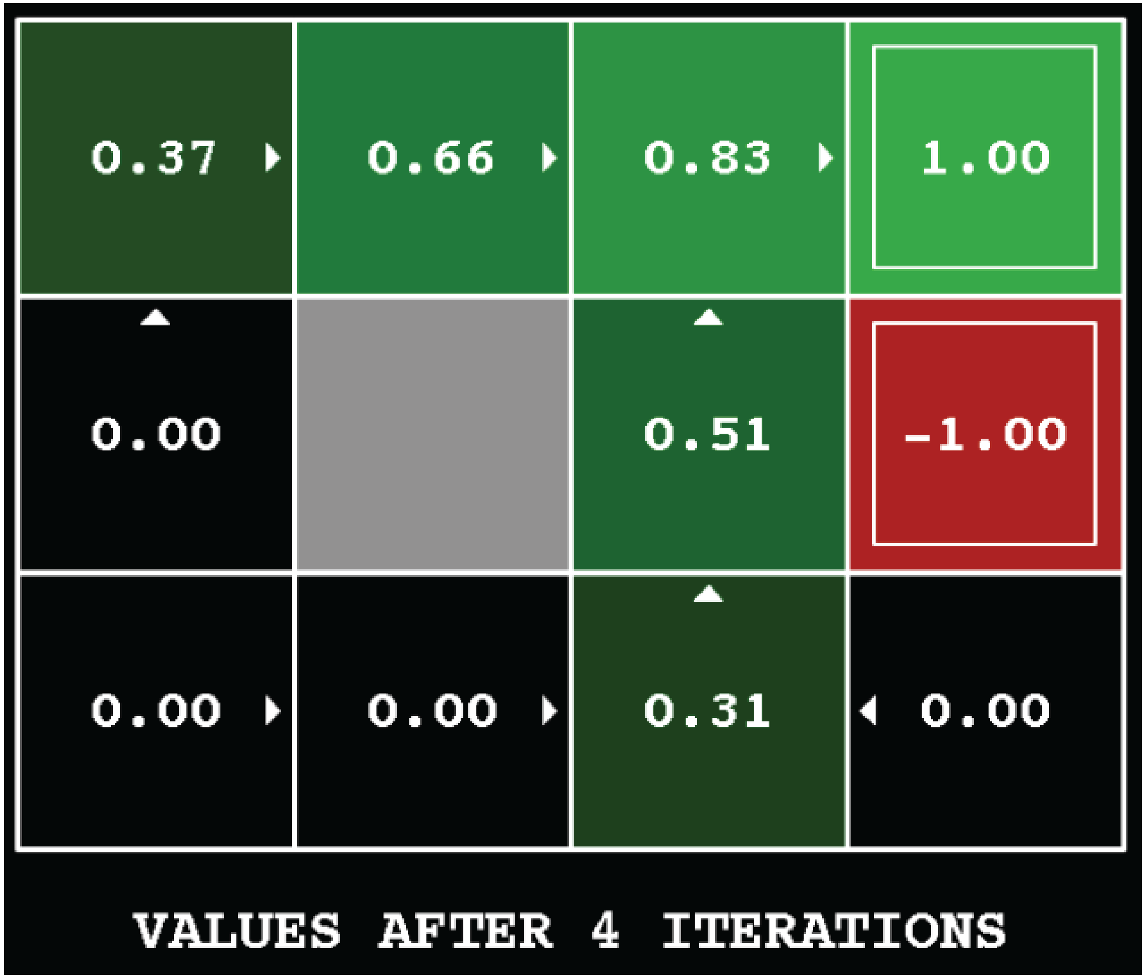
Example of Value Iteration



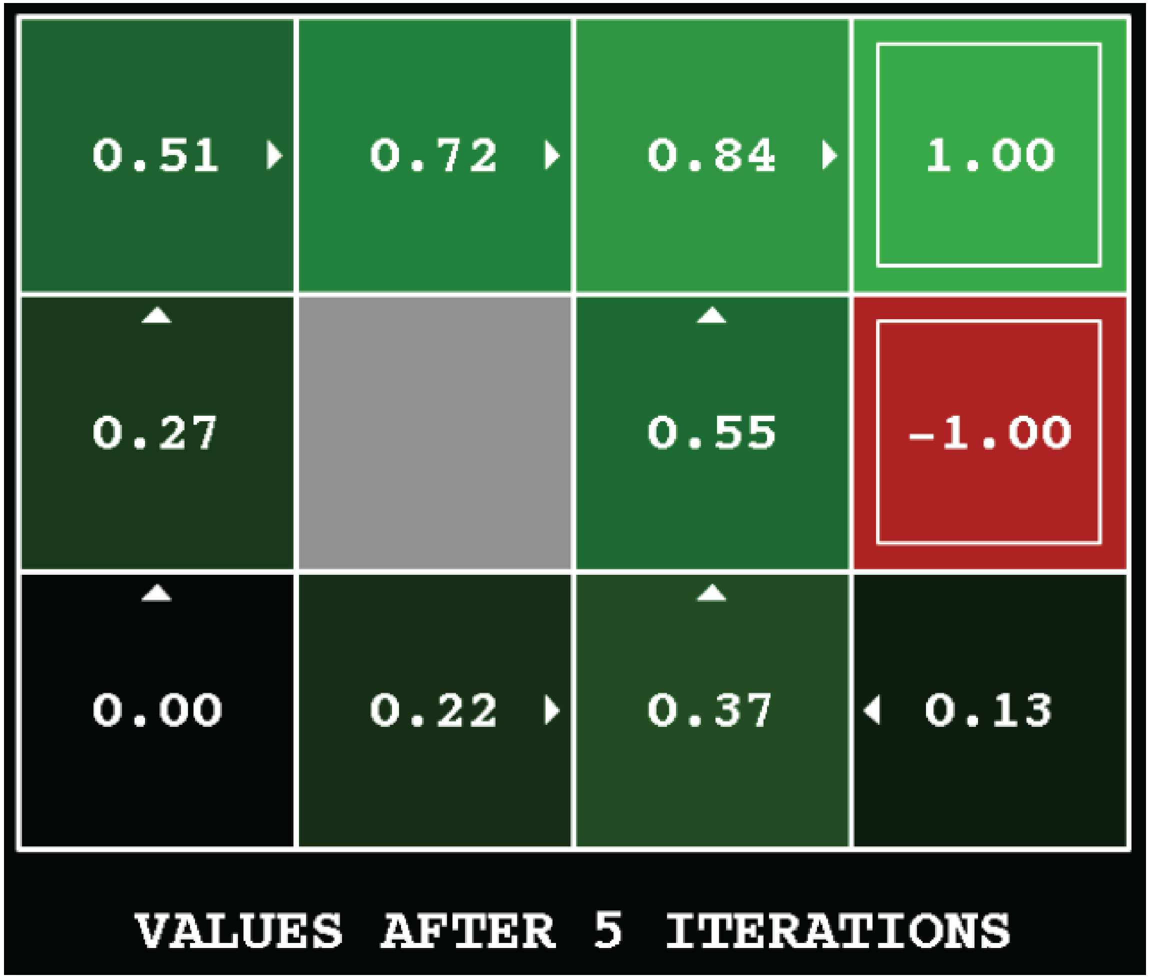
Example of Value Iteration



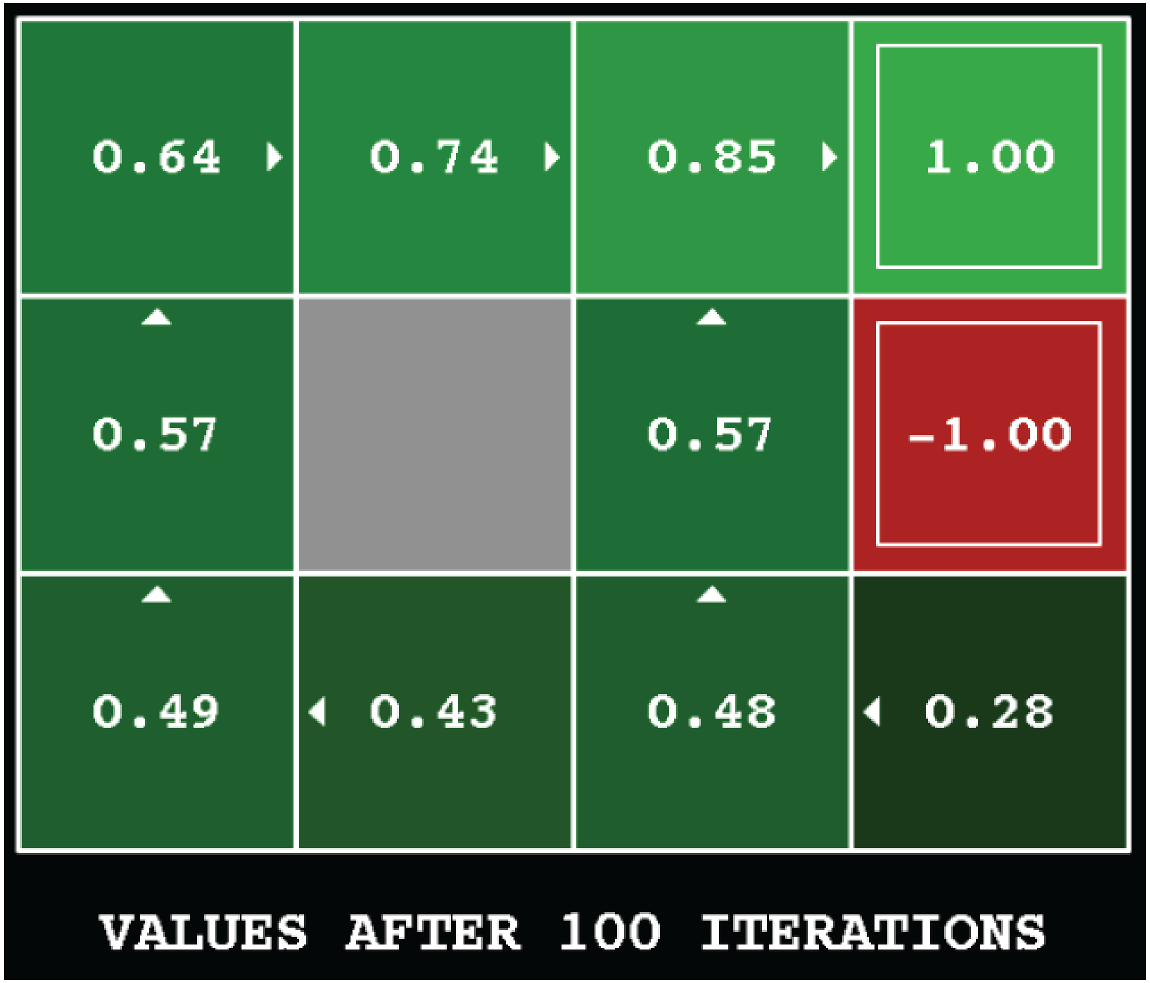
Example of Value Iteration



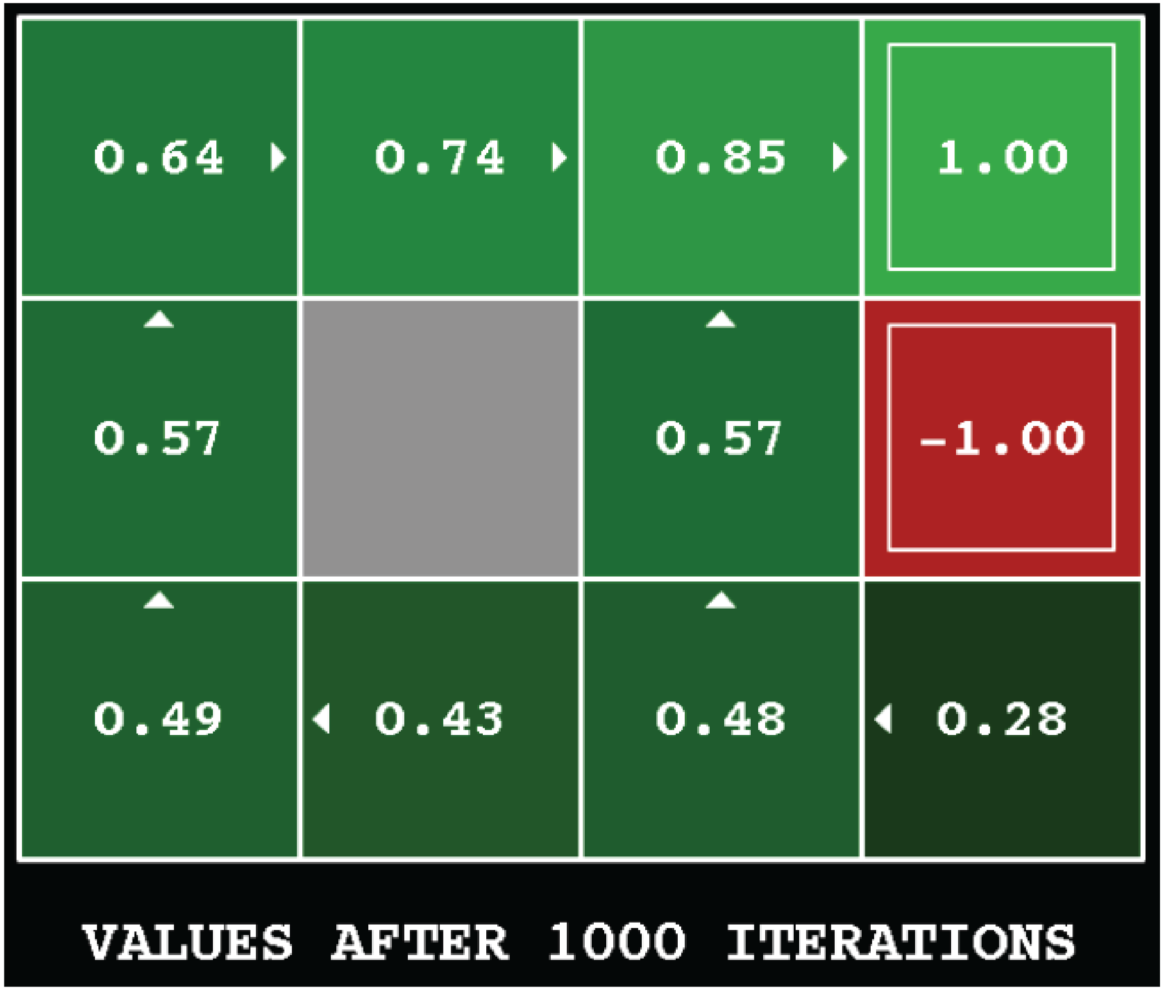
Example of Value Iteration



Example of Value Iteration



Example of Value Iteration



Policy Iteration

- Alternative way to solve for the optimal policy

- Beginning with an initial policy, alternative between:
  - **Policy Evaluation:** What is the utility of each state under the policy?
  - **Policy Improvement:** Update the policy to maximize the expected utility

- When no improvements can be made, we've converged

Policy Evaluation

- Easier than the Bellman Equations
- Policy is fixed, so no search over action
- Just solve linear system updated utilities!

$$U_i(s_t) = R(s_t) + \gamma \sum_{s_{t+1}} P(s_{t+1} \mid s_t, \pi_i(s_t)) U_i(s_{t+1})$$

Policy Update

- Modify the policy at each state

$$\pi_i(s_t) \leftarrow \arg\max_{a \in A(s_t)} \sum_{s_{t+1}} P(s_{t+1} \mid s_t, a) U_i(s_{t+1})$$

Algorithm

```
function POLICY-ITERATION(mdp) returns a policy
   inputs: mdp, an MDP with states S, transition model T
   local variables: U, U', vectors of utilities for states in S, initially zero
                    π, a policy vector indexed by state, initially random

   repeat
       U ← POLICY-EVALUATION(π, U, mdp)
       unchanged? ← true
       for each state s in S do
           if max_a Σ_{s'} T(s, a, s') U[s'] > Σ_{s'} T(s, π[s], s') U[s'] then
               π[s] ← argmax_a Σ_{s'} T(s, a, s') U[s']
               unchanged? ← false
   until unchanged?
   return π
```