

> restart;
with(Physics) :

$$\begin{aligned} &> -2 \cdot \frac{\frac{d^2}{dl^2} r(l)}{r(l)} - \left(\frac{d}{dl} r(l) \right)^2 + \frac{1}{r^2(l)} = -V(T) \cdot \left(1 + \left(\frac{d}{dl} T(l) \right)^2 \right)^{\frac{1}{2}} \\ &> -\frac{2 \left(\frac{d^2}{dl^2} r(l) \right)}{r(l)} - \frac{\left(\frac{d}{dl} r(l) \right)^2}{r(l)^2} + \frac{1}{r(l)^2} = -V(T) \sqrt{1 + \left(\frac{d}{dl} T(l) \right)^2} \end{aligned} \quad (1)$$

$$\begin{aligned} &> 2 \cdot \frac{\frac{d}{dl} \Psi(l) \cdot \frac{d}{dl} r(l)}{\Psi(l) \cdot r(l)} + \left(\frac{d}{dl} r(l) \right)^2 - \frac{1}{r^2(l)} = \frac{V(T)}{\left(1 + \left(\frac{d}{dl} T(l) \right)^2 \right)^{\frac{1}{2}}} \\ &> \frac{2 \left(\frac{d}{dl} \Psi(l) \right) \left(\frac{d}{dl} r(l) \right)}{\Psi(l) r(l)} + \frac{\left(\frac{d}{dl} r(l) \right)^2}{r(l)^2} - \frac{1}{r(l)^2} = \frac{V(T)}{\sqrt{1 + \left(\frac{d}{dl} T(l) \right)^2}} \end{aligned} \quad (2)$$

$$\begin{aligned} &> \frac{\frac{d^2}{dl^2} \Psi(l)}{\Psi(l)} + \frac{\frac{d}{dl} \Psi(l) \cdot \frac{d}{dl} r(l)}{\Psi(l) \cdot r(l)} + \frac{\frac{d^2}{dl^2} r(l)}{r(l)} = V(T) \cdot \left(1 + \left(\frac{d}{dl} T(l) \right)^2 \right)^{\frac{1}{2}} \\ &> \frac{\frac{d^2}{dl^2} \Psi(l)}{\Psi(l)} + \frac{\left(\frac{d}{dl} \Psi(l) \right) \left(\frac{d}{dl} r(l) \right)}{\Psi(l) r(l)} + \frac{\frac{d^2}{dl^2} r(l)}{r(l)} = V(T) \sqrt{1 + \left(\frac{d}{dl} T(l) \right)^2} \end{aligned} \quad (3)$$

conservation equation :

$$\begin{aligned} &> \frac{d}{dl} p_r(l) + \frac{2}{l} \cdot (p_t(l) - p_r(l)) = 0 \\ &> \frac{d}{dl} p_r(l) + \frac{2 (p_t(l) - p_r(l))}{l} = 0 \end{aligned} \quad (4)$$

$$\begin{aligned} &> eval \left((4), \left[p_r(l) = \frac{V(T)}{\sqrt{1 + \left(\frac{d}{dl} T(l) \right)^2}}, p_t(l) = V(T) \sqrt{1 + \left(\frac{d}{dl} T(l) \right)^2} \right] \right) \end{aligned}$$

(5)

$$-\frac{V(T) \left(\frac{d}{dl} T(l) \right) \left(\frac{d^2}{dl^2} T(l) \right)}{\left(1 + \left(\frac{d}{dl} T(l) \right)^2 \right)^{3/2}} + \frac{2 \left(V(T) \sqrt{1 + \left(\frac{d}{dl} T(l) \right)^2} - \frac{V(T)}{\sqrt{1 + \left(\frac{d}{dl} T(l) \right)^2}} \right)}{l} = 0 \quad (5)$$

$$> simplify((5), 'size')$$

$$-\frac{V(T) \left(\frac{d}{dl} T(l) \right) \left(-2 \left(\frac{d}{dl} T(l) \right)^3 + l \left(\frac{d^2}{dl^2} T(l) \right) - 2 \left(\frac{d}{dl} T(l) \right) \right)}{\left(1 + \left(\frac{d}{dl} T(l) \right)^2 \right)^{3/2} l} = 0 \quad (6)$$

$$> simplify((6))$$

$$\frac{V(T) \left(\frac{d}{dl} T(l) \right) \left(2 \left(\frac{d}{dl} T(l) \right)^3 - l \left(\frac{d^2}{dl^2} T(l) \right) + 2 \left(\frac{d}{dl} T(l) \right) \right)}{\left(1 + \left(\frac{d}{dl} T(l) \right)^2 \right)^{3/2} l} = 0 \quad (7)$$

$$> \left(2 \left(\frac{d}{dl} T(l) \right)^3 - l \left(\frac{d^2}{dl^2} T(l) \right) + 2 \left(\frac{d}{dl} T(l) \right) \right) = 0$$

$$2 \left(\frac{d}{dl} T(l) \right)^3 - l \left(\frac{d^2}{dl^2} T(l) \right) + 2 \left(\frac{d}{dl} T(l) \right) = 0 \quad (8)$$

$$> dsolve((8), \{ T(l) \})$$

$$T(l) = \int \frac{l^2}{\sqrt{-l^4 + _C1}} dl + _C2, T(l) = \int \left(-\frac{l^2}{\sqrt{-l^4 + _C1}} \right) dl + _C2 \quad (9)$$

$$> \frac{l^2}{\sqrt{-l^4 + _C1}}$$

$$\frac{l^2}{\sqrt{-l^4 + _C1}} \quad (10)$$

$$> int((10), l)$$

$$-\frac{1}{\sqrt{-l^4 + _C1}} \left(-C1^{3/4} \sqrt{1 - \frac{l^2}{\sqrt{-_C1}}} \sqrt{1 + \frac{l^2}{\sqrt{-_C1}}} \left(\text{EllipticF}\left(\frac{l}{-_C1^{1/4}}, 1\right) \right. \right. \quad (11)$$

$$- \text{EllipticE}\left(\frac{l}{CI^{1/4}}, I\right)\right)$$

Taking Eq One and Two :

$$\begin{aligned}
 > & \left(-\frac{2 \left(\frac{d^2}{dl^2} r(l) \right)}{r(l)} - \frac{\left(\frac{d}{dl} r(l) \right)^2}{r(l)^2} + \frac{1}{r(l)^2} \right) \cdot \left(\frac{2 \left(\frac{d}{dl} \Psi(l) \right) \left(\frac{d}{dl} r(l) \right)}{\Psi(l) r(l)} + \frac{\left(\frac{d}{dl} r(l) \right)^2}{r(l)^2} \right. \\
 & \quad \left. - \frac{1}{r(l)^2} \right) = \left(-V \cdot \sqrt{1 + \left(\frac{d}{dl} T(l) \right)^2} \right) \cdot \left(\frac{V}{\sqrt{1 + \left(\frac{d}{dl} T(l) \right)^2}} \right) \\
 & \left(-\frac{2 \left(\frac{d^2}{dl^2} r(l) \right)}{r(l)} - \frac{\left(\frac{d}{dl} r(l) \right)^2}{r(l)^2} + \frac{1}{r(l)^2} \right) \left(\frac{2 \left(\frac{d}{dl} \Psi(l) \right) \left(\frac{d}{dl} r(l) \right)}{\Psi(l) r(l)} + \frac{\left(\frac{d}{dl} r(l) \right)^2}{r(l)^2} \right. \\
 & \quad \left. - \frac{1}{r(l)^2} \right) = -V^2
 \end{aligned} \tag{12}$$

$$\begin{aligned}
 > & \text{eval}\left((12), \left[r(l) = r_0 \cdot \left(1 + \frac{l}{r_0}\right)^n, \frac{d}{dl} T(l) = \frac{l^2}{\sqrt{-l^4 + CI}}, \Psi(l) = C\right]\right) \\
 & \left(-\frac{2 \left(\frac{\left(1 + \frac{l}{r_0}\right)^n n^2}{r_0 \left(1 + \frac{l}{r_0}\right)^2} - \frac{\left(1 + \frac{l}{r_0}\right)^n n}{\left(1 + \frac{l}{r_0}\right)^2 r_0} \right)}{r_0 \left(1 + \frac{l}{r_0}\right)^n} - \frac{n^2}{\left(1 + \frac{l}{r_0}\right)^2 r_0^2} \right. \\
 & \quad \left. + \frac{1}{r_0^2 \left(\left(1 + \frac{l}{r_0}\right)^n \right)^2} \right) \left(\frac{n^2}{\left(1 + \frac{l}{r_0}\right)^2 r_0^2} - \frac{1}{r_0^2 \left(\left(1 + \frac{l}{r_0}\right)^n \right)^2} \right) = -V^2
 \end{aligned} \tag{13}$$

> `simplify((13), 'size')`

$$-\frac{1}{r_0^4 (r_0 + l)^4 \left(\left(1 + \frac{l}{r_0}\right)^n\right)^4} \left(\left(n r_0 \left(1 + \frac{l}{r_0}\right)^n + l + r_0\right) \left(-3 \left(n - \frac{2}{3}\right)\right.\right.$$

$$\left.\left. r_0^2 n \left(\left(1 + \frac{l}{r_0}\right)^n\right)^2 + (r_0 + l)^2\right) \left(-n r_0 \left(1 + \frac{l}{r_0}\right)^n + l + r_0\right)\right) = -V^2$$

> solve({ (14) }, [V])

$$\left[\begin{array}{l} V \\ \end{array} \right] \quad (15)$$

$$= \frac{1}{\left(\frac{r_0 + l}{r_0}\right)^{4n} (r_0 + l)^2 r_0^2} \left(\left(\frac{r_0 + l}{r_0}\right)^{4n} \left(3 \left(\frac{r_0 + l}{r_0}\right)^{4n} n^4 r_0^4\right.\right.$$

$$\left.-2 \left(\frac{r_0 + l}{r_0}\right)^{4n} n^3 r_0^4 - 4 \left(\frac{r_0 + l}{r_0}\right)^{2n} l^2 n^2 r_0^2 - 8 \left(\frac{r_0 + l}{r_0}\right)^{2n} l n^2 r_0^3\right)$$

$$-4 \left(\frac{r_0 + l}{r_0}\right)^{2n} n^2 r_0^4 + 2 \left(\frac{r_0 + l}{r_0}\right)^{2n} l^2 n r_0^2 + 4 \left(\frac{r_0 + l}{r_0}\right)^{2n} l n r_0^3 + 2 \left(\frac{r_0 + l}{r_0}\right)^{2n} n$$

$$\left. \left. r_0^4 + l^4 + 4 l^3 r_0 + 6 l^2 r_0^2 + 4 l r_0^3 + r_0^4\right)\right)^{1/2} \right], \left[V = \right.$$

$$-\frac{1}{\left(\frac{r_0 + l}{r_0}\right)^{4n} (r_0 + l)^2 r_0^2} \left(\left(\frac{r_0 + l}{r_0}\right)^{4n} \left(3 \left(\frac{r_0 + l}{r_0}\right)^{4n} n^4 r_0^4\right.\right.$$

$$\begin{aligned}
& -2 \left(\frac{r_0 + l}{r_0} \right)^{4n} n^3 r_0^4 - 4 \left(\frac{r_0 + l}{r_0} \right)^{2n} l^2 n^2 r_0^2 - 8 \left(\frac{r_0 + l}{r_0} \right)^{2n} l n^2 r_0^3 \\
& - 4 \left(\frac{r_0 + l}{r_0} \right)^{2n} n^2 r_0^4 + 2 \left(\frac{r_0 + l}{r_0} \right)^{2n} l^2 n r_0^2 + 4 \left(\frac{r_0 + l}{r_0} \right)^{2n} l n r_0^3 + 2 \left(\frac{r_0 + l}{r_0} \right)^{2n} n \\
& r_0^4 + l^4 + 4 l^3 r_0 + 6 l^2 r_0^2 + 4 l r_0^3 + r_0^4 \Big) \Bigg) \Bigg]^{1/2} \Bigg]
\end{aligned}$$

>

>

> V

$$\begin{aligned}
& = \frac{1}{\left(\frac{r_0 + l}{r_0} \right)^{4n} (r_0 + l)^2 r_0^2} \left(\left(\frac{r_0 + l}{r_0} \right)^{4n} \left(3 \left(\frac{r_0 + l}{r_0} \right)^{4n} n^4 r_0^4 \right. \right. \\
& \left. \left. - 2 \left(\frac{r_0 + l}{r_0} \right)^{4n} n^3 r_0^4 - 4 \left(\frac{r_0 + l}{r_0} \right)^{2n} l^2 n^2 r_0^2 - 8 \left(\frac{r_0 + l}{r_0} \right)^{2n} l n^2 r_0^3 \right. \right. \\
& \left. \left. - 4 \left(\frac{r_0 + l}{r_0} \right)^{2n} n^2 r_0^4 + 2 \left(\frac{r_0 + l}{r_0} \right)^{2n} l^2 n r_0^2 + 4 \left(\frac{r_0 + l}{r_0} \right)^{2n} l n r_0^3 + 2 \left(\frac{r_0 + l}{r_0} \right)^{2n} n \right. \right. \\
& \left. \left. r_0^4 + l^4 + 4 l^3 r_0 + 6 l^2 r_0^2 + 4 l r_0^3 + r_0^4 \right) \right) \Bigg) \Bigg)^{1/2}
\end{aligned}$$

>

(16)

$$\begin{aligned}
& = \frac{1}{\left(\frac{r_0 + l}{r_0} \right)^{4n} (r_0 + l)^2 r_0^2} \left(\left(\frac{r_0 + l}{r_0} \right)^{4n} \left(3 \left(\frac{r_0 + l}{r_0} \right)^{4n} n^4 r_0^4 \right. \right. \\
& \left. \left. - 2 \left(\frac{r_0 + l}{r_0} \right)^{4n} n^3 r_0^4 - 4 \left(\frac{r_0 + l}{r_0} \right)^{2n} l^2 n^2 r_0^2 - 8 \left(\frac{r_0 + l}{r_0} \right)^{2n} l n^2 r_0^3 \right. \right. \\
& \left. \left. - 4 \left(\frac{r_0 + l}{r_0} \right)^{2n} n^2 r_0^4 + 2 \left(\frac{r_0 + l}{r_0} \right)^{2n} l^2 n r_0^2 + 4 \left(\frac{r_0 + l}{r_0} \right)^{2n} l n r_0^3 + 2 \left(\frac{r_0 + l}{r_0} \right)^{2n} n \right. \right. \\
& \left. \left. r_0^4 + l^4 + 4 l^3 r_0 + 6 l^2 r_0^2 + 4 l r_0^3 + r_0^4 \right) \right) \Bigg) \Bigg)^{1/2}
\end{aligned}$$

>

> $\text{simplify}((16), \text{'size'})$

>

(17)

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> 
$$= \frac{1}{\left(\frac{r_0+l}{r_0}\right)^{4n} (r_0+l)^2 r_0^2} \left( \left( -4 r_0^2 \left(n - \frac{1}{2}\right) n (r_0+l)^2 \left(\frac{r_0+l}{r_0}\right)^{2n} + 3 \left(n - \frac{2}{3}\right) r_0^4 n^3 \left(\frac{r_0+l}{r_0}\right)^{4n} + (r_0+l)^4 \right) \left(\frac{r_0+l}{r_0}\right)^{4n} \right)^{1/2}$$


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$$> \frac{1}{\left(\frac{r_0+l}{r_0}\right)^{4n} (r_0+l)^2 r_0^2} \left(\left(-4 r_0^2 \left(n - \frac{1}{2}\right) n (r_0+l)^2 \left(\frac{r_0+l}{r_0}\right)^{2n} + 3 \left(n - \frac{2}{3}\right) r_0^4 n^3 \left(\frac{r_0+l}{r_0}\right)^{4n} + (r_0+l)^4 \right) \left(\frac{r_0+l}{r_0}\right)^{4n} \right)^{1/2}$$

$$\frac{1}{\left(\frac{r_0+l}{r_0}\right)^{4n} (r_0+l)^2 r_0^2} \left(\left(-4 r_0^2 \left(n - \frac{1}{2}\right) n (r_0+l)^2 \left(\frac{r_0+l}{r_0}\right)^{2n} + 3 \left(n - \frac{2}{3}\right) r_0^4 n^3 \left(\frac{r_0+l}{r_0}\right)^{4n} + (r_0+l)^4 \right) \left(\frac{r_0+l}{r_0}\right)^{4n} \right)^{1/2}$$

$$> eval(\text{(18)}, [n=0.5, r[0]=0.5])$$

$$\frac{1}{(1.000000000 + 2.000000000 l)^{2.0} (0.5 + l)^2} \left(4.000000000 ((-0.003906250002 (1.000000000 + 2.000000000 l)^{2.0} + 2.000000000 l)^{2.0} + (0.5 + l)^4) (1.000000000 + 2.000000000 l)^{2.0} \right)^{1/2} \quad (19)$$

$$> plot(\text{(19)}, l=0 .. 4)$$

NEC:

$$\begin{aligned}
 > -V \cdot \sqrt{\left(1 + \left(\frac{d}{dl} T(l)\right)^2\right)} + \frac{V}{\sqrt{\left(1 + \left(\frac{d}{dl} T(l)\right)^2\right)}} \\
 &\quad - V \sqrt{1 + \left(\frac{d}{dl} T(l)\right)^2} + \frac{V}{\sqrt{1 + \left(\frac{d}{dl} T(l)\right)^2}}
 \end{aligned} \tag{20}$$

$$\begin{aligned}
 > \text{simplify((20), 'size')} \\
 &\quad - \frac{V \left(\frac{d}{dl} T(l)\right)^2}{\sqrt{1 + \left(\frac{d}{dl} T(l)\right)^2}}
 \end{aligned} \tag{21}$$

$$\begin{aligned}
 > \text{eval} \left((21), \left[\frac{d}{dl} T(l) = \frac{l^2}{\sqrt{-l^4 + _C l}}, V \right. \right. \\
 &= \frac{\frac{1}{\left(r_0 + l\right)^4 n}}{\left(r_0 + l\right)^2 r_0^2} \left(\left(-4 r_0^2 \left(n - \frac{1}{2}\right) n (r_0 + l)^2 \left(\frac{r_0 + l}{r_0}\right)^{2n} \right. \right. \\
 &\quad \left. \left. + 3 \left(n - \frac{2}{3}\right) r_0^4 n^3 \left(\frac{r_0 + l}{r_0}\right)^{4n} + (r_0 + l)^4 \right) \left(\frac{r_0 + l}{r_0}\right)^{4n} \right)^{1/2} \right)
 \end{aligned}$$

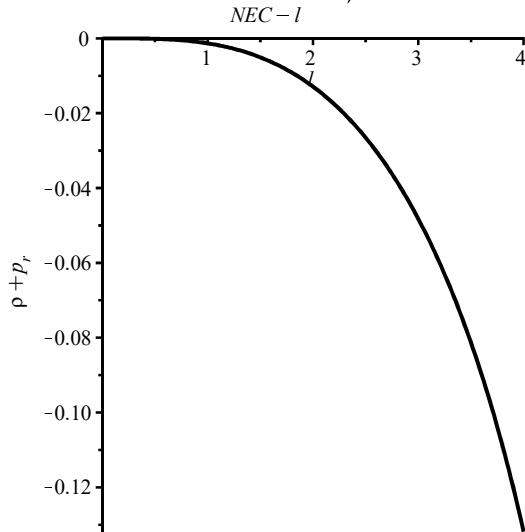
$$\begin{aligned}
 &- \left(\left(\left(-4 r_0^2 \left(n - \frac{1}{2}\right) n (r_0 + l)^2 \left(\frac{r_0 + l}{r_0}\right)^{2n} + 3 \left(n - \frac{2}{3}\right) r_0^4 n^3 \left(\frac{r_0 + l}{r_0}\right)^{4n} + (r_0 + l)^4 \right) \left(\frac{r_0 + l}{r_0}\right)^{4n} \right)^{1/2} l^4 \right) \Bigg/ \left(\left(\left(\frac{r_0 + l}{r_0}\right)^{4n} (r_0 + l)^2 r_0^2 (-l^4 \right.
 \end{aligned} \tag{22}$$

$$+ \text{CI}) \sqrt{1 + \frac{l^4}{-l^4 + \text{CI}}}$$

> eval((22), [n = 0.5, r[0] = 0.5, _CI = 1000.1])

$$\begin{aligned} & - \left(4.000000000 (((-0.003906250002 (1.000000000 + 2.000000000 l)^{2.0} + (0.5 + l)^4) (1.000000000 + 2.000000000 l)^{2.0})^{1/2} l^4) \right. \\ & \quad \left. + 2.000000000 l^{2.0} \right) \left/ \left((1.000000000 + 2.000000000 l)^{2.0} (0.5 + l)^2 (-l^4 + 1000.1) \right. \right. \\ & \quad \left. \left. \sqrt{1 + \frac{l^4}{-l^4 + 1000.1}} \right) \right) \end{aligned} \quad (23)$$

> plot((23), l = 0 .. 4, labels = [l, p + p_r], labeldirections = [HORIZONTAL, VERTICAL], color = [black], linestyle = [solid], title = [NEC - l])



PLOT of V, T vs l :

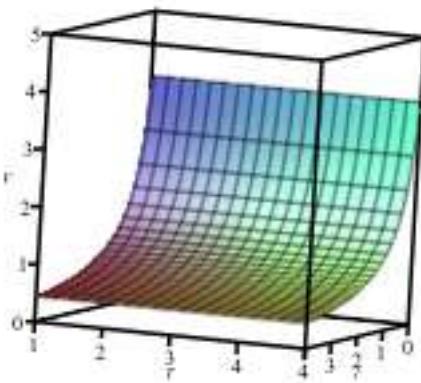
> V

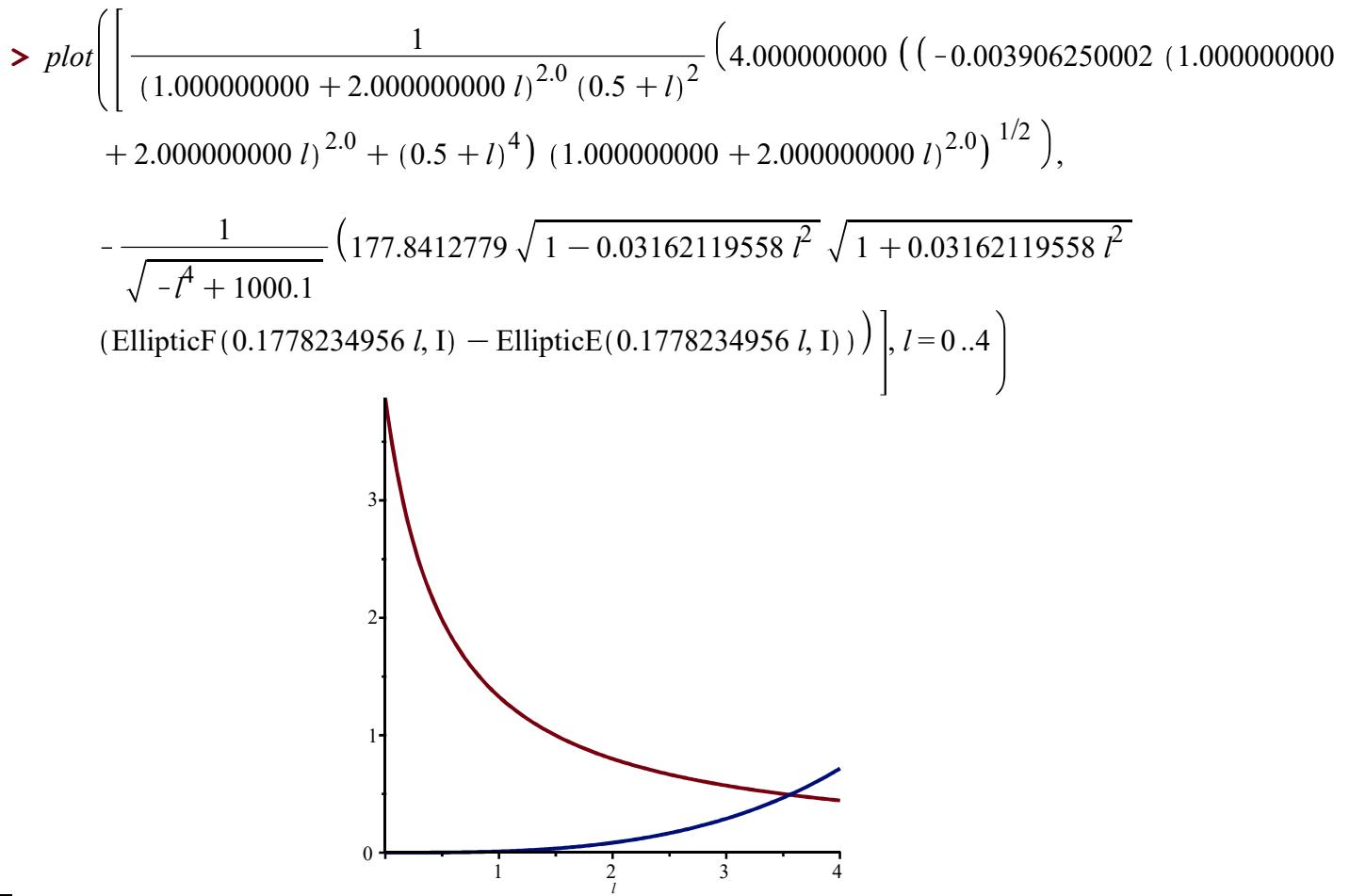
$$\begin{aligned} & = \frac{1}{\left(\frac{r_0 + l}{r_0}\right)^{4n} (r_0 + l)^2 r_0^2} \left(\left(-4 r_0^2 \left(n - \frac{1}{2}\right) n (r_0 + l)^2 \left(\frac{r_0 + l}{r_0}\right)^{2n} + 3 \left(n - \frac{2}{3}\right) r_0^4 n^3 \left(\frac{r_0 + l}{r_0}\right)^{4n} + (r_0 + l)^4 \right) \left(\frac{r_0 + l}{r_0}\right)^{4n} \right)^{1/2}, T = \\ & - \frac{1}{\sqrt{-l^4 + \text{CI}}} \left(-\text{CI}^{3/4} \sqrt{1 - \frac{l^2}{\sqrt{-\text{CI}}}} \sqrt{1 + \frac{l^2}{\sqrt{-\text{CI}}}} \left(\text{EllipticF}\left(\frac{l}{\text{CI}^{1/4}}, \text{I}\right) \right. \right. \end{aligned}$$

$$\begin{aligned}
V &= \left. -\text{EllipticE}\left(\frac{l}{CI^{1/4}}, I\right)\right) \Bigg) \\
&= \frac{1}{\left(\frac{r_0+l}{r_0}\right)^{4n} (r_0+l)^2 r_0^2} \left(\left(-4 r_0^2 \left(n - \frac{1}{2}\right) n (r_0+l)^2 \left(\frac{r_0+l}{r_0}\right)^{2n} + 3 \left(n - \frac{2}{3}\right) r_0^4 n^3 \left(\frac{r_0+l}{r_0}\right)^{4n} + (r_0+l)^4 \right) \left(\frac{r_0+l}{r_0}\right)^{4n} \right)^{1/2}, T = \right. \\
&\quad \left. -\frac{1}{\sqrt{-l^4 + CI}} \left(-CI^{3/4} \sqrt{1 - \frac{l^2}{\sqrt{CI}}} \sqrt{1 + \frac{l^2}{\sqrt{CI}}} \left(\text{EllipticF}\left(\frac{l}{CI^{1/4}}, I\right) \right. \right. \right. \\
&\quad \left. \left. \left. - \text{EllipticE}\left(\frac{l}{CI^{1/4}}, I\right)\right) \right)
\end{aligned} \tag{24}$$

$$\begin{aligned}
> op(\text{eval}([(24)], [CI = 1000.1, n = 0.5, r[0] = 0.5])) \\
V &= \frac{1}{(1.000000000 + 2.000000000 l)^{2.0} (0.5 + l)^2} \left(4.000000000 ((\right. \\
&\quad \left. -0.003906250002 (1.000000000 + 2.000000000 l)^{2.0} + (0.5 + l)^4 \right) (1.000000000 \\
&\quad + 2.000000000 l)^{2.0})^{1/2} \right), T = \\
&\quad -\frac{1}{\sqrt{-l^4 + 1000.1}} \left(177.8412779 \sqrt{1 - 0.03162119558 l^2} \sqrt{1 + 0.03162119558 l^2} \right. \\
&\quad \left. (\text{EllipticF}(0.1778234956 l, I) - \text{EllipticE}(0.1778234956 l, I)) \right)
\end{aligned} \tag{25}$$

> smartplot3d[l, T, V]([(25)])





Perturbation in delta r :

>
$$\frac{d^2}{dl^2} f(l) + 2 \cdot \frac{\frac{d}{dl}(r(l))}{r(l)} \cdot \left(\frac{d}{dl} f(l) \right) + \frac{\frac{d^2}{dl^2} r(l)}{r(l)} \cdot (f(l)) = 0$$

$$\frac{d^2}{dl^2} f(l) + \frac{2 \left(\frac{d}{dl} r(l) \right) \left(\frac{d}{dl} f(l) \right)}{r(l)} + \frac{\left(\frac{d^2}{dl^2} r(l) \right) f(l)}{r(l)} = 0 \quad (26)$$

>
$$\text{eval}\left((26), \left[r(l) = r_0 \cdot \left(1 + \frac{l}{r_0}\right)^n\right]\right)$$

$$\frac{\frac{d^2}{dl^2} f(l) + \frac{2n}{\left(1 + \frac{l}{r_0}\right)r_0} \left(\frac{d}{dl} f(l) \right) + \left(\frac{\left(1 + \frac{l}{r_0}\right)^n n^2}{r_0 \left(1 + \frac{l}{r_0}\right)^2} - \frac{\left(1 + \frac{l}{r_0}\right)^n n}{r_0 \left(1 + \frac{l}{r_0}\right)^2} \right) f(l)}{r_0 \left(1 + \frac{l}{r_0}\right)^n} = 0 \quad (27)$$

> *simplify((27), 'size')*

$$\frac{2n(r_0+l)\left(\frac{d}{dl}f(l)\right)+(n^2-n)f(l)+(r_0+l)^2\left(\frac{d^2}{dl^2}f(l)\right)}{(r_0+l)^2}=0 \quad (28)$$

> *dsolve((28), {f(l)})*

$$f(l) = _C1 (r_0 + l)^{-n} + _C2 (r_0 + l)^{-n + 1} \quad (29)$$

>

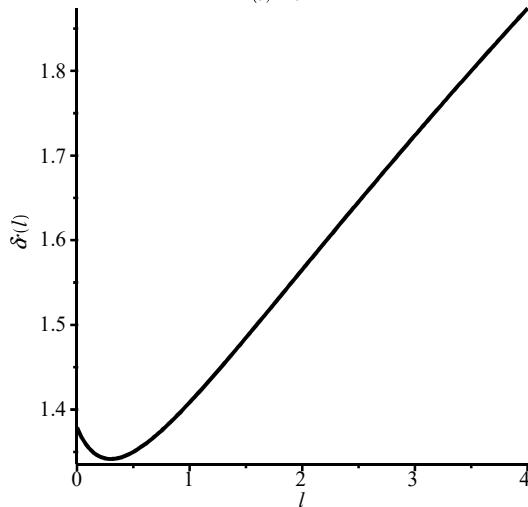
>

$$-_C1 (r_0 + l)^{-n} +-_C2 (r_0 + l)^{-n + 1} \\ \underline{-_C1 (r_0 + l)^{-n}} + \underline{-_C2 (r_0 + l)^{-n + 1}} \quad (30)$$

> *eval((30), [_C1 = 0.6, _C2 = 0.75, n = 0.5, r[0] = 0.5])*

$$\frac{0.6}{(0.5 + l)^{0.5}} + 0.75 (0.5 + l)^{0.5} \quad (31)$$

> *plot((31), l = 0 .. 4, labels = [l, δr(l)], labeldirections = [HORIZONTAL, VERTICAL], color = [black], linestyle = [solid], title = [δr(l) - l])*



>

>

> *Perturbation in delta t :*

>

$$\begin{aligned} & \frac{d}{dl} V(l) \cdot \left(1 + \left(\frac{d}{dl} T(l) \right)^2 \right) \cdot \left(2 + \left(\frac{d}{dl} T(l) \right)^2 \right) \cdot \cancel{f(l)} + V(l) \cdot \left(\frac{d}{dl} T(l) \right)^3 \cdot \frac{d}{dl} \cancel{f(l)} = 0 \\ & \left(\frac{d}{dl} V(l) \right) \left(1 + \left(\frac{d}{dl} T(l) \right)^2 \right) \left(2 + \left(\frac{d}{dl} T(l) \right)^2 \right) f(l) + V(l) \left(\frac{d}{dl} T(l) \right)^3 \left(\frac{d}{dl} f(l) \right) = 0 \end{aligned} \quad (32)$$

$$\begin{aligned} & \text{eval} \left(\text{(32)}, \left[V(l) \right] \right) \\ &= \frac{\frac{1}{\left(\frac{r_0 + l}{r_0} \right)^4 n}}{\left(r_0 + l \right)^2 r_0^2} \left(\left(-4 r_0^2 \left(n - \frac{1}{2} \right) n (r_0 + l)^2 \left(\frac{r_0 + l}{r_0} \right)^{2n} \right. \right. \\ & \quad \left. \left. + 3 \left(n - \frac{2}{3} \right) r_0^4 n^3 \left(\frac{r_0 + l}{r_0} \right)^{4n} + (r_0 + l)^4 \right) \left(\frac{r_0 + l}{r_0} \right)^{4n} \right)^{1/2}, \frac{d}{dl} T(l) \right) \end{aligned}$$

$$r_0^2 \left(n - \frac{1}{2} \right) n (r_0 + l)^2 \left(\frac{r_0 + l}{r_0} \right)^{2n} + 3 \left(n - \frac{2}{3} \right) r_0^4 n^3 \left(\frac{r_0 + l}{r_0} \right)^{4n} + (r_0 + l)^4 \right)$$

$$\left(\frac{r_0 + l}{r_0} \right)^{4n} n \Bigg) \Bigg)$$

$$\left(\left(-4 r_0^2 \left(n - \frac{1}{2} \right) n (r_0 + l)^2 \left(\frac{r_0 + l}{r_0} \right)^{2n} + 3 \left(n - \frac{2}{3} \right) r_0^4 n^3 \left(\frac{r_0 + l}{r_0} \right)^{4n} \right. \right.$$

$$+ (r_0 + l)^4 \left(\frac{r_0 + l}{r_0} \right)^{4n} \left. \left. \left(\frac{r_0 + l}{r_0} \right)^{4n} (r_0 + l)^2 r_0^2 \right) \right)^{1/2}$$

$$- \frac{1}{\left(\frac{r_0 + l}{r_0} \right)^{4n} (r_0 + l)^3 r_0^2} \left(4 \left(-4 r_0^2 \left(n - \frac{1}{2} \right) n (r_0 + l)^2 \left(\frac{r_0 + l}{r_0} \right)^{2n} \right. \right. \\ \left. \left. + 3 \left(n - \frac{2}{3} \right) r_0^4 n^3 \left(\frac{r_0 + l}{r_0} \right)^{4n} + (r_0 + l)^4 \right) \left(\frac{r_0 + l}{r_0} \right)^{4n} \right)^{1/2} n \right)$$

$$- \frac{1}{\left(\frac{r_0 + l}{r_0} \right)^{4n} (r_0 + l)^3 r_0^2} \left(2 \left(-4 r_0^2 \left(n - \frac{1}{2} \right) n (r_0 + l)^2 \left(\frac{r_0 + l}{r_0} \right)^{2n} \right. \right. \\ \left. \left. + 3 \left(n - \frac{2}{3} \right) r_0^4 n^3 \left(\frac{r_0 + l}{r_0} \right)^{4n} + (r_0 + l)^4 \right) \left(\frac{r_0 + l}{r_0} \right)^{4n} \right)^{1/2} \right) \left(1 \right.$$

$$+ \frac{t^4}{-t^4 + _C I} \Big) \left(2 + \frac{t^4}{-t^4 + _C I} \right) f(l)$$

$$\begin{aligned}
& + \frac{1}{\left(\frac{r_0+l}{r_0}\right)^{4n} (r_0+l)^2 r_0^2 (-l^4 + _{CI})^{3/2}} \left(\left(\left(-4 r_0^2 \left(n - \frac{1}{2}\right) n (r_0 + l)^2 \left(\frac{r_0+l}{r_0}\right)^{2n} + 3 \left(n - \frac{2}{3}\right) r_0^4 n^3 \left(\frac{r_0+l}{r_0}\right)^{4n} + (r_0 + l)^4 \right) \left(\frac{r_0+l}{r_0}\right)^{4n} \right)^{1/2} l^6 \left(\frac{d}{dl} f(l)\right) \right) = 0
\end{aligned}$$

> isolate((33), diff(f(l), l))

$$\frac{d}{dl} f(l) = - \left(\frac{1}{2} \left(-8 r_0^2 \left(n - \frac{1}{2}\right) n (r_0 + l) \left(\frac{r_0+l}{r_0}\right)^{2n} - 8 r_0^2 \left(n - \frac{1}{2}\right) n^2 (r_0 + l)^2 \left(\frac{r_0+l}{r_0}\right)^{2n} + \frac{12 \left(n - \frac{2}{3}\right) r_0^4 n^4 \left(\frac{r_0+l}{r_0}\right)^{4n}}{r_0 + l} + 4 (r_0 + l)^3 \right) \left(\frac{r_0+l}{r_0}\right)^{4n} \right)^{1/2} l^6 \left(\frac{d}{dl} f(l)\right)$$

$$+ \frac{1}{r_0 + l} \left(4 \left(-4 r_0^2 \left(n - \frac{1}{2}\right) n (r_0 + l)^2 \left(\frac{r_0+l}{r_0}\right)^{2n} + 3 \left(n - \frac{2}{3}\right) r_0^4 n^3 \left(\frac{r_0+l}{r_0}\right)^{4n} + (r_0 + l)^4 \right) \left(\frac{r_0+l}{r_0}\right)^{4n} n \right) \right)$$

$$\left(\left(-4 r_0^2 \left(n - \frac{1}{2}\right) n (r_0 + l)^2 \left(\frac{r_0+l}{r_0}\right)^{2n} + 3 \left(n - \frac{2}{3}\right) r_0^4 n^3 \left(\frac{r_0+l}{r_0}\right)^{4n} \right) \left(\frac{r_0+l}{r_0}\right)^{4n} n \right)$$

$$+ (r_0 + l)^4 \left(\frac{r_0 + l}{r_0} \right)^{4n} \right)^{1/2} \left(\frac{r_0 + l}{r_0} \right)^{4n} (r_0 + l)^2 r_0^2 \Bigg)$$

$$- \frac{1}{\left(\frac{r_0 + l}{r_0} \right)^{4n} (r_0 + l)^3 r_0^2} \left(4 \left(\left(-4 r_0^2 \left(n - \frac{1}{2} \right) n (r_0 + l)^2 \left(\frac{r_0 + l}{r_0} \right)^{2n} \right. \right. \right. \right. \\ \left. \left. \left. \left. + 3 \left(n - \frac{2}{3} \right) r_0^4 n^3 \left(\frac{r_0 + l}{r_0} \right)^{4n} + (r_0 + l)^4 \right) \left(\frac{r_0 + l}{r_0} \right)^{4n} \right)^{1/2} n \right) \right)$$

$$- \frac{1}{\left(\frac{r_0 + l}{r_0} \right)^{4n} (r_0 + l)^3 r_0^2} \left(2 \left(\left(-4 r_0^2 \left(n - \frac{1}{2} \right) n (r_0 + l)^2 \left(\frac{r_0 + l}{r_0} \right)^{2n} \right. \right. \right. \right. \\ \left. \left. \left. \left. + 3 \left(n - \frac{2}{3} \right) r_0^4 n^3 \left(\frac{r_0 + l}{r_0} \right)^{4n} + (r_0 + l)^4 \right) \left(\frac{r_0 + l}{r_0} \right)^{4n} \right)^{1/2} \right) \right) \left(1 \right. \\ \left. + \frac{t^4}{-t^4 + CI} \right) \left(2 + \frac{t^4}{-t^4 + CI} \right) f(l) \left(\frac{r_0 + l}{r_0} \right)^{4n} (r_0 + l)^2 r_0^2 (-t^4 + CI)^{3/2} \right)$$

$$\left(\left(\left(-4 r_0^2 \left(n - \frac{1}{2} \right) n (r_0 + l)^2 \left(\frac{r_0 + l}{r_0} \right)^{2n} \right. \right. \right. \right. \\ \left. \left. \left. \left. + 3 \left(n - \frac{2}{3} \right) r_0^4 n^3 \left(\frac{r_0 + l}{r_0} \right)^{4n} \right) \left(\frac{r_0 + l}{r_0} \right)^{4n} \right)^{1/2} \right) \right)$$

$$+ (r_0 + l)^4 \left(\left(\frac{r_0 + l}{r_0} \right)^{4n} \right)^{1/2} l^6 \Bigg)$$

> `simplify((33), 'size')`

$$\left(2 \left(-2 \left(n - \frac{1}{2} \right) (r_0 + l)^2 n \left(l^4 (-l^4 + _{CI})^2 (r_0 + l) \left(\frac{d}{dl} f(l) \right) + (n + 1) f(l) (l^4 - _{CI}) (-l^4 + _{CI})^{3/2} _{CI} \right) r_0^2 \left(\frac{r_0 + l}{r_0} \right)^{2n} + 3 n^3 \left(n - \frac{2}{3} \right) r_0^4 \left(\frac{1}{2} l^6 (l^4 - _{CI})^2 (r_0 + l) \left(\frac{d}{dl} f(l) \right) + f(l) (l^4 - 2 _{CI}) (-l^4 + _{CI})^{3/2} _{CI} \right) \left(\frac{r_0 + l}{r_0} \right)^{4n} + \left(\frac{1}{2} l^6 (l^4 - _{CI})^2 (r_0 + l) \left(\frac{d}{dl} f(l) \right) + f(l) n (l^4 - 2 _{CI}) (-l^4 + _{CI})^3 \right. \right.$$

$$\left. \left. /2 _{CI} \right) (r_0 + l)^4 \right) \right) \Bigg) \Bigg/ \left(\left(\left(-4 r_0^2 \left(n - \frac{1}{2} \right) n (r_0 + l)^2 \left(\frac{r_0 + l}{r_0} \right)^{2n} + 3 \left(n - \frac{2}{3} \right) r_0^4 n^3 \left(\frac{r_0 + l}{r_0} \right)^{4n} + (r_0 + l)^4 \left(\frac{r_0 + l}{r_0} \right)^{4n} \right)^{1/2} (r_0 + l)^3 r_0^2 (-l^4 + _{CI})^{7/2} \right) = 0$$

> `isolate((35), diff(f(l), l))`

$$\frac{d}{dl} f(l) = \left(2 r_0^4 n \left(\frac{r_0 + l}{r_0} \right)^{2n} f(l) (-l^4 + _{CI})^{3/2} _{CI}^2 - 4 r_0^4 n^3 \left(\frac{r_0 + l}{r_0} \right)^{2n} f(l) (-l^4 + _{CI})^3 \right.$$

$$\left. /2 _{CI}^2 - 2 r_0^4 n^2 \left(\frac{r_0 + l}{r_0} \right)^{2n} f(l) (-l^4 + _{CI})^{3/2} _{CI}^2 + 6 \right)$$

$$r_0^4 n^4 \left(\frac{r_0 + l}{r_0} \right)^{4n} f(l) (-l^4 + _{CI})^{3/2} _{CI}^2 - 4 _{CI}^2 f(l) (-l^4 + _{CI})^3$$

$$\left. /2 \left(\frac{r_0 + l}{r_0} \right)^{4n} n^3 r_0^4 - 6 _{CI} f(l) (-l^4 + _{CI})^{3/2} l^6 n r_0^2 - 4 _{CI} f(l) (-l^4 \right.$$

$$\left. + _{CI})^{3/2} l^5 n r_0^3 - _{CI} f(l) (-l^4 + _{CI})^{3/2} l^4 n r_0^4 + 4 r_0^3 n \left(\frac{r_0 + l}{r_0} \right)^{2n} l f(l) (\right.$$

$$\left. -l^4 + _{CI})^{3/2} _{CI}^2 - r_0^4 n \left(\frac{r_0 + l}{r_0} \right)^{2n} f(l) (-l^4 + _{CI})^{3/2} _{CI} l^4 - 3 \right)$$

$$\begin{aligned}
& r_0^4 n^4 \left(\frac{r_0 + l}{r_0} \right)^{4n} f(l) (-l^4 + _{CI})^{3/2} -_{CI} l^4 + 2 -_{CI} f(l) (-l^4 + _{CI})^3 \\
& /2 \left(\frac{r_0 + l}{r_0} \right)^{4n} l^4 n^3 r_0^4 - r_0^2 n \left(\frac{r_0 + l}{r_0} \right)^{2n} l^6 f(l) (-l^4 + _{CI})^{3/2} -_{CI} + 2 \\
& r_0^2 n \left(\frac{r_0 + l}{r_0} \right)^{2n} l^2 f(l) (-l^4 + _{CI})^{3/2} -_{CI} l^2 + 4 r_0^3 n^3 \left(\frac{r_0 + l}{r_0} \right)^{2n} l^5 f(l) (-l^4 \\
& + _{CI})^{3/2} -_{CI} + 2 r_0^3 n^2 \left(\frac{r_0 + l}{r_0} \right)^{2n} l^5 f(l) (-l^4 + _{CI})^{3/2} -_{CI} - 8 \\
& r_0^3 n^3 \left(\frac{r_0 + l}{r_0} \right)^{2n} l f(l) (-l^4 + _{CI})^{3/2} -_{CI} l^2 - 4 r_0^3 n^2 \left(\frac{r_0 + l}{r_0} \right)^{2n} l f(l) (-l^4 \\
& + _{CI})^{3/2} -_{CI} l^2 + 2 r_0^4 n^3 \left(\frac{r_0 + l}{r_0} \right)^{2n} f(l) (-l^4 + _{CI})^{3/2} -_{CI} l^4 + \\
& r_0^4 n^2 \left(\frac{r_0 + l}{r_0} \right)^{2n} f(l) (-l^4 + _{CI})^{3/2} -_{CI} l^4 + 2 r_0^2 n^3 \left(\frac{r_0 + l}{r_0} \right)^{2n} l^6 f(l) (-l^4 \\
& + _{CI})^{3/2} -_{CI} + r_0^2 n^2 \left(\frac{r_0 + l}{r_0} \right)^{2n} l^6 f(l) (-l^4 + _{CI})^{3/2} -_{CI} - 4 \\
& r_0^2 n^3 \left(\frac{r_0 + l}{r_0} \right)^{2n} l^2 f(l) (-l^4 + _{CI})^{3/2} -_{CI} l^2 - 2 r_0^2 n^2 \left(\frac{r_0 + l}{r_0} \right)^{2n} l^2 f(l) (-l^4 \\
& + _{CI})^{3/2} -_{CI} l^2 - 2 r_0^3 n \left(\frac{r_0 + l}{r_0} \right)^{2n} l^5 f(l) (-l^4 + _{CI})^{3/2} -_{CI} - 4 f(l) (-l^4 \\
& + _{CI})^{3/2} -_{CI} l^7 r_0 n + 8 f(l) (-l^4 + _{CI})^{3/2} -_{CI} l^2 l^3 r_0 n + 12 -_{CI} l^2 f(l) (-l^4 \\
& + _{CI})^{3/2} l^2 n r_0^2 + 8 -_{CI} l^2 f(l) (-l^4 + _{CI})^{3/2} l n r_0^3 + 2 -_{CI} l^2 f(l) (-l^4 + _{CI})^{3/2} n \\
& r_0^4 + 2 f(l) (-l^4 + _{CI})^{3/2} -_{CI} l^2 l^4 n - f(l) (-l^4 + _{CI})^{3/2} -_{CI} l^8 n \Big) \Bigg/ \left(\frac{5}{2} l^{18} r_0 \right. \\
& + 5 l^{17} r_0^2 + 5 l^{16} r_0^3 - -_{CI} l^{15} + \frac{1}{2} -_{CI} l^2 l^{11} - 6 r_0^3 n \left(\frac{r_0 + l}{r_0} \right)^{2n} l^{12} -_{CI} + 3 \\
& r_0^3 n \left(\frac{r_0 + l}{r_0} \right)^{2n} l^8 -_{CI} l^2 - 6 r_0^4 n \left(\frac{r_0 + l}{r_0} \right)^{2n} l^{11} -_{CI} + 3 r_0^4 n \left(\frac{r_0 + l}{r_0} \right)^{2n} l^7 -_{CI} l^2 - 2
\end{aligned}$$

$$\begin{aligned}
& r_0^5 n \left(\frac{r_0 + l}{r_0} \right)^{2n} -CI l^{10} + r_0^5 n \left(\frac{r_0 + l}{r_0} \right)^{2n} -CI^2 l^6 + 12 r_0^3 n^2 \left(\frac{r_0 + l}{r_0} \right)^{2n} l^{12} -CI - 6 \\
& r_0^3 n^2 \left(\frac{r_0 + l}{r_0} \right)^{2n} l^8 -CI^2 + 12 r_0^4 n^2 \left(\frac{r_0 + l}{r_0} \right)^{2n} l^{11} -CI - 6 r_0^4 n^2 \left(\frac{r_0 + l}{r_0} \right)^{2n} l^7 -CI^2 \\
& + 4 r_0^5 n^2 \left(\frac{r_0 + l}{r_0} \right)^{2n} -CI l^{10} - 2 r_0^5 n^2 \left(\frac{r_0 + l}{r_0} \right)^{2n} -CI^2 l^6 + r_0^2 n \left(\frac{r_0 + l}{r_0} \right)^{2n} l^9 -CI^2 \\
& - 2 r_0^2 n \left(\frac{r_0 + l}{r_0} \right)^{2n} l^{13} -CI - 3 r_0^4 n^4 \left(\frac{r_0 + l}{r_0} \right)^{4n} -CI l^{11} - 3 \\
& r_0^5 n^4 \left(\frac{r_0 + l}{r_0} \right)^{4n} -CI l^{10} + \frac{3}{2} r_0^4 n^4 \left(\frac{r_0 + l}{r_0} \right)^{4n} -CI^2 l^7 + \frac{3}{2} \\
& r_0^5 n^4 \left(\frac{r_0 + l}{r_0} \right)^{4n} -CI^2 l^6 + 2 r_0^4 n^3 \left(\frac{r_0 + l}{r_0} \right)^{4n} -CI l^{11} + 2 r_0^5 n^3 \left(\frac{r_0 + l}{r_0} \right)^{4n} -CI l^{10} \\
& - r_0^4 n^3 \left(\frac{r_0 + l}{r_0} \right)^{4n} -CI^2 l^7 - r_0^5 n^3 \left(\frac{r_0 + l}{r_0} \right)^{4n} -CI^2 l^6 + 4 r_0^2 n^2 \left(\frac{r_0 + l}{r_0} \right)^{2n} l^{13} -CI \\
& - 2 r_0^2 n^2 \left(\frac{r_0 + l}{r_0} \right)^{2n} l^9 -CI^2 - 5 -CI l^{11} r_0^4 - -CI l^{10} r_0^5 + \frac{5}{2} -CI^2 l^7 r_0^4 + \frac{1}{2} -CI^2 l^6 \\
& r_0^5 - 5 -CI l^{14} r_0 - 10 -CI l^{13} r_0^2 - 10 -CI l^{12} r_0^3 + \frac{5}{2} -CI^2 l^{10} r_0 + 5 -CI^2 l^9 r_0^2 \\
& + 5 -CI^2 l^8 r_0^3 + \frac{1}{2} l^{19} - 6 r_0^3 n^2 \left(\frac{r_0 + l}{r_0} \right)^{2n} l^{16} - 6 r_0^4 n^2 \left(\frac{r_0 + l}{r_0} \right)^{2n} l^{15} - 2 \\
& r_0^5 n^2 \left(\frac{r_0 + l}{r_0} \right)^{2n} l^{14} + r_0^2 n \left(\frac{r_0 + l}{r_0} \right)^{2n} l^{17} + 3 r_0^4 n \left(\frac{r_0 + l}{r_0} \right)^{2n} l^{15} + 3 \\
& r_0^3 n \left(\frac{r_0 + l}{r_0} \right)^{2n} l^{16} - 2 r_0^2 n^2 \left(\frac{r_0 + l}{r_0} \right)^{2n} l^{17} + r_0^5 n \left(\frac{r_0 + l}{r_0} \right)^{2n} l^{14} + \frac{3}{2} \\
& r_0^4 n^4 \left(\frac{r_0 + l}{r_0} \right)^{4n} l^{15} + \frac{3}{2} r_0^5 n^4 \left(\frac{r_0 + l}{r_0} \right)^{4n} l^{14} - r_0^4 n^3 \left(\frac{r_0 + l}{r_0} \right)^{4n} l^{15} - \\
& r_0^5 n^3 \left(\frac{r_0 + l}{r_0} \right)^{4n} l^{14} + \frac{5}{2} l^{15} r_0^4 + \frac{1}{2} l^{14} r_0^5
\end{aligned}$$

> `simplify((36), 'size')`

(37)

$$\frac{d}{dl} f(l) = - \left(2 \left(-2 \left(n - \frac{1}{2} \right) (r_0 + l)^2 (n+1) r_0^2 \left(\frac{r_0 + l}{r_0} \right)^{2n} + 3 n^2 \left(n - \frac{2}{3} \right) r_0^4 \left(\frac{r_0 + l}{r_0} \right)^{4n} + (r_0 + l)^4 \right) f(l) n (l^4 - 2 _CI) (-l^4 + _CI)^{3/2} _CI \right) \Bigg/ \left((r_0 + l)^6 (l^4 - _CI)^2 \left(-4 r_0^2 \left(n - \frac{1}{2} \right) n (r_0 + l)^2 \left(\frac{r_0 + l}{r_0} \right)^{2n} + 3 \left(n - \frac{2}{3} \right) r_0^4 n^3 \left(\frac{r_0 + l}{r_0} \right)^{4n} + (r_0 + l)^4 \right) \right)$$

>

$$\frac{d}{dl} f(l) = - \left(2 \left(-2 \left(n - \frac{1}{2} \right) (r_0 + l)^2 (n+1) r_0^2 \left(\frac{r_0 + l}{r_0} \right)^{2n} + 3 n^2 \left(n - \frac{2}{3} \right) r_0^4 \left(\frac{r_0 + l}{r_0} \right)^{4n} + (r_0 + l)^4 \right) f(l) n (l^4 - 2 _CI) (-l^4 + _CI)^{3/2} _CI \right) \Bigg/ \left((r_0 + l)^6 (l^4 - _CI)^2 \left(-4 r_0^2 \left(n - \frac{1}{2} \right) n (r_0 + l)^2 \left(\frac{r_0 + l}{r_0} \right)^{2n} + 3 \left(n - \frac{2}{3} \right) r_0^4 n^3 \left(\frac{r_0 + l}{r_0} \right)^{4n} + (r_0 + l)^4 \right) \right) \cdot \left(\frac{1}{f(l)} \right)$$

$$\frac{d}{dl} f(l) = - \left(2 \left(-2 \left(n - \frac{1}{2} \right) (r_0 + l)^2 (n+1) r_0^2 \left(\frac{r_0 + l}{r_0} \right)^{2n} + 3 n^2 \left(n - \frac{2}{3} \right) r_0^4 \left(\frac{r_0 + l}{r_0} \right)^{4n} + (r_0 + l)^4 \right) n (l^4 - 2 _CI) (-l^4 + _CI)^{3/2} _CI \right) \Bigg/ \left((r_0 + l)^6 (l^4 - _CI)^2 \left(-4 r_0^2 \left(n - \frac{1}{2} \right) n (r_0 + l)^2 \left(\frac{r_0 + l}{r_0} \right)^{2n} + 3 \left(n - \frac{2}{3} \right) r_0^4 n^3 \left(\frac{r_0 + l}{r_0} \right)^{4n} + (r_0 + l)^4 \right) \right)$$

> `simplify((38), 'size')`

$$\frac{d}{dl} f(l) = - \left(2 n (l^4 - 2 _CI) _CI \left(-2 \left(n - \frac{1}{2} \right) (r_0 + l)^2 (n+1) r_0^2 \left(\frac{r_0 + l}{r_0} \right)^{2n} + 3 n^2 \left(n - \frac{2}{3} \right) r_0^4 \left(\frac{r_0 + l}{r_0} \right)^{4n} + (r_0 + l)^4 \right) \right) \Bigg/ \left((r_0 + l) \sqrt{-l^4 + _CI} \left(-4 r_0^2 \left(n - \frac{1}{2} \right) n (r_0 + l)^2 \left(\frac{r_0 + l}{r_0} \right)^{2n} + 3 \left(n - \frac{2}{3} \right) r_0^4 n^3 \left(\frac{r_0 + l}{r_0} \right)^{4n} + (r_0 + l)^4 \right) \right)$$

$$-\frac{1}{2} \Big) n \left(r_0 + l\right)^2 \left(\frac{r_0 + l}{r_0}\right)^{2n} + 3 \left(n - \frac{2}{3}\right) r_0^4 n^3 \left(\frac{r_0 + l}{r_0}\right)^{4n} + \left(r_0 + l\right)^4 \Big) l^6 \Bigg)$$

```
> latex('39')
{\frac{{{\frac{{{\rm d}}{{{\rm d}l}}}}}{f}\left({{\left(1\right)}{\left(1\right)}}\right)}{{{\left(1\right)}{\left(1\right)}}}}=-2\,,{\frac{n\left({{\left(1\right)}^4}-2\right),{{\it \_C1}}}{{{\it \_C1}}}}{\it \_C1}}{\left({{\left(r_{\{0\}}+1\right)}{\sqrt{{{-1}}^4}+{{\it \_C1}}}}\right){{1}}^{\{6\}}}
\left({{-2}\,,\left({{n-1/2}}\right)}\right)\left({{\left(r_{\{0\}}+1\right)}^2}\right)^{2}
\left({{n+1}}\right){{r_{\{0\}}}}^2\left({{\frac{{r_{\{0\}}+1}}{{r_{\{0\}}}}}{r_{\{0\}}}}\right)
\left({{2\,,n}}\right)+{{n}}^2\left({{n-2/3}}\right)\left({{r_{\{0\}}}}\right)^4
\left({{\frac{{r_{\{0\}}+1}}{{r_{\{0\}}}}}{r_{\{0\}}}}\right)^4\left({{\frac{{r_{\{0\}}+1}}{{r_{\{0\}}}}}{r_{\{0\}}}}\right)^4
\left({{n-1/2}}\right)\left({{r_{\{0\}}+1}}\right)^2\left({{\frac{{r_{\{0\}}+1}}{{r_{\{0\}}}}}{r_{\{0\}}}}\right)^2
\left({{2\,,n}}\right)+{{n}}^3\left({{n-2/3}}\right)\left({{r_{\{0\}}}}\right)^4{{n}}^3
\left({{\frac{{r_{\{0\}}+1}}{{r_{\{0\}}}}}{r_{\{0\}}}}\right)^4\left({{\frac{{r_{\{0\}}+1}}{{r_{\{0\}}}}}{r_{\{0\}}}}\right)^{-1}}

```

$$\begin{aligned} &> - \left(2 n (\mathcal{A} - 2 _CI) _CI \left(-2 \left(n - \frac{1}{2} \right) (r_0 + l)^2 (n + 1) r_0^2 \left(\frac{r_0 + l}{r_0} \right)^{2n} + 3 n^2 \left(n - \frac{2}{3} \right) \right. \right. \\ &\quad \left. \left. r_0^4 \left(\frac{r_0 + l}{r_0} \right)^{4n} + (r_0 + l)^4 \right) \right) \Bigg/ \left((r_0 + l) \sqrt{-\mathcal{A} + _CI} \left(-4 r_0^2 \left(n - \frac{1}{2} \right) n (r_0 \right. \right. \\ &\quad \left. \left. + l)^2 \left(\frac{r_0 + l}{r_0} \right)^{2n} + 3 \left(n - \frac{2}{3} \right) r_0^4 n^3 \left(\frac{r_0 + l}{r_0} \right)^{4n} + (r_0 + l)^4 \right) l^6 \right) \\ &- \left(2 n (\mathcal{A} - 2 _CI) _CI \left(-2 \left(n - \frac{1}{2} \right) (r_0 + l)^2 (n + 1) r_0^2 \left(\frac{r_0 + l}{r_0} \right)^{2n} + 3 n^2 \left(n - \frac{2}{3} \right) \right. \right. \\ &\quad \left. \left. r_0^4 \left(\frac{r_0 + l}{r_0} \right)^{4n} + (r_0 + l)^4 \right) \right) \Bigg/ \left((r_0 + l) \sqrt{-\mathcal{A} + _CI} \left(-4 r_0^2 \left(n - \frac{1}{2} \right) n (r_0 \right. \right. \\ &\quad \left. \left. + l)^2 \left(\frac{r_0 + l}{r_0} \right)^{2n} + 3 \left(n - \frac{2}{3} \right) r_0^4 n^3 \left(\frac{r_0 + l}{r_0} \right)^{4n} + (r_0 + l)^4 \right) l^6 \right) \end{aligned} \quad (40)$$

$$\begin{aligned} &> eval(40), [_CI = 0.011000, n = 0.5, r[0] = 0.5] \\ &- (0.011000 (\mathcal{A} - 0.022000) (-0.007812500004 (1.0000000000 + 2.0000000000 l)^{2.0} + (0.5 \\ &\quad + l)^4)) / ((0.5 + l) \sqrt{-\mathcal{A} + 0.011000} (-0.003906250002 (1.0000000000 \\ &\quad + 2.0000000000 l)^{2.0} + (0.5 + l)^4) l^6) \end{aligned} \quad (41)$$

> int((41), l)

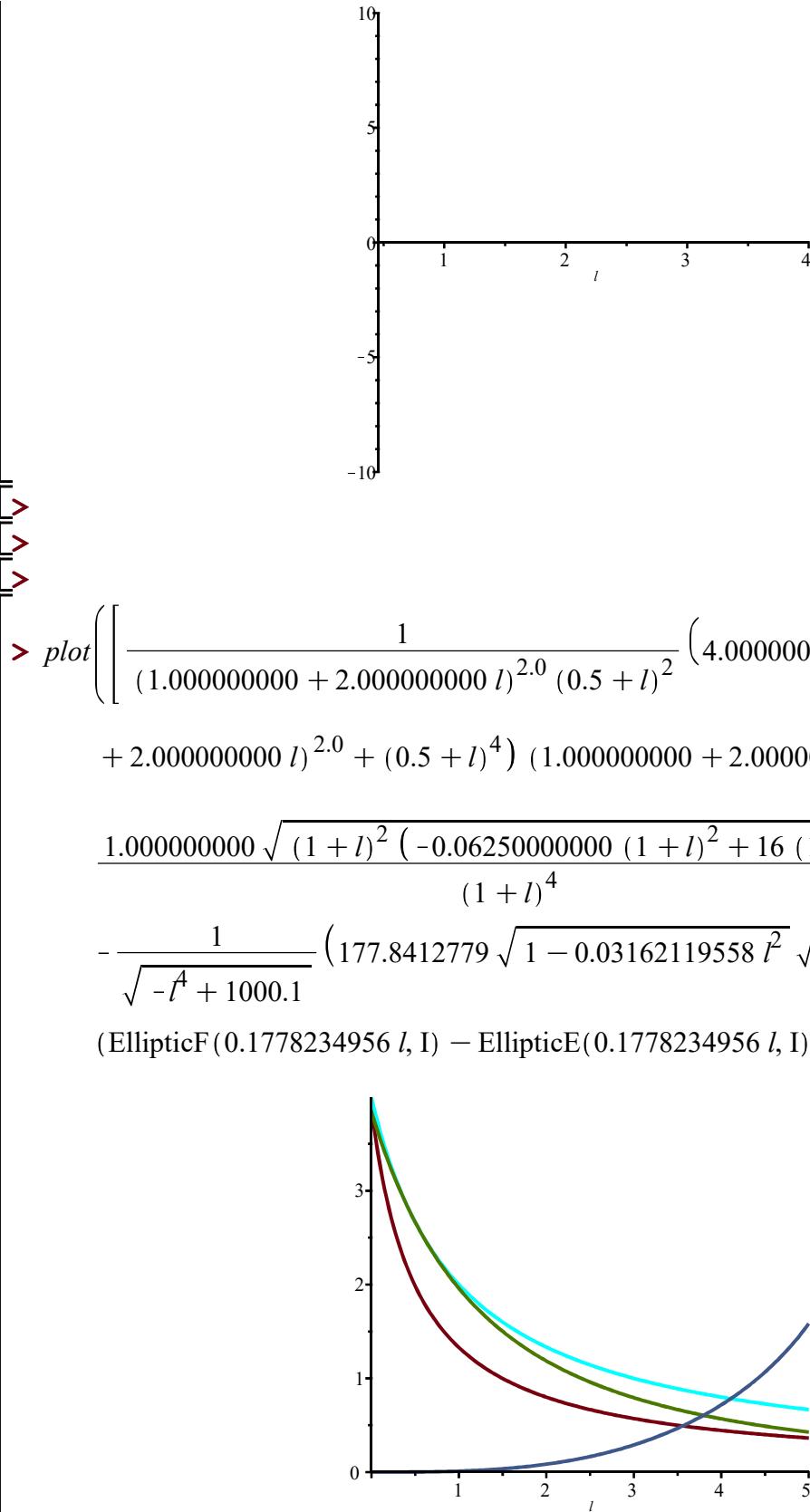
(42)

$$\begin{aligned}
& -0.02348365132 \operatorname{I}_{\text{arctanh}}\left(\frac{150.1168030 I t^2}{\sqrt{-10000. t^4 + 110.}} - \frac{11.74246992 I}{\sqrt{-10000. t^4 + 110.}}\right) \\
& - \frac{1}{\sqrt{-10000. t^4 + 110.}} \left(0.3799668875 \sqrt{1. - 9.534625892 t^2} \sqrt{1. + 9.534625892 t^2}\right. \\
& \quad \left.\operatorname{EllipticPi}(3.087818954 I, 0.7458196255, 1.000000000 I)\right) \\
& + 0.1256388834 \operatorname{I}_{\text{arctanh}}\left(\frac{0.02203263246 I (-5000. t^2 + 220.)}{\sqrt{-10000. t^4 + 110.}}\right) \\
& - 0.01601188420 \operatorname{I}_{\text{arctanh}}\left(-\frac{103.8118615 I t^2}{\sqrt{-10000. t^4 + 110.}} + \frac{2.923342021 I}{\sqrt{-10000. t^4 + 110.}}\right) \\
& + \frac{1}{\sqrt{-10000. t^4 + 110.}} \left(0.6243867036 \sqrt{1. - 9.534625892 t^2} \sqrt{1. + 9.534625892 t^2}\right. \\
& \quad \left.\operatorname{EllipticPi}(3.087818954 I, 0.2684950651, 1.000000000 I)\right) \\
& - \frac{0.0003278380245 \sqrt{-10000. t^4 + 110.}}{t^3} \\
& + \frac{1}{\sqrt{-10000. t^4 + 110.}} \left(0.008774494884 \sqrt{121. - 1153.689733 t^2}\right. \\
& \quad \left.\sqrt{121. + 1153.689733 t^2} \operatorname{EllipticF}(3.087818954 I, 1. I)\right) \\
& - \frac{0.00008213333333 \sqrt{-10000. t^4 + 110.}}{t^5} \\
& - \frac{0.001826648698 \sqrt{-10000. t^4 + 110.}}{t} \\
& + \frac{1}{\sqrt{-10000. t^4 + 110.}} \left(0.005127600430 \sqrt{121. - 1153.689733 t^2}\right. \\
& \quad \left.\sqrt{121. + 1153.689733 t^2} (\operatorname{EllipticF}(3.087818954 I, 1. I) - 1. \operatorname{EllipticE}(3.087818954 I, 1. I))\right) \\
& + \frac{0.0001740444444 \sqrt{-10000. t^4 + 110.}}{t^4} \\
& - 0.06943549755 \operatorname{arctanh}\left(\frac{10.48808848}{\sqrt{-10000. t^4 + 110.}}\right) - \frac{0.003783174312 (1000. t^4 - 11.)}{t^2 \sqrt{-10000. t^4 + 110.}} \\
& - \frac{1}{\sqrt{-10000. t^4 + 110.}} \left(3.693480793 \sqrt{1. - 9.534625892 t^2}\right. \\
& \quad \left.\sqrt{1. + 9.534625892 t^2} \operatorname{EllipticPi}(3.087818954 I, 0.4195235393, 1.000000000 I)\right) \\
& > \operatorname{simplify}(\text{(42)}, \text{'size'})
\end{aligned} \tag{42}$$

$$\begin{aligned}
& \frac{1}{\sqrt{-10000. l^4 + 110.}} l^5 \left(0.01601188420 \left(1.000000000 \operatorname{Iarctanh} \left(\frac{1}{\sqrt{-10000. l^4 + 110.}} \right) \right. \right. \\
& \left. \left. - 2.923342021 \operatorname{I} + 103.8118615 \operatorname{I} l^2 \right) \right) \sqrt{-10000. l^4 + 110.} l^5 \\
& - 1.466638843 \operatorname{Iarctanh} \left(\frac{-11.74246992 \operatorname{I} + 150.1168030 \operatorname{I} l^2}{\sqrt{-10000. l^4 + 110.}} \right) \sqrt{-10000. l^4 + 110.} l^5 \\
& - 7.846602051 \operatorname{arctan} \left(\frac{-110.1631623 l^2 + 4.847179141}{\sqrt{-10000. l^4 + 110.}} \right) \sqrt{-10000. l^4 + 110.} l^5 \\
& - 4.336497609 \operatorname{arctanh} \left(\frac{10.48808848}{\sqrt{-10000. l^4 + 110.}} \right) \sqrt{-10000. l^4 + 110.} l^5 + \\
& - 23.73030449 \operatorname{EllipticPi}(3.087818954 l, 0.7458196255, 1.000000000 \operatorname{I}) l^5 \\
& + 38.99520480 \operatorname{EllipticPi}(3.087818954 l, 0.2684950651, 1.000000000 \operatorname{I}) l^5 \\
& - 230.6712156 \operatorname{EllipticPi}(3.087818954 l, 0.4195235393, 1.000000000 \operatorname{I}) l^5 \\
& \sqrt{1. + 9.534625892 l^2} \sqrt{1. - 9.534625892 l^2} \\
& + (0.8682360637 \operatorname{EllipticF}(3.087818954 l, 1. \operatorname{I}) l^5 \\
& - 0.3202371667 l^5 \operatorname{EllipticE}(3.087818954 l, 1. \operatorname{I})) \\
& \sqrt{121. + 1153.689733 l^2} \sqrt{121. - 1153.689733 l^2} + 1140.808087 (l \\
& + 0.323853184028959) (l - 0.323853184083034) (l^2 + 0.412549175666313 l \\
& + 0.266212798019144) (l^2 + 4.732728962 10^{-11} l + 0.1048808848) (l^2 \\
& - 0.619659295751131 l + 0.168902270209559)))
\end{aligned} \tag{43}$$

> `plot((43), l=0.5 .. 4)`

Warning, unable to evaluate the function to numeric values in the region; see the plotting command's help page to ensure the calling sequence is correct



— V1 — V 2 — V 3 — T