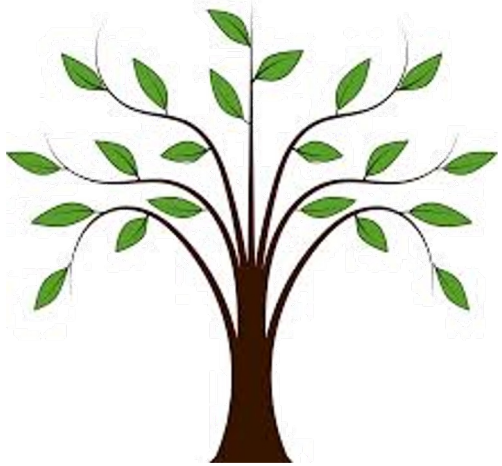
Tree 2

Kiatnarong Tongprasert

Tree

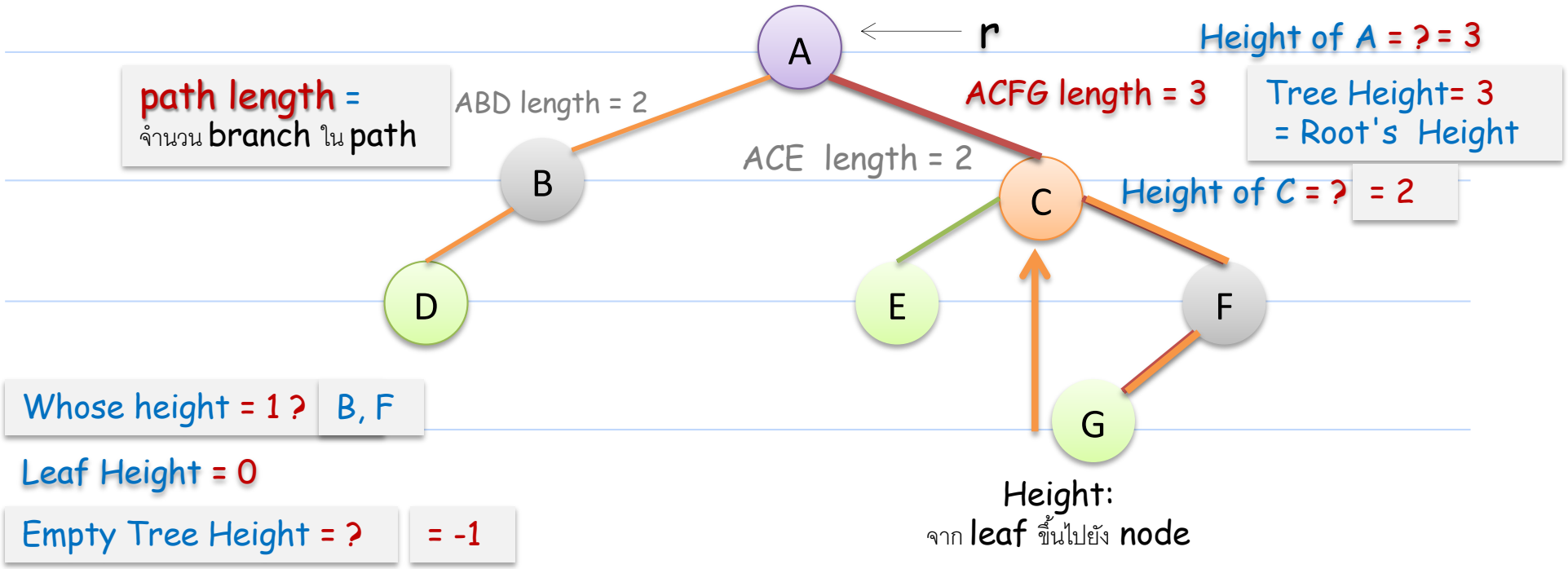
1. Tree Definitions
2. Binary Tree
   - Traversals
   - Binary Search Tree
   - Representations
   - Application : Expression Tree



3. AVL Tree
4. Which Represention ?
5. n-ary Tree
6. Generic Tree
7. Multiway Search Tree
8. B-Trees

Height

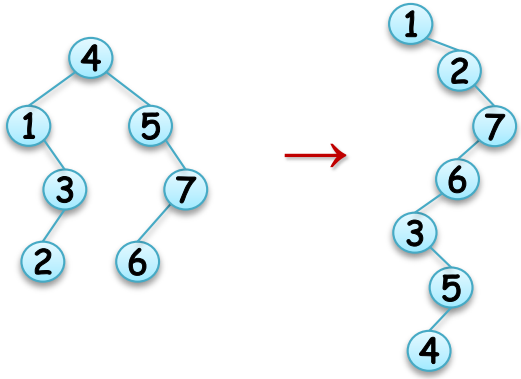
Height of node = longest path length from node to leaf
path ที่ยาวที่สุดจาก node นั้น ถึง leaf



Why Balanced Tree ?

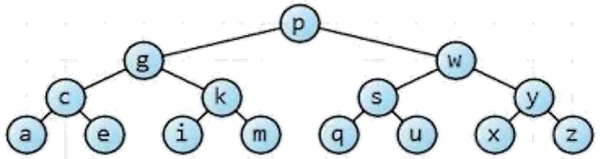
Tree Efficiency (searching, inserting, deleting,...) → O(Height)

Binary Search Tree :
บางครั้งมีการเก็บค่าเป็นเส้น



Linear Linked list or near
Linear Search O(n)

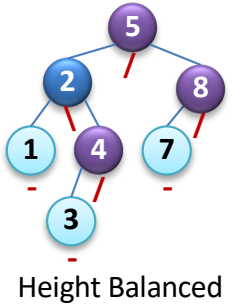
Binary Search Tree :
(Perfect) Balanced Tree
ทุกใบอยู่ที่ระดับเดียวกัน



$O(\log_2 n)$

Balanced Binary Search Tree

Binary Search Tree :
Hight Balanced Tree → AVL Maximum height <= $1.44 \log_2 n$
ในแต่ละโหนดต้นไม้ย่อยด้านซ้าย และขวา มีความสูงต่างกัน <= 1



Height Balanced

$O(\log_2 n)$

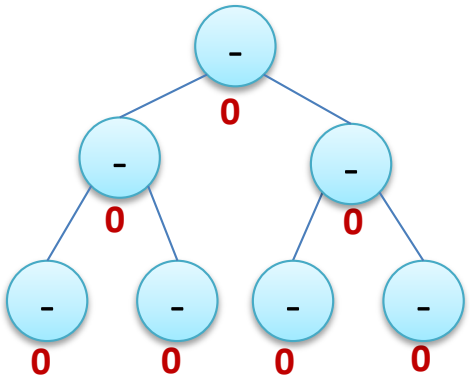
Balance Factor

Height Balanced Tree : ในแต่ละโหนดต้นไม้ย่อยด้านซ้าย และขวา มีความสูงต่างกัน <= 1

(Nearly Balanced) Balance factor ทุกโหนดมีค่า 0 หรือ 1 หรือ -1

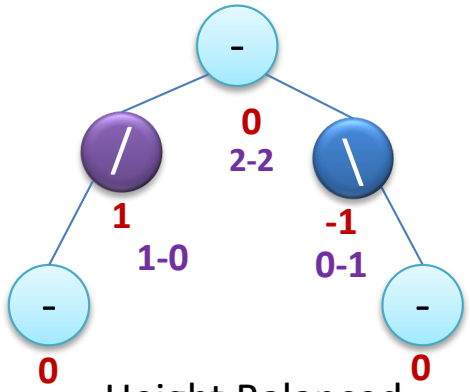
Balance factor ของแต่ละโหนด คือ ค่าความต่างระหว่างต้นไม้ย่อยด้านซ้าย และ ต้นไม้ย่อยด้านขวา

= Height(Left Subtree) – Height(Right Subtree)

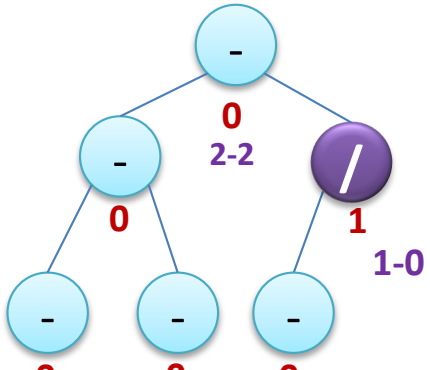


Height Balanced

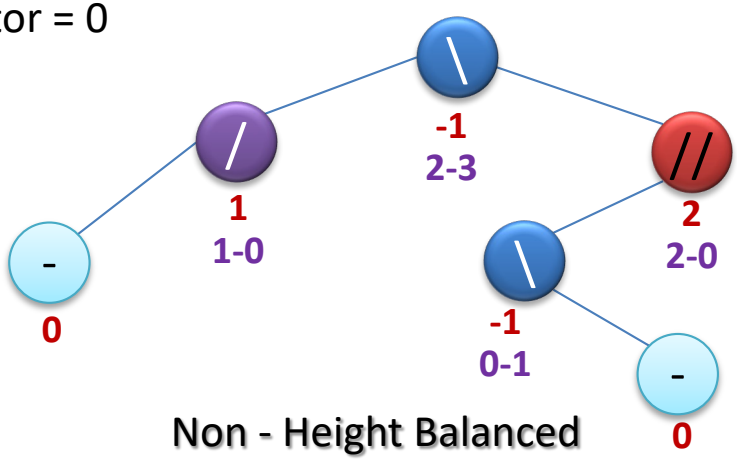
Every node's balance factor = 0



Height Balanced



Height Balanced



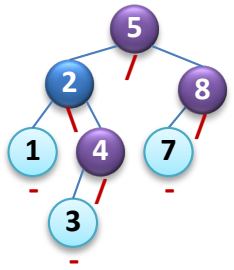
Non - Height Balanced

AVL (Height Balanced) Tree

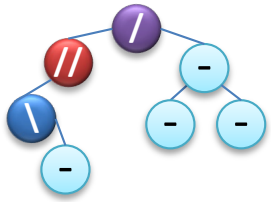
AVLTree Self- rebalancing binary search tree to keep height balance.

- Maximum height <= 1.44 $\log_2 n$: ใกล้กับ complete binary tree.
- Search, insertion, deletion (both average & worst cases) $O(\log_2 n)$
- การเพิ่ม และ การลบ อาจต้องทำการหมุน
- Worst case จะไม่ช้ากว่า binary search tree.

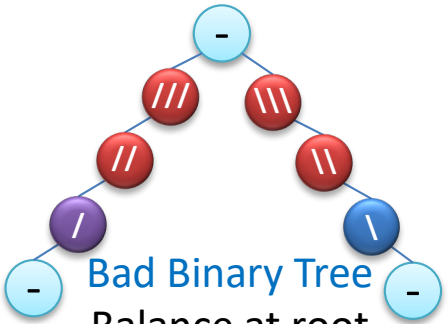

Adelson-Velskii Landis
(1962 Soviet inventors)



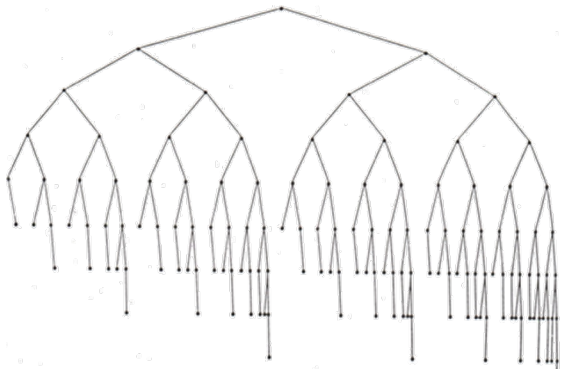
Height Balanced



Non-Height Balanced



Bad Binary Tree
Balance at root
is not enough.



Smallest AVL tree of height 9

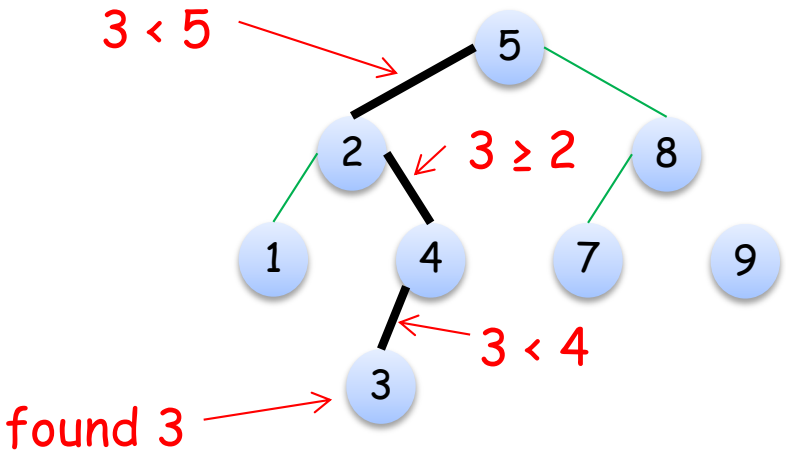
Searching in AVL = Searching in BST

AVL is a binary search tree.

Review Searching a BST (Binary Search Tree).

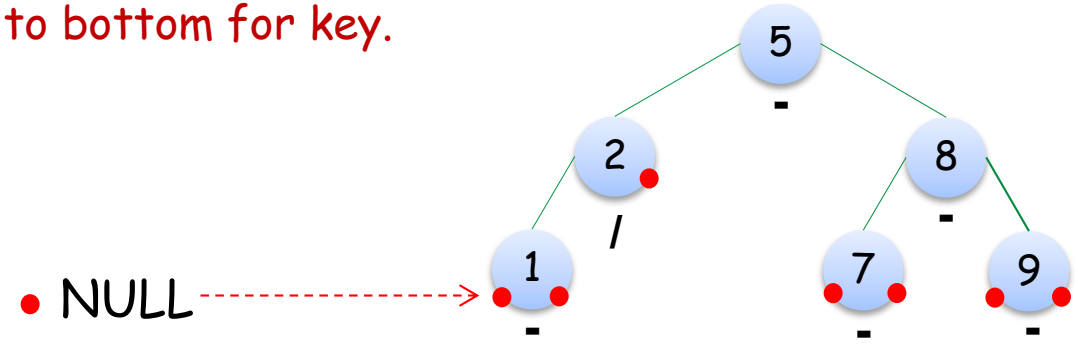
- เปรียบเทียบ **key** กับค่าในโหนดโดยเริ่มจาก **root**
- Follow associated link (recursively).
  - Left link if search key < key in node
  - Right link if search key ≥ key in node

Ex. Search for 3

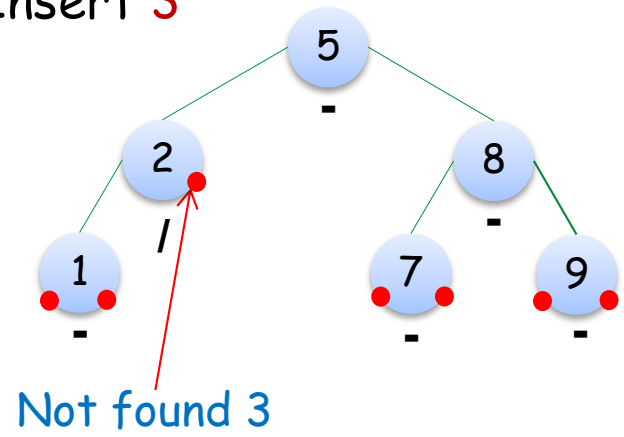


Review Inserting in BST

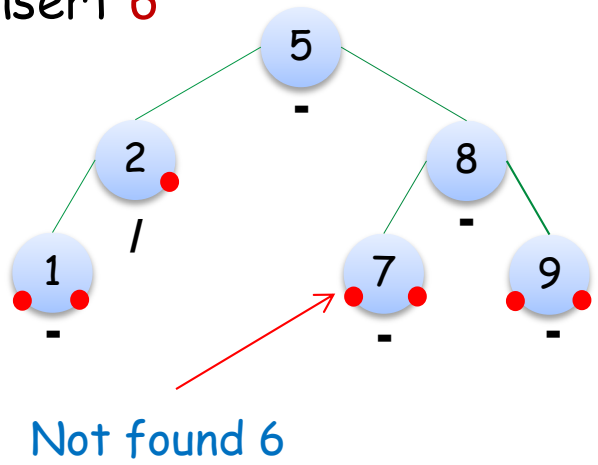
- Search to bottom for key.



Ex. Insert 3

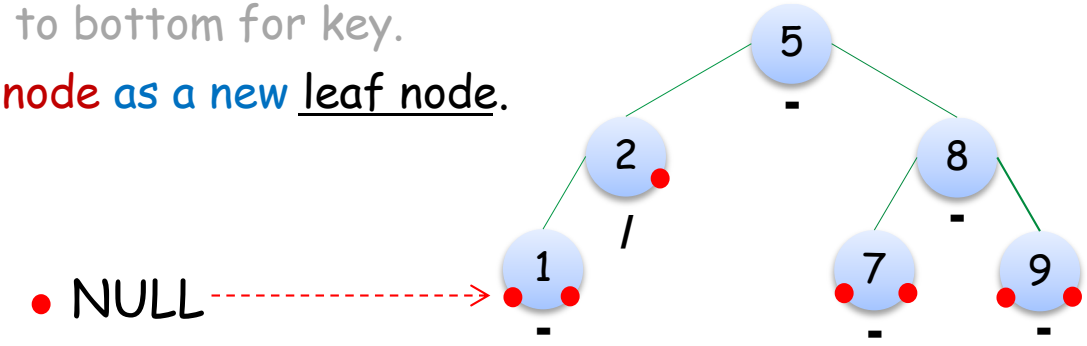


Ex. Insert 6

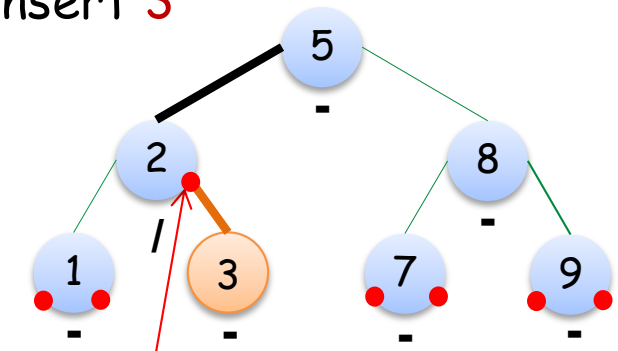


Review Inserting in BST

- Search to bottom for key.
- Insert node as a new leaf node.

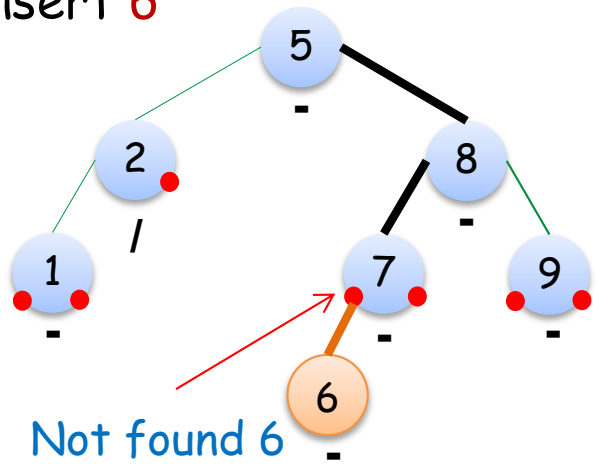


Ex. Insert 3



Not found 3
Insert 3 as a new leaf

Ex. Insert 6



Not found 6
Insert 6 as a new leaf

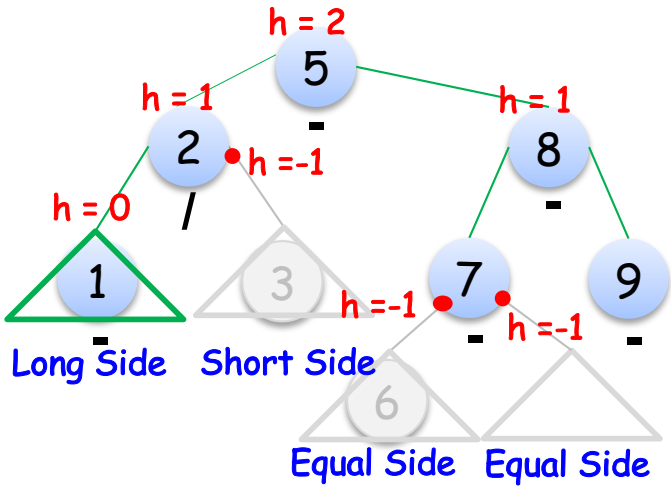
Inserting in AVL without Rebalancing

Inserting at short side or equal side

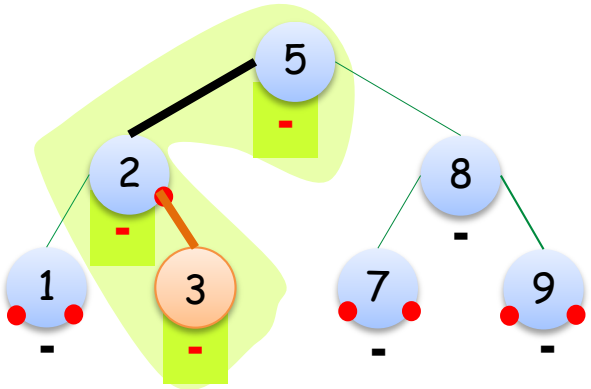
→ AVL 's still height balanced

- Insert BST new leaf node.
- การเพิ่มโหนดจะเปลี่ยนความสูงของบางโหนด (เพิ่มขึ้น 1)

จาก root มายังโหนดใหม่ และเปลี่ยนค่า balance factors

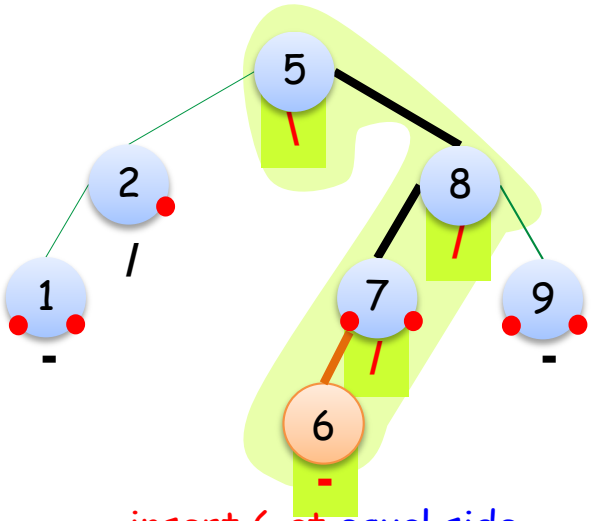


Ex. Insert 3 at short side.



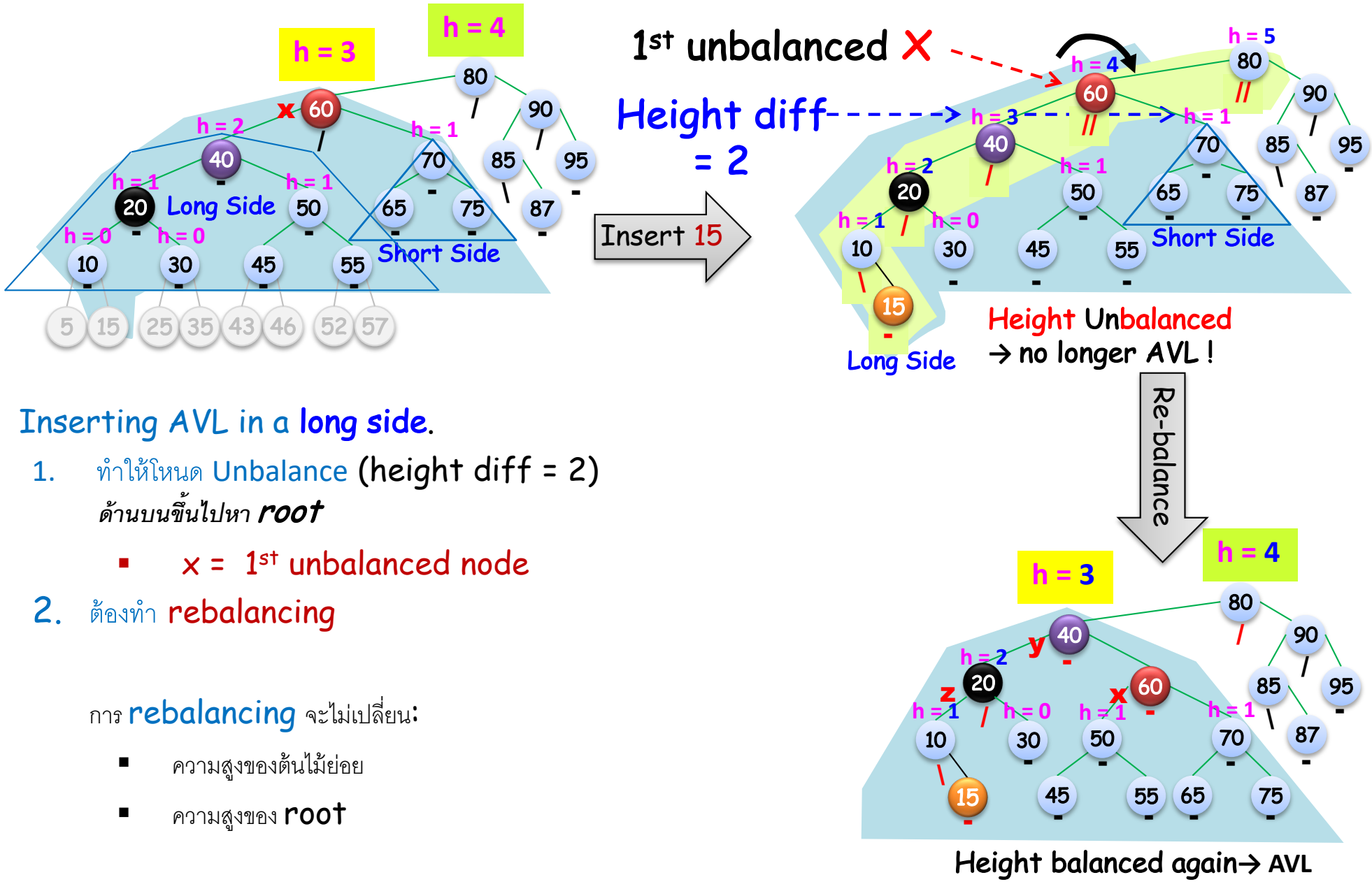
insert 3 at short side
still height balance

Ex. Insert 6 at equal side.



insert 6 at equal side
still height balance

Insertion Re-balancing in AVL



Inserting AVL in a **long side**.

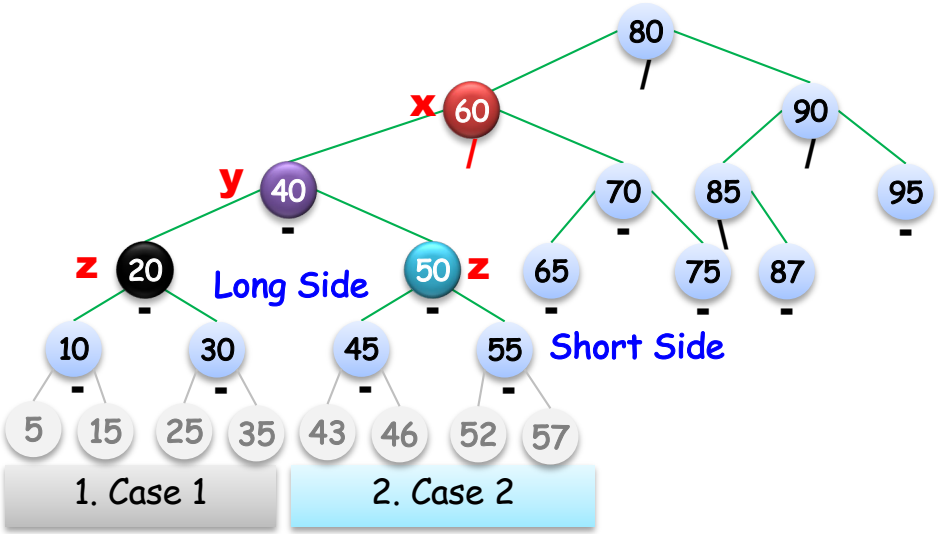
1. ทำให้โหนด Unbalance (height diff = 2)
   *ด้านบนขึ้นไปหา **root***
   - x = 1st unbalanced node
2. ต้องทำ rebalancing

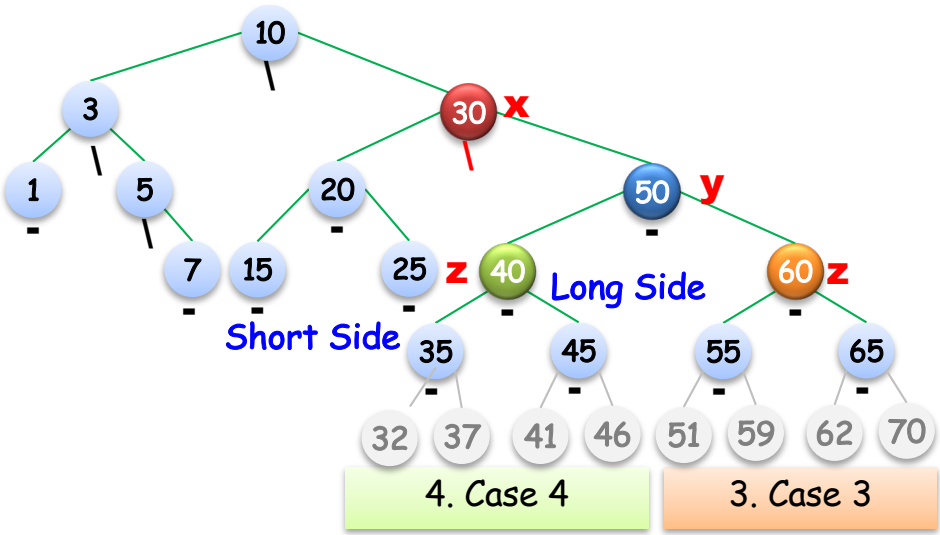
การ rebalancing จะไม่เปลี่ยน:

- ความสูงของต้นไม้ย่อย
- ความสูงของ root

4 Insertions Re-balancing in AVL

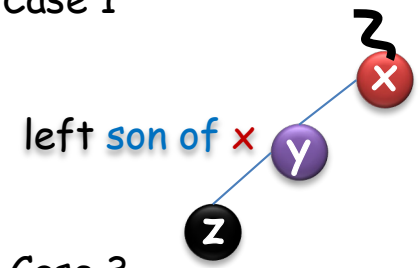
Inserting AVL in a **long side** : ให้ x = node แรกที่ไม่ balance , ตาม path ที่ insert ให้ y เป็นลูกของ x และ z เป็นลูกของ y



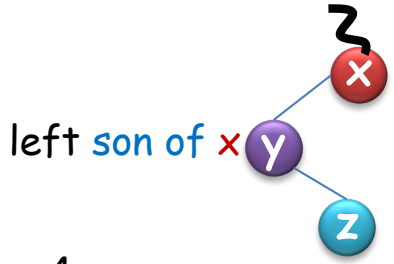


มี 4 กรณี เมื่อ **insert** ที่

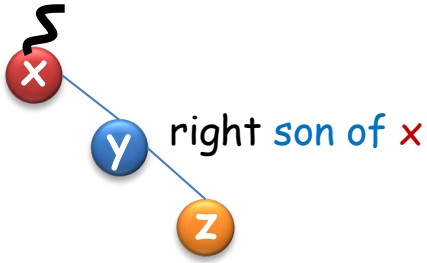
1. Case 1



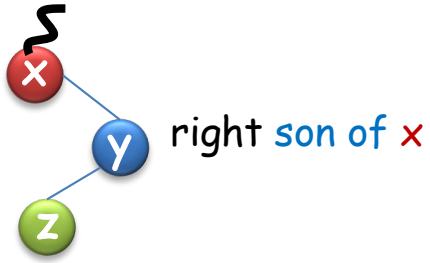
2. Case 2



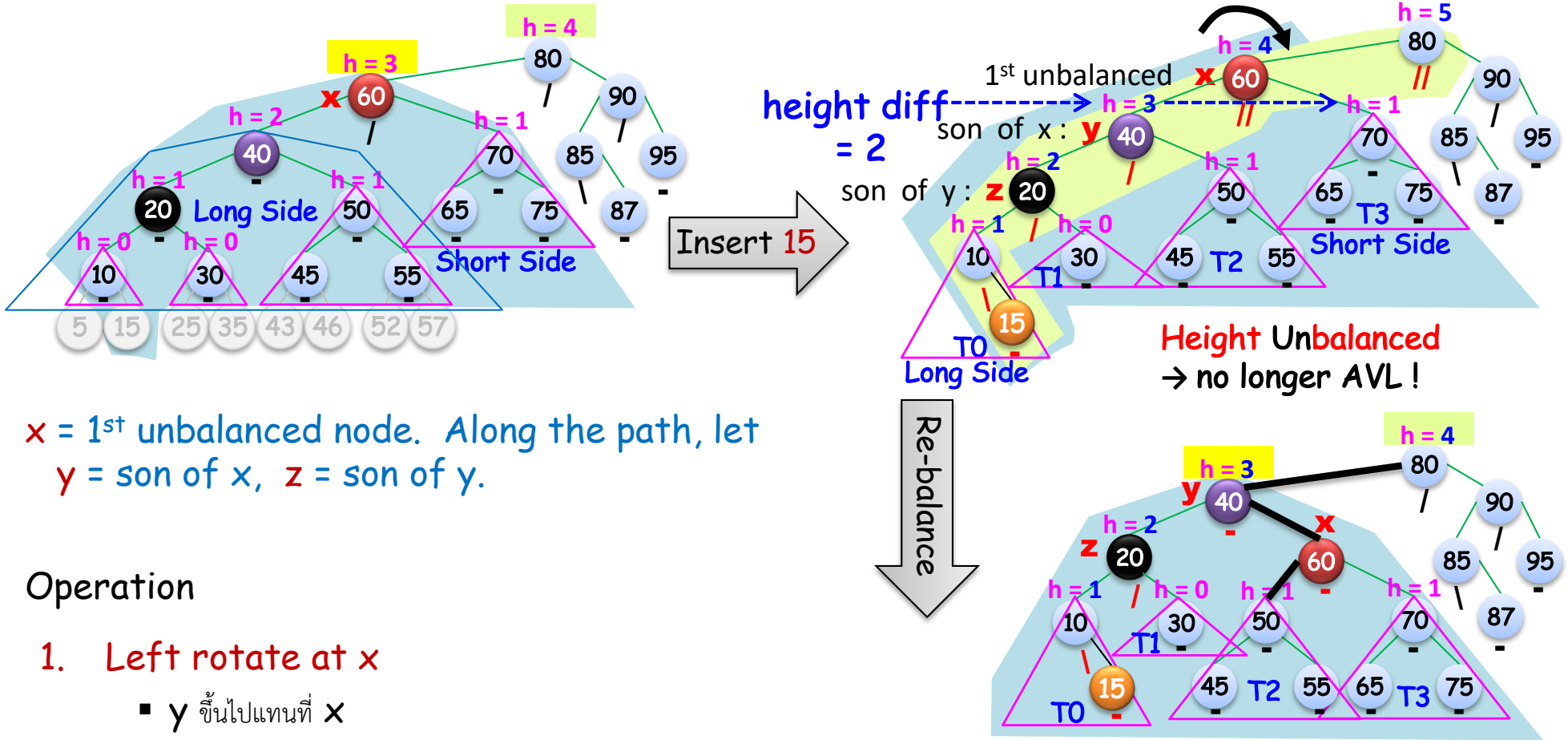
3. Case 3



4. Case 4



Rebalancing Left Subtree of Left Son of x (1st Unbalanced Node)



x = 1st unbalanced node. Along the path, let
y = son of x, z = son of y.

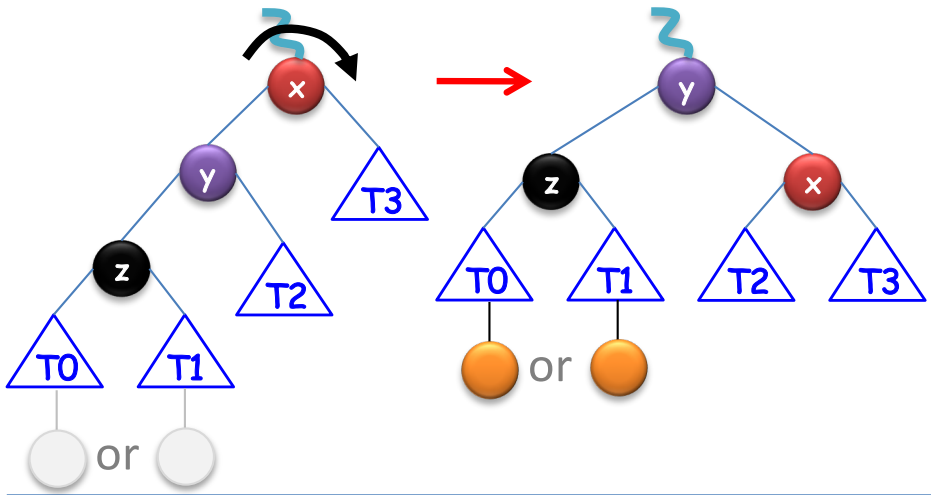
Operation

1. Left rotate at x
   - y ขึ้นไปแทนที่ x
   - x ลงมาเป็นลูกขวาของ y
2. Re-arrange T0,T1,T2,T3 to preserve the properties of BST.
   - T0 ≤ z ≤ T1 ≤ y ≤ T2 ≤ x ≤ T3
3. Fixing some part of the shaded subtree's balance facters.
4. Height of the blue shaded subtree doesn't change -> also the whole AVL's.

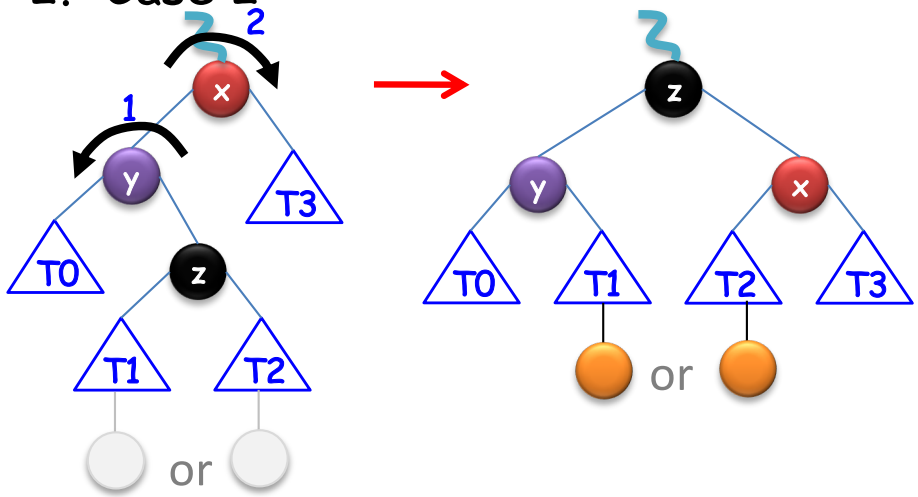
4 Kinds of Rebalancing in AVL

x = 1st unbalanced node, y = son of x, z = son of y

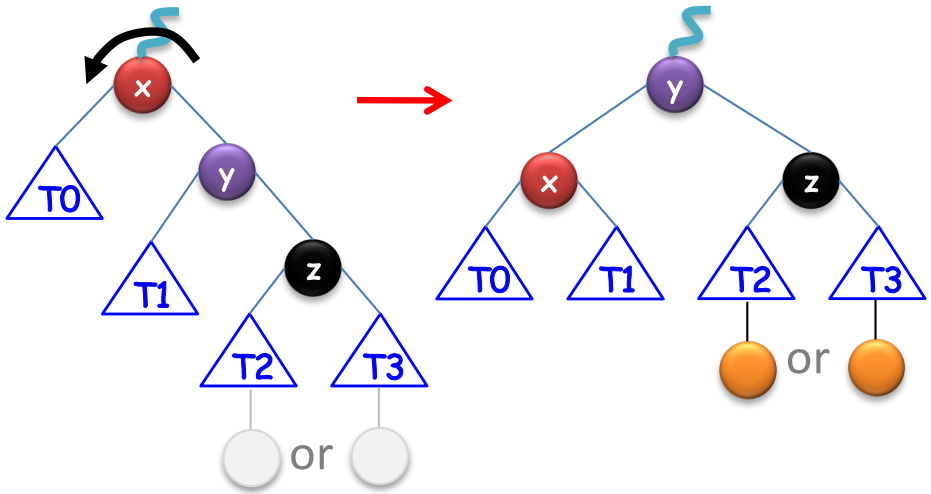
1. Case 1



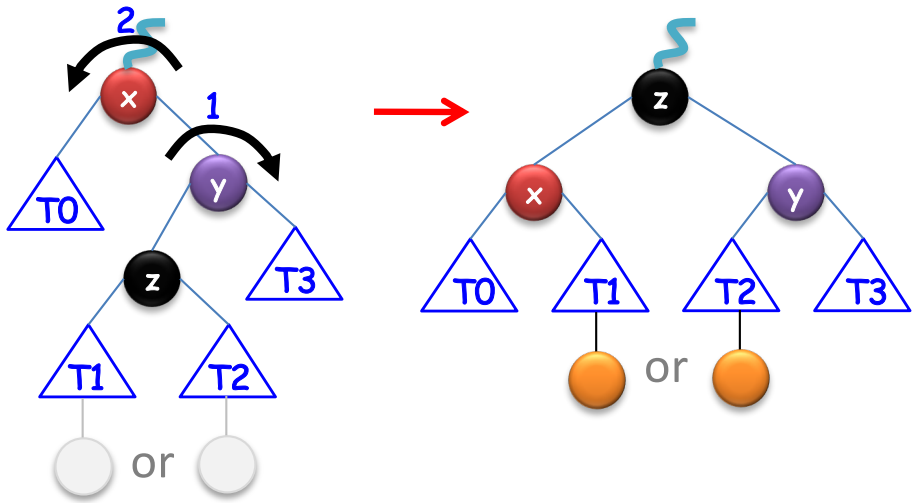
2. Case 2



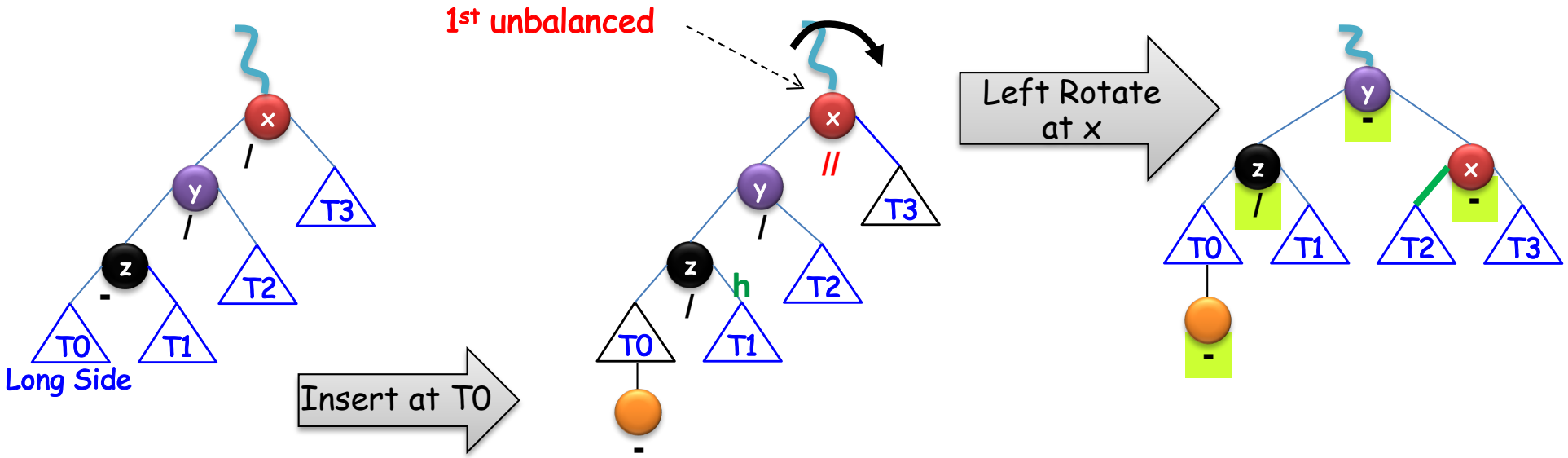
3. Case 3



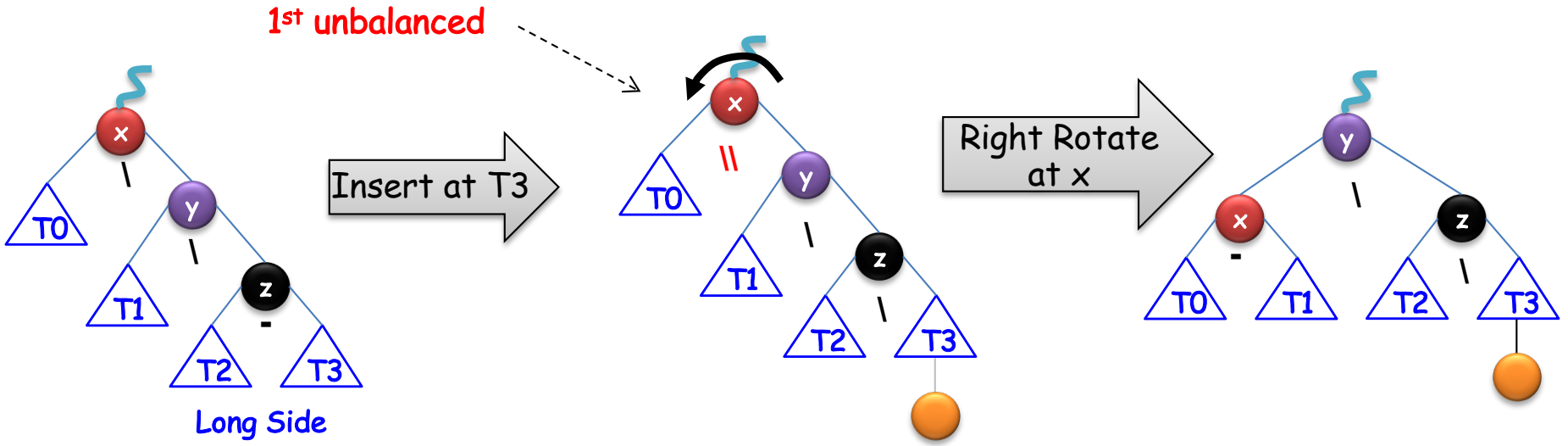
4. Case 4



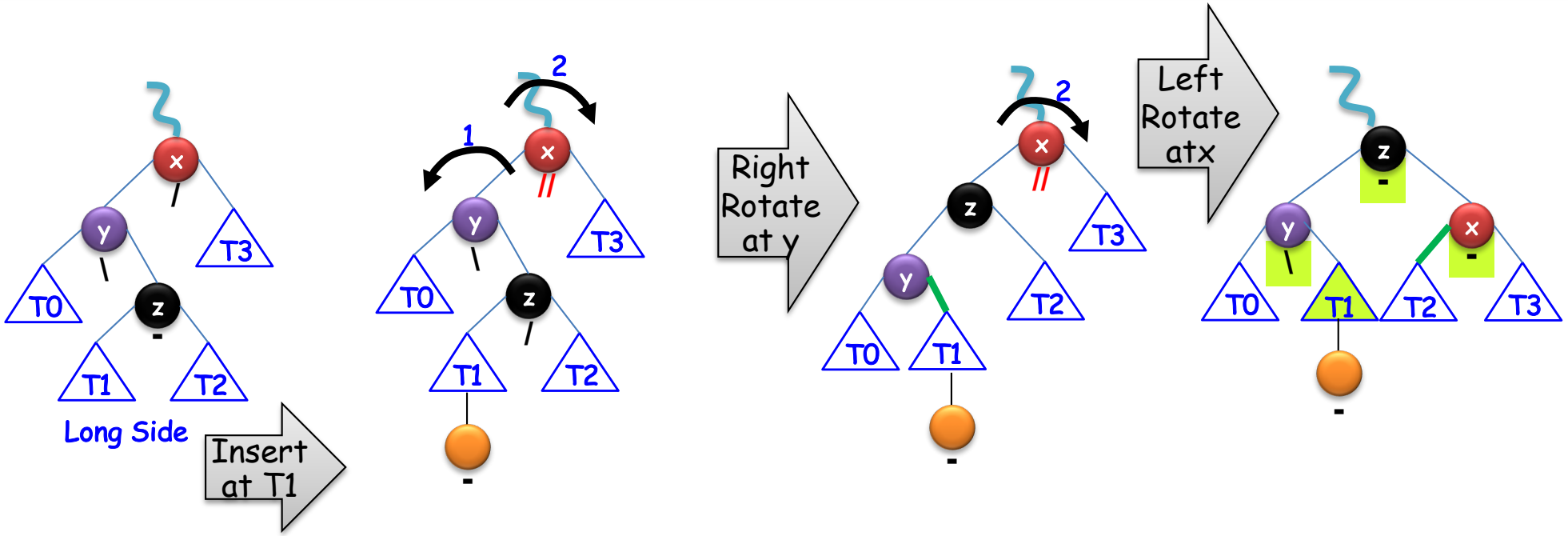
Case 1


1st unbalanced
x
y
z
-
T0
T1
T2
T3
Long Side
Insert at T0
h
Left Rotate at x

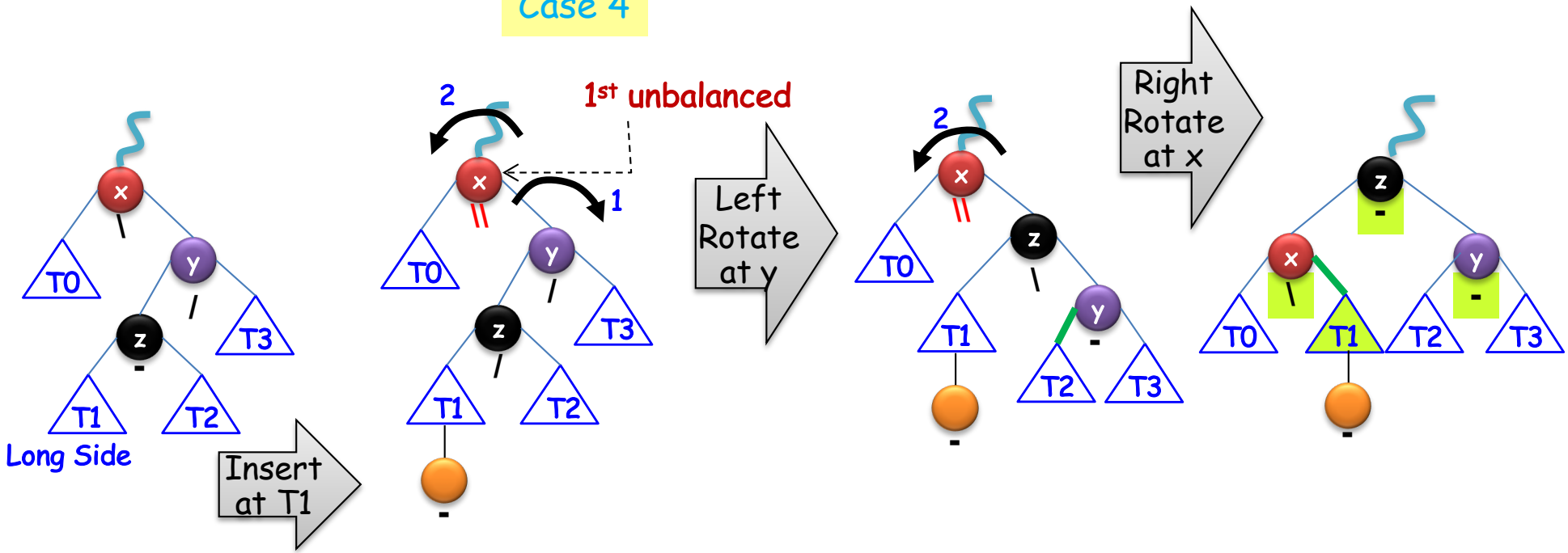
Case 3


1st unbalanced
x
y
z
-
T0
T1
T2
T3
Long Side
Insert at T3
Right Rotate at x

Case 2


x
y
z
T0
T1
T2
T3
Long Side
Insert at T1
2
1
Right Rotate at y
Left Rotate atx

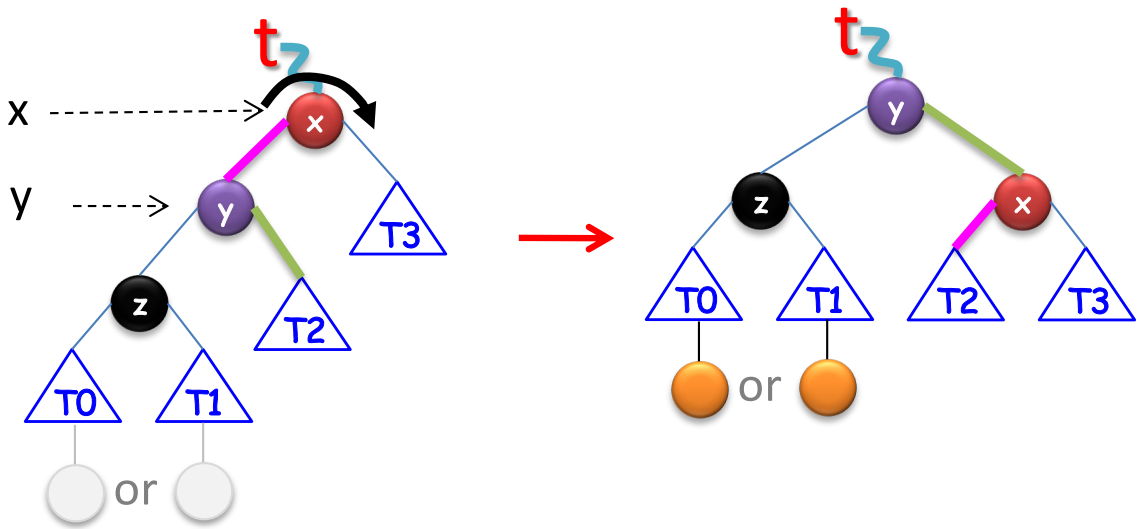
Case 4


x
y
z
T0
T1
T2
T3
Long Side
Insert at T1
2
1
1st unbalanced
Left Rotate at y
Right Rotate at x

Single Right Rotation & Single Left Rotation Algorithms

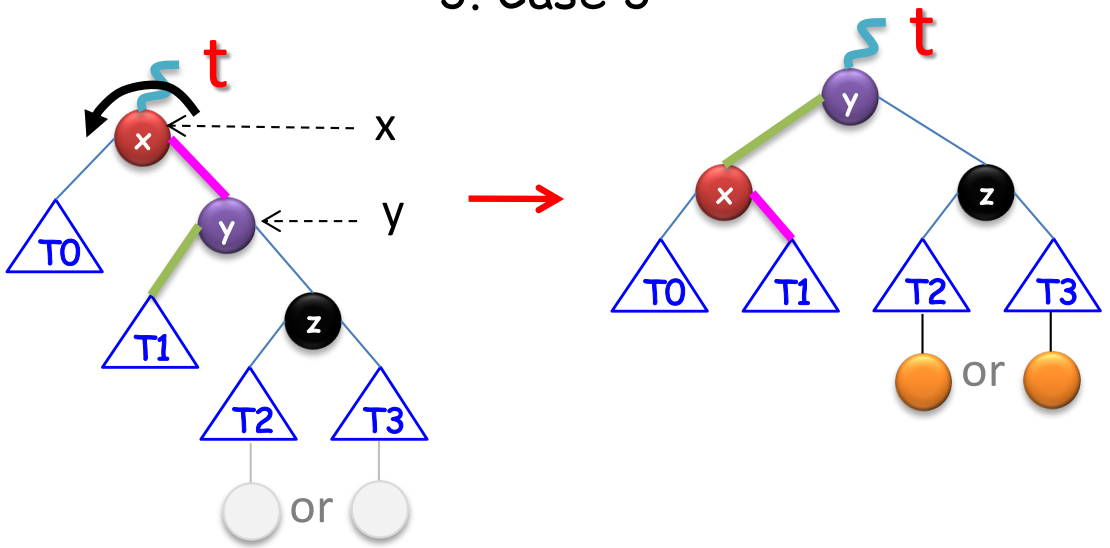
```
// Single Left Rotation
leftRotate(x)
    y = x.left
    x.left = y.right
    y.right = x
    return y
```

1. Case 1




```
// Single Right Rotation
rightRotate(x)
    y = x.right
    x.right= y.left
    y.left = x
    return y
```

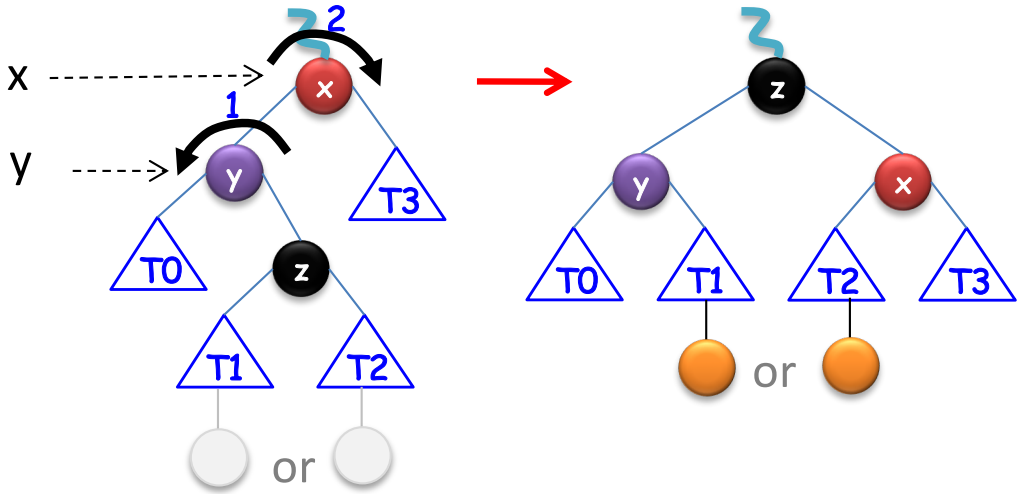
3. Case 3



Double Rotations Algorithms

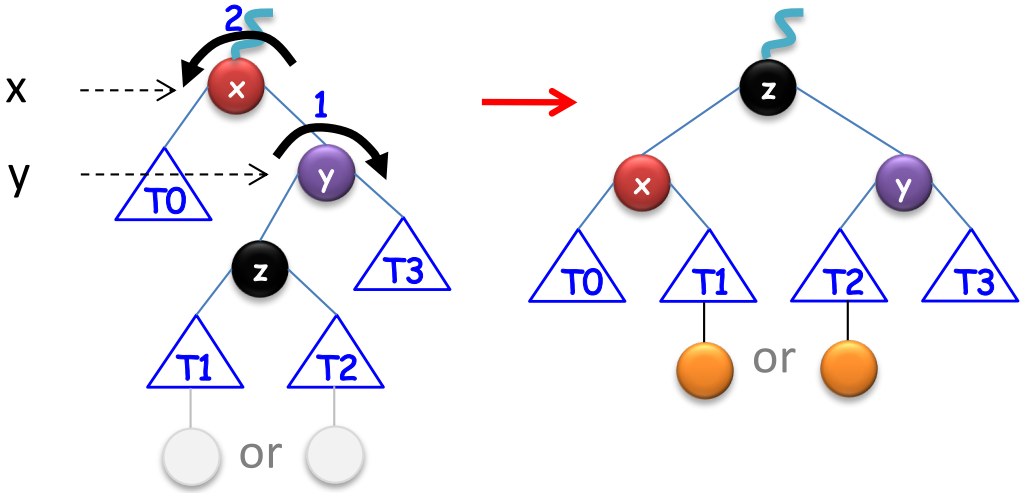
```
// Double Rotation :
// Case 2 :
doubleRotation(x)
        x.left = rightRotate(x.left)
        x = leftRotate(x)
        return x
// Case 4 :
doubleRotation (x)
        x.right = leftRotate(x.right)
        x = rightRotate(x)
        return x
```

2. Case 2




```
//Rebalance()
balance = x.balanceValue
if balance == -2 :
    if x.right.balanceValue == 1 :
        x.right = leftRotage(x.right)
    x = rightRotage(x)
elif balance == 2 :
    if x.left.balanceValue == -1:
        x.left = rightRotage(x.left)
    x = leftRotage(x)
```

4. Case 4



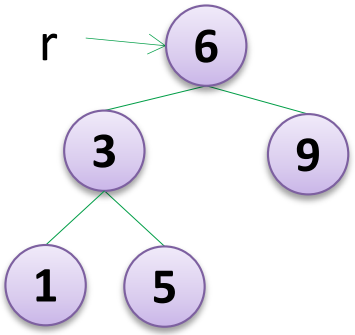
Tree 2

3. AVL Tree
   Height Balanced Tree
4. Which Representions ?
5. n-ary Tree
6. Generic Tree
7. Multiway Search Tree
8. Top Down Tree
9. B-Tree

1. Tree Definitions
2. Binary Tree
   - Traversals
   - Binary Search Tree
   - Representations
   - Application : Expression Tree

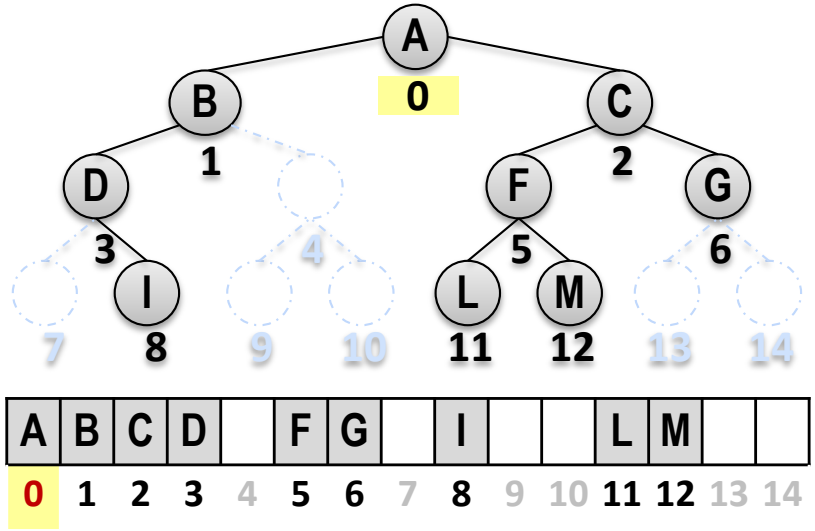
Binary Tree Representations

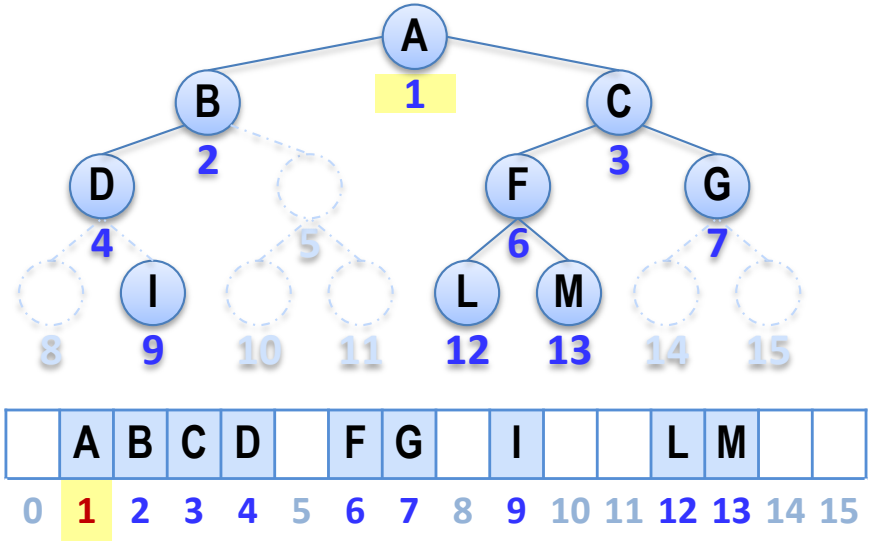
1. Dynamic



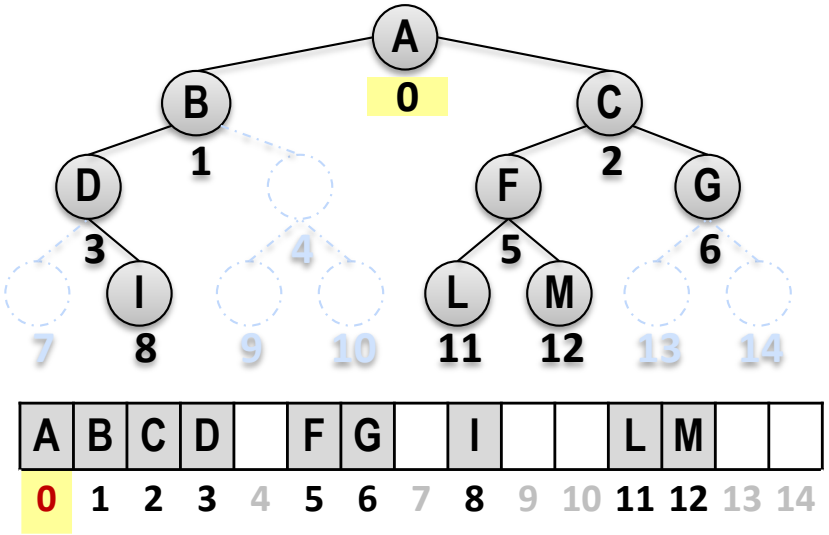

```
class node:

    def __init__(self,  data, left = None, right = None):
        self.data = data
        self.left = left if left is not None else None
        self.right = right if right is not None else None
```

2. Sequential (Implicit) Array



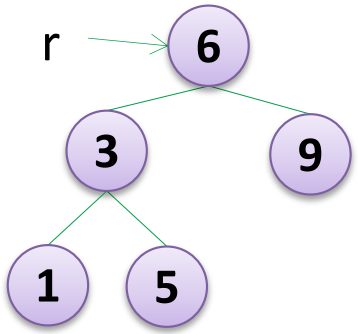


Sequential Array (Implicit Array)



- ง่าย
- ต้องกะขนาด array
- เพิ่มขึ้นอีก 1 level => array มีขนาด เพิ่มมากกว่าที่มีอยู่เดิม 1 เท่า (คิดเต็มที่ full binary tree)
- ถ้ากะขนาด array ถูก และ tree ใช้พื้นที่ส่วนใหญ่ของ array เช่น almost complete binary tree) จะ save space เพราะไม่ต้องมีฟิลด์ left, right, father

Dynamic



- จำนวน node ถูกจำกัดโดย memory

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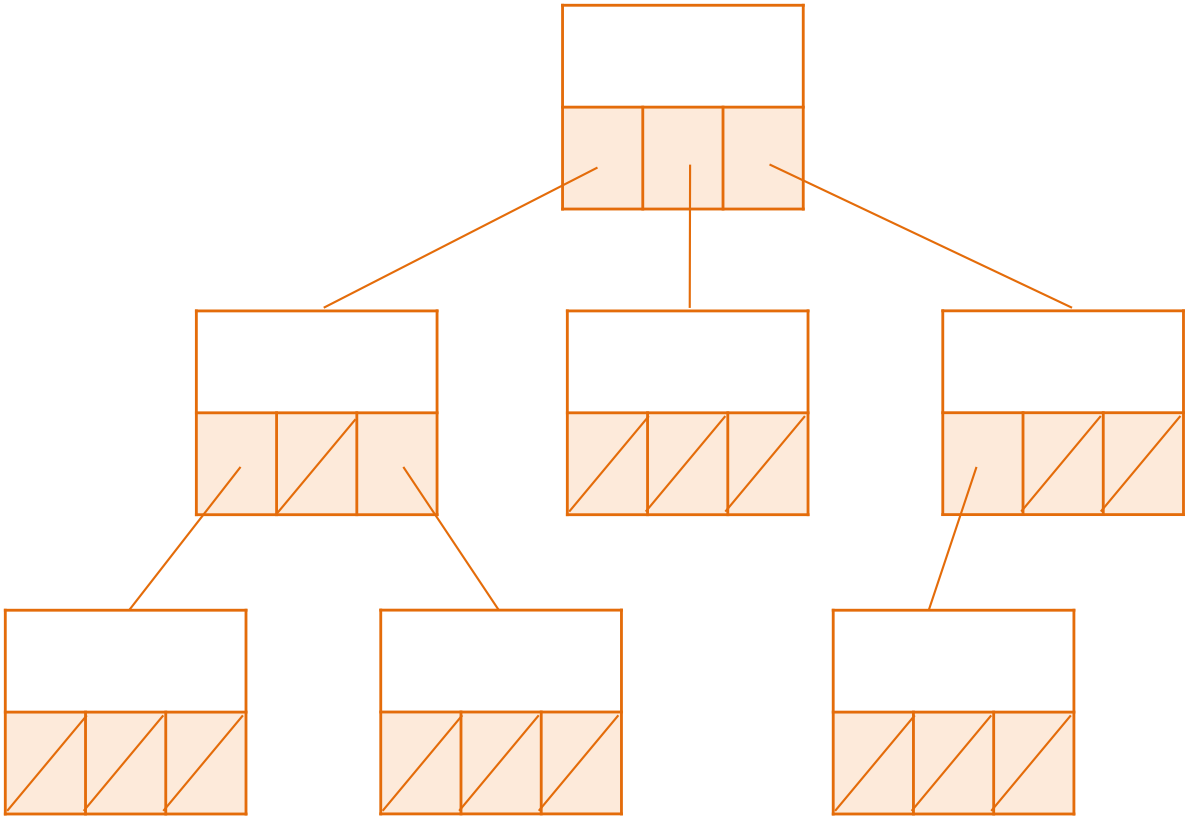
n-ary (multiway order n) Trees

bi-nary Tree แต่ละ node มีลูกได้อย่างมากที่สุด 2 ตัว

n-ary Tree แต่ละ node มีลูกได้อย่างมากที่สุด n ตัว



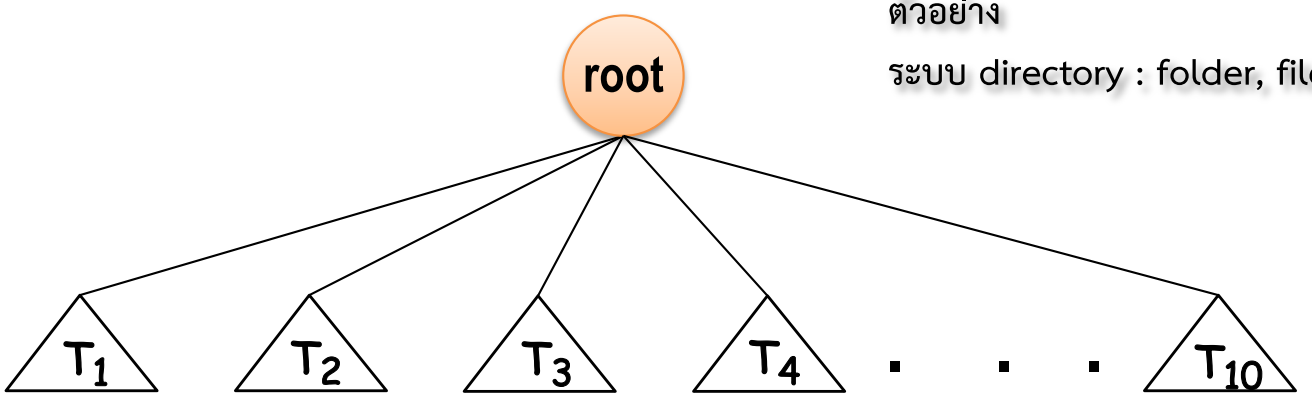
Ternary Tree n = 3

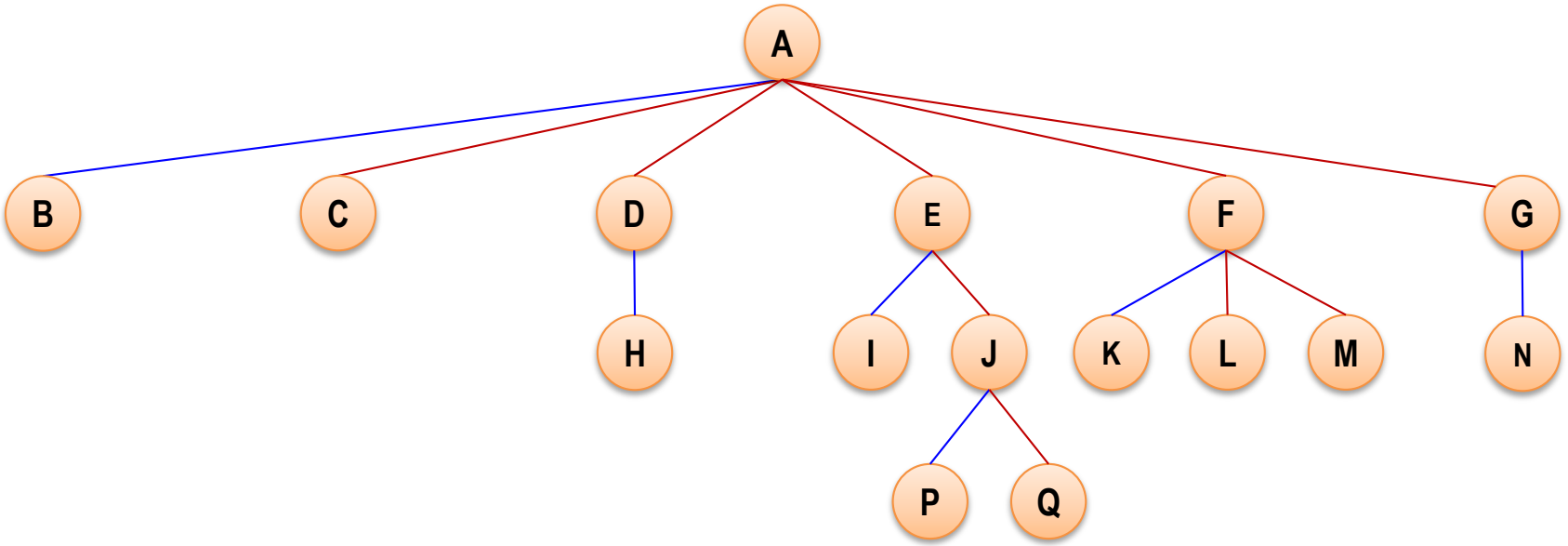
Generic Tree

Generic Tree : แต่ละ node มีลูกได้ไม่จำกัดจำนวน

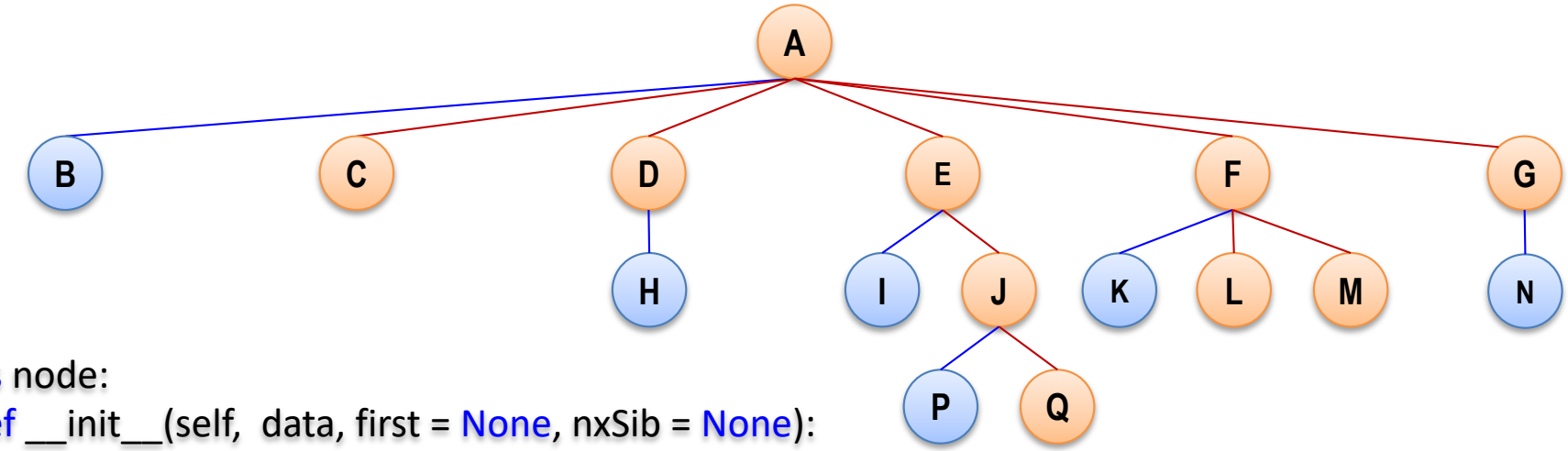
ตัวอย่าง

ระบบ directory : folder, file

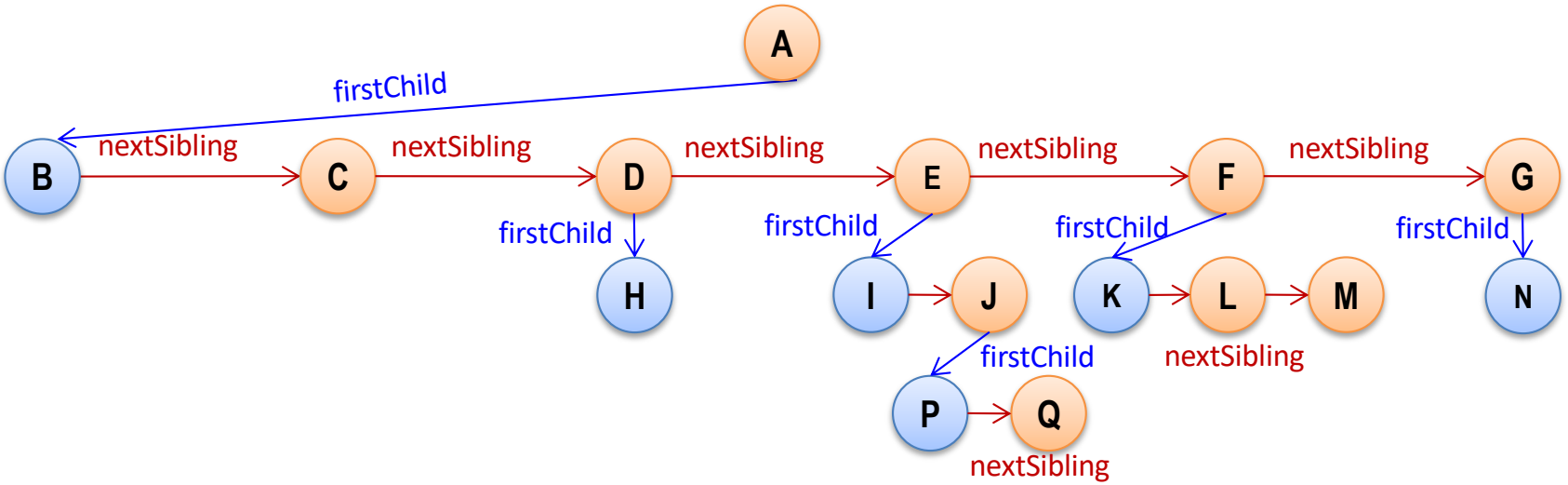




Implementation of Generic Tree




```
class node:
   def __init__(self,  data, first = None, nxSib = None):
      self.data = data
      self. firstChild = None if first is None else first
      self. nextSibling = None if nxSib is None else nxSib
```



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Multiway Search Tree

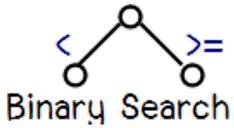
Multiway Tree order n : มีลูกได้มากที่สุด n ตัว (0, 1,... or n subtrees) (#max. sons = n)

For order n :
- #max. sons = n
- #max. keys = n-1

Multiway Search Tree : for every key K

left decendents ≤ K

right decendents > K

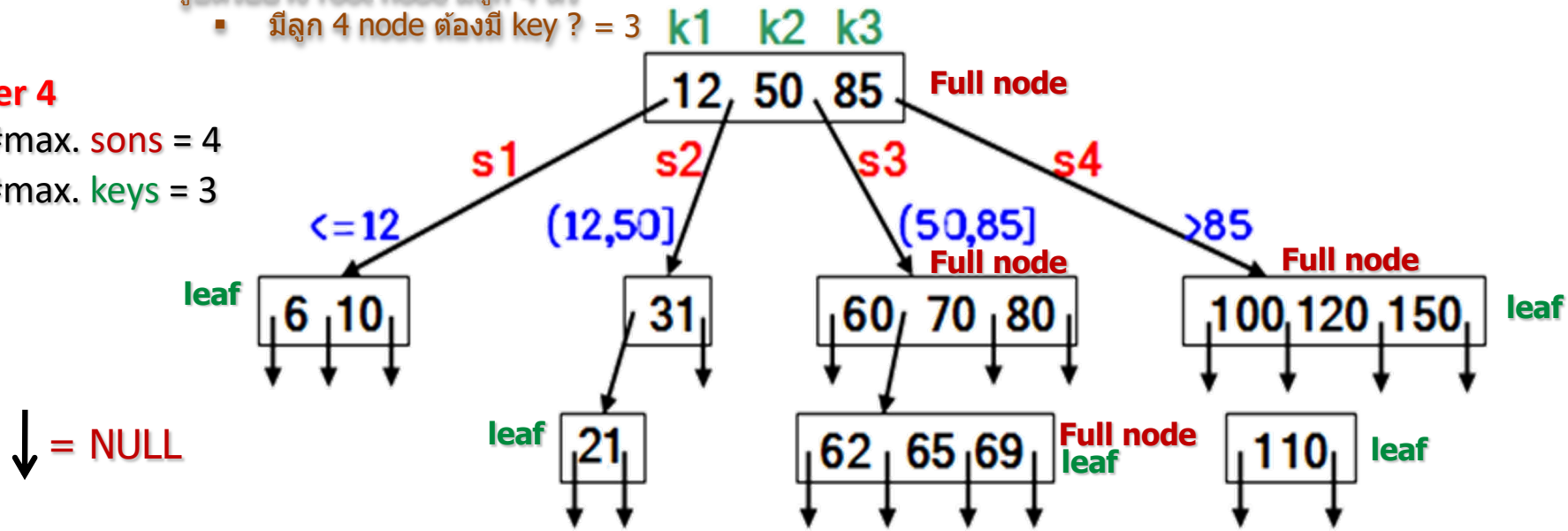


leaf : node ที่ไม่มีลูกเลย

Full node : node ที่มีจำนวน key เต็มที่

รูปตัวอย่าง root node มีลูก 4 ตัว
- มีลูก 4 node ต้องมี key ? = 3

Order 4
- #max. sons = 4
- #max. keys = 3



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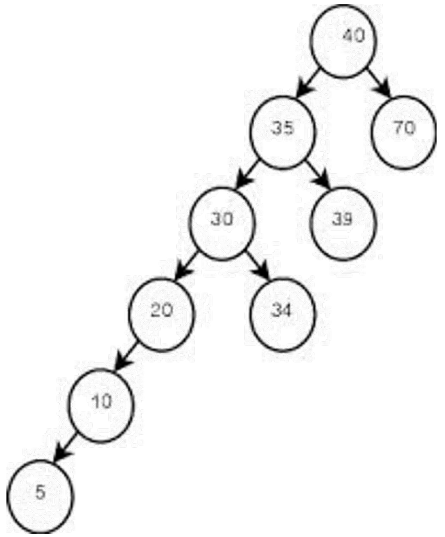
Top Down Tree

Top Down Tree : Non full node must be leaf.
Node ที่ไม่เต็มจะเป็น leaf ได้เท่านั้น เป็น internal node ไม่ได้

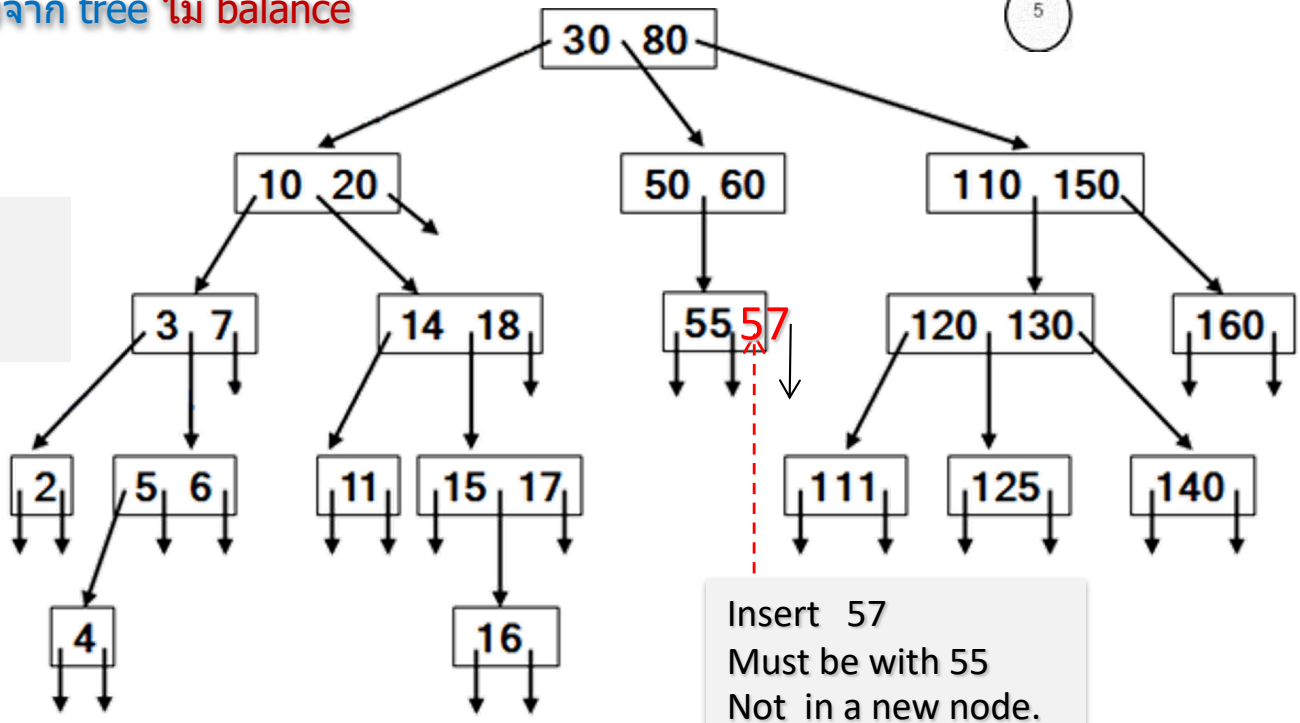
- Fill up the existing nodes before creating a new node !
  เติม node ให้เต็มก่อนค่อยแตก node ใหม่

แนวคิด : มี nodes ให้น้อยที่สุด เพราะ อยากให้ height สั้น

แม้ว่าพยามทำให้ node น้อย โดยเติมให้เต็มก่อนสร้าง node ใหม่ ก็ตาม height อาจยาวได้เนื่องจาก tree ไม่ balance



Order 3
- #max. sons = 3
- #max. keys = 2



Insert 57
Must be with 55
Not in a new node.

Tree 2

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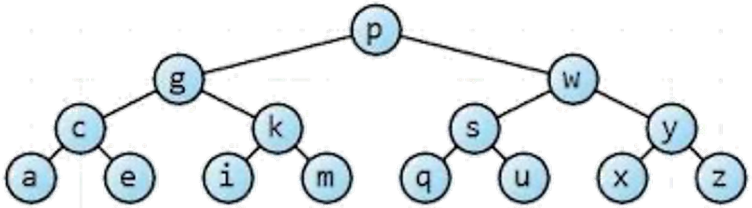
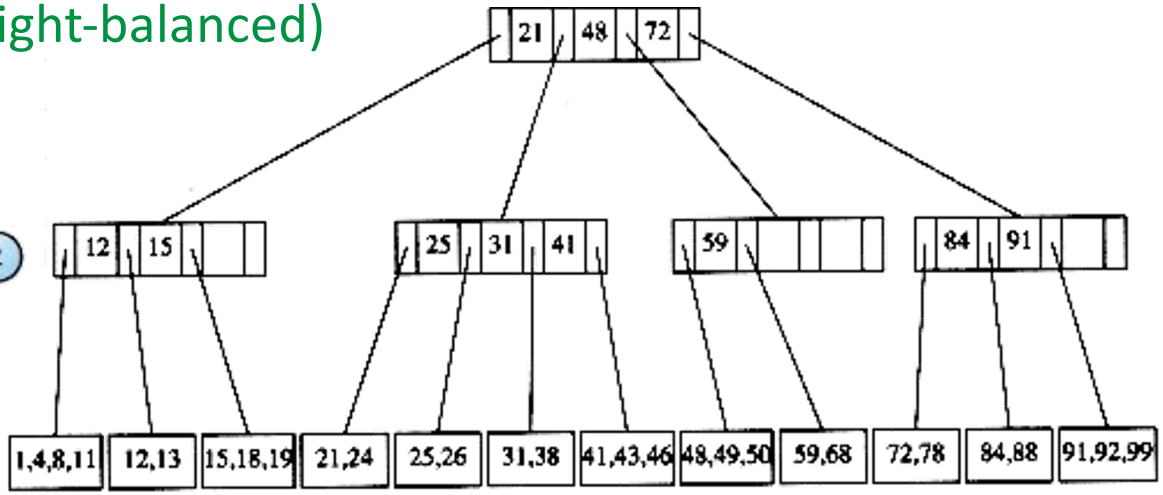
Balanced Tree

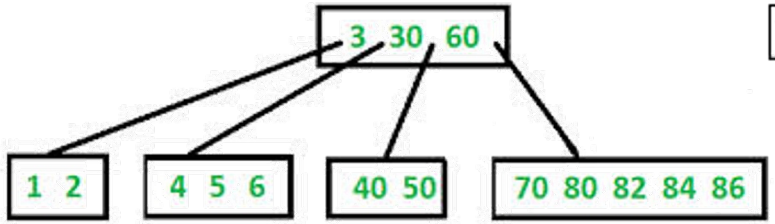
Balanced Tree : every leaf node is at the same level

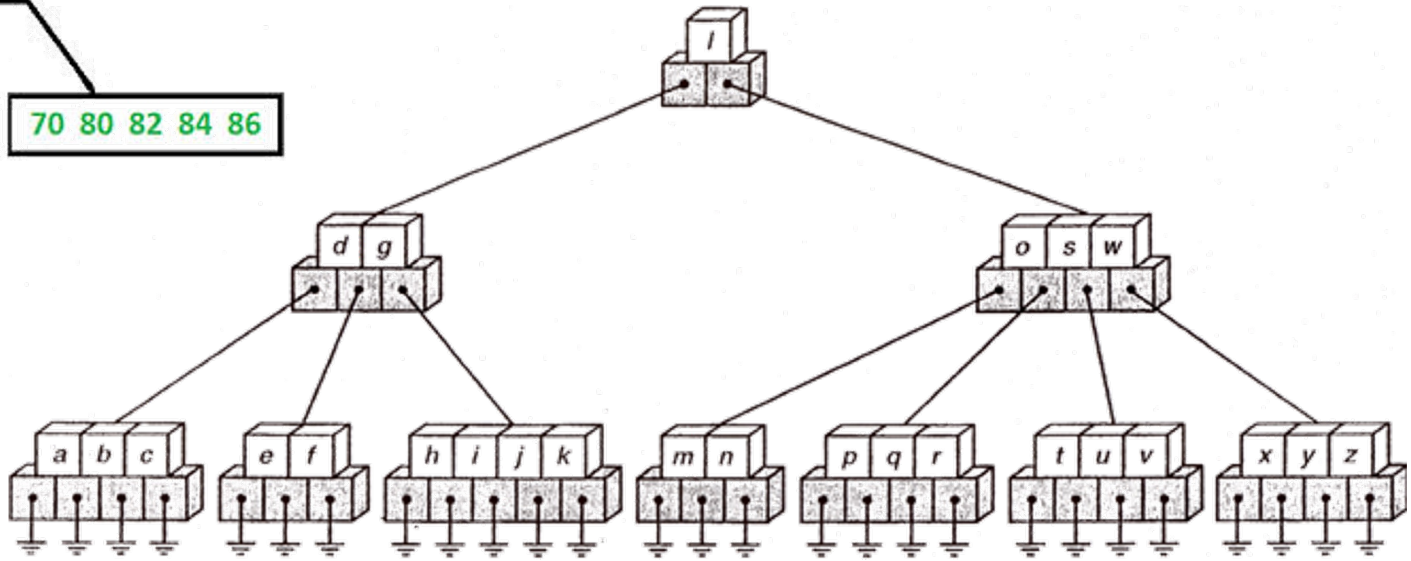
ทุก leaf อยู่ level เดียวกัน

(Perfect balance , perfectly height-balanced)







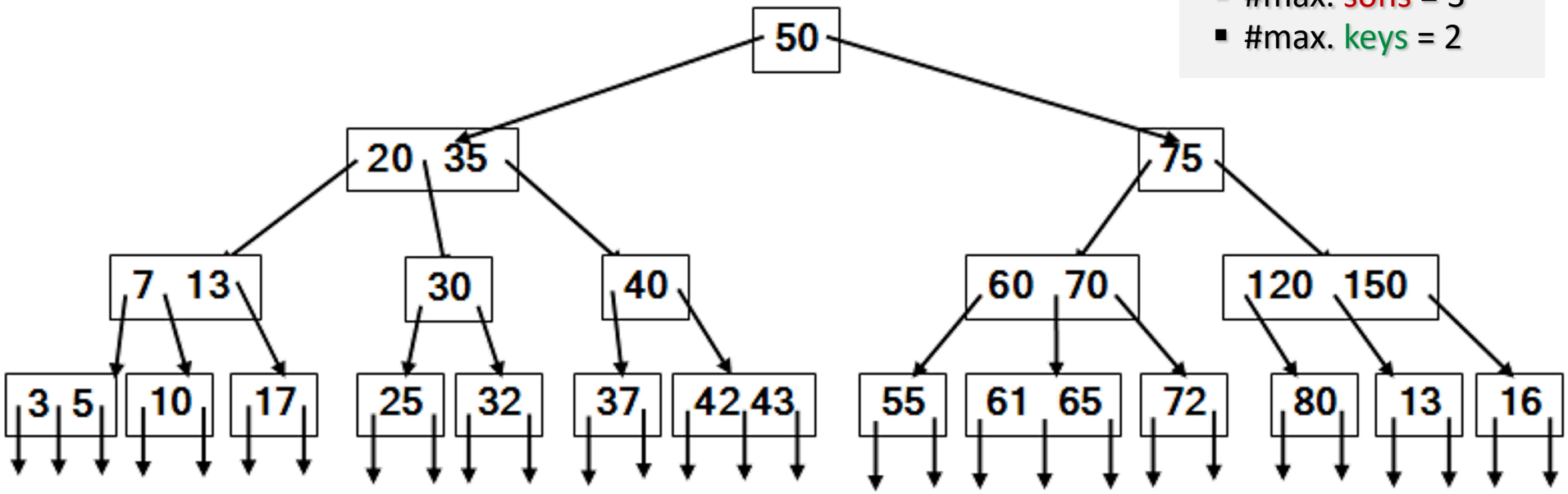


B-Trees

B-tree order n

1. Multi-way search tree order n : #max son = n, #max keys = n-1
2. Balanced tree : ทุก leaf อยู่ใน level เดียวกัน
   (Perfect balance , perfectly height-balanced)
3. => next page

Order 3
- #max. sons = 3
- #max. keys = 2



B-Trees

B-tree order n

1. Multi-way search tree order n

   #max son = n, #max keys = n-1

2. Balanced tree

3. ทุก node มีจำนวน non-empty subtree (มีลูก)อย่างน้อยที่สุด เท่ากับครึ่งหนึ่งของ n (มีน้อยกว่านี้ไม่ได้) คือ Half Full

   ยกเว้น root node ไม่ต้อง half full ก็ได้

   #min sons = $\lceil n/2 \rceil$ ยกเว้น root

   -> ไม่มี node ที่เล็กมากๆ

   -> ดังนั้น จำนวน node ทั้งหมดไม่มากนัก

   -> height ไม่ยาว

EX.

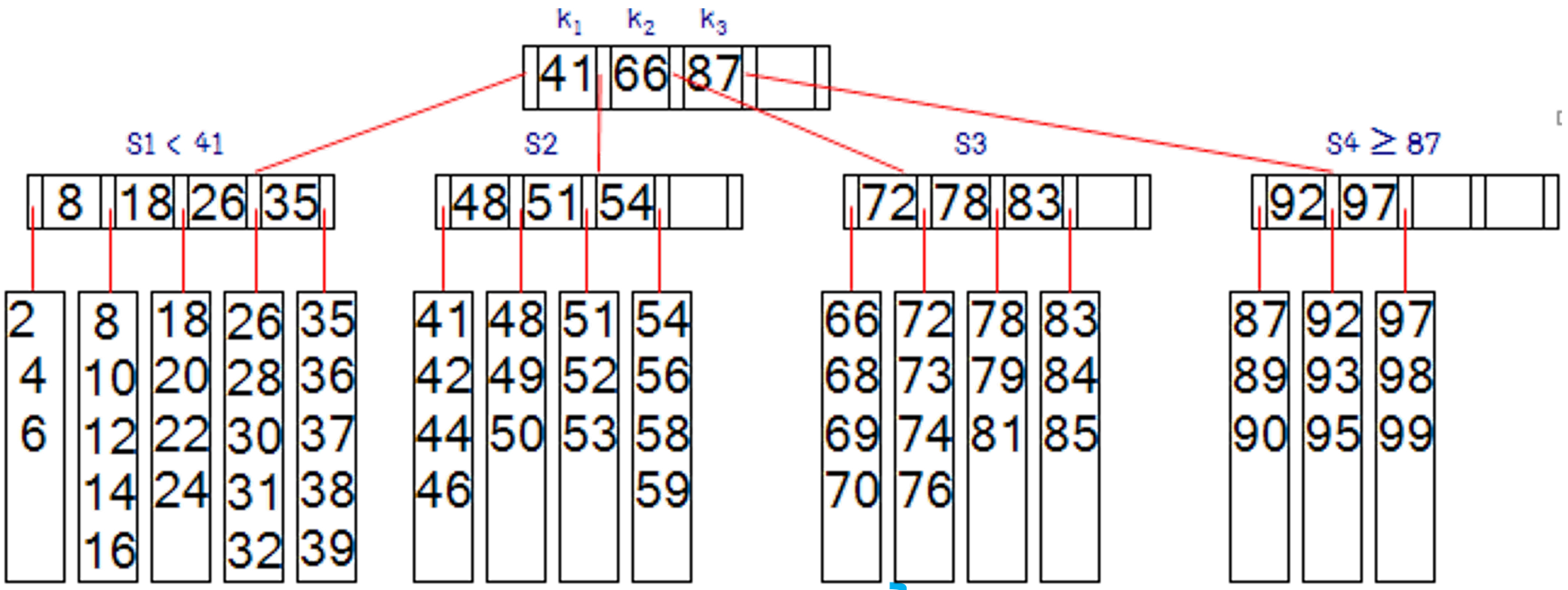
1. order 2 : #max sons =2 , #min sons = $\lceil 2/2 \rceil$ = 1

   ∴ Binary ที่เป็น (almost) complete binary tree จะเป็น B-tree

2 order 5 : #max sons = 5 , #min sons = $\lceil 5/2 \rceil$ = $\lceil 2.5 \rceil$ = 3

3 order 201 : #max sons = 201 , #min sons = $\lceil 201/2 \rceil$ = 101

B-Tree Variation



order 5 #max son = 5, #max keys = 4

- Half Full #min sons = $\lceil 5/2 \rceil = \lceil 2.5 \rceil = 3$ (ยกเว้น root node)

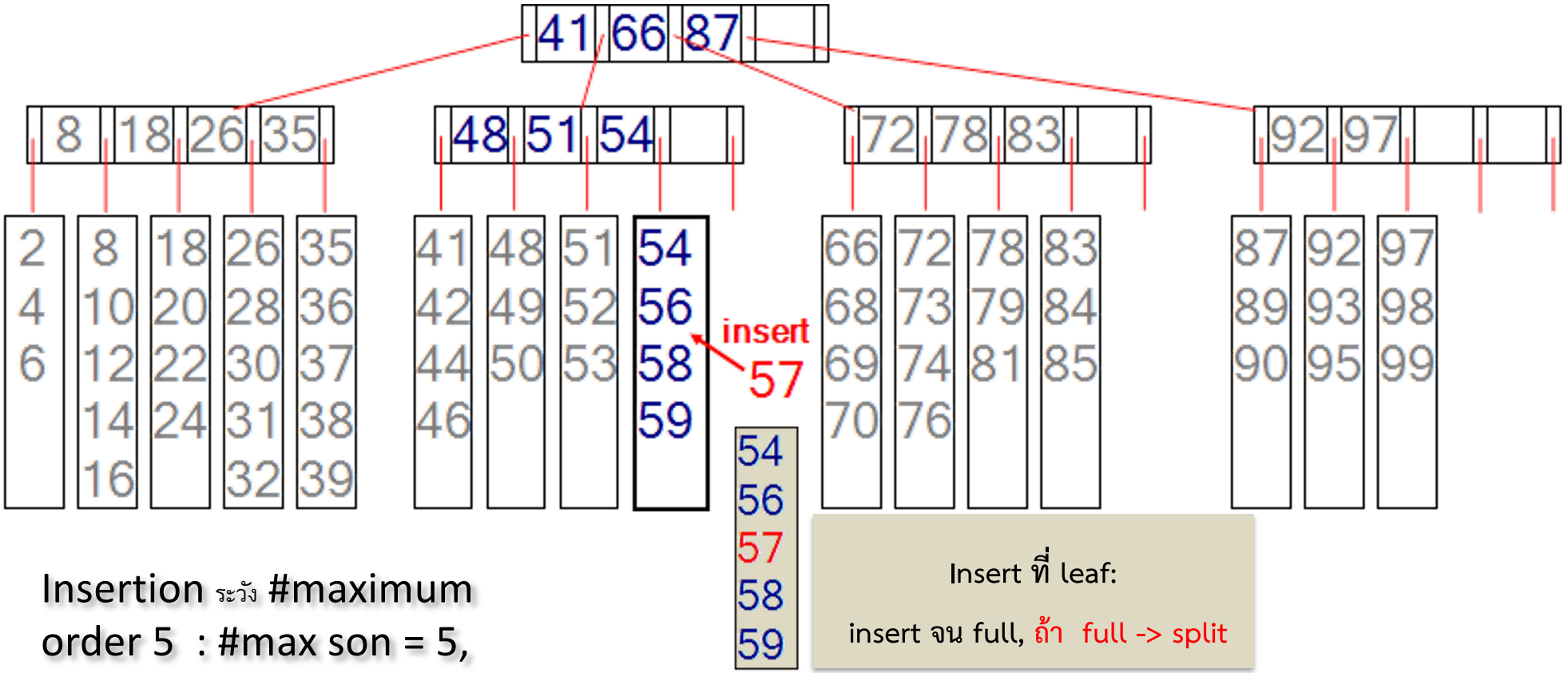
B-tree -> variations

- internal nodes : เก็บเฉพาะ key เพื่อประหยัดพื้นที่ (เก็บ data ได้มากกว่า เก็บทั้ง record), order กำหนดจำนวนลูก
- external nodes : เก็บ data ทั้ง record

ค่า L กำหนดจำนวน record/node ของ leaf

ถ้า L = 5 : #max data = 5, #min data = $\lceil 5/2 \rceil = \lceil 2.5 \rceil = 3$

B-Tree (Variation) Insertion



Insertion ระวัง #maximum

order 5 : #max son = 5,

Internal : **#max keys = 4,**

#min sons = $\lceil 5/2 \rceil = 3$

Leaf L = 5: **#max data = 5,**

#min data = $\lceil 5/2 \rceil = 3$

Insert ที่ leaf:

insert จน full, ถ้า full -> split

จะ insert ต้องระวังไม่ให้เกินจำนวนที่กำหนด (max)

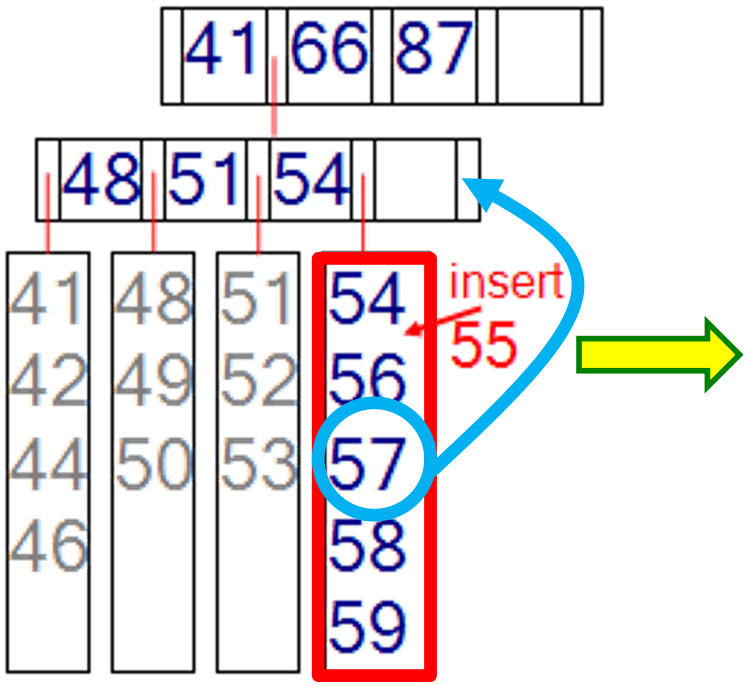
-> Insert ที่ leaf ดูค่า L

Leaf L = 5: #max data = 5

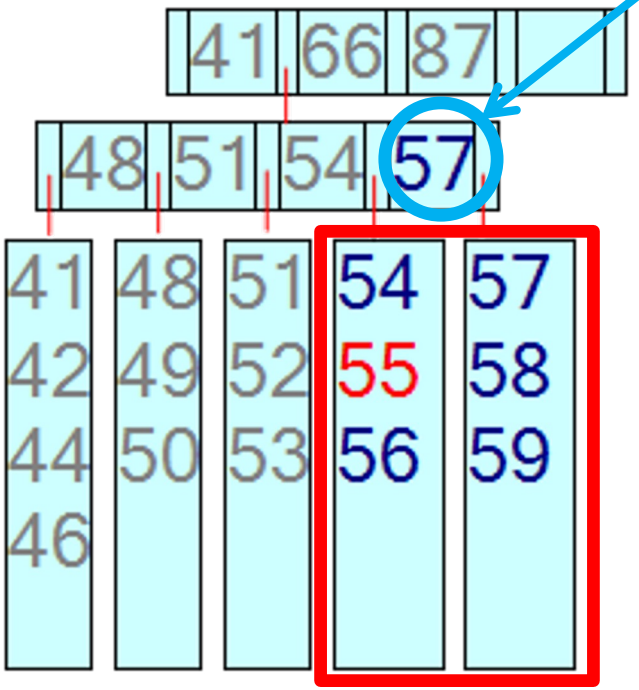
B-Tree (Variation) Insertion

order 5, #max key = 5-1=4
L=5 #leaf max data = 5

order 5 : internal node มี
#max son = 5
: **#max keys = 4,**
not full -> can insert



Insert 55
Insert ที่ leaf จน full :
L = 5: #max data = 5
Full

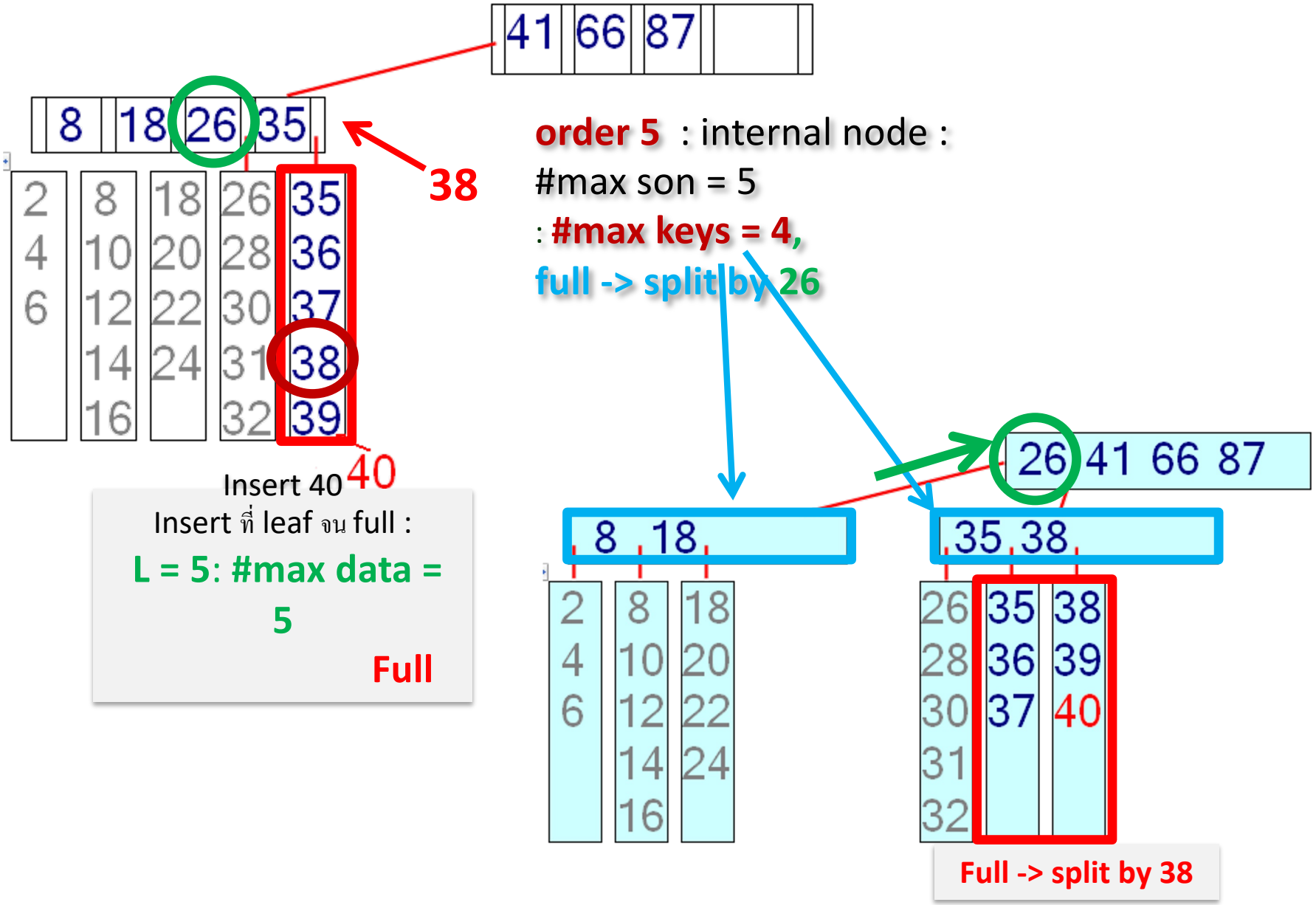


key 57 splits 2 nodes
key 57 ต้องถูก insert ใน father node

Full -> split half-half ที่ 57

B-Tree (Variation) Insertion

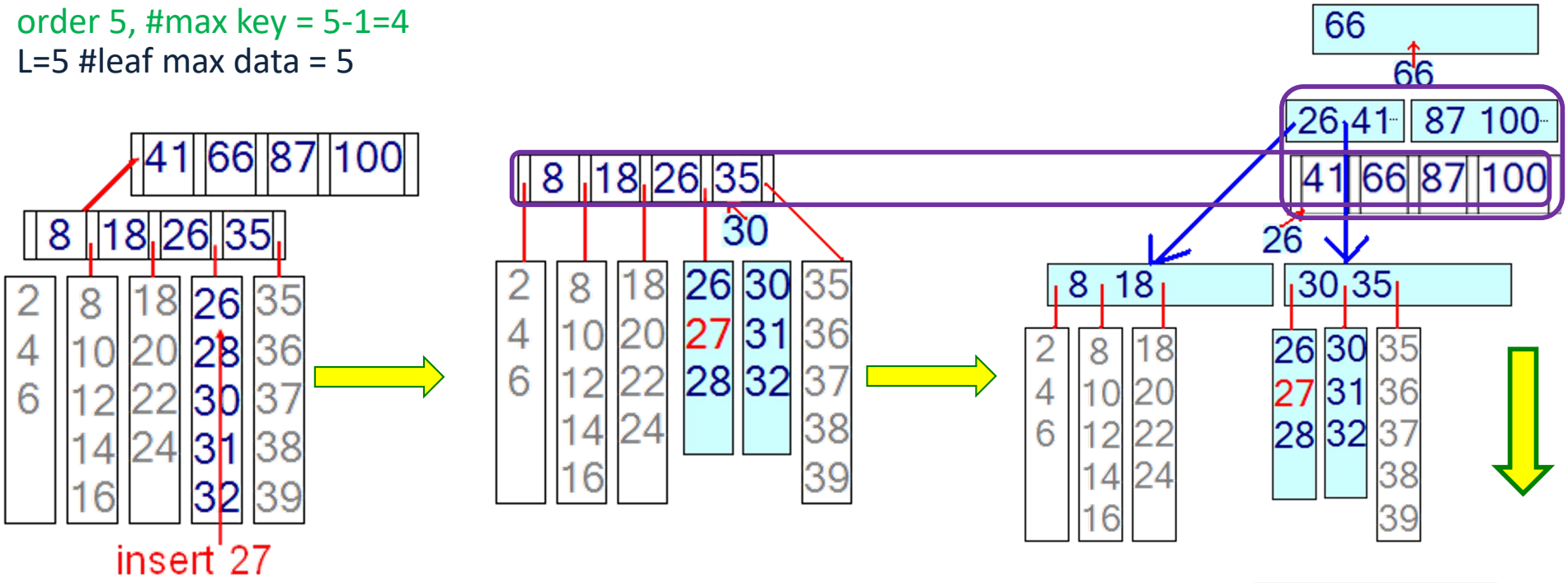
Insert at leaf:
until full, if full -> split



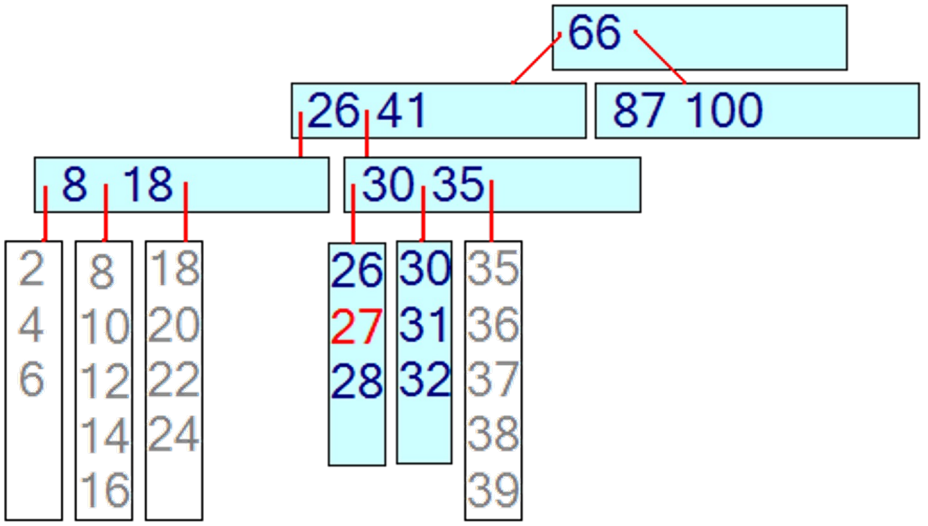
B-Tree (Variation) Insertion

Insert at leaf:
until full, if full -> split

order 5, #max key = 5-1=4
L=5 #leaf max data = 5



Insertion ระวัง #maximum
order 5 : #max son = 5,
Internal : **#max keys = 4,**
#min sons = $\lceil 5/2 \rceil = 3$
Leaf L = 5: **#max data = 5,**
#min data = $\lceil 5/2 \rceil = 3$



B-Tree (Variation) Deletion

Deletion consider
#minimum
Internal order 5 :
#min sons = $\lceil 5/2 \rceil = 3$
#min keys = 2
Leaf L = 5:
#max data = 5,
#min data = $\lceil 5/2 \rceil = 3$

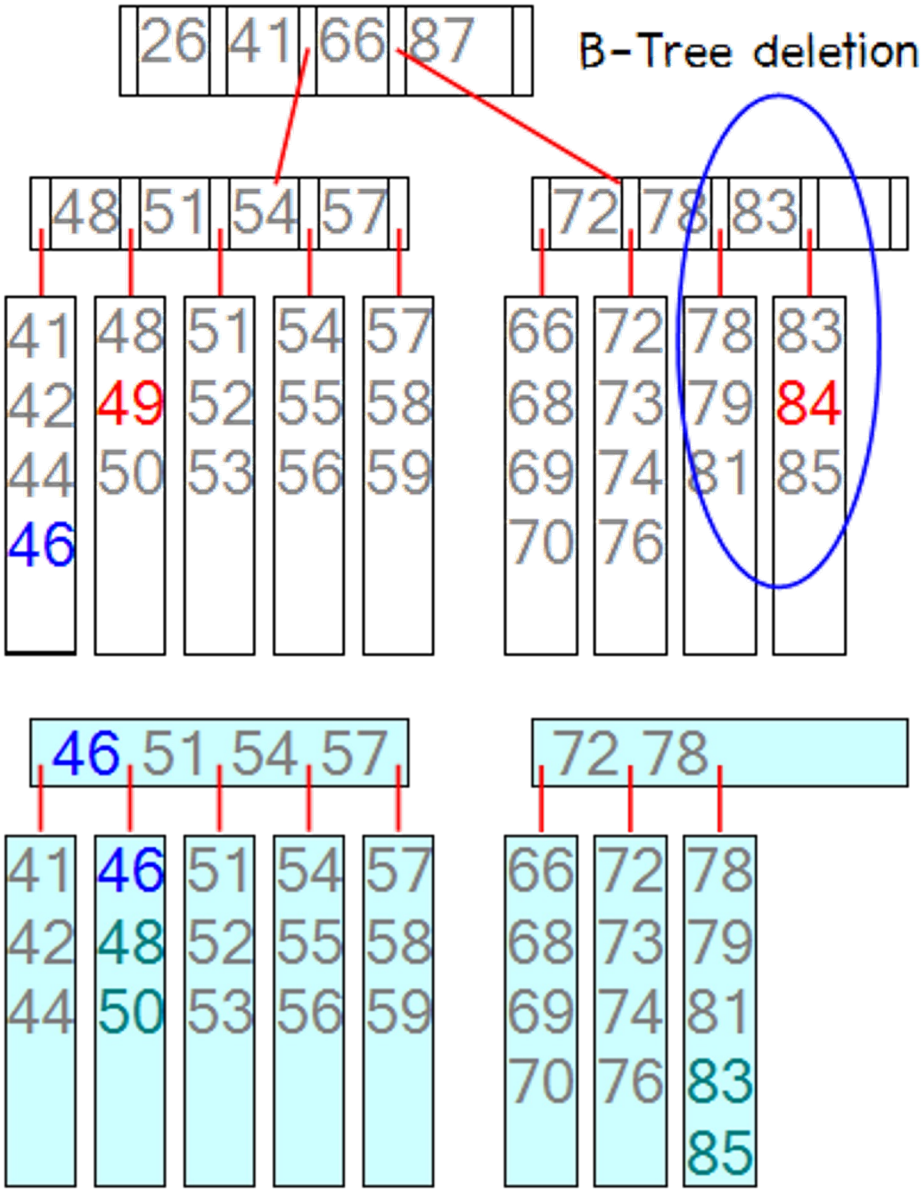
Delete at leaf:

insert ระวังไม่ให้เกิน ต้องดู **max**
delete ระวังไม่ให้ขาด ต้องดู **min**
until low, if low -> borrow

Borrowing
1. ยืมพี่น้อง(ผ่านพ่อ)
2. ยืมพ่อ ต้อง consolidate
พ่อไม่มี พ่อยืม
-พี่น้องพ่อ(ผ่านปู่)
. . .

B-Tree (Variation) Deletion

Delete at leaf:
until low, if low -> borrow



Deletion consider #minimum
Internal order 5 : #min sons = $\lceil 5/2 \rceil = 3$
#min keys = 2
Leaf L = 5: #max data = 5,
#min data = $\lceil 5/2 \rceil = 3$

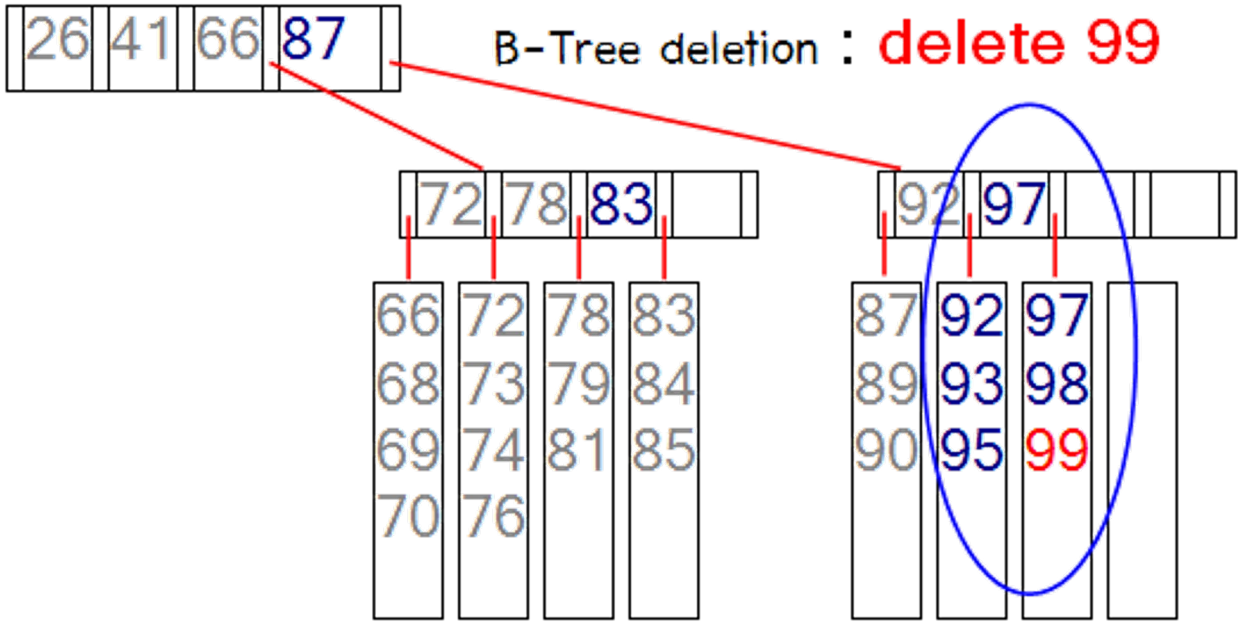
Borrowing

1. ยืมพี่น้อง(ผ่านพ่อ) เช่น del 49 เอา 46 ของพี่มา
2. ยืมพ่อ ต้อง consolidate เช่น del 84 พี่น้องไม่มีให้ยืม ยืมพ่อ โดยรวม consolidate กับพ่อ

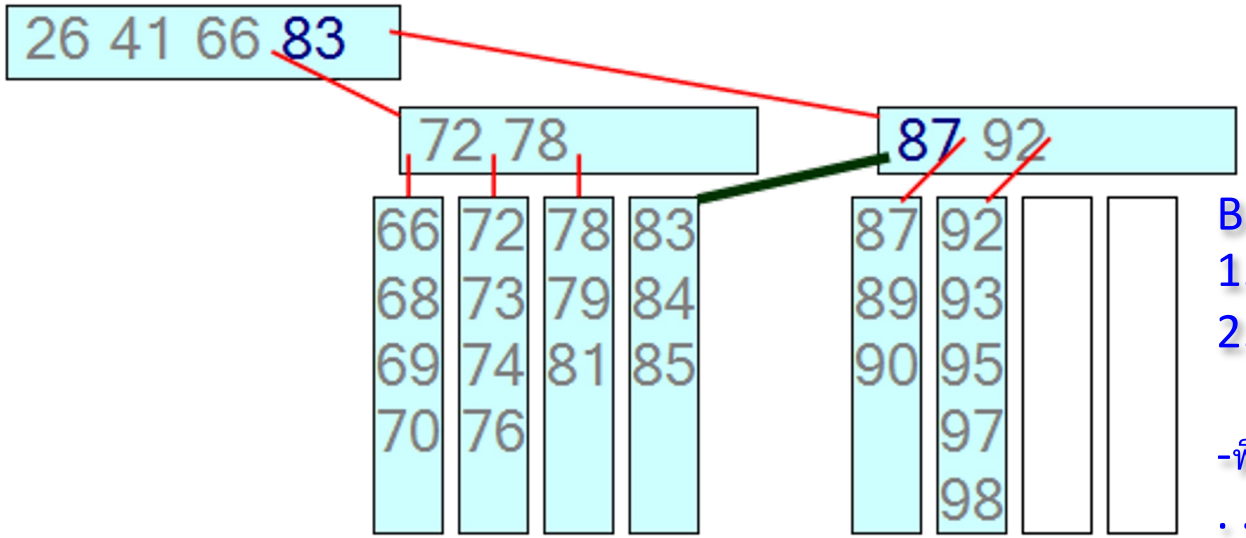
ถ้าพ่อไม่มี พ่อยืม
-พี่น้องพ่อ(ผ่านปู่)
. . .

B-Tree (Variation) Deletion

Delete at leaf:
until low, if low -> borrow



Deletion consider
#minimum
Internal order 5 :
#min sons = $\lceil 5/2 \rceil = 3$
#min keys = 2
Leaf L = 5:
#max data = 5,
#min data = $\lceil 5/2 \rceil = 3$



Borrowing
1. ยืมพี่น้อง(ผ่านพ่อ)
2. ยืมพ่อ ต้อง consolidate
พ่อไม่มี พ่อยืม
-พี่น้องพ่อ(ผ่านปู่)
. . .