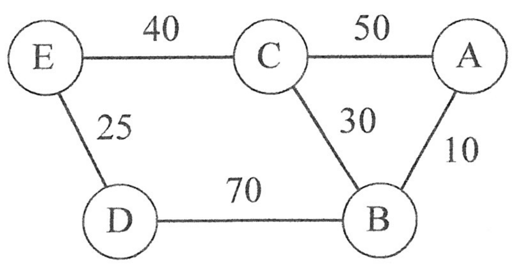
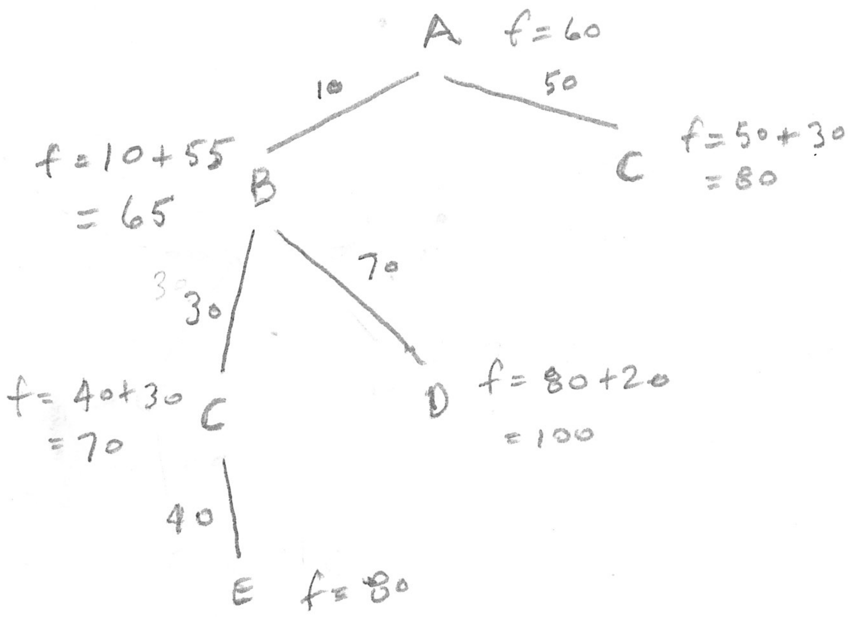
Question 1. Use **A* Search** to solve the map problem below, where the numbers give the distances between states.

- **Initial State** = A
- **Goal State** = E
- The value of the heuristic function for each node is given in the table on the left.

n	$h(n)$
A	60
B	55
C	30
D	20
E	0



(a) Draw the search tree for A*, showing the evolving frontier.



FRONTIER

A(60)
~~B(65)~~ ~~C(80)~~
~~C(70)~~ D(100)
E(80) D(100)

Note: This node replaced by C(70) in frontier once B is expanded.

(b) Indicate below the order in which nodes will be expanded (i.e. put in the explored set)

1	2	3	4	5	6	7	8	9	10
A	B	C	E						

(c) For each of the possible node expansion orders shown below, indicate which *uninformed search algorithms* could have produced the given node order when applied to the graph above. Note: Be explicit about how you are breaking ties for nodes at the same depth.

Node expansion order	Algorithm
1 A, 2 C, 3 E	BFS $A\uparrow$ & DFS $A\uparrow$
1 A, 2 B, 3 C, 4 D, 5 E	UNIFORM COST SEARCH
1 A, 2 B, 3 C, 4 E	BFS $A\downarrow$
1 A, 2 C, 3 B, 4 E	ITERATIVE DEEP DFS $A\uparrow$

Notes: Let $A\downarrow$ = BREAK TIES LOWEST LETTER

$A\uparrow$ = BREAK TIES HIGHEST LETTER

We follow convention from book: BFS checks for goal before insertion into frontier, while DFS, ITERATIVE DEEPENING DFS, and COST SENSITIVE all check for goal node on popping from frontier.

See next page for work.

(d) Show that the heuristic function for this problem satisfies the *consistency* criteria.

$$h(n) \le c(n,a,n') + h(n') \quad \forall n, n'$$

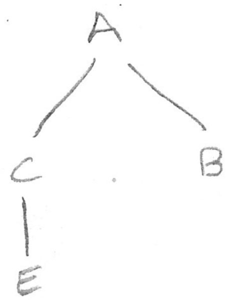
n'	n	$c+h(n')$		$h(n)$	
E	C	40		30	
E	D	25		20	
D	B	90	$\geq$	55	✓
C	A	80		60	
C	B	60		55	
B	A	65		60	

BFS A↓ VISITED: A B C E

FRONTIER

~~A~~
~~B~~ C
~~C~~ D

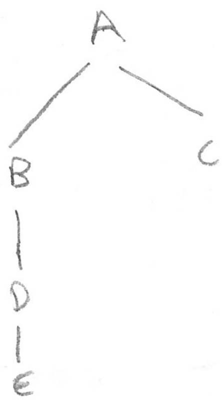
BFS A↑ VISITED: A C E



FRONTIER

~~A~~
~~C~~ B

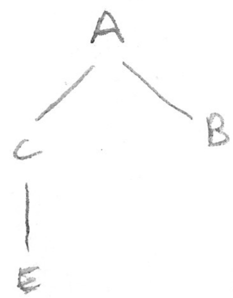
DFS A↓ VISITED: A B D E



FRONTIER

~~A~~
~~B~~ C
~~D~~ C
~~E~~ C

DFS A↑ VISITED: A C E

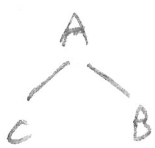


FRONTIER

~~A~~
~~C~~ B
~~E~~ B

ITER-DEEP DFS A↑ V: A C B E

ITER #1:



FRONTIER

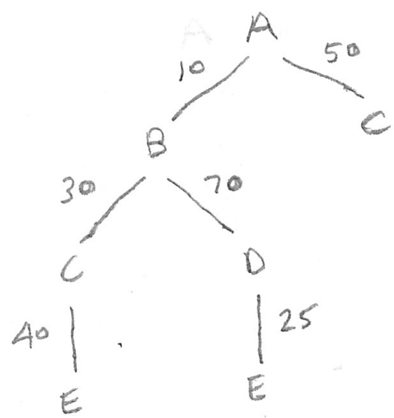
~~A~~
~~C~~ B
~~B~~

ITER #2:

FRONTIER

~~A~~
~~C~~ B
~~E~~ B

UNIFORM COST SEARCH V: A B C D E

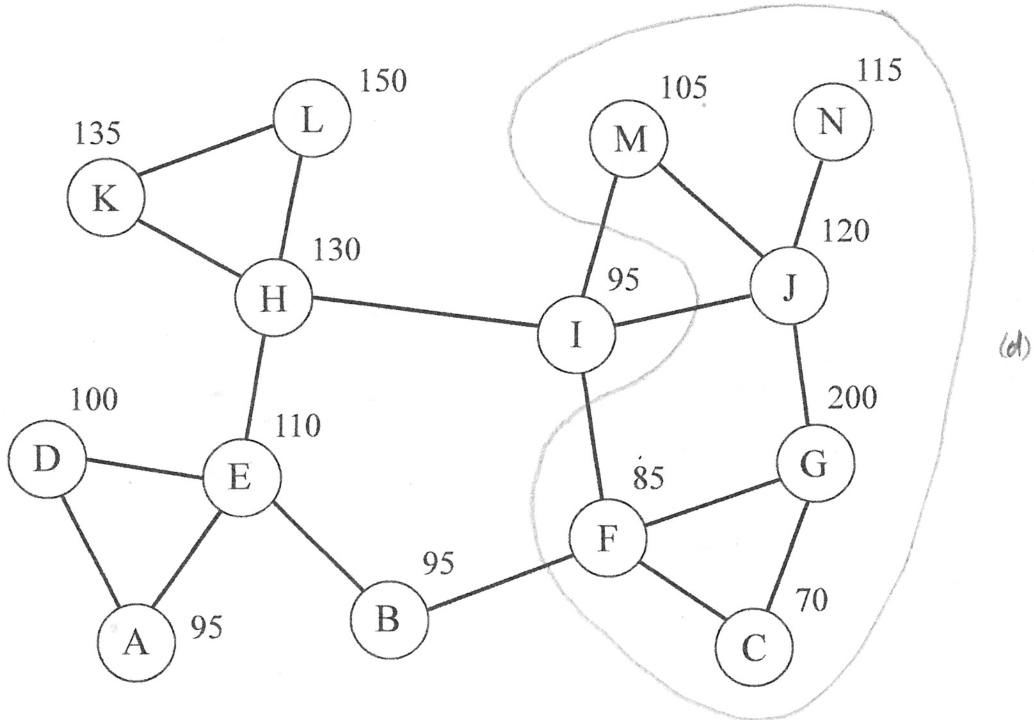


FRONTIER

A(~~0~~)
B(~~10~~) C(50)
C(~~40~~) D(80)
D(~~80~~) E(80)
E(~~80~~) E(105)

FINAL PATH:
A B C E

Question 2. In this question you will use the **Hill Climbing search algorithm** on the graph shown below to *maximize the objective function*, which is given by the number next to each state.



(a) If hill climbing search starts in state D, which adjacent state would be visited next? E

(b) Give the order in which nodes would be visited by hill climbing search, starting at D:

1	2	3	4	5	6	7	8	9	10
D	E	H	L						

(c) Which state is the global maximum of the objective function? G

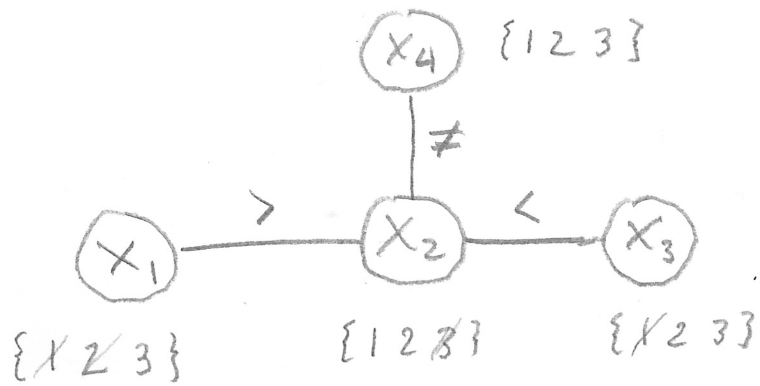
(d) List all of the starting states with the property that hill climbing search will reach the global maximum.

C F J N M G

Question 3. The following *constraint satisfaction problem* (CSP) has four state variables X_1, X_2, X_3, and X_4 each of which can take on domain values $D = \{1, 2, 3\}$. The goal is to find an assignment that satisfies the following constraints:

$$C_1\colon X_1 > X_2 \qquad C_2\colon X_3 > X_2 \qquad C_3\colon X_2 \neq X_4$$

(a) Draw the constraint graph that represents this CSP.



(b) Assume that we assign $X_1 = 3$, and then run the *Arc Consistency algorithm*. Give the resulting domain values for X_2, X_3, and X_4:

X_2	1 2
X_3	2 3
X_4	1 2 3

(c) Now what is the best variable to assign next, and why?

X_2 & X_3 TIED UNDER MINIMUM REMAINING VALUE (MRV)

BREAK TIE W/ DEGREE HEURISTIC

CHOOSE X_2

Question 4. Mark each of the following statements as *TRUE or FALSE*.
If FALSE, **rewrite the sentence** changing just a few words to make it true.

- A game of poker is an example of a stochastic, ~~fully~~-observable, multi-agent task environment.

  partially

  FALSE

- The primary difference between A* search and uniform cost search is the use of a ~~priority queue.~~ heuristic function

  FALSE

- Given two admissable heuristics a and b, the heuristic defined by $\max\{a, b\}$ dominates both of them.

  TRUE

  — $h^*(n)$

  — a — max (a,b)

  — b

- If $M(\alpha)$ is the set of models of a sentence α in a propositional logic, then if α entails β we must have $M(\alpha) \subseteq M(\beta)$.

  TRUE

  $\alpha \models \beta \Rightarrow$ Every model in which α is T, β is also T

  $\Rightarrow M(\alpha) \subseteq M(\beta)$

- An online agent ~~is a program for searching the internet.~~

  interleaves computation and action.

  FALSE

- If a sentence α in a propositional logic is valid, then it follows that $\neg\alpha$ is ~~satisfiable~~ unsatisfiable via a proof by contradiction.

  FALSE

  α VALID $\Rightarrow$ α TRUE IN ALL MODELS

  $\neg\alpha$ SATISFIABLE $\Rightarrow$ $\neg\alpha$ TRUE IN SOME MODEL

  $\Rightarrow$ α FALSE IN SOME MODEL

  CONTRADICTION!

Question 5. Conjunctive Normal Form (CNF) $\longrightarrow$ $(\cdot\cdot \vee \cdot\cdot) \wedge (\cdot\cdot \vee \cdot\cdot) \wedge \ldots$

Write each of the following expressions as a conjunction of clauses in order to put them in CNF.

(a) $A \Rightarrow (B \vee C)$

$\neg A \vee (B \vee C)$

$\neg A \vee B \vee C$ SINGLE CLAUSE

(b) $B \Leftrightarrow \neg C$

$(B \Rightarrow \neg C) \wedge (\neg C \Rightarrow B)$

$(\neg B \vee \neg C) \wedge (C \vee B)$

HC

(c) $((A \Rightarrow B) \wedge C) \Rightarrow D$

$((\neg A \vee B) \wedge C) \Rightarrow D$

$(\neg(\neg A \vee B) \vee \neg C)) \vee D$

$((A \wedge \neg B) \vee \neg C) \vee D$

$((A \vee \neg C) \wedge (\neg B \vee \neg C)) \vee D$

$(A \vee \neg C \vee D) \wedge (\neg B \vee \neg C \vee D)$

HC

(d) For each of the clauses you generated above, identify whether or not it is a Horn clause (e.g. write "HC" below each of the Horn clauses). HC = AT MOST 1 POS. LITERAL

Question 6. Resolution in Predicate Logic

In the game *Minesweeper*, the objective is to uncover hidden mines in a board by probing board locations. Each time a square is probed, the player learns the number of mines which are adjacent to that square (adjacent horizontally, vertically, or diagonally). The objective is to locate all of the mines. The following figure shows the state of the game for a simple 3 by 2 board:

3		
2	$X_{1,2}$	$X_{2,2}$
1	2	$X_{2,1}$
	1	2

The presence of a 2 in location $(1, 1)$ means that there are exactly two mines hidden in the adjacent squares $(1, 2)$, $(2, 1)$, and $(2, 2)$. Let $X_{i,j}$ be a boolean literal denoting the presence of a mine at location (i, j).

(a) For each of the following logical sentences, give the number of mines that could be adjacent to $(1, 1)$ if the sentence is true. Explain your answer.

- $R_1 : \neg X_{1,2} \vee \neg X_{2,2} \vee \neg X_{2,1}$

R1 FALSE ⇒ ALL LITERALS TRUE

IF AT LEAST 1 LIT. FALSE ⇒ R1 TRUE

2 OR FEWER MINES

- $R_2 : (X_{1,2} \vee X_{2,2}) \wedge (X_{1,2} \vee X_{2,1}) \wedge (X_{2,2} \vee X_{2,1})$

R2 FALSE ⇒ AT LEAST 2 LIT FALSE (CLAUSES CONTAIN ALL PAIRS OF PRED.)

R2 TRUE ⇒ AT MOST 1 LIT. FALSE

⇒ 2 OR MORE LIT TRUE ⇒ 2 OR MORE MINES

- $R_1 \wedge R_2$

$2 \leq X \leq 2 \Rightarrow X = 2$ MINES

(b) Using resolution, show that $\neg X_{2,2}$ entails $X_{1,2} \wedge X_{2,1}$. Note: KB $\equiv R_1 \wedge R_2 \wedge \neg X_{2,2}$

$KB \models \alpha \quad \Leftrightarrow \quad KB \wedge \neg(X_{12} \wedge X_{21})$ not satisfiable

$\alpha = X_{12} \wedge X_{21}$

$\neg(X_{12} \wedge X_{21}) = \neg X_{12} \vee \neg X_{21}$

R1: $\neg X_{12} \vee \neg X_{22} \vee \neg X_{21}$

R2: $X_{12} \vee X_{22}$ → X_{12}

$X_{12} \vee X_{21}$

$X_{22} \vee X_{21}$

$\neg X_{22}$ → X_{21}

$\neg\alpha$: $\neg X_{12} \vee \neg X_{21}$ → $\neg X_{12}$

$\square$

Question 7. Consider the following joint probability distribution in three discrete random variables A, B, C.

$P(C, A=1)$

$p(A=1, B, C) =$

	$B=1$	$B=2$	$B=3$
$C=1$	0	0.05	0.05
$C=2$	0.05	0.05	0.05
$C=3$	0.05	0	0.05

$\Rightarrow \begin{bmatrix} 0.1 \\ 0.15 \\ 0.1 \end{bmatrix}$

$C=1$

$P(C, A=2)$

$p(A=2, B, C) =$

	$B=1$	$B=2$	$B=3$
$C=1$	0.1	0.1	0.2
$C=2$	0.1	0	0
$C=3$	0	0.1	0.05

$A=2$

$\Rightarrow \begin{bmatrix} 0.4 \\ 0.1 \\ 0.15 \end{bmatrix}$

(a) You are told that $A=2$ and $C=1$. Compute $P(B|A=2, C=1)$.

$$P(B \mid A=2, C=1) = \frac{P(B, A=2, C=1)}{P(A=2, C=1)} = \frac{1}{K}\begin{bmatrix} 0.1 \\ 0.1 \\ 0.2 \end{bmatrix}$$

$$P(A=2, C=1) = \sum_{B=1}^{3} P(B, A=2, C=1)$$

$$= 0.1 + 0.1 + 0.2 = 0.4 = K$$

$$P(B \mid A=2, C=1) = [0.25 \;\; 0.25 \;\; 0.5]^T$$

(c) What is the *a priori* probability distribution over C?

$$P(C) = \sum_{A=1}^{2} P(A, C) = \begin{bmatrix} 0.5 \\ 0.25 \\ 0.25 \end{bmatrix}$$