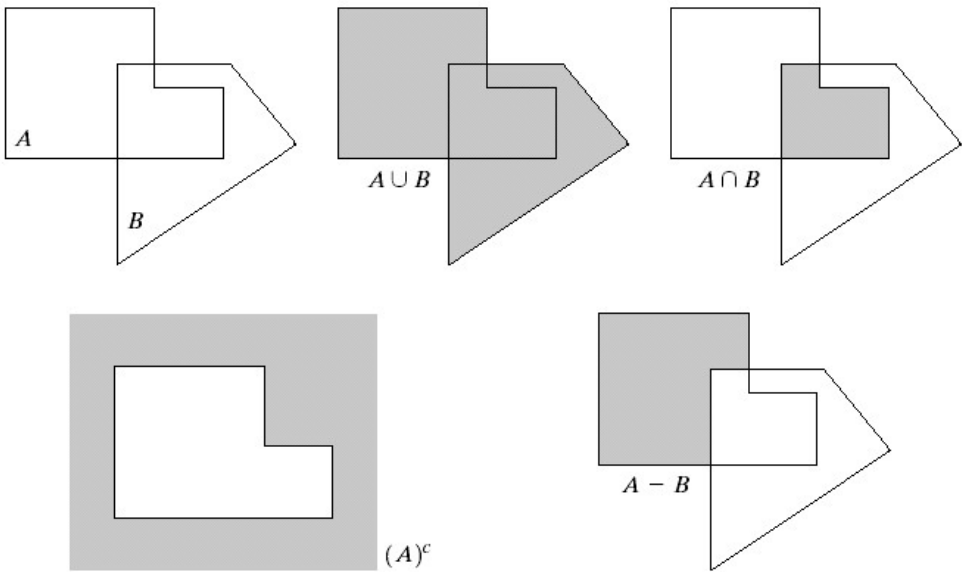
Experiment 7: Mathematical Morphology

1. Introduction:

Mathematical morphology is a tool for extracting image components that are useful in representation and description of region shape, such as boundaries, skeletons etc. It is also used for pre, post processing. The Language of mathematical morphology is set theory. Sets in mathematical morphology represent objects in an image. Eg: set of all black pixels in a binary images is a complete morphological description of the image. In binary image, the set in question are member of Z^2 (real integers) space where each element of a set is a tuple (2D vector) whose coordinates are the (x,y) coordinates of a black (or white depending on the convention) pixel in the image.

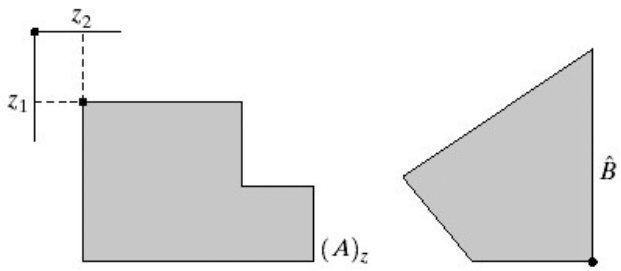
- Some basic concepts of set theory (refer to Figures on page 2)
- If a = (a1,a2) is an element of A, then $a \in A$
  - If a not an element of A, then $a \notin A$
  - Set with no elements = null or empty set ϕ
- In this exp, the elements of the set are the coordinates of the pixel representing the objects or other features of interest in an image.
  Eg: $C = \{w | w = -d, \text{ for } d \in D\}$ means set C is the set of elements w, such that w is formed by multiplying each of the two coordinates of all the elements of set D by -1
- If every element of set A is also an element of another set B, then A is a subset of B $A \subseteq B$.
- The union of two set A and B $C = A \cup B$.
- Intersection of two sets A and B $D = A \cap B$
- Two sets A and B are said to be disjoint or mutually exclusive if they have no common element $A \cap B = \varnothing$.
- Two additional definitions used in mathematical morphology but not found in basic text of set theory
  - Reflection of Set B denoted by $\hat{B}$ $\hat{B} = \{w | w = -b, \text{ for } b \in B\}$.
  - Translation of set A by point z (z1,z2) denoted by

$$(A)_z = \{c | c = a + z, \text{ for } a \in A\}.$$



a b c
d e

FIGURE 9.1
(a) Two sets A and B. (b) The union of A and B. (c) The intersection of A and B. (d) The complement of A. (e) The difference between A and B.



a b

FIGURE 9.2
(a) Translation of A by z. (b) Reflection of B. The sets A and B are from Fig. 9.1.

2. Dilation

The dilation and erosion are most basic morphological operations. Dilation adds pixels to the boundaries of objects in an image, while erosion removes pixels on object boundaries. In case of a binary image dilation grows or thicken the objects. Thickening is controlled by a shape called as structuring element. Structuring element (or Mask: M) is a square matrix with binary values which is applied on to the image. In Dilation we replace the central point of the mask M with the largest value of image covered by the mask. When the structural element is positioned at a given point and it touches the object, then this point will appear in the result of the transformation, otherwise it will not. Dilation generally increases the sizes of objects, filling in holes and broken areas, and connecting areas that are separated by spaces smaller than the size of the structuring element. With grayscale images, dilation increases the brightness of objects by taking the neighborhood maximum when passing the structuring element over the image. With binary images, dilation connects areas that are separated by spaces smaller than the structuring element and adds pixels to the perimeter of each image object.

- Mathematically, dilation is defined in terms of set theory operations. Dilation of A and B

$$A \oplus B = \{z | (\widehat{B})_z \cap A \neq \phi\}$$

i.e. Obtain a reflection of B about its origin and shift this reflection by z. The dilation of A by B is the set of all displacement z such that $\hat{B}$ and A overlap by at least one element

Programming exercise:

1) Create a 10 x 10 binary image with center pixel as 1 and other pixels as 0. Dilate this binary image with the following 1x 3 structuring element: [1, 1, 1] without using imdilate function.
   Hint: Follow the following steps to dilate a binary image:
   a) Create a matrix A of size 11x11 with all zeros.
   b) Place structuring element on the matrix A and compute logical AND.
   c) Repeat the process for each pixel of input image

2) Repeat question 1 while using the inbuilt function imdilate.
3) Create a 30 x 30 binary image where the center 3x3 pixels are 1 and other are 0 . Dilate this image with the following structuring elements
   a. 3x3 SE with all ones
   b. 3x1 SE with all ones
   c. 3x3 SE with all zeros

   Compare and justify the results obtained in each of these cases.

3) Erosion:

   Erosion generally decreases the sizes of objects and removes small anomalies by subtracting objects with a radius smaller than the structuring element. In Erosion, we replace the central point of the mask (structuring element) with the smallest value of image covered by the mask. With grayscale images, erosion reduces the brightness (and therefore the size) of bright objects on a dark background by taking the neighborhood minimum when passing the structuring element over the image. With binary images, erosion completely removes objects smaller than the structuring element and removes perimeter pixels from larger image objects.

- Erosion of A by B is defined as

$$A \ominus B = \{z|(B)_z \subseteq A\}$$

Erosion of A by B is the set of all points z such that B, translated by z, is contained in A.

Programming exercise:

4) Create a 10 x 10 binary image with center pixel as 1 and other pixels as 0. Erode this binary image with the following 1x 3 structuring element: [1, 1, 1] without using imerode function.

5) Repeat question 1 while using the inbuilt function imerode.
6) Create a 30 x 30 binary image where the center 3x3 pixels are 1 and other are 0 . Erode this image with the following structuring elements

a. 3x3 SE with all ones
b. 3x1 SE with all ones
c. 3x3 SE with all zeros

Compare and justify the results obtained in each of these cases.

7) Create a 30x30 binary image B with the center 5x5 pixels are 1 and other pixels are 0. Dilate this binary image with a 3x3 SE of all ones and subsequently erode this image with 3x3 SE with all ones. Compare the resulting image with the original image B. Justify your answer.
8) Given the following binary image B. Dilate this binary image with a 3x3 SE of all ones and subsequently erode this image with 3x3 SE with all ones. Compare the resulting image with the original image B. Justify your answer.