> restart;
with(Physics) :

>

> $-2\cdot\frac{\frac{d^2}{dl^2}r(l)}{r(l)}-\left(\frac{\frac{d}{dl}r(l)}{r(l)}\right)^2+\frac{1}{r^2(l)}=-V(T)\cdot\left(1+\left(\frac{d}{dl}T(l)\right)^2\right)^{\frac{1}{2}}$

$$-\frac{2\left(\frac{d^2}{dl^2}r(l)\right)}{r(l)}-\frac{\left(\frac{d}{dl}r(l)\right)^2}{r(l)^2}+\frac{1}{r(l)^2}=-V(T)\sqrt{1+\left(\frac{d}{dl}T(l)\right)^2} \quad (1)$$

>

>

> $2\cdot\frac{\frac{d}{dl}\psi(l)\cdot\frac{d}{dl}r(l)}{\psi(l)\cdot r(l)}+\left(\frac{\frac{d}{dl}r(l)}{r(l)}\right)^2-\frac{1}{r^2(l)}=\frac{V(T)}{\left(1+\left(\frac{d}{dl}T(l)\right)^2\right)^{\frac{1}{2}}}$

$$\frac{2\left(\frac{d}{dl}\psi(l)\right)\left(\frac{d}{dl}r(l)\right)}{\psi(l)\,r(l)}+\frac{\left(\frac{d}{dl}r(l)\right)^2}{r(l)^2}-\frac{1}{r(l)^2}=\frac{V(T)}{\sqrt{1+\left(\frac{d}{dl}T(l)\right)^2}} \quad (2)$$

> $\frac{\frac{d^2}{dl^2}\psi(l)}{\psi(l)}+\frac{\frac{d}{dl}\psi(l)\cdot\frac{d}{dl}r(l)}{\psi(l)\cdot r(l)}+\frac{\frac{d^2}{dl^2}r(l)}{r(l)}=V(T)\cdot\left(1+\left(\frac{d}{dl}T(l)\right)^2\right)^{\frac{1}{2}}$

$$\frac{\frac{d^2}{dl^2}\psi(l)}{\psi(l)}+\frac{\left(\frac{d}{dl}\psi(l)\right)\left(\frac{d}{dl}r(l)\right)}{\psi(l)\,r(l)}+\frac{\frac{d^2}{dl^2}r(l)}{r(l)}=V(T)\sqrt{1+\left(\frac{d}{dl}T(l)\right)^2} \quad (3)$$

>

> *conservation equation :*

> $\frac{d}{dl}p_r(l)+\frac{2}{l}\cdot\left(p_t(l)-p_r(l)\right)=0$

$$\frac{d}{dl}p_r(l)+\frac{2\left(p_t(l)-p_r(l)\right)}{l}=0 \quad (4)$$

> $eval\left((4), \left[p_r(l)=\frac{V(T)}{\sqrt{1+\left(\frac{d}{dl}T(l)\right)^2}}, p_t(l)=V(T)\sqrt{1+\left(\frac{d}{dl}T(l)\right)^2}\right]\right)$

(5)

$$-\frac{V(T)\left(\frac{d}{dl}T(l)\right)\left(\frac{d^2}{dl^2}T(l)\right)}{\left(1+\left(\frac{d}{dl}T(l)\right)^2\right)^{3/2}} + \frac{2\left(V(T)\sqrt{1+\left(\frac{d}{dl}T(l)\right)^2}-\frac{V(T)}{\sqrt{1+\left(\frac{d}{dl}T(l)\right)^2}}\right)}{l}=0 \tag{5}$$

> *simplify*(**(5)**, 'size')

$$-\frac{V(T)\left(\frac{d}{dl}T(l)\right)\left(-2\left(\frac{d}{dl}T(l)\right)^3+l\left(\frac{d^2}{dl^2}T(l)\right)-2\left(\frac{d}{dl}T(l)\right)\right)}{\left(1+\left(\frac{d}{dl}T(l)\right)^2\right)^{3/2}l}=0 \tag{6}$$

> *simplify*(**(6)**)

$$\frac{V(T)\left(\frac{d}{dl}T(l)\right)\left(2\left(\frac{d}{dl}T(l)\right)^3-l\left(\frac{d^2}{dl^2}T(l)\right)+2\left(\frac{d}{dl}T(l)\right)\right)}{\left(1+\left(\frac{d}{dl}T(l)\right)^2\right)^{3/2}l}=0 \tag{7}$$

> $\left(2\left(\frac{d}{dl}T(l)\right)^3-l\left(\frac{d^2}{dl^2}T(l)\right)+2\left(\frac{d}{dl}T(l)\right)\right)=0$

$$2\left(\frac{d}{dl}T(l)\right)^3-l\left(\frac{d^2}{dl^2}T(l)\right)+2\left(\frac{d}{dl}T(l)\right)=0 \tag{8}$$

> *dsolve*(**(8)**, { $T(l)$ })

$$T(l)=\int\frac{l^2}{\sqrt{-l^4+\_C1}}\,dl+\_C2,\ T(l)=\int\left(-\frac{l^2}{\sqrt{-l^4+\_C1}}\right)dl+\_C2 \tag{9}$$

>

>

> $\frac{l^2}{\sqrt{-l^4+\_C1}}$

$$\frac{l^2}{\sqrt{-l^4+\_C1}} \tag{10}$$

> *int*(**(10)**, l)

$$-\frac{1}{\sqrt{-l^4+\_C1}}\left(\_C1^{3/4}\sqrt{1-\frac{l^2}{\sqrt{\_C1}}}\sqrt{1+\frac{l^2}{\sqrt{\_C1}}}\left(\mathrm{EllipticF}\left(\frac{l}{\_C1^{1/4}},\mathrm{I}\right)\right.\right. \tag{11}$$

$$-\operatorname{EllipticE}\left(\frac{l}{\_C1^{1/4}}, \mathrm{I}\right)\Bigg)\Bigg)$$

>

>

> **Taking Eq One and Two :**

>

>

> $\left(-\frac{2\left(\frac{d^2}{dl^2} r(l)\right)}{r(l)} - \frac{\left(\frac{d}{dl} r(l)\right)^2}{r(l)^2} + \frac{1}{r(l)^2}\right) \cdot \left(\frac{2\left(\frac{d}{dl}\psi(l)\right)\left(\frac{d}{dl} r(l)\right)}{\psi(l)\, r(l)} + \frac{\left(\frac{d}{dl} r(l)\right)^2}{r(l)^2} - \frac{1}{r(l)^2}\right) = \left(-V \cdot \sqrt{1 + \left(\frac{d}{dl} T(l)\right)^2}\right) \cdot \left(\frac{V}{\sqrt{1 + \left(\frac{d}{dl} T(l)\right)^2}}\right)$

$$\left(-\frac{2\left(\frac{d^2}{dl^2} r(l)\right)}{r(l)} - \frac{\left(\frac{d}{dl} r(l)\right)^2}{r(l)^2} + \frac{1}{r(l)^2}\right)\left(\frac{2\left(\frac{d}{dl}\psi(l)\right)\left(\frac{d}{dl} r(l)\right)}{\psi(l)\, r(l)} + \frac{\left(\frac{d}{dl} r(l)\right)^2}{r(l)^2} - \frac{1}{r(l)^2}\right) = -V^2 \quad \textbf{(12)}$$

> $eval\left(\textbf{(12)}, \left[r(l) = r_0\ (1+l)^{\frac{1}{\omega}}, \frac{d}{d\,l}T(l) = \frac{l^2}{\sqrt{-l^4 + \_C1}}, \psi(l) = C\right]\right)$

$$\left(-\frac{2\left(\frac{r_0\ (1+l)^{\frac{1}{\omega}}}{\omega^2\ (1+l)^2} - \frac{r_0\ (1+l)^{\frac{1}{\omega}}}{\omega\ (1+l)^2}\right)}{r_0\ (1+l)^{\frac{1}{\omega}}} - \frac{1}{\omega^2\ (1+l)^2} + \frac{1}{r_0^2\left((1+l)^{\frac{1}{\omega}}\right)^2}\right)\left(\frac{1}{\omega^2\ (1+l)^2} - \frac{1}{r_0^2\left((1+l)^{\frac{1}{\omega}}\right)^2}\right) = -V^2 \quad \textbf{(13)}$$

> $simplify(\ \textbf{(13)}, 'size'\ )$

(14)

$$-\frac{1}{\omega^4 (1+l)^4 r_0^4 \left((1+l)^{\frac{1}{\omega}}\right)^4} \left(\left(-r_0 (1+l)^{\frac{1}{\omega}} + \omega (1+l) \right) \left(2 \left(\omega - \frac{3}{2} \right) r_0^2 \left((1+l)^{\frac{1}{\omega}}\right)^2 + \omega^2 (1+l)^2 \right) \left(r_0 (1+l)^{\frac{1}{\omega}} + \omega (1+l) \right) \right) = -V^2 \tag{14}$$

> solve({ **(14)** }, [V])

$$\left[\left[V = \frac{1}{(1+l)^{\frac{4}{\omega}} (1+l)^2 r_0^2 \omega^2} \left((1+l)^{\frac{4}{\omega}} \left(2 (1+l)^{\frac{2}{\omega}} l^2 \omega^3 r_0^2 + l^4 \omega^4 - 4 (1+l)^{\frac{2}{\omega}} l^2 \omega^2 r_0^2 + 4 (1+l)^{\frac{2}{\omega}} l \omega^3 r_0^2 + 4 l^3 \omega^4 - 8 (1+l)^{\frac{2}{\omega}} l \omega^2 r_0^2 + 2 (1+l)^{\frac{2}{\omega}} \omega^3 r_0^2 - 2 (1+l)^{\frac{4}{\omega}} \omega r_0^4 + 6 l^2 \omega^4 - 4 (1+l)^{\frac{2}{\omega}} \omega^2 r_0^2 + 3 (1+l)^{\frac{4}{\omega}} r_0^4 + 4 l \omega^4 + \omega^4 \right) \right)^{1/2} \right], \right. \tag{15}$$

$$\left[V = -\frac{1}{(1+l)^{\frac{4}{\omega}} (1+l)^2 r_0^2 \omega^2} \left((1+l)^{\frac{4}{\omega}} \left(2 (1+l)^{\frac{2}{\omega}} l^2 \omega^3 r_0^2 + l^4 \omega^4 - 4 (1+l)^{\frac{2}{\omega}} l^2 \omega^2 r_0^2 + 4 (1+l)^{\frac{2}{\omega}} l \omega^3 r_0^2 + 4 l^3 \omega^4 - 8 (1+l)^{\frac{2}{\omega}} l \omega^2 r_0^2 + 2 (1+l)^{\frac{2}{\omega}} \omega^3 r_0^2 \right.\right.$$

$$-2\,(1+l)^{\frac{4}{\omega}}\,\omega\,r_0^4+6\,l^2\,\omega^4-4\,(1+l)^{\frac{2}{\omega}}\,\omega^2\,r_0^2+3\,(1+l)^{\frac{4}{\omega}}\,r_0^4+4\,l\,\omega^4+\omega^4\Big)\Big)^{1/2}\Big]\Big]$$

>

>

> V

$$=\frac{1}{(1+l)^{\frac{4}{\omega}}\,(1+l)^2\,r_0^2\,\omega^2}\Big((1+l)^{\frac{4}{\omega}}\Big(2\,(1+l)^{\frac{2}{\omega}}\,l^2\,\omega^3\,r_0^2+l^4\,\omega^4-4\,(1+l)^{\frac{2}{\omega}}\,l^2\,\omega^2\,r_0^2+4\,(1+l)^{\frac{2}{\omega}}\,l\,\omega^3\,r_0^2+4\,l^3\,\omega^4-8\,(1+l)^{\frac{2}{\omega}}\,l\,\omega^2\,r_0^2+2\,(1+l)^{\frac{2}{\omega}}\,\omega^3\,r_0^2-2\,(1+l)^{\frac{4}{\omega}}\,\omega\,r_0^4+6\,l^2\,\omega^4-4\,(1+l)^{\frac{2}{\omega}}\,\omega^2\,r_0^2+3\,(1+l)^{\frac{4}{\omega}}\,r_0^4+4\,l\,\omega^4+\omega^4\Big)\Big)^{1/2}$$

$$V=\frac{1}{(1+l)^{\frac{4}{\omega}}\,(1+l)^2\,r_0^2\,\omega^2}\Big((1+l)^{\frac{4}{\omega}}\Big(2\,(1+l)^{\frac{2}{\omega}}\,l^2\,\omega^3\,r_0^2+l^4\,\omega^4-4\,(1+l)^{\frac{2}{\omega}}\,l^2\,\omega^2\,r_0^2+4\,(1+l)^{\frac{2}{\omega}}\,l\,\omega^3\,r_0^2+4\,l^3\,\omega^4-8\,(1+l)^{\frac{2}{\omega}}\,l\,\omega^2\,r_0^2+2\,(1+l)^{\frac{2}{\omega}}\,\omega^3\,r_0^2-2\,(1+l)^{\frac{4}{\omega}}\,\omega\,r_0^4+6\,l^2\,\omega^4-4\,(1+l)^{\frac{2}{\omega}}\,\omega^2\,r_0^2+3\,(1+l)^{\frac{4}{\omega}}\,r_0^4+4\,l\,\omega^4+\omega^4\Big)\Big)^{1/2} \qquad \textbf{(16)}$$

> *simplify*(**(16)**, '*size*')

$$V=\frac{1}{(1+l)^{\frac{4}{\omega}}\,(1+l)^2\,r_0^2\,\omega^2}\Big((1+l)^{\frac{4}{\omega}}\Big(2\,\omega^2\,r_0^2\,(1+l)^2\,(\omega-2)\,(1+l)^{\frac{2}{\omega}}-2\left(\omega-\frac{3}{2}\right)r_0^4\,(1+l)^{\frac{4}{\omega}}+\omega^4\,(1+l)^4\Big)\Big)^{1/2} \qquad \textbf{(17)}$$

>

>

> $\frac{1}{(1+l)^{\frac{4}{\omega}}\,(1+l)^2\,r_0^2\,\omega^2}\Big((1+l)^{\frac{4}{\omega}}\Big(2\,\omega^2\,r_0^2\,(1+l)^2\,(\omega-2)\,(1+l)^{\frac{2}{\omega}}-2\left(\omega-\frac{3}{2}\right)$

$$r_0^4 (1+l)^{\frac{4}{\omega}} + \omega^4 (1+l)^4 \Big)\Big)^{1/2}$$

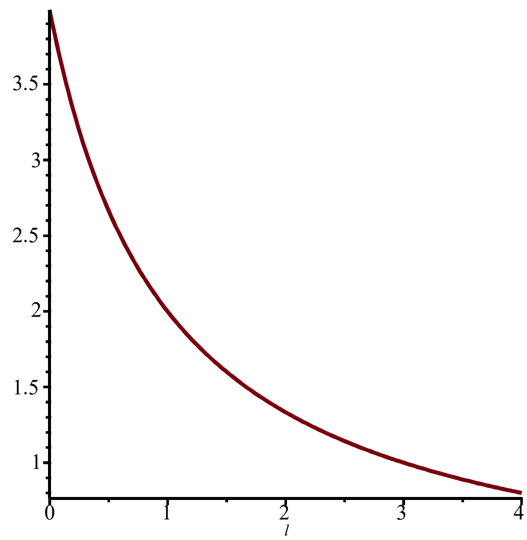
$$\frac{1}{(1+l)^{\frac{4}{\omega}} (1+l)^2 r_0^2 \omega^2} \left((1+l)^{\frac{4}{\omega}} \left(2\omega^2 r_0^2 (1+l)^2 (\omega-2) (1+l)^{\frac{2}{\omega}} - 2\left(\omega - \frac{3}{2}\right) \right.\right. \quad \textbf{(18)}$$

$$\left.\left. r_0^4 (1+l)^{\frac{4}{\omega}} + \omega^4 (1+l)^4 \right)\right)^{1/2}$$

> *eval*(**(18)**, [ω = 2, r[0] = 0.5])

$$\frac{1.000000000\sqrt{(1+l)^2 \left(-0.06250000000 (1+l)^2 + 16 (1+l)^4\right)}}{(1+l)^4} \quad \textbf{(19)}$$

> *plot*(**(19)**, *l* = 0 .. 4)



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NEC :

>

$$> \; -V\cdot\sqrt{\left(1 + \left(\frac{d}{dl} T(l)\right)^2\right)} + \frac{V}{\sqrt{\left(1 + \left(\frac{d}{dl} T(l)\right)^2\right)}}$$

(20)

$$-V\sqrt{1+\left(\frac{d}{dl}T(l)\right)^2}+\frac{V}{\sqrt{1+\left(\frac{d}{dl}T(l)\right)^2}} \tag{20}$$

> simplify(**(20)**, 'size')

$$-\frac{V\left(\frac{d}{dl}T(l)\right)^2}{\sqrt{1+\left(\frac{d}{dl}T(l)\right)^2}} \tag{21}$$

> eval(**(21)**, [$\frac{d}{dl}T(l)=\frac{l^2}{\sqrt{-l^4+\_C1}}$, V

$= \frac{1}{(1+l)^{\frac{4}{\omega}}(1+l)^2 r_0^2\,\omega^2}\left((1+l)^{\frac{4}{\omega}}\left(2\,\omega^2\,r_0^2\,(1+l)^2\,(\omega-2)\,(1+l)^{\frac{2}{\omega}}\right.\right.$

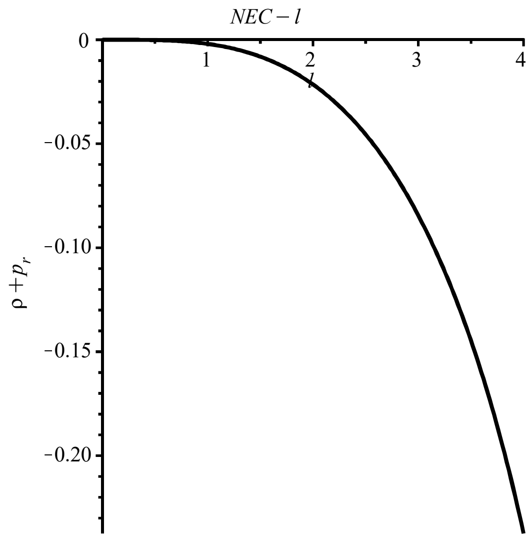
$\left.\left.-2\left(\omega-\frac{3}{2}\right)r_0^4\,(1+l)^{\frac{4}{\omega}}+\omega^4\,(1+l)^4\right)\right)^{1/2}$])

$$-\left(\left((1+l)^{\frac{4}{\omega}}\left(2\omega^2 r_0^2(1+l)^2(\omega-2)(1+l)^{\frac{2}{\omega}}-2\left(\omega-\frac{3}{2}\right)r_0^4(1+l)^{\frac{4}{\omega}}+\omega^4(1+l)^4\right)\right)^{1/2} l^4\right)\Big/\left((1+l)^{\frac{4}{\omega}}(1+l)^2 r_0^2\omega^2\left(-l^4+\_C1\right)\sqrt{1+\frac{l^4}{-l^4+\_C1}}\right) \tag{22}$$

> eval(**(22)**, [ω = 2, r[0] = 0.5, _C1 = 1000.1])

$$-\frac{1.000000000\sqrt{(1+l)^2\left(-0.06250000000\,(1+l)^2+16\,(1+l)^4\right)}\,l^4}{(1+l)^4\left(-l^4+1000.1\right)\sqrt{1+\frac{l^4}{-l^4+1000.1}}} \tag{23}$$

> plot(**(23)**, l = 0 .. 4, $labels$ = [l, $\rho+p_r$], $labeldirections$ = [$HORIZONTAL$, $VERTICAL$], $color$ = [$black$], $linestyle$ = [solid], $title$ = [NEC − l])



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>

> # *PLOT of V, T vs l :*

>

> V

$$= \frac{1}{(1+l)^{\frac{4}{\omega}} (1+l)^2 r_0^2 \omega^2} \left(\left(2\,\omega^2 r_0^2 (1+l)^2 (\omega-2) (1+l)^{\frac{2}{\omega}} - 2\left(\omega - \frac{3}{2}\right) r_0^4 (1+l)^{\frac{4}{\omega}} + \omega^4 (1+l)^4 \right) (1+l)^{\frac{4}{\omega}} \right)^{1/2}, T = -\frac{1}{\sqrt{-l^4 + \_C1}} \left(\_C1^{3/4} \sqrt{1 - \frac{l^2}{\sqrt{\_C1}}} \sqrt{1 + \frac{l^2}{\sqrt{\_C1}}} \left(\mathrm{EllipticF}\left(\frac{l}{\_C1^{1/4}}, \mathrm{I} \right) - \mathrm{EllipticE}\left(\frac{l}{\_C1^{1/4}}, \mathrm{I} \right) \right) \right)$$

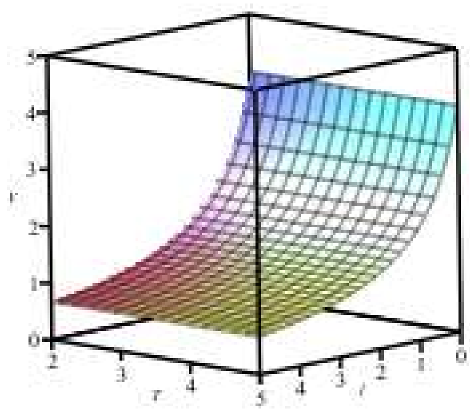
$$V = \frac{1}{(1+l)^{\frac{4}{\omega}} (1+l)^2 r_0^2 \omega^2} \left(\left(2\,\omega^2 r_0^2 (1+l)^2 (\omega-2) (1+l)^{\frac{2}{\omega}} - 2\left(\omega - \frac{3}{2}\right) r_0^4 (1+l)^{\frac{4}{\omega}} + \omega^4 (1+l)^4 \right) (1+l)^{\frac{4}{\omega}} \right)^{1/2}, T = -\frac{1}{\sqrt{-l^4 + \_C1}} \left(\_C1^{3/4} \sqrt{1 - \frac{l^2}{\sqrt{\_C1}}} \sqrt{1 + \frac{l^2}{\sqrt{\_C1}}} \left(\mathrm{EllipticF}\left(\frac{l}{\_C1^{1/4}}, \mathrm{I} \right) \right. \right. \tag{24}$$

$$-\operatorname{EllipticE}\left(\frac{l}{\_C1^{1/4}}, \mathrm{I}\right)\Bigg)\Bigg)\Bigg)$$

> $op(\,eval(\,[\mathbf{(24)}], [\_C1 = 1000.1, \omega = 2, r[0] = 0.5]))$

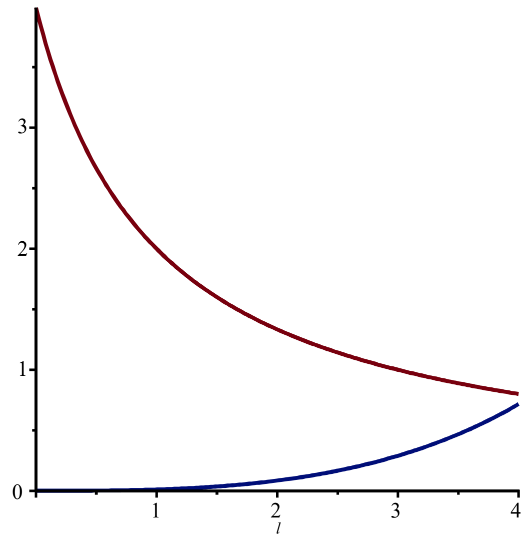
$$V = \frac{1.000000000\sqrt{(1+l)^2\left(-0.06250000000\,(1+l)^2 + 16\,(1+l)^4\right)}}{(1+l)^4}, T = -\frac{1}{\sqrt{-l^4+1000.1}}\Big(177.8412779\sqrt{1-0.03162119558\,l^2}\,\sqrt{1+0.03162119558\,l^2}\,(\operatorname{EllipticF}(0.1778234956\,l, \mathrm{I}) - \operatorname{EllipticE}(0.1778234956\,l, \mathrm{I}))\Big) \quad \mathbf{(25)}$$

> $smartplot3d[l, T, V]([\,\mathbf{(25)}\,])$



>

> $plot\left(\left[\frac{1.000000000\sqrt{(1+l)^2\left(-0.06250000000\,(1+l)^2+16\,(1+l)^4\right)}}{(1+l)^4}, -\frac{1}{\sqrt{-l^4+1000.1}}\Big(177.8412779\sqrt{1-0.03162119558\,l^2}\,\sqrt{1+0.03162119558\,l^2}\,(\operatorname{EllipticF}(0.1778234956\,l, \mathrm{I}) - \operatorname{EllipticE}(0.1778234956\,l, \mathrm{I}))\Big)\right], l = 0..4\right)$



>

> ***Perturbation* in delta *r* :**

>

> $\frac{d^2}{dl^2}f(l)+2\cdot\frac{\frac{d}{dl}(r(l))}{r(l)}\cdot\left(\frac{d}{dl}f(l)\right)+\frac{\frac{d^2}{dl^2}r(l)}{r(l)}\cdot(f(l))=0$

$$\frac{d^2}{dl^2}f(l)+\frac{2\left(\frac{d}{dl}r(l)\right)\left(\frac{d}{dl}f(l)\right)}{r(l)}+\frac{\left(\frac{d^2}{dl^2}r(l)\right)f(l)}{r(l)}=0 \tag{26}$$

> $eval\left((26), \left[r(l)=r_0\ (1+l)^{\frac{1}{\omega}}\right]\right)$

$$\frac{d^2}{dl^2}f(l)+\frac{2\left(\frac{d}{dl}f(l)\right)}{\omega\ (1+l)}+\frac{\left(\frac{r_0\ (1+l)^{\frac{1}{\omega}}}{\omega^2\ (1+l)^2}-\frac{r_0\ (1+l)^{\frac{1}{\omega}}}{\omega\ (1+l)^2}\right)f(l)}{r_0\ (1+l)^{\frac{1}{\omega}}}=0 \tag{27}$$

> $simplify((27), 'size')$

$$\frac{\omega^2\ (1+l)^2\left(\frac{d^2}{dl^2}f(l)\right)+2\,\omega\ (1+l)\left(\frac{d}{dl}f(l)\right)-f(l)\ (\omega-1)}{\omega^2\ (1+l)^2}=0 \tag{28}$$

> $dsolve((28), \{f(l)\})$

$$f(l)=\_C1\ (1+l)^{\frac{\omega-1}{\omega}}+\_C2\ (1+l)^{-\frac{1}{\omega}} \tag{29}$$

>

>

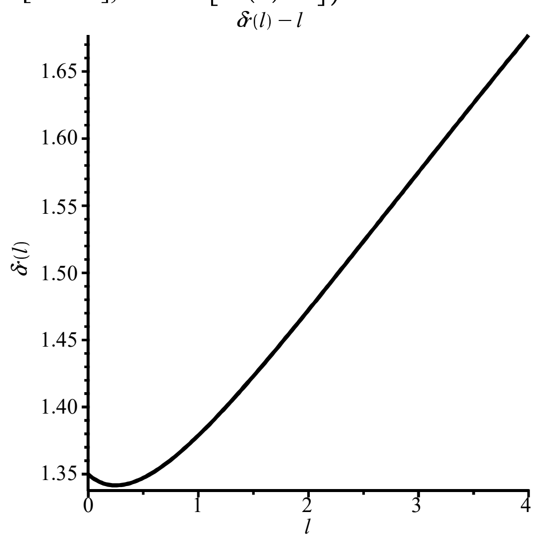
> $\_C1\ (1+l)^{\frac{\omega-1}{\omega}} + \_C2\ (1+l)^{-\frac{1}{\omega}}$

$$\_C1\ (1+l)^{\frac{\omega-1}{\omega}} + \_C2\ (1+l)^{-\frac{1}{\omega}} \quad \textbf{(30)}$$

> $eval(\ \textbf{(30)}, [\_C1=0.6, \_C2=0.75, \omega=2, r[0]=0.5])$

$$0.6\sqrt{1+l} + \frac{0.75}{\sqrt{1+l}} \quad \textbf{(31)}$$

> $plot(\textbf{(31)}, l=0\ ..\ 4, labels=[1, \delta r(1)], labeldirections=[HORIZONTAL, VERTICAL], color=[black], linestyle=[\mathrm{solid}], title=[\delta r(1)-l])$



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>

> *Perturbation* **in** delta t :

>

>

> $\frac{d}{dl}V(l)\cdot\left(1+\left(\frac{d}{dl}T(l)\right)^2\right)\cdot\left(2+\left(\frac{d}{dl}T(l)\right)^2\right)\cdot f(l) + V(l)\cdot\left(\frac{d}{dl}T(l)\right)^3\cdot\frac{d}{dl}f(l)=0$

$$\left(\frac{d}{dl}V(l)\right)\left(1+\left(\frac{d}{dl}T(l)\right)^2\right)\left(2+\left(\frac{d}{dl}T(l)\right)^2\right)f(l) + V(l)\left(\frac{d}{dl}T(l)\right)^3\left(\frac{d}{dl}f(l)\right) = 0 \quad \textbf{(32)}$$

> $eval\left(\ \textbf{(32)},\ \left[V(l)\right.\right.$

$$= \left(\left((1+l)^{\frac{4}{\omega}} \left(2\ \omega^2\ r_0^2\ (1+l)^2\ (\omega-2)\ (1+l)^{\frac{2}{\omega}} - 2\left(\omega-\frac{3}{2}\right) r_0^4\ (1+l)^{\frac{4}{\omega}} + \omega^4\ (1+l)^4 \right) \right)^{1/2} l^4 \right) \Big/ \left((1+l)^{\frac{4}{\omega}} (1+l)^2\ r_0^2\ \omega^2\ \left(-l^4 + \_C1\right) \sqrt{1+\frac{l^4}{-l^4+\_C1}} \right), \frac{\mathrm{d}}{\mathrm{d}l}\ T(l) = \frac{l^2}{\sqrt{-l^4+\_C1}} \Bigg] \Bigg)$$

$$\Bigg(-\left(4\left((1+l)^{\frac{4}{\omega}} \left(2\,\omega^2 r_0^2 (1+l)^2 (\omega-2)\,(1+l)^{\frac{2}{\omega}} - 2\left(\omega-\frac{3}{2}\right) r_0^4 (1+l)^{\frac{4}{\omega}} + \omega^4 (1+l)^4 \right) \right)^{1/2} l^4 \right) \Big/ \left((1+l)^{\frac{4}{\omega}} (1+l)^3 r_0^2\,\omega^3 \left(-l^4+\_C1\right) \sqrt{1+\frac{l^4}{-l^4+\_C1}} \right)$$

$$+ \frac{1}{2} \Bigg(l^4 \Bigg(\frac{1}{\omega\,(1+l)} \left(4\,(1+l)^{\frac{4}{\omega}} \left(2\,\omega^2 r_0^2 (1+l)^2 (\omega-2)\,(1+l)^{\frac{2}{\omega}} - 2\left(\omega-\frac{3}{2}\right) r_0^4 (1+l)^{\frac{4}{\omega}} + \omega^4 (1+l)^4 \right) \right) + (1+l)^{\frac{4}{\omega}} \Bigg(4\,\omega^2 r_0^2 (1+l)\,(\omega-2)\,(1+l)^{\frac{2}{\omega}}$$

$$+ 4\,\omega\, r_0^2 (1+l)\,(\omega-2)\,(1+l)^{\frac{2}{\omega}} - \frac{8\left(\omega-\frac{3}{2}\right) r_0^4 (1+l)^{\frac{4}{\omega}}}{\omega\,(1+l)} + 4\,\omega^4 (1+l)^3 \Bigg) \Bigg) \Bigg)$$

$$\Bigg/ \Bigg((1$$

$$+ l)$$

$$\frac{4}{\omega} \left((1+l)^{\frac{4}{\omega}} \left(2\,\omega^2\, r_0^2\, (1+l)^2\, (\omega-2)\, (1+l)^{\frac{2}{\omega}} - 2\left(\omega - \frac{3}{2}\right) r_0^4\, (1+l)^{\frac{4}{\omega}}\right.\right.$$

$$\left.\left. + \omega^4\, (1+l)^4 \right)\right)^{1/2} (1+l)^2\, r_0^2\, \omega^2\, \left(-t^4 + \_C1\right) \sqrt{1 + \frac{t^4}{-t^4 + \_C1}}\ \Bigg)$$

$$- \left(2\sqrt{(1+l)^{\frac{4}{\omega}} \left(2\,\omega^2\, r_0^2\, (1+l)^2\, (\omega-2)\, (1+l)^{\frac{2}{\omega}} - 2\left(\omega - \frac{3}{2}\right) r_0^4\, (1+l)^{\frac{4}{\omega}} + \omega^4\, (1+l)^4 \right)}\ t^4 \right)$$

$$+ \_C1) \sqrt{1 + \frac{t^4}{-t^4 + \_C1}}\ \Bigg)$$

$$+\Biggl(4\sqrt{(1+l)^{\frac{4}{\omega}}\left(2\,\omega^{2}\,r_0^{2}\,(1+l)^{2}\,(\omega-2)\,(1+l)^{\frac{2}{\omega}}-2\left(\omega-\frac{3}{2}\right)r_0^{4}\,(1+l)^{\frac{4}{\omega}}+\omega^{4}\,(1+l)^{4}\right)}\;l^{3}\Biggr)$$

$$+\_C1\bigr)\sqrt{1+\frac{l^{4}}{-l^{4}+\_C1}}\;\Biggr)$$

$$+\Biggl(4\sqrt{(1+l)^{\frac{4}{\omega}}\left(2\,\omega^{2}\,r_0^{2}\,(1+l)^{2}\,(\omega-2)\,(1+l)^{\frac{2}{\omega}}-2\left(\omega-\frac{3}{2}\right)r_0^{4}\,(1+l)^{\frac{4}{\omega}}+\omega^{4}\,(1+l)^{4}\right)}\;l^{7}\Biggr)$$

$$+\_C1\bigr)^{2}\sqrt{1+\frac{l^{4}}{-l^{4}+\_C1}}\;\Biggr)$$

$$-\frac{1}{2}\Biggl(\Biggl(\biggl((1+l)^{\frac{4}{\omega}}\Bigl(2\,\omega^{2}\,r_0^{2}\,(1+l)^{2}\,(\omega-2)\,(1+l)^{\frac{2}{\omega}}-2\left(\omega-\frac{3}{2}\right)r_0^{4}\,(1$$

$$+l)^{\frac{4}{\omega}}+\omega^{4}\,(1+l)^{4}\Bigr)\biggr)^{1/2}\,l^{4}\left(\frac{4\,l^{3}}{-l^{4}+\_C1}+\frac{4\,l^{7}}{\left(-l^{4}+\_C1\right)^{2}}\right)\Biggr)\Bigg/\Biggl((1$$

$$+l)^{\frac{4}{\omega}}\,(1+l)^{2}\,r_0^{2}\,\omega^{2}\,\left(-l^{4}+\_C1\right)\left(1+\frac{l^{4}}{-l^{4}+\_C1}\right)^{3/2}\Biggr)\Biggr)\left(1+\frac{l^{4}}{-l^{4}+\_C1}\right)\Biggl(2$$

$$+\frac{l^{4}}{-l^{4}+\_C1}\Biggr)f(l)$$

$$0\left(\frac{\mathrm{d}}{\mathrm{d}l}\,f(l)\right)\Biggr)\Bigg/\left((1+l)^{\frac{4}{\omega}}\,(1+l)^{2}\,r_0^{2}\,\omega^{2}\,\left(-l^{4}+\_C1\right)^{5/2}\sqrt{1+\frac{l^{4}}{-l^{4}+\_C1}}\right)=0$$

> *isolate*(**(33)**, *diff* ($f(l)$, l))

$$\frac{d}{dl} f(l) = - \Bigg(\Bigg($$

$$- \Bigg(4 \Bigg((1+l)^{\frac{4}{\omega}} \Bigg(2\,\omega^2\, r_0^2\, (1+l)^2 \left(\omega - 2\right) (1+l)^{\frac{2}{\omega}} - 2 \left(\omega - \frac{3}{2}\right) r_0^4\, (1+l)^{\frac{4}{\omega}}$$

$$+ \omega^4\, (1+l)^4 \Bigg) \Bigg)^{1/2} l^4 \Bigg) \Bigg/ \Bigg((1+l)^{\frac{4}{\omega}} (1+l)^3\, r_0^2\, \omega^3 \left(-l^4 \right.$$

$$\left. + \_C1 \right) \sqrt{1 + \frac{l^4}{-l^4 + \_C1}} \Bigg) + \frac{1}{2} \Bigg(l^4 \Bigg(\frac{1}{\omega\,(1+l)} \Bigg(4\,(1+l)^{\frac{4}{\omega}} \Bigg(2\,\omega^2$$

$$r_0^2\, (1+l)^2 \left(\omega - 2\right) (1+l)^{\frac{2}{\omega}} - 2 \left(\omega - \frac{3}{2}\right) r_0^4\, (1+l)^{\frac{4}{\omega}} + \omega^4\, (1+l)^4 \Bigg) \Bigg) + (1$$

$$+ l)^{\frac{4}{\omega}} \Bigg(4\,\omega^2\, r_0^2\, (1+l) \left(\omega - 2\right) (1+l)^{\frac{2}{\omega}} + 4\,\omega\, r_0^2\, (1+l) \left(\omega - 2\right) (1+l)^{\frac{2}{\omega}}$$

$$- \frac{8 \left(\omega - \frac{3}{2}\right) r_0^4 (1+l)^{\frac{4}{\omega}}}{\omega (1+l)} + 4 \omega^4 (1+l)^3 \Bigg)\Bigg)\Bigg) \Bigg/ \Bigg((1$$

$$+ l)$$

$$\frac{4}{\omega} \Bigg((1+l)^{\frac{4}{\omega}} \Bigg(2 \omega^2 r_0^2 (1+l)^2 (\omega - 2) (1+l)^{\frac{2}{\omega}} - 2 \left(\omega - \frac{3}{2}\right) r_0^4 (1+l)^{\frac{4}{\omega}}$$

$$+ \omega^4 (1+l)^4 \Bigg)\Bigg)^{1/2} (1+l)^2 r_0^2 \omega^2 \left(-l^4 + \_C1\right) \sqrt{1 + \frac{l^4}{-l^4 + \_C1}} \Bigg)$$

$$- \Bigg(2 \sqrt{(1+l)^{\frac{4}{\omega}} \left(2 \omega^2 r_0^2 (1+l)^2 (\omega - 2) (1+l)^{\frac{2}{\omega}} - 2 \left(\omega - \frac{3}{2}\right) r_0^4 (1+l)^{\frac{4}{\omega}} + \omega^4 (1+l)^4 \right)} \, l^4 \Bigg)$$

$$+ \_C1) \sqrt{1 + \frac{l^4}{-l^4 + \_C1}} \Bigg)$$

$$+\left(4\sqrt{(1+l)^{\frac{4}{\omega}}\left(2\,\omega^2\,r_0^2\,(1+l)^2\,(\omega-2)\,(1+l)^{\frac{2}{\omega}}-2\left(\omega-\frac{3}{2}\right)r_0^4\,(1+l)^{\frac{4}{\omega}}+\omega^4\,(1+l)^4\right)}\;l^3\right)$$

$$+\_C1)\sqrt{1+\frac{l^4}{-l^4+\_C1}}\right)$$

$$+\left(4\sqrt{(1+l)^{\frac{4}{\omega}}\left(2\,\omega^2\,r_0^2\,(1+l)^2\,(\omega-2)\,(1+l)^{\frac{2}{\omega}}-2\left(\omega-\frac{3}{2}\right)r_0^4\,(1+l)^{\frac{4}{\omega}}+\omega^4\,(1+l)^4\right)}\;l^7\right)$$

$$+\_C1)^2\sqrt{1+\frac{l^4}{-l^4+\_C1}}\right)$$

$$-\frac{1}{2}\left(\left((1+l)^{\frac{4}{\omega}}\left(2\,\omega^2\,r_0^2\,(1+l)^2\,(\omega-2)\,(1+l)^{\frac{2}{\omega}}-2\left(\omega-\frac{3}{2}\right)r_0^4\,(1\right.\right.\right.$$

$$\left.\left.+l)^{\frac{4}{\omega}}+\omega^4\,(1+l)^4\right)\right)^{1/2} l^4\left(\frac{4\,l^3}{-l^4+\_C1}+\frac{4\,l^7}{(-l^4+\_C1)^2}\right)\right)\Bigg/\left((1\right.$$

$$\left.\left.+l)^{\frac{4}{\omega}}\,(1+l)^2\,r_0^2\,\omega^2\,(-l^4+\_C1)\left(1+\frac{l^4}{-l^4+\_C1}\right)^{3/2}\right)\right)\left(1+\frac{l^4}{-l^4+\_C1}\right)^{3/2}\left(2\right.$$

$$\left.+\frac{l^4}{-l^4+\_C1}\right)f(l)\,(1+l)^{\frac{4}{\omega}}\,(1+l)^2\,r_0^2\,\omega^2\,(-l^4+\_C1)^{5/2}\right) \Bigg/$$

$$\left(\left(\left((1+l)^{\frac{4}{\omega}}\left(2\,\omega^2\,r_0^2\,(1+l)^2\,(\omega-2)\,(1+l)^{\frac{2}{\omega}}-2\left(\omega-\frac{3}{2}\right)r_0^4\,(1+l)^{\frac{4}{\omega}}+\omega^4\,(1+l)^4\right)\right)^{1/2} l^{10}\right)$$

> *simplify*(**(33)**, '*size*')

$$\left(2\,\_C1\,l^3\left((\omega-2)\,r_0^2\,(1+l)^2\left((1+l)\,(l^4-\_C1)^4\,l^7\,\omega\left(\frac{d}{dl}f(l)\right)+f(l)\,\_C1\,\big((\omega-1)\,l^5+2\,\omega\,l^4+(-3\,\omega+1)\,\_C1\,l-4\,\_C1\,\omega\big)\,(l^4-2\,\_C1)\,(-l^4+\_C1)^{5/2}\right)\omega\,(1+l)^{\frac{2}{\omega}}-2\,r_0^4\left(\omega-\frac{3}{2}\right)\left(\frac{1}{2}\,l^7\,(1+l)\,(l^4-\_C1)^4\left(\frac{d}{dl}f(l)\right)+(l^4-\_C1\,l-2\,\_C1)\,f(l)\,\_C1\,(l^4-2\,\_C1)\,(-l^4+\_C1)^{5/2}\right)(1+l)^{\frac{4}{\omega}}+(1+l)^4\left(\frac{1}{2}\,(1+l)\,(l^4-\_C1)^4\,l^7\,\omega\left(\frac{d}{dl}f(l)\right)+f(l)\,\_C1\,\big((\omega-1)\,l^5+\omega\,l^4+(-2\,\omega+1)\,\_C1\,l-2\,\_C1\,\omega\big)\,(l^4-2\,\_C1)\,(-l^4+\_C1)^{5/2}\right)\omega^3\right)\right)\Bigg/\left(\left(-\frac{\_C1}{l^4-\_C1}\right)^{3/2}\left((1+l)^{\frac{4}{\omega}}\left(2\,\omega^2\,r_0^2\,(1+l)^2\,(\omega-2)\,(1+l)^{\frac{2}{\omega}}-2\left(\omega-\frac{3}{2}\right)r_0^4\,(1+l)^{\frac{4}{\omega}}+\omega^4\,(1+l)^4\right)\right)^{1/2}\omega^2\,(1+l)^3\,r_0^2\,(-l^4+\_C1)^{15/2}\right)=0 \quad \textbf{(35)}$$

> *isolate*(**(35)**, *diff* (*f* (*l*), *l*))

$$\frac{d}{dl}f(l)=\Big(-6\,r_0^4\,(1+l)^{\frac{4}{\omega}}\,f(l)\,(-l^4+\_C1)^{5/2}\,\_C1^3\,l+8\,r_0^4\,(1+l)^{\frac{4}{\omega}}\,f(l)\,(-l^4+\_C1)^{5/2}\,\_C1^3\,\omega-3\,r_0^4\,(1+l)^{\frac{4}{\omega}}\,f(l)\,(-l^4+\_C1)^{5/2}\,\_C1\,l^8+12\,r_0^4\,(1+l)^{\frac{4}{\omega}}\,f(l)\,(-l^4+\_C1)^{5/2}\,\_C1^2\,l^4+16\,r_0^2\,\omega^2\,(1+l)^{\frac{2}{\omega}}\,f(l)\,(-l^4+\_C1)^{5/2}\,\_C1^3-8\,r_0^2\,\omega^3\,(1+l)^{\frac{2}{\omega}}\,f(l)\,(-l^4+\_C1)^{5/2}\,\_C1^3+3\,r_0^4\,(1+l)^{\frac{4}{\omega}}\,f(l)\,(-l^4+\_C1)^{5/2}\,\_C1^2\,l^5 \quad \textbf{(36)}$$

$$-4\,\omega^{4} f(l)\,(-l^{4}+\_C1)^{5/2}\,\_C1^{3}-12\,r_{0}^{4}\,(1+l)^{\frac{4}{\omega}} f(l)\,(-l^{4}+\_C1)^{5/2}\,\_C1^{3}-8$$

$$r_{0}^{4}\,(1+l)^{\frac{4}{\omega}} f(l)\,(-l^{4}+\_C1)^{5/2}\,\_C1^{2}\,l^{4}\,\omega+4\,r_{0}^{4}\,(1+l)^{\frac{4}{\omega}} f(l)\,(-l^{4}+\_C1)^{5}$$

$${}^{/2}\,\_C1^{3}\,l\,\omega-2\,r_{0}^{2}\,\omega\,(1+l)^{\frac{2}{\omega}} f(l)\,(-l^{4}+\_C1)^{5/2}\,\_C1\,l^{9}+6\,r_{0}^{2}\,\omega\,(1+l)^{\frac{2}{\omega}} f(l)\,($$

$$-l^{4}+\_C1)^{5/2}\,\_C1^{2}\,l^{5}-4\,r_{0}^{2}\,\omega\,(1+l)^{\frac{2}{\omega}} f(l)\,(-l^{4}+\_C1)^{5/2}\,\_C1^{3}\,l-2\,r_{0}^{2}\,\omega\,(1$$

$$+l)^{\frac{2}{\omega}} f(l)\,(-l^{4}+\_C1)^{5/2}\,\_C1\,l^{11}-4\,r_{0}^{2}\,\omega\,(1+l)^{\frac{2}{\omega}} f(l)\,(-l^{4}+\_C1)^{5/2}\,\_C1\,l^{10}$$

$$+6\,r_{0}^{2}\,\omega\,(1+l)^{\frac{2}{\omega}} f(l)\,(-l^{4}+\_C1)^{5/2}\,\_C1^{2}\,l^{7}+12\,r_{0}^{2}\,\omega\,(1+l)^{\frac{2}{\omega}} f(l)\,(-l^{4}$$

$$+\_C1)^{5/2}\,\_C1^{2}\,l^{6}-4\,r_{0}^{2}\,\omega\,(1+l)^{\frac{2}{\omega}} f(l)\,(-l^{4}+\_C1)^{5/2}\,\_C1^{3}\,l^{3}-8\,r_{0}^{2}\,\omega\,(1$$

$$+l)^{\frac{2}{\omega}} f(l)\,(-l^{4}+\_C1)^{5/2}\,\_C1^{3}\,l^{2}+11\,r_{0}^{2}\,\omega^{2}\,(1+l)^{\frac{2}{\omega}} f(l)\,(-l^{4}+\_C1)^{5/2}\,\_C1\,l^{9}$$

$$+4\,r_{0}^{2}\,\omega^{2}\,(1+l)^{\frac{2}{\omega}} f(l)\,(-l^{4}+\_C1)^{5/2}\,\_C1\,l^{8}-45\,r_{0}^{2}\,\omega^{2}\,(1+l)^{\frac{2}{\omega}} f(l)\,(-l^{4}$$

$$+\_C1)^{5/2}\,\_C1^{2}\,l^{5}-16\,r_{0}^{2}\,\omega^{2}\,(1+l)^{\frac{2}{\omega}} f(l)\,(-l^{4}+\_C1)^{5/2}\,\_C1^{2}\,l^{4}+46\,r_{0}^{2}\,\omega^{2}\,(1$$

$$+l)^{\frac{2}{\omega}} f(l)\,(-l^{4}+\_C1)^{5/2}\,\_C1^{3}\,l-r_{0}^{2}\,\omega^{3}\,(1+l)^{\frac{2}{\omega}} f(l)\,(-l^{4}+\_C1)^{5/2}\,\_C1\,l^{11}+3$$

$$r_{0}^{2}\,\omega^{2}\,(1+l)^{\frac{2}{\omega}} f(l)\,(-l^{4}+\_C1)^{5/2}\,\_C1\,l^{11}-4\,r_{0}^{2}\,\omega^{3}\,(1+l)^{\frac{2}{\omega}} f(l)\,(-l^{4}+\_C1)^{5}$$

$${}^{/2}\,\_C1\,l^{10}+10\,r_{0}^{2}\,\omega^{2}\,(1+l)^{\frac{2}{\omega}} f(l)\,(-l^{4}+\_C1)^{5/2}\,\_C1\,l^{10}-5\,r_{0}^{2}\,\omega^{3}\,(1$$

$$+l)^{\frac{2}{\omega}} f(l)\,(-l^{4}+\_C1)^{5/2}\,\_C1\,l^{9}-2\,r_{0}^{2}\,\omega^{3}\,(1+l)^{\frac{2}{\omega}} f(l)\,(-l^{4}+\_C1)^{5/2}\,\_C1\,l^{8}$$

$$+5\,r_{0}^{2}\,\omega^{3}\,(1+l)^{\frac{2}{\omega}} f(l)\,(-l^{4}+\_C1)^{5/2}\,\_C1^{2}\,l^{7}-13\,r_{0}^{2}\,\omega^{2}\,(1+l)^{\frac{2}{\omega}} f(l)\,(-l^{4}$$

$$+\_C1)^{5/2}\,\_C1^{2}\,l^{7}+18\,r_{0}^{2}\,\omega^{3}\,(1+l)^{\frac{2}{\omega}} f(l)\,(-l^{4}+\_C1)^{5/2}\,\_C1^{2}\,l^{6}-42\,r_{0}^{2}\,\omega^{2}\,(1$$

$$+ l)^{\frac{2}{\omega}} f(l) \left(-l^4 + \_C1\right)^{5/2} \_C1^2 l^6 + 21 r_0^2 \omega^3 (1+l)^{\frac{2}{\omega}} f(l) \left(-l^4 + \_C1\right)^{5/2} \_C1^2 l^5$$

$$+ 8 r_0^2 \omega^3 (1+l)^{\frac{2}{\omega}} f(l) \left(-l^4 + \_C1\right)^{5/2} \_C1^2 l^4 - 6 r_0^2 \omega^3 (1+l)^{\frac{2}{\omega}} f(l) \left(-l^4\right.$$

$$\left.+ \_C1\right)^{5/2} \_C1^3 l^3 + 14 r_0^2 \omega^2 (1+l)^{\frac{2}{\omega}} f(l) \left(-l^4 + \_C1\right)^{5/2} \_C1^3 l^3 - 20 r_0^2 \omega^3 (1$$

$$+ l)^{\frac{2}{\omega}} f(l) \left(-l^4 + \_C1\right)^{5/2} \_C1^3 l^2 + 44 r_0^2 \omega^2 (1+l)^{\frac{2}{\omega}} f(l) \left(-l^4 + \_C1\right)^{5/2} \_C1^3 l^2$$

$$- 22 r_0^2 \omega^3 (1+l)^{\frac{2}{\omega}} f(l) \left(-l^4 + \_C1\right)^{5/2} \_C1^3 l + 2 r_0^4 (1+l)^{\frac{4}{\omega}} f(l) \left(-l^4\right.$$

$$\left.+ \_C1\right)^{5/2} \_C1\, l^8 \omega - 2 r_0^4 (1+l)^{\frac{4}{\omega}} f(l) \left(-l^4 + \_C1\right)^{5/2} \_C1^2 l^5 \omega$$

$$+ 4 \omega^4 \_C1^2 f(l) \left(-l^4 + \_C1\right)^{5/2} l^9 - 4 \omega^4 \_C1^3 f(l) \left(-l^4 + \_C1\right)^{5/2} l^5$$

$$+ 20 \omega^4 \_C1^2 f(l) \left(-l^4 + \_C1\right)^{5/2} l^8 - 20 \omega^4 \_C1^3 f(l) \left(-l^4 + \_C1\right)^{5/2} l^4$$

$$+ \omega^3 f(l) \left(-l^4 + \_C1\right)^{5/2} \_C1\, l^9 - 3 \omega^3 f(l) \left(-l^4 + \_C1\right)^{5/2} \_C1^2 l^5 + 2 \omega^3 f(l) \Big($$

$$-l^4 + \_C1\Big)^{5/2} \_C1^3 l + 6 \omega^3 f(l) \left(-l^4 + \_C1\right)^{5/2} \_C1\, l^{11} + 4 \omega^3 f(l) \left(-l^4\right.$$

$$\left.+ \_C1\right)^{5/2} \_C1\, l^{10} - 18 \omega^3 f(l) \left(-l^4 + \_C1\right)^{5/2} \_C1^2 l^7 - 12 \omega^3 f(l) \left(-l^4\right.$$

$$\left.+ \_C1\right)^{5/2} \_C1^2 l^6 + 12 \omega^3 f(l) \left(-l^4 + \_C1\right)^{5/2} \_C1^3 l^3 + 8 \omega^3 f(l) \left(-l^4 + \_C1\right)^{5}$$

$$^{/2} \_C1^3 l^2 + 4 \omega^3 f(l) \left(-l^4 + \_C1\right)^{5/2} \_C1\, l^{12} - 12 \omega^3 f(l) \left(-l^4 + \_C1\right)^{5/2} \_C1^2 l^8$$

$$+ 8 \omega^3 f(l) \left(-l^4 + \_C1\right)^{5/2} \_C1^3 l^4 - 3 \omega^3 \_C1^2 f(l) \left(-l^4 + \_C1\right)^{5/2} l^9$$

$$+ \omega^3 \_C1 f(l) \left(-l^4 + \_C1\right)^{5/2} l^{13} + 2 \omega^3 \_C1^3 f(l) \left(-l^4 + \_C1\right)^{5/2} l^5$$

$$- 5 \omega^4 f(l) \left(-l^4 + \_C1\right)^{5/2} \_C1\, l^9 - \omega^4 f(l) \left(-l^4 + \_C1\right)^{5/2} \_C1\, l^8 + 20 \omega^4 f(l) \Big($$

$$-l^4 + \_C1\Big)^{5/2} \_C1^2 l^5 + 4 \omega^4 f(l) \left(-l^4 + \_C1\right)^{5/2} \_C1^2 l^4 - 20 \omega^4 f(l) \left(-l^4\right.$$

$$\left.+ \_C1\right)^{5/2} \_C1^3 l - 10 \omega^4 f(l) \left(-l^4 + \_C1\right)^{5/2} \_C1\, l^{11} - 10 \omega^4 f(l) \left(-l^4 + \_C1\right)^{5}$$

$$^{/2} \_C1\, l^{10} + 40 \omega^4 f(l) \left(-l^4 + \_C1\right)^{5/2} \_C1^2 l^7 + 40 \omega^4 f(l) \left(-l^4 + \_C1\right)^{5/2} \_C1^2 l^6$$

$$- 40 \omega^4 f(l) \left(-l^4 + \_C1\right)^{5/2} \_C1^3 l^3 - 40 \omega^4 f(l) \left(-l^4 + \_C1\right)^{5/2} \_C1^3 l^2$$

$$- 5 \omega^4 f(l) \left(-l^4 + \_C1\right)^{5/2} \_C1\, l^{12} - \omega^4 \_C1 f(l) \left(-l^4 + \_C1\right)^{5/2} l^{13}\Big) \Big/ \Big(-6$$

$$r_0^2\,\omega^2\,(1+l)^{\frac{2}{\omega}}\,l^{24}-2\,r_0^2\,\omega^2\,(1+l)^{\frac{2}{\omega}}\,l^{23}+r_0^2\,\omega^3\,(1+l)^{\frac{2}{\omega}}\,l^{26}-2\,r_0^2\,\omega^2\,(1+l)^{\frac{2}{\omega}}\,l^{26}$$
$$+3\,r_0^2\,\omega^3\,(1+l)^{\frac{2}{\omega}}\,l^{25}-6\,r_0^2\,\omega^2\,(1+l)^{\frac{2}{\omega}}\,l^{25}+3\,r_0^2\,\omega^3\,(1+l)^{\frac{2}{\omega}}\,l^{24}+r_0^2\,\omega^3\,(1$$
$$+l)^{\frac{2}{\omega}}\,l^{23}-r_0^4\,(1+l)^{\frac{4}{\omega}}\,l^{24}\,\omega-r_0^4\,(1+l)^{\frac{4}{\omega}}\,l^{23}\,\omega-6\,r_0^4\,(1+l)^{\frac{4}{\omega}}\,\_C1\,l^{20}-6$$
$$r_0^4\,(1+l)^{\frac{4}{\omega}}\,\_C1\,l^{19}+9\,r_0^4\,(1+l)^{\frac{4}{\omega}}\,\_C1^2\,l^{16}+9\,r_0^4\,(1+l)^{\frac{4}{\omega}}\,\_C1^2\,l^{15}-6\,r_0^4\,(1$$
$$+l)^{\frac{4}{\omega}}\,\_C1^3\,l^{12}-6\,r_0^4\,(1+l)^{\frac{4}{\omega}}\,\_C1^3\,l^{11}+\frac{3}{2}\,r_0^4\,(1+l)^{\frac{4}{\omega}}\,\_C1^4\,l^8+\frac{3}{2}\,r_0^4\,(1$$
$$+l)^{\frac{4}{\omega}}\,\_C1^4\,l^7-4\,r_0^2\,\omega^3\,(1+l)^{\frac{2}{\omega}}\,\_C1^3\,l^{11}+r_0^2\,\omega^3\,(1+l)^{\frac{2}{\omega}}\,\_C1^4\,l^{10}-2\,r_0^2\,\omega^2\,(1$$
$$+l)^{\frac{2}{\omega}}\,\_C1^4\,l^{10}+3\,r_0^2\,\omega^3\,(1+l)^{\frac{2}{\omega}}\,\_C1^4\,l^9-6\,r_0^2\,\omega^2\,(1+l)^{\frac{2}{\omega}}\,\_C1^4\,l^9+3\,r_0^2\,\omega^3\,(1$$
$$+l)^{\frac{2}{\omega}}\,\_C1^4\,l^8+r_0^2\,\omega^3\,(1+l)^{\frac{2}{\omega}}\,\_C1^4\,l^7+4\,r_0^4\,(1+l)^{\frac{4}{\omega}}\,\_C1\,l^{20}\,\omega+4\,r_0^4\,(1$$
$$+l)^{\frac{4}{\omega}}\,\_C1\,l^{19}\,\omega-6\,r_0^4\,(1+l)^{\frac{4}{\omega}}\,\_C1^2\,l^{16}\,\omega-6\,r_0^4\,(1+l)^{\frac{4}{\omega}}\,\_C1^2\,l^{15}\,\omega+4\,r_0^4\,(1$$
$$+l)^{\frac{4}{\omega}}\,\_C1^3\,l^{12}\,\omega+4\,r_0^4\,(1+l)^{\frac{4}{\omega}}\,\_C1^3\,l^{11}\,\omega-r_0^4\,(1+l)^{\frac{4}{\omega}}\,\_C1^4\,l^8\,\omega-r_0^4\,(1$$
$$+l)^{\frac{4}{\omega}}\,\_C1^4\,l^7\,\omega-20\,\omega^4\,\_C1\,l^{22}-20\,\omega^4\,\_C1\,l^{21}+30\,\omega^4\,\_C1^2\,l^{18}+30\,\omega^4\,\_C1^2\,l^{17}$$
$$-20\,\omega^4\,\_C1^3\,l^{14}-20\,\omega^4\,\_C1^3\,l^{13}+5\,\omega^4\,\_C1^4\,l^{10}+5\,\omega^4\,\_C1^4\,l^9-2\,\omega^4\,\_C1\,l^{24}$$
$$-10\,\omega^4\,\_C1\,l^{23}+3\,\omega^4\,\_C1^2\,l^{20}+15\,\omega^4\,\_C1^2\,l^{19}-2\,\omega^4\,\_C1^3\,l^{16}-10\,\omega^4\,\_C1^3\,l^{15}$$
$$+\frac{1}{2}\,\omega^4\,\_C1^4\,l^{12}+\frac{5}{2}\,\omega^4\,\_C1^4\,l^{11}+\frac{3}{2}\,r_0^4\,(1+l)^{\frac{4}{\omega}}\,l^{24}+\frac{3}{2}\,r_0^4\,(1+l)^{\frac{4}{\omega}}\,l^{23}$$
$$-10\,\omega^4\,\_C1\,l^{20}-2\,\omega^4\,\_C1\,l^{19}+15\,\omega^4\,\_C1^2\,l^{16}+3\,\omega^4\,\_C1^2\,l^{15}-10\,\omega^4\,\_C1^3\,l^{12}$$

$$-2\,\omega^4\,\_C1^3\,l^{11}+\frac{5}{2}\,\omega^4\,\_C1^4\,l^8+\frac{1}{2}\,\omega^4\,\_C1^4\,l^7+24\,r_0^2\,\omega^2\,(1+l)^{\frac{2}{\omega}}\,\_C1\,l^{20}+8$$

$$r_0^2\,\omega^2\,(1+l)^{\frac{2}{\omega}}\,\_C1\,l^{19}-36\,r_0^2\,\omega^2\,(1+l)^{\frac{2}{\omega}}\,\_C1^2\,l^{16}-12\,r_0^2\,\omega^2\,(1+l)^{\frac{2}{\omega}}\,\_C1^2\,l^{15}+24$$

$$r_0^2\,\omega^2\,(1+l)^{\frac{2}{\omega}}\,\_C1^3\,l^{12}-2\,r_0^2\,\omega^2\,(1+l)^{\frac{2}{\omega}}\,\_C1^4\,l^7+8\,r_0^2\,\omega^2\,(1+l)^{\frac{2}{\omega}}\,\_C1^3\,l^{11}-6$$

$$r_0^2\,\omega^2\,(1+l)^{\frac{2}{\omega}}\,\_C1^4\,l^8-4\,r_0^2\,\omega^3\,(1+l)^{\frac{2}{\omega}}\,\_C1\,l^{22}+8\,r_0^2\,\omega^2\,(1+l)^{\frac{2}{\omega}}\,\_C1\,l^{22}-12$$

$$r_0^2\,\omega^3\,(1+l)^{\frac{2}{\omega}}\,\_C1\,l^{21}+24\,r_0^2\,\omega^2\,(1+l)^{\frac{2}{\omega}}\,\_C1\,l^{21}-12\,r_0^2\,\omega^3\,(1+l)^{\frac{2}{\omega}}\,\_C1\,l^{20}-4$$

$$r_0^2\,\omega^3\,(1+l)^{\frac{2}{\omega}}\,\_C1\,l^{19}+6\,r_0^2\,\omega^3\,(1+l)^{\frac{2}{\omega}}\,\_C1^2\,l^{18}-12\,r_0^2\,\omega^2\,(1+l)^{\frac{2}{\omega}}\,\_C1^2\,l^{18}+18$$

$$r_0^2\,\omega^3\,(1+l)^{\frac{2}{\omega}}\,\_C1^2\,l^{17}-36\,r_0^2\,\omega^2\,(1+l)^{\frac{2}{\omega}}\,\_C1^2\,l^{17}+18\,r_0^2\,\omega^3\,(1+l)^{\frac{2}{\omega}}\,\_C1^2\,l^{16}+6$$

$$r_0^2\,\omega^3\,(1+l)^{\frac{2}{\omega}}\,\_C1^2\,l^{15}-4\,r_0^2\,\omega^3\,(1+l)^{\frac{2}{\omega}}\,\_C1^3\,l^{14}+8\,r_0^2\,\omega^2\,(1+l)^{\frac{2}{\omega}}\,\_C1^3\,l^{14}-12$$

$$r_0^2\,\omega^3\,(1+l)^{\frac{2}{\omega}}\,\_C1^3\,l^{13}+24\,r_0^2\,\omega^2\,(1+l)^{\frac{2}{\omega}}\,\_C1^3\,l^{13}-12\,r_0^2\,\omega^3\,(1+l)^{\frac{2}{\omega}}\,\_C1^3\,l^{12}$$

$$+\frac{5}{2}\,\omega^4\,l^{24}+\frac{1}{2}\,\omega^4\,l^{23}+5\,\omega^4\,l^{26}+5\,\omega^4\,l^{25}+\frac{1}{2}\,\omega^4\,l^{28}+\frac{5}{2}\,\omega^4\,l^{27}\Big)$$

> *simplify*(**(36)**, '*size*')

$$\frac{\mathrm{d}}{\mathrm{d}l}\,f(l)=-\Bigg(2\,f(l)\,\_C1\,\left(-l^4+\_C1\right)^{5/2}\Bigg((\omega-2)\,r_0^2\,\Big((\omega-1)\,l^5+2\,\omega\,l^4+(-3\,\_C1\,\omega+\_C1)\,l-4\,\_C1\,\omega\Big)\,(1+l)^2\,\omega\,(1+l)^{\frac{2}{\omega}}-2\,\left(l^4-\_C1\,l-2\,\_C1\right)\,r_0^4\left(\omega-\frac{3}{2}\right)(1+l)^{\frac{4}{\omega}}+(1+l)^4\Big((\omega-1)\,l^5+\omega\,l^4+(-2\,\_C1\,\omega+\_C1)\,l-2\,\_C1\,\omega\Big)\,\omega^3\Bigg)\left(l^4-2\,\_C1\right)\Bigg)\Bigg/\Bigg(\left(l^4-\_C1\right)^4(1+l)\,l^7\Bigg(2\,\omega^2\,r_0^2\,(1+l)^2\,(\omega-2)\,(1+l)^{\frac{2}{\omega}}-2\left(\omega-\frac{3}{2}\right)r_0^4\,(1+l)^{\frac{4}{\omega}}+\omega^4\,(1+l)^4\Bigg)\Bigg) \tag{37}$$

>

> $$\frac{\frac{d}{dl} f(l)}{f(l)} = -\left(2 f(l)\_C1 \left(-l^4+\_C1\right)^{5/2} \left((\omega-2) r_0^2 \left(\left((\omega-1) l^5 + 2\omega l^4 + (-3\_C1\,\omega + \_C1) l - 4\_C1\,\omega\right) (1+l)^2 \omega (1+l)^{\frac{2}{\omega}} - 2\left(l^4 - \_C1\, l - 2\_C1\right) r_0^4 \left(\omega - \frac{3}{2}\right) (1+l)^{\frac{4}{\omega}} + (1+l)^4 \left((\omega-1) l^5 + \omega l^4 + (-2\_C1\,\omega + \_C1) l - 2\_C1\,\omega\right)\omega^3\right) \left(l^4 - 2\_C1\right)\right) \Big/ \left(\left(l^4-\_C1\right)^4 (1+l) l^7 \left(2\omega^2 r_0^2 (1+l)^2 (\omega-2)(1+l)^{\frac{2}{\omega}} - 2\left(\omega-\frac{3}{2}\right) r_0^4 (1+l)^{\frac{4}{\omega}} + \omega^4 (1+l)^4\right)\right) \cdot \left(\frac{1}{f(l)}\right)$$

$$\frac{\frac{d}{dl} f(l)}{f(l)} = -\left(2\_C1 \left(-l^4+\_C1\right)^{5/2} \left((\omega-2) r_0^2 \left(\left((\omega-1) l^5 + 2\omega l^4 + (-3\_C1\,\omega + \_C1) l - 4\_C1\,\omega\right) (1+l)^2 \omega (1+l)^{\frac{2}{\omega}} - 2\left(l^4 - \_C1\, l - 2\_C1\right) r_0^4 \left(\omega - \frac{3}{2}\right) (1+l)^{\frac{4}{\omega}} + (1+l)^4 \left((\omega-1) l^5 + \omega l^4 + (-2\_C1\,\omega + \_C1) l - 2\_C1\,\omega\right)\omega^3\right) \left(l^4 - 2\_C1\right)\right) \Big/ \left(\left(l^4-\_C1\right)^4 (1+l) l^7 \left(2\omega^2 r_0^2 (1+l)^2 (\omega-2)(1+l)^{\frac{2}{\omega}} - 2\left(\omega-\frac{3}{2}\right) r_0^4 (1+l)^{\frac{4}{\omega}} + \omega^4 (1+l)^4\right)\right) \tag{38}$$

> *simplify*(**(38)**, '*size*')

$$\frac{\frac{d}{dl} f(l)}{f(l)} = -\left(2\_C1 \left(l^4 - 2\_C1\right) \left((\omega-2) r_0^2 \left(\left(l^5 + 2 l^4 - 3\_C1\, l - 4\_C1\right)\omega - l^5 + \_C1\, l\right) (1+l)^2 \omega (1+l)^{\frac{2}{\omega}} - 2\left(l^4 - \_C1\, l - 2\_C1\right) r_0^4 \left(\omega - \frac{3}{2}\right) (1+l)^{\frac{4}{\omega}} + (1+l)^4 \left((1+l)\left(l^4 - 2\_C1\right)\omega - l^5 + \_C1\, l\right)\omega^3\right)\right) \Big/ \left((1+l) l^7 \left(2\omega^2 r_0^2 (1+l)^2 (\omega-2)(1+l)^{\frac{2}{\omega}} - 2\left(\omega-\frac{3}{2}\right) r_0^4 (1+l)^{\frac{4}{\omega}} + \omega^4 (1+l)^4\right)\left(-l^4+\_C1\right)^{3/2}\right) \tag{39}$$

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