EXPERIMENT NO 4

Image Restoration

Objective:

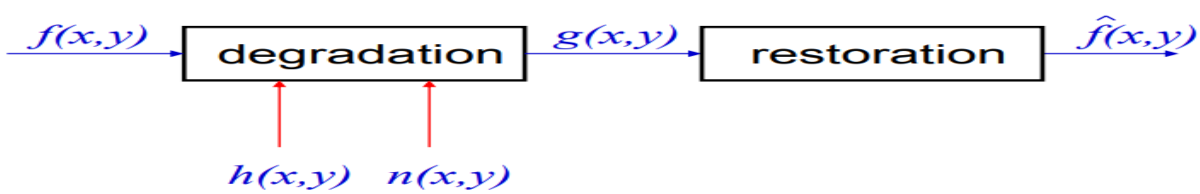
i) To restore a degraded image to its original form
ii) To get an idea of different types of image degradation
iii) To understand image restoration techniques

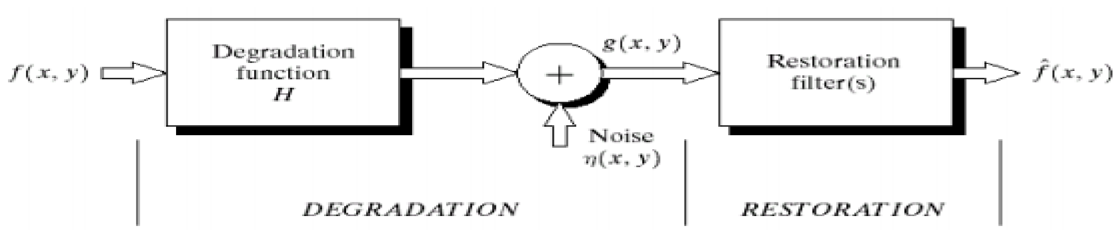
Theory

Restoration aims to reconstruct or recover an image that has been degraded by using a clear knowledge of the degrading phenomenon. Images may be corrupted by degradation such as frequency distortion, noise and blocking artifacts.

1.1 Image Degradation and Restoration model

Image degradation is the process by which the original image is blurred. To simplify the calculations, the degradation is often modelled as a linear function which is called as point-spread function.





$$g(x,y) = h(x,y) * f(x,y) + n(x,y) \Leftrightarrow G(u,v) = H(u,v)\, F(u,v) + N(u,v)$$

Where

f(x,y) – image before degradation, ‘true image’
g(x,y) – image after degradation, ‘observed image’
h(x,y) – degradation filter
f(x,y) – estimate of f(x,y) computed from g(x,y)
n(x,y) – additive noise

1.1.1 *Point spread function (PSF)*

An observed image can often be modelled as:

$$g(x,y) = \int\int h(x-x', y-y') f(x', y')\, dx'\, dy' + n(x,y)$$

where the integral is a convolution, *h* is the point spread function of the imaging system, and *n* is additive noise. In a linear model, the point-spread function *h* models the blurring of the image.
The objective of image restoration in this case is to estimate the original image f from the observed degraded image *g*.

1.2 Types of Image Blur

The degradation consists of two distinct processes:- the deterministic blur and random noise. Blurring is a form of bandwidth reduction of an ideal image owing to the imperfect image-formation process. Image blurs can be broadly classified as

a) Gauss blur occurs due to long time atmosphere exposure.
b) Out-of –focus blur due to defocussed optical system
c) Motion blur due to relative motion between the recording device and the scene

1.3 Image Noise Models

Name	PDF
Uniform	$p_z(z) = \begin{cases} \frac{1}{b-a} & \text{if } a \le z \le b \\ 0 & \text{otherwise} \end{cases}$
Gaussian	$p_z(z) = \frac{1}{\sqrt{2\pi}b} e^{-(z-a)^2/2b^2}$ $\quad -\infty < z < \infty$
Salt & Pepper	$p_z(z) = \begin{cases} P_a & \text{for } z = a \\ P_b & \text{for } z = b \\ 0 & \text{otherwise} \end{cases}$ $\quad b > a$
Lognormal	$p_z(z) = \frac{1}{\sqrt{2\pi}bz} e^{-[\ln(z)-a]^2/2b^2}$ $\quad z > 0$
Rayleigh	$p_z(z) = \begin{cases} \frac{2}{b}(z-a)e^{-(z-a)^2/b} & z \ge a \\ 0 & z < a \end{cases}$
Exponential	$p_z(z) = \begin{cases} ae^{-az} & z \ge 0 \\ 0 & z < 0 \end{cases}$

1.4 Linear Image restoration techniques

Most restoration techniques model the degradation process and attempt to apply an inverse procedure to obtain an approximation of the original image.
Classical linear techniques are deterministic method of image restoration where prior knowledge about degradation is known. Linear techniques restore the true image by filtering the observed

image using a properly designed filter. Types are (a) Inverse filter (b) pseudo-inverse filter (c) wiener filter and (d) constarined least-square filter.

1.4.1 Inverse Filter

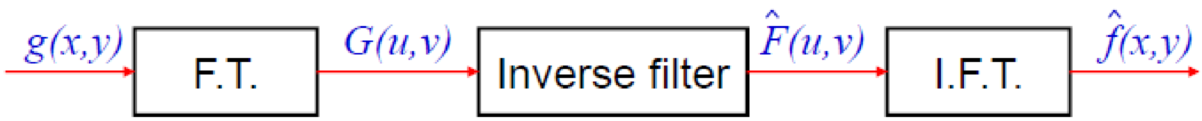
The inverse filter is a straightforward image restoration method. If we know the exact PSF model in the image degraded system and ignore the noise effect, the degraded image can be restored using inverse filter approach.

$F(u,v) = G(u,v) / H(u,v)$ *; noise ignored*

$$G(u,v) = F(u,v)H(u,v) + N(u,v) \qquad \hat{H}(u,v) = 1/H(u,v)$$

$$\hat{F}(u,v) = G(u,v)\hat{H}(u,v) = F(u,v) + \frac{N(u,v)}{H(u,v)}$$

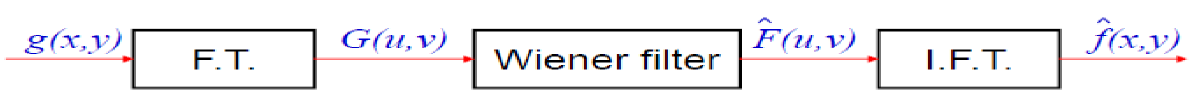
Restoration with an inverse filter



Drawback: In case of noise, if blurring filter has zeros at some frequencies (which it will since it is a low-pass filter), those frequencies will be amplified in the noise. In the presence of noise, it is better to go for a Wiener filter.

1.4.2 Wiener Filter

The Wiener filter tries to build an optimal estimate of the original image by enforcing a minimum mean-square error constraint between estimate and original image.



$$\hat{F}(u,v) = W(u,v)\ G(u,v)$$

$$W(u,v) = \frac{H^*(u,v)}{|H(u,v)|^2 + K(u,v)}$$

where

$$K(u,v) = S_\eta(u,v)/S_f(u,v)$$

$S_f(u,v) = |F(u,v)|^2$ power spectral density of $f(x,y)$

$S_\eta(u,v) = |N(u,v)|^2$ power spectral density of $\eta(x,y)$

- If $K = 0$ then $W(u,v) = 1 / H(u,v)$, i.e. an inverse filter
- If K >> $|H(u,v)|$ *for large* u,v, then high frequencies are attenuated

- $|F(u,v)|$ and $|N(u,v)|$ are often known approximately, or
- K is set to a constant scalar which is determined empirically

- A Wiener filter minimizes the least square error $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left(f(x,y) - \hat{f}(x,y)\right)^2 dxdy$

MCQs

1) Gaussian noise is referred to as

A. red noise
B. black noise
C. white noise
D. normal noise

2) PDF in image processing is called

A. probability degraded function
B. probability density function
C. probabilistic degraded function
D. probabilistic density function

3) In wiener filtering it is assumed that noise and image are

A. different
B. homogenous
C. correlated
D. uncorrelated

4) Mean filters reduce noise using

A. sharpening
B. blurring
C. restoration
D. acquisition

5) Degraded image is produced using degradation process and

A. additive noise
B. destruction
C. pixels
D. coordinates

6) Approach that incorporates both degradation function and statistical noise in restoration is called

A. inverse filtering
B. spike filtering
C. wiener filtering
D. ranking

7) Principle sources of noise arise during image

A. destruction
B. degradation
C. restoration
D. acquisition

An **example program** for removal of Additive White Gaussian Noise using Weiner filter

```
>> img=imread('sample image'); % read the photography
>> img=rgb2gray(img); % it has to be grayscale
>> img=img2double(img); % and of a type double
>> imgZ=conv2(img,h); % blurring with atmospherical noise + camera shift
>> imgZ=imgZ/imgZ(:); % scaling because of adding the noise
>> imgZN=imnoise(imgZ,'gaussian',0,.001); % add the additive noise
>> imshow(imgZN) % display the noisy image
>> imgO=imwiener(imgZN,h,0.013); % apply Wiener filter
>> imshow(imgO) % display the restored image
```

EXERCISES

1**)** Create a 512 x 512 grayscale image containing an intensity of 127 for all pixels. Add three different types of noise and plot the Histogram of the original and degraded image in all the cases. Discuss the characteristics of the histograms.

2) Download the Lenna image. Degrade the image with the Gaussian noise with a zero mean value. a) Calculate and display the auto-correlation function estimated from the degraded image (b) Determine the result of the Wiener filtering. Compare the result with the auto-correlation function calculated on the original image.

3) For the Lenna image, display the result of inverse filtering for a case of
a) Degradation without additive noise and b) degradation with additive noise. Calculate the Mean Square Error (between the reconstructed and original image) for both cases.(hint: err = mean2((img1-img2).ˆ2)to calculate MSE)