> *restart*;
> *with*(*Physics*) :

>

> $$-2\cdot\frac{\frac{d^2}{d l^2} r(l)}{r(l)} - \left(\frac{\frac{d}{d l} r(l)}{r(l)}\right)^2 + \frac{1}{r^2(l)} = -V(T)\cdot\left(1+\left(\frac{d}{d l}T(l)\right)^2\right)^{\frac{1}{2}}$$

$$-\frac{2\left(\frac{d^2}{dl^2}r(l)\right)}{r(l)} - \frac{\left(\frac{d}{dl}r(l)\right)^2}{r(l)^2} + \frac{1}{r(l)^2} = -V(T)\sqrt{1+\left(\frac{d}{dl}T(l)\right)^2} \quad \textbf{(1)}$$

>

>

> $$2\cdot\frac{\frac{d}{d l}\psi(l)\cdot\frac{d}{d l}r(l)}{\psi(l)\cdot r(l)} + \left(\frac{\frac{d}{d l}r(l)}{r(l)}\right)^2 - \frac{1}{r^2(l)} = \frac{V(T)}{\left(1+\left(\frac{d}{d l}T(l)\right)^2\right)^{\frac{1}{2}}}$$

$$\frac{2\left(\frac{d}{dl}\psi(l)\right)\left(\frac{d}{dl}r(l)\right)}{\psi(l)\, r(l)} + \frac{\left(\frac{d}{dl}r(l)\right)^2}{r(l)^2} - \frac{1}{r(l)^2} = \frac{V(T)}{\sqrt{1+\left(\frac{d}{dl}T(l)\right)^2}} \quad \textbf{(2)}$$

> $$\frac{\frac{d^2}{d l^2}\psi(l)}{\psi(l)} + \frac{\frac{d}{d l}\psi(l)\cdot\frac{d}{d l}r(l)}{\psi(l)\cdot r(l)} + \frac{\frac{d^2}{d l^2}r(l)}{r(l)} = V(T)\cdot\left(1+\left(\frac{d}{d l}T(l)\right)^2\right)^{\frac{1}{2}}$$

$$\frac{\frac{d^2}{dl^2}\psi(l)}{\psi(l)} + \frac{\left(\frac{d}{dl}\psi(l)\right)\left(\frac{d}{dl}r(l)\right)}{\psi(l)\, r(l)} + \frac{\frac{d^2}{dl^2}r(l)}{r(l)} = V(T)\sqrt{1+\left(\frac{d}{dl}T(l)\right)^2} \quad \textbf{(3)}$$

>

> ***conservation equation :***

> $$\frac{d}{d l}p_r(l) + \frac{2}{l}\cdot\left(p_t(l) - p_r(l)\right) = 0$$

$$\frac{d}{dl}p_r(l) + \frac{2\left(p_t(l)-p_r(l)\right)}{l} = 0 \quad \textbf{(4)}$$

> $$eval\left(\textbf{(4)}, \left[p_r(l) = \frac{V(T)}{\sqrt{1+\left(\frac{d}{dl}T(l)\right)^2}}, p_t(l) = V(T)\sqrt{1+\left(\frac{d}{dl}T(l)\right)^2}\right]\right)$$

(5)

$$-\frac{V(T)\left(\frac{d}{dl}T(l)\right)\left(\frac{d^2}{dl^2}T(l)\right)}{\left(1+\left(\frac{d}{dl}T(l)\right)^2\right)^{3/2}}+\frac{2\left(V(T)\sqrt{1+\left(\frac{d}{dl}T(l)\right)^2}-\frac{V(T)}{\sqrt{1+\left(\frac{d}{dl}T(l)\right)^2}}\right)}{l}=0 \qquad (5)$$

> *simplify*(**(5)**, 'size')

$$-\frac{V(T)\left(\frac{d}{dl}T(l)\right)\left(-2\left(\frac{d}{dl}T(l)\right)^3+l\left(\frac{d^2}{dl^2}T(l)\right)-2\left(\frac{d}{dl}T(l)\right)\right)}{\left(1+\left(\frac{d}{dl}T(l)\right)^2\right)^{3/2}l}=0 \qquad (6)$$

> *simplify*(**(6)**)

$$\frac{V(T)\left(\frac{d}{dl}T(l)\right)\left(2\left(\frac{d}{dl}T(l)\right)^3-l\left(\frac{d^2}{dl^2}T(l)\right)+2\left(\frac{d}{dl}T(l)\right)\right)}{\left(1+\left(\frac{d}{dl}T(l)\right)^2\right)^{3/2}l}=0 \qquad (7)$$

> $\left(2\left(\frac{d}{dl}T(l)\right)^3-l\left(\frac{d^2}{dl^2}T(l)\right)+2\left(\frac{d}{dl}T(l)\right)\right)=0$

$$2\left(\frac{d}{dl}T(l)\right)^3-l\left(\frac{d^2}{dl^2}T(l)\right)+2\left(\frac{d}{dl}T(l)\right)=0 \qquad (8)$$

> *dsolve*(**(8)**, { $T(l)$ })

$$T(l)=\int\frac{l^2}{\sqrt{-l^4+\_C1}}\,dl+\_C2,\ T(l)=\int\left(-\frac{l^2}{\sqrt{-l^4+\_C1}}\right)dl+\_C2 \qquad (9)$$

>

>

> $\frac{l^2}{\sqrt{-l^4+\_C1}}$

$$\frac{l^2}{\sqrt{-l^4+\_C1}} \qquad (10)$$

> *int*(**(10)**, l)

$$-\frac{1}{\sqrt{-l^4+\_C1}}\left(\_C1^{3/4}\sqrt{1-\frac{l^2}{\sqrt{\_C1}}}\sqrt{1+\frac{l^2}{\sqrt{\_C1}}}\left(\mathrm{EllipticF}\left(\frac{l}{\_C1^{1/4}},\mathrm{I}\right)\right.\right. \qquad (11)$$

$$- \mathrm{EllipticE}\left(\frac{l}{\_C1^{1/4}}, \mathrm{I}\right)\Big)\Big)$$

>

>

> *Taking Eq One* **and** *Two* :

>

>

$$> \left(-\frac{2\left(\frac{d^2}{dl^2} r(l)\right)}{r(l)} - \frac{\left(\frac{d}{dl} r(l)\right)^2}{r(l)^2} + \frac{1}{r(l)^2}\right) \cdot \left(\frac{2\left(\frac{d}{dl}\psi(l)\right)\left(\frac{d}{dl} r(l)\right)}{\psi(l)\, r(l)} + \frac{\left(\frac{d}{dl} r(l)\right)^2}{r(l)^2} - \frac{1}{r(l)^2}\right) = \left(-V\cdot\sqrt{1+\left(\frac{d}{dl}T(l)\right)^2}\right)\cdot\left(\frac{V}{\sqrt{1+\left(\frac{d}{dl}T(l)\right)^2}}\right)$$

$$\left(-\frac{2\left(\frac{d^2}{dl^2} r(l)\right)}{r(l)} - \frac{\left(\frac{d}{dl} r(l)\right)^2}{r(l)^2} + \frac{1}{r(l)^2}\right)\left(\frac{2\left(\frac{d}{dl}\psi(l)\right)\left(\frac{d}{dl} r(l)\right)}{\psi(l)\, r(l)} + \frac{\left(\frac{d}{dl} r(l)\right)^2}{r(l)^2} - \frac{1}{r(l)^2}\right) = -V^2 \quad \mathbf{(12)}$$

$$> eval\left(\mathbf{(12)}, \left[r(l) = t\cdot l + r_0, \frac{d}{dl}T(l) = \frac{l^2}{\sqrt{-l^4 + \_C1}}, \psi(l) = C\right]\right)$$

$$\left(-\frac{t^2}{(l\,t + r_0)^2} + \frac{1}{(l\,t + r_0)^2}\right)\left(\frac{t^2}{(l\,t + r_0)^2} - \frac{1}{(l\,t + r_0)^2}\right) = -V^2 \quad \mathbf{(13)}$$

$$> simplify(\,\mathbf{(13)}, \text{'size'}\,)$$

$$-\frac{(t^2 - 1)^2}{(l\,t + r_0)^4} = -V^2 \quad \mathbf{(14)}$$

$$> solve(\,\{\,\mathbf{(14)}\,\}, [V]\,)$$

$$\left[\left[V = \frac{t^2 - 1}{l^2\,t^2 + 2\,l\,t\,r_0 + r_0^2}\right], \left[V = -\frac{t^2 - 1}{l^2\,t^2 + 2\,l\,t\,r_0 + r_0^2}\right]\right] \quad \mathbf{(15)}$$

>

>

$$> V = -\frac{t^2 - 1}{l^2\,t^2 + 2\,l\,t\,r_0 + r_0^2}$$

(16)

$$V = -\frac{t^2 - 1}{l^2 t^2 + 2 l t r_0 + r_0^2} \qquad \textbf{(16)}$$

> *simplify*(**(16)**, '*size*')

$$V = \frac{-t^2 + 1}{(l t + r_0)^2} \qquad \textbf{(17)}$$

> *latex*('**(17)**')

```
V={\frac {-{t}^{2}+1}{ \left( lt+r_{{0}} \right) ^{2}}}
```

>

>

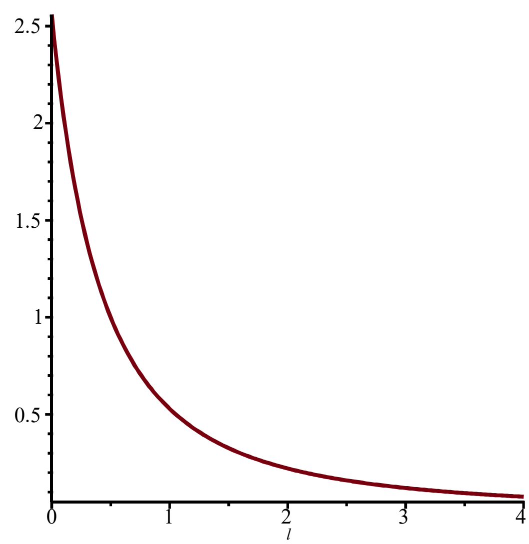
> $\frac{-t^2 + 1}{(l t + r_0)^2}$

$$\frac{-t^2 + 1}{(l t + r_0)^2} \qquad \textbf{(18)}$$

> *eval*(**(18)**, [$t = 0.6$, $r[0] = 0.5$])

$$\frac{0.64}{(0.6\, l + 0.5)^2} \qquad \textbf{(19)}$$

> *plot*(**(19)**, $l = 0\,..\,4$)



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>

> *NEC* :

>

> $-V \cdot \sqrt{\left(1+\left(\frac{d}{dl} T(l)\right)^2\right)} + \frac{V}{\sqrt{\left(1+\left(\frac{d}{dl} T(l)\right)^2\right)}}$

$$-V\sqrt{1+\left(\frac{d}{dl} T(l)\right)^2} + \frac{V}{\sqrt{1+\left(\frac{d}{dl} T(l)\right)^2}} \quad \textbf{(20)}$$

> *simplify*(**(20)**, '*size*')

$$-\frac{V\left(\frac{d}{dl} T(l)\right)^2}{\sqrt{1+\left(\frac{d}{dl} T(l)\right)^2}} \quad \textbf{(21)}$$

> *latex*('**(21)**')

```
-{\frac {V \left( {\frac {\rm d}{{\rm d}l}}T \left( l \right)
 \right) ^{2}}{\sqrt {1+ \left( {\frac {\rm d}{{\rm d}l}}T \left
( l
 \right)  \right) ^{2}}}}
```

> $eval\left(\textbf{(21)}, \left[\frac{d}{dl}T(l) = \frac{l^2}{\sqrt{-l^4+\_C1}}, V = \frac{-t^2+1}{(l\ t+r_0)^2}\right]\right)$

$$-\frac{(-t^2+1)\, l^4}{(l\,t+r_0)^2\,(-l^4+\_C1)\sqrt{1+\frac{l^4}{-l^4+\_C1}}} \quad \textbf{(22)}$$

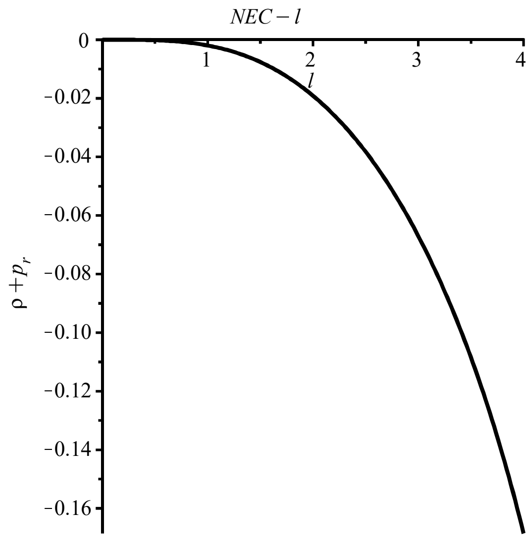
> *latex*('**(22)**')

```
-{\frac { \left( -{t}^{2}+1 \right) {l}^{4}}{ \left( lt+r_{{0}}
 \right) ^{2} \left( -{l}^{4}+{\it \_C1} \right) }{\frac {1}
{\sqrt {1+
{\frac {{l}^{4}}{-{l}^{4}+{\it \_C1}}}}}}}
```

> $eval(\textbf{(22)}, [t=0.2, r[0]=0.5, \_C1=1000.1])$

$$-\frac{0.96\, l^4}{(0.2\,l+0.5)^2\,(-l^4+1000.1)\sqrt{1+\frac{l^4}{-l^4+1000.1}}} \quad \textbf{(23)}$$

> $plot\left(\textbf{(23)}, l=0\,..\,4, labels=[l, \rho+p_r], labeldirections=[HORIZONTAL, VERTICAL], color = [black], linestyle=[\text{solid}], title=[\text{NEC} - l]\right)$



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>

> ## PLOT of V, T vs l :

>

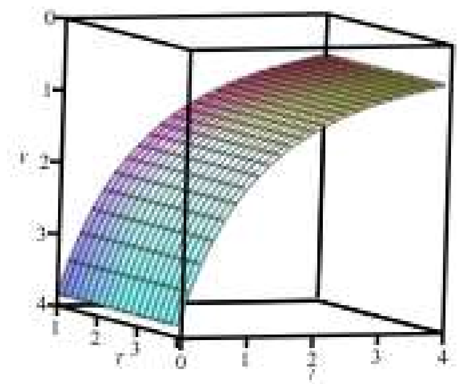
> $V = \frac{-t^2+1}{(l\,t+r_0)^2},\ T = -\frac{1}{\sqrt{-l^4+\_C1}}\left(\_C1^{3/4}\sqrt{1-\frac{l^2}{\sqrt{\_C1}}}\sqrt{1+\frac{l^2}{\sqrt{\_C1}}}\left(\mathrm{EllipticF}\left(\frac{l}{\_C1^{1/4}}, \mathrm{I}\right) - \mathrm{EllipticE}\left(\frac{l}{\_C1^{1/4}}, \mathrm{I}\right)\right)\right)$

$$V = \frac{-t^2+1}{(l\,t+r_0)^2},\ T = -\frac{1}{\sqrt{-l^4+\_C1}}\left(\_C1^{3/4}\sqrt{1-\frac{l^2}{\sqrt{\_C1}}}\sqrt{1+\frac{l^2}{\sqrt{\_C1}}}\left(\mathrm{EllipticF}\left(\frac{l}{\_C1^{1/4}}, \mathrm{I}\right) - \mathrm{EllipticE}\left(\frac{l}{\_C1^{1/4}}, \mathrm{I}\right)\right)\right) \quad \textbf{(24)}$$

> *op*(*eval*([**(24)**], [_C1 = 1000.1, t = 0.2, r[0] = 0.5]))

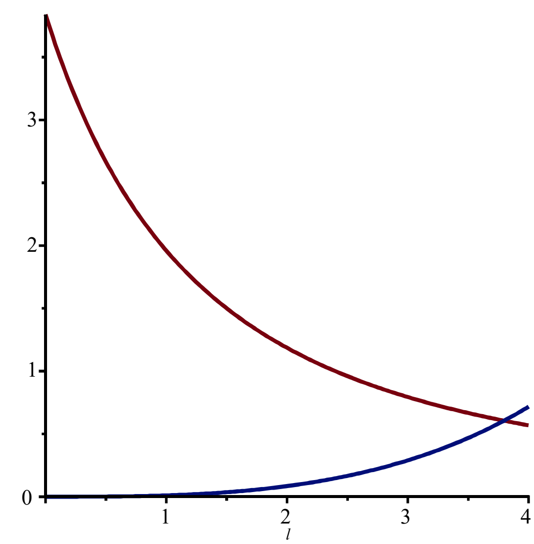
$$V = \frac{0.96}{(0.2\,l+0.5)^2},\ T = -\frac{1}{\sqrt{-l^4+1000.1}}\left(177.8412779\sqrt{1-0.03162119558\,l^2}\sqrt{1+0.03162119558\,l^2}\left(\mathrm{EllipticF}(0.1778234956\,l, \mathrm{I}) - \mathrm{EllipticE}(0.1778234956\,l, \mathrm{I})\right)\right) \quad \textbf{(25)}$$

> *smartplot3d*[l, T, V]([**(25)**])



>

> $$plot\left(\left[\frac{0.96}{(0.2\,l+0.5)^2},\ -\frac{1}{\sqrt{-l^4+1000.1}}\left(177.8412779\sqrt{1-0.03162119558\,l^2}\,\sqrt{1+0.03162119558\,l^2}\,(\mathrm{EllipticF}(0.1778234956\,l,\mathrm{I})-\mathrm{EllipticE}(0.1778234956\,l,\mathrm{I}))\right)\right],\ l=0..4,\ labels\right)$$



>

> ***Perturbation* in delta *r* :**

>

$$> \frac{d^2}{dl^2}f(l) + 2\cdot\frac{\frac{d}{dl}(r(l))}{r(l)}\cdot\left(\frac{d}{dl}f(l)\right) + \frac{\frac{d^2}{dl^2}r(l)}{r(l)}\cdot(f(l)) = 0$$

$$\frac{d^2}{dl^2}f(l) + \frac{2\left(\frac{d}{dl}r(l)\right)\left(\frac{d}{dl}f(l)\right)}{r(l)} + \frac{\left(\frac{d^2}{dl^2}r(l)\right)f(l)}{r(l)} = 0 \quad \textbf{(26)}$$

$> eval\left(\textbf{(26)}, \left[r(l) = t\cdot l + r_0\right]\right)$

$$\frac{d^2}{dl^2}f(l) + \frac{2\,t\left(\frac{d}{dl}f(l)\right)}{l\,t + r_0} = 0 \quad \textbf{(27)}$$

$>$

$> simplify(\textbf{(27)}, 'size')$

$$\frac{d^2}{dl^2}f(l) + \frac{2\,t\left(\frac{d}{dl}f(l)\right)}{l\,t + r_0} = 0 \quad \textbf{(28)}$$

$> dsolve(\textbf{(28)}, \{f(l)\})$

$$f(l) = \_C1 + \frac{\_C2}{l + \frac{r_0}{t}} \quad \textbf{(29)}$$

$>$

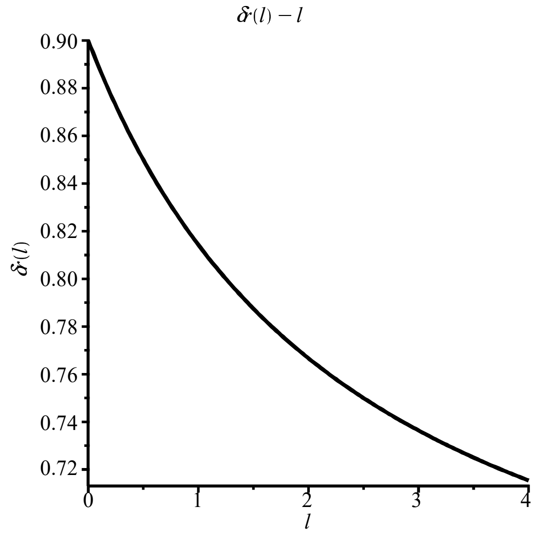
$$> \_C1 + \frac{\_C2}{l + \frac{r_0}{t}}$$

$$\_C1 + \frac{\_C2}{l + \frac{r_0}{t}} \quad \textbf{(30)}$$

$> eval\left(\textbf{(30)}, \left[\_C1 = 0.6, \_C2 = 0.75, t = 0.2, r_0 = 0.5\right]\right)$

$$0.6 + \frac{0.75}{l + 2.500000000} \quad \textbf{(31)}$$

$> plot\left(\textbf{(31)}, l = 0 \,..\, 4, labels = [l, \delta r(1)], labeldirections = [HORIZONTAL, VERTICAL], color = [black], linestyle = [\mathrm{solid}], title = [\delta r(1) - l]\right)$



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>

> *Perturbation* **in** delta t :

>

>

> $\frac{d}{dl}V(l)\cdot\left(1+\left(\frac{d}{dl}T(l)\right)^2\right)\cdot\left(2+\left(\frac{d}{dl}T(l)\right)^2\right)\cdot f(l)+V(l)\cdot\left(\frac{d}{dl}T(l)\right)^3\cdot\frac{d}{dl}f(l)=0$

$$\left(\frac{d}{dl}V(l)\right)\left(1+\left(\frac{d}{dl}T(l)\right)^2\right)\left(2+\left(\frac{d}{dl}T(l)\right)^2\right)f(l)+V(l)\left(\frac{d}{dl}T(l)\right)^3\left(\frac{d}{dl}f(l)\right)=0 \quad \textbf{(32)}$$

> $eval\left(\textbf{(32)}, \left[V(l)=\frac{-t^2+1}{(l\ t+r_0)^2}, \frac{d}{dl}\ T(l)=\frac{l^2}{\sqrt{-l^4+\_C1}}\right]\right)$

$$-\frac{2\left(-t^2+1\right)t\left(1+\frac{l^4}{-l^4+\_C1}\right)\left(2+\frac{l^4}{-l^4+\_C1}\right)f(l)}{\left(l\,t+r_0\right)^3}+\frac{\left(-t^2+1\right)l^6\left(\frac{d}{dl}f(l)\right)}{\left(l\,t+r_0\right)^2\left(-l^4+\_C1\right)^{3/2}}=0 \quad \textbf{(33)}$$

> *isolate*(**(33)**, *diff* ($f(l), l$))

$$\frac{d}{dl}f(l)=\frac{2\,t\left(1+\frac{l^4}{-l^4+\_C1}\right)\left(2+\frac{l^4}{-l^4+\_C1}\right)f(l)\left(-l^4+\_C1\right)^{3/2}}{\left(l\,t+r_0\right)l^6} \quad \textbf{(34)}$$

> *simplify*(**(33)**, '*size*')

$$-\frac{1}{\left(l\,t+r_0\right)^3\left(-l^4+\_C1\right)^{7/2}}\left(2\left(\frac{1}{2}\,l^6\left(l^4-\_C1\right)^2\left(l\,t+r_0\right)\left(\frac{d}{dl}f(l)\right)+\left(-l^4+\_C1\right)^{3/2}\_C1\left(l^4-2\,\_C1\right)t\,f(l)\right)(t+1)(t-1)\right)=0 \quad \textbf{(35)}$$

> *isolate*(**(35)**, *diff* ($f(l), l$))

$$\frac{d}{dl} f(l) = \frac{-f(l)\left(-l^4+\_C1\right)^{3/2}\_C1\, l^4\, t + 2 f(l)\left(-l^4+\_C1\right)^{3/2}\_C1^2\, t}{\frac{1}{2} l^{15} t + \frac{1}{2} l^{14} r_0 - \_C1\, l^{11} t - \_C1\, l^{10} r_0 + \frac{1}{2}\_C1^2 l^7 t + \frac{1}{2}\_C1^2 l^6 r_0} \tag{36}$$

> *simplify*(**(36)**, 'size')

$$\frac{d}{dl} f(l) = -\frac{2 t\,\_C1\left(l^4-2\,\_C1\right) f(l)\left(-l^4+\_C1\right)^{3/2}}{l^6\left(l^4-\_C1\right)^2\left(l t + r_0\right)} \tag{37}$$

> *dsolve*(**(37)**, { $f(l)$ })

$$f(l) = \_C2\, e^{\int \frac{2 t\,\_C1\left(-l^4+2\,\_C1\right)}{\sqrt{-l^4+\_C1}\, l^6\left(l t + r_0\right)}\, dl} \tag{38}$$

>

> $$\frac{\frac{d}{dl} f(l)}{f(l)} = -\left(2 f(l)\,\_C1\left(-l^4+\_C1\right)^{5/2}\left((\omega-2) r_0^2\left((\omega-1) l^5 + 2\omega l^4 + (-3\,\_C1\,\omega + \_C1) l - 4\,\_C1\,\omega\right)(1+l)^2\omega (1+l)^{\frac{2}{\omega}} - 2\left(l^4 - \_C1\, l - 2\,\_C1\right) r_0^4\left(\omega - \frac{3}{2}\right)(1+l)^{\frac{4}{\omega}} + (1+l)^4\left((\omega-1) l^5 + \omega l^4 + (-2\,\_C1\,\omega + \_C1) l - 2\,\_C1\,\omega\right)\omega^3\right)\left(l^4 - 2\,\_C1\right)\right) \Big/ \left(\left(l^4-\_C1\right)^4 (1+l)\, l^7\left(2\omega^2 r_0^2 (1+l)^2(\omega-2)(1+l)^{\frac{2}{\omega}} - 2\left(\omega-\frac{3}{2}\right) r_0^4 (1+l)^{\frac{4}{\omega}} + \omega^4 (1+l)^4\right)\right)\cdot\left(\frac{1}{f(l)}\right)$$

$$\frac{\frac{d}{dl} f(l)}{f(l)} = -\left(2\,\_C1\left(-l^4+\_C1\right)^{5/2}\left((\omega-2) r_0^2\left((\omega-1) l^5 + 2\omega l^4 + (-3\,\_C1\,\omega + \_C1) l - 4\,\_C1\,\omega\right)(1+l)^2\omega (1+l)^{\frac{2}{\omega}} - 2\left(l^4 - \_C1\, l - 2\,\_C1\right) r_0^4\left(\omega - \frac{3}{2}\right)(1+l)^{\frac{4}{\omega}} + (1+l)^4\left((\omega-1) l^5 + \omega l^4 + (-2\,\_C1\,\omega + \_C1) l - 2\,\_C1\,\omega\right)\omega^3\right)\left(l^4 - 2\,\_C1\right)\right) \Big/ \left(\left(l^4-\_C1\right)^4 (1+l)\, l^7\left(2\omega^2 r_0^2 (1+l)^2(\omega-2)(1+l)^{\frac{2}{\omega}} - 2\left(\omega-\frac{3}{2}\right) r_0^4 (1+l)^{\frac{4}{\omega}} + \omega^4 (1+l)^4\right)\right) \tag{39}$$

> *simplify*(**(39)**, 'size')

(40)

$$\frac{\frac{\mathrm{d}}{\mathrm{d}l} f(l)}{f(l)} = -\Bigg(2\,\_C1\,\left(l^4 - 2\,\_C1\right)\Bigg(\left(\omega - 2\right) r_0^2 \Big(\Big(\left(l^5 + 2\,l^4 - 3\,\_C1\,l - 4\,\_C1\right)\omega - l^5 + \_C1\,l\Big)\left(1 + l\right)^2 \omega \left(1 + l\right)^{\frac{2}{\omega}} - 2\left(l^4 - \_C1\,l - 2\,\_C1\right) r_0^4 \left(\omega - \frac{3}{2}\right)\left(1 + l\right)^{\frac{4}{\omega}} + \left(1 + l\right)^4 \Big(\left(1 + l\right)\left(l^4 - 2\,\_C1\right)\omega - l^5 + \_C1\,l\Big)\omega^3\Big)\Bigg)\Bigg) \Bigg/ \Bigg(\left(1 + l\right) l^7 \Bigg(2\,\omega^2 r_0^2 \left(1 + l\right)^2 \left(\omega - 2\right)\left(1 + l\right)^{\frac{2}{\omega}} - 2\left(\omega - \frac{3}{2}\right) r_0^4 \left(1 + l\right)^{\frac{4}{\omega}} + \omega^4 \left(1 + l\right)^4\Bigg)\left(-l^4 + \_C1\right)^{3/2}\Bigg) \tag{40}$$

>

>