Let us solve here some of the questions which are important with respect to class 10th Maths board exam.

Q.1: **Represent the following situations in the form of quadratic equations:**

(i) The area of a rectangular plot is 528 m^2. The length of the plot (in metres) is one more than twice its breadth. We need to find the length and breadth of the plot.

(ii) A train travels a distance of 480 km at a uniform speed. If the speed had been 8 km/h less, then it would have taken.

Solution:

(i) Let us consider,
The breadth of the rectangular plot is x m

Thus, the length of the plot = (2x + 1) m.

As we know,

Area of rectangle = length × breadth = 528 m^2
Putting the value of length and breadth of the plot in the formula, we get,

$(2x + 1) \times x = 528$

$\Rightarrow 2x^2 + x = 528$

$\Rightarrow 2x^2 + x - 528 = 0$

Hence, $2x^2 + x - 528 = 0$, is the required equation which represents the given situation.

(ii) Let us consider,
speed of train = x km/h
And
Time taken to travel 480 km = 480 (x) km/h
As per second situation, the speed of train = (x − 8) km/h

As given, the train will take 3 hours to cover the same distance.
Therefore, time taken to travel 480 km = 480x + 3 km/h
As we know,
Speed × Time = Distance
Therefore,
$(x - 8)(480/x + 3) = 480$
$\Rightarrow 480 + 3x - 3840/x - 24 = 480$
$\Rightarrow 3x - 3840/x = 24$
$\Rightarrow 3x^2 - 8x - 1280 = 0$

Hence, $3x^2 - 8x - 1280 = 0$ is the required representation of the problem mathematically.

Q.2: Find the roots of quadratic equations by factorisation:

(i) $\sqrt{2}\, x^2 + 7x + 5\sqrt{2} = 0$

(ii) $100x^2 - 20x + 1 = 0$

Solution:

(i) $\sqrt{2}\, x^2 + 7x + 5\sqrt{2} = 0$
Considering the L.H.S. first,

$\Rightarrow \sqrt{2}\, x^2 + 5x + 2x + 5\sqrt{2}$

$\Rightarrow x(\sqrt{2}x + 5) + \sqrt{2}(\sqrt{2}x + 5) = (\sqrt{2}x + 5)(x + \sqrt{2})$
The roots of this equation, $\sqrt{2}\, x^2 + 7x + 5\sqrt{2} = 0$ are the values of x for which $(x - 5)(x + 2) = 0$
Therefore, $\sqrt{2}x + 5 = 0$ or $x + \sqrt{2} = 0$
$\Rightarrow x = -5/\sqrt{2}$ or $x = -\sqrt{2}$

(ii) Given, $100x^2 - 20x + 1 = 0$
Considering the L.H.S. first,
$\Rightarrow 100x^2 - 10x - 10x + 1$
$\Rightarrow 10x(10x - 1) - 1(10x - 1)$
$\Rightarrow (10x - 1)^2$
The roots of this equation, $100x^2 - 20x + 1 = 0$, are the values of x for which $(10x - 1)^2 = 0$
Therefore,

$(10x - 1) = 0$

or $(10x - 1) = 0$
$\Rightarrow x = 110$ or $x = 110$

Q.3: Find two consecutive positive integers, sum of whose squares is 365.

Solutions: Let us say, the two consecutive positive integers be x and x + 1.
Therefore, as per the given questions,
$x^2 + (x + 1)^2 = 365$
$\Rightarrow x^2 + x^2 + 1 + 2x = 365$
$\Rightarrow 2x^2 + 2x - 364 = 0$
$\Rightarrow x^2 + x - 182 = 0$
$\Rightarrow x^2 + 14x - 13x - 182 = 0$
$\Rightarrow x(x + 14) - 13(x + 14) = 0$
$\Rightarrow (x + 14)(x - 13) = 0$
Thus, either, x + 14 = 0 or x − 13 = 0,
$\Rightarrow x = -14$ or $x = 13$
since, the integers are positive, so x can be 13, only.

So, x + 1 = 13 + 1 = 14
Therefore, the two consecutive positive integers will be 13 and 14.

Q.4: Find the roots of the following quadratic equations, if they exist, by the method of completing the square:

(i) $2x^2 - 7x + 3 = 0$

(ii) $2x^2 + x - 4 = 0$

Solution:

(i) $2x^2 - 7x + 3 = 0$
$\Rightarrow 2x^2 - 7x = -3$
Dividing by 2 on both sides, we get
$\Rightarrow x^2 - 7x/2 = -3/2$
$\Rightarrow x^2 - 2 \times x \times 7/4 = -3/2$
On adding $(7/4)^2$ to both sides of above equation, we get
$\Rightarrow (x)^2 - 2 \times x \times 7/4 + (7/4)^2 = (7/4)^2 - 3/2$
$\Rightarrow (x - 7/4)^2 = 49/16 - 3/2$
$\Rightarrow (x - 7/4)^2 = 25/16$
$\Rightarrow (x - 7/4) = \pm 5/4$
$\Rightarrow x = 7/4 \pm 5/4$
$\Rightarrow x = 7/4 + 5/4$ or $x = 7/4 - 5/4$

$\Rightarrow x = 12/4$ or $x = 2/4$
$\Rightarrow x = 3$ or $1/2$

(ii) $2x^2 + x - 4 = 0$
$\Rightarrow 2x^2 + x = 4$
Dividing both sides of the above equation by 2, we get
$\Rightarrow x^2 + x/2 = 2$
Now on adding $(1/4)^2$ to both sides of the equation, we get,

$\Rightarrow (x)^2 + 2 \times x \times 1/4 + (1/4)^2 = 2 + (1/4)^2$
$\Rightarrow (x + 1/4)^2 = 33/16$
$\Rightarrow x + 1/4 = \pm \sqrt{33}/4$
$\Rightarrow x = \pm \sqrt{33}/4 - 1/4$
$\Rightarrow x = \pm \sqrt{33} - 1/4$

Therefore, either $x = \sqrt{33} - 1/4$ or $x = -\sqrt{33} - 1/4$.

Q.5: The diagonal of a rectangular field is 60 metres more than the shorter side. If the longer side is 30 metres more than the shorter side, find the sides of the field.

Solutions: Let us say, the shorter side of the rectangle is x m.
Then, larger side of the rectangle = (x + 30) m
Diagonal of the rectangle = $\sqrt{[x^2 + (x+30)^2]}$ As given, the length of the diagonal is = x + 30 m
Therefore,
$\sqrt{[x^2 + (x+30)^2]} = x + 60$

Squaring on both the sides, we get;
$\Rightarrow x^2 + (x + 30)^2 = (x + 60)^2$
$\Rightarrow x^2 + x^2 + 900 + 60x = x^2 + 3600 + 120x$
$\Rightarrow x^2 - 60x - 2700 = 0$
$\Rightarrow x^2 - 90x + 30x - 2700 = 0$
$\Rightarrow x(x - 90) + 30(x - 90)$
$\Rightarrow (x - 90)(x + 30) = 0$
$\Rightarrow x = 90, -30$

Q.6 : Solve the quadratic equation $2x^2 - 7x + 3 = 0$ by using quadratic formula.

Solution: $2x^2 - 7x + 3 = 0$

On comparing the given equation with $ax^2 + bx + c = 0$, we get,

a = 2, b = -7 and c = 3
By using quadratic formula, we get,

$x = [-b \pm \sqrt{(b^2 - 4ac)}]/2a$

$\Rightarrow x = [7 \pm \sqrt{(49 - 24)}]/4$
$\Rightarrow x = [7 \pm \sqrt{25}]/4$
$\Rightarrow x = [7 \pm 5]/4$

Therefore,
$\Rightarrow x = 7+5/4$ or $x = 7-5/4$
$\Rightarrow x = 12/4$ or $2/4$
$\therefore x = 3$ or ½

Q.7: Sum of the areas of two squares is 468 m^2. If the difference of their perimeters is 24 m, find the sides of the two squares.

Solutions: Let the sides of the two squares be x m and y m.

Therefore, their perimeter will be 4x and 4y respectively

And area of the squares will be x^2 and y^2 respectively.

Given,

$4x - 4y = 24$

$x - y = 6$

$x = y + 6$

Also, $x^2 + y^2 = 468$

$\Rightarrow (6 + y^2) + y^2 = 468$

$\Rightarrow 36 + y^2 + 12y + y^2 = 468$

$\Rightarrow 2y^2 + 12y + 432 = 0$

$\Rightarrow y^2 + 6y - 216 = 0$

$\Rightarrow y^2 + 18y - 12y - 216 = 0$

$\Rightarrow y(y + 18) - 12(y + 18) = 0$

$\Rightarrow (y + 18)(y - 12) = 0$

$\Rightarrow y = -18, 12$

As we know, the side of a square cannot be negative.

Hence, the sides of the squares are 12 m and (12 + 6) m = 18

Q.8: Find the values of k for each of the following quadratic equations, so that they have two equal roots.
(i) $2x^2 + kx + 3 = 0$
(ii) $kx(x - 2) + 6 = 0$

Solutions:

(i) $2x^2 + kx + 3 = 0$

Comparing the given equation with $ax^2 + bx + c = 0$, we get,

a = 2, b = k and c = 3

As we know, Discriminant = $b^2 - 4ac$

$= (k)^2 - 4(2)(3)$

$= k^2 - 24$

For equal roots, we know,

Discriminant = 0

$k^2 - 24 = 0$

$k^2 = 24$

$k = \pm\sqrt{24} = \pm 2\sqrt{6}$

(ii) $kx(x - 2) + 6 = 0$

or $kx^2 - 2kx + 6 = 0$

Comparing the given equation with $ax^2 + bx + c = 0$, we get

a = k, b = − 2k and c = 6

We know, Discriminant = $b^2 - 4ac$

$= (-2k)^2 - 4(k)(6)$

$= 4k^2 - 24k$

For equal roots, we know,

$b^2 - 4ac = 0$

$4k^2 - 24k = 0$

$4k(k - 6) = 0$

Either 4k = 0 or k = 6 = 0

k = 0 or k = 6

However, if k = 0, then the equation will not have the terms 'x^2' and 'x'.

Therefore, if this equation has two equal roots, k should be 6 only.

Q.9: Is it possible to design a rectangular park of perimeter 80 and area 400m2? If so find its length and breadth.

Solution: Let the length and breadth of the park be L and B.

Perimeter of the rectangular park = 2 (L + B) = 80

So, L + B = 40

Or, B = 40 − L

Area of the rectangular park = $L \times B = L(40 - L) = 40L - L^2 = 400$

$L^2 - 40L + 400 = 0$,

which is a quadratic equation.

Comparing the equation with $ax^2 + bx + c = 0$, we get

a = 1, b = -40, c = 400

Since, Discriminant = $b^2 - 4ac$

$\Rightarrow (-40)^2 - 4 \times 400$

$\Rightarrow 1600 - 1600 = 0$

Thus, $b^2 - 4ac = 0$

Therefore, this equation has equal real roots. Hence, the situation is possible.

Root of the equation,

$L = -b/2a$

$L = (40)/2(1) = 40/2 = 20$

Therefore, length of rectangular park, L = 20 m

And breadth of the park, B = 40 − L = 40 − 20 = 20 m.

Q.10: Find the discriminant of the equation 3×2− 2x +1/3= 0 and hence find the nature of its roots. Find them, if they are real.
Solution : Here, a = 3, b = − 2 and c = 1/3
Since, Discriminant = $b^2 - 4ac$

= (− 2)2 − 4 × 3 × 1/3
= 4 − 4 = 0.
Hence, the given quadratic equation has two equal real roots.
The roots are -b/2a and -b/2a.

2/6 and 2/6

or

1/3, 1/3