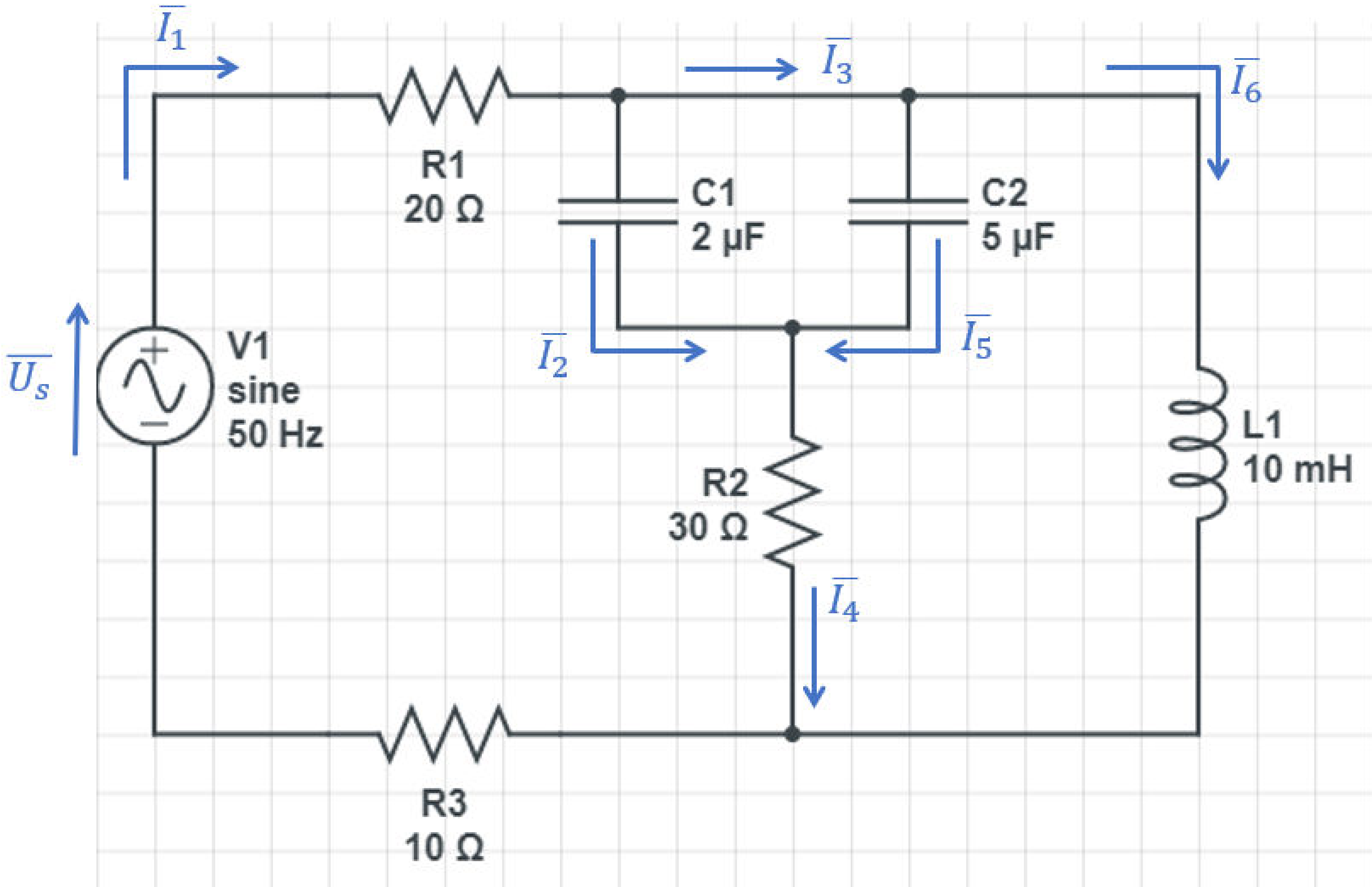
Matrix resolution of an AC circuit via Kirchhoff, including transient **

Let's imagine the AC electrical circuit below : (already solved using phasors)



with :

- Us = $5 \sin\left(\omega t + \frac{\pi}{4}\right)$ [V]
- R1 = 20 $[\Omega]$
- R2 = 30 $[\Omega]$
- R3 = 10 $[\Omega]$
- C1 = 2 $[\mu F]$
- C2 = 5 $[\mu F]$
- L = 10 [*mH*]
- $f = 50\,[\mathrm{Hz}]$

You're asked to :

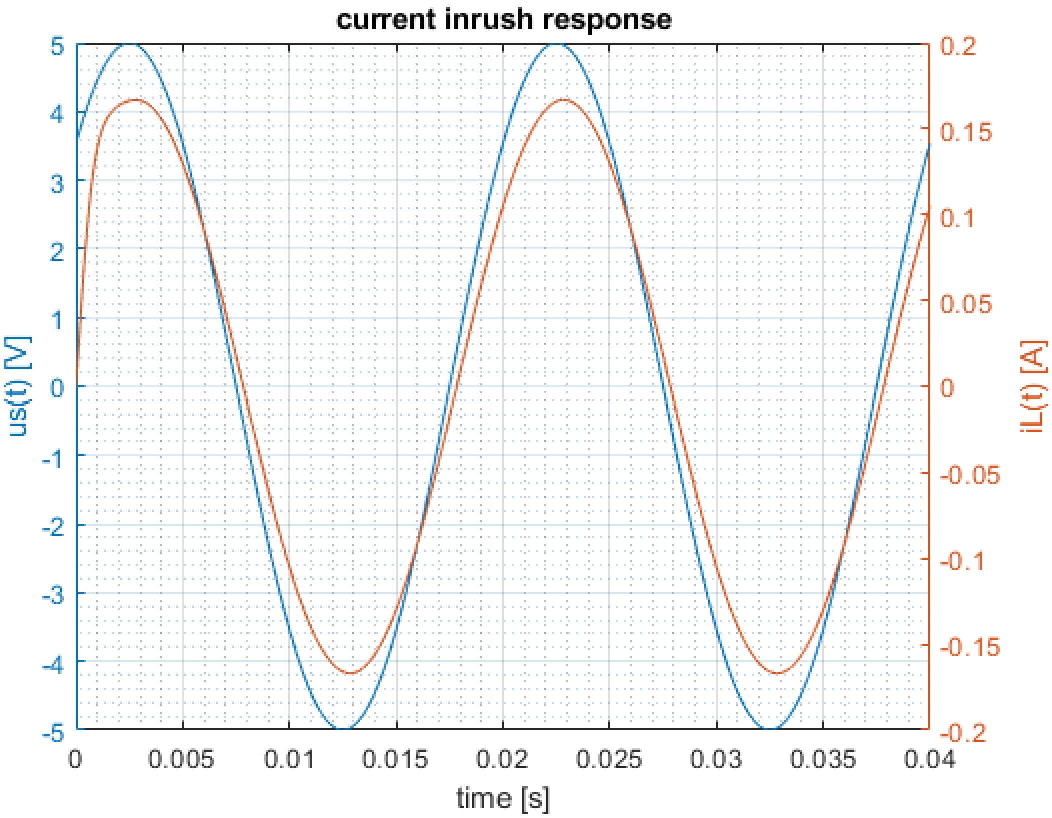
- Find the transfer function linking the current $I_6(s)$ through the inductance L to the input voltage $U_s(s)$ and check the duration of the transient. This is done on the basis of Kirchhoff's equations (previously obtained in exercise exo_circuit_phaseurs_1) and a matrix inversion but this time via dynamic systems and no longer complex impedances. For example : using phasors, the current in a capacitor is written Ic=Uc*1i*w*C; using dynamical system: ic=uc.s.C (with s; the Laplacian operator).
- Using the Bode diagram, check the steady state current obtained for the input voltage defined above.
- Represent the signal of the source voltage and that of the current (in transient and established state) in the coil as a function of time, on 2 complete periods, with 1000 points, and the same formatting as the graph below.
- Then observe the index response of this circuit (input step: us, output = 0)
- Draw the bode diagram of the answer (input: us; output: ul)

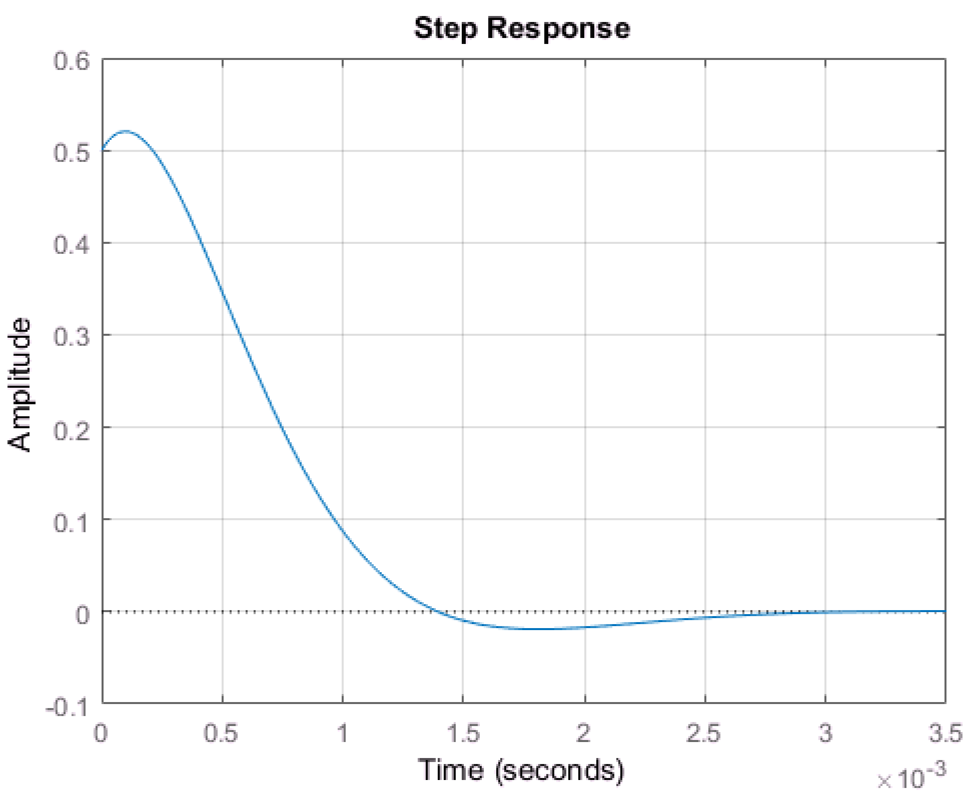
Tips : tf('s') pole lsim step bode

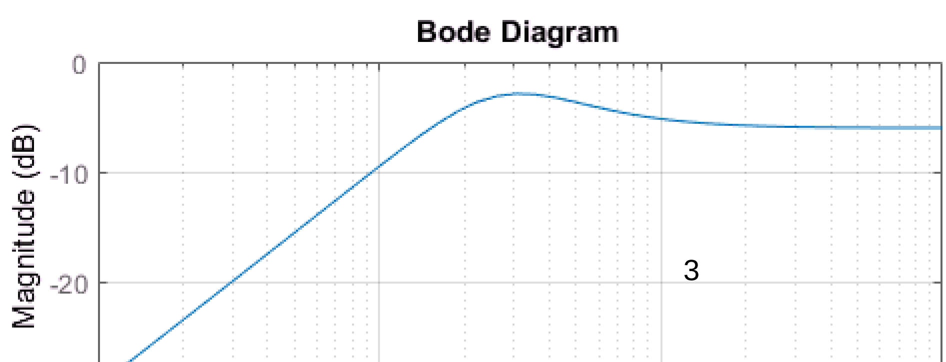
Solution:

$$H(s) = \frac{I_6(s)}{U_s(s)} = \frac{(50s + 2.381e05)}{s^2 + 3881s + 7.143e06}$$

- ***Transient duration : 2.6 ms***
- ***Stead-state current : Amplitude of 0.1669 A and phase equal to 38.95°. frequency = 100 pi***


current inrush response
us(t) [V]
iL(t) [A]
time [s]
5
4
3
2
1
0
-1
-2
-3
-4
-5
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
0
0.005
0.01
0.015
0.02
0.025
0.03
0.035
0.04


Step Response
Amplitude
Time (seconds)
×10-3
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
0
0.5
1
1.5
2
2.5
3
3.5


Bode Diagram
Magnitude (dB)
0
-10
-20

```
clear all
close all

% Encoding of all necessary parameters and creation of "impedances"
according to the variable s
R1=20;
R2=30;
R3=10;
C1=0.000002;
C2=0.000005;
L=0.010;

s=tf('s');
ZR1=R1;
ZR2=R2;
ZR3=R3;
ZC1=1/(s*C1);
ZC2=1/(s*C2);
ZL=s*L;
```

```
% First way to find the TF : thanks to matrix inversion
Z=[ZR1+ZR3  ZC1  0  ZR2   0    0;...
    0       ZC1  0   0  -ZC2   0;...
    0        0   0  ZR2  ZC2 -ZL;...
    1       -1  -1   0    0    0;...
    0        1   0  -1    1    0;...
    1        0   0  -1    0   -1];

Z1=inv(Z);    % This inverted matrix contains the TF linking the circuit's
currents to the input voltage
H=minreal(Z1(6,1))    % The TF we're interested in
```

```
H =

      50 s + 2.381e05
  -----------------------
  s^2 + 3881 s + 7.143e06

Continuous-time transfer function.
```

```
% Second way to find the TF
Z1=ZR1+ZR3;Z1=(ZR1+ZR3);
Z2=((ZC1*ZC2)/(ZC1+ZC2))+ZR2;
Zth=(Z1*Z2)/(Z1+Z2);
H=minreal((Z2/(Z1+Z2))/(Zth+ZL))    % It's the same as previously found
```

```
H =

      50 s + 2.381e05
```

```
  ----------------------
  s^2 + 3881 s + 7.143e06

Continuous-time transfer function.
```

```
% Steady-state response for a 100 rad/s frequency sine sollicitation (it's %
our input signal frequency)
[mag,ph]=bode(H,100*pi)
```

```
mag = 0.0334
ph = -6.0452
```

```
% Comparason to the previously solved exercise exo 1 : We found a current of
0.1669 A and a phase of 38.95 ° when the input voltage was 5 V and 45°.
5*mag
```

```
ans = 0.1669
```

```
45+ph
```

```
ans = 38.9548
```

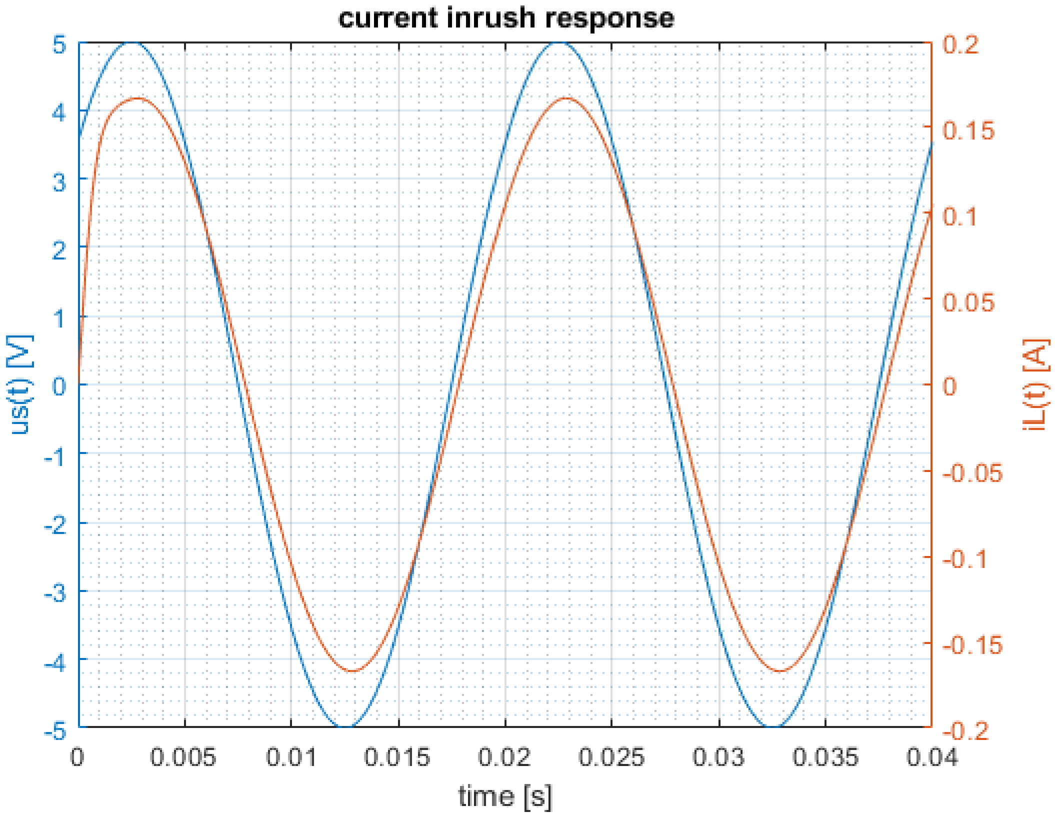
```
% Plotting the time-related response for 2 periods
f=50; w=2*pi*f; T=1/f;
t=linspace(0,2*T,1000);
us=5*sin(w*t+pi/4);

% Check the transient duration : Really fast!
p1=pole(H)
```

```
p1 = 2×1 complex
10^3 ×
  -1.9405 + 1.8378i
  -1.9405 - 1.8378i
```

```
Transitoire=5*-1/real(p1(1)) % in seconds
```

```
Transitoire = 0.0026
```

```
% PLoting the curve : one can see the transient (for about 2 ms) and the
amplitude and phase matching what we got thanks to Bode command.
iL=lsim(H,us,t);
grid, grid minor
xlabel('time [s]')
title('current inrush response')
yyaxis left, plot(t,us), ylabel('us(t) [V]')
yyaxis right, plot(t,iL), ylabel('iL(t) [A]')
```

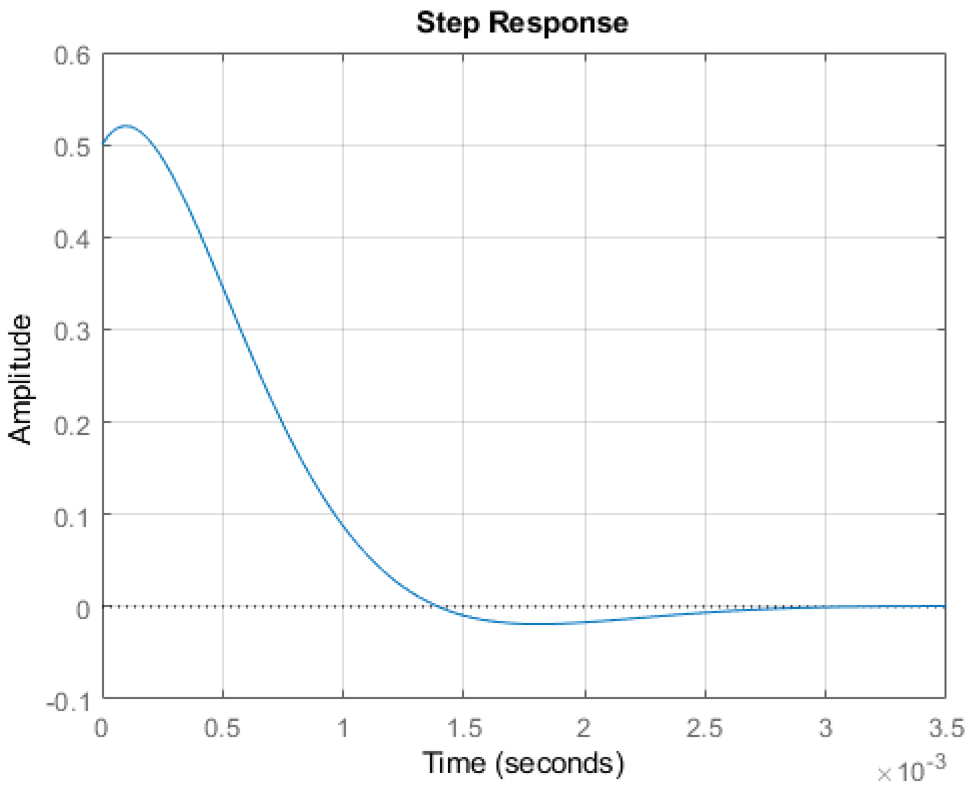



```
figure
```

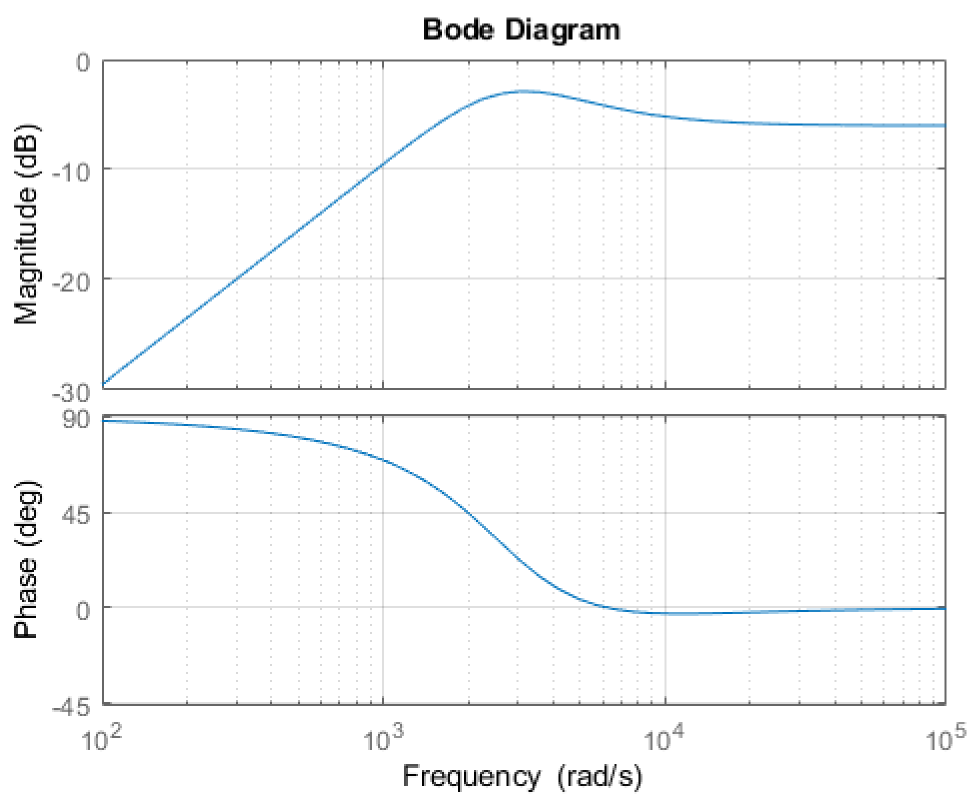
```
% New FT linking Ul to Us :
G=minreal(ZL*H)
```

```
G =

     0.5 s^2 + 2381 s
  -----------------------
  s^2 + 3881 s + 7.143e06

Continuous-time transfer function.
```

```
step(G); grid
```


Step Response
Amplitude
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
0
0.5
1
1.5
2
2.5
3
3.5
Time (seconds)
$\times 10^{-3}$

```
bode(G); grid
```


Bode Diagram
Magnitude (dB)
0
-10
-20
-30
Phase (deg)
90
45
0
-45
10^2
10^3
10^4
10^5
Frequency (rad/s)