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Assignment - 1

Ques ①. What is Computer Graphics ? And also explain types of computer graphics.

Answer. Computer Graphics is an art of drawing pictures, lines, charts etc, using computer with the help of programming. Computer graphics is made up of number of pixels. Pixel is the smaller graphical pictures or unit represented on the computer screen.

Basically, there are 2 types of computer graphics namely—

- (i) Interactive Computer Graphics
- (ii) Non-Interactive Computer Graphics.

Interactive Computer Graphics

Interactive Computer Graphics involves a two way communication between Computer & user. Here the observer is given some control over the image by providing him an input device for eg. the video game controller of the ping pong game. This helps him to signal his request to the computer.

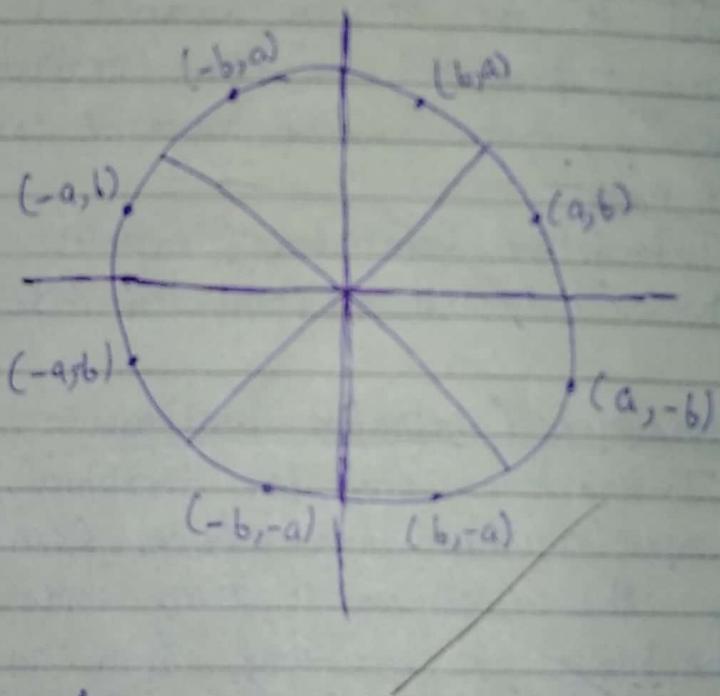
Non-Interactive Computer Graphics =

In non-interactive computer graphics otherwise known as passive computer in which user does not have any kind of control over the image. Image is nearly the product of static stored program and will work according to the instructions given the program linearly. The image is

totally under the control of program instructions not under the user

Ans: Drawing a circle on screen is little complex than drawing a line. There are two algorithms for generating a circle.

- ① Bresenham's algorithm
- ② Mid-point circle algorithm



Algorithm :-

Step ①. Get the coordinates of the center of the circle and radius and store them in x, y & R respectively set $i=0$ &

Step ② Set decision parameter $D = 3 - 2R$

Step ③. Repeat through step 8 while $R \geq P \geq Q$

Step ④. Call Draw Circle x, y, P, Q .

Step ⑤. Increment the value of P

Step ⑥. If $D < 0$ then $D = D + 4P + 6$

Step ⑦. Else set $R=R-1$, $D=D+4P+10$

Step 8. Call Draw circle x, y, r, Q .

Midpoint Algorithm

Step ①. Input radius r and circle centre (x_c, y_c) & obtain the first point on the circumference of the circle centered on the origin as $(x_0, y_0) = (0, r)$

Step ②. Calculate the initial value of decision parameter as $P_0 = 5r - r^2$

Step ③ At each x_n position at $x=0$
Perform the full test

If $P_k < 0$ then next point on circle $(0, 0)$ is (x_{k+1}, y_k) and

$$P_{k+1} = P_k + 2x_{k+1} + 1$$

else $P_{k+1} = P_k + 2x_{k+1} + 1 - 2y_{k+1}$

where, $2x_{k+1} = 2x_{k+2}$ and $2y_{k+1} = 2y_{k+2}$

Step ④. Determine the symmetry points other seven octants.

Step ⑤. Move each calculate pixel position x, y onto the circular path centered on (x_c, y_c) & plot the coordinate values

$$X = X + x_c, Y = Y + y_c$$

Step ⑥. Repeat step-3 through 5 until $x_c \geq y_c$

Disadvantage

- ① Accuracy of the generating points is an issue in this algorithm.
- ② This algorithm suffers when used to generate complex and high graphical images.

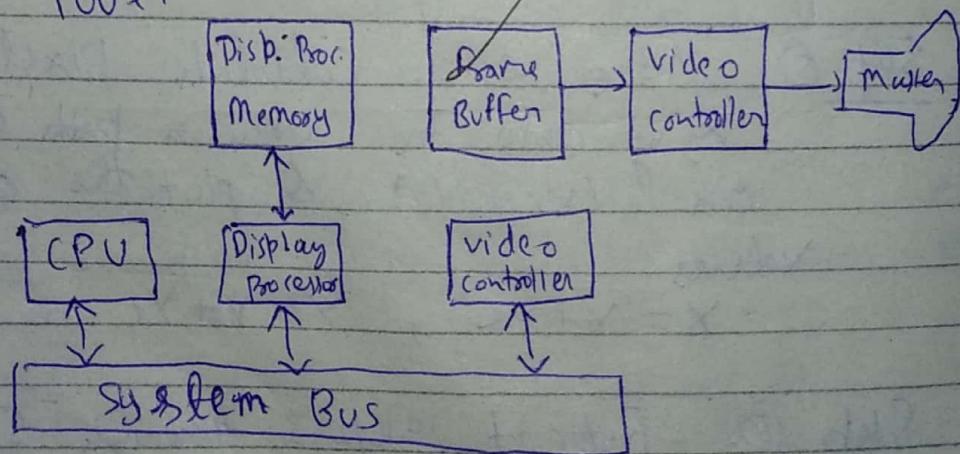
Ques ③ Explain the architecture of raster scan.

Answer ① In a raster scan display the image which has to be displayed is saved in a binary format in refresh buffer memory.

② Then a video controller is used to scan each and every line of refresh buffer memory.

③ The lines are scanned from left to right & when a line is finished again the controller moves to the next lines are scan the line from left to right scan.

④ After that the image is produced in the Cathode Ray Tube.



Ques ①. Give a brief description of various input devices used in computer ~~for~~ graphics.

Answer. Input devices

The input devices are the hardware that is used to transfer input to the computer.

Different input devices-

- Keyboard

- Mouse

- Trackball

- Spaceball

- Light Pen

- Digitizer

- Touch Panels

- Voice Recognition

- Image Scanner.

①. Keyboard - The most commonly used input device is a keyboard. The data is entered by pressing the set of keys.

②. Mouse - A mouse is a pointing device & used to position the pointer on the screen.

It is a small palm size box.

③. Trackball - It is a pointing device. It is similar to be a mouse. This is mainly used in notebooks or laptops, computers instead of mouse.

④. Spaceball - It is similar to trackball but it can move in six directions only.

The movement is recorded by the strain gauge.

Strain gauge is applied with pressure. One third of the ball is an inside box, the rest is outside.

⑤. Joystick - A joystick is also a pointing device which is used to choose cursor position on a monitor screen. Joystick is a stick having a spherical ball at its both lower & upper ends.

Ques ⑥. What are flat Panel display?

Answer. Flat Panel Displays-

A flat panel display is a television, monitor or other display ~~application~~ appliance, that uses a thin panel design instead of a traditional cathode ray tube (CRT) design. These are much lighter & thinner, and can be much more portable than traditional television and monitor they also have higher resolution than older models.

Types of Flat Panel Display

Emissive Display

Non-Emissive Display

Ques ⑦. Distinguish between raster & vector graphics methods, which method do we prefer.

Answer. The main difference between vector and raster graphics is that raster graphics is that raster graphics are composed of pixels, while vector graphics are composed of paths. A

Raster graphic such as gif or jpg. is an array of pixels of various colors, which together form a image.

Raster Graphics

- ① They are composed pixels
 - ② In Raster Graphics, refresh process is independent when the number of the complexity of the primitives in the image becomes too large.
 - ③ Graphic primitives are specified in terms of end points & must be scan converted into corresponding pixels.
 - ④ Raster Graphics is cost less.
 - ⑤ This occupy more space which depends on image quality.
 - ⑥ File extension - BMP, GIF, JPEG.
- ① They are composed of paths.
 - ② Vector displays flicker.
 - ③ Scan conversion is not required.
 - ④ Vector graphics cost more as compared to raster graphics.
 - ⑤ They occupy less space.
 - ⑥ File extension - SVG, EPS, PDF, DCF.

Ques 7. What is meant by refreshing of the screen?
Explain refresh Cathode Ray Tube.
Answer. Refreshing is needed for maintaining the picture.

on the screen. Refreshing of screen is done by keeping the phosphorus glowing to redraw the picture repeatedly, i.e., by quickly drawing the electron beam back to same points. In computer display, to refresh is to redraw the image information from memory. Computer displays have to be refreshed because they don't have the capacity to hold a stable image. Electron guns in the cathode ray tube constantly sweep across the screen, redrawing the display. The ~~RAM~~ RANDAC (Random Access memory digital to analog converter) in the graphics card determines a refresh rate. How many lines per second the information will be drawn & the image repainted. At adequate refresh rate levels, a display appears stable, but if a refresh rate is too low a display will flicker and can cause eye strain & headache. A beam of electrons, emitted by electron gun, passes through focusing deflection system that directs the beam towards specified position on the phosphorus-coated screen. This type of display is called refresh CRT.

Ques 8. Explain DDA algorithm and draw the line having end point from A(20, 10) to B(30, 18) by using the algorithm.

Answer: DDA algorithm -

- ① Accept start and end point coordinates (x_1, y_1) & (x_2, y_2)
- ② Calculate $\Delta x = x_2 - x_1$, $\Delta y = y_2 - y_1$

③ If $\text{abs}(\Delta x) > \text{abs}(\Delta y)$
Then $k = \text{abs}(\Delta x)$

else

$$k = \text{abs}(\Delta y)$$

④ Calculate $\Delta x = \frac{\Delta x}{k}$, $\Delta y = \frac{\Delta y}{k}$

⑤ Initialize $x = x_1$, $y = y_1$

⑥ Display pixel (x, y)

⑦ $x = x + \Delta x$, $y = y + \Delta y$

⑧ Display pixel round(x), round(y)

⑨ Repeat step 7, step 8, k times

• A(20, 10) to B(30, 18)

① $\Delta x = 30 - 20 = 10$

$\Delta y = 18 - 10 = 8$

② \therefore steps, $k = 10$

③ $\Delta x = \frac{10}{10} = 1$, $\Delta y = \frac{8}{10} = 0.8$

$$x = x + \Delta x \quad y = y + \Delta y$$

20 10 (20, 10)

21 10.8 (21, 11)

22 11.6 (22, 12)

23 12.4 (23, 13)

24 13.2 (24, 14)

25 14 (25, 15)

26 14.8 (26, 16)

27 15.6 (27, 17)

28 16.4 (28, 18)

29 17.2 (29, 19)

30 18 (30, 20)

Ques 5. Explain shadow mask method & how it is different from beam penetration method.

Answer. Shadow Mask method is commonly used in Raster-scan system because they produce a much wider range of colors than the beam penetration method. It is used in majority of color TV sets & monitors.

Working: The deflection system of the CRT operates on all 3 electron beam simultaneously. The 3 electron beams are deflected & focused as a group onto the shadow mask, which contains a sequence of holes aligned with the phosphor's dot pattern. When the three beams pass through a hole in the shadow mask, they activate a dotted triangle, which occurs as a small color spot on the screen. The phosphor dots in the triangles are organised so that each electron beam can activate only its corresponding color dot when it passes through the shadow mask.

Beam Penetration Methods-

1 | 2 | 3 | 4

The Beam Penetration Method has been used with random scan monitors. In this method, the CRT screen is coated with two layers of phosphorus, red & green & the displayed color depends on how far the electron beam penetrates the phosphorus layers. This method produces four colors only red, green, orange & yellow. A beam of slow electron excites the outer red layer only, hence screen shows red color only. A beam of high-speed electrons excites the inner green layer. Thus screen shows green color.

Assignment - 2

Ques ①: What do you mean by normalization transformation? Why is it needed.

Answer: Normalize transformation :-

- ↳ We have to define the picture co-ordinate in some units other than pixels and use the ~~interpolator~~ interpolator to convert these coordinate to appropriate pixel value for a particular display device.
- ↳ the device independent unit is known as normalized device co-ordinate
- ↳ The interpolators use a simple linear formula to convert ~~the~~ the normalize device co-ordinate to the actual device co-ordinate.

$$x = x_n + X_w$$

$$y = y_n + Y_h$$

where x = actual device of x co-ordinate

y = actual device of y co-ordinate.

x_n = normalized x co-ordinate

y_n = normalized y co-ordinate

X_w = width of the actual screen in pixel

Y_h = actual height of the actual screen in pixel

The transformation which maps the work coordinate to the normalized deviced coordinate is called Normalization transformation. It involve a scaling of x, y co-ordinate then it also called scaling transformation.

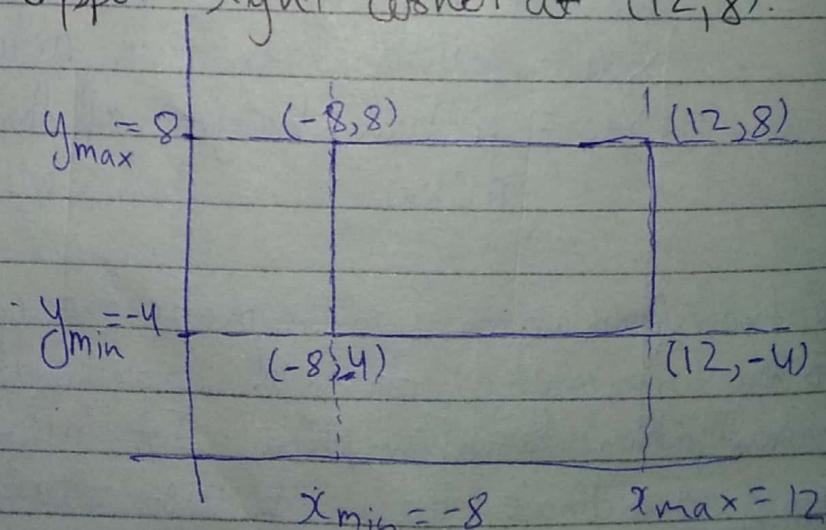
Ques② What is workstation transformation?

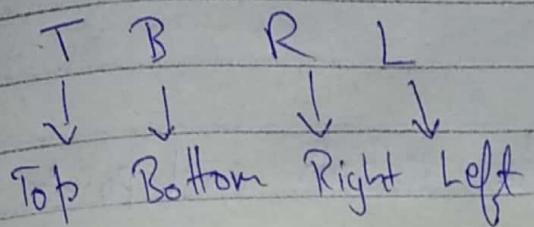
Answer: The transformation which maps the normalized device co-ordinate is known as workstation transformation.

- The window defined in world co-ordinates is first transformed into the normalized device coordinate then they are transformed into view port coordinate.
- This window to viewport transformation is called workstation transformation.
- This process is achieved by the following steps:
 - (i) The object together with its window is translated until the lower left corner of the window is at the origin.
 - (ii) Object and the window are scaled until the window has the dimension of the view port.

Ques③ Use Cohen-Sutherland line clipping method to clip ~~a line~~ a line starting from (-3, 5) and ending at (17, 11) against the window having its lower left corner at (-8, -4) and upper right corner at (12, 8).

Answer:





$T \rightarrow y > y_{\max}$, Top $\rightarrow 1$ if $y > y_{\max}$
 $B \rightarrow y < y_{\min}$, Bottom $\rightarrow 1$ if $y < y_{\min}$
 $R \rightarrow x > x_{\max}$, Right $\rightarrow 1$ if $x > x_{\max}$
 $L \rightarrow x < x_{\min}$, Left $\rightarrow 1$ if $x < x_{\min}$

(i) generate TBRL (Region code) Code \rightarrow 4 bit code

Pseudo Code

1). Assign the region code for 2 end points of a given line.

2). If both have region codes 0000 then line accepted completely.

3). else

perform logical AND operation for both region codes.

(a). If result $\neq 0000$, line is outside

(b). else line is partially inside.

(i). choose an endpoint of the line that is outside the given rectangle

(ii). find intersection point.

(iii). Replace endpoint with the intersection point and update region code.

(iv). Repeat step 2 until line is totally accepted or totally rejected.

4). Repeat from Step-1 for other line,

Algorithm

- (i). assign Region code to both end points say C_0 and C_1 .
- (ii). if $C_0 \text{ OR } C_1 = 0000$ then accepted completely (inside window)
- else if $C_0 \text{ Logical AND } C_1 \neq 0000$, Reject it

else

Clip if line crossed X_{\min} or X_{\max}
Then

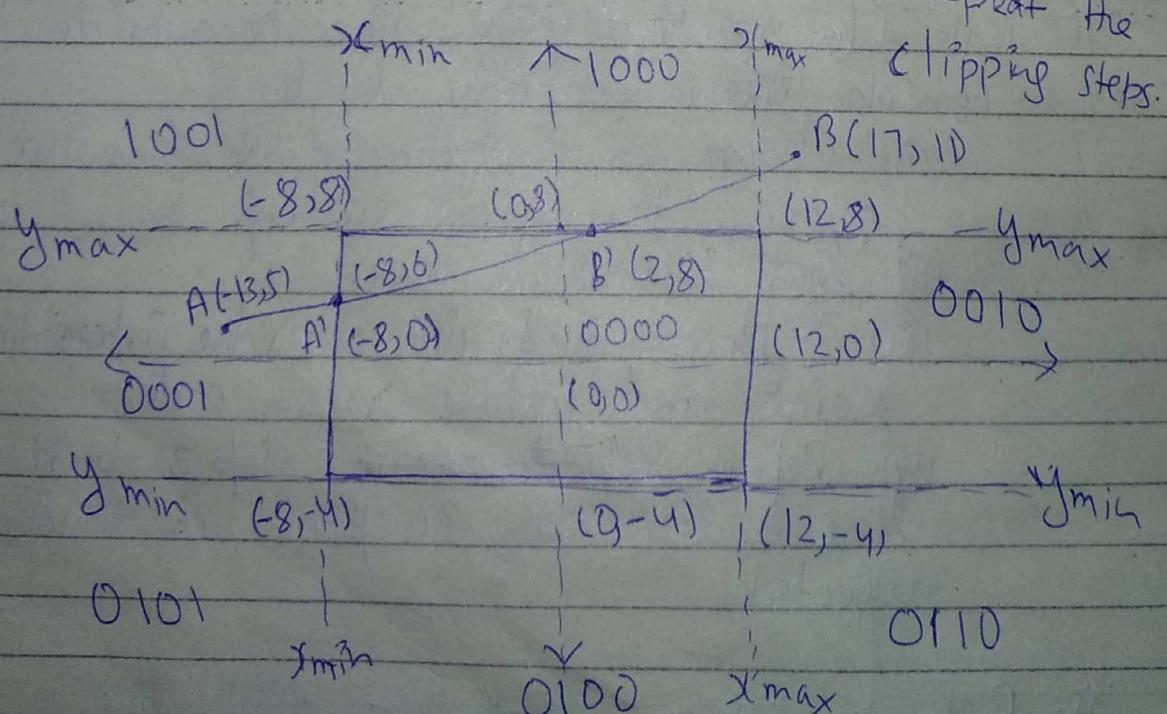
$$Y = Y_1 + m(X - X_1)$$

else

$$X = X_1 + \frac{1}{m}(Y - Y_1)$$

$\boxed{\begin{array}{l} X = X_{\max} \\ \text{or} \\ X_{\min} \\ Y = Y_{\max} \\ \text{or} \\ Y_{\min} \end{array}}$

- (iii) Verify $X_{\min} \leq X \leq X_{\max}$ $Y_{\min} \leq Y \leq Y_{\max}$ If it does not satisfy then repeat the



① b Line AB, A(-13, 5), B(17, 11)
 Rectangular $(x_{\min}, y_{\min}) \Rightarrow (-8, -4)$
 $(x_{\max}, y_{\max}) \Rightarrow (12, 8)$.

For point A

$$A(-13, 5), x_1 = -13, y_1 = 5$$

$$x_{\min} \leq x_1 \leq x_{\max} \quad TBRL$$
 ~~$-8 \leq -13 \leq 12$~~

$$0001$$

$$y_{\min} \leq y_1 \leq y_{\max} \quad R_A = 0001$$

$$-4 \leq 5 \leq 8$$

For point B

$$B(17, 11), x_2 = 17, y_2 = 11$$

$$x_{\min} \leq x_2 \leq x_{\max} \quad y_{\min} \leq y_2 \leq y_{\max}$$

$$-8 \leq 17 \leq 12 \quad -4 \leq 11 \leq 8$$

$$TBRL \rightarrow R_B = 1010$$

② If $R_A = 0000$ and $R_B = 0000$
 Then accepted line

③ Else

$$R_A \text{ AND } R_B$$

$$0001 \text{ AND } 1010 \Rightarrow 0000$$

$$\textcircled{4}. \frac{y - 5}{x + 13} = \frac{11 - 5}{17 - (-13)}$$

$$\frac{y - 5}{x + 13} = \frac{-6}{-30} = \frac{1}{5}$$

$$5(y-5) = x+13$$

$$5y - 25 = x + 13$$

$$5y = x + 13 + 25$$

$$5y = x + 38$$

$$\boxed{y = \frac{1}{5}x + \frac{38}{5}}$$

$$x = 5y - 38$$

(i). Left ($x_{\min}, \frac{1}{5}x_{\min} + \frac{38}{5}$) \checkmark

$$(-8, \frac{1}{5} \times (-8) + \frac{38}{5})$$

$$(-8, \frac{-8+38}{5}) = (-8, 6)$$

$$\Rightarrow \text{Left } (-8, 6)$$

(ii). Right ($x_{\max}, \frac{1}{5}x_{\max} + \frac{38}{5}$) \times

$$(12, \frac{1}{5} \times 12 + \frac{38}{5})$$

$$(12, \frac{50}{5})$$

$$\Rightarrow \text{Right } (12, 10)$$

(iii). Top ~~$5y_m$~~ Top ($5y_{\min} - 38, y_{\max}$)

$$\text{Top } (5 \times 8 - 38, 8)$$

$$\text{Top } (40 - 38, 8)$$

$$\text{Top } (2, 8)$$

 \checkmark

(iv). Bottom ($5y_{\min} - 38, y_{\max}$)

$$\text{Bottom } (5 \times (-4) - 38, -4)$$

$$\text{Bottom } (-58, -4)$$

 \times

Clipped line $(A'B') \Rightarrow (-8, 6) \text{ to } (2, 8)$

Ques ④ What are the limitation of Sutherland-Hodgeman polygon clipping algorithm?

Answer- The Sutherland - Hodgeman algorithm is used for clipping polygons. It works by, extending each line of the convex clip polygon in turn and selecting only vertices from the subject polygon that are on the visible side.

Limitations of Sutherland-Hodgeman polygon clipping algorithm.

- ① It clips to each window boundary one at a time.
- ② It has a "Random" edge choice.
- ③ It has Redundant edge-line-cross calculations.

* This method requires a considerable amount of memory. The first of all polygons are stored in original form.

Then clipping against left edge done and output is stored. Then clipping against right edge done, then top edge. Finally, the bottom edge is clipped. Results of all these operations are stored in memory. So wastage of memory for storing intermediate polygons.

Ques ⑤ Explain and state the advantages of Weiler Atherton polygon clipping algorithm.

Answer. The Weiler - Atherton is a polygon-clipping algorithm. It is used in the areas like computer graphics, games development and others where clipping of Polygon is needed.

→ It allows clipping of a subject or candidate polygon by an arbitrarily shaped clipping

Polygon / area / region.

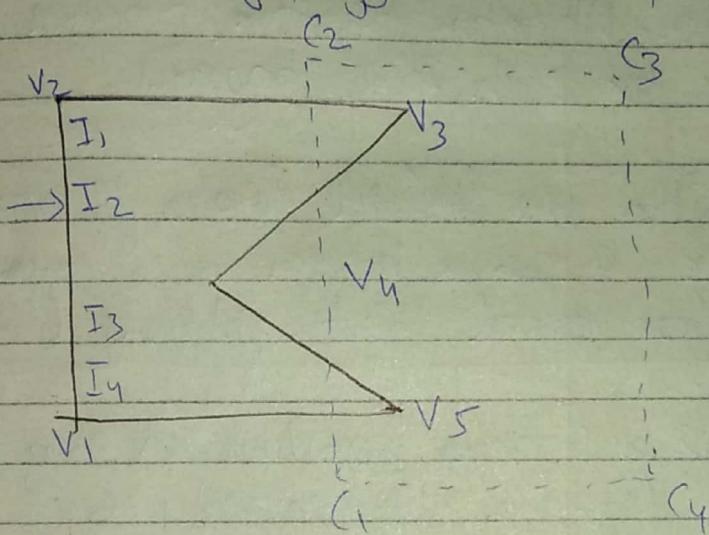
→ It is generally applicable only in 2D. However it can be used in 3D through visible surface determination and with improved efficiency through Z-ordering.

Given polygon A as the clipping region and polygon B as the subject polygon to be clipped, the algorithm consists of the following steps :-

- ① List the vertices of the clipping-region polygon A and those of the subject polygon B.
- ② Label the listed vertices of subject polygon B as either inside or outside of clipping region A.
- ③ Find all the polygon intersections and insert them into both lists, linking the lists at the intersections.
- ④ Generate a list of "in bound" intersections - the intersections where the vector from the intersection to the subsequent vertex of subject polygon B begins inside the clipping region.
- ⑤ Follow each intersection clockwise around the linked lists until the start position is found.

→ This algorithm is used for clipping concave polygons
Here, V_1, V_2, V_3, V_4, V_5 are the vertices of the polygon.

C_1, C_2, C_3, C_4 are the vertices of the clip polygon
and I_1, I_2, I_3, I_4 are the intersection points
points of polygon and clip polygon.



In this algorithm, we take a starting vertex like I_1 and traverse the polygon like I_1, V_3, I_2 . At occurrence of leaving intersection the algorithm follows the clip polygon vertex list from leaving vertex in downward direction.

At occurrence of entering intersection the algorithm follows subject polygon vertex. This process is repeated till we get starting vertex. This process has to be repeated for all remaining entering intersections which are not included in the previous traversing of vertex list. Since I_3 was not included in first traversal, hence, we start the second traversal from I_3 . Therefore, first traversal gives polygon as I_1, V_3, I_2, I_1 and second traversal gives polygon as:-
 I_3, V_5, I_4, I_3 .

Advantages of Weiler-Atherton polygon clipping algorithm.

- ① When we have non-convex Polygon then the algorithm above ~~will~~ might produce polygons with concave edges.
- ② The Weiler-Atherton algorithm produces separate polygons for each visible fragment.
- ③ This algorithm was developed clip a fill area that is either a convex polygon or a concave polygon.
- ④ Work for general input polygons (concave & convex).
- ⑤ Handles a clipping window with any polygon shape (concave & convex).
- ⑥ Can be extended to 3D.

Ques 6: Prove that the scaling transformation commutes i.e. $S_1 S_2 = S_2 S_1$

Answer: Let S_1 is scaling by factor $|m|$ about origin and S_2 is a scaling by factor $|n|$ after S_1

$$S_1 = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ and } S_2 = \begin{bmatrix} n & 0 & 0 \\ 0 & n & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$S_1 S_2 = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} n & 0 & 0 \\ 0 & n & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$S_1 S_2 = \begin{bmatrix} mn & 0 & 0 \\ 0 & mn & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{Now, } S_2 S_1 = \begin{bmatrix} n & 0 & 0 \\ 0 & n & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$S_2 S_1 = \begin{bmatrix} mn & 0 & 0 \\ 0 & mn & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$S_2 S_1 = S_1 S_2$$

Ques 7 Reflect the object with vertices A(5,5), B(4,0) and C(7,5) about-

(i) x-axis (iii) y-axis

Using appropriate transformation matrices

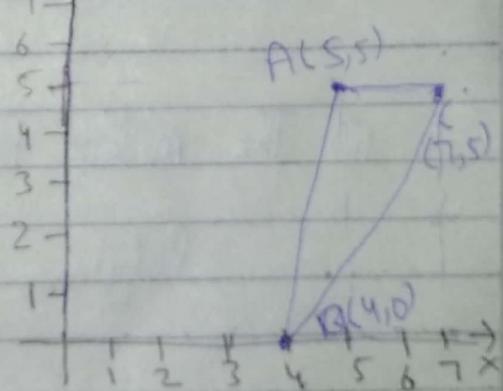
Answer (i) The matrix for $R_c(x)$ is

$$R_c(x) = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

\Rightarrow The 'A' point coordinates after reflection

$$[x, y] = [5, 5] \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



$$[x, y] = [5, -5]$$

\Rightarrow The 'B' point coordinates after reflection

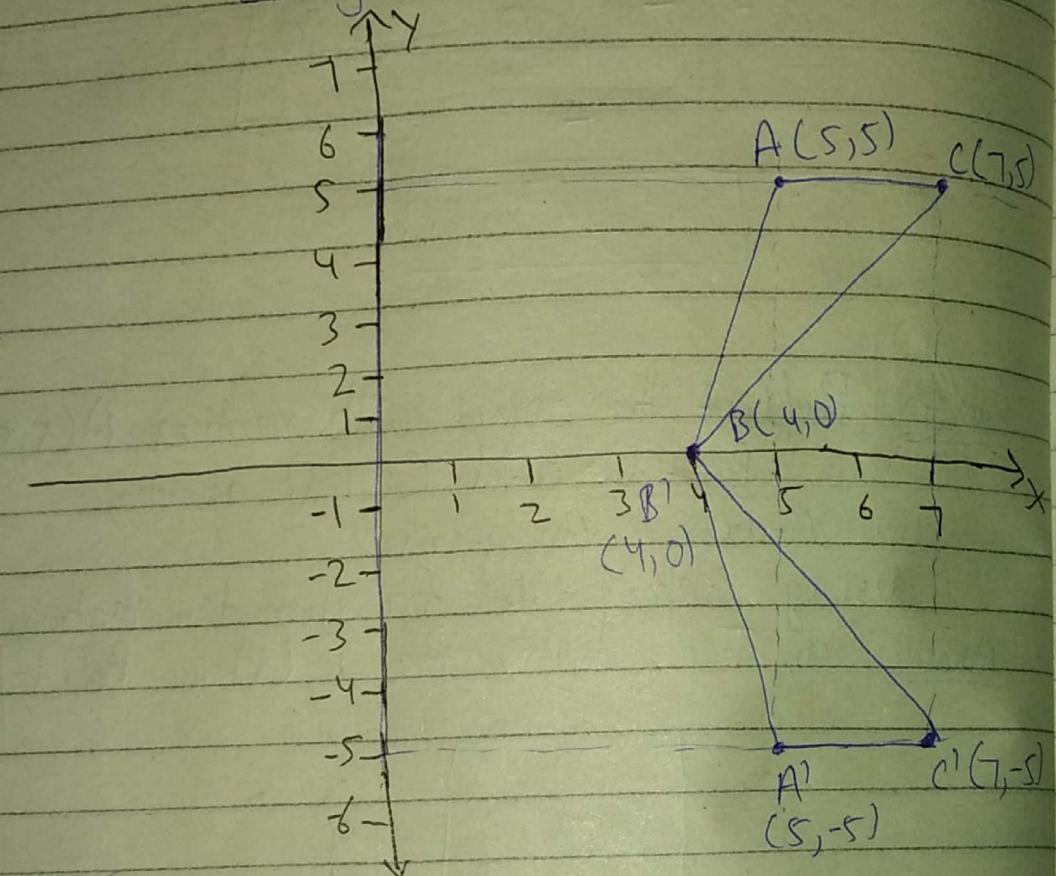
$$[x, y] = [4, 0] \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[x, y] = [4, 0]$$

\Rightarrow The 'C' point coordinates after reflection.

$$[x \ y] = [7 \ 5] \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[x \ y] = [7 \ -5]$$



(ii) The matrix for $Re(y)$ is

$$Re(y) = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

\Rightarrow The 'A' point coordinates after reflection

$$[x \ y] = [5 \ 5] \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$[x \ y] = [-5 \ 5]$$

The 'B' point coordinates after reflection.

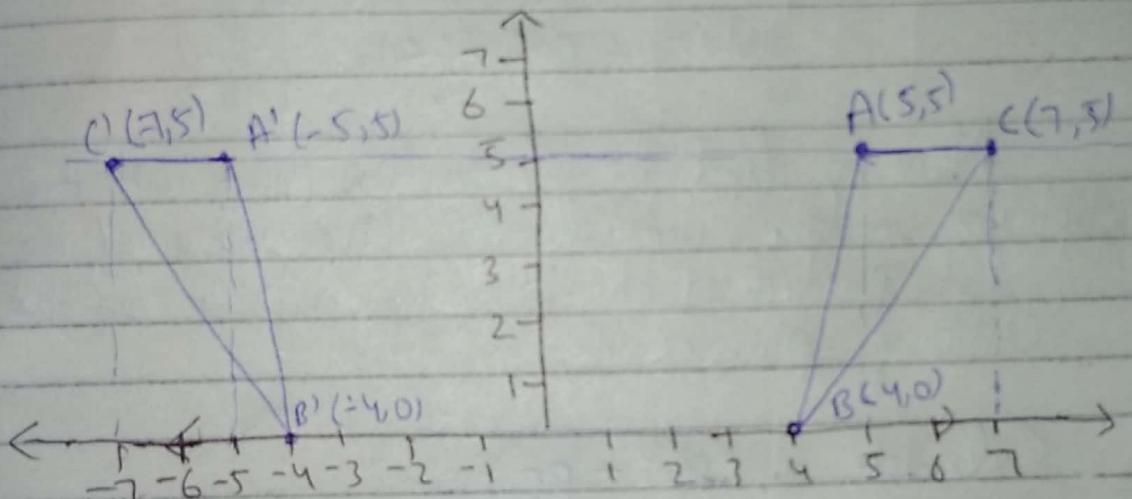
$$[x \ y] = [4 \ 0] \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$[x \ y] = [-4 \ 0]$$

The 'C' point coordinates after reflection.

$$[x \ y] = [7 \ 5] \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$[x \ y] = [-7 \ 5]$$



Ques Q. Write short notes on:-

(a) Generalised Clipping.

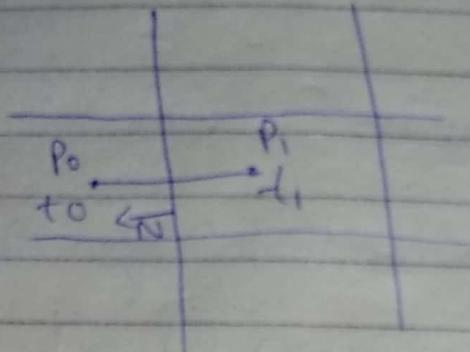
(b) Multiple Windowing.

Answer to Generalised Clipping:-

The Cyrus-Beck algorithm is a generalized line clipping algorithm. It was designed to be more efficient than the Cohen Sutherland algorithm, which uses repetitive clipping.

Cyrus-Beck is a general algorithm and can be used with a convex polygon clipping window,

unlike Sutherland-Cohen, which can be used
only on a rectangular clipping area



Here the parametric equation of a line in
view plan is

$$P(t) = tP_1 + (1-t)P_0$$

where $0 \leq t \leq 1$

Now to find the intersection point with the
clipping window, we calculate the value of the
dot product. Let P_E be a point on the
clipping plane E.

Calculate $n(P(t) - P_E)$

if < 0 , vector pointed towards interior

if $= 0$, vector pointed parallel to plane
containing P .

if > 0 , vector pointed away from interior

Here n stands for normal of the current
clipping plane (pointed away from interior).
By this we select the point of intersection
of line and clipping window where dot
product is 0 and hence clip the
line.

(b). Multiple Windowing

A multiple windowing system is a system for sharing a computer's graphical display presentation resources among multiple applications at the same time.

A windowing system uses a window manager to keep track of where each window is located on the display screen and its size and status.

Assignment -3

Ques Define 3D transformation. Derive relation about x-axis y-axis & z-axis matrix in 3D with proper diagram.

Ans 3D transformation In a three dimensional homogeneous coordinate representation, a point is translate form position $p(x, y, z)$ to position $p'(x', y', z')$ with matrix operation.

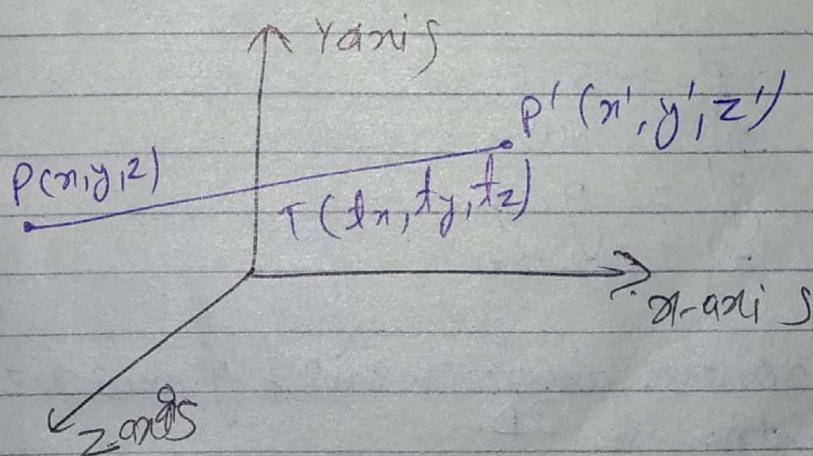
$$\textcircled{a} \begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$p' = t p$$

$$x' = x + t_x$$

$$y' = y + t_y$$

$$z' = z + t_z$$



Rotations In 3D rotation transformation a point is rotated from $p(x, y, z)$ to $p'(x', y', z')$ and require an angle of rotation and an axis of rotation, matrix representation of rotation about z-axis as given as.

$$\begin{aligned}x' &= x \cos \theta - y \sin \theta \\y' &= x \sin \theta + y \cos \theta \\z' &= z\end{aligned}$$

$$R_z(\theta) = \begin{bmatrix} \cos \theta & -\sin \theta & 0 & 0 \\ \sin \theta & \cos \theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Scaling & The matrix representation for the scaling transformation of a position $P(x, y, z)$ related to the coordinate origin can be written as

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$P' = S \cdot P$$

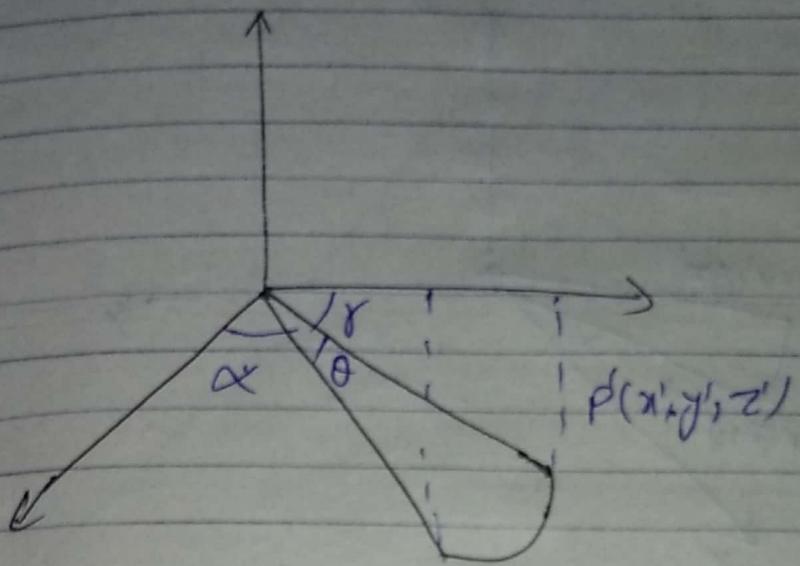
$$x' = x \cdot S_x$$

$$y' = y \cdot S_y$$

$$z' = z \cdot S_z$$

Derive rotation about x-axis, y-axis & z-axis in 3D.

① Rotation about x-axis &



$$x' = x$$

$$y' = r \cos \theta (\alpha + \theta)$$

$$z' = r \cos \alpha \cos \theta - r \sin \alpha \sin \theta$$

$$r \cos \alpha = y \text{ and } r \sin \alpha = z \quad \text{---} ①$$

$$y' = y \cos \theta - z \sin \theta$$

$$z' = r \sin(\alpha + \theta)$$

$$\begin{aligned} r \sin \alpha \cos \theta + r \cos \alpha \sin \theta \\ = z \cos \theta + y \sin \theta \end{aligned}$$

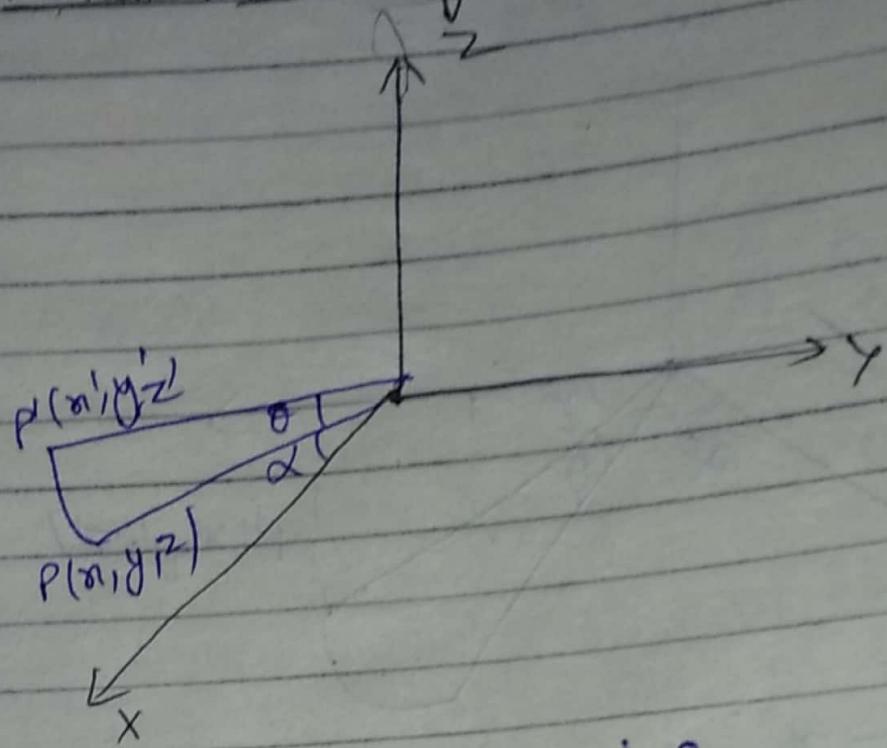
$$R_\alpha = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta \\ 0 & \sin \theta & \cos \theta \end{pmatrix}$$

$$P'(x', y', z') = R_\alpha P(x, y, z)$$

making given form as

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta & 0 \\ 0 & \sin \theta & \cos \theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

② Rotation about y-axis by θ



$$x' = x \cos \theta + z \sin \theta$$

$$y' = y$$

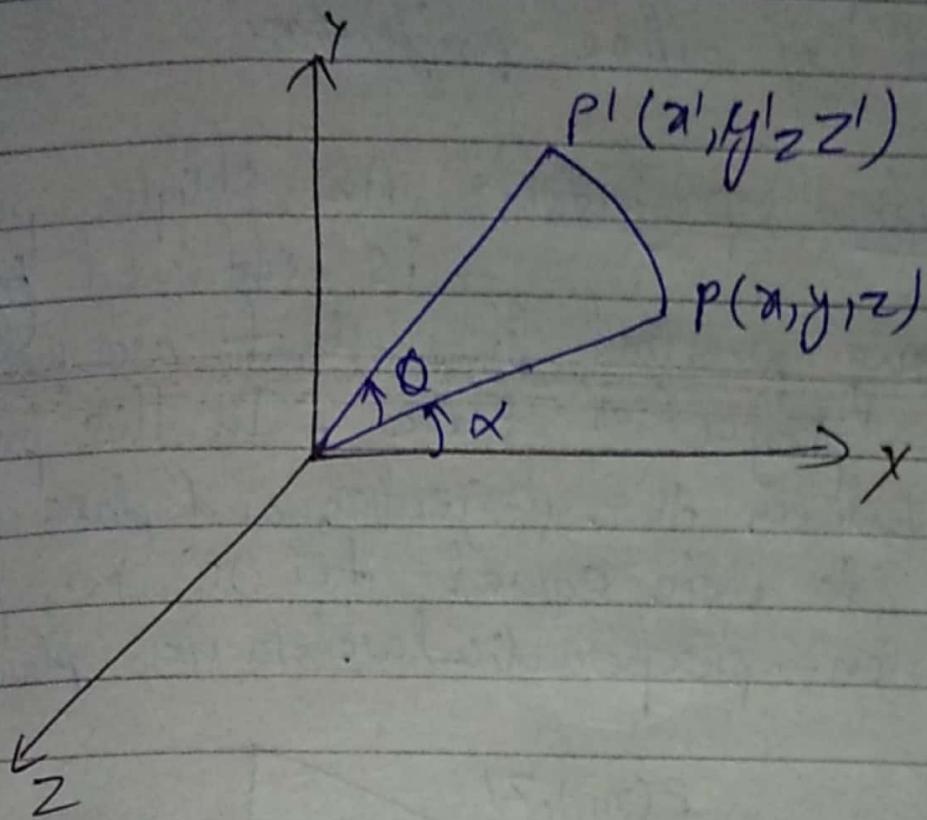
$$z' = x \sin \theta + z \cos \theta$$

$$RQ_1 = \begin{bmatrix} \cos \theta & 0 & \sin \theta \\ 0 & 1 & 0 \\ -\sin \theta & 0 & \cos \theta \end{bmatrix}$$

Matrix given as

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \cos \theta & 0 & \sin \theta & 0 \\ 0 & 1 & 0 & 0 \\ \sin \theta & 0 & \cos \theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

③ Rotation about the z-axis



$$x' = x \cos \theta - y \sin \theta$$

$$y' = x \sin \theta + y \cos \theta$$

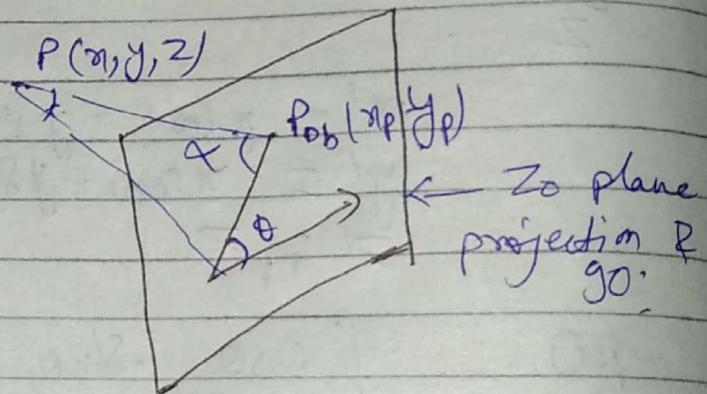
$$z' = z$$

$$\text{RQ.} = \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta & 0 & 0 \\ \sin \theta & \cos \theta & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Ques 2- what is oblique projection and derive oblique projection matrix? Beside some advantages and types of oblique projection.

Oblique projection & An oblique projection is obtained by projecting points along parallel lines that are not perpendicular to the projection plane. In this projection angle between the projection and plane of projection is not equal to 90° i.e., projections are non-perpendicular to view plane.



We can express the projection coordinates in terms of x, y, L and θ as

$$x_p = x + L \cos \theta$$

$$y_p = y + L \sin \theta$$

length L depends on the angle α and the Z coordinate of the point to be projected

$$\tan \alpha = \frac{Z}{L} \Rightarrow L = \frac{Z}{\tan \alpha} = Z L_1$$

$$\left[\text{where } L_1 = \frac{1}{\tan \alpha} \right]$$

when $z=1, L_1=L$

Now we can write the oblique projection

as,

$$x_p = x + z(L, \cos\theta)$$

$$y_p = x + z(L, \sin\theta)$$

$$z_p = 0, \text{ we get}$$

Considering

$$\begin{bmatrix} x_p \\ y_p \\ z_p \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & L, \cos\theta & 0 \\ 0 & 1 & L, \sin\theta & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

This is the standard matrix for oblique projection onto $z=0$ plane.

Ques Differentiate between parallel projection and perspective projection.

Ans Parallel Projection

Perspective projection

① In parallel projection, parallel lines from each vertex on the object which are extended until they intersect the view plane.

② Lines of projection are parallel.

③ These are linear transform

① Perspective projection produces realistic view

Line-of projection are not parallel.

③ These are non-linear transform.

Ques find the matrix for parallel projection onto the plane $3x + y + 4z + 1 = 0$, when

- ① an orthographic projection is used.
- ② Oblique projection is used.

Ans Here $(a, b, c) = (P, q, r) = (3, 1, 4)$
parametric equation of direction line

$$x_p = x_0 + 3t \quad - \textcircled{1}$$

$$y_p = y_0 + t \quad - \textcircled{2}$$

$$z_p = z_0 + 4t \quad - \textcircled{3}$$

$$\text{From equation } t = \frac{ax_0 + by_0 + cz_0 + d}{ap + bq + cr}$$

$$= \frac{3x_0 + y_0 + 4z_0 + 1}{(3)^2 + (1)^2 + (4)^2} = \frac{3x_0 + y_0 + 4z_0 + 1}{26}$$

Put the value of t in the parametric equation $\textcircled{1}, \textcircled{2}, \textcircled{3}$ we get.

$$x_p = \frac{1}{26} (17x_0 - 3y_0 - 12z_0 - 3)$$

$$y_p = \frac{1}{26} (-3x_0 + 25y_0 - 4z_0 - 1)$$

$$z_p = \frac{1}{26} (-12x_0 - y_0 + 10z_0 - 4)$$

The matrix for orthographic parallel projection is given by.

$$\begin{pmatrix} x_p \\ y_p \\ z_p \\ 1 \end{pmatrix} = \frac{1}{26} \begin{pmatrix} 17 & -3 & -12 & -3 \\ -3 & 25 & -4 & -1 \\ -12 & -4 & 10 & -4 \\ 0 & 0 & 0 & -26 \end{pmatrix} \begin{pmatrix} x_0 \\ y_0 \\ z_0 \\ 1 \end{pmatrix}$$

① Now, here $P = q = r = 1$
 $a = 3, b = 1, c = 4$ and $d = 1$

Parametric equation are

$$x_p = x_0 + f$$

$$y_p = y_0 + f$$

$$z_p = z_0 + f$$

From extract from equation

$$f = \frac{ax_0 + by_0 + cz_0 + d}{ap + bq + cr}$$

$$= \frac{3x_0 + y_0 + 4z_0 + 1}{8}$$

Put the value of f we get.

$$x_p = \frac{1}{8} (-5x_0 - y_0 - 4z_0 - 1)$$

$$y_p = \frac{1}{8} (-3x_0 - 7y_0 - 4z_0 - 1)$$

$$z_p = \frac{1}{8} (-3x_0 - y_0 - 4z_0 - 1)$$

The matrix of all of oblique parallel projection is given by

$$\begin{bmatrix} x_p \\ y_p \\ z_p \\ 1 \end{bmatrix} = \frac{1}{8} \begin{bmatrix} -5 & -1 & -7 & -1 \\ -3 & -7 & -4 & -1 \\ -3 & -1 & -4 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Now find the matrix for mirror reflection with respects to the plane passing through the origin and having a normal vector whose direction is $N = i\hat{j} + k\hat{k}$.

Now we know $N = n_1\hat{i} + n_2\hat{j} + n_3\hat{k}$ with

$$|N| = \sqrt{n_1^2 + n_2^2 + n_3^2} \quad \Rightarrow |N| = \sqrt{n_2^2 + n_3^2}$$

Now, $N = \hat{i} + \hat{j} + \hat{k}$
 so $|N| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$

$$N = \sqrt{1^2 + 1^2} = \sqrt{2} \text{ and } P_0(0, 0, 0)$$

$$\gamma_V = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\gamma_V^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$v = \hat{i} + \hat{j} + \hat{k}$

then,

$$AN =$$

$$\begin{vmatrix} \frac{\lambda}{|N|} & \frac{-m_1 m_2}{\lambda |N|} & \frac{-m_1 m_3}{\lambda |N|} & 0 \\ 0 & \frac{m_3}{\lambda} & \frac{-m_2}{\lambda} & 0 \\ \frac{m_1}{|N|} & \frac{m_2}{|N|} & \frac{m_3}{|N|} & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} \frac{\sqrt{2}}{\sqrt{3}} & \frac{-1}{\sqrt{2}\sqrt{3}} & \frac{-1}{\sqrt{2}\sqrt{3}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

and,

$$A^{-1}N = \begin{vmatrix} \frac{\sqrt{2}}{\sqrt{3}} & 0 & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{2}\sqrt{3}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{2}\sqrt{3}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

Motion for mirror reflection with xy plane
in homogeneous form is

$$M_{xy} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

the final reflector matrix is

$$TV^{-1}AN^{-1} \text{ May } AN \cdot TV$$

$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} \frac{\sqrt{2}}{\sqrt{3}} & 0 & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{2}\sqrt{3}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{2}\sqrt{3}} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{3}} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{3} & -\frac{2}{3} & -\frac{2}{3} & 0 \\ -\frac{2}{3} & \frac{1}{3} & -\frac{2}{3} & 0 \\ -\frac{2}{3} & -\frac{2}{3} & \frac{1}{3} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Aux Define axonometric projections and types of axonometric projection in details.

Aux Axonometric Projection & Axonometric projection are orthographic projections in which the direction of projection is not parallel to any of three principle axis. The construction of an axonometric projection is done by using rotation and translation to manipulate the objects such that at least three adjacent faces are shown. The foreshortening factor is the ratio of the projected length of a line to its true length.

The sub-categories of axonometric projection are :-

- a) Isometric Projection & The direction of projection makes equal angles with all three principle axis.
- ⑥ Dimetric Projection & The direction of projection makes unequal angles with all three principle axis.
- ⑦ Trimetric Projection & The direction of projection makes unequal angles with the three principle axis.

Ques Establish and write Cyrus-Beck clipping algorithm.

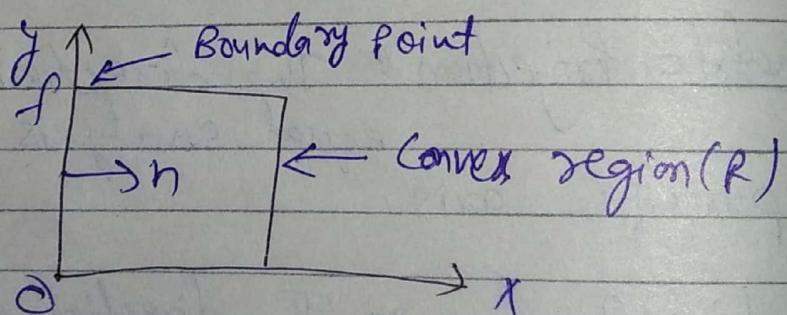
Ans Cyrus-Beck algorithm is applicable for arbitrary convex region. This algorithm uses a parametric equation of line segment to find the intersection points of a line with the clipping edges.

The parametric equation of a line segment from P_1 to P_2 is:

$$P(t) = P_1 + (P_2 - P_1)t \quad 0 \leq t \leq 1$$

where t is a parameter, $t=0$ at P_1 and $t=1$ at P_2 .

Consider a convex clipping region R , f is a boundary point of the convex region R and n is an inner normal for one of its boundaries as shown in



we evaluate the dot product $n \cdot [P(t) - f]$ and check the following condition as

- a) if dot product $n \cdot [P(t) - f] < 0$
then the vector $P(t) - f$ is pointed away from the interior of R .

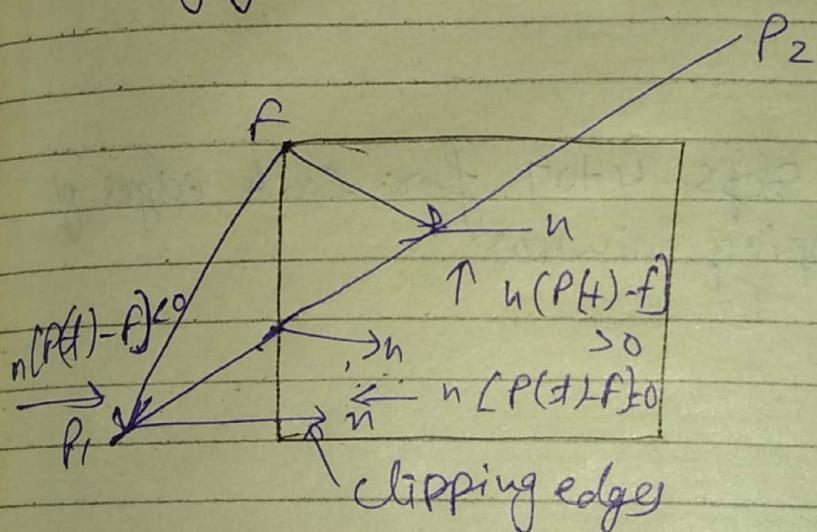
⑥ if dot products is zero, i.e.,

$$n \cdot [P(t) - f] = 0$$

⑦ if dot products i.e.,

$$n \cdot [P(t) - f] > 0$$

then the vector $P(t) - f$ is pointed towards the interior or R, as shown in fig.



Cross back links clipping algorithm

① Read two endpoint of the P_1 and P_2

② Read vertex coordinates of the clipping window.

③ Calculate $D = P_2 - P_1$

④ Assign boundary point (f) with particular edge find their normal vectors for corresponding edges

- ⑤ Find linear normal vector for coordinate edges
- ⑥ Calculate D_n and $w \cdot p_i - F$
- ⑦ if $D_n > 0$

$$f_1 = -\frac{w_n}{D_n}$$

else,

$$f_0 = -\frac{w \cdot n}{D \cdot n}$$

end-if.

- ⑧ Repeat steps 4 to 7 for each edges of the clipping window.