

> restart;  
with(Physics) :

$$\begin{aligned} & -2 \cdot \frac{\frac{d^2}{dl^2} r(l)}{r(l)} - \left( \frac{\frac{d}{dl} r(l)}{r(l)} \right)^2 + \frac{1}{r^2(l)} = -V(T) \cdot \left( 1 + \left( \frac{d}{dl} T(l) \right)^2 \right)^{\frac{1}{2}} \\ & - \frac{2 \left( \frac{d^2}{dl^2} r(l) \right)}{r(l)} - \frac{\left( \frac{d}{dl} r(l) \right)^2}{r(l)^2} + \frac{1}{r(l)^2} = -V(T) \sqrt{1 + \left( \frac{d}{dl} T(l) \right)^2} \end{aligned} \quad (1)$$

$$\begin{aligned} & 2 \cdot \frac{\frac{d}{dl} \psi(l) \cdot \frac{d}{dl} r(l)}{\psi(l) \cdot r(l)} + \left( \frac{\frac{d}{dl} r(l)}{r(l)} \right)^2 - \frac{1}{r^2(l)} = \frac{V(T)}{\left( 1 + \left( \frac{d}{dl} T(l) \right)^2 \right)^{\frac{1}{2}}} \\ & \frac{2 \left( \frac{d}{dl} \psi(l) \right) \left( \frac{d}{dl} r(l) \right)}{\psi(l) r(l)} + \frac{\left( \frac{d}{dl} r(l) \right)^2}{r(l)^2} - \frac{1}{r(l)^2} = \frac{V(T)}{\sqrt{1 + \left( \frac{d}{dl} T(l) \right)^2}} \end{aligned} \quad (2)$$

$$\begin{aligned} & \frac{\frac{d^2}{dl^2} \psi(l)}{\psi(l)} + \frac{\frac{d}{dl} \psi(l) \cdot \frac{d}{dl} r(l)}{\psi(l) \cdot r(l)} + \frac{\frac{d^2}{dl^2} r(l)}{r(l)} = V(T) \cdot \left( 1 + \left( \frac{d}{dl} T(l) \right)^2 \right)^{\frac{1}{2}} \\ & \frac{\frac{d^2}{dl^2} \psi(l)}{\psi(l)} + \frac{\left( \frac{d}{dl} \psi(l) \right) \left( \frac{d}{dl} r(l) \right)}{\psi(l) r(l)} + \frac{\frac{d^2}{dl^2} r(l)}{r(l)} = V(T) \sqrt{1 + \left( \frac{d}{dl} T(l) \right)^2} \end{aligned} \quad (3)$$

**conservation equation :**

$$\begin{aligned} & \frac{d}{dl} p_r(l) + \frac{2}{l} \cdot (p_t(l) - p_r(l)) = 0 \\ & \frac{d}{dl} p_r(l) + \frac{2 (p_t(l) - p_r(l))}{l} = 0 \end{aligned} \quad (4)$$

$$\text{eval} \left( (4), \left[ p_r(l) = \frac{V(T)}{\sqrt{1 + \left( \frac{d}{dl} T(l) \right)^2}}, p_t(l) = V(T) \sqrt{1 + \left( \frac{d}{dl} T(l) \right)^2} \right] \right)$$

(5)

$$\begin{aligned}
& - \frac{V(T) \left( \frac{d}{dl} T(l) \right) \left( \frac{d^2}{dl^2} T(l) \right)}{\left( 1 + \left( \frac{d}{dl} T(l) \right)^2 \right)^{3/2}} \\
& + \frac{2 \left( V(T) \sqrt{1 + \left( \frac{d}{dl} T(l) \right)^2} - \frac{V(T)}{\sqrt{1 + \left( \frac{d}{dl} T(l) \right)^2}} \right)}{l} = 0
\end{aligned} \tag{5}$$

> simplify( (5), 'size' )

$$- \frac{V(T) \left( \frac{d}{dl} T(l) \right) \left( -2 \left( \frac{d}{dl} T(l) \right)^3 + l \left( \frac{d^2}{dl^2} T(l) \right) - 2 \left( \frac{d}{dl} T(l) \right) \right)}{\left( 1 + \left( \frac{d}{dl} T(l) \right)^2 \right)^{3/2} l} = 0 \tag{6}$$

> simplify( (6) )

$$\frac{V(T) \left( \frac{d}{dl} T(l) \right) \left( 2 \left( \frac{d}{dl} T(l) \right)^3 - l \left( \frac{d^2}{dl^2} T(l) \right) + 2 \left( \frac{d}{dl} T(l) \right) \right)}{\left( 1 + \left( \frac{d}{dl} T(l) \right)^2 \right)^{3/2} l} = 0 \tag{7}$$

$$> \left( 2 \left( \frac{d}{dl} T(l) \right)^3 - l \left( \frac{d^2}{dl^2} T(l) \right) + 2 \left( \frac{d}{dl} T(l) \right) \right) = 0$$

$$2 \left( \frac{d}{dl} T(l) \right)^3 - l \left( \frac{d^2}{dl^2} T(l) \right) + 2 \left( \frac{d}{dl} T(l) \right) = 0 \tag{8}$$

> dsolve( (8), { T(l) } )

$$T(l) = \int \frac{l^2}{\sqrt{-l^4 + \_CI}} dl + \_C2, T(l) = \int \left( - \frac{l^2}{\sqrt{-l^4 + \_CI}} \right) dl + \_C2 \tag{9}$$

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$$> \frac{l^2}{\sqrt{-l^4 + \_CI}}$$

$$\frac{l^2}{\sqrt{-l^4 + \_CI}} \tag{10}$$

> int((10), l)

$$- \frac{1}{\sqrt{-l^4 + \_CI}} \left( -\_CI^{3/4} \sqrt{1 - \frac{l^2}{\sqrt{\_CI}}} \sqrt{1 + \frac{l^2}{\sqrt{\_CI}}} \left( \text{EllipticF} \left( \frac{l}{\_CI^{1/4}}, I \right) \right. \right. \tag{11}$$

$$- \text{EllipticE}\left(\frac{l}{\sqrt{-CI^{1/4}}}, I\right)\right)$$

**Taking Eq One and Two :**

$$\begin{aligned} & \left( -\frac{2 \left( \frac{d^2}{dl^2} r(l) \right)}{r(l)} - \frac{\left( \frac{d}{dl} r(l) \right)^2}{r(l)^2} + \frac{1}{r(l)^2} \right) \cdot \left( \frac{2 \left( \frac{d}{dl} \psi(l) \right) \left( \frac{d}{dl} r(l) \right)}{\psi(l) r(l)} + \frac{\left( \frac{d}{dl} r(l) \right)^2}{r(l)^2} \right. \\ & \left. - \frac{1}{r(l)^2} \right) = \left( -V \cdot \sqrt{1 + \left( \frac{d}{dl} T(l) \right)^2} \right) \cdot \left( \frac{V}{\sqrt{1 + \left( \frac{d}{dl} T(l) \right)^2}} \right) \\ & \left( -\frac{2 \left( \frac{d^2}{dl^2} r(l) \right)}{r(l)} - \frac{\left( \frac{d}{dl} r(l) \right)^2}{r(l)^2} + \frac{1}{r(l)^2} \right) \left( \frac{2 \left( \frac{d}{dl} \psi(l) \right) \left( \frac{d}{dl} r(l) \right)}{\psi(l) r(l)} + \frac{\left( \frac{d}{dl} r(l) \right)^2}{r(l)^2} \right. \\ & \left. - \frac{1}{r(l)^2} \right) = -V^2 \end{aligned} \quad (12)$$

$$\begin{aligned} & \text{eval}\left( (12), \left[ r(l) = r_0 \cdot \left( 1 + \frac{l}{r_0} \right)^n, \frac{d}{dl} T(l) = \frac{l^2}{\sqrt{-l^4 + CI}}, \psi(l) = C \right] \right) \\ & \left( -\frac{2 \left( \frac{\left( 1 + \frac{l}{r_0} \right)^n n^2}{r_0 \left( 1 + \frac{l}{r_0} \right)^2} - \frac{\left( 1 + \frac{l}{r_0} \right)^n n}{\left( 1 + \frac{l}{r_0} \right)^2 r_0} \right)}{r_0 \left( 1 + \frac{l}{r_0} \right)^n} - \frac{n^2}{\left( 1 + \frac{l}{r_0} \right)^2 r_0^2} \right. \\ & \left. + \frac{1}{r_0^2 \left( \left( 1 + \frac{l}{r_0} \right)^n \right)^2} \right) \left( \frac{n^2}{\left( 1 + \frac{l}{r_0} \right)^2 r_0^2} - \frac{1}{r_0^2 \left( \left( 1 + \frac{l}{r_0} \right)^n \right)^2} \right) = -V^2 \end{aligned} \quad (13)$$

**simplify( (13), 'size' )**

$$- \frac{1}{r_0^4 (r_0 + l)^4 \left( \left( 1 + \frac{l}{r_0} \right)^n \right)^4} \left( \left( n r_0 \left( 1 + \frac{l}{r_0} \right)^n + l + r_0 \right) \left( -3 \left( n - \frac{2}{3} \right) \right. \right. \\ \left. \left. r_0^2 n \left( \left( 1 + \frac{l}{r_0} \right)^n \right)^2 + (r_0 + l)^2 \right) \left( -n r_0 \left( 1 + \frac{l}{r_0} \right)^n + l + r_0 \right) \right) = -V^2 \quad (14)$$

> solve( { (14) }, [V] )

$$\left[ \left[ V \right] \right] \quad (15)$$

$$= \frac{1}{\left( \frac{r_0 + l}{r_0} \right)^{4n} (r_0 + l)^2 r_0^2} \left( \left( \frac{r_0 + l}{r_0} \right)^{4n} \left( 3 \left( \frac{r_0 + l}{r_0} \right)^{4n} n^4 r_0^4 \right. \right. \\ \left. \left. - 2 \left( \frac{r_0 + l}{r_0} \right)^{4n} n^3 r_0^4 - 4 \left( \frac{r_0 + l}{r_0} \right)^{2n} l^2 n^2 r_0^2 - 8 \left( \frac{r_0 + l}{r_0} \right)^{2n} l n^2 r_0^3 \right. \right. \\ \left. \left. - 4 \left( \frac{r_0 + l}{r_0} \right)^{2n} n^2 r_0^4 + 2 \left( \frac{r_0 + l}{r_0} \right)^{2n} l^2 n r_0^2 + 4 \left( \frac{r_0 + l}{r_0} \right)^{2n} l n r_0^3 + 2 \left( \frac{r_0 + l}{r_0} \right)^{2n} n \right. \right. \\ \left. \left. r_0^4 + l^4 + 4 l^3 r_0 + 6 l^2 r_0^2 + 4 l r_0^3 + r_0^4 \right) \right)^{1/2} \left[ V = \right.$$

$$\left. - \frac{1}{\left( \frac{r_0 + l}{r_0} \right)^{4n} (r_0 + l)^2 r_0^2} \left( \left( \frac{r_0 + l}{r_0} \right)^{4n} \left( 3 \left( \frac{r_0 + l}{r_0} \right)^{4n} n^4 r_0^4 \right. \right. \right.$$

$$\begin{aligned} & -2 \left( \frac{r_0 + l}{r_0} \right)^{4n} n^3 r_0^4 - 4 \left( \frac{r_0 + l}{r_0} \right)^{2n} l^2 n^2 r_0^2 - 8 \left( \frac{r_0 + l}{r_0} \right)^{2n} l n^2 r_0^3 \\ & -4 \left( \frac{r_0 + l}{r_0} \right)^{2n} n^2 r_0^4 + 2 \left( \frac{r_0 + l}{r_0} \right)^{2n} l^2 n r_0^2 + 4 \left( \frac{r_0 + l}{r_0} \right)^{2n} l n r_0^3 + 2 \left( \frac{r_0 + l}{r_0} \right)^{2n} n \\ & \left. \left. \left. r_0^4 + l^4 + 4 l^3 r_0 + 6 l^2 r_0^2 + 4 l r_0^3 + r_0^4 \right) \right)^{1/2} \right] \end{aligned}$$

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> V

$$\begin{aligned} & = \frac{1}{\left( \frac{r_0 + l}{r_0} \right)^{4n} (r_0 + l)^2 r_0^2} \left( \left( \frac{r_0 + l}{r_0} \right)^{4n} \left( 3 \left( \frac{r_0 + l}{r_0} \right)^{4n} n^4 r_0^4 \right. \right. \\ & - 2 \left( \frac{r_0 + l}{r_0} \right)^{4n} n^3 r_0^4 - 4 \left( \frac{r_0 + l}{r_0} \right)^{2n} l^2 n^2 r_0^2 - 8 \left( \frac{r_0 + l}{r_0} \right)^{2n} l n^2 r_0^3 \\ & - 4 \left( \frac{r_0 + l}{r_0} \right)^{2n} n^2 r_0^4 + 2 \left( \frac{r_0 + l}{r_0} \right)^{2n} l^2 n r_0^2 + 4 \left( \frac{r_0 + l}{r_0} \right)^{2n} l n r_0^3 + 2 \left( \frac{r_0 + l}{r_0} \right)^{2n} n \\ & \left. \left. \left. r_0^4 + l^4 + 4 l^3 r_0 + 6 l^2 r_0^2 + 4 l r_0^3 + r_0^4 \right) \right)^{1/2} \right) \end{aligned}$$

V

(16)

$$\begin{aligned} & = \frac{1}{\left( \frac{r_0 + l}{r_0} \right)^{4n} (r_0 + l)^2 r_0^2} \left( \left( \frac{r_0 + l}{r_0} \right)^{4n} \left( 3 \left( \frac{r_0 + l}{r_0} \right)^{4n} n^4 r_0^4 \right. \right. \\ & - 2 \left( \frac{r_0 + l}{r_0} \right)^{4n} n^3 r_0^4 - 4 \left( \frac{r_0 + l}{r_0} \right)^{2n} l^2 n^2 r_0^2 - 8 \left( \frac{r_0 + l}{r_0} \right)^{2n} l n^2 r_0^3 \\ & - 4 \left( \frac{r_0 + l}{r_0} \right)^{2n} n^2 r_0^4 + 2 \left( \frac{r_0 + l}{r_0} \right)^{2n} l^2 n r_0^2 + 4 \left( \frac{r_0 + l}{r_0} \right)^{2n} l n r_0^3 + 2 \left( \frac{r_0 + l}{r_0} \right)^{2n} n \\ & \left. \left. \left. r_0^4 + l^4 + 4 l^3 r_0 + 6 l^2 r_0^2 + 4 l r_0^3 + r_0^4 \right) \right)^{1/2} \right) \end{aligned}$$

=

> simplify( (16), 'size' )

V

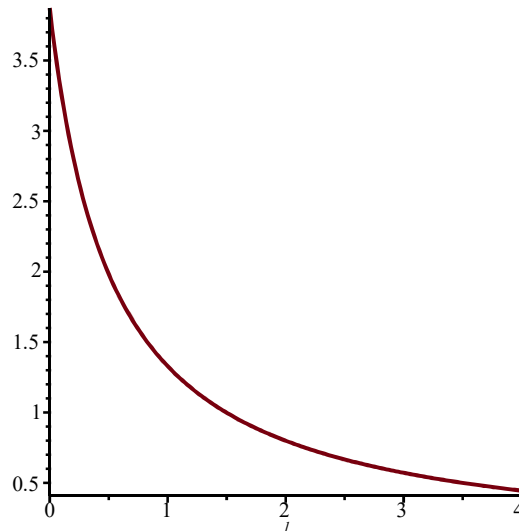
(17)

$$= \frac{1}{\left(\frac{r_0 + l}{r_0}\right)^{4n} (r_0 + l)^2 r_0^2} \left( \left( -4 r_0^2 \left(n - \frac{1}{2}\right) n (r_0 + l)^2 \left(\frac{r_0 + l}{r_0}\right)^{2n} + 3 \left(n - \frac{2}{3}\right) r_0^4 n^3 \left(\frac{r_0 + l}{r_0}\right)^{4n} + (r_0 + l)^4 \right) \left(\frac{r_0 + l}{r_0}\right)^{4n} \right)^{1/2}$$

$$\begin{aligned} &> \frac{1}{\left(\frac{r_0 + l}{r_0}\right)^{4n} (r_0 + l)^2 r_0^2} \left( \left( -4 r_0^2 \left(n - \frac{1}{2}\right) n (r_0 + l)^2 \left(\frac{r_0 + l}{r_0}\right)^{2n} + 3 \left(n - \frac{2}{3}\right) r_0^4 n^3 \left(\frac{r_0 + l}{r_0}\right)^{4n} + (r_0 + l)^4 \right) \left(\frac{r_0 + l}{r_0}\right)^{4n} \right)^{1/2} \\ &\frac{1}{\left(\frac{r_0 + l}{r_0}\right)^{4n} (r_0 + l)^2 r_0^2} \left( \left( -4 r_0^2 \left(n - \frac{1}{2}\right) n (r_0 + l)^2 \left(\frac{r_0 + l}{r_0}\right)^{2n} + 3 \left(n - \frac{2}{3}\right) r_0^4 n^3 \left(\frac{r_0 + l}{r_0}\right)^{4n} + (r_0 + l)^4 \right) \left(\frac{r_0 + l}{r_0}\right)^{4n} \right)^{1/2} \end{aligned} \quad (18)$$

$$\begin{aligned} &> \text{eval}((18), [n=0.5, r[0]=0.5]) \\ &\frac{1}{(1.000000000 + 2.000000000 l)^{2.0} (0.5 + l)^2} \left( 4.000000000 \left( (-0.003906250002 (1.000000000 + 2.000000000 l)^{2.0} + (0.5 + l)^4) (1.000000000 + 2.000000000 l)^{2.0} \right)^{1/2} \right) \end{aligned} \quad (19)$$

> plot((19), l=0 .. 4)



NEC :

> 
$$-V \cdot \sqrt{\left(1 + \left(\frac{d}{dl} T(l)\right)^2\right)} + \frac{V}{\sqrt{\left(1 + \left(\frac{d}{dl} T(l)\right)^2\right)}} \\ -V \sqrt{1 + \left(\frac{d}{dl} T(l)\right)^2} + \frac{V}{\sqrt{1 + \left(\frac{d}{dl} T(l)\right)^2}} \quad (20)$$

> simplify( (20), 'size' )

$$-\frac{V \left(\frac{d}{dl} T(l)\right)^2}{\sqrt{1 + \left(\frac{d}{dl} T(l)\right)^2}} \quad (21)$$

> eval  $\left( (21), \left[ \frac{d}{dl} T(l) = \frac{l^2}{\sqrt{-l^4 + Cl}}, V \right. \right.$

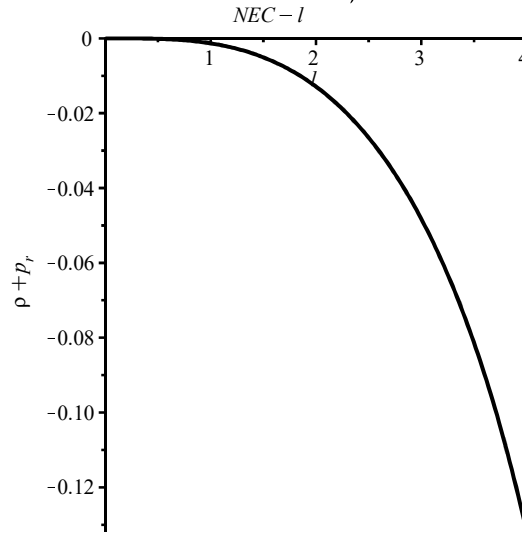
$$= \frac{1}{\left(\frac{r_0 + l}{r_0}\right)^{4n} (r_0 + l)^2 r_0^2} \left( \left( -4 r_0^2 \left(n - \frac{1}{2}\right) n (r_0 + l)^2 \left(\frac{r_0 + l}{r_0}\right)^{2n} \right. \right. \\ \left. \left. + 3 \left(n - \frac{2}{3}\right) r_0^4 n^3 \left(\frac{r_0 + l}{r_0}\right)^{4n} + (r_0 + l)^4 \right) \left(\frac{r_0 + l}{r_0}\right)^{4n} \right)^{1/2} \Bigg) \\ - \left( \left( \left( -4 r_0^2 \left(n - \frac{1}{2}\right) n (r_0 + l)^2 \left(\frac{r_0 + l}{r_0}\right)^{2n} + 3 \left(n - \frac{2}{3}\right) r_0^4 n^3 \left(\frac{r_0 + l}{r_0}\right)^{4n} + (r_0 + l)^4 \right) \left(\frac{r_0 + l}{r_0}\right)^{4n} \right)^{1/2} l^4 \right) / \left( \left(\frac{r_0 + l}{r_0}\right)^{4n} (r_0 + l)^2 r_0^2 (-l^4 \right. \right. \quad (22)$$

$$+_{CI}) \sqrt{1 + \frac{f^A}{-f^A +_{CI}}}$$

> eval( (22), [n=0.5, r[0]=0.5, \_CI=1000.1])

$$- \left( 4.000000000 \left( (-0.003906250002 (1.000000000 + 2.000000000 l)^{2.0} + (0.5 + l)^4) (1.000000000 + 2.000000000 l)^{2.0} \right)^{1/2} f^A \right) / \left( (1.000000000 + 2.000000000 l)^{2.0} (0.5 + l)^2 (-f^A + 1000.1) \sqrt{1 + \frac{f^A}{-f^A + 1000.1}} \right) \quad (23)$$

> plot((23), l=0 .. 4, labels=[l, ρ + p<sub>r</sub>], labeldirections=[HORIZONTAL, VERTICAL], color=[black], linestyle=[solid], title=[NEC - l])



**PLOT of V, T vs l :**

> V

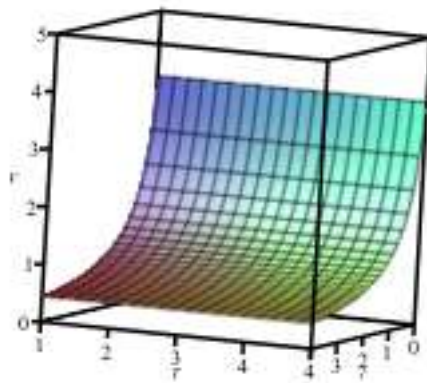
$$= \frac{1}{\left(\frac{r_0 + l}{r_0}\right)^{4n} (r_0 + l)^2 r_0^2} \left( \left( -4 r_0^2 \left(n - \frac{1}{2}\right) n (r_0 + l)^2 \left(\frac{r_0 + l}{r_0}\right)^{2n} + 3 \left(n - \frac{2}{3}\right) r_0^4 n^3 \left(\frac{r_0 + l}{r_0}\right)^{4n} + (r_0 + l)^4 \right) \left(\frac{r_0 + l}{r_0}\right)^{4n} \right)^{1/2}, T =$$

$$- \frac{1}{\sqrt{-f^A +_{CI}}} \left( -CI^{3/4} \sqrt{1 - \frac{f^2}{\sqrt{-CI}}} \sqrt{1 + \frac{f^2}{\sqrt{-CI}}} \left( \text{EllipticF}\left(\frac{l}{-CI^{1/4}}, I\right) \right) \right)$$

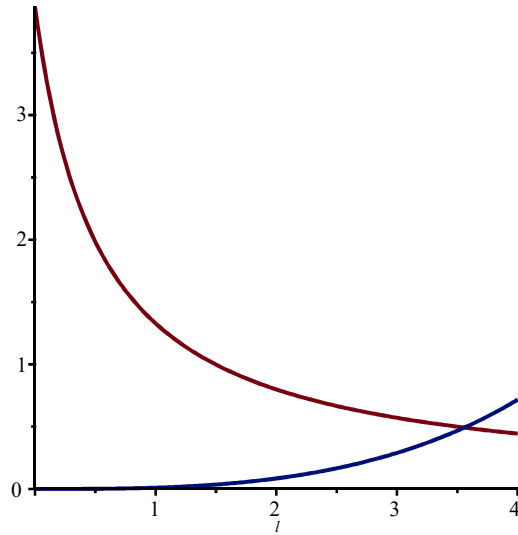
$$\begin{aligned}
 & - \text{EllipticE}\left(\frac{l}{-CI^{1/4}}, I\right) \Bigg) \\
 & = \frac{1}{\left(\frac{r_0 + l}{r_0}\right)^{4n} (r_0 + l)^2 r_0^2} \left( \left( -4 r_0^2 \left(n - \frac{1}{2}\right) n (r_0 + l)^2 \left(\frac{r_0 + l}{r_0}\right)^{2n} + 3 \left(n - \frac{2}{3}\right) r_0^4 n^3 \left(\frac{r_0 + l}{r_0}\right)^{4n} + (r_0 + l)^4 \right) \left(\frac{r_0 + l}{r_0}\right)^{4n} \right)^{1/2}, T = \\
 & - \frac{1}{\sqrt{-l^4 + -CI}} \left( -CI^{3/4} \sqrt{1 - \frac{l^2}{\sqrt{-CI}}} \sqrt{1 + \frac{l^2}{\sqrt{-CI}}} \left( \text{EllipticF}\left(\frac{l}{-CI^{1/4}}, I\right) - \text{EllipticE}\left(\frac{l}{-CI^{1/4}}, I\right) \right) \right)
 \end{aligned}
 \tag{24}$$

$$\begin{aligned}
 & \text{op( eval( [(24)], [-CI = 1000.1, n = 0.5, r[0] = 0.5] ) )} \\
 & = \frac{1}{(1.000000000 + 2.000000000 l)^{2.0} (0.5 + l)^2} \left( 4.000000000 \left( -0.003906250002 (1.000000000 + 2.000000000 l)^{2.0} + (0.5 + l)^4 \right) (1.000000000 + 2.000000000 l)^{2.0} \right)^{1/2}, T = \\
 & - \frac{1}{\sqrt{-l^4 + 1000.1}} \left( 177.8412779 \sqrt{1 - 0.03162119558 l^2} \sqrt{1 + 0.03162119558 l^2} \right. \\
 & \left. (\text{EllipticF}(0.1778234956 l, I) - \text{EllipticE}(0.1778234956 l, I)) \right)
 \end{aligned}
 \tag{25}$$

> smartplot3d[l, T, V]([ (25) ])



$$\begin{aligned}
 &> \text{plot} \left( \left[ \frac{1}{(1.0000000000 + 2.0000000000 \, l)^{2.0} (0.5 + l)^2} \left( 4.0000000000 \left( (-0.003906250002 (1.0000000000 \right. \right. \right. \right. \\
 &\quad \left. \left. \left. + 2.0000000000 \, l)^{2.0} + (0.5 + l)^4 \right) (1.0000000000 + 2.0000000000 \, l)^{2.0} \right)^{1/2} \right), \\
 &\quad - \frac{1}{\sqrt{-l^4 + 1000.1}} \left( 177.8412779 \sqrt{1 - 0.03162119558 \, l^2} \sqrt{1 + 0.03162119558 \, l^2} \right. \\
 &\quad \left. \left( \text{EllipticF}(0.1778234956 \, l, 1) - \text{EllipticE}(0.1778234956 \, l, 1) \right) \right) \right], l = 0..4 \Big)
 \end{aligned}$$



**Perturbation in delta  $r$  :**

$$> \frac{d^2}{dl^2} f(l) + 2 \cdot \frac{\frac{d}{dl} (r(l))}{r(l)} \cdot \left( \frac{d}{dl} f(l) \right) + \frac{\frac{d^2}{dl^2} r(l)}{r(l)} \cdot (f(l)) = 0$$

$$\frac{d^2}{dl^2} f(l) + \frac{2 \left( \frac{d}{dl} r(l) \right) \left( \frac{d}{dl} f(l) \right)}{r(l)} + \frac{\left( \frac{d^2}{dl^2} r(l) \right) f(l)}{r(l)} = 0$$

**(26)**

$$> \text{eval} \left( (26), \left[ r(l) = r_0 \cdot \left( 1 + \frac{l}{r_0} \right)^n \right] \right)$$

$$\frac{d^2}{dl^2} f(l) + \frac{2n \left( \frac{d}{dl} f(l) \right)}{\left( 1 + \frac{l}{r_0} \right) r_0} + \frac{\left( \frac{\left( 1 + \frac{l}{r_0} \right)^n n^2}{r_0 \left( 1 + \frac{l}{r_0} \right)^2} - \frac{\left( 1 + \frac{l}{r_0} \right)^n n}{\left( 1 + \frac{l}{r_0} \right)^2 r_0} \right) f(l)}{r_0 \left( 1 + \frac{l}{r_0} \right)^n} = 0 \quad (27)$$

> simplify( (27), 'size' )

$$\frac{2n(r_0 + l) \left( \frac{d}{dl} f(l) \right) + (n^2 - n) f(l) + (r_0 + l)^2 \left( \frac{d^2}{dl^2} f(l) \right)}{(r_0 + l)^2} = 0 \quad (28)$$

> dsolve( (28), {f(l)} )

$$f(l) = \_C1 (r_0 + l)^{-n} + \_C2 (r_0 + l)^{-n+1} \quad (29)$$

>

>

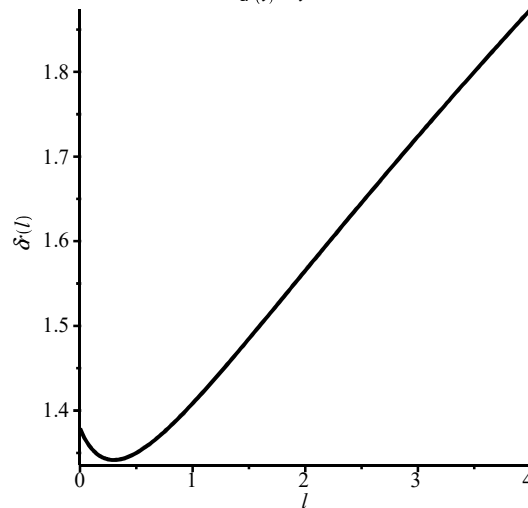
$$\_C1 (r_0 + l)^{-n} + \_C2 (r_0 + l)^{-n+1}$$

$$\_C1 (r_0 + l)^{-n} + \_C2 (r_0 + l)^{-n+1} \quad (30)$$

> eval( (30), [\_C1=0.6, \_C2=0.75, n=0.5, r[0]=0.5] )

$$\frac{0.6}{(0.5 + l)^{0.5}} + 0.75 (0.5 + l)^{0.5} \quad (31)$$

> plot( (31), l=0 .. 4, labels=[1,  $\delta r(1)$ ], labeldirections=[HORIZONTAL, VERTICAL], color=[black], linestyle=[solid], title=[ $\delta r(1) - l$ ])



>

>

> **Perturbation in delta t :**

>

>

$$\begin{aligned}
& \rightarrow \frac{d}{dl} V(l) \cdot \left( 1 + \left( \frac{d}{dl} T(l) \right)^2 \right) \cdot \left( 2 + \left( \frac{d}{dl} T(l) \right)^2 \right) \cdot f(l) + V(l) \cdot \left( \frac{d}{dl} T(l) \right)^3 \cdot \frac{d}{dl} f(l) = 0 \\
& \left( \frac{d}{dl} V(l) \right) \left( 1 + \left( \frac{d}{dl} T(l) \right)^2 \right) \left( 2 + \left( \frac{d}{dl} T(l) \right)^2 \right) f(l) + V(l) \left( \frac{d}{dl} T(l) \right)^3 \left( \frac{d}{dl} f(l) \right) \\
& = 0
\end{aligned} \tag{32}$$

$$\begin{aligned}
& \rightarrow eval \left( (32), \left[ V(l) \right. \right. \\
& = \frac{1}{\left( \frac{r_0 + l}{r_0} \right)^{4n} (r_0 + l)^2 r_0^2} \left( \left( -4 r_0^2 \left( n - \frac{1}{2} \right) n (r_0 + l)^2 \left( \frac{r_0 + l}{r_0} \right)^{2n} \right. \right. \\
& + 3 \left( n - \frac{2}{3} \right) r_0^4 n^3 \left( \frac{r_0 + l}{r_0} \right)^{4n} + (r_0 + l)^4 \left. \right) \left( \frac{r_0 + l}{r_0} \right)^{4n} \Big)^{1/2}, \frac{d}{dl} T(l) \\
& = \frac{l^2}{\sqrt{-l^A + \_CI}} \Bigg) \\
& \left( \frac{1}{2} \left( \left( -8 r_0^2 \left( n - \frac{1}{2} \right) n (r_0 + l) \left( \frac{r_0 + l}{r_0} \right)^{2n} - 8 r_0^2 \left( n - \frac{1}{2} \right) n^2 (r_0 + l) \left( \frac{r_0 + l}{r_0} \right)^{2n} \right. \right. \right. \\
& + \frac{12 \left( n - \frac{2}{3} \right) r_0^4 n^4 \left( \frac{r_0 + l}{r_0} \right)^{4n}}{r_0 + l} + 4 (r_0 + l)^3 \left. \right) \left( \frac{r_0 + l}{r_0} \right)^{4n} + \frac{1}{r_0 + l} \left( 4 \left( -4 \right. \right. \\
& \left. \left. r_0^2 \left( n - \frac{1}{2} \right) n (r_0 + l)^2 \left( \frac{r_0 + l}{r_0} \right)^{2n} + 3 \left( n - \frac{2}{3} \right) r_0^4 n^3 \left( \frac{r_0 + l}{r_0} \right)^{4n} + (r_0 + l)^4 \right) \right. \\
& \left. \right) \tag{33}
\end{aligned}$$

$$\left(\left(\frac{r_0+l}{r_0}\right)^{4n}n\right)\Bigg)\Bigg/\Bigg/$$

$$\left(\left(\left(\left(-4r_0^2\left(n-\frac{1}{2}\right)n\left(r_0+l\right)^2\left(\frac{r_0+l}{r_0}\right)^{2n}+3\left(n-\frac{2}{3}\right)r_0^4n^3\left(\frac{r_0+l}{r_0}\right)^{4n}\right.\right.\right.\right.$$

$$\left.\left.\left.\left.+ \left(r_0+l\right)^4\right)\left(\frac{r_0+l}{r_0}\right)^{4n}\right)^{1/2}\left(\frac{r_0+l}{r_0}\right)^{4n}\left(r_0+l\right)^2r_0^2\right)\right)$$

$$-\frac{1}{\left(\frac{r_0+l}{r_0}\right)^{4n}\left(r_0+l\right)^3r_0^2}\left(4\left(\left(\left(-4r_0^2\left(n-\frac{1}{2}\right)n\left(r_0+l\right)^2\left(\frac{r_0+l}{r_0}\right)^{2n}\right.\right.\right.\right.$$

$$\left.\left.\left.\left.+3\left(n-\frac{2}{3}\right)r_0^4n^3\left(\frac{r_0+l}{r_0}\right)^{4n}+\left(r_0+l\right)^4\right)\left(\frac{r_0+l}{r_0}\right)^{4n}\right)^{1/2}n\right)\right)$$

$$-\frac{1}{\left(\frac{r_0+l}{r_0}\right)^{4n}\left(r_0+l\right)^3r_0^2}\left(2\left(\left(\left(-4r_0^2\left(n-\frac{1}{2}\right)n\left(r_0+l\right)^2\left(\frac{r_0+l}{r_0}\right)^{2n}\right.\right.\right.\right.$$

$$\left.\left.\left.\left.+3\left(n-\frac{2}{3}\right)r_0^4n^3\left(\frac{r_0+l}{r_0}\right)^{4n}+\left(r_0+l\right)^4\right)\left(\frac{r_0+l}{r_0}\right)^{4n}\right)^{1/2}\right)\right)\left(1\right.$$

$$\left.+\frac{f^A}{-f^A+_CI}\right)\left(2+\frac{f^A}{-f^A+_CI}\right)f^{(l)}$$

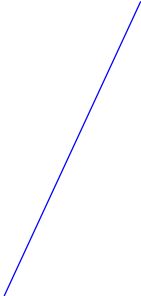
$$+ \frac{1}{\left(\frac{r_0+l}{r_0}\right)^{4n} (r_0+l)^2 r_0^2 (-l^4 + Cl)^{3/2}} \left( \left( \left( -4 r_0^2 \left(n - \frac{1}{2}\right) n (r_0 + l)^2 \left(\frac{r_0+l}{r_0}\right)^{2n} + 3 \left(n - \frac{2}{3}\right) r_0^4 n^3 \left(\frac{r_0+l}{r_0}\right)^{4n} + (r_0+l)^4 \right) \left(\frac{r_0+l}{r_0}\right)^{4n} \right)^{1/2} f^6 \left( \frac{d}{dl} f(l) \right) \right) = 0$$

> isolate( (33), diff(f(l), l) )

$$\begin{aligned} \frac{d}{dl} f(l) = - \left( \frac{1}{2} \left( \left( -8 r_0^2 \left(n - \frac{1}{2}\right) n (r_0+l) \left(\frac{r_0+l}{r_0}\right)^{2n} - 8 r_0^2 \left(n - \frac{1}{2}\right) n^2 (r_0 + l) \left(\frac{r_0+l}{r_0}\right)^{2n} + \frac{12 \left(n - \frac{2}{3}\right) r_0^4 n^4 \left(\frac{r_0+l}{r_0}\right)^{4n}}{r_0+l} + 4 (r_0+l)^3 \right) \left(\frac{r_0+l}{r_0}\right)^{4n} \right. \right. \\ \left. \left. + \frac{1}{r_0+l} \left( 4 \left( -4 r_0^2 \left(n - \frac{1}{2}\right) n (r_0+l)^2 \left(\frac{r_0+l}{r_0}\right)^{2n} + 3 \left(n - \frac{2}{3}\right) r_0^4 n^3 \left(\frac{r_0+l}{r_0}\right)^{4n} + (r_0+l)^4 \right) \left(\frac{r_0+l}{r_0}\right)^{4n} n \right) \right) \right) / \end{aligned} \quad (34)$$

$$\left( \left( \left( -4 r_0^2 \left(n - \frac{1}{2}\right) n (r_0+l)^2 \left(\frac{r_0+l}{r_0}\right)^{2n} + 3 \left(n - \frac{2}{3}\right) r_0^4 n^3 \left(\frac{r_0+l}{r_0}\right)^{4n} \right. \right. \right.$$

$$\begin{aligned}
& + (r_0 + l)^4 \left( \left( \frac{r_0 + l}{r_0} \right)^{4n} \right)^{1/2} \left( \left( \frac{r_0 + l}{r_0} \right)^{4n} (r_0 + l)^2 r_0^2 \right) \\
& - \frac{1}{\left( \frac{r_0 + l}{r_0} \right)^{4n} (r_0 + l)^3 r_0^2} \left( 4 \left( \left( -4 r_0^2 \left( n - \frac{1}{2} \right) n (r_0 + l)^2 \left( \frac{r_0 + l}{r_0} \right)^{2n} \right. \right. \right. \\
& + 3 \left( n - \frac{2}{3} \right) r_0^4 n^3 \left( \frac{r_0 + l}{r_0} \right)^{4n} + (r_0 + l)^4 \left. \left. \left( \frac{r_0 + l}{r_0} \right)^{4n} \right)^{1/2} n \right) \\
& - \frac{1}{\left( \frac{r_0 + l}{r_0} \right)^{4n} (r_0 + l)^3 r_0^2} \left( 2 \left( \left( -4 r_0^2 \left( n - \frac{1}{2} \right) n (r_0 + l)^2 \left( \frac{r_0 + l}{r_0} \right)^{2n} \right. \right. \right. \\
& + 3 \left( n - \frac{2}{3} \right) r_0^4 n^3 \left( \frac{r_0 + l}{r_0} \right)^{4n} + (r_0 + l)^4 \left. \left. \left( \frac{r_0 + l}{r_0} \right)^{4n} \right)^{1/2} \right) \right) \left( 1 \right. \\
& \left. + \frac{f^A}{-f^A + \_CI} \right) \left( 2 + \frac{f^A}{-f^A + \_CI} \right) f^{(l)} \left( \frac{r_0 + l}{r_0} \right)^{4n} (r_0 + l)^2 r_0^2 (-f^A + \_CI)^{3/2} \Bigg)
\end{aligned}$$



$$\left( \left( \left( \left( -4 r_0^2 \left( n - \frac{1}{2} \right) n (r_0 + l)^2 \left( \frac{r_0 + l}{r_0} \right)^{2n} + 3 \left( n - \frac{2}{3} \right) r_0^4 n^3 \left( \frac{r_0 + l}{r_0} \right)^{4n} \right. \right. \right. \right.
\right.$$

$$+ (r_0 + l)^4 \left( \left( \frac{r_0 + l}{r_0} \right)^{4n} \right)^{1/2} l^6 \Bigg)$$

> simplify( (33), 'size' )

$$\begin{aligned} & \left( 2 \left( -2 \left( n - \frac{1}{2} \right) (r_0 + l)^2 n \left( l^6 (l^4 - \_CI)^2 (r_0 + l) \left( \frac{d}{dl} f(l) \right) + (n + 1) f(l) (l^4 \right. \right. \quad (35) \\ & \quad \left. \left. - 2 \_CI) (-l^4 + \_CI)^{3/2} \_CI \right) r_0^2 \left( \frac{r_0 + l}{r_0} \right)^{2n} + 3 n^3 \left( n - \frac{2}{3} \right) r_0^4 \left( \frac{1}{2} l^6 (l^4 \right. \right. \\ & \quad \left. \left. - \_CI)^2 (r_0 + l) \left( \frac{d}{dl} f(l) \right) + f(l) (l^4 - 2 \_CI) (-l^4 + \_CI)^{3/2} \_CI \right) \left( \frac{r_0 + l}{r_0} \right)^{4n} \right. \\ & \quad \left. + \left( \frac{1}{2} l^6 (l^4 - \_CI)^2 (r_0 + l) \left( \frac{d}{dl} f(l) \right) + f(l) n (l^4 - 2 \_CI) (-l^4 + \_CI)^3 \right. \right. \\ & \quad \left. \left. ^{1/2} \_CI \right) (r_0 + l)^4 \right) \Bigg) / \\ & \left( \left( \left( \left( -4 r_0^2 \left( n - \frac{1}{2} \right) n (r_0 + l)^2 \left( \frac{r_0 + l}{r_0} \right)^{2n} + 3 \left( n - \frac{2}{3} \right) r_0^4 n^3 \left( \frac{r_0 + l}{r_0} \right)^{4n} \right. \right. \right. \right. \\ & \quad \left. \left. + (r_0 + l)^4 \right) \left( \frac{r_0 + l}{r_0} \right)^{4n} \right)^{1/2} (r_0 + l)^3 r_0^2 (-l^4 + \_CI)^{7/2} \Bigg) = 0 \end{aligned}$$

> isolate( (35), diff( f(l), l) )

$$\begin{aligned} \frac{d}{dl} f(l) = & \left( 2 r_0^4 n \left( \frac{r_0 + l}{r_0} \right)^{2n} f(l) (-l^4 + \_CI)^{3/2} \_CI^2 - 4 r_0^4 n^3 \left( \frac{r_0 + l}{r_0} \right)^{2n} f(l) (-l^4 + \_CI)^3 \right. \quad (36) \\ & \left. ^{1/2} \_CI^2 - 2 r_0^4 n^2 \left( \frac{r_0 + l}{r_0} \right)^{2n} f(l) (-l^4 + \_CI)^{3/2} \_CI^2 + 6 \right. \\ & \left. r_0^4 n^4 \left( \frac{r_0 + l}{r_0} \right)^{4n} f(l) (-l^4 + \_CI)^{3/2} \_CI^2 - 4 \_CI^2 f(l) (-l^4 + \_CI)^3 \right. \\ & \left. ^{1/2} \left( \frac{r_0 + l}{r_0} \right)^{4n} n^3 r_0^4 - 6 \_CI f(l) (-l^4 + \_CI)^{3/2} l^6 n r_0^2 - 4 \_CI f(l) (-l^4 \right. \\ & \quad \left. + \_CI)^{3/2} l^5 n r_0^3 - \_CI f(l) (-l^4 + \_CI)^{3/2} l^4 n r_0^4 + 4 r_0^3 n \left( \frac{r_0 + l}{r_0} \right)^{2n} l f(l) ( \right. \\ & \quad \left. - l^4 + \_CI)^{3/2} \_CI^2 - r_0^4 n \left( \frac{r_0 + l}{r_0} \right)^{2n} f(l) (-l^4 + \_CI)^{3/2} \_CI l^4 - 3 \right. \end{aligned}$$

$$\begin{aligned}
& r_0^4 n^4 \left( \frac{r_0 + l}{r_0} \right)^{4n} f(l) (-l^A + \_CI)^{3/2} \_CI l^A + 2 \_CI f(l) (-l^A + \_CI)^3 \\
& {}^{1/2} \left( \frac{r_0 + l}{r_0} \right)^{4n} l^A n^3 r_0^4 - r_0^2 n \left( \frac{r_0 + l}{r_0} \right)^{2n} l^6 f(l) (-l^A + \_CI)^{3/2} \_CI + 2 \\
& r_0^2 n \left( \frac{r_0 + l}{r_0} \right)^{2n} l^2 f(l) (-l^A + \_CI)^{3/2} \_CI l^2 + 4 r_0^3 n^3 \left( \frac{r_0 + l}{r_0} \right)^{2n} l^5 f(l) (-l^A \\
& + \_CI)^{3/2} \_CI + 2 r_0^3 n^2 \left( \frac{r_0 + l}{r_0} \right)^{2n} l^5 f(l) (-l^A + \_CI)^{3/2} \_CI - 8 \\
& r_0^3 n^3 \left( \frac{r_0 + l}{r_0} \right)^{2n} l f(l) (-l^A + \_CI)^{3/2} \_CI l^2 - 4 r_0^3 n^2 \left( \frac{r_0 + l}{r_0} \right)^{2n} l f(l) (-l^A \\
& + \_CI)^{3/2} \_CI l^2 + 2 r_0^4 n^3 \left( \frac{r_0 + l}{r_0} \right)^{2n} f(l) (-l^A + \_CI)^{3/2} \_CI l^A + \\
& r_0^4 n^2 \left( \frac{r_0 + l}{r_0} \right)^{2n} f(l) (-l^A + \_CI)^{3/2} \_CI l^A + 2 r_0^2 n^3 \left( \frac{r_0 + l}{r_0} \right)^{2n} l^6 f(l) (-l^A \\
& + \_CI)^{3/2} \_CI + r_0^2 n^2 \left( \frac{r_0 + l}{r_0} \right)^{2n} l^6 f(l) (-l^A + \_CI)^{3/2} \_CI - 4 \\
& r_0^2 n^3 \left( \frac{r_0 + l}{r_0} \right)^{2n} l^2 f(l) (-l^A + \_CI)^{3/2} \_CI l^2 - 2 r_0^2 n^2 \left( \frac{r_0 + l}{r_0} \right)^{2n} l^2 f(l) (-l^A \\
& + \_CI)^{3/2} \_CI l^2 - 2 r_0^3 n \left( \frac{r_0 + l}{r_0} \right)^{2n} l^5 f(l) (-l^A + \_CI)^{3/2} \_CI - 4 f(l) (-l^A \\
& + \_CI)^{3/2} \_CI l^7 r_0 n + 8 f(l) (-l^A + \_CI)^{3/2} \_CI l^2 l^3 r_0 n + 12 \_CI l^2 f(l) (-l^A \\
& + \_CI)^{3/2} l^2 n r_0^2 + 8 \_CI l^2 f(l) (-l^A + \_CI)^{3/2} l n r_0^3 + 2 \_CI l^2 f(l) (-l^A + \_CI)^{3/2} n \\
& r_0^4 + 2 f(l) (-l^A + \_CI)^{3/2} \_CI l^2 l^A n - f(l) (-l^A + \_CI)^{3/2} \_CI l^8 n \Big) \Bigg/ \left( \frac{5}{2} l^{18} r_0 \right. \\
& \left. + 5 l^{17} r_0^2 + 5 l^{16} r_0^3 - \_CI l^{15} + \frac{1}{2} \_CI l^2 l^{11} - 6 r_0^3 n \left( \frac{r_0 + l}{r_0} \right)^{2n} l^{12} \_CI + 3 \right. \\
& \left. r_0^3 n \left( \frac{r_0 + l}{r_0} \right)^{2n} l^8 \_CI l^2 - 6 r_0^4 n \left( \frac{r_0 + l}{r_0} \right)^{2n} l^{11} \_CI + 3 r_0^4 n \left( \frac{r_0 + l}{r_0} \right)^{2n} l^7 \_CI l^2 - 2 \right.
\end{aligned}$$

$$\begin{aligned}
& r_0^5 n \left( \frac{r_0 + l}{r_0} \right)^{2n} \_CI l^{10} + r_0^5 n \left( \frac{r_0 + l}{r_0} \right)^{2n} \_CI^2 l^6 + 12 r_0^3 n^2 \left( \frac{r_0 + l}{r_0} \right)^{2n} l^{12} \_CI - 6 \\
& r_0^3 n^2 \left( \frac{r_0 + l}{r_0} \right)^{2n} l^8 \_CI^2 + 12 r_0^4 n^2 \left( \frac{r_0 + l}{r_0} \right)^{2n} l^{11} \_CI - 6 r_0^4 n^2 \left( \frac{r_0 + l}{r_0} \right)^{2n} l^7 \_CI^2 \\
& + 4 r_0^5 n^2 \left( \frac{r_0 + l}{r_0} \right)^{2n} \_CI l^{10} - 2 r_0^5 n^2 \left( \frac{r_0 + l}{r_0} \right)^{2n} \_CI^2 l^6 + r_0^2 n \left( \frac{r_0 + l}{r_0} \right)^{2n} l^9 \_CI^2 \\
& - 2 r_0^2 n \left( \frac{r_0 + l}{r_0} \right)^{2n} l^{13} \_CI - 3 r_0^4 n^4 \left( \frac{r_0 + l}{r_0} \right)^{4n} \_CI l^{11} - 3 \\
& r_0^5 n^4 \left( \frac{r_0 + l}{r_0} \right)^{4n} \_CI l^{10} + \frac{3}{2} r_0^4 n^4 \left( \frac{r_0 + l}{r_0} \right)^{4n} \_CI^2 l^7 + \frac{3}{2} \\
& r_0^5 n^4 \left( \frac{r_0 + l}{r_0} \right)^{4n} \_CI^2 l^6 + 2 r_0^4 n^3 \left( \frac{r_0 + l}{r_0} \right)^{4n} \_CI l^{11} + 2 r_0^5 n^3 \left( \frac{r_0 + l}{r_0} \right)^{4n} \_CI l^{10} \\
& - r_0^4 n^3 \left( \frac{r_0 + l}{r_0} \right)^{4n} \_CI^2 l^7 - r_0^5 n^3 \left( \frac{r_0 + l}{r_0} \right)^{4n} \_CI^2 l^6 + 4 r_0^2 n^2 \left( \frac{r_0 + l}{r_0} \right)^{2n} l^{13} \_CI \\
& - 2 r_0^2 n^2 \left( \frac{r_0 + l}{r_0} \right)^{2n} l^9 \_CI^2 - 5 \_CI l^{11} r_0^4 - \_CI l^{10} r_0^5 + \frac{5}{2} \_CI^2 l^7 r_0^4 + \frac{1}{2} \_CI^2 l^6 \\
& r_0^5 - 5 \_CI l^{14} r_0 - 10 \_CI l^{13} r_0^2 - 10 \_CI l^{12} r_0^3 + \frac{5}{2} \_CI^2 l^{10} r_0 + 5 \_CI^2 l^9 r_0^2 \\
& + 5 \_CI^2 l^8 r_0^3 + \frac{1}{2} l^{19} - 6 r_0^3 n^2 \left( \frac{r_0 + l}{r_0} \right)^{2n} l^{16} - 6 r_0^4 n^2 \left( \frac{r_0 + l}{r_0} \right)^{2n} l^{15} - 2 \\
& r_0^5 n^2 \left( \frac{r_0 + l}{r_0} \right)^{2n} l^{14} + r_0^2 n \left( \frac{r_0 + l}{r_0} \right)^{2n} l^{17} + 3 r_0^4 n \left( \frac{r_0 + l}{r_0} \right)^{2n} l^{15} + 3 \\
& r_0^3 n \left( \frac{r_0 + l}{r_0} \right)^{2n} l^{16} - 2 r_0^2 n^2 \left( \frac{r_0 + l}{r_0} \right)^{2n} l^{17} + r_0^5 n \left( \frac{r_0 + l}{r_0} \right)^{2n} l^{14} + \frac{3}{2} \\
& r_0^4 n^4 \left( \frac{r_0 + l}{r_0} \right)^{4n} l^{15} + \frac{3}{2} r_0^5 n^4 \left( \frac{r_0 + l}{r_0} \right)^{4n} l^{14} - r_0^4 n^3 \left( \frac{r_0 + l}{r_0} \right)^{4n} l^{15} - \\
& r_0^5 n^3 \left( \frac{r_0 + l}{r_0} \right)^{4n} l^{14} + \frac{5}{2} l^{15} r_0^4 + \frac{1}{2} l^{14} r_0^5 \Big)
\end{aligned}$$

> simplify( (36), 'size' )

$$\frac{d}{dl} f(l) = - \left( 2 \left( -2 \left( n - \frac{1}{2} \right) (r_0 + l)^2 (n+1) r_0^2 \left( \frac{r_0 + l}{r_0} \right)^{2n} + 3 n^2 \left( n - \frac{2}{3} \right) r_0^4 \left( \frac{r_0 + l}{r_0} \right)^{4n} + (r_0 + l)^4 \right) f(l) n (l^A - 2\_CI) (-l^A +\_CI)^{3/2} \_CI \right) / \left( (r_0 + l) l^6 (l^A -\_CI)^2 \left( -4 r_0^2 \left( n - \frac{1}{2} \right) n (r_0 + l)^2 \left( \frac{r_0 + l}{r_0} \right)^{2n} + 3 \left( n - \frac{2}{3} \right) r_0^4 n^3 \left( \frac{r_0 + l}{r_0} \right)^{4n} + (r_0 + l)^4 \right) \right) \quad (37)$$

$$\begin{aligned} & \frac{d}{dl} f(l) \\ & \frac{d}{f(l)} f(l) = - \left( 2 \left( -2 \left( n - \frac{1}{2} \right) (r_0 + l)^2 (n+1) r_0^2 \left( \frac{r_0 + l}{r_0} \right)^{2n} + 3 n^2 \left( n - \frac{2}{3} \right) r_0^4 \left( \frac{r_0 + l}{r_0} \right)^{4n} + (r_0 + l)^4 \right) f(l) n (l^A - 2\_CI) (-l^A +\_CI)^{3/2} \_CI \right) / \left( (r_0 + l) l^6 (l^A -\_CI)^2 \left( -4 r_0^2 \left( n - \frac{1}{2} \right) n (r_0 + l)^2 \left( \frac{r_0 + l}{r_0} \right)^{2n} + 3 \left( n - \frac{2}{3} \right) r_0^4 n^3 \left( \frac{r_0 + l}{r_0} \right)^{4n} + (r_0 + l)^4 \right) \right) \cdot \left( \frac{1}{f(l)} \right) \end{aligned}$$

$$\frac{d}{dl} f(l) = - \left( 2 \left( -2 \left( n - \frac{1}{2} \right) (r_0 + l)^2 (n+1) r_0^2 \left( \frac{r_0 + l}{r_0} \right)^{2n} + 3 n^2 \left( n - \frac{2}{3} \right) r_0^4 \left( \frac{r_0 + l}{r_0} \right)^{4n} + (r_0 + l)^4 \right) n (l^A - 2\_CI) (-l^A +\_CI)^{3/2} \_CI \right) / \left( (r_0 + l) l^6 (l^A -\_CI)^2 \left( -4 r_0^2 \left( n - \frac{1}{2} \right) n (r_0 + l)^2 \left( \frac{r_0 + l}{r_0} \right)^{2n} + 3 \left( n - \frac{2}{3} \right) r_0^4 n^3 \left( \frac{r_0 + l}{r_0} \right)^{4n} + (r_0 + l)^4 \right) \right) \quad (38)$$

> simplify( (38), 'size' )

$$\frac{d}{f(l)} f(l) = - \left( 2 n (l^A - 2\_CI) \_CI \left( -2 \left( n - \frac{1}{2} \right) (r_0 + l)^2 (n+1) r_0^2 \left( \frac{r_0 + l}{r_0} \right)^{2n} + 3 n^2 \left( n - \frac{2}{3} \right) r_0^4 \left( \frac{r_0 + l}{r_0} \right)^{4n} + (r_0 + l)^4 \right) \right) / \left( (r_0 + l) \sqrt{-l^A +\_CI} \left( -4 r_0^2 \left( n \right. \right. \right) \quad (39)$$

$$-\frac{1}{2} \Big) n (r_0 + l)^2 \left( \frac{r_0 + l}{r_0} \right)^{2n} + 3 \left( n - \frac{2}{3} \right) r_0^4 n^3 \left( \frac{r_0 + l}{r_0} \right)^{4n} + (r_0 + l)^4 \Big) l^6 \Big)$$

> latex( '(39)' )

```
{\frac {{\frac {\rm d}{{\rm d}l}}f \left( l \right) }{f \left( l \right) }}=-2\,{\frac {n \left( l^4-2\,{\it \_C1} \right) }{{\it \_C1}}}{\left( r_{0}+l \right) \sqrt {-l^4+{\it \_C1}}}{l}^{\frac {1}{6}}
\left( -2\,{\left( n-1/2 \right) \left( r_{0}+l \right) ^2} \right.
\left. \left( n+1 \right) {r_{0}}^2 \left( {\frac {r_{0}+l}{r_{0}}} \right)^{2n} \right.
\left. \left( n-2/3 \right) r_0^4 n^3 \left( \frac{r_0+l}{r_0} \right)^{4n} + (r_0+l)^4 \right) l^6
\left( {\frac {r_{0}+l}{r_0}} \right)^{4n} + (r_0+l)^4 \Big) l^6 \Big)
```

$$\begin{aligned} & - \left( 2n (l^4 - 2\_C1) \_C1 \left( -2 \left( n - \frac{1}{2} \right) (r_0 + l)^2 (n+1) r_0^2 \left( \frac{r_0 + l}{r_0} \right)^{2n} + 3n^2 \left( n - \frac{2}{3} \right) \right. \right. \\ & \quad \left. \left. r_0^4 \left( \frac{r_0 + l}{r_0} \right)^{4n} + (r_0 + l)^4 \right) \right) / \left( (r_0 + l) \sqrt{-l^4 + \_C1} \left( -4r_0^2 \left( n - \frac{1}{2} \right) n (r_0 \right. \right. \\ & \quad \left. \left. + l)^2 \left( \frac{r_0 + l}{r_0} \right)^{2n} + 3 \left( n - \frac{2}{3} \right) r_0^4 n^3 \left( \frac{r_0 + l}{r_0} \right)^{4n} + (r_0 + l)^4 \right) l^6 \right) \\ & - \left( 2n (l^4 - 2\_C1) \_C1 \left( -2 \left( n - \frac{1}{2} \right) (r_0 + l)^2 (n+1) r_0^2 \left( \frac{r_0 + l}{r_0} \right)^{2n} + 3n^2 \left( n - \frac{2}{3} \right) \right. \right. \\ & \quad \left. \left. r_0^4 \left( \frac{r_0 + l}{r_0} \right)^{4n} + (r_0 + l)^4 \right) \right) / \left( (r_0 + l) \sqrt{-l^4 + \_C1} \left( -4r_0^2 \left( n - \frac{1}{2} \right) n (r_0 \right. \right. \\ & \quad \left. \left. + l)^2 \left( \frac{r_0 + l}{r_0} \right)^{2n} + 3 \left( n - \frac{2}{3} \right) r_0^4 n^3 \left( \frac{r_0 + l}{r_0} \right)^{4n} + (r_0 + l)^4 \right) l^6 \right) \end{aligned} \quad (40)$$

> eval( (40), [\_C1=0.011000, n=0.5, r[0]=0.5])

$$\begin{aligned} & - (0.0110000 (l^4 - 0.022000) (-0.007812500004 (1.000000000 + 2.000000000 l)^{2.0} + (0.5 \\ & \quad + l)^4)) / ((0.5 + l) \sqrt{-l^4 + 0.011000} (-0.003906250002 (1.000000000 \\ & \quad + 2.000000000 l)^{2.0} + (0.5 + l)^4) l^6) \end{aligned} \quad (41)$$

> int((41), l)

(42)

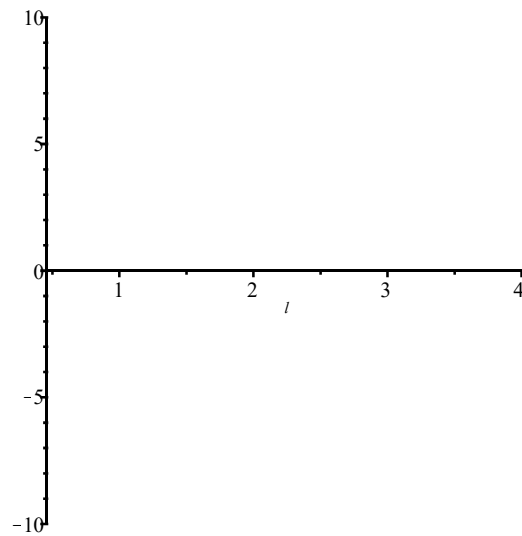
$$\begin{aligned}
& -0.02348365132 \operatorname{Iarctanh} \left( \frac{150.1168030 \operatorname{I} t^2}{\sqrt{-10000. t^4 + 110.}} - \frac{11.74246992 \operatorname{I}}{\sqrt{-10000. t^4 + 110.}} \right) \\
& - \frac{1}{\sqrt{-10000. t^4 + 110.}} \left( 0.3799668875 \sqrt{1. - 9.534625892 t^2} \sqrt{1. + 9.534625892 t^2} \right. \\
& \operatorname{EllipticPi}(3.087818954 \operatorname{I}, 0.7458196255, 1.000000000 \operatorname{I}) \\
& + 0.1256388834 \operatorname{Iarctanh} \left( \frac{0.02203263246 \operatorname{I} (-5000. t^2 + 220.)}{\sqrt{-10000. t^4 + 110.}} \right) \\
& - 0.01601188420 \operatorname{Iarctanh} \left( - \frac{103.8118615 \operatorname{I} t^2}{\sqrt{-10000. t^4 + 110.}} + \frac{2.923342021 \operatorname{I}}{\sqrt{-10000. t^4 + 110.}} \right) \\
& + \frac{1}{\sqrt{-10000. t^4 + 110.}} \left( 0.6243867036 \sqrt{1. - 9.534625892 t^2} \sqrt{1. + 9.534625892 t^2} \right. \\
& \operatorname{EllipticPi}(3.087818954 \operatorname{I}, 0.2684950651, 1.000000000 \operatorname{I}) \\
& - \frac{0.0003278380245 \sqrt{-10000. t^4 + 110.}}{t^3} \\
& + \frac{1}{\sqrt{-10000. t^4 + 110.}} \left( 0.008774494884 \sqrt{121. - 1153.689733 t^2} \right. \\
& \sqrt{121. + 1153.689733 t^2} \operatorname{EllipticF}(3.087818954 \operatorname{I}, 1. \operatorname{I}) \\
& - \frac{0.00008213333333 \sqrt{-10000. t^4 + 110.}}{t^5} \\
& - \frac{0.001826648698 \sqrt{-10000. t^4 + 110.}}{t} \\
& + \frac{1}{\sqrt{-10000. t^4 + 110.}} \left( 0.005127600430 \sqrt{121. - 1153.689733 t^2} \right. \\
& \sqrt{121. + 1153.689733 t^2} (\operatorname{EllipticF}(3.087818954 \operatorname{I}, 1. \operatorname{I}) - 1. \operatorname{EllipticE}(3.087818954 \operatorname{I}, \\
& 1. \operatorname{I})) + \frac{0.0001740444444 \sqrt{-10000. t^4 + 110.}}{t^4} \\
& - 0.06943549755 \operatorname{arctanh} \left( \frac{10.48808848}{\sqrt{-10000. t^4 + 110.}} \right) - \frac{0.003783174312 (1000. t^4 - 11.)}{t^2 \sqrt{-10000. t^4 + 110.}} \\
& - \frac{1}{\sqrt{-10000. t^4 + 110.}} \left( 3.693480793 \sqrt{1. - 9.534625892 t^2} \right. \\
& \left. \sqrt{1. + 9.534625892 t^2} \operatorname{EllipticPi}(3.087818954 \operatorname{I}, 0.4195235393, 1.000000000 \operatorname{I}) \right)
\end{aligned}$$

> simplify( (42), 'size' )

$$\begin{aligned}
& \frac{1}{\sqrt{-10000. l^4 + 110. l^5}} \left( 0.01601188420 \left( 1.0000000000 l \operatorname{arctanh} \left( \frac{1}{\sqrt{-10000. l^4 + 110. l^5}} \right. \right. \right. \\
& \quad \left. \left. \left. - 2.923342021 l + 103.8118615 l^2 \right) \right) \sqrt{-10000. l^4 + 110. l^5} \right. \\
& \quad - 1.466638843 l \operatorname{arctanh} \left( \frac{-11.74246992 l + 150.1168030 l^2}{\sqrt{-10000. l^4 + 110. l^5}} \right) \sqrt{-10000. l^4 + 110. l^5} \\
& \quad - 7.846602051 \operatorname{arctan} \left( \frac{-110.1631623 l^2 + 4.847179141}{\sqrt{-10000. l^4 + 110. l^5}} \right) \sqrt{-10000. l^4 + 110. l^5} \\
& \quad - 4.336497609 \operatorname{arctanh} \left( \frac{10.48808848}{\sqrt{-10000. l^4 + 110. l^5}} \right) \sqrt{-10000. l^4 + 110. l^5} + ( \\
& \quad - 23.73030449 \operatorname{EllipticPi}(3.087818954 l, 0.7458196255, 1.0000000000 l) l^5 \\
& \quad + 38.99520480 \operatorname{EllipticPi}(3.087818954 l, 0.2684950651, 1.0000000000 l) l^5 \\
& \quad - 230.6712156 \operatorname{EllipticPi}(3.087818954 l, 0.4195235393, 1.0000000000 l) l^5) \\
& \quad \sqrt{1. + 9.534625892 l^2} \sqrt{1. - 9.534625892 l^2} \\
& \quad + (0.8682360637 \operatorname{EllipticF}(3.087818954 l, 1. l) l^5 \\
& \quad - 0.3202371667 l^5 \operatorname{EllipticE}(3.087818954 l, 1. l) ) \\
& \quad \sqrt{121. + 1153.689733 l^2} \sqrt{121. - 1153.689733 l^2} + 1140.808087 (l \\
& \quad + 0.323853184028959) (l - 0.323853184083034) (l^2 + 0.412549175666313 l \\
& \quad + 0.266212798019144) (l^2 + 4.732728962 \cdot 10^{-11} l + 0.1048808848) (l^2 \\
& \quad - 0.619659295751131 l + 0.168902270209559) ) )
\end{aligned} \tag{43}$$

> plot((43), l=0.5 .. 4)

Warning, unable to evaluate the function to numeric values in the region; see the plotting command's help page to ensure the calling sequence is correct



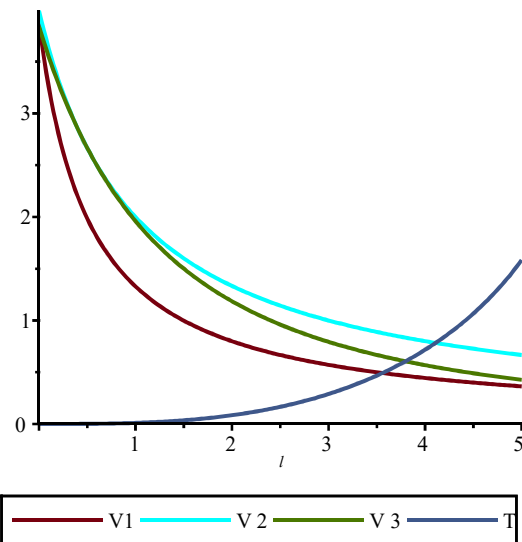
>  
>  
>  
>

> plot  $\left( \left[ \frac{1}{(1.0000000000 + 2.0000000000 l)^{2.0} (0.5 + l)^2} \left( 4.0000000000 \left( (-0.003906250002 (1.0000000000 + 2.0000000000 l)^{2.0} + (0.5 + l)^4 \right) (1.0000000000 + 2.0000000000 l)^{2.0} \right)^{1/2} \right), \right.$

$\frac{1.0000000000 \sqrt{(1 + l)^2 (-0.06250000000 (1 + l)^2 + 16 (1 + l)^4)}}{(1 + l)^4}, \frac{0.96}{(0.2 l + 0.5)^2},$

$-\frac{1}{\sqrt{-l^4 + 1000.1}} \left( 177.8412779 \sqrt{1 - 0.03162119558 l^2} \sqrt{1 + 0.03162119558 l^2} \right.$

$\left. \left. \left( \text{EllipticF}(0.1778234956 l, I) - \text{EllipticE}(0.1778234956 l, I) \right) \right) \right], l = 0 .. 5 \right)$



>