Midterm Review

AIMA CH 2-7

Chapter 1&2

Environments

* Observable / Partially Observable
* Deterministic / Stochastic
* Episodic / Sequential
* Static / Dynamic
* Discrete / Continuous
* Single-agent / Multi-agent

Chapter 1&2

Agent Designs

* Reflex
* Model-based
* Goal-based
* Utility

Chapter 3

Classical Search

* Problem Formulation
* Tree Search
* Uninformed Strategies
* Informed Strategies

Problem Formulation

* The design decision of how to represent the agent's: **actions**, **states**, **costs**

Problem Solving Steps

Problem → Search Algorithm → Solution

* Formulate the Goal
* Formulate the States, Actions, Costs
* Find Solution
* Execute sequence of actions

Tree Search

* Root node is init state
* Transition model tells next states
* Goal test? yes -> done, no->expand more

Search Strategy: Which leaf node to expand 1st?

Tree Search

What's in my node representation?

* **State** this node represents
* **Parent** node that generated this
* **Action** that generated this
* **Cost** of path from init to here
* **Depth** of path from init to here

Node != State
represents path
to a state

General Tree Search

TreeSearch(problem) **returns** solution

frontier={init-state}

loop:

1. **if** frontier empty **return** failure
2. **choose leaf node, remove from** frontier
3. **if** node.contains(goal) **return** node.solution
4. node.expand(), **add children to** frontier

How they get added into queue is important

Evaluating Search Strategies

* Strategy = order of node expansion
  * Complete: finds solution if one exists
  * Optimal: always finds least cost solution
  * Time Complexity: # nodes generated
  * Space Complexity: max nodes ever in memory

Uninformed Search

Use only the info in the problem definition

* Breadth-first search
* Uniform-cost search
* Depth-first search
* Depth-limited search
* Iterative-deepening search

Comparison of Algs

Criterion	Breadth-First	Uniform-Cost	Depth-First	Depth-Limited	Iterative Deepening
Complete?	Yes*	Yes*	No	Yes, if $l \geq d$	Yes
Time	b^d	$b^{\lceil C^*/\epsilon \rceil}$	b^m	b^l	b^d
Space	b^d	$b^{\lceil C^*/\epsilon \rceil}$	bm	bl	bd
Optimal?	Yes*	Yes	No	No	Yes*

- BFS vs. DFS
  - memory
- DFS vs. D-limited vs. Iterative-D
  - time

Summary for Uninformed

* Variety of search strategies
* Iterative deepening uses only linear space and not much more time than other algorithms
* Graph search can be exponentially more efficient than tree search

Informed Search

* What if you know more...
  * Designer knows something about the problem to help the agent
  * Domain knowledge
* Use this to expand the **BEST** node first

Best-First Search

* Tree search + Evaluation Function
* $f(n)$ = desirability of node n

Search Strategy: How to define eval function

Best-First Search

* Heuristic function h(n)
  * estimated cheapest path, n to goal
  * estimated future path cost from n

Greedy Best-First

* $f(n) = h(n)$: expand node that looks closest from here
* Example -- a common heuristic for route planning: straight line distance to goal

A* Search

* Most widely known Best-First alg
* $f(n) = g(n) + h(n)$
  * $g(n)$ = cost to get to this node
  * $h(n)$ = estimated cost from here
* Minimizes total solution cost

Admissible Heuristic

* h(n) =
  * under-estimate of cost to goal
  * zero for any goal state
  * non-zero for all others
* Makes A* Optimal & Complete!

Dominance

If $h_2(n) \geq h_1(n)$ for all n (both admissible)
then h_2 dominates h_1 and is better for search

Typical search costs:

$d = 14$ IDS = 3,473,941 nodes
A$^*(h_1)$ = 539 nodes
A$^*(h_2)$ = 113 nodes
$d = 24$ IDS $\approx$ 54,000,000,000 nodes
A$^*(h_1)$ = 39,135 nodes
A$^*(h_2)$ = 1,641 nodes

Given any admissible heuristics h_a, h_b,

$$h(n) = \max(h_a(n), h_b(n))$$

is also admissible and dominates h_a, h_b

Chapter 4

Local Search

* Classical Search
    * Solution = path to goal state
* Local Search
    * Solution = goal state itself
    * How to search for a solution when the path doesn't matter?

Local Search

- **Complete** = always finds a goal state if there is one
- **Optimal** = always find the global max

Local Search Algs

* Hill-Climbing
* Simulated Annealing
* Local Beam Search
* Genetic Algorithms

Online Search

* **Offline:** simulate the world and reason about a plan to get to a goal
* **Online:** solve the search problem while executing actions
* Interleaves search and execution

Chapter 6

Constraint Satisfaction Problems

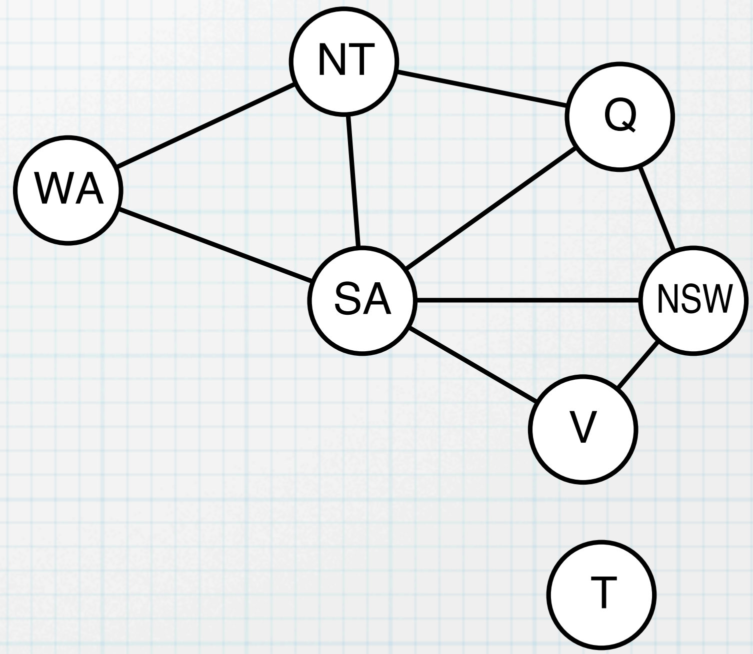
* Problem Formulation
* Backtracking Search
    * Variable order heuristics
    * Value order heuristics
    * Constraint Propagation

Constraint Satisfaction Problem

* State: variables X_i, values from domain D_i
* Goal Test: **constraints** specifying allowable combinations of values for variables
* **Formal Representation Language**
* Allows **general-purpose** algorithms with more power than standard search algs

Constraint Graph

* **Nodes** are variables
* **Arcs** show constraints
* Binary CSP: each constraint relates only 2 variables
* CSP algorithms use the graph structure to speed up search for a goal state configuration



Search for CSP Solution

* **Init State** = {}
* **Successor()** = assign value (consistent with constraints) to any unassigned variable
* **Goal Test** = all vars are assigned
* Fail if no legal assignment to do
* **Same formulation** for every CSP problem, yea!
* Problem: **n** vars, with **d** values, branch factor at root is **nd**, then **(n-1)d** ...terrible!
* Order doesn't matter, consider 1 var at a time

Backtracking Search

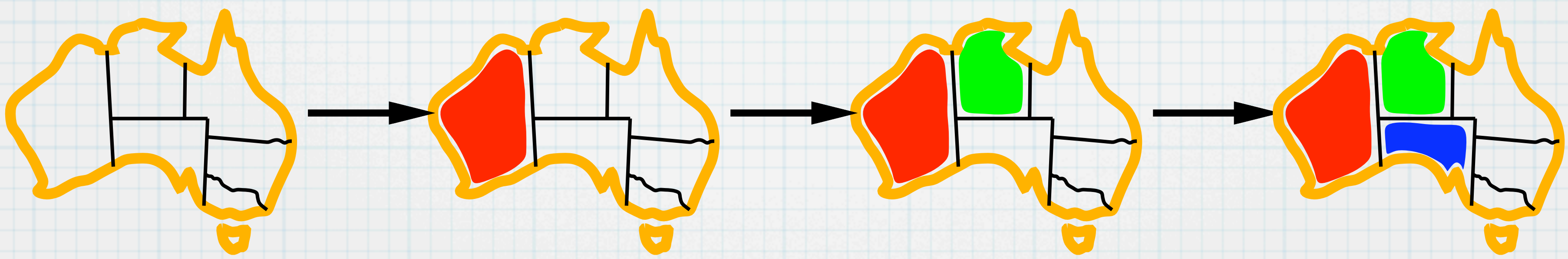
```
function BACKTRACKING-SEARCH(csp) returns solution/failure
    return RECURSIVE-BACKTRACKING({ }, csp)

function RECURSIVE-BACKTRACKING(assignment, csp) returns soln/failure
    if assignment is complete then return assignment
    var ← SELECT-UNASSIGNED-VARIABLE(VARIABLES[csp], assignment, csp)
    for each value in ORDER-DOMAIN-VALUES(var, assignment, csp) do
        if value is consistent with assignment given CONSTRAINTS[csp] then
            add {var = value} to assignment
            result ← RECURSIVE-BACKTRACKING(assignment, csp)
            if result ≠ failure then return result
            remove {var = value} from assignment
    return failure
```

* Repeatedly choose value for unassigned var, return fail if inconsistency detected

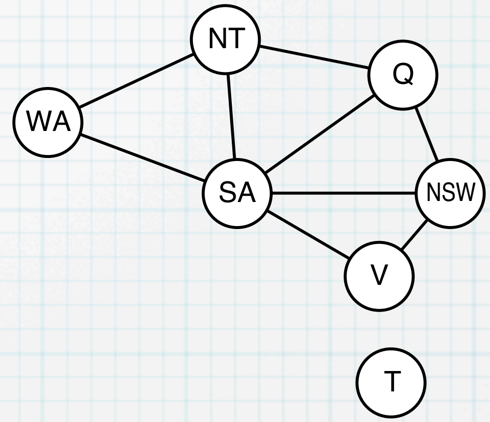
Minimum Remaining Values (MRV)

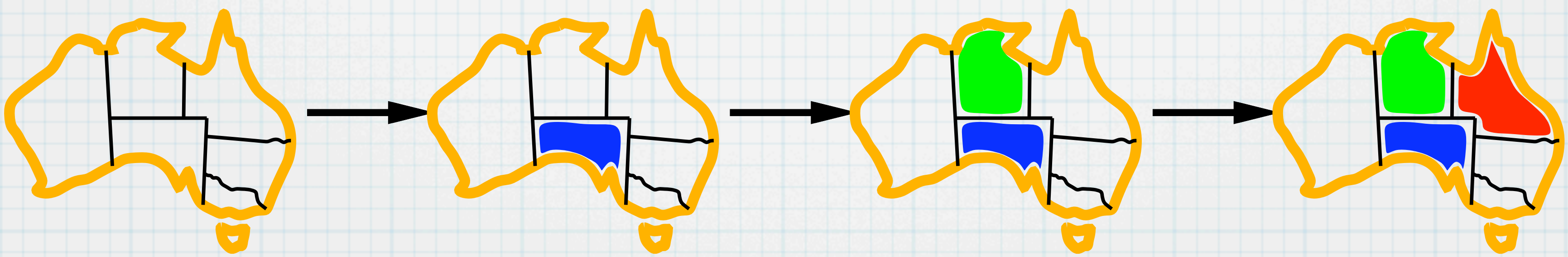
* What variable to do next?
* Choose var with the fewest legal values



Degree Heuristic

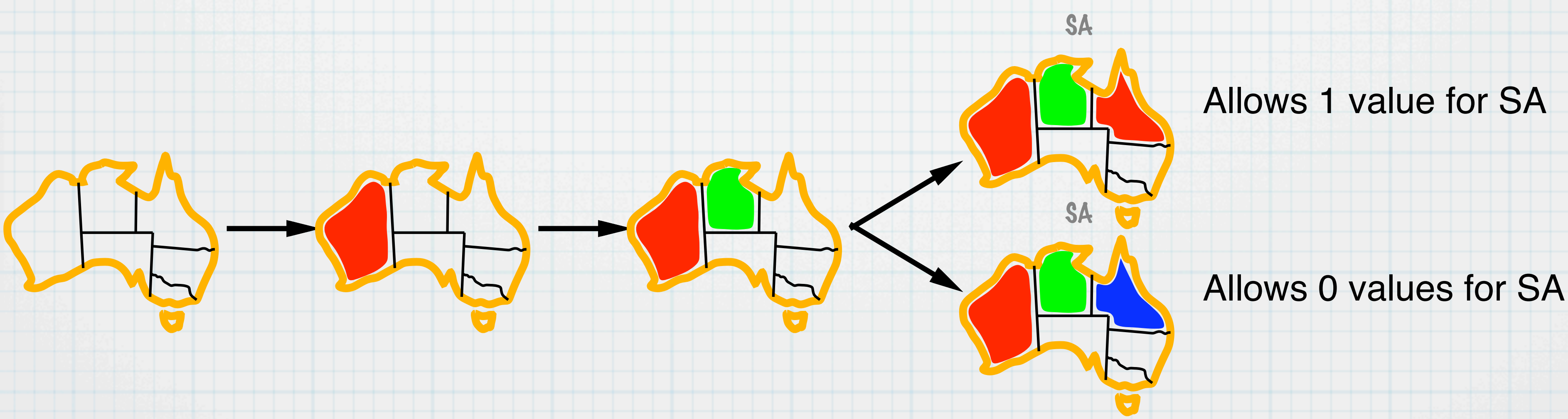
* Tie breaker for MRV
* Choose var with most constraints on remaining variables





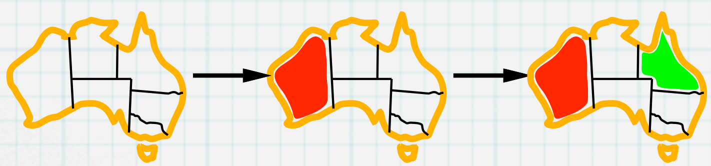
Least Constraining Value

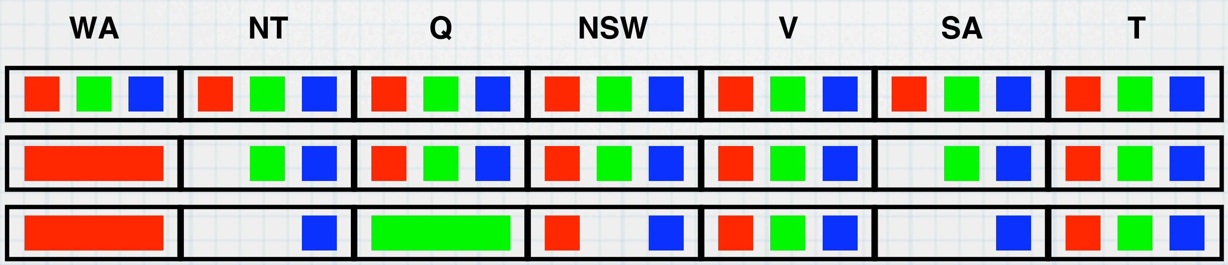
* What value to try next?
* Given a variable, choose value that rules out the least values in remaining vars



Constraint Propagation

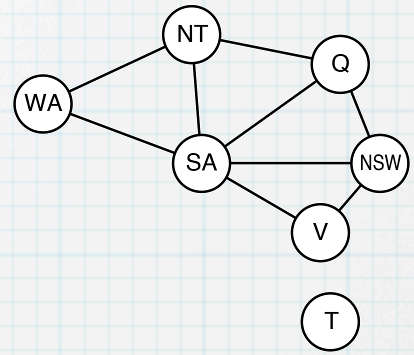
- Can stop a branch even earlier by propagating constraints and values
- After deleting neighbors run constraints





NT and SA cannot both be blue!

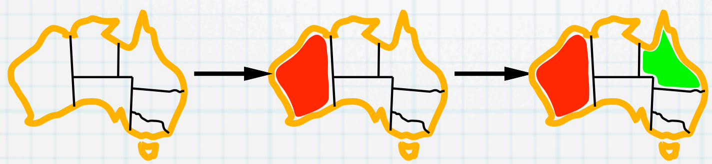
Arc Consistency

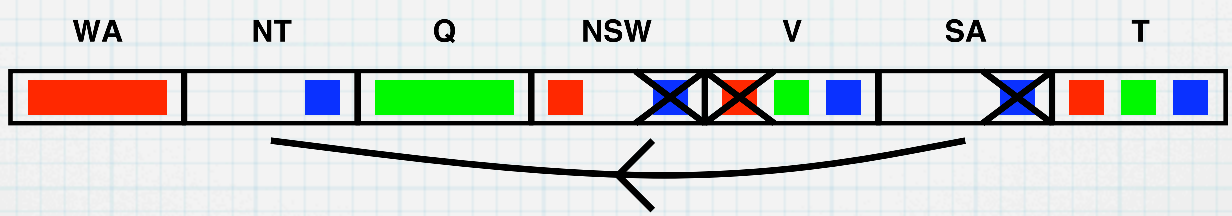


Simplest form of propagation makes each arc consistent

$X \rightarrow Y$ is consistent iff

for **every** value x of X there is **some** allowed y





If X loses a value, neighbors of X need to be rechecked

Arc consistency detects failure earlier than forward checking

Can be run as a preprocessor or after each assignment

Chapter 7

Logical Agents

* Propositional Logic
* Inference in Propositional Logic
  * by Enumeration
  * Forward/Backward Chaining
  * Resolution

Simple KB-agent

function KB-AGENT(*percept*) **returns** an *action*
    static: *KB*, a knowledge base
            t, a counter, initially 0, indicating time

    TELL(*KB*, MAKE-PERCEPT-SENTENCE(*percept*, *t*))
    $action \leftarrow$ ASK(*KB*, MAKE-ACTION-QUERY(*t*))
    TELL(*KB*, MAKE-ACTION-SENTENCE(*action*, *t*))
    $t \leftarrow t + 1$
    return *action*

The agent must be able to:

- Represent states, actions, etc.
- Incorporate new percepts
- Update internal representations of the world
- Deduce hidden properties of the world
- Deduce appropriate actions

Propositional Logic: Syntax

Propositional logic is the simplest logic—illustrates basic ideas

The proposition symbols P_1, P_2 etc are sentences

If S is a sentence, $\neg S$ is a sentence (negation)

If S_1 and S_2 are sentences, $S_1 \wedge S_2$ is a sentence (conjunction)

If S_1 and S_2 are sentences, $S_1 \vee S_2$ is a sentence (disjunction)

If S_1 and S_2 are sentences, $S_1 \;\Rightarrow\; S_2$ is a sentence (implication)

If S_1 and S_2 are sentences, $S_1 \;\Leftrightarrow\; S_2$ is a sentence (biconditional)

Truth Table for Connectives

Model		Truth value w.r.t. given Model				
P	Q	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \Rightarrow Q$	$P \Leftrightarrow Q$
false	*false*	*true*	*false*	*false*	*true*	*true*
false	*true*	*true*	*false*	*true*	*true*	*false*
true	*false*	*false*	*false*	*true*	*false*	*false*
true	*true*	*false*	*true*	*true*	*true*	*true*

Inference by Enumeration

```
function TT-ENTAILS?(KB, α) returns true or false
    inputs: KB, the knowledge base, a sentence in propositional logic
            α, the query, a sentence in propositional logic

    symbols ← a list of the proposition symbols in KB and α
    return TT-CHECK-ALL(KB, α, symbols, [ ])
```

```
function TT-CHECK-ALL(KB, α, symbols, model) returns true or false
    if EMPTY?(symbols) then
        if PL-TRUE?(KB, model) then return PL-TRUE?(α, model)
        else return true
    else do
        P ← FIRST(symbols); rest ← REST(symbols)
        return TT-CHECK-ALL(KB, α, rest, EXTEND(P, true, model)) and
                   TT-CHECK-ALL(KB, α, rest, EXTEND(P, false, model))
```

* DFS enumeration of all variables
* Checking if query T everywhere KB is T

Truth Tables for Inference

Model / **KB sentences**

$B_{1,1}$	$B_{2,1}$	$P_{1,1}$	$P_{1,2}$	$P_{2,1}$	$P_{2,2}$	$P_{3,1}$	R_1	R_2	R_3	R_4	R_5	KB
false	false	false	false	false	false	false	true	true	true	true	false	false
false	false	false	false	false	false	true	true	true	false	true	false	false
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
false	true	false	false	false	false	false	true	true	false	true	true	false
false	true	false	false	false	false	true	true	true	true	true	true	true
false	true	false	false	false	true	false	true	true	true	true	true	true
false	true	false	false	false	true	true	true	true	true	true	true	true
false	true	false	false	true	false	false	true	false	false	true	true	false
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
true	true	true	true	true	true	true	false	true	true	false	true	false

Enumerate rows (different assignments to symbols),
if KB is true in row, check that α is too

Inference as Search

* **Init State**: initial KB
* **Transition model**: all logical inference rules and resulting additions to the KB
* **Goal**: KB that contains the sentence we are trying to prove

Forward and Backward Chaining

Horn Form (restricted)

KB = **conjunction** of **Horn clauses**

Horn clause =

- ♢ proposition symbol; or
- ♢ (conjunction of symbols) $\Rightarrow$ symbol

E.g., $C \wedge (B \Rightarrow A) \wedge (C \wedge D \Rightarrow B)$

Modus Ponens (for Horn Form): complete for Horn KBs

$$\frac{\alpha_1, \ldots, \alpha_n, \qquad \alpha_1 \wedge \cdots \wedge \alpha_n \Rightarrow \beta}{\beta}$$

Can be used with forward chaining or backward chaining.
These algorithms are very natural and run in **linear** time

Forward Chaining

```
function PL-FC-ENTAILS?(KB, q) returns true or false
  inputs: KB, the knowledge base, a set of propositional Horn clauses
          q, the query, a proposition symbol
  local variables: count, a table, indexed by clause, initially the number of premises
                   inferred, a table, indexed by symbol, each entry initially false
                   agenda, a list of symbols, initially the symbols known in KB

  while agenda is not empty do
      p ← POP(agenda)
      unless inferred[p] do
          inferred[p] ← true
          for each Horn clause c in whose premise p appears do
              decrement count[c]
              if count[c] = 0 then do
                  if HEAD[c] = q then return true
                  PUSH(HEAD[c], agenda)
  return false
```

Resolution

Conjunctive Normal Form (CNF—universal)

$\textbf{conjunction}$ of $\underbrace{\textbf{disjunctions}\text{ of }\textbf{literals}}_{\textbf{clauses}}$

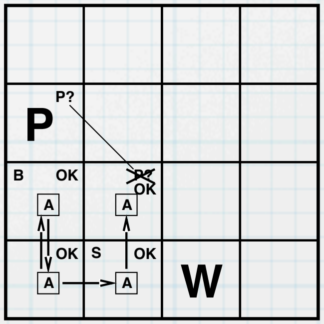
E.g., $(A \vee \neg B) \wedge (B \vee \neg C \vee \neg D)$

Resolution inference rule (for CNF): complete for propositional logic

$$\frac{\ell_1 \vee \cdots \vee \ell_k, \qquad m_1 \vee \cdots \vee m_n}{\ell_1 \vee \cdots \vee \ell_{i-1} \vee \ell_{i+1} \vee \cdots \vee \ell_k \vee m_1 \vee \cdots \vee m_{j-1} \vee m_{j+1} \vee \cdots \vee m_n}$$

where ℓ_i and m_j are complementary literals. E.g.,

$$\frac{P_{1,3} \vee P_{2,2}, \qquad \neg P_{2,2}}{P_{1,3}}$$



Resolution is sound and complete for propositional logic

Example CNF Convert

* Great! Can we make everything CNF?
* Convert sentences to Conjunctive Normal Form with our rules of logical equivalence

$$
\begin{array}{rcll}
(\alpha \land \beta) & \equiv & (\beta \land \alpha) & \text{commutativity of } \land \\
(\alpha \lor \beta) & \equiv & (\beta \lor \alpha) & \text{commutativity of } \lor \\
((\alpha \land \beta) \land \gamma) & \equiv & (\alpha \land (\beta \land \gamma)) & \text{associativity of } \land \\
((\alpha \lor \beta) \lor \gamma) & \equiv & (\alpha \lor (\beta \lor \gamma)) & \text{associativity of } \lor \\
\neg(\neg\alpha) & \equiv & \alpha & \text{double-negation elimination} \\
(\alpha \Rightarrow \beta) & \equiv & (\neg\beta \Rightarrow \neg\alpha) & \text{contraposition} \\
(\alpha \Rightarrow \beta) & \equiv & (\neg\alpha \lor \beta) & \text{implication elimination} \\
(\alpha \Leftrightarrow \beta) & \equiv & ((\alpha \Rightarrow \beta) \land (\beta \Rightarrow \alpha)) & \text{biconditional elimination} \\
\neg(\alpha \land \beta) & \equiv & (\neg\alpha \lor \neg\beta) & \text{De Morgan} \\
\neg(\alpha \lor \beta) & \equiv & (\neg\alpha \land \neg\beta) & \text{De Morgan} \\
(\alpha \land (\beta \lor \gamma)) & \equiv & ((\alpha \land \beta) \lor (\alpha \land \gamma)) & \text{distributivity of } \land \text{ over } \lor \\
(\alpha \lor (\beta \land \gamma)) & \equiv & ((\alpha \lor \beta) \land (\alpha \lor \gamma)) & \text{distributivity of } \lor \text{ over } \land
\end{array}
$$

Example CNF Convert

$$B_{11} \Leftrightarrow (P_{12} \vee P_{21})$$

Biconditional Elimination

$$B_{11} \Rightarrow (P_{12} \vee P_{21}) \wedge (P_{12} \vee P_{21}) \Rightarrow B_{11}$$

Implication Elimination

$$(\neg B_{11} \vee P_{12} \vee P_{21}) \wedge (\neg(P_{12} \vee P_{21}) \vee B_{11})$$

Move neg. in, DeMorgans

$$(\neg B_{11} \vee P_{12} \vee P_{21}) \wedge ((\neg P_{12} \wedge \neg P_{21}) \vee B_{11})$$

Distribute over and/or

$$(\neg B_{11} \vee P_{12} \vee P_{21}) \wedge (\neg P_{12} \vee B_{11}) \wedge (\neg P_{21} \vee B_{11})$$

Resolution Algorithm

Proof by contradiction, i.e., show $KB \wedge \neg\alpha$ unsatisfiable

function PL-RESOLUTION(KB, α) **returns** *true* or *false*
  inputs: KB, the knowledge base, a sentence in propositional logic
    α, the query, a sentence in propositional logic

  clauses ← the set of clauses in the CNF representation of $KB \wedge \neg\alpha$
  new ← { }
  loop do
    for each C_i, C_j **in** *clauses* **do**
      resolvents ← PL-RESOLVE(C_i, C_j)
      if *resolvents* contains the empty clause **then return** *true*
      new ← *new* ∪ *resolvents*
    if *new* ⊆ *clauses* **then return** *false*
    clauses ← *clauses* ∪ *new*