> *restart*;
> *with*(*Physics*) :

> $ds^2 := \frac{(r-2\,m)\,dt^2}{r} - \frac{r\,dr^2}{r-2\,m} - r^2\left(d\theta^2 + \sin(\theta)^2\,d\phi^2\right)$

$$ds^2 := \frac{(r-2\,m)\,dt^2}{r} - \frac{r\,dr^2}{r-2\,m} - r^2\left(d\theta^2 + \sin(\theta)^2\,d\phi^2\right) \tag{1}$$

> $ds^2 := -\,e^{\nu(r)}dt^2 + \frac{dr^2}{1-\frac{b(r)}{r}} + r^2\left(d\theta^2 + \sin(\theta)^2\,d\phi^2\right)$

$$ds^2 := -\,e^{\nu(r)}\,dt^2 + \frac{dr^2}{1-\frac{b(r)}{r}} + r^2\left(d\theta^2 + \sin(\theta)^2\,d\phi^2\right) \tag{2}$$

> *Setup*(*coordinates* = *spherical*, *signature* = \`+ + + -\`, *metric* = ds^2)

Default differentiation variables for d_, D_ and dAlembertian are: $\{X = (r, \theta, \phi, t)\}$

Systems of spacetime coordinates are: $\{X = (r, \theta, \phi, t)\}$

$$\left[coordinatesystems = \{X\},\ metric = \left\{(1,1) = \frac{1}{1-\frac{b(r)}{r}},\ (2,2) = r^2,\ (3,3) = r^2\sin(\theta)^2,\ (4,4) = -\,e^{\nu(r)}\right\},\ signature = +\,+\,+\,-\right] \tag{3}$$

> g_[ ]

$$g_{\mu,\nu} = \begin{bmatrix} \frac{r}{r-b(r)} & 0 & 0 & 0 \\ 0 & r^2 & 0 & 0 \\ 0 & 0 & r^2\sin(\theta)^2 & 0 \\ 0 & 0 & 0 & -\,e^{\nu(r)} \end{bmatrix} \tag{4}$$

> g_[~]

$$g^{\mu,\nu} = \begin{bmatrix} \frac{r-b(r)}{r} & 0 & 0 & 0 \\ 0 & \frac{1}{r^2} & 0 & 0 \\ 0 & 0 & \frac{1}{r^2\sin(\theta)^2} & 0 \\ 0 & 0 & 0 & -\,e^{-\nu(r)} \end{bmatrix} \tag{5}$$

> g_[mu, ~nu, *matrix*]

$$\delta_{\mu}^{\ \nu} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{6}$$

> $g\_[lineelement]$

$$\frac{r\,\mathbf{d}(r)^2}{r - b(r)} + r^2\,\mathbf{d}(\theta)^2 + r^2 \sin(\theta)^2\,\mathbf{d}(\phi)^2 - e^{\nu(r)}\,\mathbf{d}(t)^2 \tag{7}$$

> $Christoffel[\mu, \alpha, \beta] = convert(Christoffel[\mu, \alpha, \beta], g\_)$

$$\Gamma_{\mu,\alpha,\beta} = \frac{\partial_{\beta}(g_{\alpha,\mu})}{2} + \frac{\partial_{\alpha}(g_{\beta,\mu})}{2} - \frac{\partial_{\mu}(g_{\alpha,\beta})}{2} \tag{8}$$

> seq(Christoffel[~mu, alpha, beta, *matrix*], ~mu = [~1, ~2, ~3, ~0])

$$\Gamma^{1}_{\ \alpha,\beta} = \left[\left[\frac{\left(\frac{d}{dr} b(r)\right) r - b(r)}{2\,r\,(r - b(r))}, 0, 0, 0\right],\right.$$
$$[0, -r + b(r), 0, 0],$$
$$[0, 0, (-r + b(r)) \sin(\theta)^2, 0],$$
$$\left.\left[0, 0, 0, -\frac{(-r + b(r))\left(\frac{d}{dr}\nu(r)\right) e^{\nu(r)}}{2\,r}\right]\right], \Gamma^{2}_{\ \alpha,\beta}$$
$$= \begin{bmatrix} 0 & \frac{1}{r} & 0 & 0 \\ \frac{1}{r} & 0 & 0 & 0 \\ 0 & 0 & -\sin(\theta)\cos(\theta) & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \Gamma^{3}_{\ \alpha,\beta} = \begin{bmatrix} 0 & 0 & \frac{1}{r} & 0 \\ 0 & 0 & \frac{\cos(\theta)}{\sin(\theta)} & 0 \\ \frac{1}{r} & \frac{\cos(\theta)}{\sin(\theta)} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \Gamma^{4}_{\ \alpha,\beta}$$
$$= \begin{bmatrix} 0 & 0 & 0 & \frac{\frac{d}{dr}\nu(r)}{2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{\frac{d}{dr}\nu(r)}{2} & 0 & 0 & 0 \end{bmatrix} \tag{9}$$

> $Christoffel[nonzero]$

$$\Gamma_{\alpha,\mu,\nu} = \left\{(1,1,1) = \frac{\left(\frac{d}{dr}\, b(r)\right) r - b(r)}{2\,(r - b(r))^2},\ (1,2,2) = -r,\ (1,3,3) = -r\sin(\theta)^2,\ (1,4,4) = \frac{\left(\frac{d}{dr}\, \nu(r)\right) e^{\nu(r)}}{2},\ (2,1,2) = r,\ (2,2,1) = r,\ (2,3,3) = -r^2\sin(\theta)\cos(\theta),\ (3,1,3) = r\sin(\theta)^2,\ (3,2,3) = r^2\sin(\theta)\cos(\theta),\ (3,3,1) = r\sin(\theta)^2,\ (3,3,2) = r^2\sin(\theta)\cos(\theta),\ (4,1,4) = -\frac{\left(\frac{d}{dr}\, \nu(r)\right) e^{\nu(r)}}{2},\ (4,4,1) = -\frac{\left(\frac{d}{dr}\, \nu(r)\right) e^{\nu(r)}}{2}\right\} \tag{10}$$

> $Riemann[definition]$

$$R_{\alpha,\beta,\mu,\nu} = g_{\alpha,\lambda}\left(\partial_{\mu}\left(\Gamma^{\lambda}{}_{\beta,\nu}\right) - \partial_{\nu}\left(\Gamma^{\lambda}{}_{\beta,\mu}\right) + \Gamma^{\lambda}{}_{\upsilon,\mu}\,\Gamma^{\upsilon}{}_{\beta,\nu} - \Gamma^{\lambda}{}_{\upsilon,\nu}\,\Gamma^{\upsilon}{}_{\beta,\mu}\right) \tag{11}$$

> $Riemann[nonzero]$

$$R_{\alpha,\beta,\mu,\nu} = \left\{(1,2,1,2) = \frac{-\left(\frac{d}{dr}\, b(r)\right) r + b(r)}{-2\,r + 2\,b(r)},\ (1,2,2,1) = \frac{-\left(\frac{d}{dr}\, b(r)\right) r + b(r)}{2\,r - 2\,b(r)},\ (1,3,1,3) = \frac{\left(-\left(\frac{d}{dr}\, b(r)\right) r + b(r)\right)\sin(\theta)^2}{-2\,r + 2\,b(r)},\ (1,3,3,1) = -\frac{\left(-\left(\frac{d}{dr}\, b(r)\right) r + b(r)\right)\sin(\theta)^2}{-2\,r + 2\,b(r)},\ (1,4,1,4) = \frac{1}{4\,r\,(r - b(r))}\left(\left(2\,r\,(r - b(r))\left(\frac{d^2}{dr^2}\, \nu(r)\right) + \left(\frac{d}{dr}\, \nu(r)\right)\left(r\,(r - b(r))\left(\frac{d}{dr}\, \nu(r)\right) - \left(\frac{d}{dr}\, b(r)\right) r + b(r)\right)\right) e^{\nu(r)}\right),\ (1,4,4,1) = -\frac{1}{4\,r\,(r - b(r))}\left(\left(2\,r\,(r - b(r))\left(\frac{d^2}{dr^2}\, \nu(r)\right) + \left(\frac{d}{dr}\, \nu(r)\right)\left(r\,(r - b(r))\left(\frac{d}{dr}\, \nu(r)\right) - \left(\frac{d}{dr}\, b(r)\right) r + b(r)\right)\right) e^{\nu(r)}\right),\ (2,1,1,2) = \frac{-\left(\frac{d}{dr}\, b(r)\right) r + b(r)}{2\,r - 2\,b(r)},\ (2,1,2,1) = \frac{-\left(\frac{d}{dr}\, b(r)\right) r + b(r)}{-2\,r + 2\,b(r)},\ (2,3,2,3)\right.$$

(12)

$$= r\sin(\theta)^2 b(r),\ (2,3,3,2) = -r\sin(\theta)^2 b(r),\ (2,4,2,4) = -\frac{(-r+b(r))\left(\frac{d}{dr}\nu(r)\right)e^{\nu(r)}}{2},\ (2,4,4,2) = \frac{(-r+b(r))\left(\frac{d}{dr}\nu(r)\right)e^{\nu(r)}}{2},\ (3,1,1,3) = -\frac{\left(-\left(\frac{d}{dr}b(r)\right)r+b(r)\right)\sin(\theta)^2}{-2r+2b(r)},\ (3,1,3,1) = \frac{\left(-\left(\frac{d}{dr}b(r)\right)r+b(r)\right)\sin(\theta)^2}{-2r+2b(r)},\ (3,2,2,3) = -r\sin(\theta)^2 b(r),\ (3,2,3,2) = r\sin(\theta)^2 b(r),\ (3,4,3,4) = -\frac{\sin(\theta)^2(-r+b(r))\left(\frac{d}{dr}\nu(r)\right)e^{\nu(r)}}{2},\ (3,4,4,3) = \frac{\sin(\theta)^2(-r+b(r))\left(\frac{d}{dr}\nu(r)\right)e^{\nu(r)}}{2},\ (4,1,1,4) = -\frac{1}{4r(r-b(r))}\left(\left(2r(r-b(r))\left(\frac{d^2}{dr^2}\nu(r)\right)+\left(\frac{d}{dr}\nu(r)\right)\left(r(r-b(r))\left(\frac{d}{dr}\nu(r)\right)-\left(\frac{d}{dr}b(r)\right)r+b(r)\right)\right)e^{\nu(r)}\right),\ (4,1,4,1) = \frac{1}{4r(r-b(r))}\left(\left(2r(r-b(r))\left(\frac{d^2}{dr^2}\nu(r)\right)+\left(\frac{d}{dr}\nu(r)\right)\left(r(r-b(r))\left(\frac{d}{dr}\nu(r)\right)-\left(\frac{d}{dr}b(r)\right)r+b(r)\right)\right)e^{\nu(r)}\right),\ (4,2,2,4) = \frac{(-r+b(r))\left(\frac{d}{dr}\nu(r)\right)e^{\nu(r)}}{2},\ (4,2,4,2) = -\frac{(-r+b(r))\left(\frac{d}{dr}\nu(r)\right)e^{\nu(r)}}{2},\ (4,3,3,4) = \frac{\sin(\theta)^2(-r+b(r))\left(\frac{d}{dr}\nu(r)\right)e^{\nu(r)}}{2},\ (4,3,4,3) = -\frac{\sin(\theta)^2(-r+b(r))\left(\frac{d}{dr}\nu(r)\right)e^{\nu(r)}}{2}\Big\}$$

> *Ricci[definition]*

$$R_{\mu,\nu} = \partial_{\alpha}\left(\Gamma^{\alpha}{}_{\mu,\nu}\right) - \partial_{\nu}\left(\Gamma^{\alpha}{}_{\mu,\alpha}\right) + \Gamma^{\beta}{}_{\mu,\nu}\Gamma^{\alpha}{}_{\beta,\alpha} - \Gamma^{\beta}{}_{\mu,\alpha}\Gamma^{\alpha}{}_{\nu,\beta} \quad \textbf{(13)}$$

> *Ricci*[*nonzero*]

$$R_{\mu,\nu} = \left\{ (1,1) = \frac{1}{4\,r^2\,(r-b(r))} \left(-2\,r^2\,(r-b(r)) \left(\frac{d^2}{dr^2}\,\nu(r) \right) - r^2\,(r-b(r)) \left(\frac{d}{dr}\,\nu(r) \right)^2 + r \left(\left(\frac{d}{dr}\,b(r) \right) r - b(r) \right) \left(\frac{d}{dr}\,\nu(r) \right) + 4 \left(\frac{d}{dr}\,b(r) \right) r - 4\,b(r) \right), (2,2) = \frac{-r\,(r-b(r)) \left(\frac{d}{dr}\,\nu(r) \right) + \left(\frac{d}{dr}\,b(r) \right) r + b(r)}{2\,r}, (3,3) = -\frac{\sin(\theta)^2 \left(r\,(r-b(r)) \left(\frac{d}{dr}\,\nu(r) \right) - \left(\frac{d}{dr}\,b(r) \right) r - b(r) \right)}{2\,r}, (4,4) = \frac{1}{4\,r^2} \left(\left(2\,r\,(r-b(r)) \left(\frac{d^2}{dr^2}\,\nu(r) \right) + \left(\frac{d}{dr}\,\nu(r) \right) \left(r\,(r-b(r)) \left(\frac{d}{dr}\,\nu(r) \right) - \left(\frac{d}{dr}\,b(r) \right) r + 4\,r - 3\,b(r) \right) \right) e^{\nu(r)} \right) \right\} \tag{14}$$

> *Ricci*[*scalars*]

$$\Phi_{00} = \frac{e^{-\nu(r)}\,e^{\nu(r)} \left(r\,(r-b(r)) \left(\frac{d}{dr}\,\nu(r) \right) + \left(\frac{d}{dr}\,b(r) \right) r - b(r) \right)}{4\,r^3}, \Phi_{01} = 0, \Phi_{02} = 0, \Phi_{11} = \frac{1}{16\,r^3} \left(\left(2\,r^2\,(r-b(r)) \left(\frac{d^2}{dr^2}\,\nu(r) \right) + r^2\,(r-b(r)) \left(\frac{d}{dr}\,\nu(r) \right)^2 + \left(-\left(\frac{d}{dr}\,b(r) \right) r^2 + b(r)\,r \right) \left(\frac{d}{dr}\,\nu(r) \right) + 4\,b(r) \right) e^{-\nu(r)}\,e^{\nu(r)} \right), \Phi_{12} = 0, \Phi_{22} = \frac{e^{-\nu(r)}\,e^{\nu(r)} \left(r\,(r-b(r)) \left(\frac{d}{dr}\,\nu(r) \right) + \left(\frac{d}{dr}\,b(r) \right) r - b(r) \right)}{4\,r^3}, \Lambda = \frac{1}{48\,r^2} \left(-2\,r\,(r-b(r)) \left(\frac{d^2}{dr^2}\,\nu(r) \right) - r\,(r-b(r)) \left(\frac{d}{dr}\,\nu(r) \right)^2 + \left(\left(\frac{d}{dr}\,b(r) \right) r - 4\,r + 3\,b(r) \right) \left(\frac{d}{dr}\,\nu(r) \right) + 4\,\frac{d}{dr}\,b(r) \right) \tag{15}$$

> *Einstein*[*definition*]

$$G_{\mu,\nu} = R_{\mu,\nu} - \frac{g_{\mu,\nu}\,R_{\alpha}^{\ \alpha}}{2} \tag{16}$$

> *Einstein*[*nonzero*]

$$G_{\mu,\nu} = \left\{ (1,1) = \frac{r\,(r-b(r))\left(\frac{d}{dr}\,\nu(r)\right) - b(r)}{r^2\,(r-b(r))},\ (2,2) = \frac{1}{4\,r}\left(2\,r^2\,(r-b(r))\left(\frac{d^2}{dr^2}\,\nu(r)\right) + \left(r\,(r-b(r))\left(\frac{d}{dr}\,\nu(r)\right) - \left(\frac{d}{dr}\,b(r)\right)r + b(r)\right)\left(\left(\frac{d}{dr}\,\nu(r)\right)r + 2\right)\right),\ (3,3) = \frac{1}{4\,r}\left(\sin(\theta)^2\left(2\,r^2\,(r-b(r))\left(\frac{d^2}{dr^2}\,\nu(r)\right) + \left(r\,(r-b(r))\left(\frac{d}{dr}\,\nu(r)\right) - \left(\frac{d}{dr}\,b(r)\right)r + b(r)\right)\left(\left(\frac{d}{dr}\,\nu(r)\right)r + 2\right)\right)\right),\ (4,4) = \frac{e^{\nu(r)}\left(\frac{d}{dr}\,b(r)\right)}{r^2} \right\} \tag{17}$$

>