Logical Agents

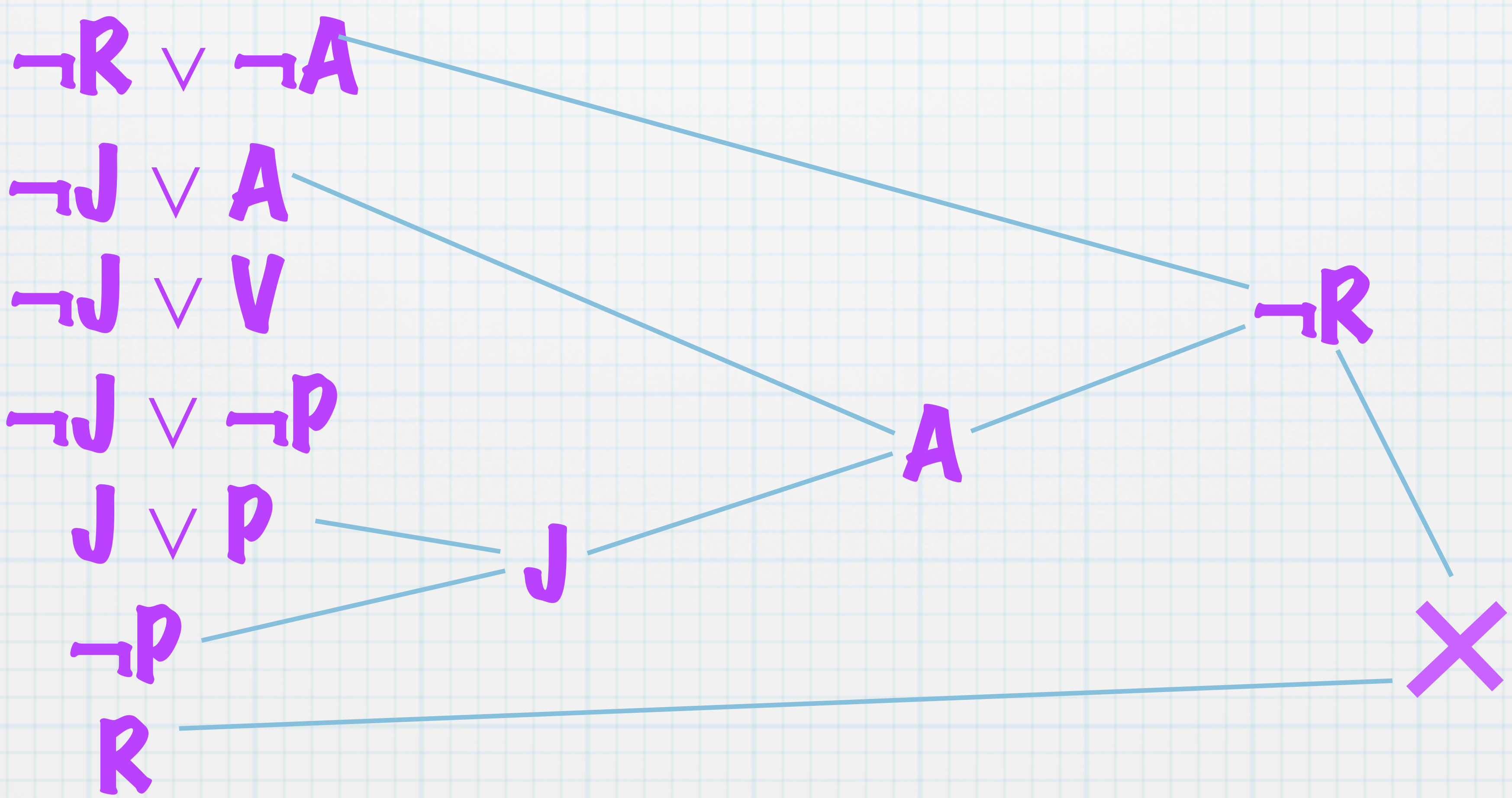
CH 8

Introduction to First Order Logic

Propositional Logic Practice

Proof by Resolution:

Start with KB and ¬α, look for contradiction



First Order Logic

Pros/Cons of Propositional Logic

- Propositional logic is **declarative**: pieces of syntax correspond to facts
- Propositional logic allows partial/disjunctive/negated information (unlike most data structures and databases)
- Propositional logic is **compositional**: meaning of $B_{1,1} \wedge P_{1,2}$ is derived from meaning of $B_{1,1}$ and of $P_{1,2}$
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- ☹ Propositional logic has very limited expressive power (unlike natural language)
  E.g., cannot say "pits cause breezes in adjacent squares"
  except by writing one sentence for each square

First Order Logic

Whereas propositional logic assumes world contains **facts**,
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brother of, bigger than, inside, part of, has color, occurred after, owns, comes between, ...
- Functions: father of, best friend, third inning of, one more than, end of ...

Syntax of FOL

Constants	$KingJohn,\ 2,\ UCB, \ldots$
Predicates	$Brother,\ >, \ldots$
Functions	$Sqrt,\ LeftLegOf, \ldots$
Variables	$x,\ y,\ a,\ b, \ldots$
Connectives	$\wedge\ \ \vee\ \ \neg\ \ \Rightarrow\ \ \ \Leftrightarrow$
Equality	$=$
Quantifiers	$\forall\ \exists$

Atomic Sentences

$$\begin{aligned}\text{Atomic sentence} \;&=\; \mathit{predicate}(\mathit{term}_1, \ldots, \mathit{term}_n) \\ &\quad\;\text{or } \mathit{term}_1 = \mathit{term}_2 \\[1em] \text{Term} \;&=\; \mathit{function}(\mathit{term}_1, \ldots, \mathit{term}_n) \\ &\quad\;\text{or } \mathit{constant} \text{ or } \mathit{variable}\end{aligned}$$

Atomic Sentences

* T if their statement is T in the world
  * Sister (Mary, Sally)
  * HasColor (Brown, Fido)
  * Mother (Mary) = Jane

Complex Sentences

- Complex sentences are made from atomic sentences using connectives ($\neg$, $\Rightarrow$, $\Leftrightarrow$, $\wedge$, $\vee$)
  - Father (Mary,John) $\wedge$ Father(Sally,John) $\Rightarrow$ Sister(Mary,Sally)

Truth in First Order Logic

Sentences are true with respect to a model and an interpretation

Model contains ≥ 1 objects (domain elements) and relations among them

Interpretation specifies referents for

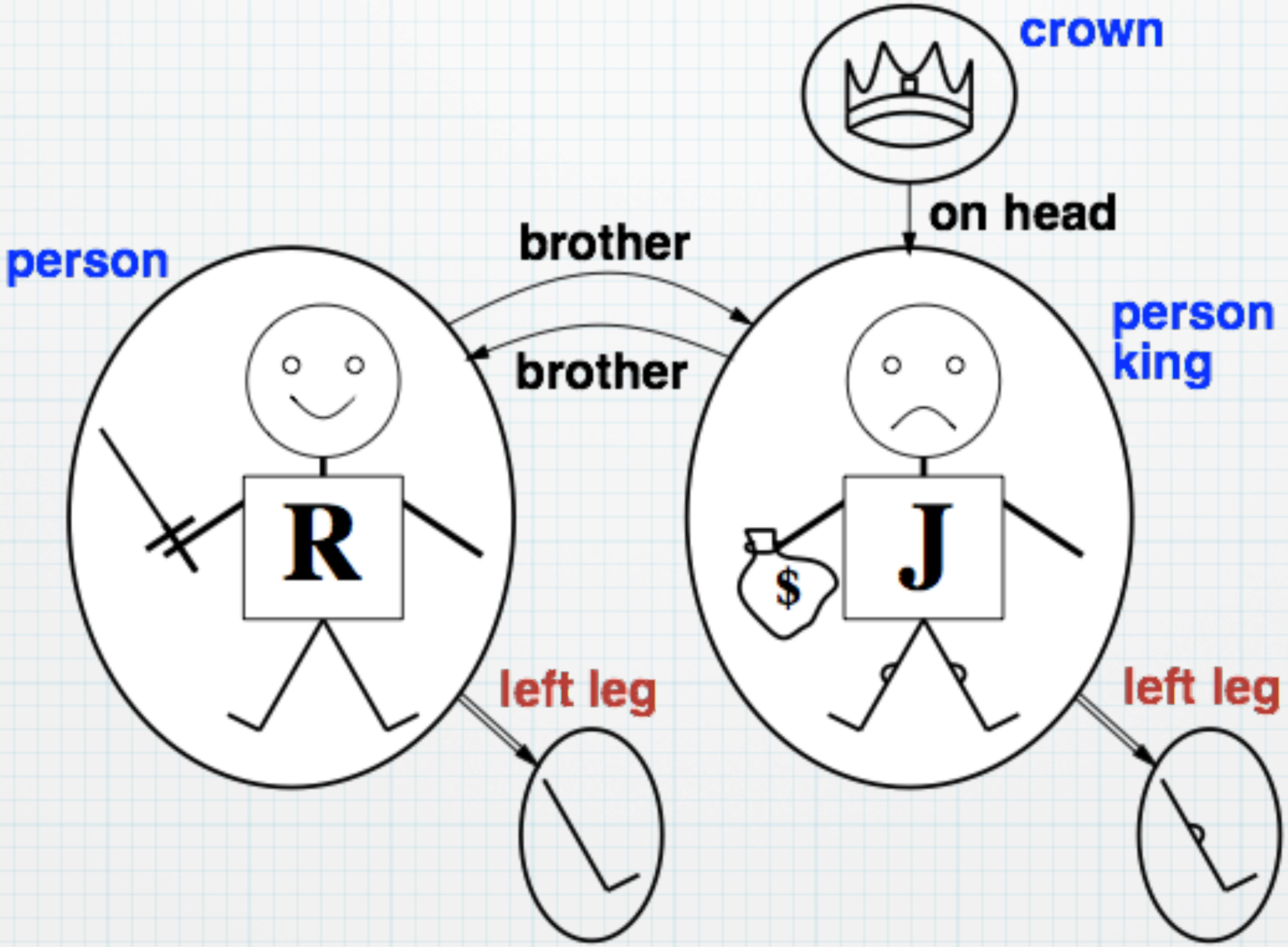
constant symbols $\rightarrow$ objects

predicate symbols $\rightarrow$ relations

function symbols $\rightarrow$ functional relations

An atomic sentence $predicate(term_1, \ldots, term_n)$ is true
iff the objects referred to by $term_1, \ldots, term_n$
are in the relation referred to by $predicate$

Royal Family FOL example



Models for FOL...lots!

Entailment in propositional logic can be computed by enumerating models

We **can** enumerate the FOL models for a given KB vocabulary:

For each number of domain elements n from 1 to ∞
  For each k-ary predicate P_k in the vocabulary
    For each possible k-ary relation on n objects
      For each constant symbol C in the vocabulary
        For each choice of referent for C from n objects . . .

Computing entailment by enumerating FOL models is not easy!

Quantifiers

Quantifiers

* Quantifiers are a key element of FOL
* They let us talk about sets of objects
* Universal quantifier: $\forall x$
* Existential quantifier: $\exists x$

Universal Quantifier

* All Dogs are mammals
  * $\forall_x$ dog(x) $\Rightarrow$ mammal (x)
* Everything is a dog and a mammal
  * $\forall_x$ dog(x) $\wedge$ mammal (x)
* P is true for all objects
  * $\forall_x$ P

Existential Quantifier

- P is true for at least one object
  - $\exists_x P$
- Some students like pizza
  - $\exists_x \text{student}(x) \wedge \text{likesPizza}(x)$

Nested Quantifiers

$\forall x \ \forall y$ is the same as $\forall y \ \forall x$ (why??)

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Quantifier duality: each can be expressed using the other

$\forall x \ \ Likes(x, IceCream) \qquad \neg\exists x \ \ \neg Likes(x, IceCream)$

$\exists x \ \ Likes(x, Broccoli) \qquad \neg\forall x \ \ \neg Likes(x, Broccoli)$

Fun with sentences

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$\forall x, y \;\; Sibling(x, y) \;\Leftrightarrow\; Sibling(y, x)$.

One's mother is one's female parent

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One's mother is one's female parent

$\forall x, y \;\; Mother(x, y) \;\Leftrightarrow\; (Female(x) \land Parent(x, y))$.

A first cousin is a child of a parent's sibling

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A first cousin is a child of a parent's sibling

$\forall x, y \;\; FirstCousin(x, y) \;\Leftrightarrow\; \exists\, p, ps \;\; Parent(p, x) \land Sibling(ps, p) \land Parent(ps, y)$