Date / /

Page no.

ASSIGNMENT-3.

(1) Define 3D transformation. Derive rotation about x-axis, y-axis and z-axis matrix in 3D with proper Diagram.

- 3D transformation :- In a three dimensional homogenous coordinate representation, a point is translate from position $P(x, y, z)$ to position $P(x', y', z')$ with matrix operation.

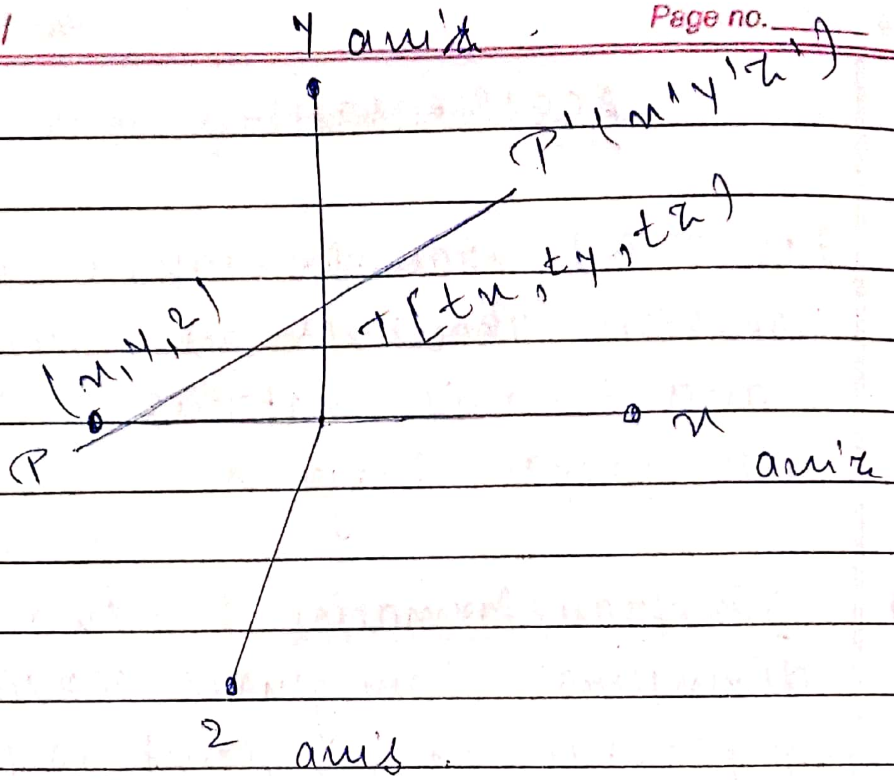
$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & tx \\ 0 & 1 & 0 & ty \\ 0 & 0 & 1 & tz \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$P' = TP$$

$$x' = x + tx$$

$$y' = y + ty$$

$$z = z + tz$$



Rotation:- In 3D rotation transformation a point is rotated from P(x, y, z) to P'(x', y', z') and require and angle of rotation and an axis of rotation.
matrix representation of rotation about z axis as given as

$$x' = x\cos\theta - y\sin\theta$$
$$y' = x\sin\theta + y\cos\theta$$
$$z' = z$$

$$R_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

Date / / Page no.

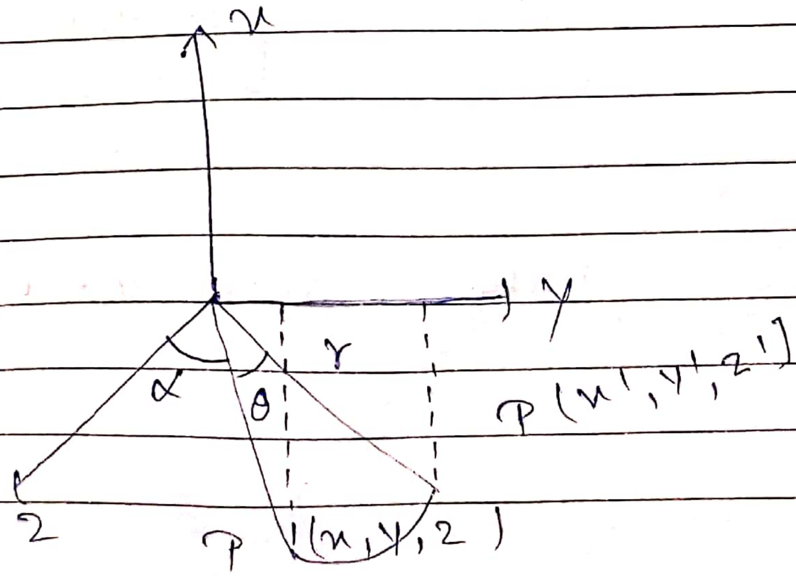
Scaling :- The matrix representation for the scaling transformation of position P (x, y, z) related to the co-ordinate original can write as.

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} Sx & 0 & 0 & 0 \\ 0 & Sy & 0 & 0 \\ 0 & 0 & Sz & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$P' = S \cdot P$$
$$x' = x \cdot Sx.$$
$$y' = y \cdot Sx.$$
$$z' = z \cdot Sz$$

Derive rotation about x-axis, y and z-axis matrix in 3D :-

1. Rotation about x-axis :-



$x' = x$

$y' = r\cos(\alpha + \theta)$

$y' = r\cos\alpha\cos\theta - r\sin\theta\alpha\sin\theta$

$r\cos\alpha = y$ and $r\sin\alpha = z$ — (1)

$y' = y\cos\theta - z\sin\theta$

$z' = r\sin(\alpha + \theta)$

$r \cdot \sin\alpha\cos\theta + r\cos\alpha\sin\theta$

$= z\cos\theta + y\sin\theta$

$$R\theta = \begin{bmatrix} 1 & 0 & 0 \\ 1 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

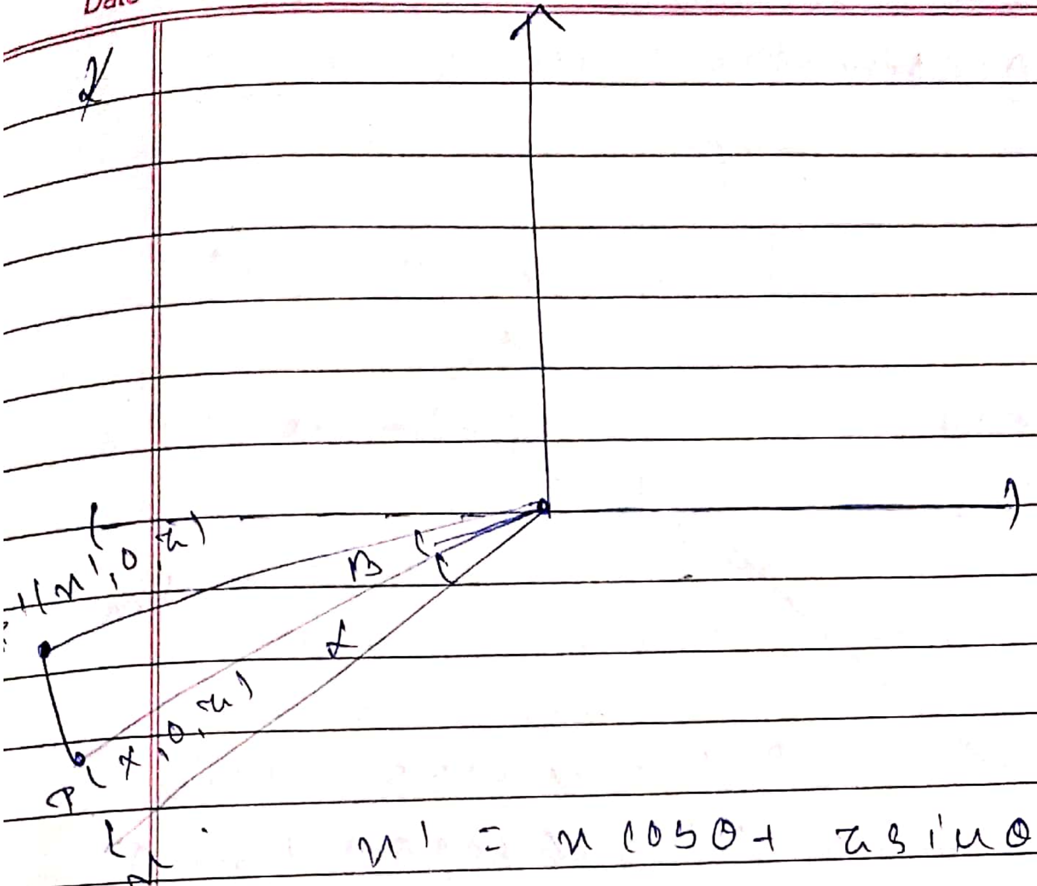
$P'(x', y', z') = R\theta, P(x, y, z)$

matrix given form as.

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

(2) Rotation about y - axis :-

Date / /   Page no.



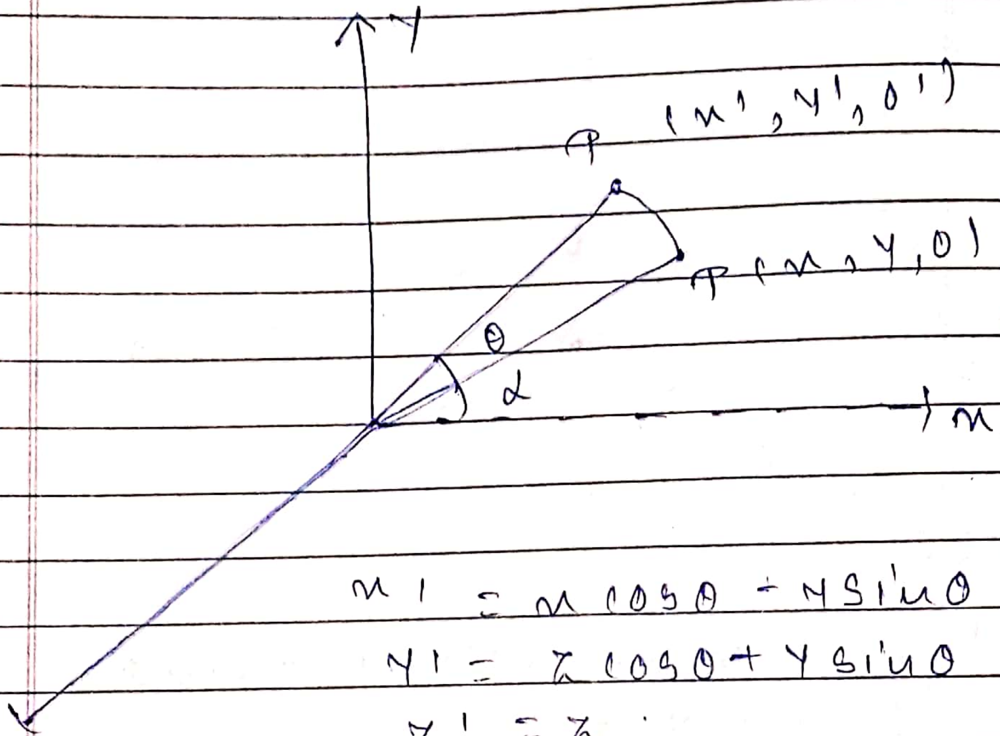
$$x' = x\cos\theta + z\sin\theta$$
$$y' = y$$
$$z' = x\sin\theta + z\cos\theta$$

$$R_{\theta} = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

Matrix form given as :-

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Rotation about the z-axis:-



$$x' = x\cos\theta - y\sin\theta$$
$$y' = x\cos\theta + y\sin\theta$$
$$z' = z$$

$$R_\theta = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

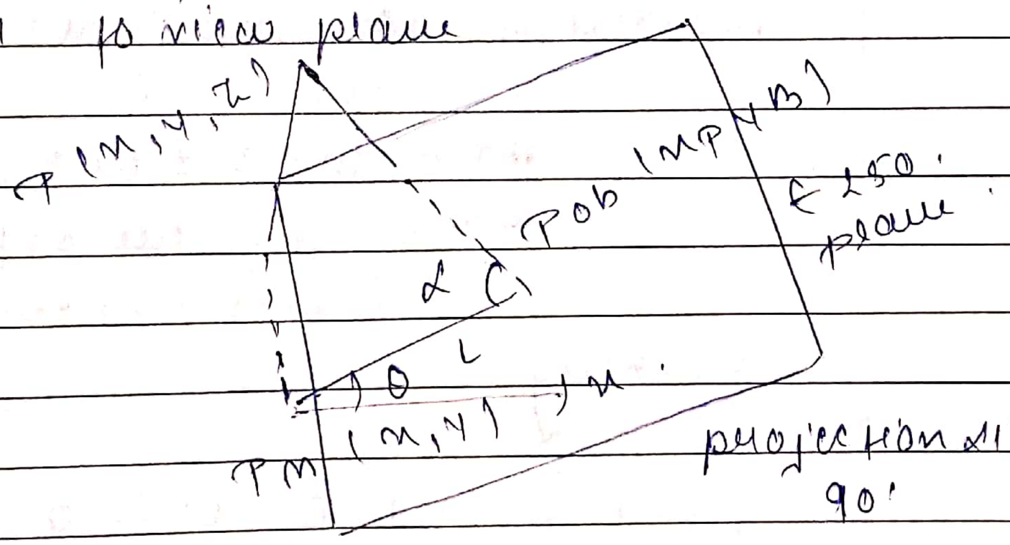
$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Date / /

Page no.

(2) what is oblique projection and Derive oblique projection matrix? provide some example and types of oblique projection.

- oblique projection:- An oblique projection is obtained by projection point along parallel lines that are not perpendicular to projection plane.
In this projection angle between the projection and plane of projection is not equal to equal to 90° i.e projection are not perpendicular to view plane



we can express the projection co-ordinate in term of x, y, L and θ as.

Date / / Page no.

$$xp = x + L \cos\theta$$
$$yp = x + L \sin\theta$$

Length L depends on the angle α and the z co-ordinate of the point to be projected

$$\tan\alpha = \frac{z}{L} \Rightarrow L = \frac{z}{\tan\alpha} = zL_1$$

$$\left[\text{where } L_1 = \frac{1}{\tan\alpha} \right]$$

when $z = 1$, $L_1 = L$

Now we can write the oblique projection as $xp = x + z(L_1 \cos\theta)$

$$yp = x + z(L_1 \sin\theta)$$
$$zp = 0, \text{ we get}$$

considering

$$\begin{bmatrix} xp \\ yp \\ zp \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & L_1 \cos\theta & 0 \\ 0 & 1 & L_1 \sin\theta & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Date / / Page no.

this is the standard matrix for oblique projection onto $z=0$ plane.

(3) Find the matrix for parallel projection onto the plane $3x+y+4z+1=0$ when,

(1) an orthographic projection
(2) oblique projection is used.

:- Here $(a,b,c) = (p,q,r) = (3,1,4)$
parametric equation of direction line.

$$x_P = x_0 + 3t \quad -(1)$$
$$y_P = y_0 + 1t \quad -(2)$$
$$z_P = z_0 + 4t \quad -(3)$$

From equation :-

$$t = \frac{ax_0 + by_0 + cz_0 + d}{ap + bq + cr}.$$

$$\Rightarrow \frac{3x_0 + y_0 + 7z_0 + 1}{3^2 + 1^2 + 4^2}$$

$$\Rightarrow \frac{3x_0 + y_0 + 4z_0 + 1}{26}.$$

put the value of t in parametric equation (1) (2) and (3) we get

$$x_P = \frac{1}{26}\left(17x_0 - 3y_0 - 12z_0 - 3\right)$$

$$y_P = \frac{1}{26}\left(-3x_0 + 25y_0 - 4z_0 - 1\right)$$

$$z_P = \frac{1}{26}\left(-12x_0 - 4y_0 + 10z_0 - 4\right)$$

The matrix for orthographics parallel projection is given by :-

$$\begin{bmatrix} x_P \\ y_P \\ z_P \\ 1 \end{bmatrix} = \frac{1}{26}\begin{bmatrix} 17 & -3 & -12 & -3 \\ -3 & 25 & -4 & -1 \\ -12 & -4 & +10 & -4 \\ 0 & 0 & 0 & 26 \end{bmatrix}\begin{bmatrix} x_0 \\ y_0 \\ z_0 \\ 1 \end{bmatrix}$$

Now here $p = q = r = 1$

$a = 3$ $b = 1$, $c = 2$

and $d = 1$

parametric equation are

$x_P = x_0 + t$

$y_P = y_0 + t$

$z_P = z_0 + t$.

Date / / Page no.

From equation $t = \frac{ax_0 + by_0 + cz_0 + d}{ap + bq + cr}$.

$$\Rightarrow \frac{3x_0 + y_0 + 4z_0 + 1}{8}$$

put the value of t we get

$$x_P = \frac{1}{8}(-5x_0 - y_0 - 4z_0 - 1)$$

$$y_P = \frac{1}{8}(-3x_0 - 7y_0 - 4z_0 - 1)$$

$$z_P = \frac{1}{8}(-3x_0 - y_0 - 4z_0 - 1)$$

The matrix all of oblique parallel projection is given by

$$\begin{bmatrix} x_P \\ y_P \\ z_P \\ 1 \end{bmatrix} = \frac{1}{8}\begin{bmatrix} -5 & -1 & -4 & 1 \\ -3 & -7 & -4 & -1 \\ -3 & -1 & -4 & -1 \\ 0 & 0 & 0 & 8 \end{bmatrix}\begin{bmatrix} x_0 \\ y_0 \\ z_0 \\ 1 \end{bmatrix}$$

(4) Differentiate between parallel projection and prespective projection?

Date / /

Page no.

parallel projection	perspective projection
① In parallel projection parallel lines from each vertex on the object are extends until they intersect the view plane.	① perspective projection produces realistic view
② Lines of projection are parallel.	② Lines of projection are not parallel
③ these are Linear transform.	③ these are non-Linear transformation.

⑤ Establish and write cyrus-Beck 3D line clipping algorithm.

:- cyrus-Beck algorithm is applicable to an arbitrary convex region. This algorithm uses a parametric equation of line segment to find the intersection point of a line with a clipping edges

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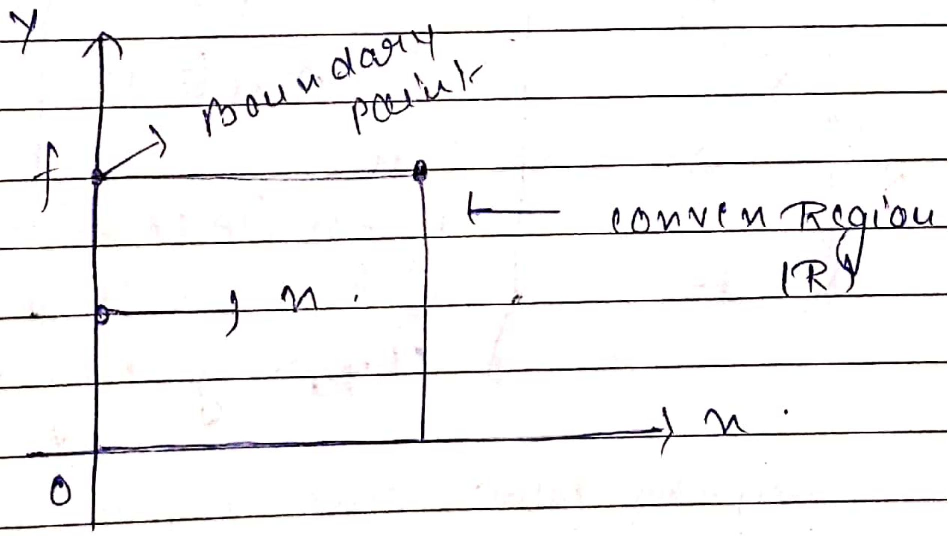
Date / / Page no.

The parametric equation of a line segment from P1 to P2 is

$$P(t) = P_1 + (P_2 - P_1)t \quad 0 \le t \le 1$$

where t is parameter, t = 0 at P1 and t = 1 at P2

Consider a convex clipping region R, f is the boundary point of the convex region R and n is an inner normal for one of its boundaries as shown in.



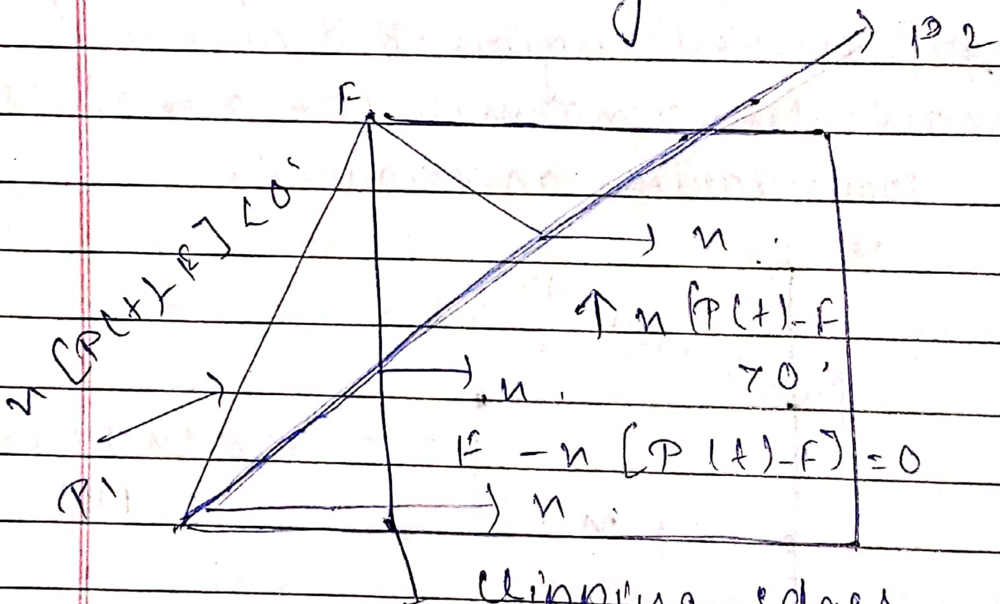
We evaluate Dot product $n \cdot [P(t) - f]$ and check the following condition as.

* If a dot product $n[P(t) - F] < 0$ then the vector $P(t) - F$ is pointed away from the inner of R

* If dot products is zero, i.e.

$$n \cdot [P(t) - F] > 0$$

then the vector $P(t) - F$ is pointed towards the interior of R as shown in Figure.



– Cyrus Beck Line clipping algorithm

* Read two end point of the line. P_1 and P_2
* Read the vertex co-ordinate of clipping window
* calculate $D = P_2 - P_1$

Date / / Page no.

* Assign boundary point(f) with particularly edge find Inner normal vector for corresponding edges.
* Find Inner normal vector for corresponding edges and co-ordinate edges
* Find inner normal vector for co-ordinate edges
* calculate D.N and $W = P1 - f$.
* If $D \cdot N > 0$.

$$t_l = -\frac{W \cdot N}{D \cdot N}$$

else.

$$t_u = -\frac{W \cdot N}{D \cdot N}$$

end if

* Repeat step 4 to 7 for each edges of clipping window.

(6) Find the matrix. for mirror reflection with respect to plan passing through the origin and having a normal vector. whose direction is $N = \hat{i} + \hat{j} + \hat{k}$

Ans) we know $N = n_1\hat{i} + n_2\hat{j} + n_3\hat{k}$
with $|N| = \sqrt{n_1^2 + n_2^2 + n_3^2}$ and $J = \sqrt{n_2^2 + n_3^2}$

Here $N = \hat{i} + \hat{j} + \hat{k}$ so $|N| = \sqrt{1^2+1^2+1^2} = \sqrt{3}$

$\lambda = \sqrt{1^2+1^2} = \sqrt{2}$ (m+Po (0,0,0))

$$T_v = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_v^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$v = 0\hat{i} + 0\hat{j} + 0\hat{k}$

then
$$A_N = \begin{bmatrix} \frac{\lambda}{|N|} & -\frac{n_1 n_2}{\lambda|N|} & -\frac{n_1 n_3}{\lambda|N|} & 0 \\ 0 & \frac{n_3}{\lambda} & -\frac{n_2}{\lambda} & 0 \\ \frac{n_1}{|N|} & \frac{n_2}{|N|} & \frac{n_3}{|N|} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \frac{\sqrt{2}}{\sqrt{2}} & \frac{-1}{\sqrt{2}\sqrt{3}} & \frac{-1}{\sqrt{2}\sqrt{3}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

and
$$A^{-1}_N = \begin{bmatrix} \frac{\sqrt{2}}{\sqrt{3}} & 0 & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{2}\sqrt{3}} & \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{2}\sqrt{3}} & \frac{-1}{\sqrt{2}} & \frac{-1}{\sqrt{3}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Matrix for mirror reflection with ray point in homogenous from is

$$M_{xy} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The final reflection matrix is

$$TV^{-1} \; AN^{-1} \; M_{xy} \cdot AN \cdot TV$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \frac{\sqrt{2}}{\sqrt{3}} & 0 & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{2}\sqrt{3}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{2}\sqrt{3}} & \frac{-1}{\sqrt{2}} & \frac{-1}{\sqrt{3}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$* \begin{bmatrix} \frac{\sqrt{2}}{\sqrt{3}} & \frac{-1}{\sqrt{2}\sqrt{3}} & \frac{-1}{\sqrt{2}\sqrt{3}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1/3 & -2/3 & -2/3 & 0 \\ -2/3 & 1/3 & -2/3 & 0 \\ -2/3 & -2/3 & 1/3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Date / /   Page no.

(7) Define anonmetric projection and types of anonmetric projection in detail?

Anonmetric Projection :- Anonmetri-c projection are the ~~art~~ orthographics projection in which the direction of projection is not parallel to any of three principle axis to construct of an anonmetric projection is done by using rotation and transaction to manupulate the object such that atleast three adjence face are shown the forestoartening factor is the ratio of the projecterined length of a line to its true length

the sub-categories of anonmetric projection are

* Isometric projection :- the direction of projection makes equal angles with all three principle axis.

Date / /

Page no.

* **Dimetric projection** :- the directio-n makes equal angles with two not all three principle axis.

* **Thrimetric projection** :- the direction of projection makes an equal with the three principle axis.

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