System response to a sine wave sollicitation both in transient & steady state. ***

Let's suppose the next system $H(s) = \dfrac{135}{s^2 + 5s + 13}$

One would like to know its time-related response to a input built as the sum of the 3 next sine waves :

$$u_1(t) = 3\sin\left(\frac{4}{3}\pi\, t\right)$$

$$\mathrm{u}_2(t) = 5\sin\left(4\,\pi\, t + \frac{\pi}{3}\right)$$

$$\mathrm{u}_3(t) = 0.5\sin\left(8\,\pi\, t - \frac{\pi}{2}\right)$$

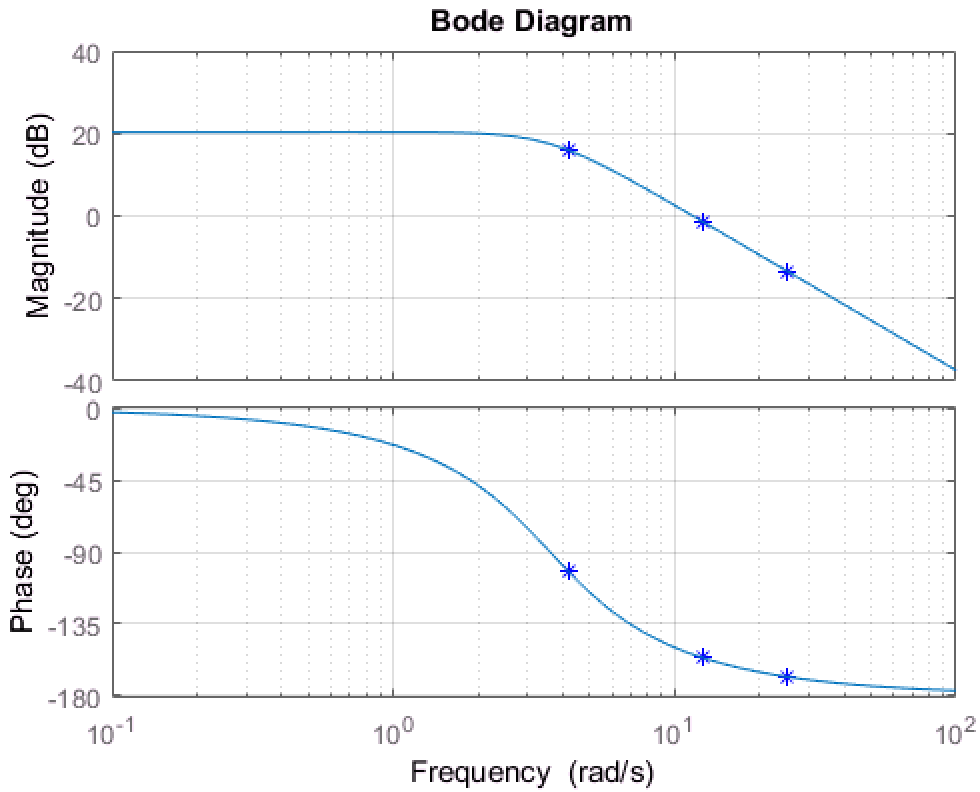
You're asked to :

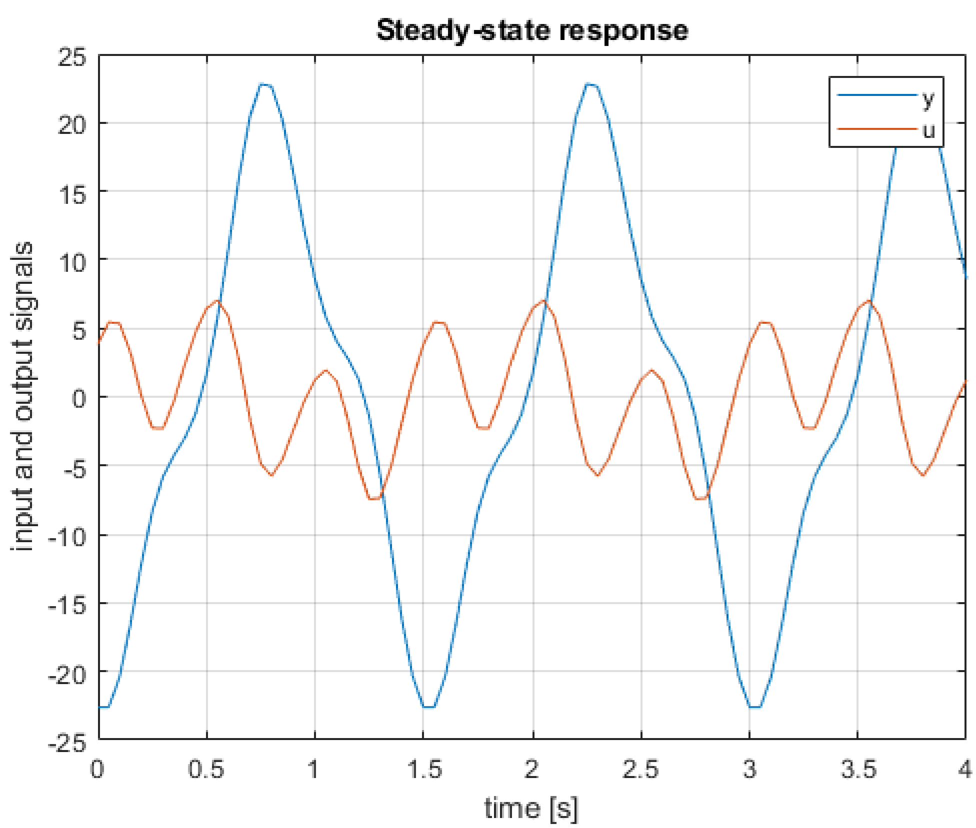
- Plot the system's Bode diagram and highlight the relevant frequencies using asteriks.
- Use this Bode's diagram to plot the system's steady state response using a sample time of 50 ms. Remember the system is LTI and thus repects the superposition principle, so the global answer is the sum of the answers to each sine wave.
- Compute the total transient duration, on basis of the system pole's location.
- Simulate the time-related response (as well transient as steady state) using the relevant Matlab command (lsim). Ensure the total simulation duration allows to have the whole transient as well as at least one complete period of the steady-state answer.
- In the 2 previous results, isolate one period of the fondamental wave in steady state, and superimpose the 2 curves. (FYI, method one is "bode" and method 2 is "lsim" commands).

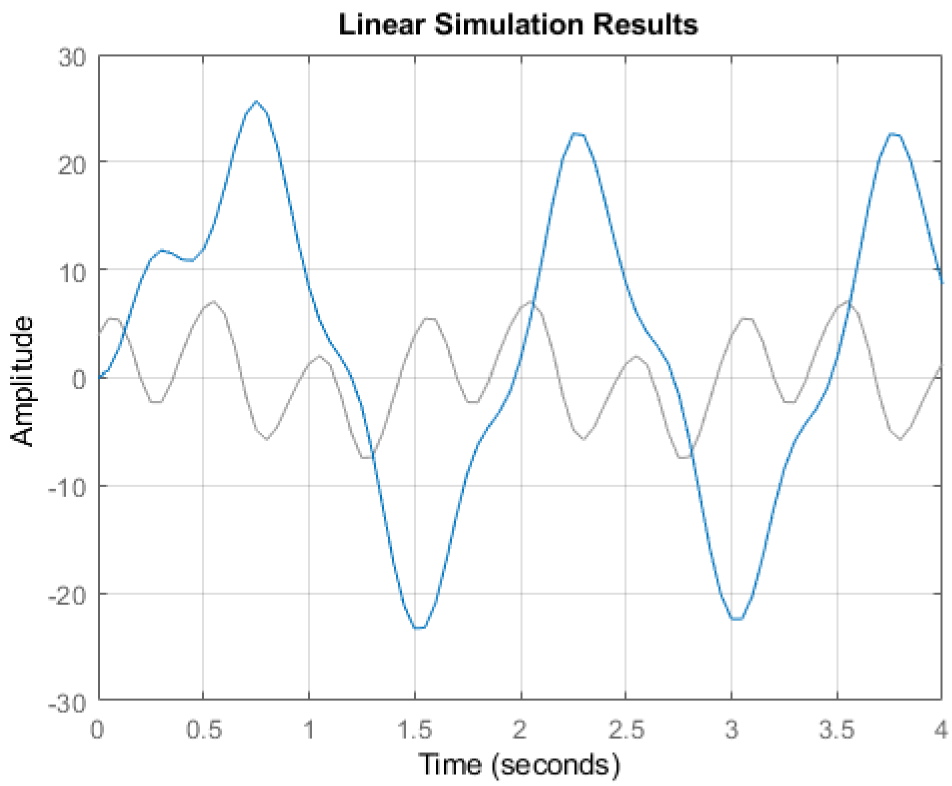
Tips : tf('s') pole lsim bode

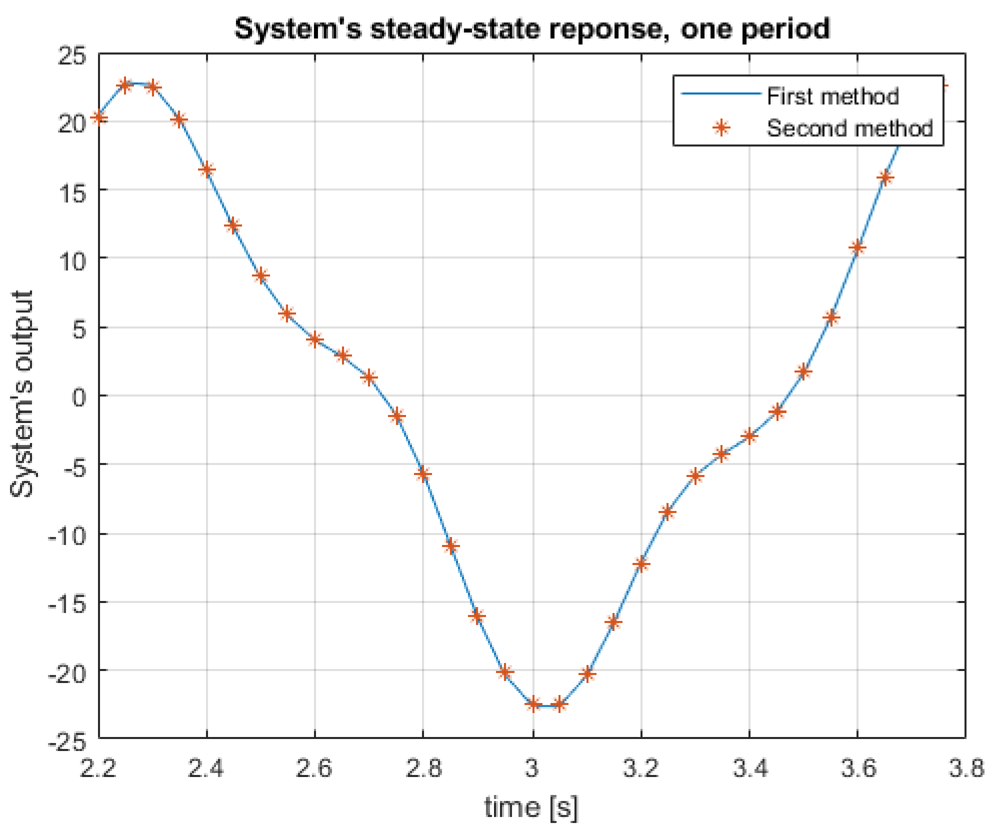
Solution :

Total transient duration : 2 seconds.


Bode Diagram
40
20
0
-20
-40
Magnitude (dB)
0
-45
-90
-135
-180
Phase (deg)
10^{-1}
10^{0}
10^{1}
10^{2}
Frequency (rad/s)


Steady-state response
25
20
15
10
5
0
-5
-10
-15
-20
-25
input and output signals
y
u
0
0.5
1
1.5
2
2.5
3
3.5
4
time [s]






```
% Creation of the required variables
s=tf('s');                  % Laplace Variable
```

```
H=135/(s^2+5*s+13);     % Transfert function
pole(H)                 % Check the pole location in order to compute the
transient duration
```

```
ans =
  -2.5000 + 2.5981i
  -2.5000 - 2.5981i
```

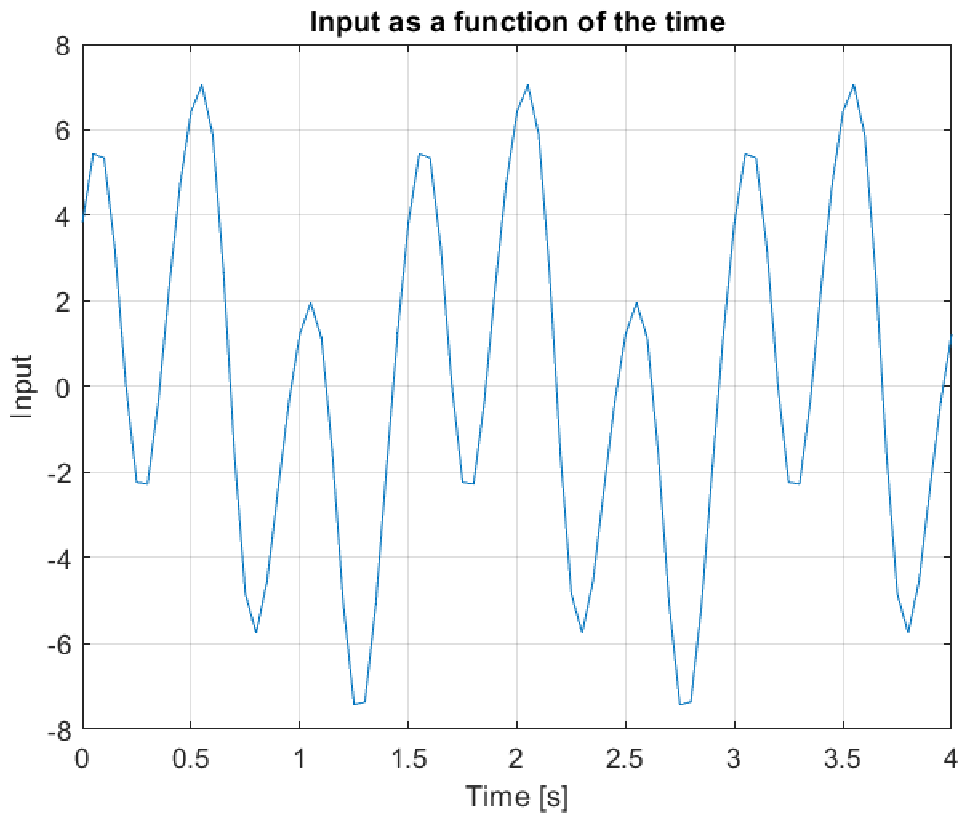
```
Time_constant=-1/real(pole(H));
Transient=5*Time_constant(1)
```

```
Transient = 2.0000
```

```
w=4*pi/3; f=w/(2*pi);     % Fondamental of the sine wave
Tf=1/f                    % Related period
```

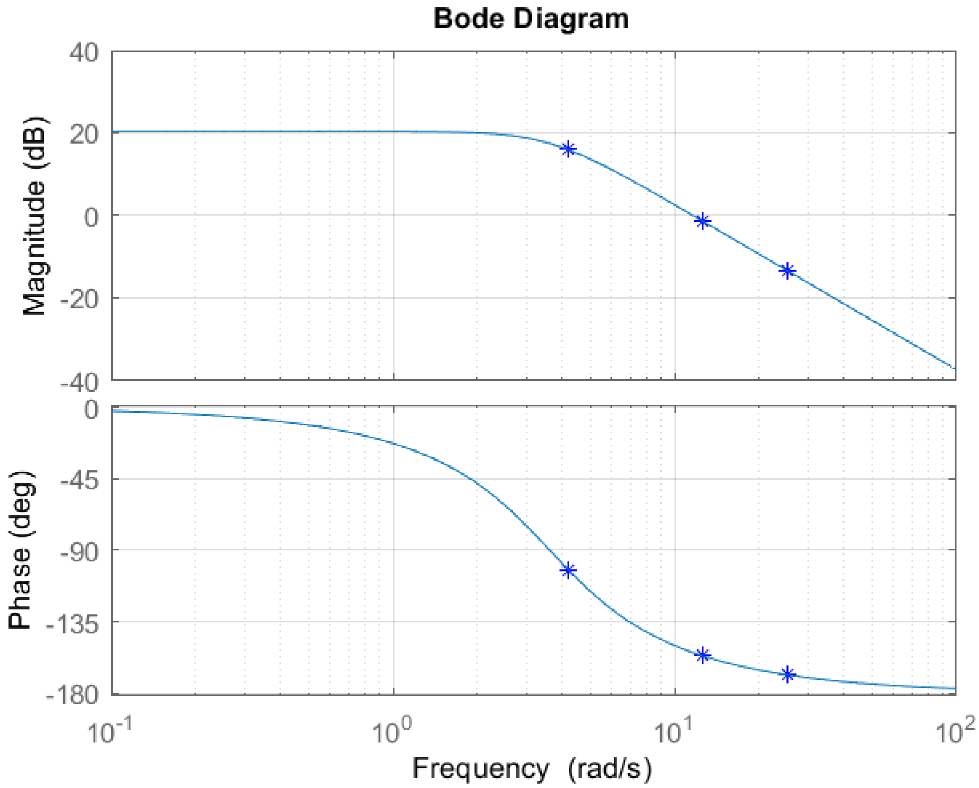
```
Tf = 1.5000
```

```
t=0:0.05:4;          % Time vector creation, so that the transient and a bit
more than one complete fondamental's period are both covered
u1=3*sin(4/3*pi*t);
u2=5*sin(4*pi*t+pi/3);
u3=0.5*sin(8*pi*t-pi/2);
u=u1+u2+u3;             % Input signal as a sum of the 3 waves
plot(t,u); grid; xlabel('Time [s]'), ylabel('Input'), title ('Input as a
function of the time');
```

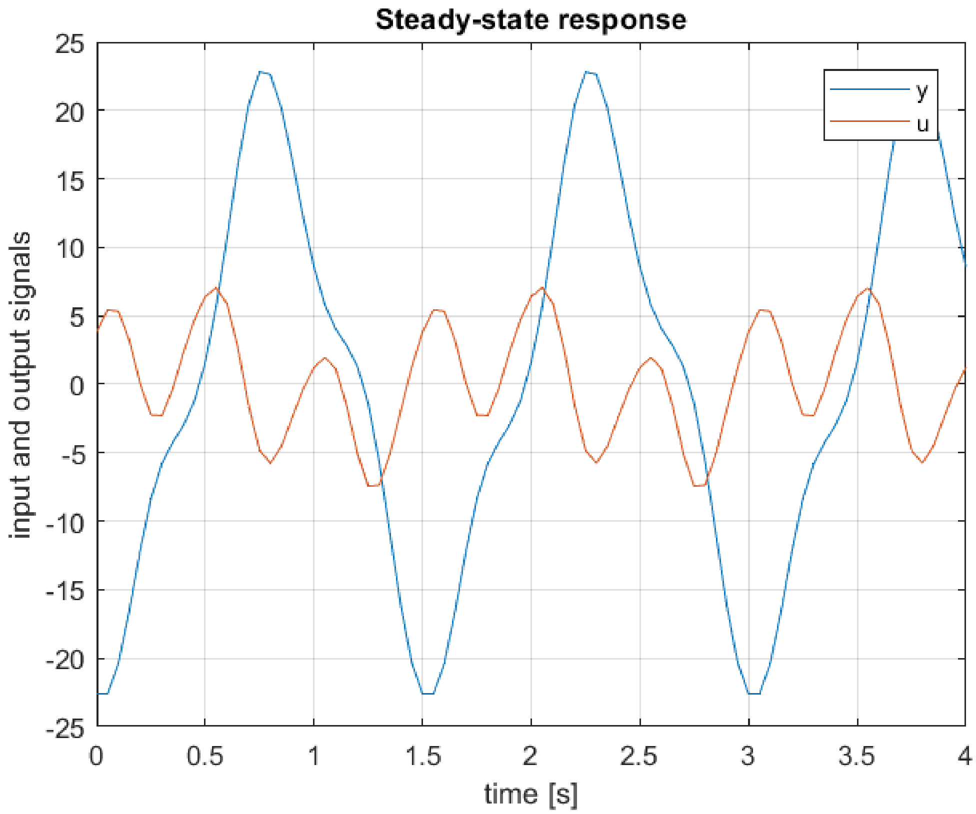



```
% First method, using Bode plot
bode(H,[4/3*pi 4*pi 8*pi],'*')
```

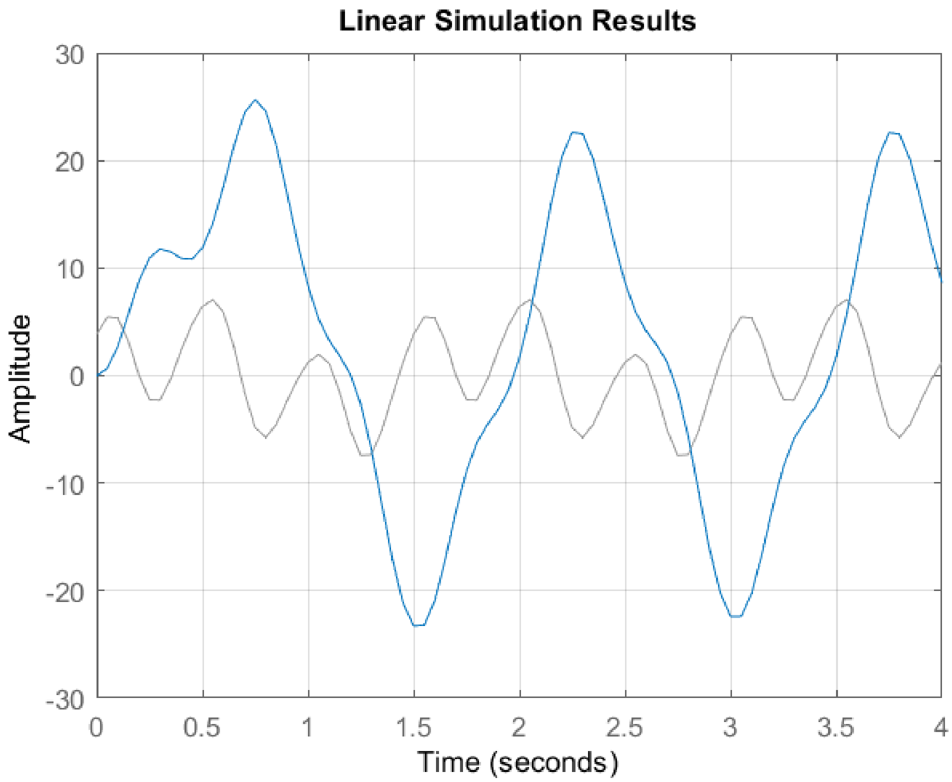
```
hold on
bode(H); grid
hold off
```



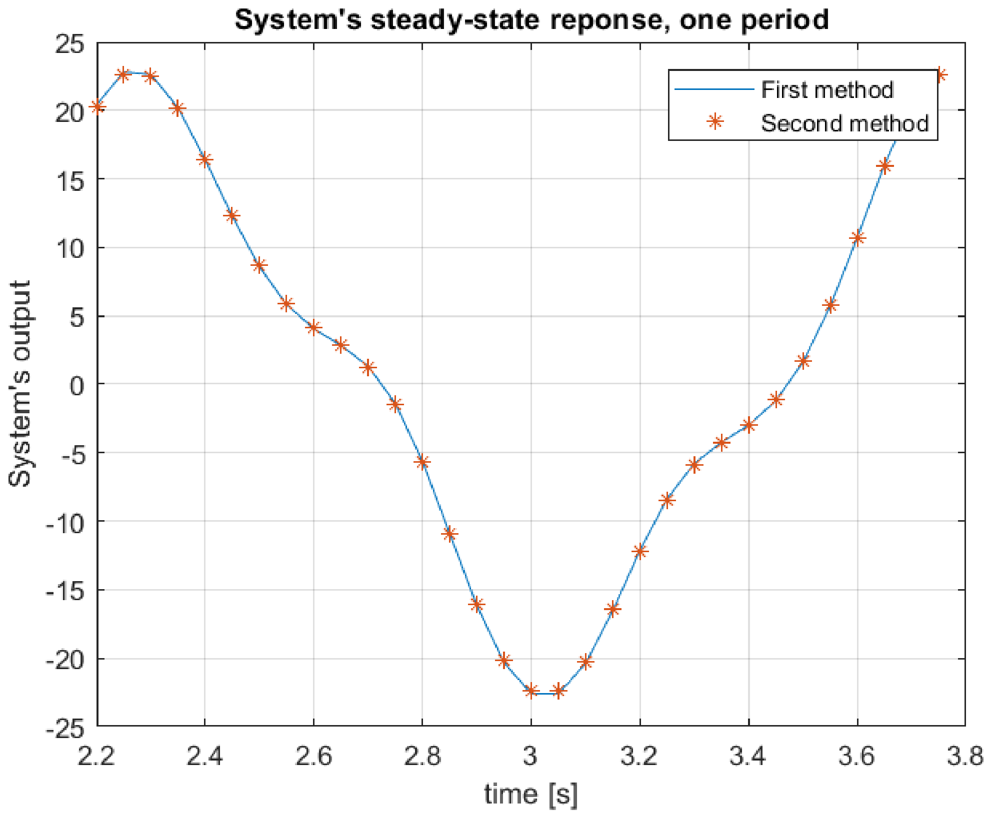

```
[mag1,phase1]=bode(H,4/3*pi);
[mag2,phase2]=bode(H,4*pi);
[mag3,phase3]=bode(H,8*pi);
y=mag1*3*sin(4/3*pi*t+deg2rad(phase1))+mag2*5*sin(4*pi*t+deg2rad(phase2)+(pi/
3))+mag3*0.5*sin(8*pi*t+deg2rad(phase3)-(pi/2));    % Superposition

plot(t,y,t,u)
legend('y','u')
grid
xlabel('time [s]')
ylabel('input and output signals')
title('Steady-state response')
```




```
% Second method, using lsim
y2=lsim(H,u,t);
lsim(H,u,t)
grid
```




```
% 3. Check : Let's extract one signal period in steady-state in the 2
responses
t1=t(find(t==2.2):find(t>2.2+Tf)); % 2 seconds (transient) + 10% to make
sure it's ok.
y1=y(find(t==2.2):find(t>2.2+Tf));
y3=y2(find(t==2.2):find(t>2.2+Tf));
plot(t1,y1,t1,y3,'*')
grid
legend('First method','Second method')
xlabel('time [s]')
ylabel('System''s output')
title('System''s steady-state reponse, one period')
```




```
sum(y1)-sum(y3)     % Should be close to 0
```

```
ans = -0.2878
```