Chapter

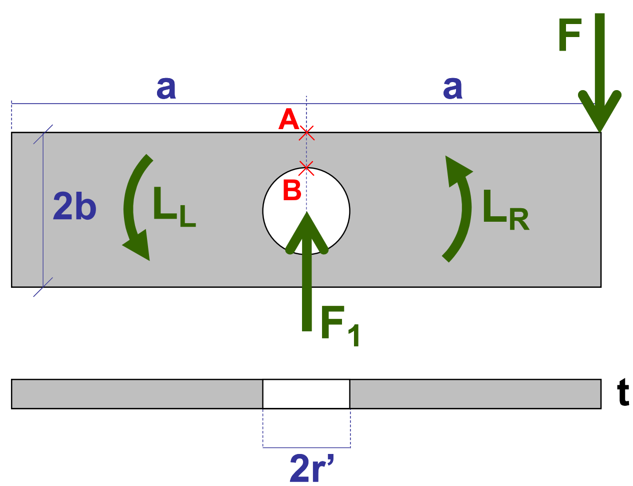
6

Weapon Design

In this chapter, we'll address several specific issues concerning weapon design. The idea is to show that mechanics calculations based on basic physics concepts and common sense can help a lot in the design of powerful and robust weapon systems.

6.1. Spinning Bar Design

It is not difficult to specify the dimensions of the bar of a spinner robot using basic stress analysis and a very simplified impact model. Consider the bar on the right, made out of hardened steel with length 2·a, width 2·b and thickness t, with a central hole of radius r'. During the impact, the angular momentum of the left and right hand sides of the spinning bar, L_L and L_R, will cause an average reaction force F from hitting the opponent, as seen in the figure. Since the bar is symmetric, it is easy to see that $L_L = L_R$.



If we assume that the chassis of the spinner robot is much heavier than its bar, we can say that the average reaction force F_1 from the weapon shaft is approximately equal to F, therefore $F_1 \cong F$ (we'll see later a better model that will allow different values for F_1 and F). We will also assume that the opponent is much heavier than the bar, and that the impact is inelastic, making the bar stop spinning after the brief time interval Δt of the impact. Therefore, the average torque F·a with respect to the weapon shaft, caused by the force F, must be able to bring the initial value of the angular momentum $(L_L + L_R)$ of the bar to zero during this Δt, resulting in $F \cdot a = (L_L + L_R) / \Delta t$, which gives $L_L / \Delta t = L_R / \Delta t = F \cdot a/2$. The average bending moment M_{max} in the middle of the bar, the region where bending is maximum, can then be calculated, $M_{max} = L_L / \Delta t = F \cdot a - L_R / \Delta t = F \cdot a/2$.

The stress at point A (see figure) due to bending is $\sigma_A = 3 \cdot M_{max} \cdot b/[2 \cdot (b^3 - r'^3) \cdot t]$. The stress at point B, theoretically, would be smaller than in A, however the hole acts as a stress raiser. It amplifies the stresses close to its border. In the geometry and loading of this example, it multiplies the stress by a factor of approximately 2, in other words, the stress concentration factor is 2 (this value is obtained from specific stress concentration factor tables [8]). Therefore, the stress acting at B is $\sigma_B = 2 \times 3 \cdot M_{max} \cdot r' / [2 \cdot (b^3 - r'^3) \cdot t]$.

If $\sigma_A > \sigma_B$, then if the bar breaks it will be from the outside in, beginning to fracture in A (where the stress is higher) and propagating a crack abruptly until point B. The bar will then break in two because the residual ligament on the other side of the hole (region below the hole in the figure) will be overloaded and break. All this happens in a split second – in metals, the fracture propagates at a speed of about 2 to 3km/s (1.24 to 1.86 miles per second), therefore a typical middleweight spinner bar would take about 0.01ms to fracture.

On the other hand, if $\sigma_B > \sigma_A$, the bar will break from the inside out, beginning fracturing from point B to point A. This was how the 5160 steel bar from our middleweight spinner *Ciclone* broke during the Winter Challenge 2005 competition, from B to A. That was because the diameter $2 \cdot r'$ of the hole of *Ciclone*'s bar was large with respect to its width $2 \cdot b$, penalizing point B.

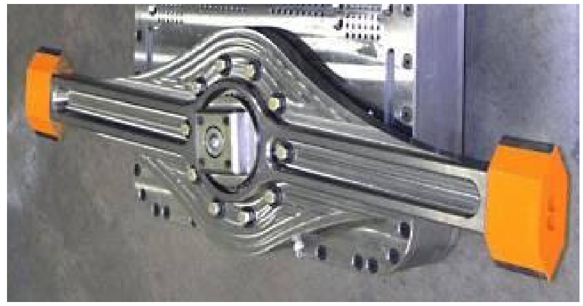
A good design choice would be to try to make point A at least as resistant as point B. For that, it is enough to equate $\sigma_A = \sigma_B$. After a little algebra with the previous expressions, we get $b = 2 \cdot r'$. Therefore, design your spinning bar (and the weapon shaft that will hold it) so that its width $2 \cdot b$ is at least twice the diameter $2 \cdot r'$ of its center hole. The bar from our middleweight spinner Titan was designed having this in mind.

And how much force would the bar support? Consider, for instance, $2 \cdot a = 1000$mm, $2 \cdot b = 80$mm, $2 \cdot r' = 2 \cdot b/2 = 40$mm, and the thickness $t = 12$mm. The steel bar, with average density $\rho = 7800 kg/m^3$, would have a mass of, approximately (without considering the hole), $\rho \cdot (2 \cdot a) \cdot (2 \cdot b) \cdot t = 7800 kg/m^3 \cdot 1m \cdot 0.080m \cdot 0.012m = 7.5kg$ (16.5lbs), which is a reasonable value for a middleweight – from the 30-30-25-15 rule, a middleweight would have 16.3kg (36lbs) for the weapon system, leaving in that example 16.3 – 7.5 = 8.8kg (more than 19lbs) for the weapon shaft, bearings, transmission and motor, an also reasonable value. A hardened steel with 45 Rockwell C (unit that measures how hard the material is, see chapter 3) tolerates a maximum stress of about $34 \times 45 = 1530 N/mm^2$ before breaking (this 34 factor is only valid for steels, estimating well the ultimate strength from the Rockwell C hardness).

Making both stresses at A and B equal to $1530 N/mm^2$, then $\sigma_A = \sigma_B = 3 \cdot M_{max} \cdot b / [2 \cdot (b^3 - r'^3) \cdot t] = 1530 N/mm^2$, where the bending moment $M_{max} = F \cdot a/2$, resulting in $F = 68{,}544N$, equivalent to almost 7 metric tons! Now it is necessary to guarantee that the weapon shaft and the rest of the robot can tolerate such average 7 tons, which can be made using the same philosophy presented above, from basic stress analyses. Approximate calculations can be very efficient if there is common sense and some familiarity with the subject.

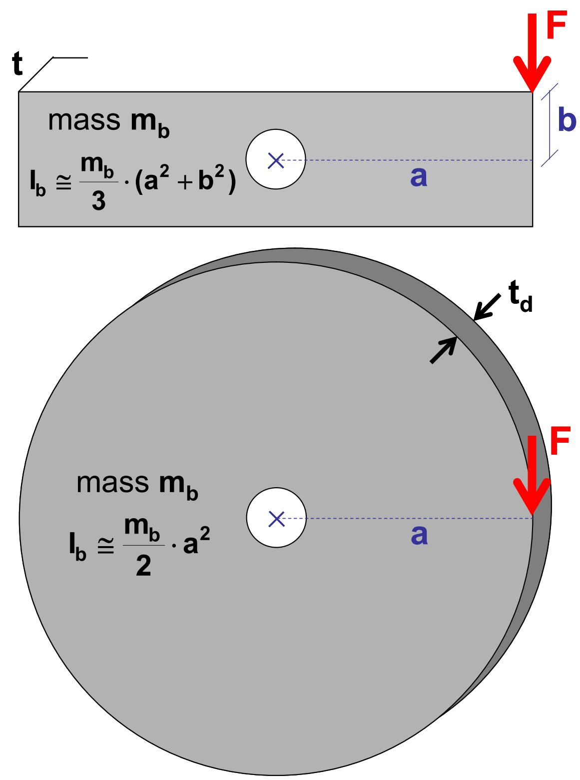
Clearly, bars with $b > 2 \cdot r'$ will result in even more strength, because a higher width $2 \cdot b$ will decrease both σ_A and σ_B values. But don't exaggerate, otherwise you'll have to decrease too much its thickness t not to go over the weight limit, compromising the strength in the out-of-plane bending direction.

Another solution would be to have a variable width bar, with a wider middle section, as pictured to the right. Note how the bar shape is optimized, with an increasingly wider middle section to withstand high bending moments, and sharp and heavy inserts at its tips to guarantee a high moment of inertia in the spin direction. Note also the ribs milled in the bar to increase its bending strength without adding too much weight.



6.2. Spinning Disk Design

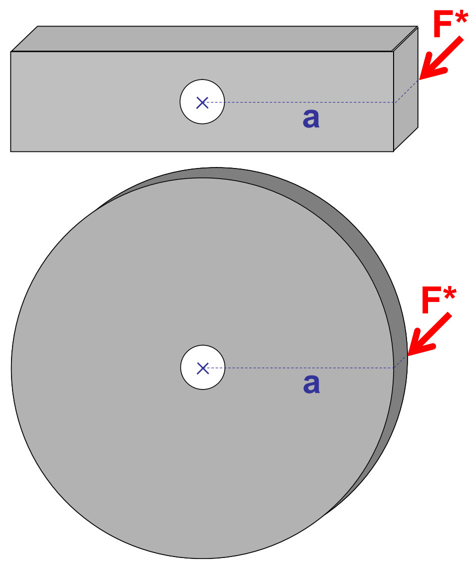
There has always been a great debate whether bars or disks make the best spinning weapons. Consider that the robot design allows a spinning weapon with mass m_b, and its reach from the weapon shaft must have a length a. Then let's compare a bar with length $2 \cdot a$ to a disk with radius a, pictured to the right. Both weapons, with same mass m_b, would be originally spinning until suffering an impact force F from hitting the opponent. If made out of the same material with a mass density ρ, then the thicknesses t and t_d of the bar and disk would be approximately $t \cong m_b/(\rho \cdot 4 \cdot a \cdot b)$ and $t_d \cong m_b/(\rho \cdot \pi \cdot a^2)$.



Concerning moment of inertia (I_b), it is easy to see from the values in the figure that the disk is a better choice, unless the half-width b of the bar is very large, above $0.707 \cdot a$. The moment of inertia of a narrow bar (with b much smaller than a) would be roughly 66.7% of the value of a disk with same mass m_b and length a.

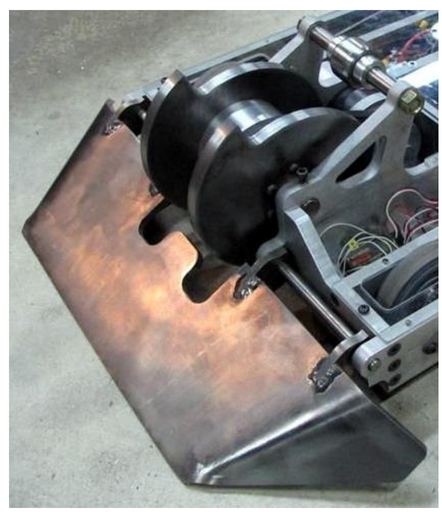
Let's take a look now at the stresses. If the width $2 \cdot b$ of the bar is much higher than twice the diameter of the center hole, then the maximum stress due to the force F is approximately $\sigma_{bar} \cong 3 \cdot F \cdot a/(4 \cdot t \cdot b^2)$. And assuming the disk diameter is much larger than its hole diameter, then the maximum stress would be $\sigma_{disk} \cong 3 \cdot F/(4 \cdot t_d \cdot a)$. It is easy to show from these equations and the expressions for t and t_d that, assuming b smaller than a, any bar would see higher maximum stresses than the disk.

So, disks are a better choice concerning both stresses (while delivering an impact) and moment inertia. But they have a major drawback. If a vertical disk is hit by a horizontal spinner, or a horizontal disk is hit by a drum or vertical spinner, it will see a force F* perpendicular to its plane, as pictured to the right, which will cause a maximum out-of-plane bending stress of approximately $\sigma_{disk}^* \cong 3 \cdot F^*/t_d^2$. The same perpendicular force would cause on the bar a maximum out-of-plane bending stress of $\sigma_{bar}^* \cong 3 \cdot F^* \cdot a/[(b - r') \cdot t^2]$, where r' is the radius of the hole. It is easy to show that any disk would result in higher maximum out-of-the-plane stresses than a bar.



For instance, a bar with b = 2·r' would only see $\pi^2/32 \cong 31\%$ of the stresses found on an equivalent disk. The above equations, together with estimates for F and F*, are very useful to find out values for the bar width 2·b that will meet requirements for maximum allowable stresses σ_{bar} and σ_{bar}^*, as well as to decide whether a disk would be an acceptable option despite its high resulting σ_{disk}^*.

Therefore, we conclude that horizontal bars are a better choice than horizontal disks against drums, vertical spinners, and wedges (which can deflect a hit and also cause high out-of-plane bending stresses). Horizontal disks would be better against all other types of opponents, due to their higher in-plane bending strength and also higher moment of inertia. And vertical disks would be a better choice than vertical bars against most robots, except against horizontal spinners, which will most likely warp or break the disk with a powerful out-of-plane hit. You can get away with a vertical disk against a horizontal spinner, but you should either limit the disk radius (having a lower radius a to decrease σ_{bar}^*), or protect it against out-of-plane hits having it recessed into the chassis or using a wedge, as seen in the lightweight K2 on the right.



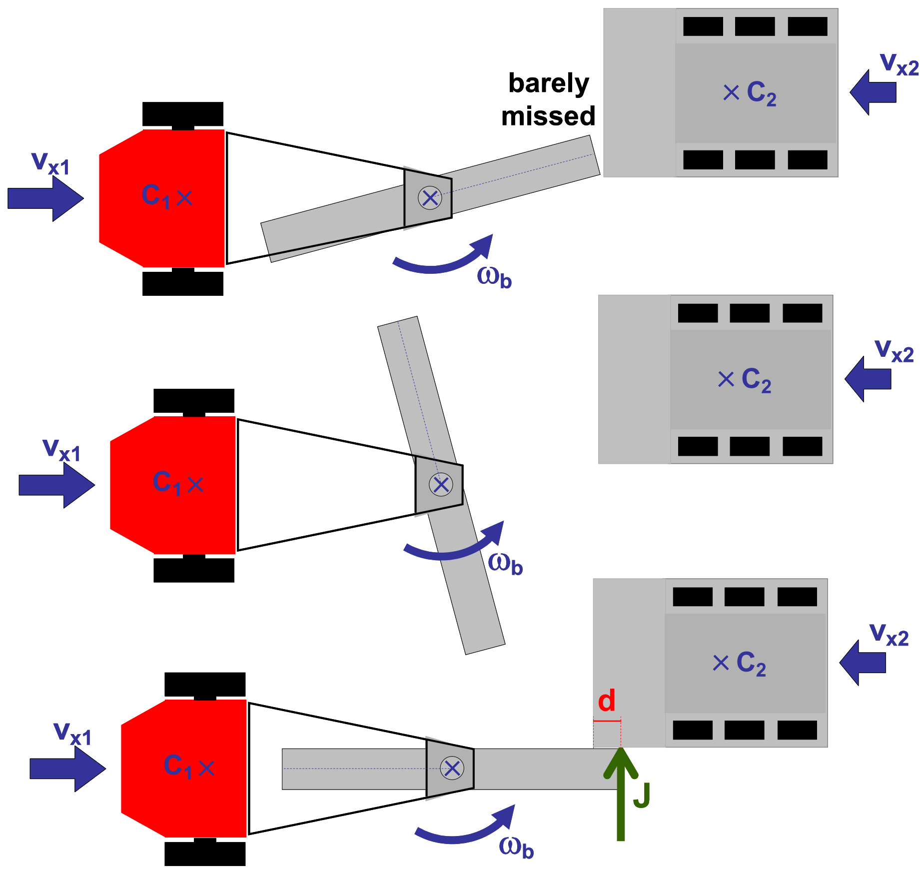
Note that the calculations above assumed solid bars and disks, without considering any shape optimization. But the conclusions would still hold if comparing an optimized bar to an optimized disk. Shape optimization can also generate hybrids between disks and bars, trying to get the best of both worlds. The drum teeth from the middleweight Angry Asp (pictured to the right) are a good example of that, with their wide disk-like mid-section and elongated bar-like overall shape.

6.3. Tooth Design

One important issue when designing spinning weapons such as disks, bars, drums and shells is regarding the number of teeth and their height. Too many teeth on a spinning disk, for instance, will make the spinner chew out the opponent instead of grabbing it to deliver a full blow. Everyone who's used a circular saw knows that fewer teeth means a higher chance of the saw binding to the piece being cut, which is exactly what we want in combat.

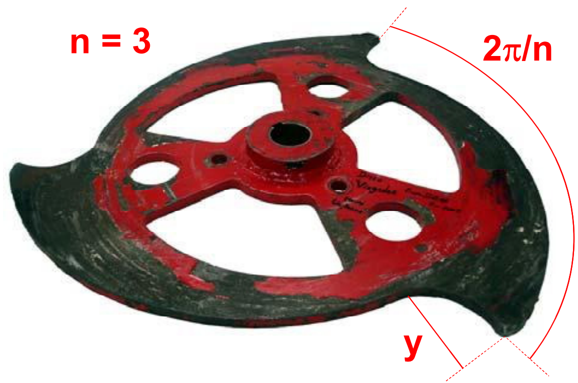
6.3.1. Tooth Height and Bite

Before we continue this analysis, we need to define the tooth bite d. The tooth bite is a distance that measures how much the tips/teeth of the spinner weapon will get into the opponent before hitting it. For instance, if two robots are moving towards each other with speeds v_{x1} and v_{x2}, one of them having a bar spinning with an angular speed ω_b (in radians per second), as pictured to the right, then the highest bite d = d_{max} would happen if the bar barely missed the opponent before turning 180 degrees to finally hit it. The time interval the bar takes to travel 180° (equal to π radians) is $\Delta t = \pi/\omega_b$, during which both robots would approach each other by $d_{max} = (v_{x1}+v_{x2})\cdot\Delta t = (v_{x1}+v_{x2})\cdot\pi/\omega_b$. So, the tooth bite d could reach values up to d_{max}.



Small values for d mean that the spinner will have a very small contact area with the opponent, most probably chewing its armor instead of binding and grabbing it. So, a spinner needs to maximize d to deliver a more effective blow. This is why an attack with the drive system at full speed is more effective, since a higher speed v_{x1} will result in a higher d. And this is why very fast spinning weapons have a tough time grabbing an opponent, their very high ω_b ends up decreasing the tooth bite d.

The maximum obtainable tooth bite $d = d_{max}$ can also be generalized for a weapon with n teeth. In this case, the teeth are separated by $2 \cdot \pi / n$ radians, as pictured to the right, resulting in $\Delta t = 2 \cdot \pi / (n \cdot \omega_b)$, and therefore $d_{max} = (v_{x1}+v_{x2}) \cdot \Delta t = (v_{x1}+v_{x2}) \cdot 2 \cdot \pi / (n \cdot \omega_b)$. Since the tooth bite cannot be higher than d_{max}, there is no reason to make the tooth height $y > d_{max}$ (see picture), which would decrease its strength due to higher bending moments. Therefore, the optimal value for the tooth height y is some value $y < (v_{x1}+v_{x2}) \cdot 2 \cdot \pi / (n \cdot \omega_b)$.



Using the maximum values of both v_{x1} and v_{x2} speeds will probably result in large values for y, so it is a reasonable idea to assume $v_{x2} = 0$. Most attacks will happen at full speed v_{x1} but without the opponent moving towards you. Besides, a spinner doesn't know beforehand the value of v_{x2} of all of its possible opponents. So, a tooth height $y = v_{x1,max} \cdot 2 \cdot \pi / (n \cdot \omega_{b,max})$ is usually more than enough. Note that this height assumes the weapon at full speed, if you want to deal with lower ω_b speeds before it fully accelerates then the value of y should be increased accordingly.

The tooth height calculated above can still be reduced if necessary without compromising much the tooth bite d. This is because the above estimates assumed that one tooth barely misses the opponent, until the next tooth is able to grab it at a distance d. But, if instead of barely missing the opponent, the previous tooth had barely hit it, it would have hit it with a distance much smaller than d. It is a matter of probability, the tooth bite can be any value between 0 and d, with equal chance (constant probability density). So, in 50% of the attacks at full speed the travel distance d will unluckily be between 0 and $d_{max}/2$, and in the other 50% it will luckily be between $d_{max}/2$ and d_{max}. An (unlucky) hit with d very close to zero probably won't grab the opponent, and it will significantly reduce the attacker speed v_{x1} until the next tooth is able to turn $2 \cdot \pi / n$ radians, decreasing the distance d of subsequent hits. If v_{x1} gets down to zero without grabbing the opponent, you'll probably end up grinding it. If this happens, the best option is to back up, and then charge again trying to reach $v_{x1,max}$ and hoping for a high d.

The chance of d being exactly d_{max} is zero, because it is always smaller than that, so if you want you can make the tooth height $y < d_{max}$. If you choose, for instance, $y = d_{max}/2 = v_{x1,max} \cdot \pi / (n \cdot \omega_{b,max})$, your robot won't notice any difference with this lower height in 50% of the hits, when $d < d_{max}/2$, while on the other 50% (where d would be higher than $d_{max}/2$) the opponent will touch the body of the drum/disk before being hit by a tooth, resulting in $d = y = d_{max}/2$. As long as this $d_{max}/2$ value is high enough to grab the opponent instead of grind it, it is a good choice.

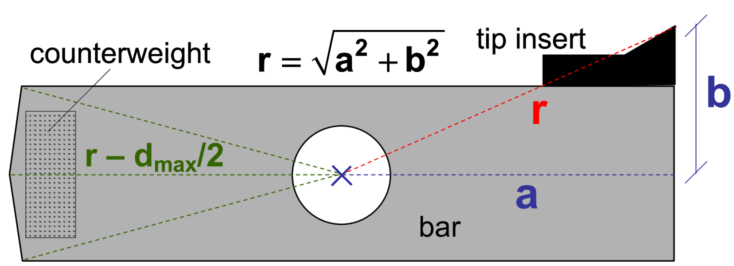
For instance, the 2008 version of our featherweight Touro Feather had a drum with n = 2 teeth (pictured to the right) spinning up to $\omega_{b,max}$ = 13,500 RPM (1413.7 rad/s). Since the robot top speed is $v_{x1,max}$ = 14.5mph (equal to 23.3km/h or

6.48m/s), then $d_{max} = 6.48 \cdot 2 \cdot \pi/(2 \cdot 1413.7) \cong 0.014$m = 14mm. Since the overall height of the drum needed to be smaller than 4" by design, a tooth height y = 14mm would result in a drum body with low diameter. We then chose y = 10mm for the tooth to stick out of the drum body. This 10mm height is usually enough to grab an opponent. Also, in 10mm/14mm = 71% of the hits at full speed, the tooth height y will be higher than the tooth bite d. The opponent will only touch the drum body in the remaining 29% of the hits, when the next tooth will be able to hit the opponent with its full 10mm height (unless the opponent had bounced off immediately after hitting the drum body).

But beware with a frontal collision between two vertical spinning weapons, because the opponent may be able to grab your drum or disk body with its teeth before you can grab it. In this case, it is a game of chance. The robot with higher teeth will have a better chance of grabbing the opponent, as long as it spins fast enough. Since a vertical spinning bar does not have a round inner body, it basically behaves as if its "tooth height" y was equal to the bar radius. So, usually a powerful vertical bar will have an edge in weapon-to-weapon hits against drums or vertical disks.

6.3.2. Number of Teeth

An important conclusion from the previous analyses is that you must aim for a minimum number of teeth, n. The lower the n, the higher the value of d. Disks with n = 3 or more teeth are not a good option. The best choice is to go for n = 2, as with bars or two-toothed disks. Even better is to try to develop a one-toothed spinning weapon, such as the disk of the vertical spinner Professor Chaos, but this requires a careful calculation to avoid unbalancing by using, for instance, a counterweight diametrically opposite to that tooth.



Note that a one-toothed weapon does not have to be too much asymmetric, nor will it need heavy counterweights, if you do your math right. For instance, the one-toothed bar pictured to the right can be made out of a symmetrical bar, as long as the short end is chamfered to reach a maximum radius $r - d_{max}/2$, where r is the effective radius of the long end including the insert, calculated from a and b as shown in the figure, and the maximum tooth bite d_{max} is calculated for n = 1 tooth. In this way, with the bar at full speed, even if the long end barely misses the opponent, the short end won't touch it because during a half turn it would approach at most half of d_{max}. After the full turn it would have approached up to d_{max}, hitting for sure with the long end. With such n = 1, it is possible to move twice as much into the opponent before hitting it, transferring more impact energy.

With this proposed one-toothed bar geometry, the counterweight wouldn't have to be much heavy, because its mass would only have to account for the mass of the tip insert plus the removed mass from the chamfers. This bar is also relatively easy to fabricate, with very little material loss. In fact, for wide bars with large inserts, which increase the value of b, it is even possible to design the bar such as $a = r - d_{max}/2$, making it almost symmetrical even after chamfering. In addition, if you perform some shape optimization removing some material from the long end, it is even possible to remove the counterweight, but be careful not to compromise the bar strength at its most stressed region.

In our experience, to bind well to the opponent, the tooth bite should not be below 1/4", no matter if the robot is a hobbyweight or a super heavyweight. We've tested different tooth heights with our drumbot hobbyweight Touro Jr and featherweight Touro Feather, and values below 1/4" made the robot grind instead of grab the used deadweights. With this in mind, it is possible to generate a small table with estimated maximum weapon speeds to avoid the grinding problem. We only need to make sure that in at least 50% of the hits at full speed the tooth will be able to travel at least 1/4" (0.00635m), thus $\omega_{b,max}$ can be found from 1/4" = $d_{max}/2 = v_{x1,max} \cdot \pi / (n \cdot \omega_{b,max})$, see the table to the right. Of course these are just rough estimates, because tooth sharpness and armor hardness also play a hole helping or avoiding dents that bind with the opponent.

number of teeth n	drivetrain speed $v_{x1,max}$	maximum $\omega_{b,max}$ to avoid grinding
3	5mph (8km/h)	3520RPM
	10mph (16km/h)	7040RPM
	15mph (24km/h)	10560RPM
2	5mph (8km/h)	5280RPM
	10mph (16km/h)	10560RPM
	15mph (24km/h)	15840RPM
1	5mph (8km/h)	10560RPM
	10mph (16km/h)	21120RPM
	15mph (24km/h)	31680RPM

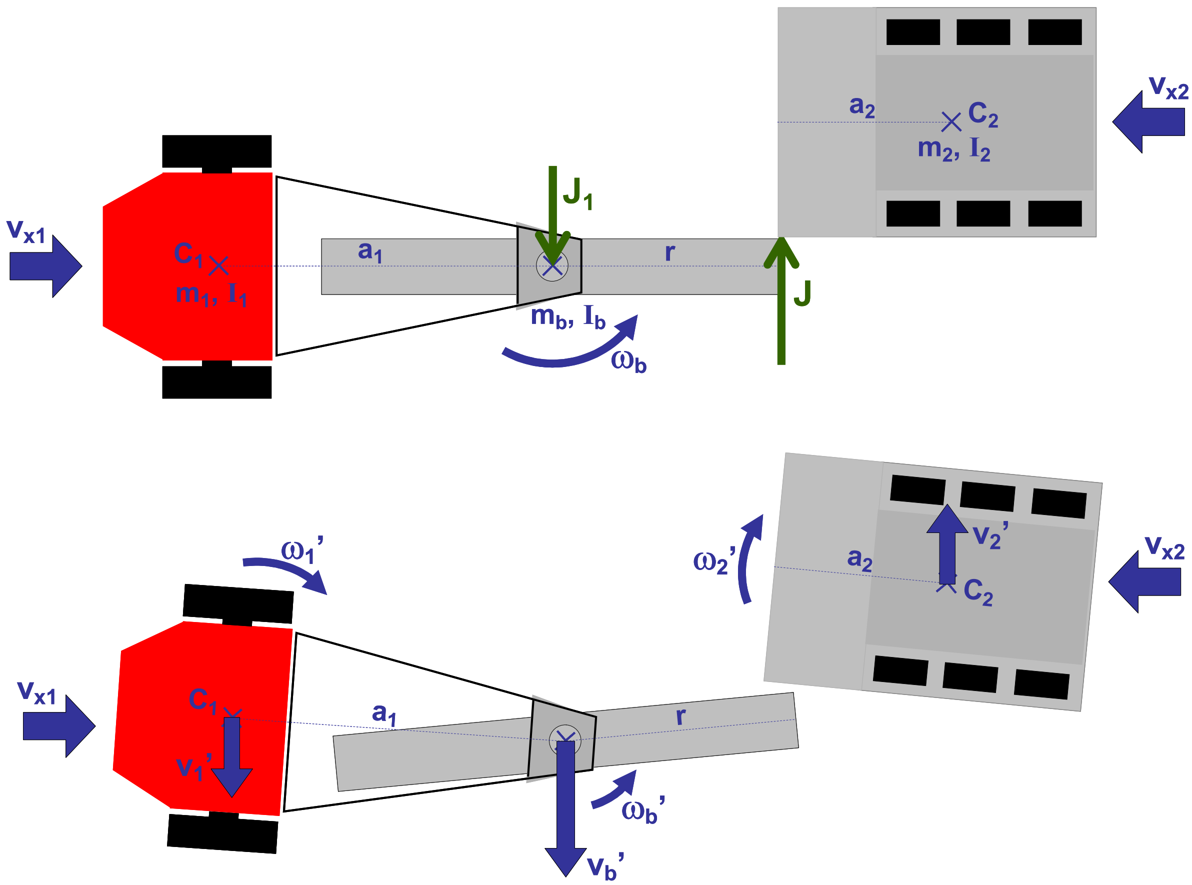
6.4. Impact Theory

In the previous sections, we've used very simplified models to describe the impact of a spinner weapon on another robot. We'll extend these models here, to get a deeper understanding of the physics behind these impacts, and hopefully design a better spinner.

6.4.1. Impact Equations

We'll consider the problem of a bar spinner hitting a generic opponent (pictured in the next page), during the impact and right after it. The spinning bar has a length $2 \cdot r$, a mass m_b and moment of inertia I_b with respect to its center in the spin direction. It is initially spinning with an angular speed ω_b. The chassis of the spinner robot, without its bar, has a mass m_1 and a moment of inertia I_1 in the spin direction with respect to the chassis center of mass C_1. The opponent has mass m_2 and moment of inertia I_2 in the spin direction with respect to its center of mass C_2. We'll assume that the opponent does not have vertical spinning weapons, which could cause gyroscopic effects.

We'll assume that the impact impulse J that the bar inflicts on the opponent is in a direction perpendicular to their approach speed ($v_{x1} + v_{x2}$), and the distance between C_2 and the vector J is a_2. If the impact force was constant during the time interval Δt of the impact, then the impulse J could be simply calculated multiplying this force times Δt, otherwise we'd have to integrate the force over time to get J. The bar will also generate a reaction impulse J_1 over the weapon shaft of the spinner. This impulse J_1 is at a distance a_1 from the chassis center of mass C_1. The distance a_1 is usually greater than r for an offset spinner when hitting as shown in the picture, or very close to zero for traditional spinners that have their weapon shaft close to C_1.



The picture also shows the moment right after the impact, where the opponent will gain a speed v_2' in the direction of J, when it will start spinning with an angular speed ω_2'. Note that both initial speeds v_{x1} and v_{x2} remain unchanged, because we assumed the impulse J perpendicular to them (we're implicitly assuming that there is no friction during the impact). The bar ends up with a slower angular speed ω_b' after the impact, while its center moves with a speed v_b' as a result of the reaction impulse J_1. The spinner robot chassis will gain a speed v_1' in the direction of J_1, and it will start spinning with an angular speed ω_1'. Note that ω_b' is measured with respect to the arena, and not with respect to the (now spinning) chassis.

If we assume that no debris is released from either robot during the impact, that the spinner bar has a perfect clutch system (which does not transmit any torque during the impact), and that the opponent does not have any spinning weapons that might cause some gyroscopic effect (studied later in this chapter), then basic physics equations of conservation of linear and angular momentum can show that

$$v_2' = \frac{J}{m_2}, \quad \omega_2' = \frac{J \cdot a_2}{I_2}, \quad v_1' = \frac{J_1}{m_1}, \quad \omega_1' = \frac{J_1 \cdot a_1}{I_1}, \quad v_b' = \frac{J - J_1}{m_b}, \text{ and } \omega_b' = \omega_b - \frac{J \cdot r}{I_b}$$

To find the values of J and J_1, we need to know the coefficient of restitution (COR) of the impact, defined by e, with $0 \le e \le 1$. The COR is the relative speed between the bar and the opponent after the impact divided by the relative speed before the impact. A purely elastic impact, where no energy is dissipated, would have e = 1. A purely inelastic impact, where a good part of the impact energy (but not all) is dissipated, would have e = 0. All other cases would have 0 < e < 1. Note that this dissipated energy is not only due to damage to the opponent in the form of plastic deformation or fracture, it also accounts for absorbed energy by the opponent's shock mounts, vibrations, sounds, and even damage to the weapon system of the spinner.

For very high speed impacts, as we see in combat, the COR e is usually close to zero, simply because no material can absorb in an elastic way all the huge impact energy. For instance, a bullet (taken from its cartridge) might have up to e = 0.9 when dropped from a 1" height against a metal surface. But the very same bullet, when fired to hit the same surface at very high speeds, will plastically deform and probably become embedded into the surface, resulting in e = 0. So, the value of e depends not only on the materials involved, but also on the impact speed.

In this problem, the approach speed before the impact is only due to the speed of the tip of the spinning bar, namely $v_{tip} = \omega_b \cdot r$. The relative speed between the bar tip and the opponent right after the impact (the departure speed) is due to several terms, such as the linear and angular speeds of the spinner and opponent robots, as well as the remaining angular speed of the bar, resulting in

$$e = \frac{v_{departure}}{v_{approach}} = \frac{v_2' + \omega_2' \cdot a_2 + v_1' + \omega_1' \cdot a_1 - \omega_b' \cdot r}{\omega_b r}$$

The speed v_b' of the center of the bar can be obtained from the linear and angular speeds of the spinner chassis, resulting in $v_b' = v_1' + \omega_1' \cdot a_1$. Solving all the previous equations, we finally obtain the values of the impulses J and J_1

$$J = M \cdot (1+e) \cdot v_{tip} \qquad \text{and} \qquad J_1 = \frac{J \cdot I_1 m_1}{I_1 m_1 + m_b \cdot (I_1 + m_1 a_1^2)}$$

where M is the effective mass of both robots, obtained from the effective masses M_1 from the spinner and M_2 from the opponent, namely

$$\frac{1}{M} = \frac{1}{M_1} + \frac{1}{M_2}, \quad \text{where} \quad \frac{1}{M_1} = \frac{1}{m_b + m_1 \cdot \dfrac{I_1}{I_1 + m_1 a_1^2}} + \frac{r^2}{I_b} \quad \text{and} \quad \frac{1}{M_2} = \frac{1}{m_2} + \frac{a_2^2}{I_2}$$

With these values of J and J_1, we can now calculate all the speeds after the impact. For instance, let's check a few limit cases to better understand the equations.

6.4.2. Limit Cases

If the spinner, instead of hitting an opponent, hits a very light debris very close to its center of mass (therefore $a_2 \cong 0$), then the debris has an effective mass $M_2 = m_2$. Since m_2 is much smaller than m_1 and m_b, we find that M_2 is much smaller than M_1, which leads to $M \cong M_2$. So, the effective mass of the entire system is only $M = m_2$, resulting in a small impulse $J = m_2 \cdot (1+e) \cdot v_{tip}$ that will accelerate the debris to a speed $v_2' = J/m_2 = (1+e) \cdot v_{tip}$.

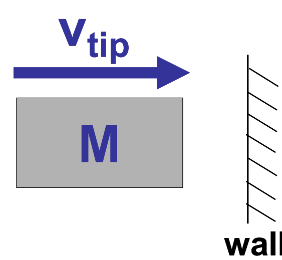
This means that if the debris is a little lump of clay (inelastic impact with $e \cong 0$), it will be thrown with basically the same speed v_{tip} of the bar tip. On the other hand, if it is a very tough rubber ball that won't burst due to the impact, its $e \cong 0.8$ will allow it to be thrown at 1.8 times the speed of the bar tip. Also, since m_2 is very small, the equations predict that the speeds of the spinner robot will almost remain unchanged, which makes sense since very little energy was transferred to the debris.

The other limit case is the spinner hitting a very heavy arena wall. The wall is so much heavier than the robot that we can assume that $m_2 \rightarrow \infty$ and $I_2 \rightarrow \infty$, resulting in $M_2 \rightarrow \infty$ and therefore the system effective mass is $M \cong M_1$, the effective mass of the spinner robot. This will result in the maximum impact that the spinner can deliver, $J = M_1 \cdot (1+e) \cdot v_{tip}$. This value is twice the impact that would be delivered to an opponent with $M_2 = M_1$. This is why it is a much tougher test to hit an arena wall than an opponent with similar mass. And, of course, the equations will tell that the speeds of the arena after the impact will be approximately zero.

6.4.3. Impact Energy

Before the impact, we'll assume that the attacking robot (such as a spinner) will have an energy E_b stored in its weapon. For the spinner impact problem presented above, $E_b = I_b \cdot \omega_b^2 / 2$. The impact usually lasts only a few milliseconds, but it can be divided into two phases: the deformation and the restitution phases.

In the deformation phase, a portion E_d of the stored energy E_b is used to deform both robots (such as bending the spinner bar or compressing the opponent's armor), while the remaining portion E_v is used to change the speeds of both robots and weapon. It is not difficult to prove using the presented equations that $E_d = M \cdot v_{tip}^2 / 2$ for the spinner impact problem. Interestingly, this would be the deformation energy E_d that a mass M with a speed v_{tip} would generate if hitting a very heavy wall, as pictured to the right. So, the higher the effective mass M, the higher the E_d. We'll see later in this chapter how an attacking robot can manage to maximize M to increase the inflicted damage to the opponent.



After the deformation reaches its peak, the restitution phase starts. A portion E_k of the deformation energy E_d was stored as elastic deformation, which is then retrieved during the restitution phase to change even more the speeds of both robots. The remaining portion E_c of E_d (where $E_d = E_k + E_c$) is the dissipated energy, transformed into permanent deformations, fractures, vibration, noise, as well as damped by the robot structure and shock mounts. We can show that, for an impact with COR equal to e, $E_k = E_d \cdot e^2$ and $E_c = E_d \cdot (1-e^2)$.

So, a perfectly elastic impact ($e = 1$) would have no dissipated energy ($E_c = 0$), and a perfectly inelastic impact ($e = 0$) would dissipate all its deformation energy ($E_c = E_d$). Note that inelastic impact does not mean that the entire energy of the system (which originally is E_b) is dissipated, it only means that the portion E_d is completely dissipated.

In summary, $E_b = E_v + E_d = E_v + E_k + E_c$, where the energy $(E_v + E_k)$ will account for the changes in linear and angular speeds of the robots and weapon, and E_c will be dissipated.

6.4.4. Example: Last Rites vs. Sir Loin

Let's solve the impact equations for an example inspired on the heavyweight match between the offset spinner Last Rites (pictured to the right) and the eggbeater-drumbot Sir Loin, at RoboGames 2008. The mass of each robot is assumed as $m = m_1 + m_b = m_2 = 220$lb. We'll estimate the weight of the bar with the steel inserts as $m_b = 44$lb $= m/5$, therefore $m_1 = 220 - 44 = 176$lb $= 4 \cdot m/5$. The bar is assumed to have a length $2 \cdot r = 40$", spinning at $\omega_b = 2000$RPM (209.4 rad/s) before the impact, with an offset length $a_1 = 30$". If the opponent robot is assumed to have a square shape with side length $2 \cdot a_2 = 30$", then the value of a_2 for the studied impact situation is about $a_2 = 15$".

The speed of the bar tip is $v_{tip} = \omega_b \cdot r = 209.4$rad/s $\cdot$ 20" = 106.4m/s (equal to 383km/h or 238mph). The moment of inertia of the bar is approximated as $I_b = m_b \cdot r^2/3 = 5867$lb·in^2 (1.72kg·m^2). The moment of inertia I_2 of the second robot is roughly estimated assuming it is a 30" square with uniform density, as seen from above, resulting in $I_2 = 220$lb·2·15"^{2}/3 = 33,000lb·in^2. The value of I_1 for the bar spinner chassis is roughly estimated as $I_1 = I_2 \cdot m_1/m = I_2 \cdot 4/5 =$ 26,400lb·in^2. The effective mass of both robots is then

$$M_1 = \{\frac{1}{44 + 175 \cdot \frac{26400}{26400 + 175 \cdot 30^2}} + \frac{20^2}{5867}\}^{-1} = 12.1 \text{ lb} \quad \text{and} \quad M_2 = \{\frac{1}{220} + \frac{15^2}{33000}\}^{-1} = 88 \text{ lb}$$

and the effective mass of the system is $M = \{M_1^{-1} + M_2^{-1}\}^{-1} = 10.64$lb (4.825kg). So, even though the spinning bar had a kinetic energy of $E_b = I_b \cdot \omega_b^2 / 2 = 1.72$kg·m^2·(209.4rad/s)^2 / 2 = 37,654J, the deformation energy involved in the impact (distributed to both robots) is only $E_d = M \cdot v_{tip}^2 / 2 =$ 4.825kg·(106.4m/s)^2 / 2 = 27,309J (72.5% of E_b).

If the impulse vector was aligned with the center of mass C_2 of the other robot, then the distance a_2 would be equal to zero. In this case, the effective mass M_2 would be much higher, equal to the robot mass $m_2 = 220$lb. Despite this much higher M_2, the effective M would not increase too much, resulting in $M = \{M_1^{-1} + M_2^{-1}\}^{-1} = 11.47$lb (5.202kg) and $E_d = 29{,}445$J.

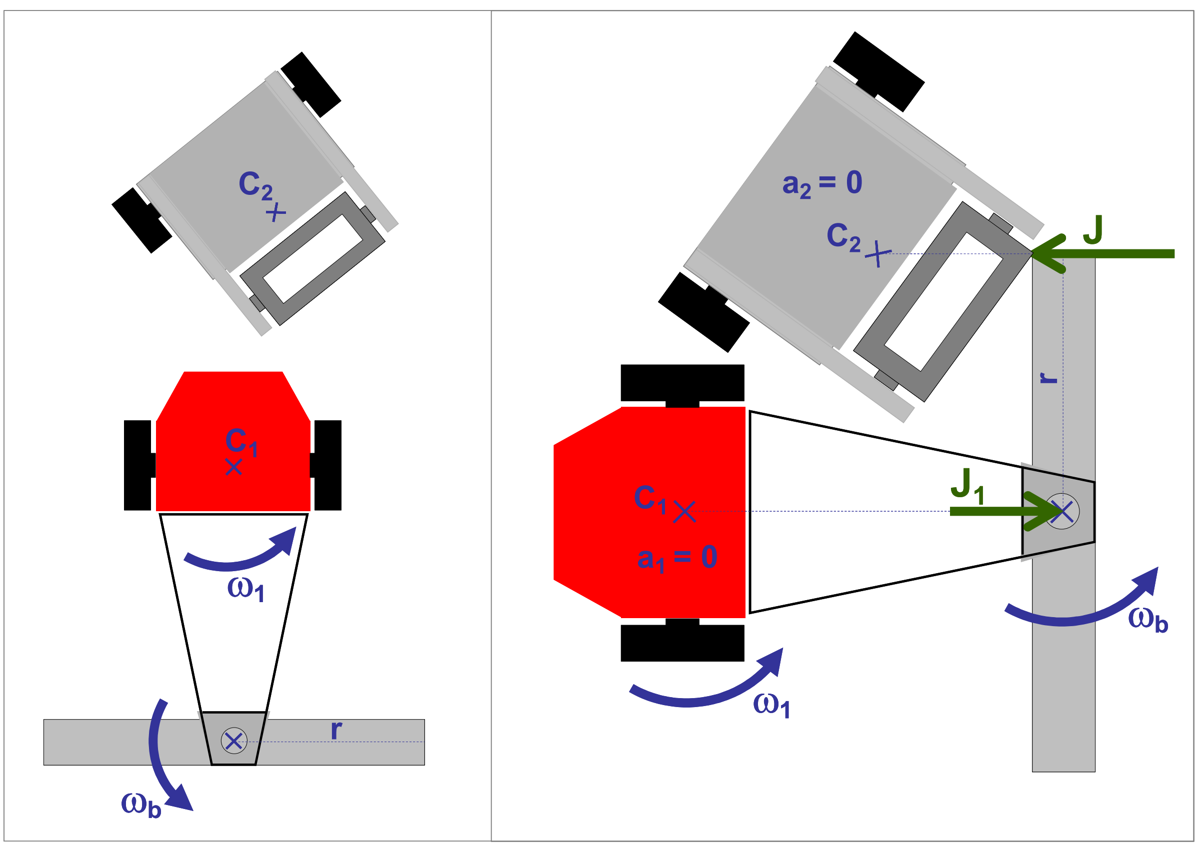
For offset spinners such as Last Rites or The Mortician, there's a way to increase even more the deformation energy caused by the impact. For frontal impacts, part of the energy of its bar is wasted making the offset spinner gain an angular speed ω_1', as shown in the next picture for The Mortician. To avoid that, the impulse J should be parallel to the line joining the chassis center of mass C_1 and the center of mass of the bar. In this case, the resulting impulse vector J_1 on the weapon shaft would be aligned with C_1, therefore its distance to C_1 would be $a_1 = 0$.

This configuration can be achieved with a special maneuver adopted by Ray Billings, the builder and driver of Last Rites. Last Rites starts facing away from the opponent, and then it turns 90 degrees to hit in the way shown below. In the best case scenario (for Last Rites), the impulse J_1 would be aligned with C_1, making $a_1 = 0$, and J would be aligned with C_2, resulting in

$$M_1 = \{\frac{1}{44+175} + \frac{20^2}{5867}\}^{-1} = 13.7 \text{ lb} \quad \text{and} \quad M_2 = \{\frac{1}{220} + \frac{0^2}{33000}\}^{-1} = 220 \text{ lb}$$

Then, $M = \{M_1^{-1} + M_2^{-1}\}^{-1}$ = 12.94lb (5.868kg). Note that the angular speed ω_1 of the spinner chassis before the impact would help to slightly increase v_{tip}, but the effect is usually negligible, because ω_b is much higher than ω_1.

So, it is not ω_1 that makes a difference here, but the alignment of the impact, which significantly increases M. If we neglect the effect of ω_1 on v_{tip}, then E_d = 33,215J, so the maneuver could increase the impact deformation energy in almost 22%.



But remember that both robots have to absorb parts of the energy E_d, so you must make sure that the attacker can also withstand the higher impact from the maneuver. In the sequence shown to the right, Last Rites won the fight after performing the described maneuver against Sir Loin, dishing out enough energy to fracture the opponent's chassis, which was already damaged from previous hits, and breaking off the eggbeater.

To calculate the speeds after the impact, we would need to know the COR e. A high speed video of the fight, for instance, could provide an estimate of the angular speed ω_b' of the bar right after the impact. If we assume that ω_b' = 0, and that the impact happened in the ideal way drawn above (with $a_1 = a_2 = 0$), then

$$\omega_b' = 0 = \omega_b - \frac{J \cdot r}{I_b} = \omega_b - \frac{M \cdot (1+e) \cdot (\omega_b \cdot r) \cdot r}{m_b \cdot r^2 / 3} = \omega_b - \frac{12.94 \cdot (1+e) \cdot \omega_b}{44/3} \quad \Rightarrow \quad e \cong 0.13$$

This low value for e is reasonable, considering the high energy of the impact. The impulse values are then J = 705.5Ns and J_1 = 564.4Ns, accelerating both robots to v_1' = v_2' = 7.07m/s (25.5km/h or 15.8mph), while keeping ω_1 unchanged (ω_1' = ω_1). Note that this v_2' value assumed that Sir Loin didn't lose its eggbeater weapon. Since the eggbeater broke off, it is expected that it was flung with a speed above v_2', while the heavier remaining chassis acquired a speed lower than v_2'.

During the first phase of the impact, while the spinning bar was compressing the opponent's chassis, the original energy E_b = 37,654J was used in part to deform both robots, with E_d = 33,215J, and the remaining $E_v = E_b - E_d$ = 4,439J was used to change their speeds. Since the COR is small, a very small part of the deformation energy is elastically stored, namely $E_k = E_d \cdot e^2$ = 33,215J·0.13^2 = 561J. The remaining energy $E_c = E_d \cdot (1-e^2)$ = 32,654J is dissipated by both robots, either by their structural parts and shock mounts in the form of vibration and sound, or transformed into plastic (permanent) deformations or fractures.

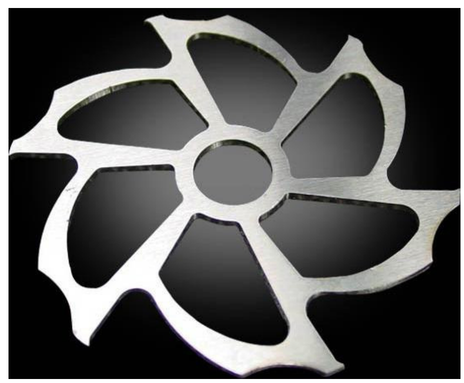
During the second part of the impact, the small stored elastic energy E_k = 561J is restituted to the system, further accelerating both robots. Therefore, from the original E_b = 37,654J, 86.7% (E_c = 32,654J) is dissipated while 13.3% ($E_v + E_k$ = 5,000J) is used to change the speeds of both robots and spinning bar.

6.5. Effective Mass

Let’s study a little bit more the concept of effective masses for an impact problem. As we’ve seen above, these masses are very important to find out how much energy is dissipated during the impact, potentially causing damages to the opponent. Therefore, an attacking robot should aim for high M_1 values. We’ll assume below that the masses of the robot chassis and weapon are m_1 and m_b, respectively. The weapon mass ratio is then defined as $x \equiv m_b/(m_1+m_b)$, it measures how much of the robot mass is spent on its weapon. We’ll also define a normalized effective mass, $M_1' \equiv M_1/(m_1+m_b)$, to make it easier to present the results.

6.5.1. Effective Mass of Horizontal Spinners

A spinning bar with length $2 \cdot r$ has a moment of inertia I_b of at least $m_b \cdot r^2/3$. If it has a large width, or if its shape is optimized, as discussed before, then the value of I_b can reach significantly higher values. It is easy to show from this result that a horizontal bar spinner without an offset shaft (therefore $a_1 = 0$) has normalized effective mass $M_1' \geq x/(3+x)$. An offset bar spinner would have a lower M_1' due to its a_1, which is usually greater than zero, unless it performs the maneuver described before to make $a_1 = 0$.



A spinning disk with radius r has I_b of at least $m_b \cdot r^2/2$. Shape optimization can increase this value, concentrating most of the mass m_b on the disk perimeter (as pictured to the right), trying to reach the (unreachable) value of $I_b = m_b \cdot r^2$. A horizontal disk spinner with $a_1 = 0$ has then $M_1' \geq x/(2+x)$, while an offset disk spinner would have a lower M_1'. These values are higher than the ones for bars.

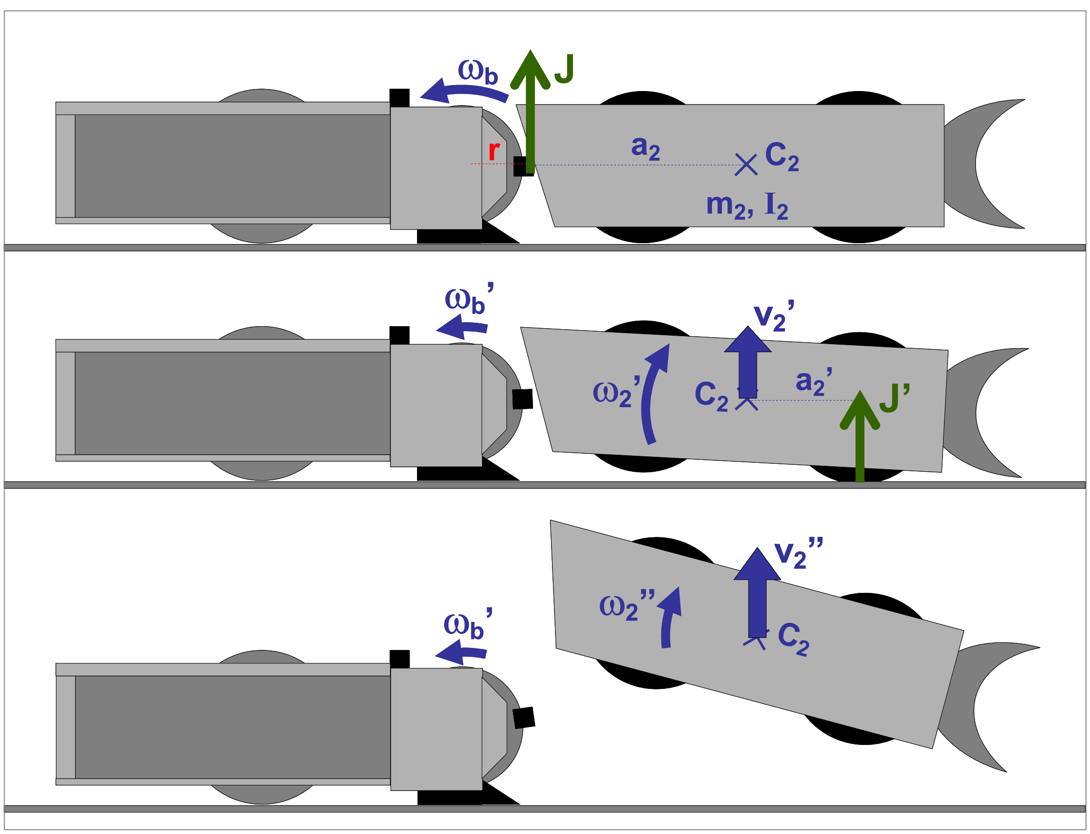
A few robots have successfully implemented a horizontal spinning ring, supported by rollers, such as the hobbyweight Ingor (pictured to the right). The advantage of a ring-shaped weapon is its high I_b, which can reach up to $m_b \cdot r^2$ for a ring with external radius r. This results in M_1' up to $x/(1+x)$, better than bars and disks.

A horizontal shell spinner, on the other hand, can have different shell shapes. If the shell is shaped like a disk, with uniformly distributed mass, then we can estimate $I_b \cong m_b \cdot r^2/2$ and then $M_1' \cong x/(2+x)$. But if its shape is optimized to concentrate most of its mass at its perimeter, then it behaves as a ring, therefore I_b can reach values up to $m_b \cdot r^2$, with M_1' up to $x/(1+x)$.

A disk-shaped thwackbot, with radius r, is basically a full-body spinner. It spins is entire mass (m_1+m_b), therefore its I_b is at least $(m_1+m_b) \cdot r^2/2$, achieving higher values if its weight is more concentrated on its perimeter. Assuming that the spin axis coincides with the robot center of gravity (otherwise it would become unbalanced), then the offset $a_1 = 0$, resulting in M_1' equal to at least 1/3 (33.3%).

6.5.2. Effective Mass of Vertical Spinners and Drumbots

All previous analyses were based on horizontal spinners, which can suffer changes in their angular speed and in the speed in the direction of the impact. But the chassis of vertical spinning robots such as drumbots or vertical spinners does not accelerate during an impact, if the impulse J is vertical in the upwards direction, as pictured below. As long as the spinning drum, disk or bar has solid ground supports that will transmit the entire impulse J without allowing the robot to tilt forward after the attack, the chassis vertical speed v_1' and angular speed ω_1' should remain equal to zero. Obviously, the arena floor won't let the attacking robot move down. Therefore, drumbots and vertical spinners that spin their weapon upwards behave as if they had a chassis with infinite inertia, with $m_1 = \infty$ and $I_1 = \infty$. Note that I_1, I_b and I_2 here are the values in the weapon spin direction, which is horizontal, not vertical. The weapon can change its angular speed, ending up with a slower ω_b' after the impact, so its moment of inertia I_b in the horizontal spin direction is still considered. But the weapon speed v_b' in the vertical direction must remain equal to zero, behaving as if $m_b = \infty$. Note that, since there is no restriction for the opponent to be launched upwards, it will gain a speed v_2' in the direction of J, and it will start spinning with an angular speed ω_2', calculated from the previously presented equations.



For m_1, m_b and I_1 tending to infinity, we have $M_1 = I_b/r^2$. So, a vertical bar spinner will have I_b of at least $m_b \cdot r^2/3$, resulting in $M_1 > m_b/3$, therefore $M_1' = M_1/(m_1+m_b) \geq x/3$. Similarly, a vertical disk spinner will have I_b of at least $m_b \cdot r^2/2$, resulting in $M_1' \geq x/2$. And a drumbot, which has I_b between $m_b \cdot r^2/2$ (for a solid homogeneous drum) and $m_b \cdot r^2$ (for a hollow drum with thin walls), ends up with a normalized effective mass M_1' between x/2 and x.

But the opponent almost always suffers a second impulse J' immediately after the impact, at the wheel (or skid, or some other ground support) that is farther away from the location of the first impact. This happens because this wheel develops, right after the first impact, a downward speed $v_{wheel}' = \omega_2' \cdot a_2' - v_2'$, where a_2' is the horizontal distance between the wheel and C_2, see the picture above. This v_{wheel}' is almost always positive, pushing the wheel down against the ground, which reacts with the vertical impulse J'. This second impulse makes v_2' increase to a value v_2'', and ω_2' decrease to ω_2'', following the linear and angular momentum equations $J' = m_2 \cdot (v_2'' - v_2')$ and $J' \cdot a_2' = I_2 \cdot (\omega_2' - \omega_2'')$, where I_2 is the moment of inertia of the opponent at C_2 in the spin direction of the weapon. These new values can be calculated if the coefficient of restitution (COR) e' between the wheel and the ground is known. The resulting equations are quite lengthy, but not difficult to obtain, as seen next.

6.5.3. Example: Drumbot Impact

Let's solve an example for a special case where $I_2 = m_2 \cdot a_2^2/3$ and $a_2 = a_2' = 0.3m$, typical of a very low profile opponent with four wheels located near its perimeter, resulting in $M_2 = m_2/4$. Remember that I_2 here is the value in the horizontal spin direction, not in the vertical one as in the horizontal spinner impact calculations. If, for instance, the opponent robot is hit by a solid drum with mass $m_b = m_2/6$ and tip speed $v_{tip} = 32m/s$ (115km/h or 71.6mph), then the drumbot's effective mass for the first impact is $M_1 = I_b/r^2 \cong m_b/2 = m_2/12$, resulting in $M = m_2/16$. Powerful weapon impacts are nearly inelastic so, if we can assume that e = 0.2, then $J = M \cdot (1+e) \cdot v_{tip} = m_2 \cdot 1.2 \cdot v_{tip}/16$, leading to $v_2' = J/m_2 = 2.4m/s$ and $\omega_2' = J \cdot a_2/I_2 = 3 \cdot 2.4/a_2 = 24rad/s$ (229RPM). Note that these values are only valid if no debris is released from either robot, and if the opponent does not have any horizontal spinning weapons that might cause some gyroscopic effect (studied later in this chapter).

Right after the first impact, there would be a downward wheel speed $v_{wheel}' = \omega_2' \cdot a_2' - v_2' = 4.8m/s$. The wheel will depart from the ground after the second impact with a speed $v_{wheel}'' = v_{wheel}' \cdot e' = 4.8 \cdot e' = v_2'' - \omega_2'' \cdot a_2'$. The other equation relating the unknowns ω_2'' and v_2'' comes from the second impulse $J' = m_2 \cdot (v_2'' - v_2') = I_2 \cdot (\omega_2' - \omega_2'')/a_2'$, resulting in an increased $v_2'' = (3.6 + 1.2 \cdot e')m/s$ and a decreased $\omega_2'' = 12 \cdot (1-e')rad/s$.

So, a purely inelastic wheel impact (e' = 0) would result in $v_2'' = 3.6m/s$ and $\omega_2'' = 12rad/s$, an angular speed more than enough to flip the opponent. And a purely elastic wheel impact (e' = 1) would lead to $v_2'' = 4.8m/s$ and $\omega_2'' = 0rad/s$, launching the opponent without flipping it at all. So, interestingly, by using rubber wheels, the opponent makes it more difficult to get flipped over because of their high COR, up to e' = 0.85 for relatively slow impacts, with low v_{wheel}'. High speed impacts, however, tend to decrease the value of e'.

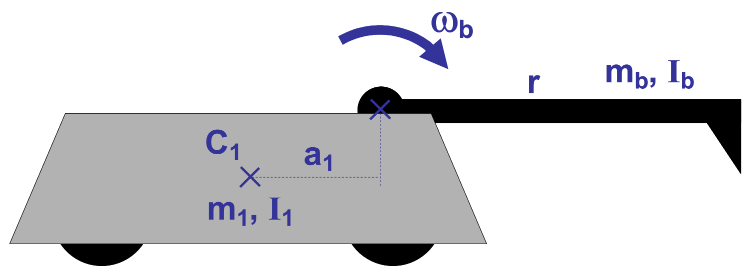
If e’ = 0.75 could be used for the considered speed v_{wheel}’ = 4.8m/s, then the resulting speeds after the second impact would be v_2” = 4.5m/s and ω_2” = 3rad/s (28.6RPM). These launching speeds would make the opponent reach a height of $v_2”^2/(2g)$ = 1.03m, where g = 9.81m/s^2 is the acceleration of gravity. The flight time would be approximately $\Delta t = 2 \cdot v_2”/g$ = 0.92s, but it can be a little less than that if the opponent lands vertically on its nose, instead of flat on the ground. During this flight time, the opponent flips 3rad/s·0.92s = 2.76rad = 158^o, more than enough to get flipped over.

Note that it is not unusual to see an opponent being spun, for instance, by 540^o before touching the ground after a powerful hit from a horizontal spinner. But it is very difficult for a vertical spinner or drumbot to cause a 540^o flip while launching an opponent. The reason for that is the second impact, which only happens in a vertical launch. In the example above, it was able to decrease the opponent’s angular speed from ω_2’ = 229RPM to only ω_2” = 28.6RPM. If the second impact hadn’t happened, the original ω_2’ and v_2’ after the first impact would have made the opponent flip 673^o, instead of only 158^o. But, because of the second impact, the drumbot from this example would have to spin its drum with v_{tip} = 59.2m/s to launch the opponent 3.53m into the air to make it flip 540^o. Even if the drumbot had the required energy and the arena was tall enough, some part of the opponent would probably break off and prevent it from reaching such height. This is why we don’t usually see drumbots or vertical spinners flipping opponents beyond 180^o.

6.5.4. Effective Mass of Hammerbots

Technically, hammerbots are not spinners, however their impact behavior can be directly obtained from the previous equations if we consider them as bar spinners that only rotate 180 degrees before hitting the opponent.

If the hammer is a homogeneous bar with length r, without a hammer head, then its I_b is $m_b \cdot r^2/3$. If it has a hammer head, then part of its mass m_b will be concentrated at its tip, increasing I_b.



Differently from vertical spinners that spin upwards, hammerbots hit downwards, so their chassis is subject to being launched, and m_1, m_b and I_1 cannot be assumed as infinite. So, their model is similar to the one for horizontal (and not vertical) bar spinners, including the effects of m_1, m_b and I_1. If the hammer pivot coincides with the chassis center of mass, then the offset a_1 is zero, and the resulting normalized effective mass is the same as the one from the bar spinner, $M_1' \geq x/(3+x)$. Otherwise, if a_1 is different than zero, as in the picture to the right, then it must be modeled as an offset horizontal bar spinner.

Note that, similarly to a drumbot’s opponent, the hammerbot will probably receive a second impact immediately after the first impact. This second impact, which happens on its back wheels, will be discussed later in this chapter.

6.5.5. Full Body, Shell and Ring Drumbots

Curiously, shell drums are not very popular, even though they have one of the highest possible effective masses. Shell drums are the vertical equivalent of shell spinners, they spin their entire armor to try to launch the opponents. The heavyweight Barber-ous II (pictured to the right) is an example of a shell drum, it uses two drive motors for the wheels and a separate motor for the drum, which doubles as its armor. The shell drum is supported on a shaft that is aligned with the wheel axis. Alternatively, if the drum was a cylinder mounted on rollers, it should be called a ring drum, the vertical equivalent of a ring spinner.

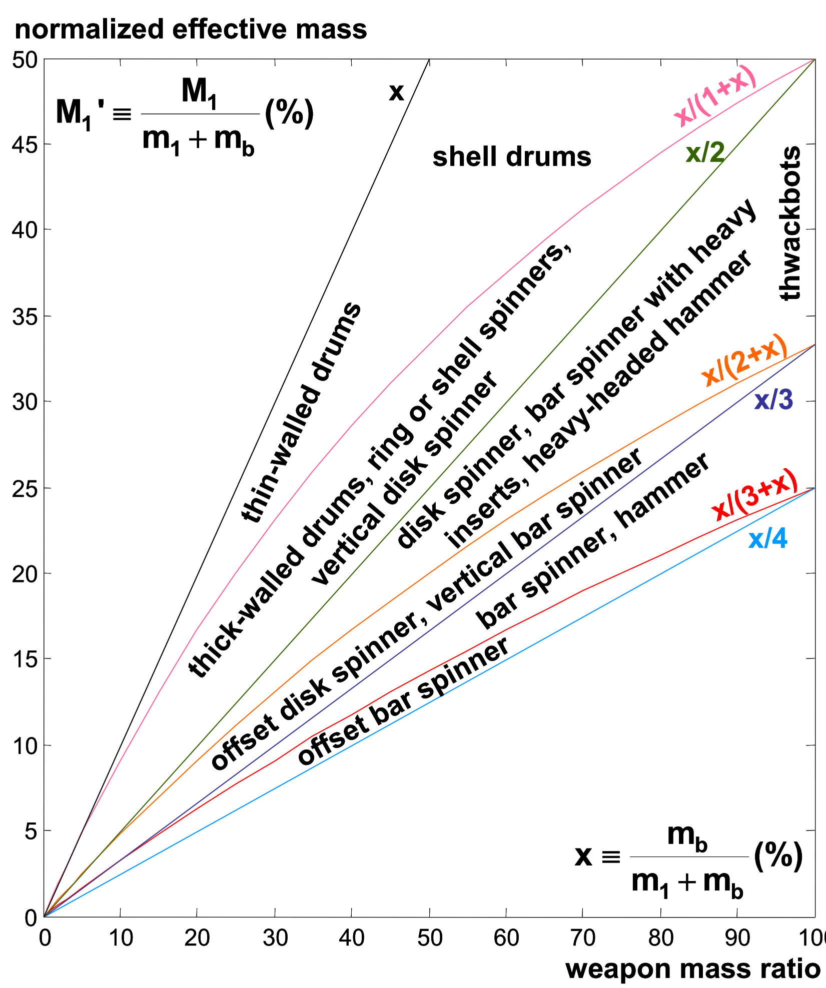
A robot type that might have never been tried is a full-body drum, which is basically an overhead thwackbot without a long rod. This robot would use the power of its two wheel motors to spin its entire chassis (and not only its armor) as if it were a big drum, maximizing its moment of inertia. It would not need a separate motor for the weapon. The challenge would be to implement at each wheel an independent braking system that would allow the chassis to spin up without moving the robot around. After reaching full speed, the braking system would be released, and the robot would be driven by slightly accelerating or braking each wheel motor. With a clever gearing system to make each wheel turn in the opposite sense of each motor, it would be possible to implement a "kamikaze attack": with the chassis/drum spinning at full speed while facing the opponent, both motors could be shorted out or reversed, directing part of the drum energy straight to the wheels to move the robot towards the opponent with very high acceleration. The combination of high robot speed, high drum angular speed and moment of inertia, and high effective mass, would result in a devastating blow to the opponent. In theory, if its wheels were very light, the weapon mass ratio $x \equiv m_b/(m_1+m_b)$ would be very close to 1 (100%), allowing the normalized effective mass M_1' of full-body drums to get very close to the absolute maximum $M_1' = 1$ (100%).

Unfortunately, all these shell, ring or full-body drumbots have a major drawback: they are easily launched by their own drum energy if attacked from behind, where the spin direction would be downwards. In addition, similarly to shell, ring and full-body horizontal spinners, their internal components need to be very well shock-mounted to avoid self-destruction.

6.5.6. Effective Mass Summary

The table in the next page summarizes the values of the weapon moment of inertia I_b and normalized effective mass M_1', as a function of weapon mass ratio x, for the robot types discussed above. The results are also presented in a graph, which shows a mapping with the ranges of the values of x and M_1' for different robot types.

robot type	weapon moment of inertia	normalized effective mass
offset bar spinner	$I_b \geq m_b \cdot r^2/3$	$x/4 \leq M_1' \leq x/(3+x)$
bar spinner	$I_b \geq m_b \cdot r^2/3$	$M_1' \geq x/(3+x)$
offset disk spinner	$I_b \geq m_b \cdot r^2/2$	$x/3 \leq M_1' \leq x/(2+x)$
disk spinner	$I_b \geq m_b \cdot r^2/2$	$M_1' \geq x/(2+x)$
shell spinner	$m_b \cdot r^2/2 \leq I_b < m_b \cdot r^2$	$x/(2+x) \leq M_1' < x/(1+x)$
ring spinner	$I_b < m_b \cdot r^2$	$M_1' < x/(1+x)$
vertical bar spinner	$I_b \geq m_b \cdot r^2/3$	$M_1' \geq x/3$
vertical disk spinner	$I_b \geq m_b \cdot r^2/2$	$M_1' \geq x/2$
drumbot	$m_b \cdot r^2/2 \leq I_b < m_b \cdot r^2$	$x/2 \leq M_1' < x$
hammerbot	$I_b \geq m_b \cdot r^2/3$	$M_1' \geq x/(3+x)$
thwackbot	$I_b \geq (m_1+m_b) \cdot r^2/2$	$M_1' \geq 1/3$



As seen on the graph, for a given weapon mass ratio x, drumbots have the highest effective mass, while horizontal and offset bar spinners have the lowest. This does not necessarily mean that drumbots are better than horizontal bar spinners, because the impact impulse and energy also depends on the weapon tip speed v_{tip}.

Drums, for instance, cannot have a very large radius r without reducing their thickness and possibly compromising their strength. They also cannot compensate their lower radius r with an arbitrarily high angular speed ω_b to achieve high v_{tip}, because they would lower their tooth bite (as explained before), ending up grinding instead of grabbing the opponent. So, despite their excellent M_1, drumbots have limitations in their achievable v_{tip}.

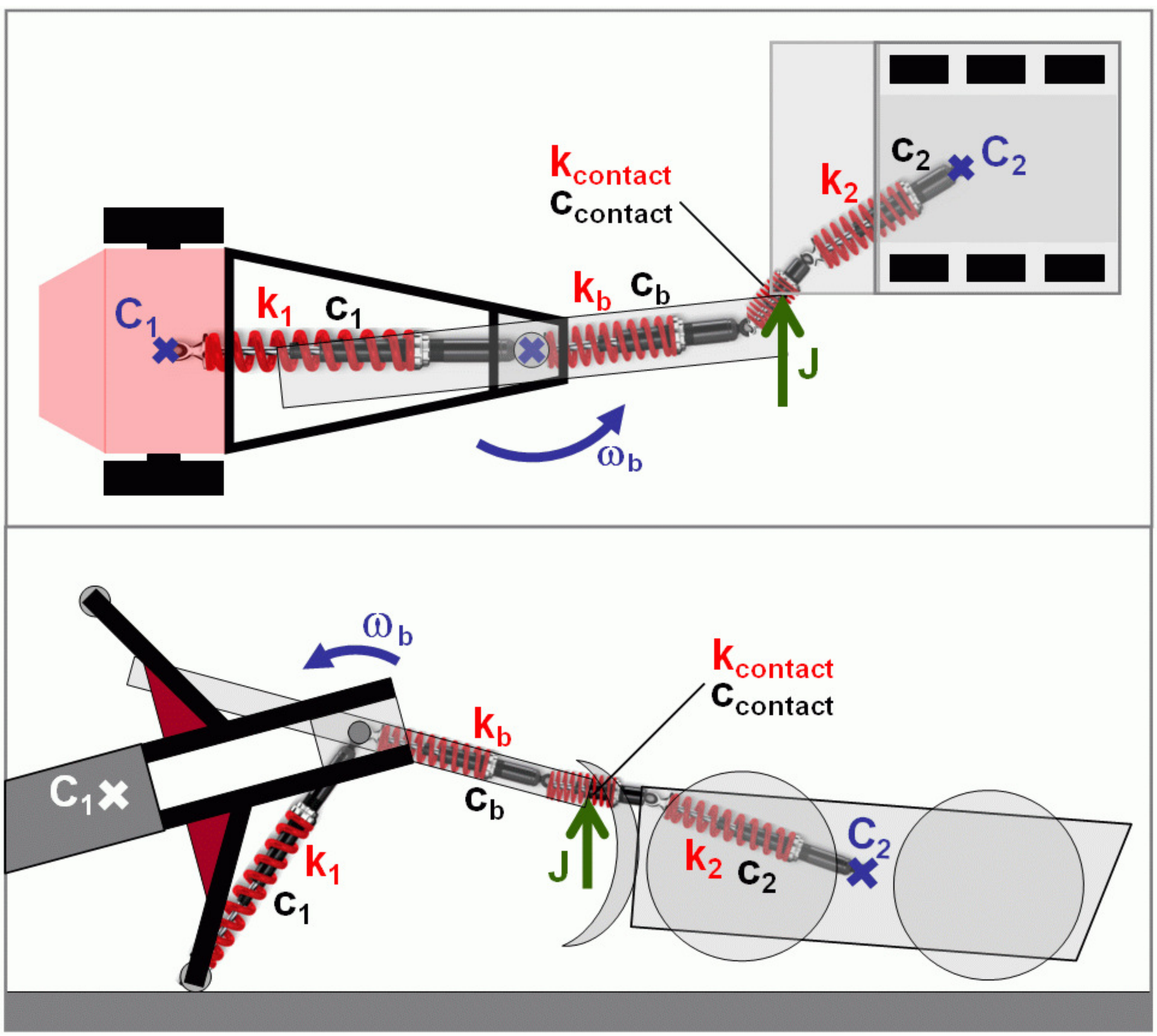
Spinning bars, on the other hand, can achieve a very large radius r without compromising strength. Their angular speed ω_b does not need to be too high to generate a very fast v_{tip}. So, they make up for their poor M_1 with their amazing v_{tip} speeds. In summary, all robot types have their advantages and disadvantages, fortunately there is no single superior design, guaranteeing diversity.

6.6. Effective Spring and Damper

We've learned that the effective mass of the impact determines how much of the weapon energy will be used to compress and deform both robots. But how is this energy distributed between the two robots? To find that out, we need to evaluate the stiffness and damping properties of the system. The stiffness is responsible for storing the elastic energy E_k, while the system damping is related to the dissipated impact energy E_c.

6.6.1. A Simple Spring-Damper Model

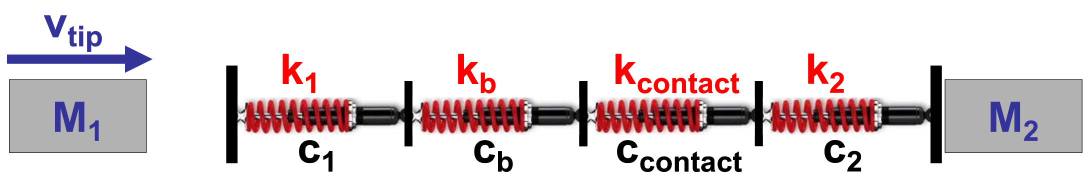
A very simple model would consider stiffness and damping coefficients for the structure of the attacking robot (k_1 and c_1), for its weapon (k_b and c_b), for the contact region between the weapon tip and the opponent's armor ($k_{contact}$ and $c_{contact}$), and for the opponent's structure (k_2 and c_2). The picture below schematically shows virtual effective spring-dampers with these stiffness and damping coefficients, for a horizontal spinner impact and for a vertical spinner impact.



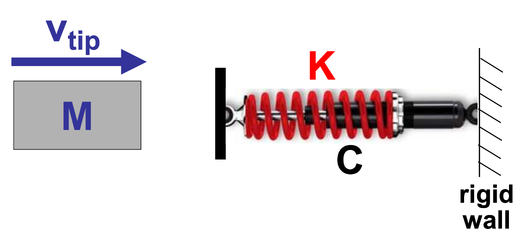
The k_2 and c_2 coefficients come from the stiffness and damping between the impact point and the center of mass C_2 of the opponent robot. The $k_{contact}$ and $c_{contact}$ coefficients are due to the localized contact (compression) between the weapon tip and the opponent's armor, achieving high values for a blunt tip and low ones for a sharp tip. The k_b and c_b coefficients represent the weapon properties, which in the figure above would be the bending stiffness and damping of the bars.

For a horizontal spinner (or hammer), the coefficients k_1 and c_1 would represent the stiffness and damping properties of the attacking robot between its center of mass C_1 and the weapon shaft (or pivot). On the other hand, for a drumbot or vertical spinner (that does not tilt forward during the impact), these k_1 and c_1 properties would reflect the stiffness and damping of the path between the weapon shaft and the ground supports, not necessarily passing through C_1, because the reaction forces from the opponent are transmitted directly to the ground.

This relatively complex impact problem, which deals with 3 different bodies (the attacker chassis, its weapon, and the opponent), involving translations and rotations, can be analyzed as a very simple impact problem, pictured below. It is equivalent to an effective mass M_1, moving at a speed v_{tip}, hitting an effective mass M_2 through a compliant interface, made out of 4 spring-damper systems in series.



The system can be simplified even more, to a single effective mass M hitting a rigid and heavy wall through an effective spring-damper system with stiffness K and damping C, see the figure to the right. The values of K and C are obtained from the equations of springs in series and dampers in series,



$$\frac{1}{K} = \frac{1}{k_1} + \frac{1}{k_b} + \frac{1}{k_{contact}} + \frac{1}{k_2} \quad \text{and} \quad \frac{1}{C} = \frac{1}{c_1} + \frac{1}{c_b} + \frac{1}{c_{contact}} + \frac{1}{c_2}$$

6.6.2. Spring and Damper Energy

When the springs are fully compressed, we define their elastically stored energies as E_{k1}, E_{kb}, $E_{kcontact}$ and E_{k2}, where $E_{k1} + E_{kb} + E_{kcontact} + E_{k2} = E_k$. Similarly, the energies dissipated by each of the dampers are called E_{c1}, E_{cb}, $E_{ccontact}$ and E_{c2}, where $E_{c1} + E_{cb} + E_{ccontact} + E_{c2} = E_c$. The individual energies are obtained from

$$\begin{cases} E_{k1} = E_k \cdot \dfrac{K}{k_1}, \quad E_{kb} = E_k \cdot \dfrac{K}{k_b}, \quad E_{kcontact} = E_k \cdot \dfrac{K}{k_{contact}}, \quad E_{k2} = E_k \cdot \dfrac{K}{k_2} \\ E_{c1} = E_c \cdot \dfrac{C}{c_1}, \quad E_{cb} = E_c \cdot \dfrac{C}{c_b}, \quad E_{ccontact} = E_c \cdot \dfrac{C}{c_{contact}}, \quad E_{c2} = E_c \cdot \dfrac{C}{c_2} \end{cases}$$

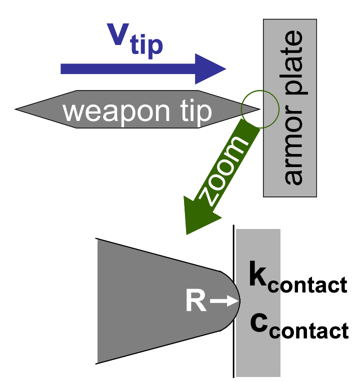
The above equations are such that K is always smaller than the smallest stiffness coefficient, and C is smaller than the smallest damping coefficient. Also, for instance, if k_2 and c_2 are much smaller than the other stiffness and damping coefficients, then $K \cong k_2$ and $C \cong c_2$, and the energies E_k and E_c are almost entirely stored into or damped by the opponent robot, because in this case we would have $E_{k2} \cong E_k$ and $E_{c2} \cong E_c$. It may seem strange, but the above equations show that the component in a series connection that has the lowest damping or stiffness coefficients is the one that will damp or elastically store the most amount of energy. This is analogous to what we see in electric circuits with resistors or capacitors connected in parallel: the resistor with lowest resistance will dissipate more energy than the others, while the capacitor with lowest elastance (the inverse of capacitance) will store more energy.

6.6.3. Offensive Strategies

From the equations above, we conclude that the strategy for the attacking robot is not only to maximize the impact energies E_c and E_k, but also to concentrate them on the opponent robot's structure (maximizing E_{c2} and E_{k2}) or contact surface (maximizing $E_{ccontact}$ and $E_{kcontact}$). To achieve that, the attacker must have c_1, k_1, c_b and k_b much higher than the opponent's c_2 and k_2 (to maximize E_{c2} and E_{k2}), or try to make the contact values $c_{contact}$ and $k_{contact}$ as low as possible (to maximize $E_{ccontact}$ and $E_{kcontact}$).

This is why the attacking robot must have a very stiff and robust weapon, with very high c_b and k_b. A very flexible weapon would end up vibrating a lot after the impact and dissipating most of its energy, instead of transferring it to the opponent. A horizontal spinner, in special an offset spinner, also needs to have a very stiff and robust connection between its weapon shaft and its center of mass, to maximize c_1 and k_1, as we can see for instance in the rigid trussed weapon support from Last Rites. And a vertical spinner or drumbot needs a very stiff and robust structure linking its weapon shaft with the ground supports, to maximize its c_1 and k_1. Here's the reason why robots with active weapons should not have their structure made out of plastic (such as UHMW): their probably low c_1 and k_1 would make the plastic structure deform and absorb most of the impact energy, instead of delivering it to the opponent.

The contact behavior between the weapon tip and the opponent's armor can be understood using a simplified model (adapted from the Hertz contact theory between two solids). The $c_{contact}$ and $k_{contact}$ of the contact between a sharp blade, with a small tip radius R, and an armor plate (pictured to the right) is proportional to the square root of R. This is why it is good to have a razor-sharp weapon, its tip radius R can reach values below one thousandth of a millimeter, lowering $c_{contact}$ and $k_{contact}$ to concentrate most of the impact energy on $E_{ccontact}$ and $E_{kcontact}$, penetrating the opponent's armor. But, to keep this sharpness, the weapon tip must be made out of a very hard material. Also, the lower the tip radius R, the more often you'll need to resharpen the weapon edge due to chipping and blunting.



6.6.4. Defensive Strategies

The values of the contact coefficients $k_{contact}$ and $c_{contact}$ are proportional to, respectively, the stiffness (measured from the Young modulus E, see chapter 3) and the hardness of the armor material. If the weapon tip has very high stiffness and hardness, then $k_{contact}$ and $c_{contact}$ will not depend much on the material properties of the weapon, they will mostly depend on the material properties of the armor.

There is a very old hardness test, using a testing instrument called a Scleroscope, where a diamond tipped hammer (which would be analogous to the weapon tip) is vertically dropped from a 10” height onto the surface of the material under test (analogous to the armor plate). A low hardness material results in a low $c_{contact}$, which causes large indentations that absorb most of the impact energy due to the high $E_{ccontact}$, lowering the height of the rebound of the hammer. So, the higher the material hardness, the higher will be the rebound height, resulting in less damping.

We can conclude then that the attacked robot has three different strategies to defend itself from a sharp blade:

1) absorb the energy at the contact – this strategy involves using an ablative armor, made out of materials with very low hardness (such as aluminum or magnesium alloys, see chapter 3), which will make sure that $c_{contact}$ will be much lower than the c_2 and k_2 coefficients of the attacked robot structure, directing most of the impact energy to $E_{ccontact}$ to be dissipated in the ablation process. With this strategy, the attacked robot structure will only need to deal with relatively small residual energies E_{c2} and E_{k2}. But make sure that your ablative armor is thick enough not to get pierced.

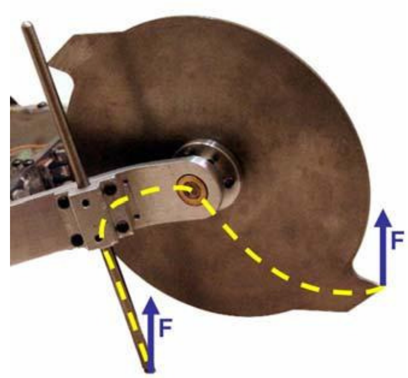
2) absorb the energy at the shock mounts – this strategy involves using a shock-mounted armor. The armor is usually of the traditional type, very hard, but an ablative armor would also work. The shock mounts make the c_2 and k_2 coefficients become very low. The high resulting energies E_{c2} and E_{k2} do not damage the attacked robot structure because they are almost entirely dissipated or stored by the shock mounts. The challenge here is to make sure that the shock mounts won’t rupture while absorbing such high amounts of energy.

3) break the weapon – this is the strategy of very aggressive rammers. It involves having a very hard and stiff traditional armor mounted to a very stiff chassis, without any shock mount in between. Shock mounts should only be used for critical internal components. The goal here is to reach high $c_{contact}$ and $k_{contact}$ (due to the traditional armor) as well as high c_2 and k_2 (from the stiff chassis without shock mounts). If these coefficients end up much higher than c_b, k_b, c_1 and k_1, then most of the impact energy will be diverted back to the attacker robot, either breaking its weapon (if E_{cb} and E_{kb} become high) or its structure (if E_{c1} and E_{k1} become high).

In summary, stiffness and damping are key properties for both attacking and attacked robots, so always design your robot keeping this in mind.

6.6.5. Case Study: Vertical Spinner Stiffness and Damping

The following robots exemplify the application of several of the presented concepts. The middleweight vertical spinner Docinho (pictured to the right) is a high power vertical disk spinner driven by 2 pairs of wheels in an ingenious invertible design. Despite its high power, it seemed to have some trouble launching the opponents. Its disk usually grinds the opponent instead of launching it, mainly because it has 3 teeth spinning at high speeds ($n = 3$, instead of better values such as 1 or 2), and also because the teeth are not hard enough to keep their sharpness. Another reason for not living up to its potential is the use of compliant wheels supporting the robot under its disk. These compliant wheels act as dampers. They significantly lower the k_1 and c_1 coefficients, ultimately making $K \cong k_1$ and $C \cong c_1$. This makes most of the deformation energy E_d go to E_{k1} and E_{c1}, permanently deforming the wheels, instead of transferring the energy to the opponent. Also, the relatively thin disk (due to its large diameter) and the Lexan armor makes it vulnerable to powerful horizontal bar spinners.

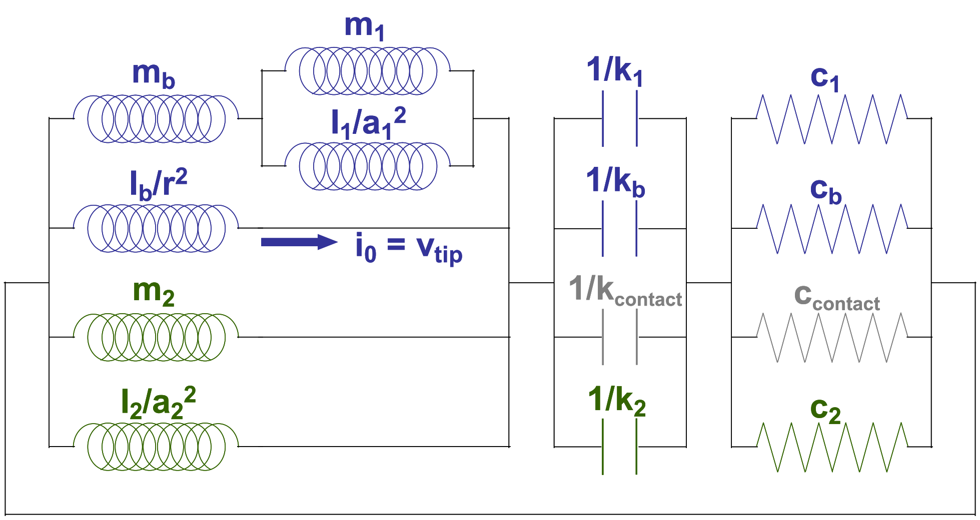


The beetleweight Altitude (pictured on the right) has addressed most of these issues. Its single-piece disk has only 2 teeth, with high hardness to prevent them from getting blunt. It is supported under the disk by skids made out of solid steel bars, which are much stiffer than rubber wheels, resulting in high k_1 and c_1 to deliver much more impact energy and peak forces. It is very important to keep in mind the force path in the robot, as seen in the picture as a dashed line. This path must only have components with high strength and stiffness to be able to guarantee high c_1, k_1, c_b and k_b and thus deliver high energy blows. Also, you must avoid any sharp notches (which are stress raisers), especially along this critical force path. The middleweight Terminal Velocity (pictured below) is another example of a vertical spinner with a stiff force path. It also has rigid skids to support its vertical spinning bar, using roller bearings to minimize sliding friction, as pictured to the right, without compromising stiffness.

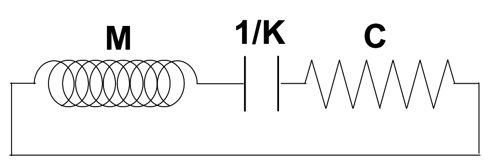
Finally, avoid installing any sensitive components, such as receivers or other electronic parts, close to the force path between the weapon tip and the ground (for drumbots or vertical spinners) or between the weapon tip and the chassis center of mass (for horizontal spinners). The impact vibrations along this path are very high, causing the sensitive components to malfunction if not shock mounted. In most drumbot and vertical spinner designs, the weapon motor ends up very close to this force path. To avoid a broken weapon motor due to impact vibrations, you can either shock mount it to the robot structure or, if possible, move it a little further towards the back of the robot.

6.6.6. Equivalent Electric Circuit

For those of you more electrically inclined, the entire impact problem has exactly the same dynamic equations as the resonator circuit below, if we consider all masses and inertial terms as if they were inductors, the elastic terms as capacitors, the damping terms as resistances, and the electric current as speeds. All blue components would come from the attacking robot, the green ones from the opponent, and the grey ones from the mechanical contact between them. The inductances would have the same numerical values (but with different physical units, of course) as the masses m_b, m_1, m_2, and inertial terms I_b/r^2, I_1/a_1^2 and I_2/a_2^2. The stiffnesses k_1, k_b, $k_{contact}$ and k_2 would be numerically equal to the elastance (the inverse of capacitance) of each capacitor, while the damping coefficients c_1, c_b, $c_{contact}$ and c_2 would be the resistances.



It is easy to see that the equivalent inductance of all blue inductors (attacker inertia) has exactly the same equation as the effective mass M_1, while the equivalent inductance of all green inductors (from the opponent) is M_2. The equivalent inductance of the entire circuit, as expected, would be M. Similarly, the equivalent elastance would have the same equation as the effective stiffness K, while the equivalent resistance would be numerically equal to the effective damping C. So, the circuit behavior would be similar to the one from the equivalent circuit pictured to the right.



Before digital calculators and digital computers were available, the first circuit shown above would be useful as an analog computer to calculate all the speeds and energy values of the impact problem.

After building the circuit, the I_b/r^2 inductor, which represents the spinning weapon, would have to be initially energized with an electric current i_0 numerically equal to the speed v_{tip} of the weapon tip, while all other components would be shorted out, with the capacitors discharged. The stored energy in this inductor would be numerically equal to the initial kinetic energy E_b of the bar. If the international system of units (SI) was used, the initial energy of both mechanical and electrical systems would be exactly the same, in Joules.

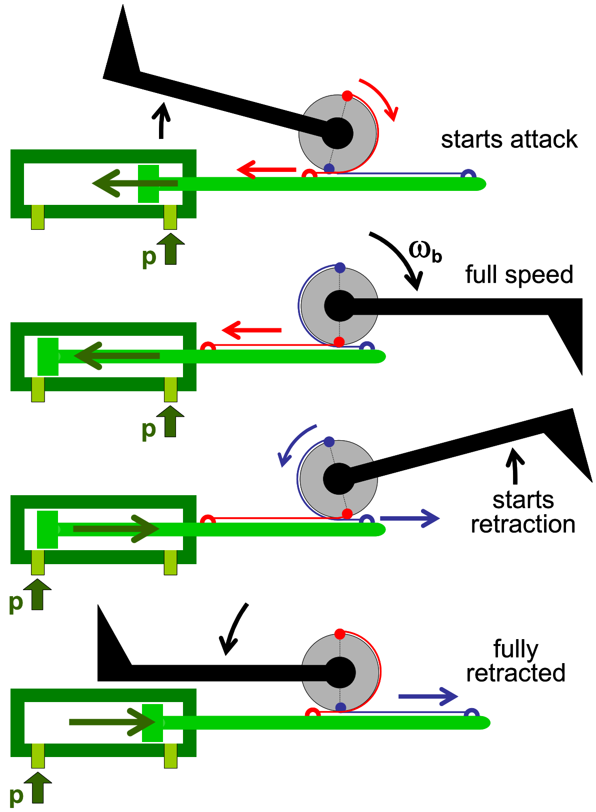
The circuit would then be connected as it was shown in the figure. During the first part of this simulated impact, the capacitors would be charged, equivalent to the compression between both robots, accumulating the same energy E_k that the equivalent mechanical system would elastically store. During the second part, the capacitors would be discharged, giving back the energy E_k to the system. As soon as the capacitors are first discharged, the electric currents in all inductors need to be immediately measured, and the simulation ends. The resonator circuit will continue to cyclically charge and discharge the capacitors, but only the first cycle is relevant to our simulation, because the impact ends after that. The subsequent cycles would only make sense if both robots would get stuck together after the impact, which is unlikely.

The initially energized inductor would contribute with E_v to the final energy of all inductors, making the total energy of all inductors at the end of the simulation equal to $(E_v + E_k)$. And the dissipated energy in the resistors during the entire cycle, which can also be measured, would be E_c. Needless to say, these energy values E_v, E_k and E_c would be the same as the ones from the mechanical system.

And how about the speeds after the impact? Well, if you measure the electric currents in all inductors at the end of the first resonator cycle, immediately after the capacitors are first discharged, then you'll see that the currents in the inductors m_1 and m_2 would be numerically equal to the attacker and opponent chassis speeds v_1' and v_2', respectively. The speed of the weapon tip after the impact would be the current going through the equivalent inductor M_1, while the current through m_b would give the speed v_b' of the center of the weapon.

6.7. Hammerbot Design

Hammers usually need to be pneumatically powered to be effective. This is because they have to reach their maximum speed in only 180 degrees. Since most pneumatic actuators are linear cylinders, you'll need some type of transmission to convert linear into rotary motion. This can be done in several ways. One of the lightest solutions, adopted by the super heavyweight The Judge, is implemented using a pair of opposing heavy-duty chains, colored in red and blue in the figure in the next page. When the right port of the cylinder in the figure is pressurized, it makes the piston move to the left and pull the red chain, which generates a rotary motion in the hammer.



The hammer could have a spring mechanism to move back to its starting position after an attack. But the best solution is to have a double acting cylinder to retract the hammer at high speeds, with the aid of the blue chain shown in the picture. This allows the hammer to get ready in less time for the next attack. Also, and most importantly, it guarantees enough torque to the hammer in both directions to work as a self-righting mechanism in case the robot gets flipped upside down.

6.7.1. Hammer Energy

No matter which mechanism you use to generate a rotary motion, it is not difficult to estimate the energy and the top angular speed of the hammer in a pneumatic robot. If we assume no energy loss due to friction or pneumatic leaks, then the energy E_b delivered by the cylinder is approximately equal to its operating pressure p times its internal volume V, so $E_b \cong p \cdot V$. If the hammer has much more inertia than the cylinder piston and the transmission mechanisms, then we can say that this energy is entirely converted into kinetic energy of the hammer, $E_b \cong I_b \cdot \omega_b^2/2$, where I_b is the hammer moment of inertia and ω_b is its top angular speed right before hitting the opponent.

For instance, assume your hammerbot uses a pneumatic cylinder pressurized at 1000psi, with a 4” diameter bore and an 8” stroke. The hammer should be able to hit the arena floor before using its entire 8” stroke. So, when hitting a tall opponent, the piston will surely have traveled significantly less than 8”. If, for instance, the actual useful stroke during an attack is 6.5”, then the useful cylinder internal volume is $V = 6.5” \cdot (\pi \cdot 4”^2/4) \cong 81.68\text{in}^3$. Since $p = 1000\text{psi} = 1000\text{lbf/in}^2$ (pounds-force per square inch), we get $E_b = p \cdot V = 81{,}680\text{lbf} \cdot \text{in} \cong 9{,}229\text{J}$. The piston force would be $F = p \cdot A = 1000\text{psi} \cdot (\pi \cdot 4”^2/4) = 12{,}566\text{lbf}$ (5,700kgf or 55,896N), where A is the internal cross-section area of the cylinder.

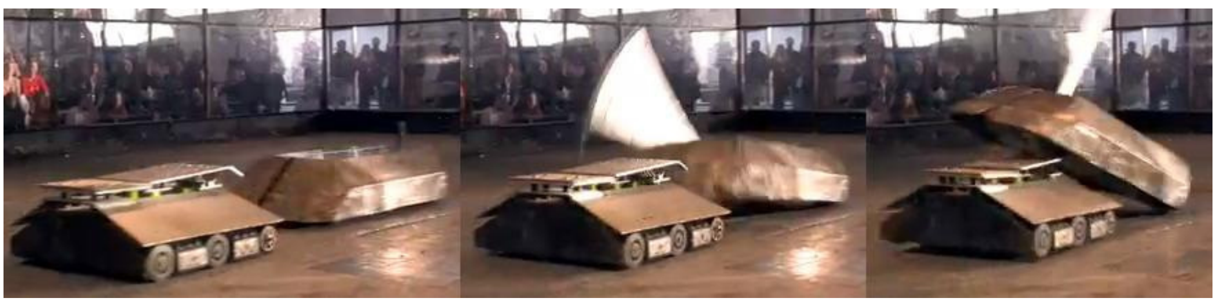
Note that this energy would be obtained while pushing the piston. When it is pulled, the energy is slightly lower, because you have to subtract the piston rod volume when calculating V. If, for instance, the piston rod has a 1.25” diameter, then $V = 6.5” \cdot [\pi \cdot (4”^2 - 1.25”^2)/4] \cong 73.70\text{in}^3$, and therefore $E_b = 73{,}700\text{lbf} \cdot \text{in} \cong 8{,}327\text{J}$. The force would also be slightly smaller, due to the smaller area $A = \pi \cdot (4”^2 - 1.25”^2)/4 \cong 11.34\text{in}^2$, resulting in $F = p \cdot A = 11{,}340\text{lbf}$ (5,144kgf or 50,443N).

So, it is slightly better to design the transmission system such that the hammer hits when the piston is extended. But, depending on the transmission design, this might place the cylinder in the front of the robot, more exposed to attacks, and limiting the reach of the hammer head.

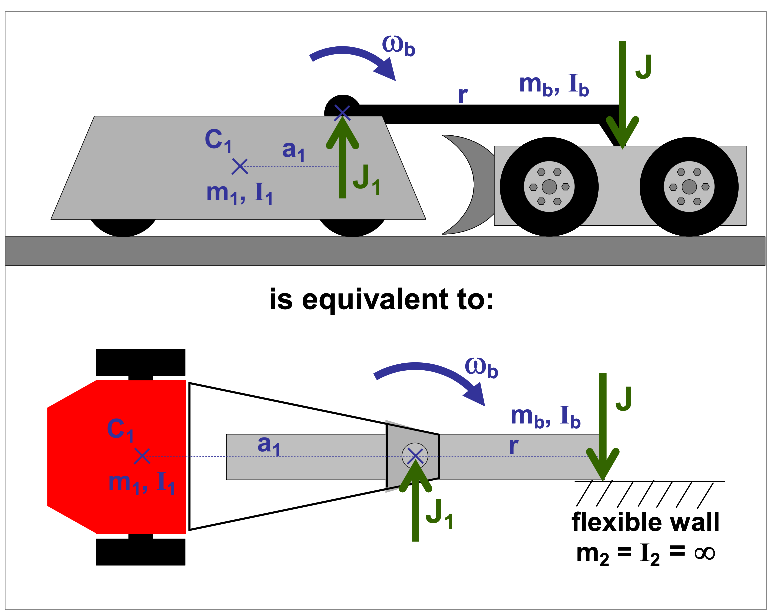
6.7.2. Hammer Impact

If in our example above the hammer handle is 36” (0.91m) long with a mass of 15lb (6.8kg), with a 10lb (4.5kg) hammer head, then its moment of inertia is $I_b \cong 6.8 \cdot 0.91^2/3 + 4.5 \cdot 0.91^2 \cong 5.6 kg \cdot m^2$. If we place the cylinder in the back of the robot, using the mechanism from the previous figure, then the energy $E_b = 8{,}327J \cong I_b \cdot \omega_b^2/2$ from the pulling motion would accelerate the hammer up to $\omega_b = 54.5rad/s = 521RPM$, resulting in a hammer head speed of $54.5rad/s \cdot 0.91m = 49.6m/s$ (179km/h or 111mph).

Note that the robot will tend to move backwards during the acceleration of the hammer, therefore it needs to compensate for that by braking its wheels. The chassis will also tend to tilt backwards, from the reaction force of the hammer accelerating forward. Powerful hammerbots may even see their front wheels lift off the ground because of that, as seen in the middle picture below, which shows The Judge tilting backwards right before it even touches the opponent. Excessive tilting may leave it vulnerable to wedges or launchers that might sneak in underneath (as shown in the picture below to the right, right before The Judge was launched by Ziggy). To avoid that, it is a good idea to move forward the center of mass of the hammerbot.



The picture above to the right shows that the tilting angle of the chassis is increased even more after the hit, due to the reaction impulse J_1 from the impact, pictured to the right. The speeds after the impact and all the involved impact energies can be calculated from the very same equations used for spinners. Since the attacked robot is hammered against the arena floor, it usually does not move its center of mass, it only deforms due to the attack, so the impact problem is similar to an offset bar spinner hitting a flexible but very heavy wall, as shown in the picture to the right.

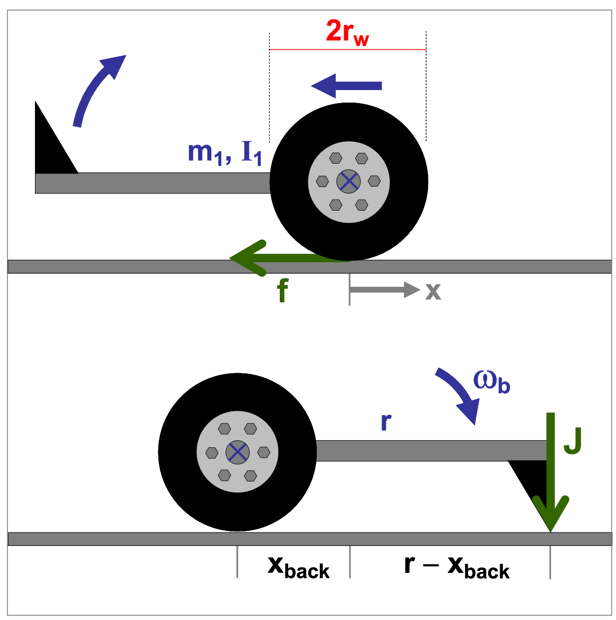


The attacked robot would then have an infinite effective mass M_2, while the hammerbot's M_1 would have the same equation from an offset spinner, where m_1 and I_1 are the chassis mass and moment of inertia in the direction that the hammer rotates, m_b and I_b are the corresponding values for the hammer, and a_1 is the horizontal offset between the chassis center of mass C_1 and the hammer pivot. The speeds of the attacked robot after the hammering are then $v_2' = \omega_2' = 0$, while the hammerbot chassis gains a vertical speed v_1' in the direction of J_1, and it may spin backwards, if $a_1 > 0$, with an angular speed ω_1' calculated from the spinner equations.

Note that, if the back wheels of the hammerbot are still in contact with the ground immediately after the impact against the opponent, then a second impact will probably occur. With the back wheels gaining a downward speed after the impulse J_1, they will press against the arena floor and receive a vertical reaction impulse J'. This back wheel impulse J' is good for the hammerbot, because it avoids its chassis from tilting too much backwards. The final linear and angular speeds v_1'' and ω_2'' of the hammerbot chassis can be calculated using the very same equations from the second impact that happens when a robot is hit by a drumbot or vertical spinner, as studied before.

6.8. Overhead Thwackbot Design

Overhead thwackbots need to be well balanced with respect to the wheel axis, otherwise they won't be able to have enough torque to lift the weapon to strike. This balancing can be done using counterweights opposite to the weapon, to place the center of mass C_1 of the entire robot on the wheel axis.



Overhead thwackbots have a few similarities with hammerbots. The main difference is that they use the drivetrain power to accelerate the weapon. This limits the weapon top speed, because a high gearmotor torque would end up making the wheels slip. Both wheels usually bear altogether a ground normal force equal to the robot weight $m_1 \cdot g$, where m_1 is its mass and g is the acceleration of gravity. If μ is the coefficient of friction between the tires and the ground, then the maximum traction force f that the tires can generate together is $f = f_{max} = \mu_t \cdot m_1 \cdot g$, see the picture to the right.

If the wheels have a radius r_w, then the maximum torque τ that both wheels can generate altogether to accelerate the weapon is $\tau_{max} = f_{max} \cdot r_w = \mu_t \cdot m_1 \cdot g \cdot r_w$. The wheel gearmotors need to be able to provide altogether this torque τ_{max}. Less than that would result in a slower weapon impact speed, while more torque would make the wheels slip. If using DC motors, it is a good idea to have a current controller (instead of a voltage controller from most speed control electronics), to guarantee a constant current to deliver a constant τ_{max} after the gear reduction, independently of the speed of the motors.

Let's assume now that the robot center of mass C_1 is located along the wheel axis. If the mass and moment of inertia of the entire robot (chassis plus weapon) in the wheel axis direction with respect to C_1 are m_1 and I_1, then the torque τ_{max} would generate an angular acceleration $\alpha = \tau_{max}/I_1$. Note that most of the value of I_1 will come from the weapon, because the rest of the chassis is very close to the wheel axis and do not contribute much to the moment of inertia. For the robot to strike, it needs to turn about 180 degrees (π radians), therefore $\pi = \alpha \cdot t^2/2$, where t is the short time the robot takes to strike.

If the robot is initially at rest, then its chassis will start moving backwards, due to the linear acceleration $f/m_1 = \mu_t \cdot g$ caused by the ground force f. During the strike period t, the distance x_{back} the chassis moves backwards is then $x_{back} = \mu_t \cdot g \cdot t^2/2$. Eliminating t and α from these equations we get $x_{back} = \pi \cdot I_1/(m_1 \cdot r_w)$.

So, if its weapon has a radius r, then it will hit a spot at a distance $(r - x_{back})$ from its position when it started the attack, see the pictures above. The driver has to get a feeling of this distance after some practice, otherwise the weapon will hit short of the opponent's position. Note that x_{back} does not depend on the coefficient of friction μ_t, it is a constant for each robot.

But x_{back} can be compensated for, if the robot starts striking when it is moving with some initial forward speed v_{x1}, not at rest as before. Then $x_{back} = \mu_t \cdot g \cdot t^2/2 - v_{x1} \cdot t$, which would be equal to zero right at the end of the attack if

$$v_{x1,ideal} = \frac{\mu_t \cdot g}{2} \cdot t = \frac{\mu_t \cdot g}{2} \cdot \sqrt{\frac{2\pi}{\alpha}} = \frac{\mu_t \cdot g}{2} \cdot \sqrt{\frac{2\pi \cdot I_1}{\mu_t \cdot m_1 \cdot g \cdot r_w}} = \sqrt{\frac{\mu_t \cdot g}{2} \cdot \frac{\pi \cdot I_1}{m_1 \cdot r_w}}$$

So, if the robot is initially moving towards the opponent at this $v_{x1,ideal}$ speed, it will hit exactly at the distance r from its initial position. If moving slower than that, it will hit short of that distance. If moving faster, it will hit beyond that. This is why overhead thwackbots are so difficult to drive, it is up to the driver to get a feeling of the attack distance as a function of the attack speed. Note that, because this function depends on μ_t, a dirtier arena (leading to a lower μ_t) will result in a lower $v_{x1,ideal}$, which needs to be adaptively controlled by the driver. Poor driver.

Finally, the weapon maximum angular speed ω_b and energy E_b can be calculated from α and t, resulting in

$$\omega_b = \alpha \cdot t = \sqrt{2\pi\alpha} = \sqrt{\frac{2\pi \cdot \mu_t \cdot m_1 \cdot g \cdot r_w}{I_1}} \quad \text{and} \quad E_b = \frac{1}{2} I_1 \omega_b^2 = \pi \cdot \mu_t \cdot m_1 \cdot g \cdot r_w$$

So, since m_1 is basically the mass of the weight class and g is a constant, to maximize the attack energy E_b you must have rubber wheels with large radius r_w, and also with large width to maximize its coefficient of friction μ_t. And, of course, the drivetrain gearmotors should be able to provide altogether at least $\tau_{max} = \mu_t \cdot m_1 \cdot g \cdot r_w$ to reach these maximum ω_b and E_b values.

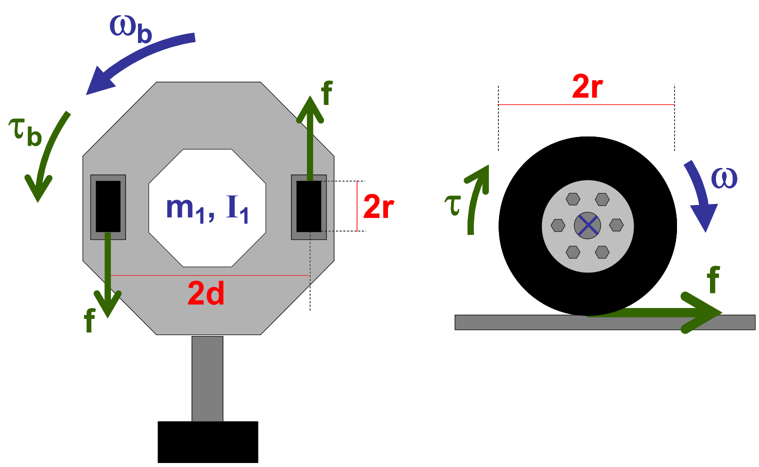
But be careful, because horizontal spinners love large wheels. It's their favorite breakfast.

6.9. Thwackbot Design

A thwackbot can be thought of as a full body spinner. It uses the power of its two (or more) wheels to spin up its entire body. As seen before, its normalized effective mass M_1' can reach very high values, from 1/3 for disk shaped designs tending towards 1/2 for ring shaped designs, being able to store a lot of kinetic energy.

6.9.1. Thwackbot Equations

Let's consider a thwackbot with mass m_1, moment of inertia I_1 in its spin direction with respect to its center of mass, and two wheels with radius r separated by a distance 2·d, as shown in the picture to the right. We'll assume that the robot center of mass is located in the middle of the line between the wheel centers, otherwise the robot would be unbalanced, compromising its maximum angular speed and drivability. So, if it has some asymmetrical feature such as a single hammer, as pictured above, then the wheel axis location must be carefully calculated to guarantee that it ends up balanced.



Each wheel has a variable angular speed ω, with a maximum value ω_{max}, and it receives a torque τ from the gearmotor output. This wheel torque will cause a traction force f, which is equal to τ/r if the wheel does not slip. Assuming that each wheel bears half of the robot weight, the maximum possible value for f is $\mu_t \cdot g \cdot m/2$, where μ_t is the coefficient of friction between the wheel and the ground, and g the acceleration of gravity. If τ/r is greater than $\mu_t \cdot g \cdot m/2$, then the wheel will slip.

Note that it is not a good idea to increase the width of the tires to increase μ_t, because tires with large width tend to waste a lot of energy while making sharp turns, due to the slipping that always occurs along their width. This slip happens because the inner surface of a wheel with width w is closer to the center of the robot, moving along a circle with radius (d – w/2), while the outer surface moves along a circle with radius (d + w/2). The mid-section of the wheel, which moves along a circle with radius d, makes the inner surface waste some energy by slipping forward to catch up with the mid-section, while the outer surface wastes energy slipping backwards. So, while overhead thwackbots should have wide tires, thwackbots should use instead relatively thin tires to avoid this energy loss.

The wheel torque τ used in the above equations depends on the wheel speed ω, as seen in chapter 5 for DC motors. We can then define an effective wheel torque function τ(ω) that includes this dependency, and which is also limited by the value $\mu_t \cdot g \cdot m \cdot r/2$ which would make the wheel slip. This effective torque τ(ω) depends not only on ω, but also on the motor and battery properties (such as K_t, K_v, I_{stall}, I_{no_load} and R_{system} for DC motors, see chapter 5), gear ratio n:1, and the value of $\mu_t \cdot g \cdot m \cdot r/2$. For instance, if min(x,y) is the function that returns the minimum value between x and y, then DC motors would result in an effective wheel torque function

$$\tau(\omega) = \min\left\{ n \cdot K_t \cdot (I_{stall} - I_{no\_load} - \frac{\omega \cdot n}{K_v \cdot R_{system}}), \quad \frac{\mu_t \cdot m \cdot g \cdot r}{2} \right\}$$

Let's first calculate the time Δt_{drive} the robot takes to accelerate to, for instance, 95% of its top speed, while driving on a straight line. In this case, the traction forces f of both wheels would be directed towards the same direction, contrary to the figure above. The forward speed of the robot would be $v = \omega \cdot r$, with a maximum value $v_{max} = \omega_{max} \cdot r$. So, the forward acceleration of the robot dv/dt would be equal to the angular acceleration $d\omega/dt$ of each wheel multiplied by the wheel radius r, thus $dv/dt = r \cdot d\omega/dt$. Both forward forces f cause the robot to accelerate, following the equation $2 \cdot f = m \cdot dv/dt$. We can then use $f = \tau(\omega)/r$ and $dv/dt = r \cdot d\omega/dt$ in this equation, to obtain

$$2 \cdot f = 2 \cdot \frac{\tau(\omega)}{r} = m \cdot r \cdot \frac{d\omega}{dt} \quad \Rightarrow \quad \Delta t_{drive} = \int dt = \frac{m \cdot r^2}{2} \cdot \int_0^{0.95\omega_{max}} \frac{d\omega}{\tau(\omega)}$$

The integral shown above is not difficult to calculate, it was obtained for DC motors in chapter 5.

Let's now calculate the spin up time Δt_{weapon} until the robot reaches 95% of its top weapon speed. The chassis angular speed ω_b is equal to the wheel linear speed $\omega \cdot r$ divided by d, so we have $\omega_b = \omega \cdot r/d$. Thus, their angular accelerations are related by $d\omega_b/dt = (d\omega/dt) \cdot r/d$, and the maximum angular speed of the chassis is $\omega_{b,max} = \omega_{max} \cdot r/d$.

The traction forces f are now in opposite directions, as shown in the figure above, spinning up the chassis with a torque $\tau_b = 2 \cdot f \cdot d = 2 \cdot \tau(\omega) \cdot d/r$, which is equal to the robot moment of inertia I_1 in the spin direction times its angular acceleration $d\omega_b/dt$. We can then use $d\omega_b/dt = (d\omega/dt) \cdot r/d$ to obtain

$$2 \cdot f \cdot d = 2 \cdot \frac{\tau(\omega) \cdot d}{r} = I_1 \cdot \frac{d\omega_b}{dt} = I_1 \cdot \frac{r}{d} \cdot \frac{d\omega}{dt} \quad \Rightarrow \quad \Delta t_{weapon} = \int dt = \frac{I_1 \cdot r^2}{2 \cdot d^2} \cdot \int_0^{0.95\omega_{max}} \frac{d\omega}{\tau(\omega)}$$

So, it is easy to see that decreasing the distance $2 \cdot d$ between the wheels increases the maximum weapon speed $\omega_{b,max}$, but it also increases the weapon spin up time Δt_{weapon}. It is not a good idea to have Δt_{weapon} much higher than 4 to 8 seconds, otherwise the thwackbot won't stand a chance against a very aggressive rammer or wedge, so choose wisely your distance $2 \cdot d$.

Since both Δt_{weapon} and Δt_{drive} depend on the same integral, they are related by

$$\Delta t_{weapon} = \frac{I_1 \cdot r^2}{2 \cdot d^2} \cdot \int_0^{0.95\omega_{max}} \frac{d\omega}{\tau(\omega)} = \frac{I_1 \cdot r^2}{2 \cdot d^2} \cdot \frac{2 \cdot \Delta t_{drive}}{m \cdot r^2} = \Delta t_{drive} \cdot \frac{I_1}{m \cdot d^2}$$

So, if you've already calculated the acceleration time Δt_{drive} of the drive system, as described in chapter 5, then the weapon spin up time is easily obtained from the above equation.

If your thwackbot has its wheels close to its perimeter, it is very likely that I_1 is equal to or a little lower than $m \cdot d^2$, therefore Δt_{weapon} and Δt_{drive} would be approximately equal. This is not good, because the low Δt_{drive} required to make the robot agile would lower the spin up time Δt_{weapon}, probably resulting in a low weapon energy. Even if you upgrade your drive motors, they won't be able to deliver a very high power to spin up the weapon in a short Δt_{weapon}, because the wheel forces are limited to $\mu_t \cdot g \cdot m/2$ (for a two wheeled thwackbot), they would slip beyond that. And a larger Δt_{weapon} would naturally result in a large Δt_{drive}, compromising the drivetrain acceleration. So, wheels with a large distance $2 \cdot d$ usually result in a relatively slow robot, or one with low weapon energy.

One way to avoid this is to decrease the wheel distance $2 \cdot d$. But they can't be too close together, otherwise any unbalancing in the robot or any attack from a wedge could make the spinning chassis touch the ground and launch itself. Using casters near the perimeter could help making the thwackbot stable, but there's a good chance they'll be knocked off or broken during an angled impact.

Another way to increase the weapon energy without compromising the drivetrain acceleration is to maximize the value of I_1. This is done by concentrating most of its mass in its outer perimeter, trying to approach the upper limit 1/2 of its normalized effective mass M_1', as done in the ring-shaped heavyweight Cyclonebot (pictured to the right).

6.9.2. Melty Brain Control

The main drawback of a thwackbot is the required complexity of its drive system to enable it to move around in a controlled way while spinning. The idea, referred to as *translational drift*, is to somehow oscillate the speed or the steering of each wheel with the same frequency that the entire robot spins. The speed oscillation solution is usually called "melty brain" or "tornado drive" control, while the steering oscillation has been called "wobbly drive" or "NavBot steering" control.

Despite its name, the math behind "melty brain" control is not hard enough to melt someone's brain. For instance, when the chassis of a two-wheeled thwackbot is spinning at ω_b, at every time period $T = 2 \cdot \pi / \omega_b$ one wheel will be facing the desired direction to which you want to move, while the other will be facing the opposite direction. If at this moment the first wheel has a slightly larger speed than the other, then the robot will end up moving in that desired direction. The angular speed of each wheel would need to be $\omega = \omega_b \cdot d/r \pm (v/r) \cdot \cos(2 \cdot \pi \cdot t/\omega_b + \varphi)$, with the plus sign for one wheel and the minus sign for the other, where t is time, v is the desired linear speed, r is the wheel radius, and φ is a phase angle that will define the direction of the movement.

"Melty brain" control is not easy to implement, because the time period T is very short. You need a very fast acting control system, powerful drive motors to be able to change the wheel speeds in such high frequency, and some way to measure both the robot angular speed, to obtain ω_b, and its orientation at the end of each time period, to define φ. In theory, wheel encoders or other angular position sensors could be used to estimate ω_b and φ, using dead reckoning, but they do not work in practice because there is wheel slip. Digital compasses do not work well because the high motor currents usually affect the readings of the Earth magnetic field.

Most successful implementations of “melty brain” control require that the driver emits some light beam, usually infra-red or laser, in the direction of the thwackbot. This beam is detected by a sensor on the periphery of the robot, allowing it to estimate ω_b from the time period T between two sensor readings, $\omega_b = 2\cdot\pi/T$. The robot can also estimate its orientation at each reading, which would be the one facing the driver’s light source. It is a good idea to use two light sensors close together instead of one, to minimize the chance of both picking up random reflections of the beam from the arena or the opponent robot, which would confuse the control system.

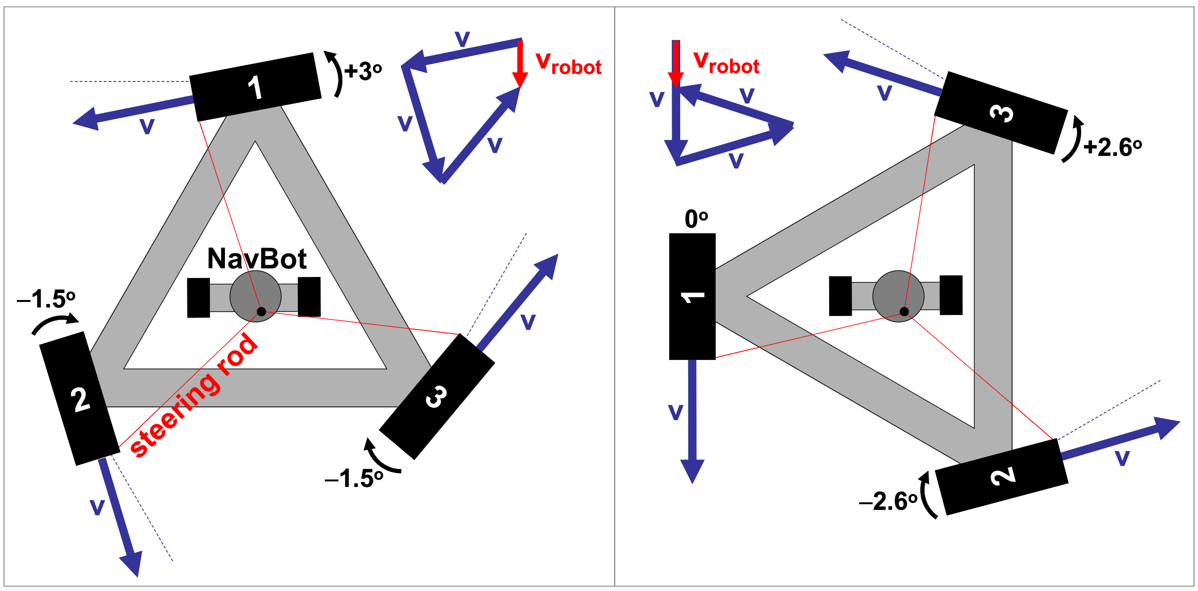
Other successful implementations of “melty brain” control use accelerometers or gyroscopes to measure or infer ω_b, and a led on its periphery that blinks with a period $T = 2\cdot\pi/\omega_b$, calculated in real time from the current ω_b. If the ω_b measurement is accurate enough, the led will only blink once per revolution, apparently at the same position in space, which would point to a nominal direction. It is then up to the driver to look at the position of the led light and use the radio control to change accordingly the phase angle φ of the robot software, allowing the thwackbot to move around.

A few robots have successfully implemented “melty brain” control through electronics, such as the heavyweight Cyclonebot, the middleweight Blade Runner, the lightweight Herr Gepöunden, the featherweight Scary-Go-Round, and the antweight Melty B, as pointed out by Kevin Berry in his “melty brain” Servo magazine articles from February and March 2008.

6.9.3. NavBot Control

Other robots, such as the middleweight WhyNot (pictured below to the left) and the heavyweight Y-Pout (to the right), have followed a mechanical approach to implement steering control. These robots have three wheels in a 120^o configuration, driven by high power motors.

The steering is performed by making each of their three active wheels slightly steer in and out about 3 degrees at every robot revolution. The steering angle of each wheel is approximately given by $\theta_1 = 3^o \cdot \cos(2\cdot\pi\cdot t/\omega_b + \varphi)$, $\theta_2 = 3^o \cdot \cos(2\cdot\pi\cdot t/\omega_b + 2\pi/3 + \varphi)$ and $\theta_3 = 3^o \cdot \cos(2\cdot\pi\cdot t/\omega_b + 4\pi/3 + \varphi)$, mechanically implemented using three steering rods connected to a cam mechanism on a small independent radio-controlled two-wheeled robot called NavBot, as seen in the figures below. The phase angle φ that defines the direction of the movement is mechanically controlled by the direction of the NavBot. The NavBot has only one motor, a low power one, which works as a differential drive. When this motor is locked, a worm gear makes sure that both wheels turn together to make the NavBot move straight, pulled by the main robot, keeping constant the phase angle φ. And when this motor is powered, following a radio control signal, one of the small wheels starts spinning with a different speed than the other, making the NavBot turn, which will change the phase angle φ and thus make the entire robot move in the new direction of the NavBot. The NavBot will then be pulled in this new direction that it is facing.



The figures above show that the differential steering of the wheels make the robot move with a constant speed $v_{robot} = v \cdot \sin 3^o$, where v is the linear speed of each wheel. The direction of v_{robot} does not change while the robot is spinning, as shown in the figures, as long as the NavBot keeps its orientation. The orientation of the cam mechanism must be setup in such a way that the directions of v_{robot} and of the two small NavBot wheels always coincide, allowing the NavBot to easily follow the robot path while being pulled.

Note that these robots can be categorized as a thwackbot, if we consider that its 3-wheeled drive system makes the entire robot spin, with the NavBot being a secondary robot (almost as a Multibot). Or they can be categorized as ring or shell spinners, if we consider that the NavBot is the main robot, with a two-wheeled drivetrain, and the spinning triangle is the ring or shell, powered by three weapon motors embedded in it. It's just a matter of point of view.

The main advantage of this system is that it is possible to implement a thwackbot (or ring or shell spinner) with a normalized effective mass M_1' close to 1/2, using the same high power motors for both the weapon and drive system. Using high diameter wheels, it might be even possible to make the robot invertible, but the NavBot would be exposed to hammer attacks without a top cover.

This system, however, has a few disadvantages, because the drive speed of the robot is a function of the weapon speed, due to the relation $v_{robot} = v \cdot \sin 3^o$. So, when the weapon is at full speed, with the wheels at full linear speed v_{max}, the robot will have to move around at a full speed $v_{robot,max} = v_{max} \cdot \sin 3^o$, all the time. It won't be able to slow down or stop its drive system, it will have to keep moving until it hits something, slowing down the weapon. And, most importantly, after a major hit, the weapon will probably be spinning so slowly that the robot won't be able to move around. An aggressive rammer would only need to survive the first hit, and then keep ramming the thwackbot to prevent its weapon from spinning up. The thwackbot wouldn't be able to run away from the rammer to try to spin up its weapon, because its drive speed would be too slow due to the slow weapon speed.

6.10. Launcher Design

Launchers need to deliver a huge amount of energy during a very brief time. Because of that, they're almost invariably powered by high pressure pneumatic systems. A very simplified estimate can show that a cylinder with piston cross section area A, stroke d, with pressure p, can accelerate a total mass m (including the mass of its piston) with an average power of up to $(p \cdot A)^{1.5} \cdot (0.5 \cdot d/m)^{0.5}$. For instance, a 4" bore cylinder with 8" stroke pressurized at 1000psi would accelerate a 220lb mass with an average power of about 566HP.

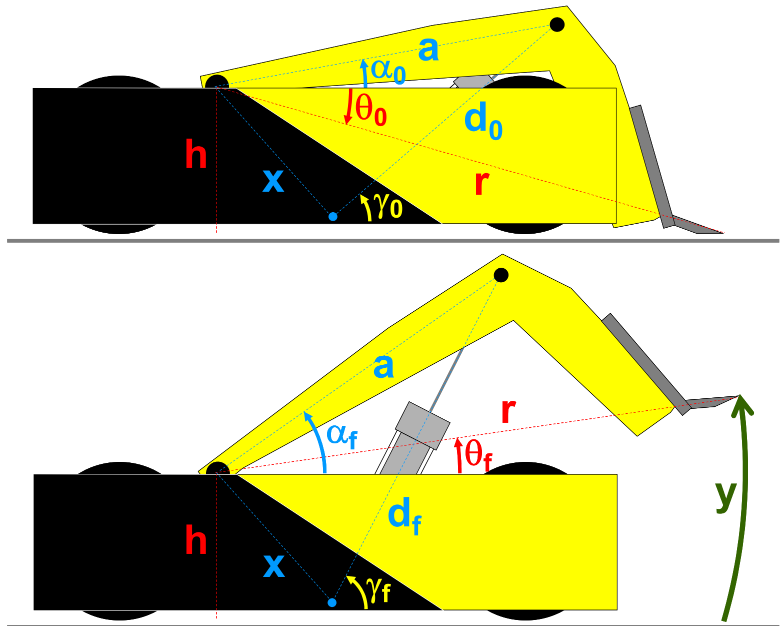
Of course, this power is only delivered during a very brief time, but a light weight electric motor or internal combustion engine cannot supply that. Unless the motor is used to store kinetic energy in a flywheel during a few seconds, with an ingenious and very strong mechanism that suddenly transfers this energy to the launcher arm, as done by the Warrior SKF robot (pictured to the right). But such sturdy mechanism is not simple to build.

Hydraulic systems are not good options either for launchers. They can deliver huge forces and accelerations, but their top speed is relatively low.

Most launchers try to either maximize the height or the range of the throw. The "height launchers" try to launch the opponent as high as possible, trying to flip it while causing damage when it hits the ground. The "range launchers" try to launch the opponent as far as possible, not necessarily high, trying to throw it out of bounds to the arena dead zone.

Against horizontal spinners, especially undercutters, there is a strategy used not only by launchers, but also by lifters and wedges, which is to tilt the spinner chassis so that its spinning weapon hits the arena floor, usually launching it. The picture to the right shows the middleweight launcher Sub Zero tilting the chassis of the spinner The Mortician, to launch it with the help of the additional energy from the spinning bar hitting the ground.

6.10.1. Three-Bar Mechanisms

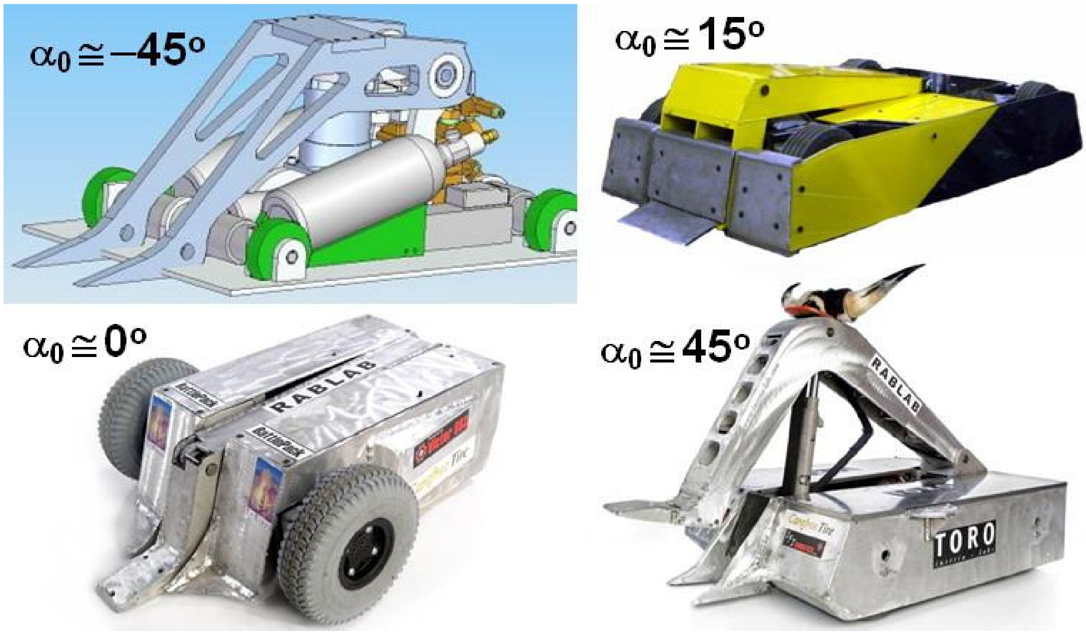


A very popular launcher design uses a so-called three-bar mechanism, as pictured to the right. The "three bars" are the pneumatic cylinder, with total length varying from d_0 to d_f during the launch, the main structure of the launcher arm, with constant length a, and the part of the robot chassis that connects the arm and cylinder pivots, with constant length x. The launcher arm tip features a wedge-like scoop, at a constant distance r from the arm pivot. The initial and final angles of the main structure of the arm are defined as α_0 and α_f, while the angles for the arm tip are θ_0 (which is negative for the particular robot shown in the picture) and θ_f. The angle variation during the launch is then $\alpha_f - \alpha_0$, which is equal to $\theta_f - \theta_0$. The initial and final angles of the cylinder are γ_0 and γ_f, as shown in the picture.

During the launch, the arm tip follows the green circular path shown above, with an arc length y that is usually between h and 2·h, where h is the height of the launcher chassis. The direction of this path is, in average, equal to 90° plus the average angle θ between θ_0 and θ_f, which is roughly the direction of the launching force. In the picture above, $\theta = (\theta_0 + \theta_f)/2$ is approximately zero, leaving in average an almost vertical (90°) force. Note also that a negative θ_0 is a good idea, it makes the arm tip move forward in the beginning of its path, helping to properly scoop the opponent.

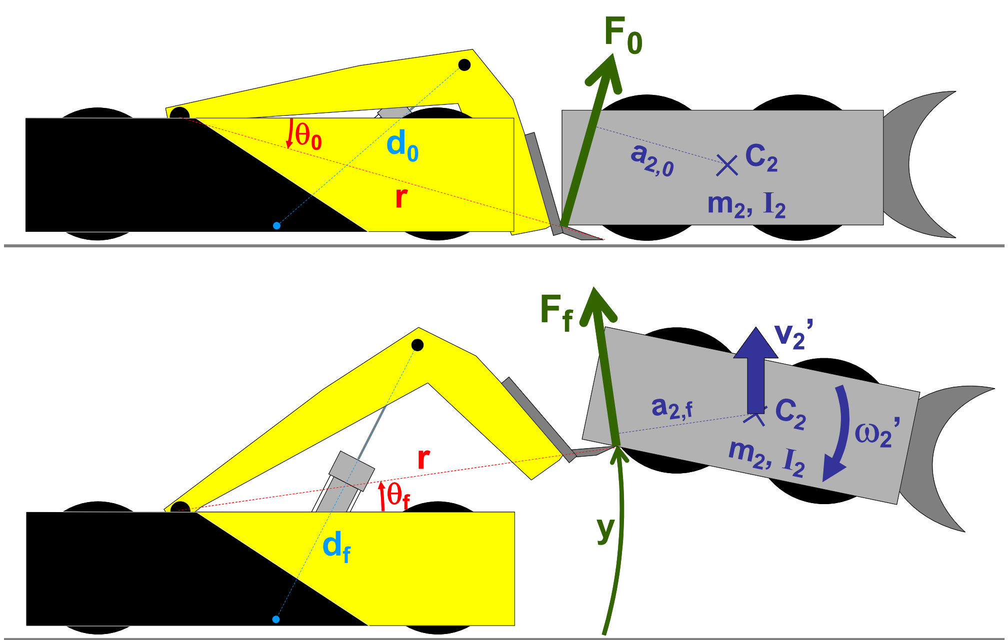
The figures in the next page show 4 different launcher configurations. The lightweight Rocket has both α_0 and θ_0 a little above –45°, making it a good "range launcher" due to the average 45° force it delivers. The only problem with this design is that it requires a high chassis to get a sufficiently long arm with θ_0 = –45°, decreasing its stability and making it more vulnerable to spinners.

The "height launchers" Bounty Hunter and T-Minus were able to lower their height when the arms are retracted, due to their low α_0, about 15° and 0°, respectively. Their average θ during the launch is close to zero, leading to an almost vertical force that allows them to throw the opponents very high in the air. But their low initial cylinder angle γ_0, below 45°, puts a lot of stress on the back pivot joint of the arm, initially trying to push forward the arm with almost the same force used to launch the opponent. This forward force, which tries to rip off the back pivot, is not necessarily wasted because it does not produce work. But this added force increases the friction losses in the joints. This is the price to pay for a low profile launcher.



Toro (pictured above), on the other hand, has γ_0 close to 90°, not stressing too much the arm pivot. It is also a "height launcher" because its average θ is close to zero. But to be able to accommodate its relatively long cylinders with such high γ_0, it needed to increase its α_0 to about 45°, as shown in the picture, making its tall launcher arm vulnerable to horizontal spinners. Note the curved strap under Toro's arm that limits the stroke of the cylinders, avoiding their self-destruction when reaching their maximum stroke.

The launching calculations are more complicated than in the impact problem, because the arm does not hit the opponent, it shoves it. Therefore, the contact point between the launcher arm and the opponent may move during the launch. The figure below shows an opponent being launched.



When the launching starts, the contact point is usually located further in the back of the arm scoop (at a distance smaller than r from the back pivot of the arm), and very far from the opponent's center of mass C_2. The initial launching force F_0 might have a direction θ_0 with respect to the vertical, defining the distance $a_{2,0}$ to C_2. When the contact ends, the contact point of the final force F_f will probably have shifted to the tip of the arm scoop (at a distance r from the back pivot), defining a different distance $a_{2,f}$ to C_2.

The intensities of F_f and F_0 are probably different, even if the force in the cylinder is constant during the entire launch, because of the different mechanism configurations for angles θ_0 and θ_f. Their directions are also different, F_f might have a direction that makes the angle θ_f with respect to the vertical if there's enough contact friction at the arm tip, or it might be perpendicular to the opponent's bottom plate if there's no contact friction, or it may have some direction in between.

Also, finding out at which value of the path length y the contact will end is not simple, it depends a lot on the opponent's mass m_2 and moment of inertia I_2 in the direction it ends up spinning. Only one thing is certain: it is that the kinetic energy you may induce in the opponent cannot be higher than the energy delivered by the pneumatic cylinder, $E_b = p \cdot A \cdot (d_f - d_0)$. In practice, this theoretical value is not reached because of friction and pneumatic losses, gravity effects, and because of the inertia of the launcher arm, which needs to be accelerated as well.

Also, since the opponent usually tends to rotate away from the launcher arm during the launch, it is more efficient to maximize the forces than the path length y, to increase the delivered work. A very large path y is not effective, because the contact between the robots will probably be lost before the end of the stroke of the pneumatic cylinder. It is better to have a cylinder with large diameter and high pressure, to increase the force, than to have a cylinder with very large stroke. The ideal stroke would be the one that ends slightly after the contact between the robots is lost. The straps, or other system that limits the cylinder movement, should also be dimensioned in this way.

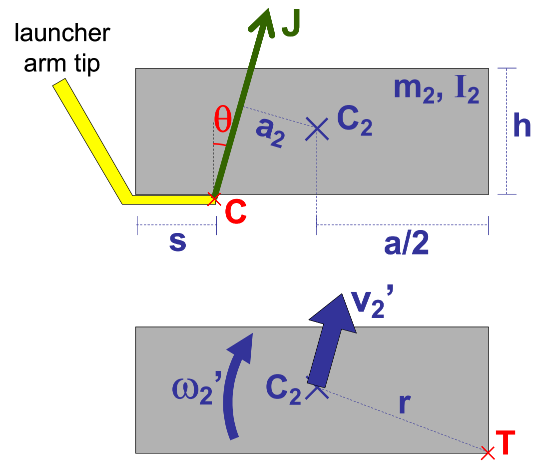
However, a short stroke cylinder does not necessarily mean it has a short overall length. For instance, a typical industrial 4" bore hydraulic cylinder, which can be adapted to pneumatic applications, has an overall retracted length of 9.625" plus the stroke length. Even if the cylinder stroke is only 1", its overall retracted and extended lengths 10.625" and 11.625" are relatively high. This short ratio between stroke and total length would not be effective in a three bar mechanism.

6.10.2. Launcher Equations

To properly simulate the launch, it is likely that you'll need some dynamic simulation software. Or you can use the spreadsheet from www.hassockshog.co.uk/flipper_calculator.htm, which has nice launcher models, as pointed out by Kevin Berry in the March 2009 edition of Servo Magazine. But you can also use a simple approximation to get a feeling of what happens during the launch.

Consider that the impulse J that the launcher inflicts on the opponent has an average direction θ with respect to the vertical, as pictured in the next page. The opponent is assumed as a homogeneous rectangular block with width a and height h. This value of a can be either the opponent's length or width, depending on whether you're launching it from its front/back or from its sides.

We also consider that the contact point between the launcher arm tip and the opponent does not shift during the launch, remaining fixed at a point C. This point is located by the distance s shown in the figure to the right, which is related to the length of the arm scoop.



This s value is also increased if the launcher is able to get under the opponent, as shown before in the action shots from the Ziggy vs. The Judge fight.

It is not difficult to calculate the distance a_2 between the impulse vector J and the opponent's center of mass C_2, and the distance r between C_2 and the edge T, they are obtained from

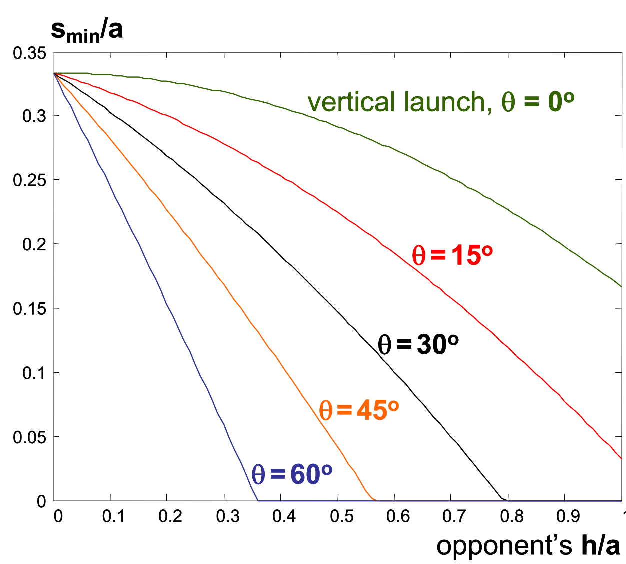
$$a_2 = (\frac{a}{2} - s)\cdot\cos\theta - \frac{h}{2}\cdot\sin\theta \quad \text{and} \quad r = \sqrt{(\frac{a}{2})^2 + (\frac{h}{2})^2}$$

If the opponent's mass is m_2, then its moment of inertia I_2, in the direction it spins due to the launch, can be estimated from the moment of inertia of rectangular bars, $I_2 = m_2 \cdot r^2/3$. If the point T in the figure does not touch the ground during the launch, and if the launching force is much higher than gravity, then we can estimate that the opponent's speed v_2', parallel to J, reaches J/m_2, and the angular speed results in $\omega_2' = J \cdot a_2/I_2$, leading to the relation $\omega_2' = v_2' \cdot a_2 \cdot m_2/I_2$ (see the figure above).

The vertical component of the speed at point T, equal to $v_2' \cdot \cos\theta - \omega_2' \cdot a/2$, must be positive to validate this analysis, making sure that this point does not touch the ground during the launch. So, from the previous equations, the point T does not touch the ground if s is greater or equal than a minimum value s_{min}, where

$$a_2 \le \frac{2 \cdot I_2 \cdot \cos\theta}{m_2 \cdot a} \quad \Rightarrow \quad s \ge s_{min} = \frac{a}{3} - \frac{h^2}{6a} - \frac{h}{2}\cdot\tan\theta$$

This resulting relationship between s_{min}/a as a function of the opponent's aspect ratio h/a is plotted to the right. Note from the graph that, to launch an opponent without making it touch the ground, a "height launcher" ($\theta \cong 0^o$) would need a higher scoop length s than a "range launcher" (which usually has $\theta = 45^o$).



In theory, from the gravity potential energy equation, the maximum height an opponent would achieve would be, in theory, $H_{max} = E_b/(m_2 \cdot g)$, where E_b is the launching energy discussed before and g is the acceleration of gravity. But this height could only be reached if the opponent was vertically launched (thus $\theta = 0^o$) without spinning ($\omega_2' = 0$). If, ideally, the entire energy E_b is transformed into the opponent's kinetic energy (translational and rotational kinetic energies), we can show from the relation $\omega_2' = v_2' \cdot a_2 \cdot m_2 / I_2$, valid if T does not touch the ground, that

$$E_b = \frac{1}{2} m_2 \cdot v_2'^2 + \frac{1}{2} I_2 \cdot \omega_2'^2 = \frac{1}{2} m_2 \cdot \left(\frac{I_2 + m_2 a_2^2}{I_2} \right) \cdot v_2'^2, \quad \text{if } s \geq s_{min}$$

6.10.3. Height Launcher Equations

A "height launcher" with $\theta = 0^o$ would have $a_2 = a/2 - s$, while v_2' would be in the vertical direction. Since the maximum height H of a vertical launch is $v_2'^2/(2g)$, the opponent would reach

$$H = \frac{E_b}{m_2 \cdot g} \cdot \left(\frac{I_2}{I_2 + m_2 a_2^2} \right) = H_{max} \cdot \left(\frac{r^2}{r^2 + 3 \cdot (a/2 - s)^2} \right), \quad \text{if } s \geq s_{min}$$

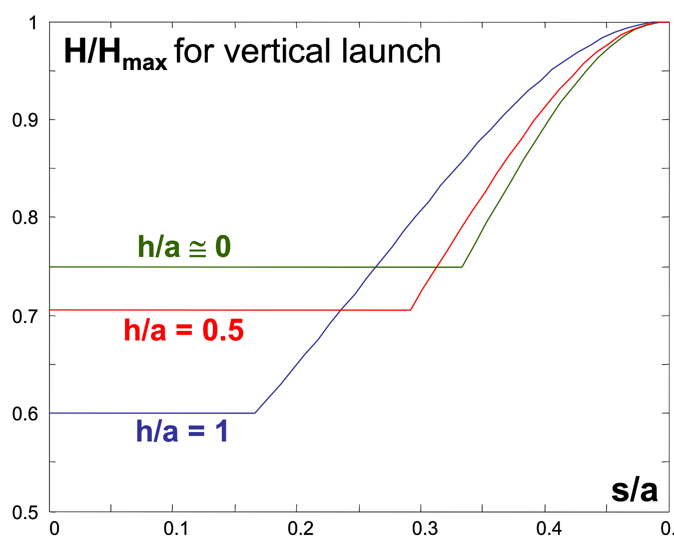
The above expressions are only valid if $s \geq s_{min}$. But if $s < s_{min}$, the point T touches the ground, which makes its vertical speed $v_2' \cdot \cos\theta - \omega_2' \cdot a/2$ equal to zero at the end of the impulse, leading to the relation $\omega_2' = 2 \cdot v_2' \cdot \cos\theta / a$.

In this case, v_2' may not be parallel to J, its direction will depend on the contact friction between point T and the ground. A frictionless contact would keep the horizontal component of v_2' unchanged, while a very high friction could significantly reduce it, making the direction of v_2' becomes steeper than the direction of the impulse J.

A "height launcher" with $s < s_{min}$ would have then $\omega_2' = 2 \cdot v_2' / a$, leading to

$$v_2'^2 = \frac{2 \cdot E_b}{m_2} \cdot \left(\frac{m_2 a^2/4}{I_2 + m_2 a^2/4} \right) \quad \Rightarrow \quad H = \frac{v_2'^2}{2 \cdot g} = H_{max} \cdot \left(\frac{3}{4 + h^2/a^2} \right), \quad \text{if } s < s_{min}$$

Note that, for such small s, the launch height H only depends on the opponent aspect ratio h/a.



The graph to the right shows the H/H_{max} ratio as a function of the normalized length s/a, against opponents with aspect ratios h/a. Note that the horizontal lines are the values obtained for $s < s_{min}$, while the curved ones reflect the results from $s \geq s_{min}$.

The horizontal lines suggests that scooping the opponent with $s/a = 0.15$ results in the same height as if s/a was close to zero 0. This is true, at least for the simple model we're using, as long as the contact between the robots is maintained during the launch, which depends not only on s/a but also on the path direction and length y of the arm tip. Obviously, $s/a = 0.15$ would be better to maintain this contact than s/a close to zero.

So, from the graph we conclude that the most effective launch happens when s is close to a/2. One way to do that is to try to launch the opponent from the direction where it is shortest. So, for instance, against a narrow robot, try to launch it from its side, making the distance a become its width instead of length. Getting under the opponent is also one way to increase s, as done by Ziggy with its front wedge.

A long scoop at the end of the launcher arm (as seen in the middleweight Sub Zero, pictured to the right) also helps to increase the distance s. A long scoop will also make sure that the contact between the robots won't be lost during the entire stroke of the launcher arm. But be careful, a very long scoop will be vulnerable to drumbots and undercutters, which may bend it until it loses functionality, not being able to get under robots with low ground clearance. In addition, the previous graph showed that, against very low profile opponents (small h/a), there's no point in having a very long scoop to increase H unless it has s/a greater than 0.35 or if the weapon tip path y is too large. If below 0.35, the value of s/a would only have to be large enough not to lose contact with the opponent during the launch.

Note that the above calculations assumed that the launcher didn't tilt forward too much during the launch, which could make it get unstable. The requirements to guarantee launcher stability are presented in section 6.10.6.

6.10.4. Range Launcher Equations

A similar analysis can be made for a "range launcher" as follows. If the opponent does not touch the ground while it is launched, which happens when $s \geq s_{min}$, then the launch speed v_2' is parallel to the impulse J. The launch angle, with respect to the horizontal, is then $90^o - \theta$. The horizontal range R of the launch is then

$$R = \frac{v_2'^2 \cdot \sin[2(90^o - \theta)]}{g} = \frac{2E_b \sin 2\theta}{m_2 \cdot g} \cdot \left(\frac{I_2}{I_2 + m_2 a_2^2} \right) = R_{max} \cdot \sin 2\theta \cdot \left(\frac{r^2}{r^2 + 3 \cdot a_2^2} \right), \quad \text{if } s \geq s_{min}$$

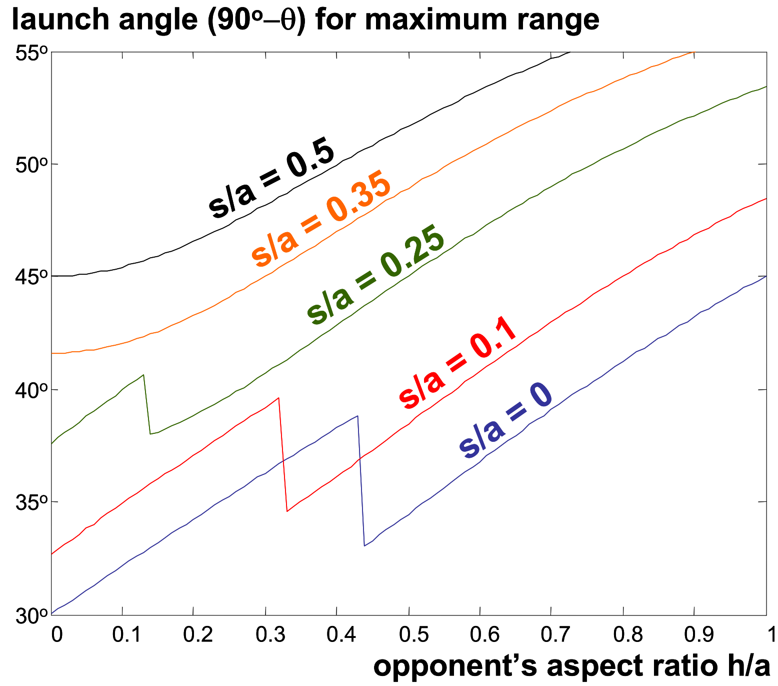
where $R_{max} = 2E_b/(m_2 \cdot g)$ is the maximum possible launch range, which only happens when $a_2 = 0$ (the impulse vector passes through C_2) and $\theta = 45^o$. The launch angle that maximizes R when a_2 is different than zero must be calculated with a numeric method, because a_2 depends on a, s, h and also θ.

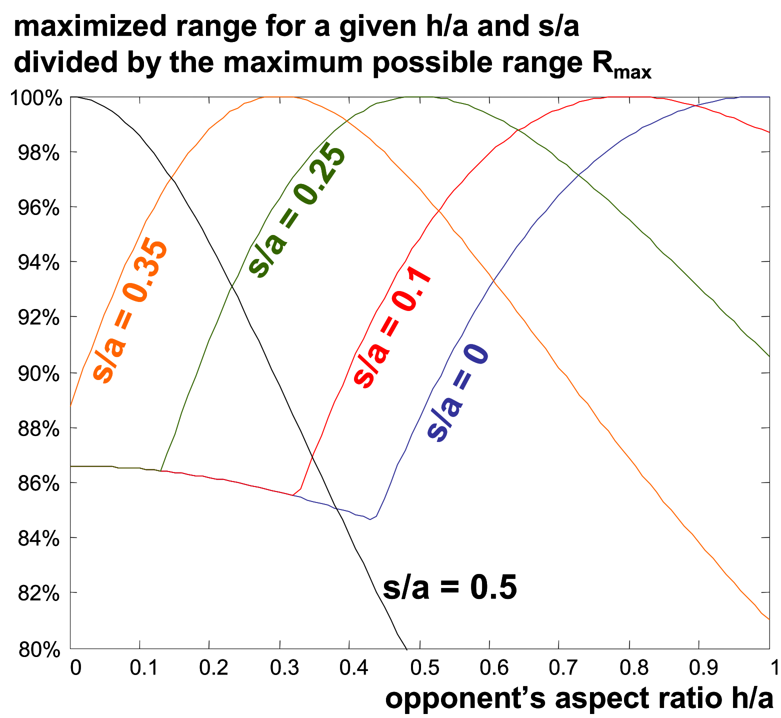
On the other hand, if the opponent touches the ground at point T while it is launched (which happens when $s < s_{min}$), then the equations get much more complicated, because the ground reaction at T will cause a vertical impulse, and also a horizontal one if there's ground friction. This will change not only the magnitude of v_2' but also the launch angle. Even without considering ground friction, the equations are too lengthy to be shown here.

But the results are seen in the graphs to the right, obtained neglecting the effect of ground friction at point T during the launch. The neglected friction effect would only be significant if both θ and a_2 were large and if both s and h were very small, therefore it does not significantly influence the conclusions that are presented next.

The top graph shows the ideal launch angle (90°–θ) of the arm to maximize the range for a given s/a and aspect ratio h/a of the opponent. Against very low profile opponents (h/a close to zero), the best launch angle is 45° if you're able to reach s/a = 0.5, as expected, reaching 100% of the maximum possible range R_{max}, as seen on the bottom graph.

But if you can't get under such very low profile opponent and your arm has a very short scoop (s/a close to 0), then the best launch angle to maximize the reach would be 30° (with respect to the horizontal). This might seem strange but it makes sense: in this case, the ground impulse at point T, which happens due to the small s/a, will add up to the launcher's 30° impulse to effectively launch the opponent at 43.7°. As seen in the bottom graph, this best 30° angle will achieve 86.6% of R_{max}, which is still pretty good considering that s/a is so small. It would be impossible to reach 100% of R_{max} with s/a = 0. Unless the opponent was very tall, with h/a = 1, but then the best launch angle would be 45°.





Note that there is a step in the top graph for the curves with s/a < 0.33. This change (or step) is associated with the opponent touching the ground (values to the left of the step) or not touching it (values to the right of the step) during the launch. For instance, if s/a = 0, then an opponent with aspect ratio h/a = 0.4 (a value to the left of the step in the s/a = 0 curve) would be thrown further away if launched at 38^o, using the ground to help in its launch.

But if h/a = 0.5 (to the right of the step), then a shallower angle of 34^o would decrease the angular speed of the opponent, making it not touch the ground, resulting in the optimal throw in this case. So, depending on h/a and s/a, it may be good or not to use the help of the ground to launch the opponent with maximized range. This conclusion is not trivial at all.

Since most combat robots tend to have a low profile, with 0.2 < h/a < 0.4, to keep low their center of gravity, we can draw several conclusions about "range launchers" as follows. If the launcher can only provide a small s/a, then it is better to set its arm such that its impulse is in average at about $(90^o-\theta) = 36^o$ from the horizontal, to reach a maximized range of about 85% of R_{max} (in this case, the opponent would touch the ground during the launch, because $s < s_{min}$).

If the arm scoop allows the launcher to reach s/a = 0.25, then a steeper $(90^o-\theta) = 41^o$ angle would be a better option, typically launching the opponent between 91% and 99% of R_{max} (in this case, the opponent does not touch the ground because $s > s_{min}$).

Finally, if the launcher has a wedge to get under the opponent, or if you decide to use a very long scoop in its arm, then the best choice would be to use s/a = 0.35 and a $(90^o-\theta) = 45^o$ launch angle. With these values, you can launch typical opponents between 99% and 100% of R_{max}. This s/a and $(90^o-\theta)$ combination makes the distance a_2 become very close to zero for most robots, spending most of the launch energy throwing the opponent instead of making it spin.

Note, however, that these ideal launch angles to maximize range are only achievable in practice if they do not cause the launcher to be pushed too much backwards by the reaction force. The requirements to avoid this are presented in section 6.10.6.

Combinations of s/a, h/a and θ that lead to high values of a_2 should be avoided, because they waste too much energy making the opponent spin forward. Also, it is not a good idea to go beyond s/a = 0.35, you'll probably end up with a negative value of a_2, which will waste energy spinning the opponent backwards. The pictures in the next page show three different launch situations. The first one shows the super heavyweight Ziggy launching an opponent with a high average distance a_2 from C_2, resulting in a forward spin. In the second situation, Ziggy is able to launch The Judge with an impulse vector very close to C_2 (therefore a_2 close to zero), resulting in a high range due to the much lower resulting spin. Finally, the lightweight Rocket is able to launch the opponent with a backward spin, because the contact point was beyond the opponent's C_2, making a_2 become negative. Ideally, you should try to keep the opponent's spin as low as possible to maximize the launch range.

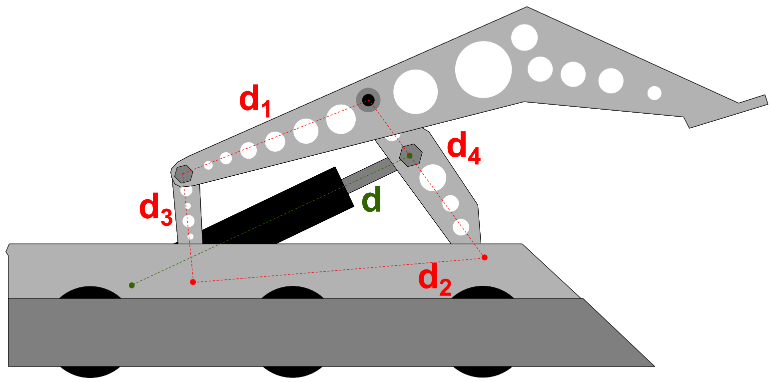
Launching with forward spin, due to the high a_2

High range launch because of the almost neutral spin, due to a_2 close to zero

Launching with backward spin, due to a negative a_2

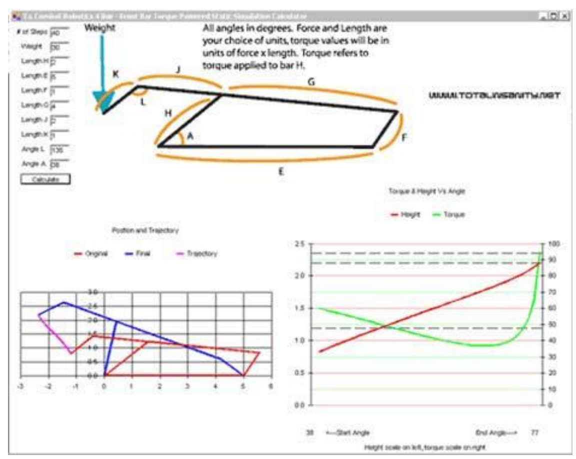
6.10.5. Four-Bar Mechanisms

As discussed before, the problem with most “range launchers” with three-bar mechanisms is that they usually end up with a tall chassis if they want to throw opponents at the ideal angles ($90^o – \theta$) from 36^o to 45^o. The tall chassis is a result of their mechanisms, which would need an average θ between 54^o and 45^o.



One alternative is to use four-bar mechanisms, as pictured to the right. The four bars consist of part of the launch arm (d_1), part of the chassis (d_2) and two auxiliary links (d_3 and d_4). Technically, these launchers have a five-bar mechanism, because the pneumatic cylinder (d) counts as a fifth link.

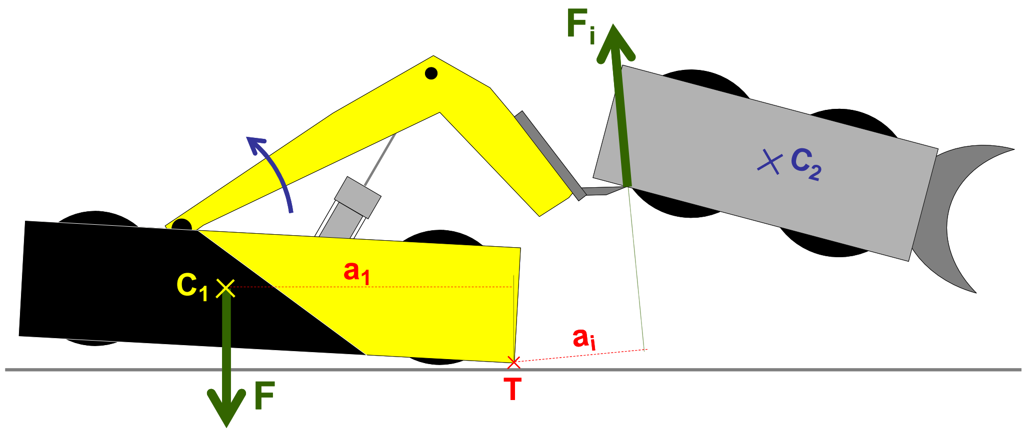
Four-bar mechanisms have two advantages. First, if well designed, they can be completely retracted inside the robot, allowing the use of a low profile chassis. And, if the constant lengths d_1, d_2, d_3 and d_4 are appropriately defined, it is possible to generate optimal trajectories for the arm tip. You can, for instance, make the arm tip trajectory become almost a straight line, with some desired optimal angle (which for “range launchers” would probably be between 36^o to 45^o with respect to the horizontal). The four-bar mechanism calculations are too lengthy to be shown here, but you can make them using, for instance, a free static simulation program (screenshot pictured to the right) that can be found on the tutorials in http://www.totalinsanity.net.



6.10.6. Launcher Stability

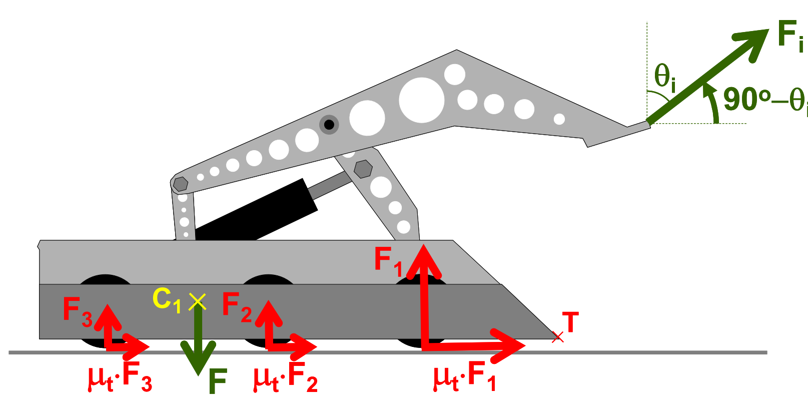
During the launch, a launcher should neither tilt forward too much, nor be pushed backwards, otherwise it will lose its effectiveness.

Due to the very high forces involved during the launch, it is very likely that “height launchers” will tilt forward until they touch the ground at their foremost point T, as shown in the figure in the next page as well as in the action shot to the right, featuring Sub Zero launching The Mortician. To avoid tilting forward even more and becoming unstable, it is necessary to locate the launcher’s center of gravity C_1 as far back as possible, to maximize the horizontal distance a_1 to point T, as pictured below. If F is the launcher weight, then the force F_i at any moment during the launch must satisfy $F_i \cdot a_i < F \cdot a_1$, where a_i is the distance between T and the line that contains the vector F_i. It is advisable that this condition is satisfied during the entire launch, for all values of F_i between F_0 and F_f, multiplied by their respective distances to point T.



Another concern with "height launchers" is with the stiffness and damping properties of their front wheels. During the launch, these wheels are very much compressed against the ground, storing a great deal of elastic energy. Towards the end of the launch, when the contact with the opponent is almost lost, these wheels will spring back. If their damping is low, as in foam-filled rubber wheels, they may launch the launcher and even make it flip backwards. The action shot to the right shows the launcher Sub Zero off the ground as soon as it loses contact with its opponent The Mortician.

"Range launchers" may tilt forward as well, but it is very unlikely that they lose stability in this way. This is because the line that contains the launch force vector F_i usually meets the ground within the launcher footprint, or very close to is foremost point T, due to the shallower launch angles ($90^o - \theta_i$) involved, see the picture to the right.



But "range launchers" have a problem with very shallow launch angles, because the horizontal component $F_i \cdot \cos(90^o - \theta_i)$ may become too large for the wheel friction to bear, pushing it backwards. As seen in the figure, if F is the launcher weight, F_1, F_2 and F_3 are normal ground forces on each wheel pair, and μ_t is the coefficient of friction between the tires and the ground that cause the maximum friction forces $\mu_t \cdot F_1$, $\mu_t \cdot F_2$ and $\mu_t \cdot F_3$, then

$$\left.\begin{aligned} F_1 + F_2 + F_3 = F + F_i \cdot \sin(90^o - \theta_i) \\ \mu_t \cdot F_1 + \mu_t \cdot F_2 + \mu_t \cdot F_3 \geq F_i \cdot \cos(90^o - \theta_i) \end{aligned}\right\} \Rightarrow \mu_t \geq \frac{F_i \cdot \cos(90^o - \theta_i)}{F + F_i \cdot \sin(90^o - \theta_i)}$$

This equation shows that tire friction is very important to avoid "range launchers" from being pushed backwards, probably decreasing the contact time with the opponent and the effectiveness of the launch. A typical high traction tire with $\mu_t = 0.9$ is usually enough for a $(90^o - \theta_i) = 45^o$ launch angle. But for the 36^o or 41^o angles, which maximize the launch range for small s/a values, you might need a higher μ_t. If this higher μ_t is not achievable, then the best option is to adopt the lowest launch angle $(90^o - \theta_i)$ that satisfies the μ_t condition above.

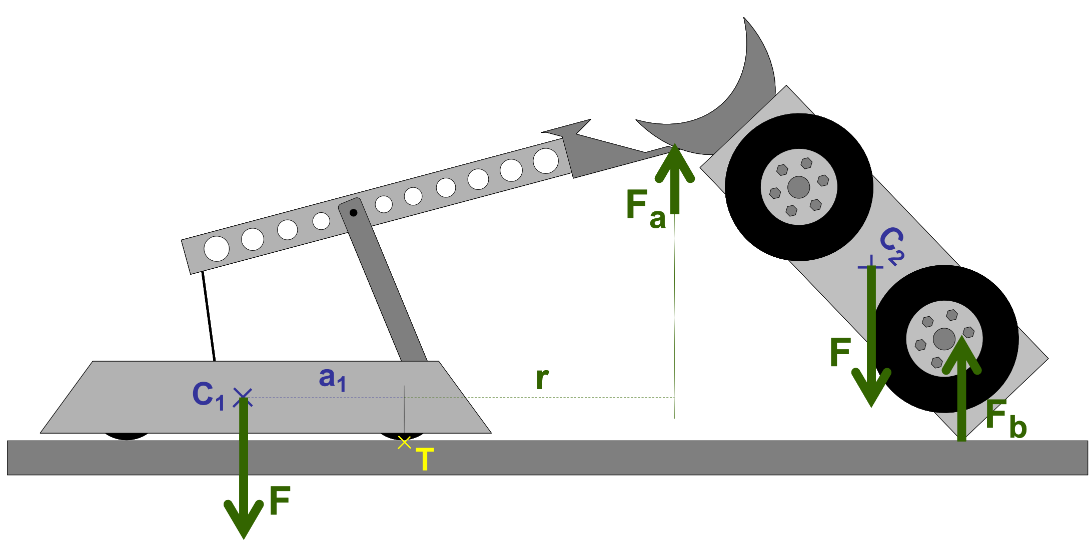
Finally, note that it is also a good idea for "range launchers" to locate C_1 as far back as possible, to prevent them from even touching their foremost point T on the ground, because this point will probably have a coefficient of friction with the ground lower than μ_t. If some tilting is inevitable, then it might be a good idea to install some anti-sliding material on the bottom of the robot beneath point T, such as a rubber strip, to increase friction.

6.11. Lifter Design

Lifters and launchers have a few common design features. The main difference is that lifters have relatively slow lifting mechanisms, allowing them to use, for instance, highly geared electric motors. The use of electric motors instead of pneumatic systems usually results in less weight to the weapon system, allowing them to improve their armor or drivetrain.

Lifters work better against non-invertible robots, or on arena with hazards where they can shove the opponent to. Clearly, they must have enough wheel traction to be able to push the opponent around the arena while it is lifted.

Similarly to launchers, a four-bar mechanism (as pictured below) is a good option to allow the lifting arm to retract inside the robot chassis, becoming less vulnerable to spinners.



To be able to lift their opponents without losing stability, it is important to locate their center of gravity C_1 as far back as possible, to maximize the distance a_1 to its foremost ground support (point T in the figure), usually located under the front wheels.

If F is the lifter weight, r is the maximum horizontal distance between point T and the tip of the lifter arm, and if F_a is the lifting force it applies on the opponent, then the gravity torque $F \cdot a_1$ with respect to point T must be higher than $F_a \cdot r$ to prevent it from tilting forward. The force F_a is a function of the weight of the opponent robot (also assumed as F, since both robots should be from the same weight class), of the force F_b from the opponent's ground support, and of the relative horizontal distances among them.

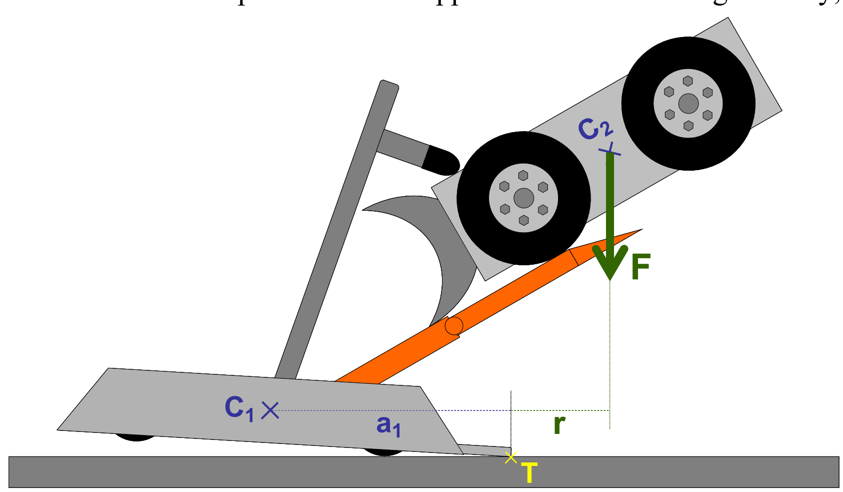
For a symmetric opponent, with a center of mass C_2 at the center of the chassis, it is easy to see that $F_a = F_b = F/2$ when the lifting begins, with the opponent in a horizontal position. As the opponent is lifted, F_b is increased while F_a decreases, as suggested by the picture. This is most noticed on tall opponents, which become easier to lift as they are lifted. So, the worst case scenario would be to consider that F_a is equal to its maximum value F/2, resulting in

$$F \cdot a_1 > \frac{F}{2} \cdot r \quad \Rightarrow \quad r < 2 \cdot a_1$$

It is not difficult to satisfy the above condition for the maximum horizontal reach r, so tilting forward is not a major concern for most lifters. Note that a lot of weight will be concentrated on the front wheels, up to 1.5 times the lifter weight in this example. So, make sure that the front wheels have high torque motors, to prevent them from stalling while pushing around the lifted opponent.

6.12. Clamper Design

Clampers are similar to lifters, except that they need to lift the entire weight F of the opponent, instead of just about half of it. To be able to clamp and lift their opponents without losing stability, they should also locate their center of mass C_1 as far back as possible. They usually need an extension on their front to act at point T as their foremost ground support, to increase the distance a_1 shown in the figure to the right.



If the lifter and opponent have same weight F, and r is the horizontal distance between point T and the opponent's center of mass C_2, then the gravity torque $F \cdot a_1$ with respect to point T must be higher than $F \cdot r$ to prevent the lifter from tilting forward, resulting in

$$F \cdot a_1 > F \cdot r \quad \Rightarrow \quad r < a_1$$

The above condition is much harder to meet than the stability condition for lifters, because here the distance r is not to the clamper tip, but to C_2. This distance to C_2, which is maximum when the opponent is starting to be lifted, can be very large for a long opponent. In addition, r must be smaller than a_1 for clampers, instead of $2 \cdot a_1$ as found for lifters. This is why tilting forward is a major concern for clampers, usually forcing them to use front extensions to increase a_1.

Similarly to lifters, clampers also need high torque on their front wheels to be able to drive around while carrying the opponent. Their front wheels have to bear up to twice the clamper weight, so make sure that their drive system is very sturdy and powerful.

6.13. Rammer Design

Rammers are usually nicknamed BMW, because they're basically made out of Batteries, Motors and Wheels. They must have a lot of traction to shove other robots around, and high top speeds to be effective as a ram.

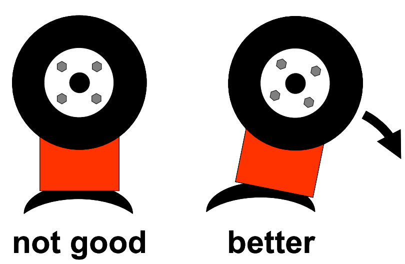
Its shield or armor is usually made out of hard materials, used in traditional armors. Ablative materials would also work, however they would have to be changed more often.

There are two design strategies to make them resistant to spinner attacks. Defensive rammers use shock mounts to attach their shield, trying to absorb and dissipate the energy of the attack. They can also use ablative shields for that.

Offensive rammers, on the other hand, have very hard shields rigidly attached to a stiff chassis, trying to divert the impact energy back to the attacker and break its weapon system. But remember to shock mount internal critical components.

Needless to say that taking the hit is not necessarily the best strategy: if you're able to push around a spinner without getting hit by its spinning weapon, it may self-destruct after hitting the arena wall.

Rammers should always be invertible. A few of them, such as the middleweight Ice Cube (pictured to the right), are even capable of righting themselves using only the power of their wheels, similarly to an overhead thwackbot.



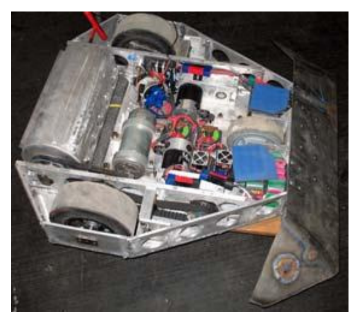
One issue with rammers with large shields is to avoid getting "stuck on their nose" (as pictured to the right). Ice Cube faces this problem, even though it is able to rock back and forth using the inertia of its wheels to flip back on its feet after a few seconds (we wish we had known that before the RoboGames 2006 semifinals!).

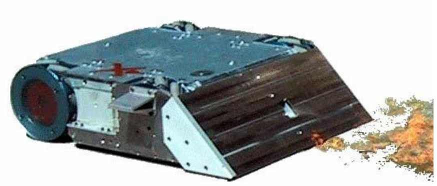
But, during the seconds it is rocking, the (orange) chassis gets exposed to attacks. Also, if the rammer is gently pushed while on its nose against the arena walls, it won't be able to get unstuck by rocking. One possible solution to avoid getting stuck is to mount the shield in a slightly asymmetrical position, as pictured above. With the ground projection of the rammer center of gravity closer to (or beyond) the edge of its shield, it becomes much easier to flip back.

6.14. Wedge Design

Wedges have a very simple and effective design, especially against spinners. They try to use the opponent's energy against it, with the aid of a ground support and an inclined plane (their wedge). The inclined plane idea is so simple and effective that it was used on one of the first combat wedges in history, Leonardo Da Vinci's tank, pictured to the right. Its 35^o sloped conical armor, covered with steel armor plates, would work as a wedge to deflect enemy fire from all sides. It was not built back in 1495 because Italian battlefields were not flat enough to allow it to move without getting stuck. We had to wait 500 years, in 1995, to see a combat wedge on a flat "battlefield", with La Machine (pictured to the right), coincidently with the same slope as Leonardo's tank, taken from his original XV century drawing.

The wedge concept is not limited to wedges. As pictured below, it can be seen in vertical spinners (such as K2), horizontal spinners (Hazard), drumbots (Stewie), lifters (Biohazard), launchers (Ziggy), spearbots (Rammstein), hammerbots (The Judge), overhead thwackbots (Toe Crusher), and even combined with flamethrowers (Alcoholic Stepfather).

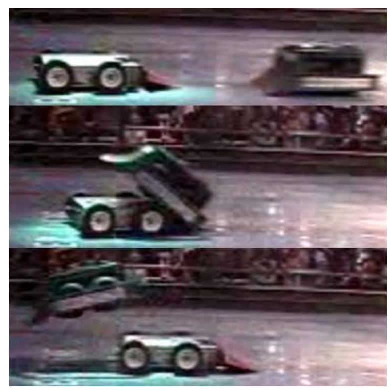




6.14.1. Wedge Types and Shapes

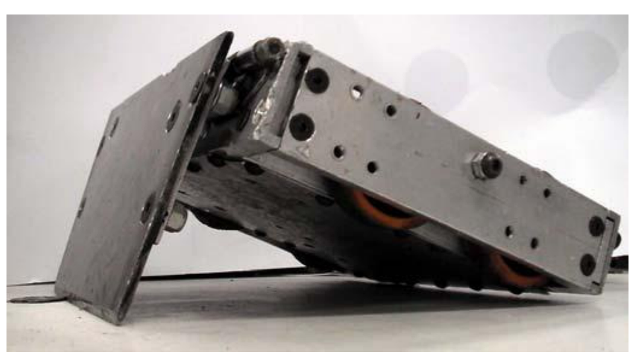
Fixed wedges, without any articulation, can either have some ground clearance, or they can be supported by the ground, scraping it. The first type might work well against most horizontal and vertical spinners, but it is vulnerable to undercutters or to a lower wedge.

The fixed wedges that scrape the floor, on the other hand, do not have this vulnerability, but they need to have a very sharp edge to stay flush to the ground. If they carry a significant portion of the robot's weight, they'll be much more difficult to get under due to the increased downward pressure. But, on the other hand, the robot might lose traction due to the decreased weight under the active wheels. Another way to increase the downward pressure is to decrease the ground contact area of the wedge, as done by the middleweight Emily with its narrow frontal titanium insert (pictured to the right).



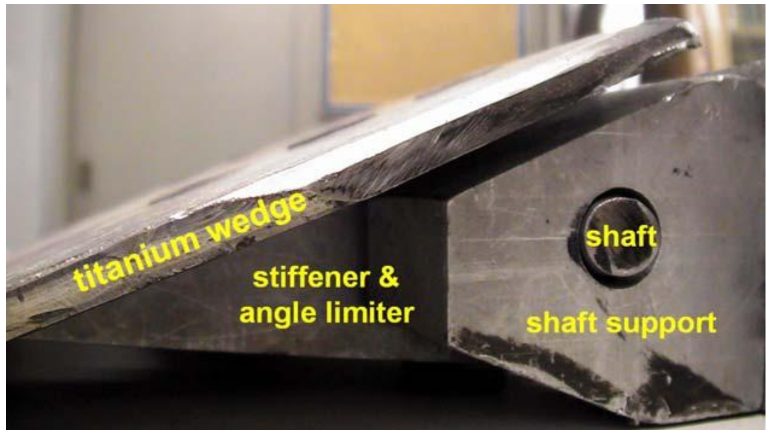
Articulated wedges are probably the most popular, resulting in a virtually zero ground clearance if their edge is sharpened. Floppy wedges, which have articulations that are not actively powered, should be heavy enough to increase the downward pressure at their sharp edge. On the other hand, active wedges, which have a powered articulation to make them work as lifters, can have their downward pressure increased by their motors just before they hit the opponent. This strategy has worked very well for the heavyweight lifter Sewer Snake, as pictured to the right.

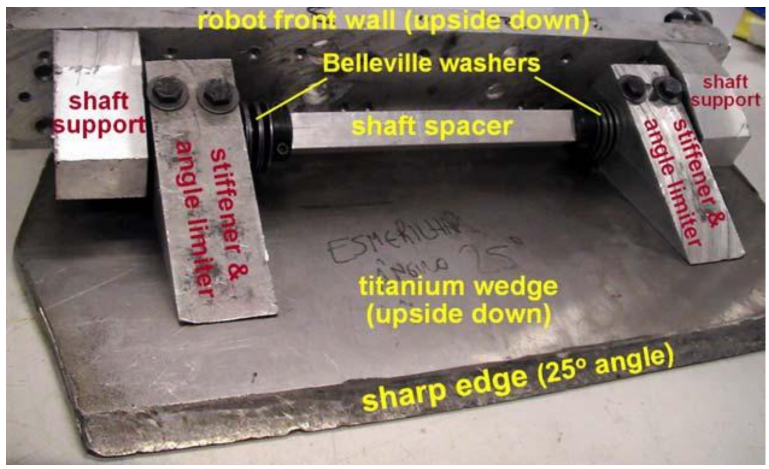
Downward pressure can also be achieved by mounting springs to the robot's walls, keeping the wedges spring loaded flush to the ground. The picture to the right shows a nice wedge from the spinner Hazard. Note that, besides the spring, there are also two triangular-shaped supports underneath the wedge. These supports work as angle limiters, preventing the wedge from articulating too much and lifting the robot's own wheels off the ground, as well as working as stiffener brackets. Note also the rubber sandwich mounts used as dampers, improving the resistance to spinner impacts.



Wedge angle limiters are very important, especially if the robot has internal wheels. The picture to the right shows that, if the angle limiters from our hobbyweight Puminha are removed, its titanium wedge may get stuck and prevent the internal rear wheels from touching the ground, immobilizing the robot.

Both pictures to the right show a sturdy Ti-6Al-4V titanium articulated wedge used by our middleweight horizontal bar spinner Titan. Besides the high strength aircraft aluminum angle limiter, which also works as a stiffener, the wedge features Belleville washers on its titanium articulation shaft that work as shock mounts against lateral impacts.





Note that this 32^{o} wedge has a shallower 25^{o} sharp edge, to make sure that only the tip of the edge touches the ground, increasing the downward pressure. Note also that this wedge wouldn't be very effective if upside down, as shown in the bottom picture. This is why we only use it in non-invertible robots such as Titan.

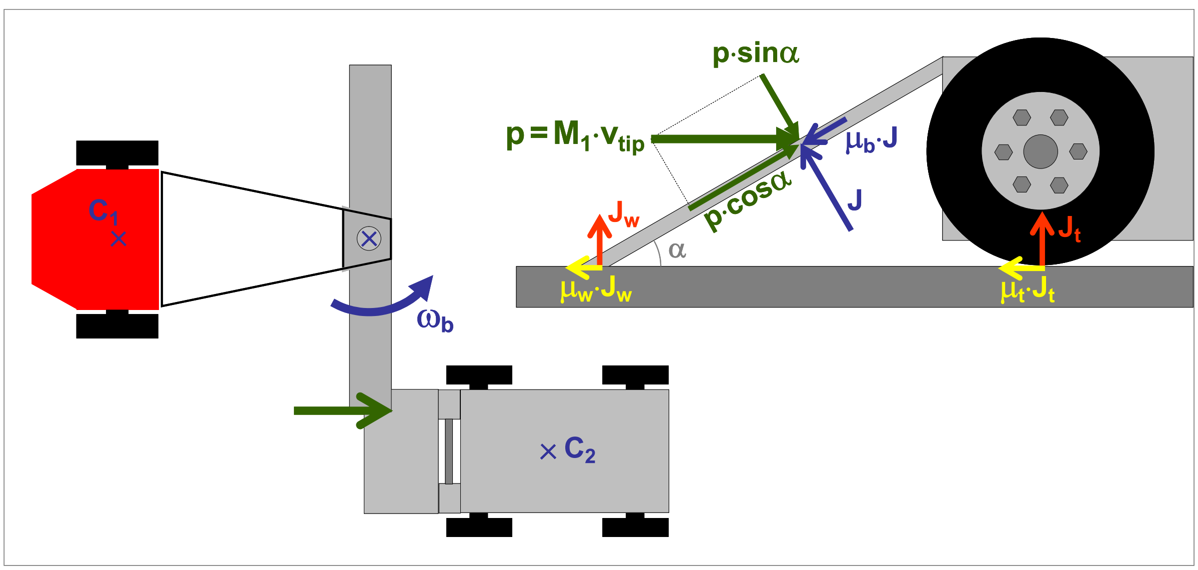
For invertible robots that don't have self-righting mechanisms, it is a good idea to have a symmetric wedge, which has the same effectiveness on either side. For instance, the heavyweight Original Sin has a hollow wedge made out of two rectangular plates separated by triangular spacers/stiffeners (pictured to the right), resulting in a symmetric wedge with a high stiffness to weight ratio.

But, unless your robot has external wheels such as Original Sin, or its wedge is actively powered, avoid using rectangular wedges, they can easily get your robot stuck resting on its side. Instead, use trapezoid-shaped wedges, either with a narrower edge (as pictured to the right) or with a wider edge. It is very unlikely that an internal-wheeled robot gets stuck resting on its side if it has a trapezoid-shaped wedge. A trapezoid-shaped wedge with a narrower edge will probably allow the robot to get unstuck and fall back, as long as there are angle limiters (which were removed before taking the picture to the right). A trapezoid wedge with wider edge will also work, but there's a greater chance that the robot will fall back upside down, which would be a problem if it does not have an invertible design.

Finally, wedges with blunt edges should be avoided, because they're usually vulnerable to sharp anti-wedge skids, such as the S7 steel ones that support the drum of our hobbyweight Tourinho (pictured to the right). These narrow skids are able to concentrate a significant amount of downward pressure on the ground, in special if they're properly sharpened, easily getting under a blunt wedge.

6.14.2. Wedge Impact

One of the most important features of a wedge is its slope, represented by an angle α with respect to the horizontal. To find optimum α values, it is necessary to study the effect of an impact caused by a weapon that has an effective horizontal linear momentum $p = M_1 \cdot v_{tip}$, where M_1 is the effective mass of the attacker robot and v_{tip} is the speed of its weapon tip. Initially, let's assume that the momentum p is perpendicular to the direction of the wedge's edge, as if facing a horizontal spinner or thwackbot at the side where the weapon tip approaches the wedge, as pictured below, or if frontally attacked by a spearbot.



The impact of the horizontal spinner (or thwackbot or spearbot) will cause a reaction impulse J normal to the wedge surface, in response to the $p \cdot \sin\alpha$ component, as shown in the figure. If the coefficient of restitution (COR) of the impact is e, and assuming that v_{tip} is much higher than the speed of the wedge (either before or after the impact), then $J \cong (1+e) \cdot p \cdot \sin\alpha$.

The wedge also responds with a friction impulse $\mu_b \cdot J$ parallel to its surface, decreasing the $p \cdot \cos\alpha$ component as the weapon slides during the impact, where μ_b is the friction coefficient between the weapon tip and the wedge. So, instead of $p \cdot \cos\alpha$, the wedge will only feel $\mu_b \cdot (1+e) \cdot p \cdot \sin\alpha$ parallel to its surface. Because of that, the wedge will effectively receive a horizontal impulse J_x that is smaller than p. It will also respond with a vertical impulse J_y that will try to launch the horizontal spinner, where

$$\begin{cases} J_x = J \cdot \sin\alpha + \mu_b \cdot J \cdot \cos\alpha = (1+e) \cdot p \cdot \sin\alpha \cdot (\sin\alpha + \mu_b \cdot \cos\alpha) \\ J_y = J \cdot \cos\alpha - \mu_b \cdot J \cdot \sin\alpha = (1+e) \cdot p \cdot \sin\alpha \cdot (\cos\alpha - \mu_b \cdot \sin\alpha) \end{cases}$$

As seen from these equations, it is very important that the wedge is very smooth, to decrease μ_b and thus minimize the backward impulse J_x, while maximizing the "launch impulse" J_y. And the wedge material must also be very hard to avoid dents, which could stick to the weapon tip and make the wedge suffer along its surface the entire component $p \cdot \cos\alpha$ instead of only $\mu_b \cdot (1+e) \cdot p \cdot \sin\alpha$.

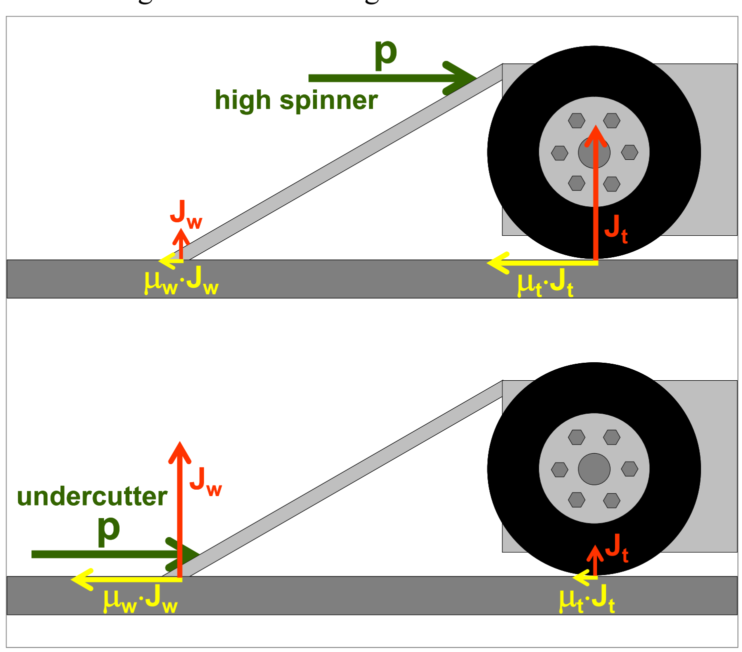
Hardened steels would be a good choice to avoid dents, however their stiffness-to-weight and toughness-to-weight ratios are not nearly as good as Ti-6Al-4V titanium, as seen in chapter 3. Since this grade 5 titanium is also relatively resistant to dents, due to its medium-high hardness, it is the material of choice for wedges. A hardened steel weapon tip would have $\mu_b \cong 0.3$ against a very smooth Ti-6Al-4V wedge, or up to $\mu_b \cong 0.5$ against a very rough and battle-battered Ti-6Al-4V wedge.

6.14.3. Defensive Wedges

A defensive wedge has the objective to resist the attack without being thrown backwards, even if it is standing still and not charging the attacker. This can happen if the horizontal impulse J_x that tries to push the wedge backwards is smaller than the vertical impulse J_y multiplied by the coefficient of friction μ with the ground, resulting in

$$J_x < \mu \cdot J_y \quad \Rightarrow \quad (\sin\alpha + \mu_b \cdot \cos\alpha) < \mu \cdot (\cos\alpha - \mu_b \cdot \sin\alpha) \quad \Rightarrow \quad \tan\alpha < \frac{\mu - \mu_b}{1 + \mu \cdot \mu_b}$$

Note that we've neglected above the effect of the robot weight on the friction impulse, because the impact is usually so fast that its forces are much higher than such weight.



If attacked by a "high spinner" as shown in the picture to the right, with the linear momentum p in the upper part of the wedge, then most of the reaction impulse will be provided by the front tires, resulting in $J_x \cong \mu_t \cdot J_t$ and $J_y \cong J_t$, where μ_t is the coefficient of friction between the front tires and the ground, and J_t is the vertical impulse from the ground to both tires altogether.

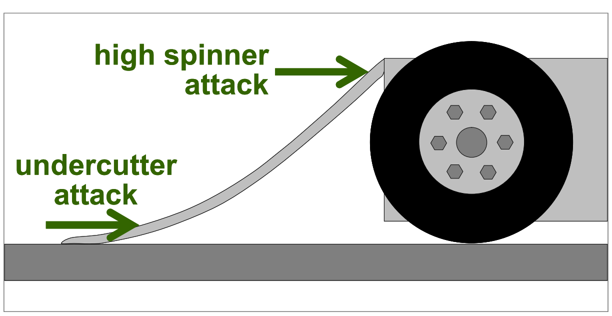
Assuming a typical high traction tire with $\mu = \mu_t = 0.9$, then a smooth titanium wedge with $\mu_b \cong 0.3$ would need to have about $\alpha < 25^o$ to be considered as a defensive wedge, while a rough wedge with $\mu_b \cong 0.5$ would only be defensive if its slope $\alpha < 15^o$. Note, however, that it is not a good idea to use a very small α, such as $\alpha < 20^o$, because it would lower too much the average thickness (and therefore the strength) of the sharp edge of the wedge.

On the other hand, against an undercutter, the linear momentum p is very close to the wedge's edge, making $J_x \cong \mu_w \cdot J_w$ and $J_y \cong J_w$, where μ_w is the coefficient of friction between the wedge and the ground, and J_w is the vertical impulse from the ground to the wedge's edge, as seen in the figure. Assuming a coefficient of friction $\mu = \mu_w = 0.35$ between the titanium edge and the soft steel arena floor, the equation shows that any $\alpha > 3^o$ would allow the robot to be thrown backwards, even if it had a very smooth wedge.

So, in theory, no wedge can be considered defensive against an undercutter: the wedge robot can defend itself from the first attack, but it will be thrown backwards or get spun, making it vulnerable to an immediate second attack. It is up to the wedge driver to keep facing the undercutter at all times.

But it is possible to have a defensive wedge against undercutters. Choosing a very small α is not a good idea, because the resulting low average thickness of the edge would allow the wedge to be easily torn apart by the undercutter, which would hit it at its weakest spot. Perhaps a good idea would be to spread some anti-slip product, such as anti-slip v-belt spray, under the edge where the wedge touches the ground, significantly increasing μ_w. Clearly, the spray should only be applied before matches against undercutters.

Another idea is to use a wedge with variable slope, similar to a scoop, with a lower α near the edge and a higher α near the top, to be effective against both undercutters and high spinners, as pictured below. Due to the variable slope, it would be possible to have an edge with a low α without compromising is thickness and strength.



A defensive wedge may be a good option if the robot also has some active weapon. The wedge could be used to defend the robot from opponents' attacks, or to slow down their spinning weapon, until the active weapon had a chance to get in action against them.

But a defensive wedge by itself would have a hard time winning a fight by knockout. You would need then an offensive wedge.

6.14.4. Offensive Wedges

Offensive wedges have the objective to launch their opponents, as pictured to the right. To most effectively accomplish that, they need to maximize the vertical impulse J_y, which happens for some $\alpha = \alpha_{launch}$ the makes the derivative $dJ_y/d\alpha = 0$, resulting in

$$\frac{dJ_y}{d\alpha} = (1+e)\cdot p\cdot\frac{d}{d\alpha}[\sin\alpha\cdot(\cos\alpha-\mu_b\cdot\sin\alpha)] = 0 \;\Rightarrow \tan(2\cdot\alpha_{launch}) = \frac{1}{\mu_b}$$

For this optimum angle $\alpha = \alpha_{launch}$, the horizontal impulse given by $J_x = J_{x,launch}$ and the maximized vertical impulse $J_y = J_{y,launch}$ would result in

$$J_{x,launch} = \frac{(1+e)\cdot p}{2} \quad \text{and} \quad J_{y,launch} = \frac{(1+e)\cdot p}{2}\cdot[\sqrt{1+\mu_b^2}-\mu_b]$$

For a very smooth titanium wedge with $\mu_b \cong 0.3$, the optimum angle to launch the opponent is $\alpha_{launch} \cong 37^o$, resulting in a maximum $J_{y,launch} = 0.37\cdot(1+e)\cdot p$, while a battle-battered titanium wedge with $\mu_b \cong 0.5$ would have $\alpha_{launch} \cong 32^o$ and $J_{y,launch} = 0.31\cdot(1+e)\cdot p$. Curiously, a hard steel wedge against a hard steel weapon would have $\mu_b \cong 0.4$ and therefore $\alpha_{launch} \cong 34^o$, very close to the slope angle from Leonardo's steel-plated tank. This might not be a coincidence: Leonardo Da Vinci was known for performing simple experiments in several areas before proposing a new design.

From the calculations for defensive wedges, a wedge robot with mass m_2 and an optimum angle $\alpha = \alpha_{launch}$, with initial speed equal to zero, would be thrown backwards with a speed v_2' such that

$$J_{x,launch} - \mu\cdot J_{y,launch} = m_2 v_2' \quad \Rightarrow \quad v_2' = \frac{(1+e)\cdot p}{2\cdot m_2}\cdot[1+\mu\cdot\mu_b-\mu\cdot\sqrt{1+\mu_b^2}]$$

where $\mu = \mu_t$ against high spinners, and $\mu = \mu_w$ against undercutters, as defined before. So, to avoid being thrown backwards, the offensive wedge just needs to charge with a speed of at least v_2' before the impact.

6.14.5. Example: Offensive Wedge vs. Horizontal Spinner

In the calculation example for Last Rites against Sir Loin, the effective linear momentum before the impact was $p = M_1\cdot v_{tip}$ = 6.21kg·106.4m/s = 661Ns. If Sir Loin had a smooth 37^o sloped titanium wedge aligned with J, with $\mu_b = 0.3$, and if the impact had as well a COR $e \cong 0.13$ with the same effective mass M_1 = 6.21kg calculated before (which is not necessarily true, since the calculated M_1 and the measured e were not obtained for an angled impact), then it would only have to take a horizontal impulse $J_x = J_{x,launch}$ = (1+0.13)·661/2 = 373Ns, while the arena floor would provide the vertical reaction impulse $J_y = J_{y,launch}$ = 0.37·(1+0.13)·661 = 276Ns to launch Last Rites.

Assuming Last Rites has a mass of 220lb (99.8kg), then this $J_{y,launch}$ would launch its center of mass with a speed v = 276Ns/99.8kg = 2.77m/s, reaching a height of $v^2/(2g) = (2.77)^2/(2\cdot 9.81)$ = 0.39m (about 15 inches). In addition, this vertical impulse $J_{y,launch}$ would also make Last Rites roll, with an angular speed that would depend on its moment of inertia in the roll direction and on the gyroscopic effect of the weapon. This roll movement could make its spinning bar touch the ground, probably launching it even higher than that.

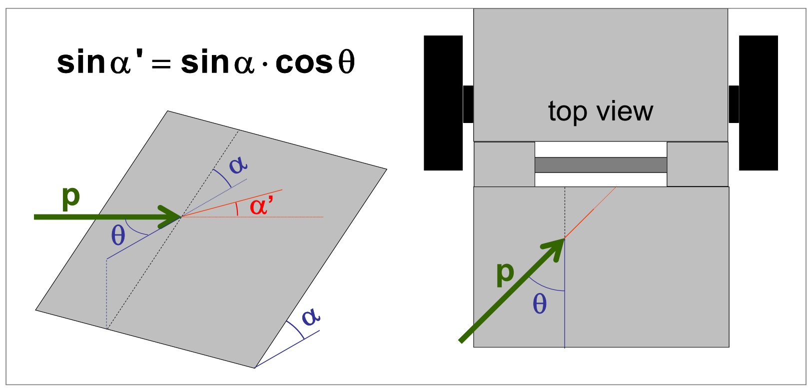
If the wedge version of Sir Loin had almost no speed before the impact, it would be thrown backwards with a speed v_2', calculated next. If the spinning bar from Last Rites was very low to the ground, then assuming $\mu = \mu_w = 0.35$ and $m_2 = 220$lb (99.8kg) we would get

$$v_2' = \frac{(1+0.13)\cdot 661}{2\cdot 99.8}\cdot[1+0.35\cdot 0.3-0.35\cdot\sqrt{1+0.3^2}] = 2.77\text{m/s}$$

So, to avoid being thrown backwards, the wedge Sir Loin would only need to charge before the impact with a forward speed of at least 2.77m/s (10km/h or 6.2mph), which is not a big deal for most combots. On the other hand, if the bar was spinning higher off the ground (which happens when Last Rites is flipped upside down), hitting near the top of the wedge, then $\mu = \mu_t = 0.9$ would lower v_2' to 1.24m/s (4.5km/h or 2.8mph), which is even easier to reach by Sir Loin.

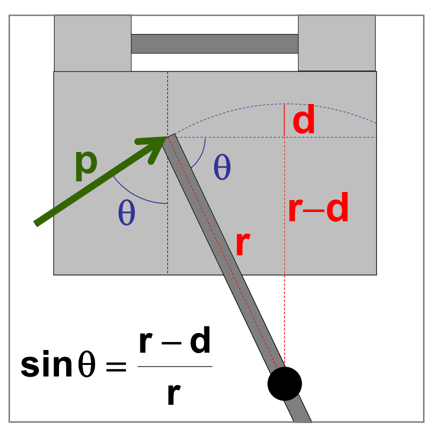
6.14.6. Angled Impacts

The previous equations assumed that the impact direction was perpendicular to the direction of the wedge's edge, meaning $\theta = 0^o$ in the figure to the right.



For angled impacts, with θ between 0^o and 90^o, the wedge works as if it had an effective slope angle α' smaller than α, where $\sin\alpha' = \sin\alpha \cdot \cos\theta$. All previous equations would remain valid, as long as α is replaced by this effective α'.

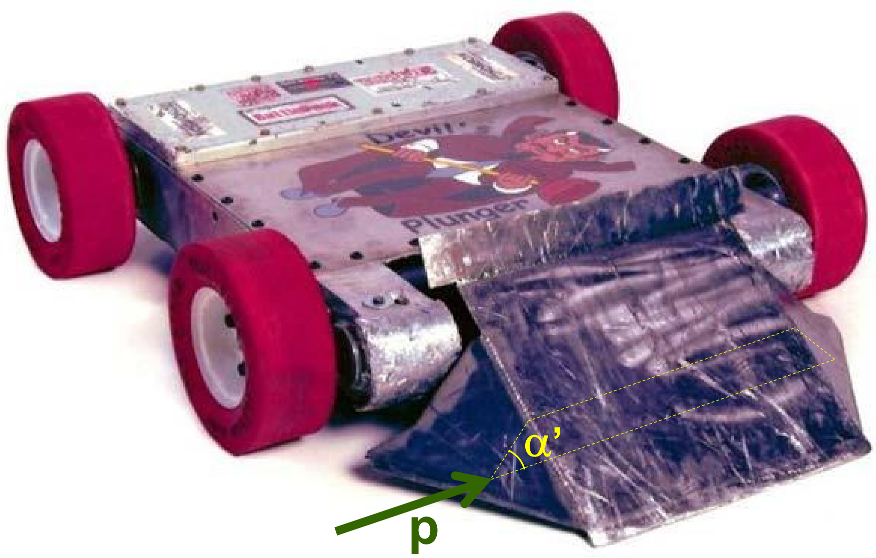
For instance, in the figure below to the left, the wedge from Pirinah 3 is not able to launch the horizontal bar spinner The Mortician, because an aligned frontal attack with a low forward speed has $\theta \cong 90^o$, resulting in $\alpha' \cong 0^o$ and therefore $J_y \cong 0$ and $J_x \cong 0$. As long as the wedge is sufficiently smooth, without dents or bolt heads sticking out, it will work as a defensive wedge if properly aligned to the attacking robot.



But if the robots are moving forward at high speeds, then the tooth travel distance d (related to a tooth bite $d \cdot \sin\alpha$) can be large enough to significantly lower the attack angle θ, because of the relation $\sin\theta = (r-d)/r$, where r is the radius of the spinning weapon, as shown in the previous picture to the right. This lower attack angle θ can then increase α', which can be enough to launch the spinner. So, for an aligned frontal attack of an offensive wedge, forward speed is fundamental.

Another approach is to try to hit the spinner with an offset to the side where the spinning weapon approaches the wedge, as it was assumed in the previous examples, trying to make θ close to zero. But, against a skillful driver, it might be difficult to make the wedge hit the spinner with such offset. The spinner will probably be trying to face the wedge robot at all times.

This is one of the reasons why several wedges have angled side edges, such in the floppy wedge from the middleweight Devil's Plunger (pictured below). Its wedge has angled sides usually fabricated through bending or welding. These angled sides can turn even a perfectly aligned hit at a low forward speed, which would have $\theta \cong 90^o$ leading to $\alpha' \cong 0^o$, into a spinner-launching hit. The effective slope angle α' is a design parameter for the side edges, measured on a vertical section of the wedge as shown in the picture. In addition, these side edges protect the wedge from being knocked off due to a side hit.



A suggested value for the side angle α' to launch spinners would be, for a smooth titanium wedge, equal to $\alpha' = \alpha_{launch} = 37^o$. And, for a hardened steel wedge, which has a larger coefficient of friction than smooth titanium, a suggested side angle α' to launch spinners would be Leonardo Da Vinci's $\alpha_{launch} = 34^o$.

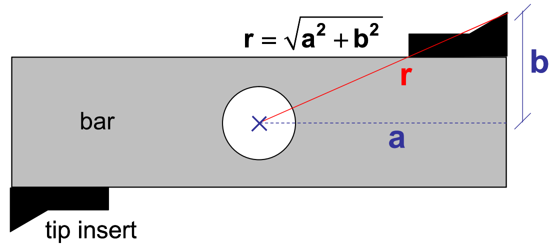
6.14.7. Wedge Design Against Vertical Spinners

One final concern regarding wedge design is to make sure it will be effective against vertical spinners.

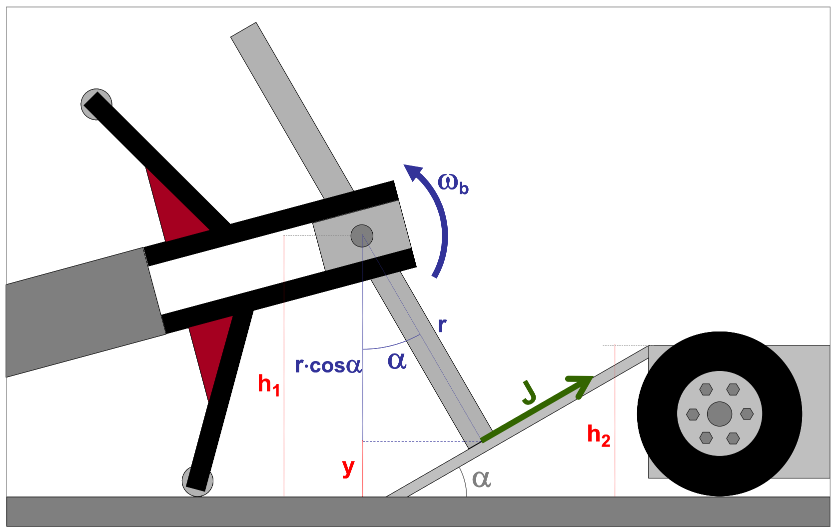
As seen in the picture in the next page, a vertical spinner with weapon radius r and weapon shaft height h_1 will most likely hit a wedge with height h_2 and slope angle α at a height $y = h_1 - r \cdot \cos\alpha$. Clearly, the height h_2 must be larger than y, so α cannot be too high to make sure that

$$y = h_1 - r \cdot \cos\alpha < h_2 \quad \Rightarrow \quad \cos\alpha > \frac{h_1 - h_2}{r}$$

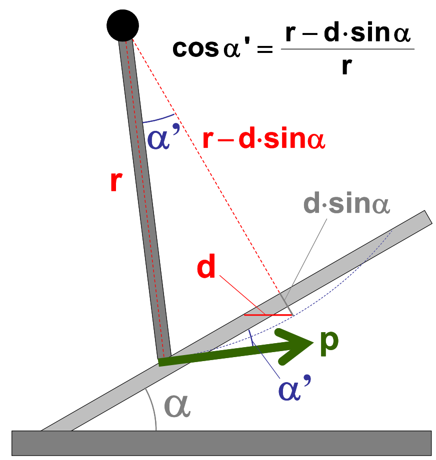
Note that if the spinning weapon has large tip inserts, or if it is a very wide bar, then the radius r used in the above equations must be calculated as in the picture to the right, considering the bar width and tip insert dimensions in the value of b, as well as the bar length 2·a.



But even if the above condition for α is satisfied, there is no guarantee that the vertical spinner won't bounce off and end up hitting the top plate of the wedge robot. This situation is very likely to happen when low profile wedges charge forward at high speeds against tall vertical spinners. To avoid that, the top of the wedge should have an overhanging section, such as the small one on Devil's Plunger or the large one on Pipe Wench, pictured below. A large overhung section is very effective against vertical spinners, working as a scoop, as shown in the action shots below featuring Pipe Wench vs. Terminal Velocity.



Note from the action shots that the wedge should be charging forward towards the vertical spinner to most effectively launch it and eventually flip it. This is because the vertical spinner weapon always hits tangentially to the wedge surface. If the robots are not moving forward and the wedge is sufficiently smooth, you should only see sparks and both robots repelling each other, but no major hits.



But if the robots are moving forward at high speeds, then the tooth travel distance d will result in a tooth bite d·sinα, due to the slope of the wedge. This tooth bite will result in an effective angle α' ≠ 0 between the speed of the weapon tip and the wedge surface, as pictured to the right, where r is the radius of the spinning weapon and cosα' = (r – d·sinα)/r. So, the higher the forward charge speed, the higher will be the tooth travel distance d and the angle α', resulting in a higher impact that might launch the vertical spinner.

6.15. Gyroscopic Effect

An interesting characteristic of robots with spinning weapons is their gyroscopic effect. Vertical spinners and drums tend to lift off their sides (tilting) when making sharp turns with their weapon turned on. The picture to the right shows our drumbot *Touro*, which is able to make turns with only one wheel touching the ground. This wheel lift-off, besides impressing well judges and audience, works as an excellent "victory dance" at the end of a match. However, if the robot's wheels lift too much off the ground it can be a disadvantage, because you'll have a hard time making turns, risking being flipped over. But what causes the gyroscopic effect?

The gyroscopic effect comes from the fact that bodies tend to remain in their state of motion, as stated by Newton's first law. In this case, they tend to maintain their angular momentum. When *Touro* tries to turn with the weapon turned on, it is forcing the drum to change its spinning orientation, making it harder to maneuver.

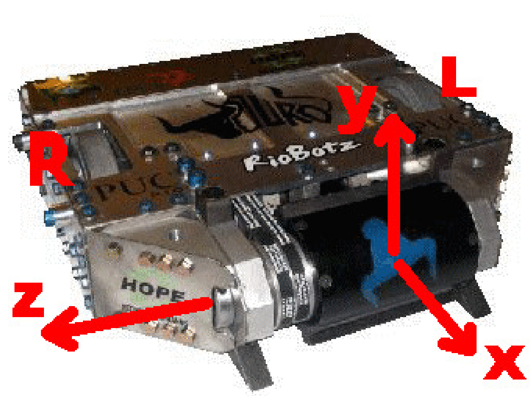
Horizontal spinners don't have this problem, because when turning they don't change the orientation of the weapon axis, which remains vertical. However, spinners have a small difficulty when turning in the opposite direction of the one that its weapon spins, and they turn more easily in the same direction. This doesn't have anything to do with the gyroscopic effect, it is simply caused by the friction between the weapon and the robot structure, which tends to turn the robot in the same sense of the weapon. This effect is usually small. The gyroscopic effect in the horizontal spinners appears when an opponent tries to flip them, because the high speed of the spinning weapon provides them with a certain stability that helps them to remain horizontal. Our spinner *Ciclone* escaped from several potential flips because its weapon was turned on.

There are spinners that have weapon spinning axis that are not vertical, such as the robot Afterthought, pictured to the right. They have a small slope forward, intended to hit lower opponents because the weapon tip gets closer to the ground. Those tilted spinners suffer a little from the gyroscopic effect, which is proportional to the sine of the slope angle between the weapon spinning axis and the vertical. The smaller the angle, the smaller the effect.

This tilted spinner type has a serious problem: there is a chance that the robot gets flipped over during its own attack. If for instance the weapon spins in the sense shown in the picture above, it won't have problems when its opponent is right in front at its left side (the "good" side shown in the figure, which hits like an uppercut). But if the opponent is on its right side (the "bad" side), then the tilted spinner may be flipped over when hitting from top to bottom.

The gyroscopic effect, besides making it harder for vertical spinners and drumbots to make turns, also causes the phenomenon called precession, the same that explains why a spinning top can have its spin axis sloped without falling over. This phenomenon explains the wheel lift off (tilting) during sharp turns. The precession of a robot's weapon can be calculated from the principle of angular momentum (which results in what is known as Euler's equations), which depends on the rotational moment of inertia of the weapon in the spin direction (horizontal, in this case) I_{zz}, and on the moments of inertia in the 2 other directions, I_{xx} and I_{yy}. The weapons of those robots usually have axial symmetry (as in the disks of the vertical spinners or the cylinders of the drums), and therefore $I_{xx} = I_{yy}$.

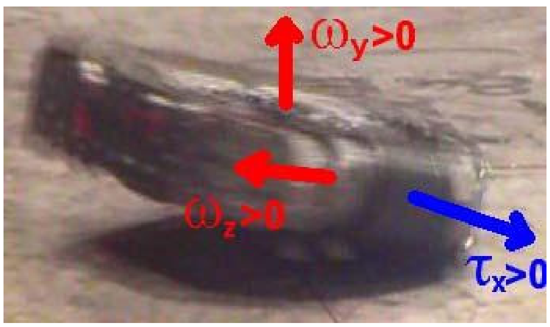
In the figure to the right, the z-axis was chosen in the direction that the weapon spins, with angular speed ω_z, from ground up in such a way to throw the opponents into the air. Notice that the robot's right and left wheels are represented by the letters R and L respectively. The y-axis was chosen as the vertical one, and the x-axis is directed horizontally towards the front of the robot. When the robot is turning around its y-axis with angular speed ω_y and with its weapon turned on spinning with ω_z, the principle of the angular momentum results in

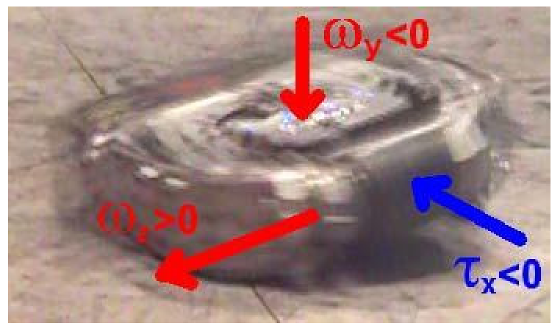


$$\tau_x \cong I_{zz} \cdot \omega_z \cdot \omega_y$$

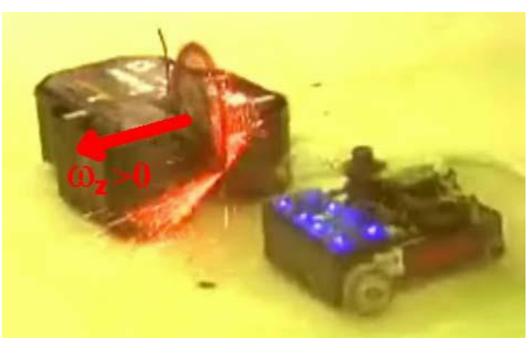
where τ_x is an external torque applied in the direction of x. This equation is a good approximation if the tilting angle of the robot is small.

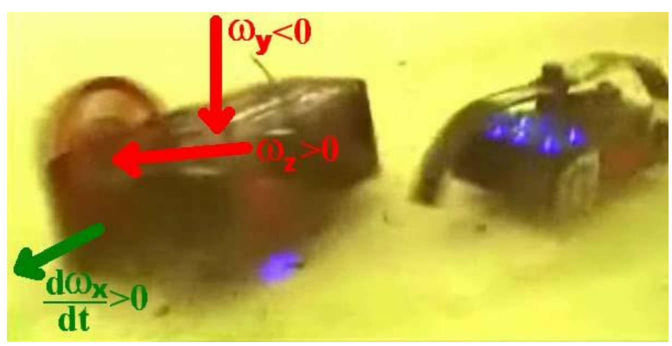
If the robot turns left, then $\omega_y > 0$, and therefore from the above equation we get $\tau_x > 0$. This means that the gravity force needs to generate a positive torque in the x direction to keep the system in balance, which happens when the right wheel (R in the figure above) lifts off the ground, see the left figure below. Similarly, if the robot turns right, then $\omega_y < 0$ and therefore $\tau_x < 0$. This negative torque that the gravity force needs to generate is obtained when the left wheel (L in the figure above) lifts off the ground, see the right figure below.





Those results are the same for drumbots as well as for vertical spinners. For instance, during the final match of the RoboCore Winter Challenge 2005 competition, the middleweight vertical spinner *Vingador* was spinning its disk with $\omega_z > 0$. After receiving an impact from our horizontal bar spinner *Ciclone* (left figure below), *Vingador* began to twirl with a clockwise speed $\omega_y < 0$. To balance this movement, a negative torque $\tau_x < 0$ would be necessary. However, even lifting its left wheel, the gravity force wasn't able to generate enough torque. *Vingador* continued tilting (with an angular acceleration $d\omega_x/dt > 0$, see the right figure below) until it ended up capsizing over its right side.



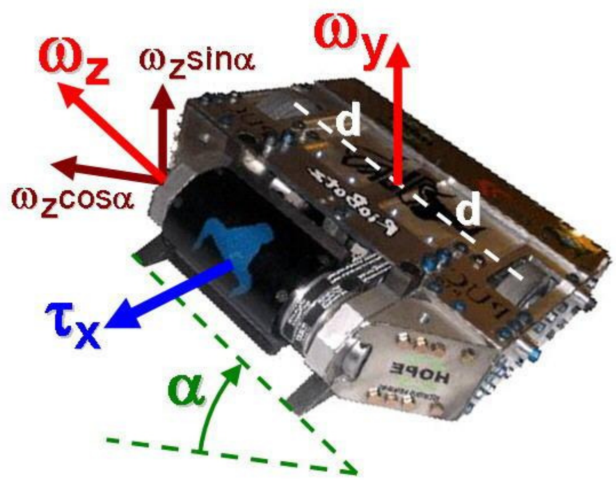


Vertical spinners have more problems with the gyroscopic effect than drumbots. The reason for that is because the gyroscopic effect is proportional to the angular speed of the weapon ω_z, while the kinetic energy depends on ω_z^2. Vertical spinners usually have lower weapon speed ω_z and higher moment of inertia I_{zz} than drums.

Therefore, for instance, a vertical spinner that spins a solid disk of mass m_b = 10kg (22lb) and radius r = 0.3m (almost 1ft) at ω_z = 1,000RPM = 105rad/s has a weapon with moment of inertia $I_{zz} = m_b \cdot r^2/2 = 10 \cdot 0.3^2/2 = 0.45 kg \cdot m^2$ and with kinetic energy equal to $E = I_{zz} \cdot \omega_z^2 / 2 = 0.45 \cdot 105^2 / 2$ = 2,481J. A drumbot that spins a cylinder with the same mass m_b = 10kg (22lb) and external and internal radii r = 0.06m and r_i = 0.04m at ω_z = 4,173RPM = 437rad/s has $I_{zz} = m_b \cdot (r^2 + r_i^2)/2 = 10 \cdot (0.06^2 + 0.04^2)/2 = 0.026 kg \cdot m^2$, and the kinetic energy of the weapon is $E = I_{zz} \cdot \omega_z^2 / 2 = 0.026 \cdot 437^2 / 2$ = 2,483J, practically the same energy of the vertical spinner. Therefore, both robots have similar destruction power.

However, the angular momentum of the vertical spinner weapon is $I_{zz} \cdot \omega_z = 0.45 \cdot 105 = 47.25$, much larger than the one from the drum, $I_{zz} \cdot \omega_z = 0.026 \cdot 437 = 11.36$. Because the gyroscopic effect depends on the product $I_{zz} \cdot \omega_z$, a drumbot usually tilts much less than a vertical spinner while making turns.

It is possible to get a better estimate of the gyroscopic effect, explicitly considering the tilt angle α with respect to the horizontal (as pictured to the right), which had been assumed to be very small in the previous calculations. As the robot turns with speed ω_y and with its weapon spinning with ω_z, the robot tilts by the angle α. The projection of the vector ω_z onto the vertical, $\omega_z \cdot \sin\alpha$, doesn't change direction, but the horizontal projection $\omega_z \cdot \cos\alpha$ does, rotating around the y-axis with speed ω_y, which is responsible for the gyroscopic effect.



The gravity torque τ_x is equal to $m \cdot g \cdot d \cdot \cos\alpha$, where m is the mass of the entire robot, g is the acceleration of gravity, and d is the distance between each wheel and the robot's center of mass, as pictured above. Assuming that ω_z is much larger than ω_y (because the weapon spins much faster than the robot turns), the principle of angular momentum states that the tilting movement is in equilibrium if $\omega_y = \omega_{y,critical}$, where

$$\tau_x = m \cdot g \cdot d \cdot \cos\alpha = I_{zz} \cdot (\omega_z \cos\alpha) \cdot \omega_{y,critical}$$

Canceling the $\cos\alpha$ term from the equation, we get

$$\omega_{y,critical} = \frac{m \cdot g \cdot d}{I_{zz} \cdot \omega_z}$$

In other words, if you turn with speed ω_y equal to $\omega_{y,critical}$, the robot will tilt with an arbitrary angle α (the robot stability does not depend on α, at least in the considered model approximation). If ω_y is smaller than $\omega_{y,critical}$, the robot doesn't lift any wheel off the ground. And if ω_y gets larger than $\omega_{y,critical}$, the robot tilting will become unstable, capsizing over its side. At the final match of the RoboCore Winter Challenge 2005, *Ciclone*'s impact made the robot *Vingador* twirl with a speed ω_y larger than its critical value $\omega_{y,critical}$, capsizing it.

In the previous example, which compared a vertical spinner and a drumbot with same weapon system energy, assuming that m = 55kg (about 120lb) and d = 0.2m (7.9”), the above equation would result in the conclusion that the vertical spinner would not be able to make turns faster than $\omega_{y,critical}$ = 2.28rad/s = 21.8RPM without lifting its wheel and risking capsizing. On the other hand, the considered drumbot would be able to turn even at $\omega_{y,critical}$ = 9.5rad/s = 90.7RPM without lifting off the ground, a much more reasonable value.

Finally, to avoid that the robot capsizes on its side, it is necessary that $\omega_{y,critical}$ has the largest possible value. Therefore, it is important that the base of a vertical spinner or drumbot is wide, because a larger distance 2·d between the wheels increases $\omega_{y,critical}$, as seen in the above equation. This explains, for instance, the reason for the large distance between the wheels of the fairyweight vertical spinner Nano Nightmare, pictured to the right.

If the vertical spinner from the previous example had a wider base with d = 0.3m (due to a distance 2·d = 600mm between the wheels, about 23.6”), then the calculations would result in allowable turning speeds of up to 33RPM. In other words, it would be able to turn 180 degrees in less than 1 second, a reasonable value to keep facing the opponent.

6.16. Summary

In this chapter, it was shown that weapon design can benefit a lot from basic physics calculations. It was concluded that spinning disks have better inertia and in-plane bending strength than bars with same weight, however they suffer from a lower out-of-plane bending strength. The concepts of tooth bite, as well as effective mass, stiffness and damping, were introduced, showing that the effectiveness of an impact depends on properties of both attacker and attacked robot. It was shown that drumbots have one of the highest effective masses M_1, however they usually suffer limitations regarding the speed v_{tip} of the weapon tip. Impact equations were presented for several robot types, including spinners and hammerbots. The difference between defensive and offensive rammers was discussed, including information on how to setup their shields. It was shown why thwackbots and overhead thwackbots are difficult to steer to try to hit an opponent. Lifter and clamper stability equations were also presented. It was seen that “height launchers” can benefit from a long scoop, while “range launchers” should choose average impulse angles between 36^o and 45^o, depending on the opponent’s aspect ratio, as long as they have enough tire friction not to be pushed backwards. It was found that defensive wedges have a slope angle of at most 25^o, while an ideal offensive wedge would be made out of Ti-6Al-4V titanium with a smooth surface and a 37^o slope. And, finally, gyroscopic effect equations were presented to help in the design of vertical spinners. In the next chapter, the main electronic concepts to power such weapon systems and the robot drivetrain are introduced.