Filtering

Objective:

- **i)** To understand filtering in spatial and frequency domain
- **ii)** To get an idea of smoothening & sharpening of an image

Theory:

Filtering is a *neighborhood operation,* in which the value of any given pixel in the output image is determined by applying some algorithm to the values of the pixels in the neighborhood of the corresponding input pixel. A pixel's neighborhood is some set of pixels, defined by their locations relative to that pixel. Image processing operations implemented with filtering include smoothing, sharpening, removing noise, and edge detection.

A filter is defined by a kernel, which is a small array applied to each pixel and its neighbors within an image. The process used to apply filters to an image is known as convolution and may be applied in either the spatial or frequency domain. The filter masks are sometimes called convolution masks or convolution kernels.

1. **Spatial Filtering**

   It is the filtering operations that are performed directly on the pixels of an image. Spatial filtering involves passing a weighted mask or kernel over the image and replacing the original image pixel value corresponding to the center of the kernel with the sum of the original pixel values in the region corresponding to the kernel multiplied by the kernel weights.

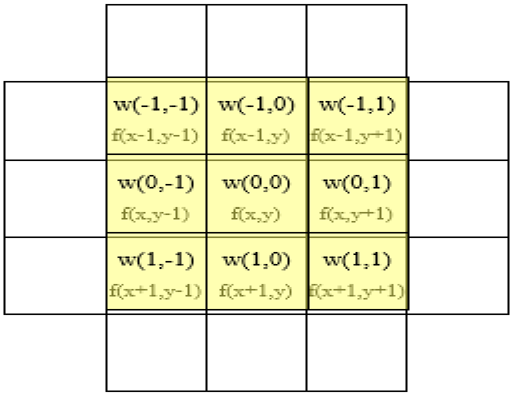
1.1 Low Pass Filtering

A low pass filter preserves the smooth region in the image and it removes sharp variations leading to blurring effect. The blurring effect will be more with the increase in the size of the mask. Normally, the size of the mask will be odd 3x3, 5x5, 7x7…

1.1.1 Linear Filtering:

Linear filtering is filtering in which the value of an output pixel is a linear combination of the values of the pixels in the input pixel's neighborhood. Linear filtering of an image is accomplished through an operation called *convolution*.

Convolution is a neighborhood operation in which each output pixel is the weighted sum of neighboring input pixels. The matrix of weights is called the *convolution kernel*, also known as the *filter.*



Pixel of an Image

w(-1,-1)	w(-1,0)	w(-1,1)
w(0,-1)	w(0,0)	w(0,1)
w(1,-1)	w(1,0)	w(1,1)

Kernel

The result is the sum of products of the mask coefficients with the corresponding pixels directly under the mask.

$$
\begin{aligned}
f(x,y) = \; & w(-1,-1)\,f(x-1,y-1) + w(-1,0)\,f(x-1,y) + w(-1,1)\,f(x-1,y+1) + \\
& w(0,-1)\,f(x,y-1) + w(0,0)\,f(x,y) + w(0,1)\,f(x,y+1) + \\
& w(1,-1)\,f(x+1,y-1) + w(1,0)\,f(x+1,y) + w(1,1)\,f(x+1,y+1)
\end{aligned}
$$

In general, linear filtering of an image f of size MxN with a filter mask of size mxn is given by the expression

$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t)\, f(x+s, y+t)$$

i) Mean or Averaging Filter

The mean filter replaces each pixel by the average of all the pixel values in the local neighbourhood. The 3 by 3 spatial mask which can perform the averaging operation is given by

$$\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

It is noted that the sum of the elements is equal to 1.

The general implementation for filtering a MxN image with a weighted averaging filter of size mxn is given by the expression

$$g(x,y) = \frac{\sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t)\, f(x+s, y+t)}{\sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t)}$$

Excersise:

1) Read an image, convolve the image with the 3x3 mask and show that it performs averaging operation which results in blurring of the image.

   Algorithm:
   i) Read an image
   ii) Create a 3x3 mask
   iii) Convolve image with a mask (hint: use *for loop* function)
   iv) Observe the output image

   Also, analyze the impact of increasing the size of the mask to 5x5.

MCQ's

1. Averaging filters is also known as ____________ filter.
   (a)Low pass (b)High pass (c)Band Pass (d) None of the mentioned

2. What is the undesirable side effects of Averaging filters?
   (a) No side effects (b) Blurred edges (c) Blurred image (d) Loss of sharp transitions

3. Which type of enhancement operations are used to modify pixel values according to the value of the pixel's neighbors?

4. Which of the following is best suited for salt-and-pepper noise elimination?
   (a) Average filter (b) Max filter (c) Box filter (d) Median filter

5. At which of the following scenarios averaging filters is/are used?
   (a) To reduce noise (b)In the reduction of irrelevant details in the image (c) to reduce sharp transition in grey levels (d) All of the mentioned

6. In linear spatial filtering, what is the pixel of the image under mask corresponding to the mask coefficient w (1, -1), assuming a 3*3 mask?
   (a) f (x, -y) (b) f (x + 1, y) (c) f (x, y – 1) (d) f (x + 1, y – 1)

(ii) Gaussian Filter

Gaussian flters are a class of linear smoothing filters with the weights chosen according to the shape of a Gaussian function. Guassian filter is very good filter for removing noise drawn from a normal distribution. The Guassain smoothing is a particular class of averaging, in which the kernel is a 2D Guassian.

In 2-D, an isotropic (*i.e.* circularly symmetric) Gaussian has the form:

$$G(x, y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

where σ is the standard deviation of the distribution which gives the degree of smoothening.

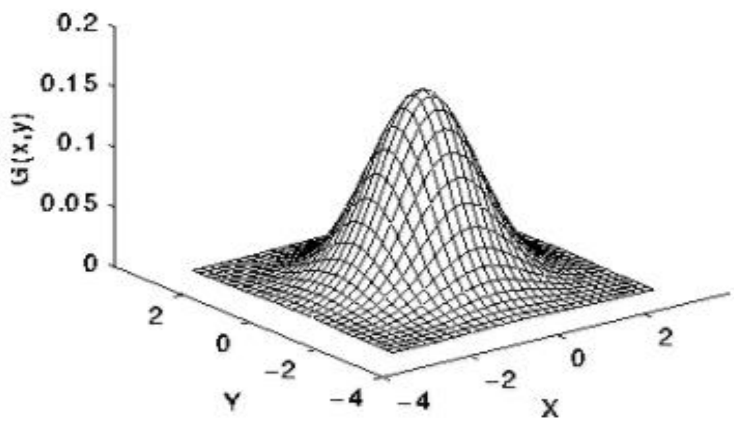


Fig 1: 2D Gaussian distribution with mean (0,0) and σ =1

A Gaussian kernel can be generated from PASCAL's triangle. Once a suitable kernel has been calculated, then the Gaussian smoothing can be performed using standard convolution methods. The 2-D convolution can be performed by first convolving with a 1-D Gaussian in the *x* direction, and then convolving with another 1-D Gaussian in the *y* direction.

Excersise:

1. Read an image and llustrate the effect of smoothing with successively larger and larger Gaussian filters (varyize of the mask & σ) (Hint: Use fspecial inbuilt function to get Gaussian mask)

MCQ's

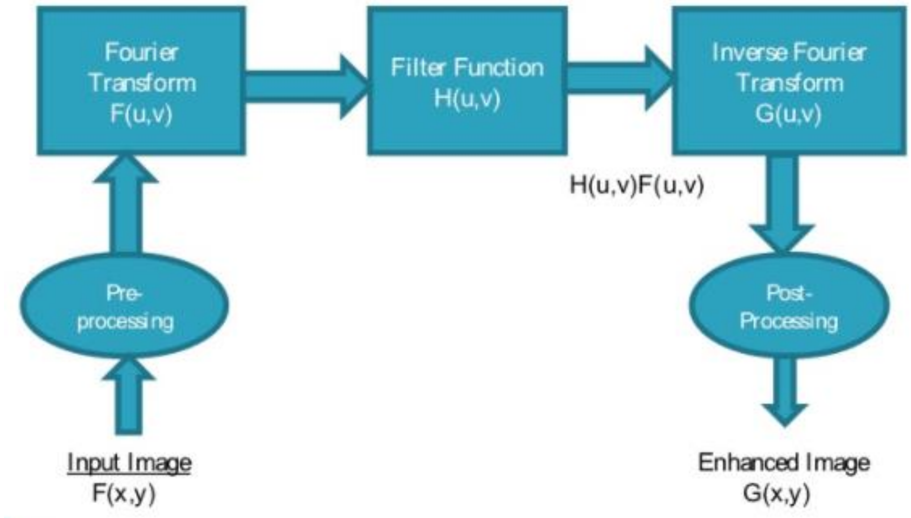
1. The standard deviation controls **___________** of the bell (2-D Gaussian function of bell shape).
(a) Size (b) Curve (c) Tightness (d) None of the Mentioned

2. An example of a continuous function of two variables is __________
(a) Intensity function (b) Contrast stretching (c) Gaussian function (d) None of the mentioned

3. What is required to generate an M X N linear spatial filter?
(a) MN mask coefficients (b) M+N coordinates (c) MN spatial coefficients (d) None of the mentioned

FILTERING IN THE FREQUENCY DOMAIN

1) Basic Process.



Methods

The general idea is that the image (***f(x,y)*** of size ***M*** x ***N***) will be represented in the frequency domain (***F(u,v)***). The equation for the two-dimensional discrete Fourier transform (DFT) is:

$$F(u,v) = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)e^{-i2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)}$$

The inverse of the above discrete Fourier transform is given by the following equation:

$$f(x,y) = \frac{1}{MN}\sum_{u=0}^{M-1}\sum_{v=0}^{N-1} F(u,v)e^{i2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)}$$

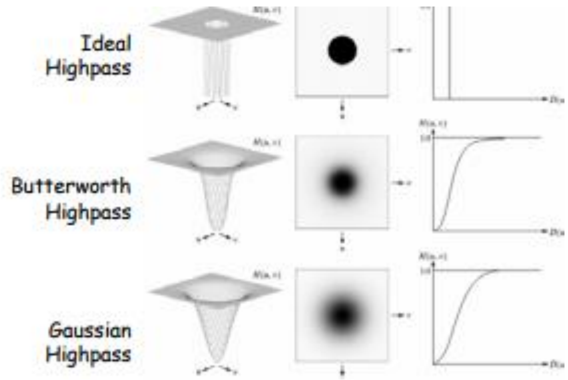
MATLAB has the function fft2 useful for images to compute the DFT.

Algorithm:

1) Convert image i into the frequency domain using fft function.(I)
2) Get the frequency response of the filter H.
3) Filtered image G=I x H
4) Get the filtered image in spatial domain g= ifft(G).

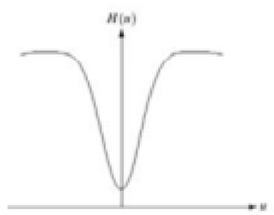
SHARPENING FILTERS

Image sharpening can be done by passing the image through any high pass filter.Two examples are given below.



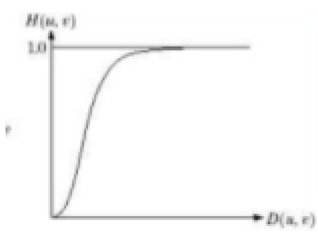
Order = n and cutoff frequency D_0.

a) Gaussian Filter



$$H(u,v) = 1 - e^{-D^2(u,v)/2D_0^2}.$$

b)Butterworth Filter



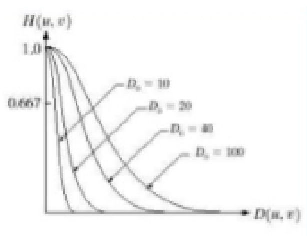
$$H(u,v) = \frac{1}{1 + [D_0 / D(u,v)]^{2n}}$$

Ideal	Butterworth	Gaussian
$H(u,v) = \begin{cases} 0 & \text{if } D(u,v) \le D_0 \\ 1 & \text{if } D(u,v) > D_0 \end{cases}$	$H(u,v) = \dfrac{1}{1 + [D_0/D(u,v)]^{2n}}$	$H(u,v) = 1 - e^{-D^2(u,v)/2D_0^2}$

SMOOTHING FILTERS

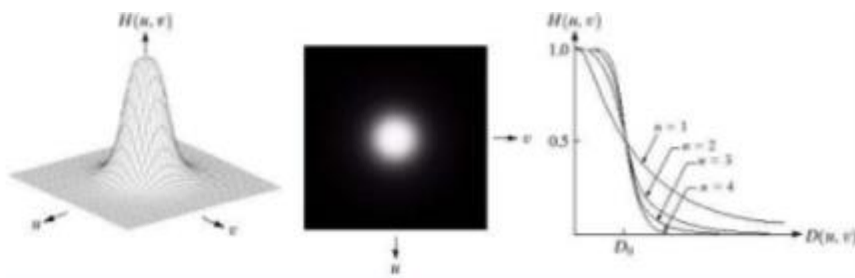
Smoothing is often used to reduce noise within an image or to produce a less pixelated image. Most smoothing methods are based on low pass filters.

1) Gaussian low pass filter



$$H(u,v) = e^{-D^2(u,v)/2D_0^2}.$$

2) Butterworth low pass filter



$$H(u,v) = \frac{1}{1+[D(u,v)/D_0\ ]^{2n}}$$

MCQ

1) Product of two functions in spatial domain is what, in frequency domain

A. correlation
B. convolution
C. Fourier transform
D. fast Fourier transform

2) High pass filters are used for image

a) contrast
b) sharpening

c) blurring
d) resizing

3) Low pass filters are used for image

a) contrast
b) sharpening
c) blurring
d) resizing

4) To remove "salt-and-pepper" noise without blurring we use

a) Max Filter

b) Median Filter

c) Min Filter

d) Smoothing Filter

5) Edge detection in images is commonly accomplished by performing a spatial ------of the image field.

a) Smoothing Filter

b) Integration

c) Differentiation

d) Min Filter

6) Both the -------- and -------- filters are used to enhance horizontal edges (or vertical if transposed).

a) Prewitt and Sobel

b) Sobel and Gaussian

c) Prewitt and Laplacian

d) Sobel and Laplacian

7) One of the following filters is nonlinear

a) Gaussian Filter

b) Averaging Filter

c) Laplacian Filter

d) Median

EXERCISE

a) i) Write a MATLAB program to corrupt a greyscale image with "SALT AND PEPPER" noise.
ii) Calculate PSNR between the original image and the corrupted image.
iii) Enhance the image using any 3 filter of your choice. Compare the results
iv) Calculate PSNR between the original image and the enhanced image, in each case.

b) Read a Grey scale image and enhance the edges using any of the filters.