Simulation and Analysis of The Solar System and 'Oumuamau

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April 1, 2024

Abstract

This report discusses a simulation of the Solar System in which the Verlet integration scheme is applied, this discretises time thus iterating through time steps. The simulation was employed primarily to gather observables of each body within the system, namely the apsides, energy and orbital periods. The report accurately produces the gravitational interactions between bodies of the Solar System (and further bodies) based of literature data collected from 23rd May 2023. Through convergence analysis, a time step of $dt = 0.3815$ was determined then used to produce observables within 0.5% accuracy. Furthermore, the simulation introduced 'Oumuamua as another body in order to determine the date, distance and velocity at its perihelion. This was achieved by running the simulation in reverse, leading to measurements accurate with that of literature.

1 Introduction

The simulation used applies the Verlet time integration scheme to a set of particles which discretely iterates through time and updates positions, forces and velocities of each particle in the following steps:

1. Update positions using the current forces f_i
   $x_{i+1} = x_i + v_i dt + f_i dt^2/2m$
2. Calculate forces at the new positions f_{i+1}
3. Update the velocities using forces $\frac{1}{2}(f_i + f_{i+1})$
   $v_{i+1} = v_i + (f_i + f_{i+1})dt/2m$

The interactions simulated between particles are purely gravitational. This was implemented through Newtons Laws, where the gravitational constant was approximated as $6.67430 \times 10^{11} m^3 kg^{-1} s^{-2}$ in SI units, equivalent to $G = 8.888 \times 10^{-10} AU^3 M_{earth}^{-1} day^-2$ in units used within the simulation. The equations below represent the calculation of forces and energy in the simulation:

$$\mathbf{F} = -\frac{G \cdot m_1 \cdot m_2}{r^2} \frac{\mathbf{r}}{|\mathbf{r}|}$$

$$U_{12} = -\frac{G \cdot m_1 \cdot m_2}{|\mathbf{r}_{12}|}$$

The simulation was then applied to a set of particles resembling the Solar System and multiple observables of this was measured and analysed (Time period, Perihelion and Aphelion).

2 Convergence

The Verlet Time Integration scheme discretises time to simulate Newtons laws of gravitation as stated in the Introduction. The time-step or dt is one of the key factors which determines the accuracy of the simulation, where, as dt tends to 0 the simulation increases in accuracy though also increases in runtime.

It was chosen to use results of the simulation for a very small dt as this would more accurately determine a *true* value for the simulation as real life data will have other factors in effect which will likely deviate the laws of the simulation.

A value for dt was found such that the simulation converged. This was done using a *Mini Solar System* which comprised of the Sun, Earth, Mercury and the Moon. Initially a value of each body (bodies which orbit others : Earth, Mercury, Moon) was calculated with $dt = 1 \times 10^{-4} days$. This lead to the results in Table 1.

Body	Period /days	Perihelion /AU	Aphelion /AU
Earth	365.2673	0.9833	1.0167
Moon	27.3207	0.0024	0.0027
Mercury	87.9689	0.3075	0.4667

Table 1: *Mini Solar System* values calculated from simulation where $dt = 1 \times 10^{-4} days$

These results represent the *true* values towards which the simulation converged. The simulation was run repeatedly with decreasing dt whilst gathering the observables from each body and compared to the *true* values (Table 1) until each observable fell within 0.5%. The time-step where this occurred was to be the time-step where convergence was determined, this lead to $dt = 0.3815$ days.

The simulation was run for 10 years with an initial $dt = 10$ days, whilst reporting which observables had been considered converged for each simulation to the command line. This revealed that the Moon was the limiting factor, as its observables took the longest to converge, this is due to it having a smaller orbit relative to the other bodies in the system. Another factor limiting the accuracy of the moons observables is its orbit around the Earth, as its position also undergoes approximate motion. Logging data from the simulation ran by the set of dt's used led to Figures 1 and 2.

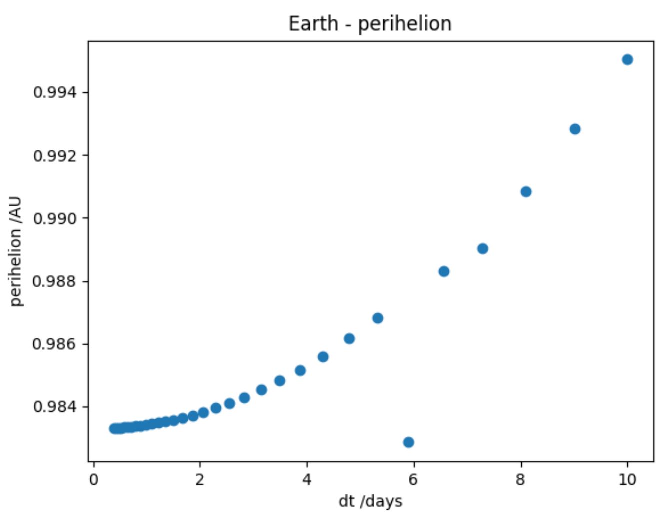


Figure 1: Plot of Earth's Perihelion with varying dts

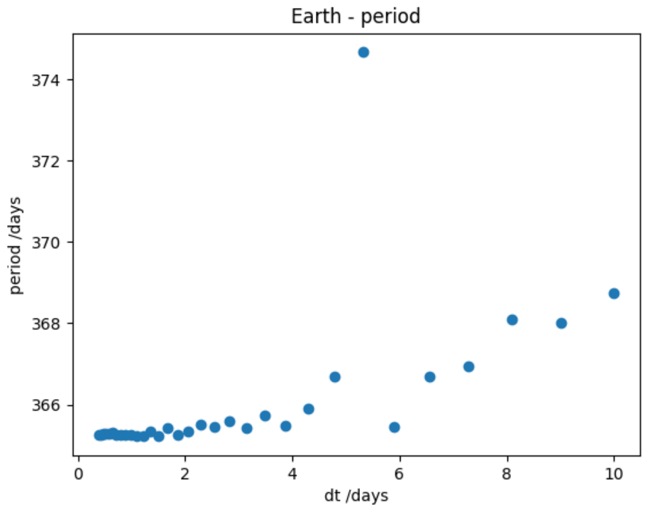


Figure 2: Plot of Earth's period with varying dts

Figures 1 and 2 show anomalies at approximately $4 < dt < 6$ days, excluding values greater than $dt = 4$ days leads to Figures 3, 4 and 5, similarly for other orbiting bodies. It can be seen there is convergence with all the observables for this body although that of the perihelion and aphelion are much more conclusive relative to the period.

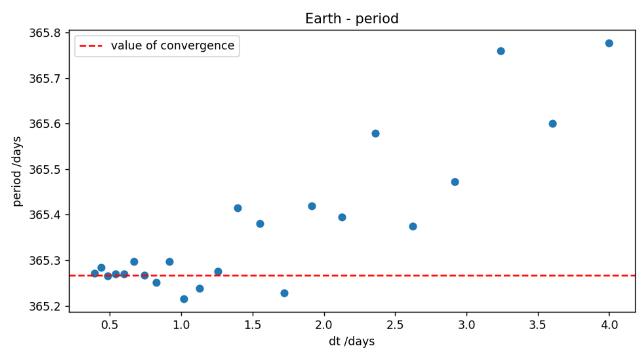


Figure 3: Plot of Earth's period with varying dts

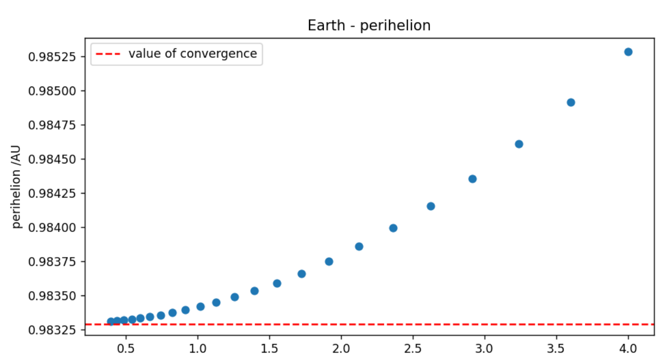


Figure 4: Plot of Earth's perihelion with varying dts

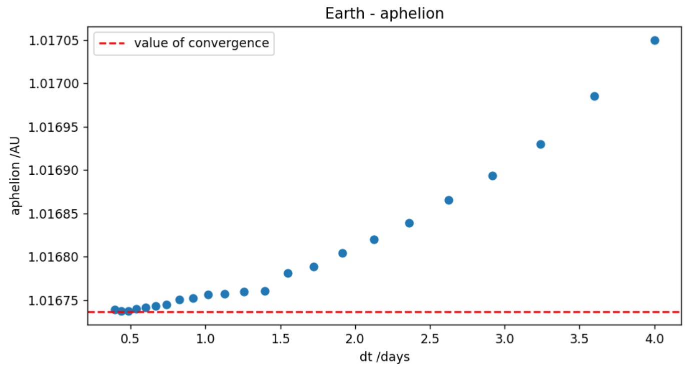


Figure 5: Plot of Earth's perihelion with varying dts

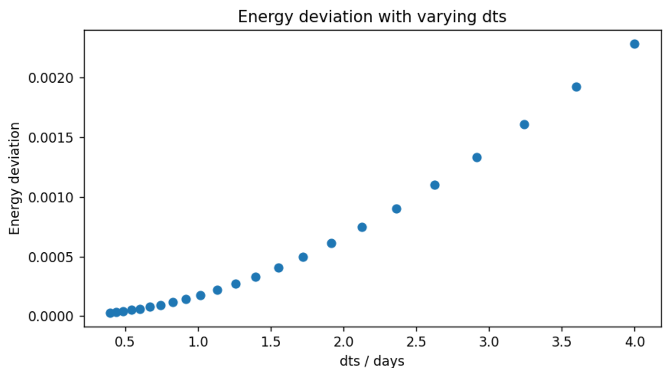
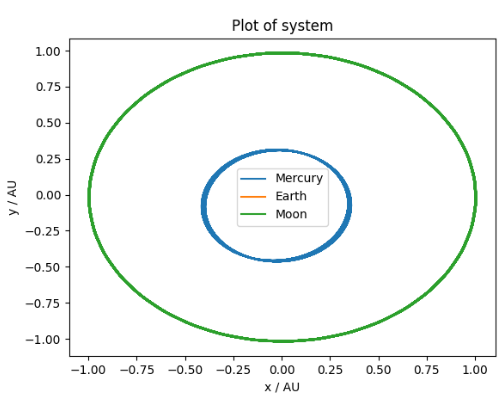


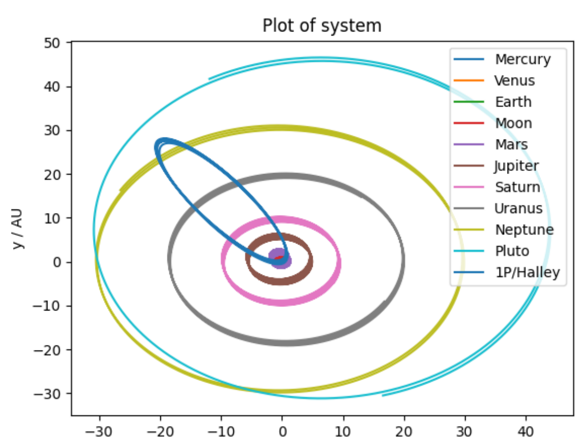
Figure 6: Plot of systems energy deviation with varying dts

Figure 6 shows the energy is converging with minute values and functions as expected. Note that further plots of each orbiting body and their observables prove similar to that of Earth, i.e. illustrating exponential decay.

3 Results



(a) One year plot of Mini System



(b) Ten year plot of Solar System

Figure 7: Plots of systems showing orbits of each body in 2 dimensions where $dt = 0.3815$ days

Body	Period /days	Perihelion /AU	Aphelion /AU
Mercury	87.9689	0.3075	0.4667
Venus	224.7090	0.7184	0.7283
Earth	365.2673	0.9833	1.0167
Moon	27.3207	0.0024	0.0027
Mars	686.9814	1.3806	1.6667
Jupiter	4332.6717	4.9444	5.4602
Saturn	10758.5775	9.0084	10.07981
Uranus	30677.6867	18.2585	20.1216
Neptune	60208.3300	29.7982	30.3445
Pluto	90599.7645	29.6453	49.3506
1P/Halley	27071.5452	0.5901	35.1435

Table 2: *Mini Solar System* values calculated from simulation where $dt = 0.3815$ days

Figure 7 and Table 2 show that the simulation is functioning as expected, 7(a) shows a one year simulation of the *toy solar system* with Earth and Mercury producing circular orbits as expected. 7(b) shows the Solar System with Halley's comet in a 10 year simulation producing results as expected.

Table 2 consists of observables gathered from the same simulation as Figure 7. Comparing these to literature values from *NASA* [1] we can see that the orbital periods are accurate and within the chosen convergence factor of 0.5%, although it was chosen to avoid quantitatively comparing these to values observed through the simulation as the simulation will not exactly reproduce that of real-life measurements (hence using *true* values of a simulation of small dt as explained in Section 2). Comparing Halley's comet to literature values, its apsides fall within 0.5% of the simulations measurements and its period falls within 1% [2].

'Oumuamua was the mini task selected, the simulation was run with the initial conditions from 23rd May 2023, the simulation was run with 'Oumuamua and a handful of observables were measured:

- The date and distance of the perihelion : The simulation measured a perihelion to have occurred on the 9th of September 2017 which is consistent with literature values. The perihelion was measured to be 0.2560 AU, this is also relatively consistent with literature values of approximately 0.2474 AU (within 5% accuracy). [3]

- Velocity at perihelion compared to escape velocity : Using $v_{escape} = \sqrt{2GM/r}$ and with the perihelion calculated, the escape velocity was found to be $v_{escape} = 0.0044$ AU/day. Through the simulation the velocity at the perihelion was measured to be $v_{perihelion} = 0.0153$ AU / day. This finding is considered valid since 'Oumuamua's trajectory in the simulation and real data is not expected to return to the solar system.

  This is again considered to be valid as 'Oumuamua is not to return to the Solar System within the simulation. Literature values reported a velocity of $v_{perihelion} = 0.0221$ AU / day [4], the values are not identical although comparable. There was no error found with this calculation and it was assumed to be general uncertainty even though most other values proved to be much more consistent with that of literature.

- Date of arrival within the Solar System (within Neptune's orbit) : For this calculation Neptune's orbit was approximated to have a perfectly circular orbit around the sun, with a radius taken as the average of its apsides. Another approximation made for this was that 'Oumuamua's trajectory was within the plane of Neptune's orbit, this was done by only considering the $x - y$ plane. The date of entry was measured as the 20th November 2022.

4 Conclusions

The Verlet time integration scheme was applied used here to produce a simulation of the solar system. The simulation was successful as it was able to produce realistic recreations of the Solar System and produce valid observables consistent with literature values for periods and apsides. Furthermore it was functioned as expected when used to produce simulations of 'Oumuamua leading to values comparable to literature values.

There is room for improvement within multiple aspects of the simulation.

- Firstly, the majority of the observables were printed to the command line which lead to unorganised printing particularly for repeated simulations when computing for convergence. A fix for this could be to take more system arguments or parameters which allow the user to chose which observable to print or the data could be written to a csv/text file.
- With respect to efficiency, there are a handful of aspects which reduce this, notably the positions of the entire system being written to a text file, this could also be fixed with a boolean system argument letting the user determine when/if to save the data.
- The script for simulating and computing observables for 'Oumuamua was written in another file which improved organisation although a lot of functions from other files were needed, but were repeated elsewhere as when it was attempted to import these files to use the functions, the source files were ran each time a function from it was called even when using the *if __name__ == "__main__": main()* idiom.

  For simplicity, this simulation of the system was run once. The timestep at which the perihelion occured was noted during the initial simulation run. This timestep was then passed through the main function and the simulation was re-run to store the necessary data for analysis. Then the timestep of which the perihelion occured was noted and then passed through main and run again such that the needed data was stored and then again analysed. Whilst this approach is functional, it is highly inefficient necessitating the simulation to be run twice to obtain the required results. There are many possible solutions for this, namely a function can be run (if 'Oumuamua is present) during the simulation comparing the distance of 'Oumuamua to the Sun to previous values, and when the distance begins to increase as opposed to decrease the perihelion will have occured. This can be stored then computed first time.

5 Appendix

Though the code was functioning prior to these changes, they were made with intent to improve the overall quality of the code.

- Firstly the main function was adapted such that there were parameters passed through it and observables returned as needed. This was for repeatability utilised when finding the convergence of the system.
- Further, the code took an extra parameter for the 'Oumuamua script where a timestep of importance ('Oumuamua's perihelion) was passed such that the script stores the needed parameters of Neptune and 'Oumuamua into a dictionary which then is returned and computed externally.
- Lastly, a for loop was created to create the plots rather than have them hard coded.

References

[1] NASA, "Planetary fact sheet," 2018.

[2] EarthSky, "Comet halley, parent of 2 meteor showers." https://earthsky.org/astronomy-essentials/comet-halley-parent-of-2-meteor-showers/, May 2022.

[3] K. C. l. updated, "'oumuamua: A guide to the 1st known interstellar visitor," Mar 2019.

[4] NASA Science, "'oumuamua," n.d.