Final Report

MECH 393 – Design 2: Machine Element Design

Preliminary Design of an Auxiliary Transmission Box for a Small Aircraft

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Abstract

This report outlines the design process and final design for the transmission gearbox of an airplane intended for geodesic observation missions. The transmission system includes gears, shafts, keys and bearings, and is based on a double-branch, double-reduction configuration. Key parameters such as the required reduction ratio, turbine output power, and operational conditions (RPM, power, and duration) for the plane's mission stages were given constraints, along with the maximum diameter and loads from the propeller.

An analysis was first conducted, using independent variables such as geometry and material properties to derive expressions for the relevant stresses in the gears, shafts, and keys. Standard factors from AGMA were applied to stresses and strengths for each component and factors of safety were then calculated to ensure the design's resilience against potential failure modes, including bending, surface contact and shear fatigue failure or bearing failure. The goal was to create a design that meets all specified requirements while ensuring safety and minimizing weight. The final design weighs 831lb.

The design process began with the sizing of the gears and shafts system, followed by the design of keys and selection of appropriate bearings from the SKF catalog. To structure the design, Excel's capabilities were used to build a detailed model, using formulas and data links. Since not all system dynamics can be represented as continuous functions, particularly when working with discrete data (such as tables or catalogs), a trial-and-error approach to iterate and refine the design was followed.

Lastly, SolidWorks was used to create an assembly model of the gearbox design and generate the necessary engineering drawings for both the assembly and individual components.

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Introduction

The following report covers the preliminary design of a reduction gearbox between the gas turbine and the propeller of a geodesic observation plane. The gearbox will be a double-reduction double-branch system as suggested by the aircraft manufacturer. In the proposed installation, a 15 000-rpm turbine engine outputs power to the gearbox, which reduces it before transmitting it to the aircraft's propeller with a rotation speed of 1 000 rpm. The take-off engine power is 1 050 hp. To fulfill its function, the aircraft has to undergo two different missions, for low and high-altitude observation. Table 1 below summarizes the different mission specifications.

Table 1: Mission specifications

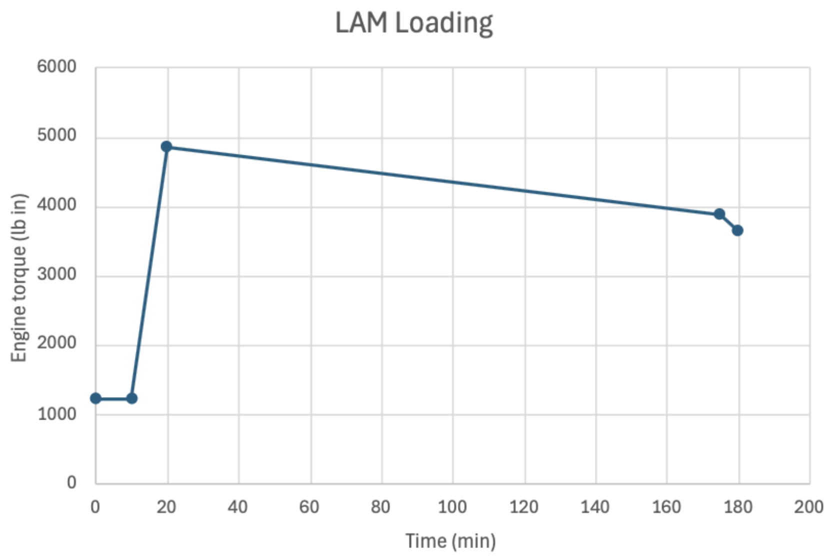
Low altitude (LAM)			
Power mode	By gas turbine	# of missions per cycle	10
Phase	Duration (min)	% of max engine power	% of rotation speed
On the tarmac	10	25	40
Take off	10	100	100
Observation	155	80	100
Landing	5	75	75
High altitude (HAM)			
Power mode	Gliding	# of missions per cycle	1
Phase	Duration (min)	% of max engine power	% of rotation speed
On the tarmac	10	25	40
Take off	10	100	100
Observation	-	-	-
Landing	5	75	75

Using the given values for the engine and propeller specifications, summarized in Table 22 in the Appendix, the following Table 2 was made, calculating the actual power, rotation speed and engine output torque at the different stages of each mission.

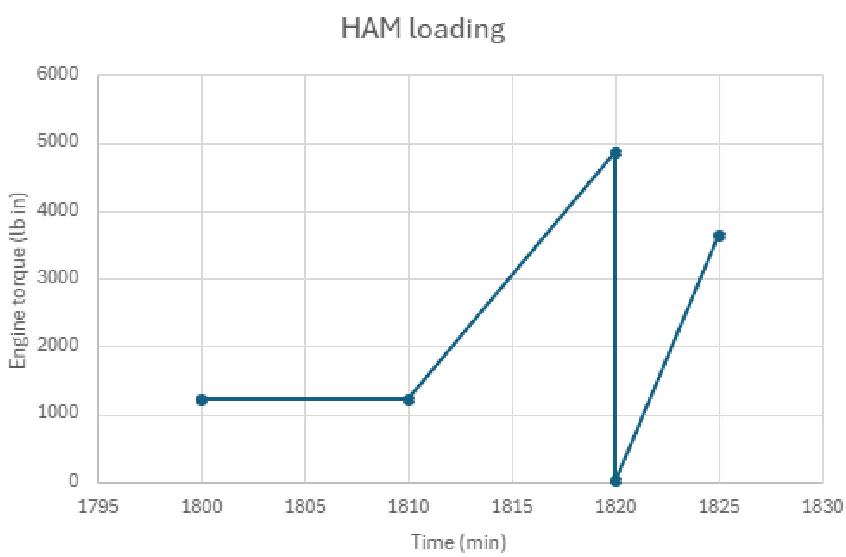
Table 2: Detailed mission specifications

Missions							
Phase	**Duration(min)**	**% of max engine power**	**Actual power (hp)**	**% of rotation speed**	**Actual rotation speed**	**# of cycles (engine output)**	**Engine torque (lb in)**
LAM							
on tarmac	10	25	288.75	40	6000	60000	1213.23125
take off	10	100	1155	100	15000	150000	4852.925
observation	155	80	924	100	15000	2325000	3882.34
landing	5	75	866.25	75	11250	56250	3639.69375
HAM							
on tarmac	10	25	288.75	40	6000	60000	1213.23125
take off	10	100	1155	100	15000	150000	4852.925
observation	0	0	0	0	0	0	0
landing	5	75	866.25	75	11250	56250	3639.69375

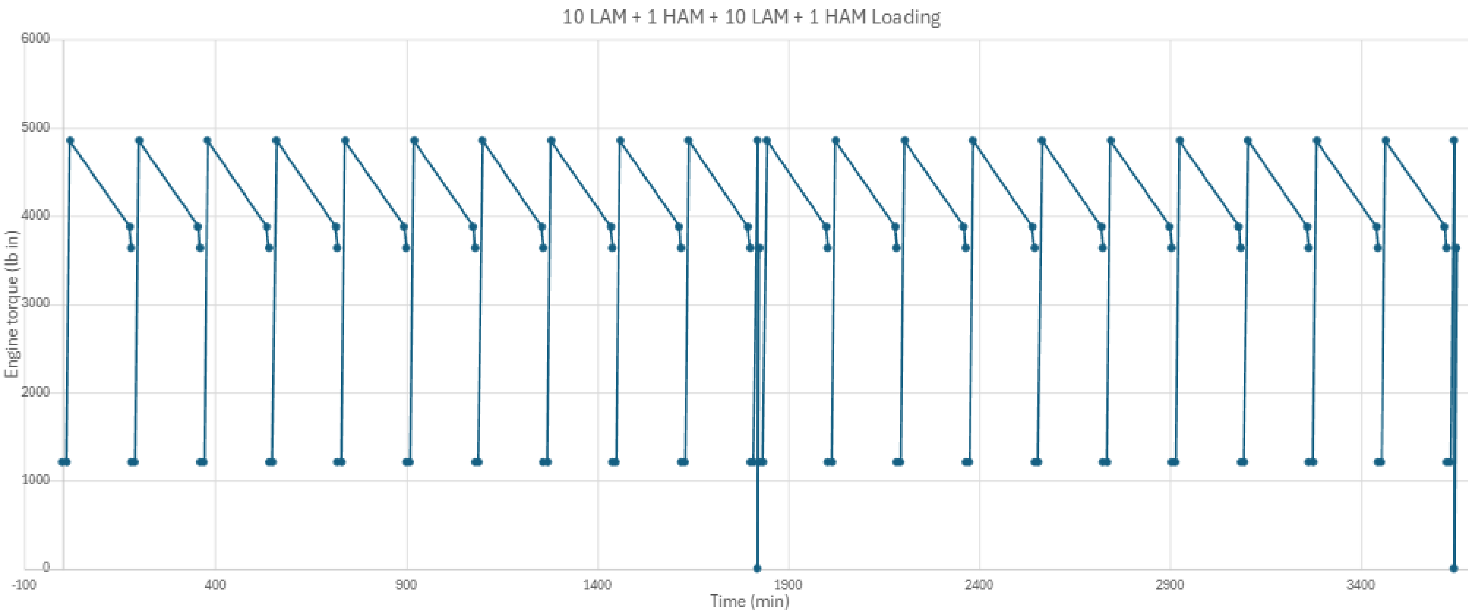
Table 23 in the Appendix outlines the torques at each stage of the gearbox and the axial load from the propeller for the different phases of the missions, from which Figure 1 was generated. It shows which load cycles the gearbox goes through for a 10 LAM – 1 HAM – 10 LAM – 1 HAM sequence.



a.   One LAM cycle



b.   One HAM cycle



c. One 10-1-10-1 sequence

Figure 1: Load cycles in the gearbox

The geartrain designed in this report will satisfy the design requirements outlined here for both missions while minimizing the weight of the gear box. The gears, the shaft, the keys and the bearings will be designed following AGMA standards for spur gears, and any assumptions will be mentioned in this report. The following design factors will have to be considered: a 5% load factor for each branch of the gearbox, a 10% power increase to accommodate potential future changes to the aircraft, and an additional 12% overload to account for the accumulation of tolerances from manufacturing. Cost will not be taken into account in this preliminary design, but a limited set of tooling for spur gears is available, only for 10, 12 or 16 diametral pitch and an angle of pressure of 20 degrees. Gears are expected to have a 30 000-hour life, while bearings have to withstand 10 000 hours. The propeller is connected to the gear train by a shaft, it weighs 150 lb and generates a maximum axial force of 600lb, proportional to the engine power.

The following guidelines will be followed to achieve the requested preliminary design. First, the missions specifications will be studied: critical parameters like the gear ratio will be defined and load cases will be derived, like the torques on the different shafts. Then, the gears will be analyzed and sized, followed by the shafts, making sure to iterate through the changes that are interdependent (like the diameter of the shaft and the inner diameter of the gears). Once this is achieved, keys will be designed and shafts will be checked for failure, due to the stress concentrations arising from the keyways, and re-sized if needed. Finally, bearings will be selected from the SKF catalog. A 3D model of the gearbox will be made in SolidWorks which will be used to generate technical drawings. Assumptions will be explained in detail through this report and all sample calculations can be found in the Appendix.

Design Process

Gears

Gear ratios and teeth number

Carefully designing each gear in the gearbox is essential to balance weight and safety factors, while ensuring proper meshing and efficient transmission of speed and torque in line with the design requirements. To evaluate each gear effectively, iterative methods were created using Python, following the AMGA method, to determine the characteristics of each gear, all while minimizing weight and preserving the integrity of the design criteria.

To start the design, the overall gear ration of the gear train would need to be determined. To achieve this, the ratio between the maximum input over the maximum output is needed.

$$m_{ideal} = \frac{\omega_{\max @ \, input}}{\omega_{\max @ \, output}} = \frac{\omega_{engine\ speed}}{\omega_{propeller\ speed}} = \frac{15000\ rpm}{1000\ rpm} = 15$$

The maximum angular velocities of the propeller and engine occur at takeoff. The maximum input of the motor will not exceed 15000 rpm while the maximum output of the gearbox will not exceed 1000 rpm. Thus, the ideal gear ratio is 15:1, and the designed pinion and gear combinations must meet this overall gear ratio. The design is a double-branch double-reduction gearbox. Consequently, while there are 6 gears in total, there are only 4 different types of gears, as the two on either side of the first gear are the same and the two on either side of the fourth gear are the same. The double-branch design divides the loads between the two sides while providing the same transmission power, but in opposite directions. The gear ratio of one branch should therefore be equal to the ideal overall gear ratio stated above.

$$m_{total} = \frac{N_2}{N_1} \cdot \frac{N_4}{N_3} = 15$$

Table 3 below summarizes the chosen gear ratios.

Table 3: Final gear ratios

Gear ratios			
Overall Train Ratio	150	10	15
Input Pinion to Driven Gears	30	5	6
Driven Pinions to Output Gear	5	2	2.5
Torque Increase from input to driven gear	6		
driven pinion to output	2.5		

Furthermore, there are a couple of other constraints that the gears designed must adhere to. The nacelle width, 30 in, restricts the size of the gears. The combined diameters of the gears must not exceed that maximum allowable width. Moreover, due to the double-branch double-reduction structure, it is essential that the pinion and gear sets are designed so that the shafts align between the two gear sets. The following geometric constraints must be adhered to:

$$\frac{1}{2}(d_1 + d_2) = \frac{1}{2}(d_3 + d_4) \;\rightarrow\; \frac{N_1 + N_2}{P_d} = \frac{N_3 + N_4}{P_d}$$

$$d_1 + 2 \cdot d_2 \leq 30\ in$$

$$d_4 + 2 \cdot d_3 \leq 30\ in$$

Figure 2 below illustrates the point above and explains the nomenclature for the gears that will be used in the rest of this report.

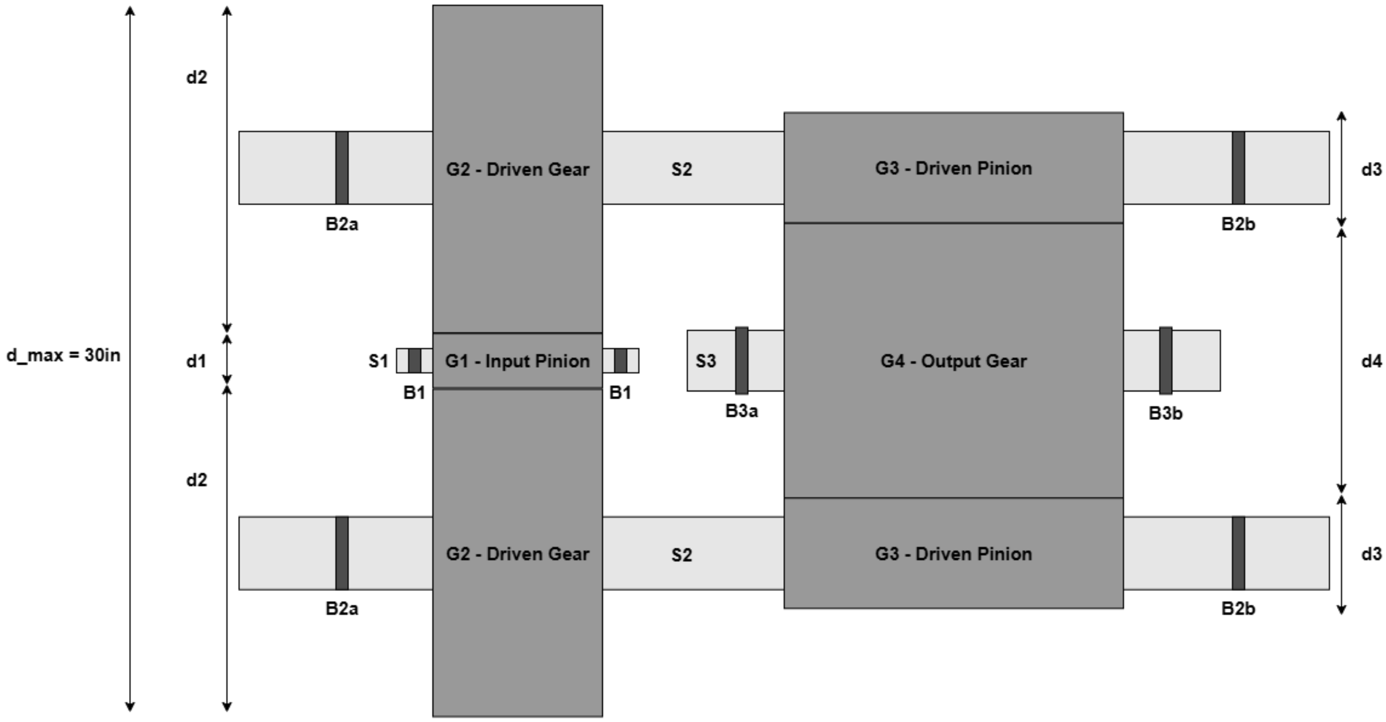


Figure 2: Geometric constraints in the geartrain

As mentioned in the Introduction, there are 3 possible diametral pitches available: 10, 12 and 16, further expanding the possible gear combinations that could fit within the space. A pressure angle of 20º is imposed onto the design, restricting the possible gear train combinations. Due to the 20º pressure angle, the minimum number of teeth allowed is 18. Indeed, as seen on AGMA table 12-8, less than 18 teeth would lead to undercutting and interference between the pinion teeth and the gear teeth. The following table 4 was used to iterate through teeth numbers for the three available diametral pitches while making sure the previously stated geometric constraints were followed. A Python script, that can be found in the Appendix, was also used to make sure the number of teeth chosen was optimal.

Table 4: Final gear sizing

Gear Sizing								
	Input Pinion	Driven Gear	Driven Pinion	Output Gear	Input side, Total Width (in)	Output side, Total Width (in)	1st Sum of Radiii (in)	2nd Sum of Radii (in)
Number of Teeth N	36	216	72	180				
Pitch Diameter d_p (in), pd=10,	3.6	21.6	7.2	18	46.8	32.4	12.6	12.6
Pitch Diameter d_p (in), pd=12	3	18	6	15	39	27	10.5	10.5
Pitch Diameter d_p (in), pd=16	2.25	13.5	4.5	11.25	29.25	20.25	7.875	7.875

The final teeth number chosen are respectively 36, 216, 72 and 180 for gears 1, 2, 3 and 4, with a p_d of 16, which respects all geometry constraints. The next step was to size the face width, considering gears' safety factors.

Power and torque transmission

The force and torque transmitted through each gear must be precisely calculated to determine the exact dimensions needed to maintain appropriate safety factors. In the double-branch gearbox design, the transmitted torque on the second stage branch gears is increased from the input. To obtain the torque on shaft 2, the torque from the input shaft 1 is multiplied by 6 due to the first gear ratio of 6:1, but it is also divided by 2 since the input pinion transmits its torque to both branches. Similarly, from shaft 2 to shaft 3: the input torque from shaft 3 is multiplied by 2.5 due to the 5:2 gear ratio. However, this time the torque is multiplied by 2 since shaft 3 receives torque from both sides. From the torques on each shaft, the tangential force Wt in each meshing can be computed.

The table below displays the transmitted forces and torques for each gear. These values are crucial for stress analysis and safety factor calculations. Gear 1 refers to the gear on the motor input shaft, with gear 2 meshing with it. On the same shaft as gear 2, gear 3 drives gear 4, which is connected to the output shaft. These outlined values of Wt were used in computing the stresses on the gears.

Table 5: Transmitted torque and applied tangential force

	Gear 1	Gear 2	Gear 3	Gear 3
Torque (lb.in)	4852.8	14558.5	14558.5	11646.8
W_t (lb)	2156.8	2156.8	6470.5	6470.5

Material, face width sizing and safety factors

Selecting an appropriate material is critical, as it directly impacts both safety factors and weight considerations. The AMGA method for determining bending fatigue and surface stresses relies on the Brinell Hardness of the chosen material. Additionally, surface and bending fatigue strengths are adjusted based on the environmental conditions experienced throughout the material's service life. These correction factors account for temperature, lifespan, and reliability, further safety factor estimations. As stated in the project description, to narrow down the unknowns of this gearbox design, a common and effective gear material should be chosen. AISI 4140 was elected, a steel often used in these applications for their machinability and corrosion resistance.

Once a material was chosen and initial gear sizing parameters were set, AGMA methods were used to determine the safety factors with reasonable assumptions to achieve reliable accuracy. To calculate the bending stress on the gear teeth, many factors were taken into account. The quality factor, Q_v, was assumed to be 12, considering the gears of a gearbox would be manufactured of the most professional grade. Thus, the dynamic factor, K_v, is 1. Moreover, the size factor, K_s, is always set to 1, the idler factor, K_i, is set to 1 because none of the gears are idlers and are not subject to fully reversed stresses; and the rim thickness factor, K_b is also set to 1 as will be explained later on. As mentioned in the project description, the gear design must account for an additional 12% overload to account for the accumulation of tolerances from manufacturing. Setting the application factor, $K_a = 1.12$ will account for any discrepancy. The geometric factor for bending, J, is dependent on the number of teeth present on the pinion or gear, the pressure angle and the loading type. In this case, the pressure angle is 20º and the full-depth teeth undergo tip-loading, thus the geometric factor will be determined accordingly using the AGMA table 12-8. Furthermore, the face width, F, is also a changing variable as it will depend on the gear's final number of teeth. The loading distribution factor, K_m, will be influenced by the final face width of each gear based on the AGMA table 12-16. When determining the face width of the gear teeth, it is important to consider that the width of the gears within one gear set must be the same. Thus, the face width of gears 1 and 2 must be the same, and the face width of gears 3 and 4 must be the same to ensure proper meshing. Moreover, it is suggested that $\frac{8}{P_d} < F < \frac{16}{P_d}$, however as this report will explore, maintaining F between these ratios is almost impossible.

To determine the bending fatigue strength of each gear, more correction factors must be applied according to the AMGA method. The uncorrected fatigue strength is a material-specific constant determined on AGMA table 12-20. The temperature factor, K_t, is 1 as the operating conditions are not extreme and the operating temperature is assumed to not exceed 250ºF. Likewise, the reliability factor, K_r, is also 1 according to the AGMA table 12-19 with an assumed reliability of 99%. To determine the life factor, K_L, the number of cycles must be taken into account. Since the number of load cycles is greater than $1x10^7$, either the top or bottom slope of AGMA 12-24 must be used. This gearbox was assumed to have a commercial application, so the top slope was chosen and K_L was solved for.

Table 6 below outlines all the factors used in the gears' bending stress computation and their safety factors.

Table 6: Gear bending calculations

Gear Calculations												
Bending												
Stresses												
	N (teeth number)	P_d	F (face width, in)	J	K_a	K_m	K_v	K_s	K_b	K_i	sigma_b (psi), 5% extra	W_t (lb)
Gear 1 input pinion	36	16	7.8	0.26	1.12	1.8	1	1	1	1	36020.19249	2156.821
Gear 2 driven gear	216	16	7.8	0.26	1.12	1.8	1	1	1	1	36020.19249	2156.821
Gear 3 driven pinion	72	16	19.1	0.28	1.12	1.8	1	1	1	1	40977.34688	6470.464
Gear 4 output gear	180	16	19.1	0.29	1.12	1.8	1	1	1	1	39564.33492	6470.464

Strength						Safety Factor
	Sfb' (psi)	K_L	K_T	K_r	Sfb (psi)	N_b
Gear 1	45000	0.87399	1	1	39329.37	1.091869961
Gear 2	45000	0.91351	1	1	41108	1.141248827
Gear 3	45000	0.91351	1	1	41108	1.003188482
Gear 4	45000	0.91715	1	1	41271.61	1.043151777

Like the bending stress of the gear teeth, the contact surface stresses can also be calculated using the AGMA method. Factors such as the dynamic factor, the load distribution factor, the application factor and the size factor are equivalent to the ones obtained for the bending stresses. The elastic coefficient accounts for differences in pinion and gear materials and its value can be determined using the AGMA table 12-18. All of the gears here are made of AISI 4140 steel. For the last correction factor, the surface finish factor, C_f, there is no AGMA standard yet, so it is assumed to be 1 for the purposes of this project. Finally, the surface geometry factor, I, is a function of the pinion pitch diameter, the pressure angle and the radii of curvature of the pinion and gear teeth. Detailed calculations are available in the Appendix.

Finally, to calculate the contact surface strengths of the gear teeth, the same correction factors will be used as the ones used for the bending strength calculation. The hardness ratio factor is added to the calculation. Since the pinion and gears are made of the same material, the hardness ratio between the two is 1, so the hardness ratio factor, C_H, is also 1. The uncorrected surface-fatigue strength of the specific material used for the gears can be obtained on AGMA table 12-21, from which we derive the corrected surface-fatigue strength S_{fc}. Once compared with the stresses on the gears, surface contact safety factors are obtained. The factors for contact stress and the gears' safety factors are presented in table 7 below.

Table 7: Gear contact calculations

Gear Calculations																
Contact																
Stresses																
	N	C_p (psi)	W_t (lb)	F (in)	I	ϕ (pressure angle, rad)	r_p	x_p	P_d	C	d_p	Ca	Cm	Cv	Cs	Cf	sigma_c (psi), 5% extra
Gear 1	36	2300	2156.8	7.8	0.13774	0.34906585	1.125	0	16	16	2.25	1.12	1.8	1	1	1	102423.7249
Gear 2	216	2300	2156.8	7.8	0.13774	0.34906585	6.75	0	16	16	13.5	1.12	1.8	1	1	1	41814.31059
Gear 3	72	2300	6470.5	19.1	0.045913	0.34906585	2.25	0	16	16	4.5	1.12	1.8	1	1	1	138847.3005
Gear 4	180	2300	6470.5	19.1	0.045913	0.34906585	5.625	0	16	16	11.25	1.12	1.8	1	1	1	87814.74332

Strength							Safety Factor
	Sfc' (psi)	C_L	C_T	C_r	C_H	Sfc (psi)	N_c
Gear 1	180000	0.874	1	1	1	157317.4647	1.535947505
Gear 2	180000	0.9135	1	1	1	164432.0097	3.932433835
Gear 3	180000	0.9135	1	1	1	164432.0097	1.184265082
Gear 4	180000	0.9171	1	1	1	165086.4251	1.879939733

Face widths were manually iterated through to find the smallest ones that would provide safety factors above 1 for both bending and contact, keeping in mind that face widths must be consistent across two meshing gears. Small face widths are desired, as they allow for lighter gears and shorter shafts, that thus undergo lower bending moments. The final chosen dimensions for them are 7.8in for gears 1-2 and 19.1 in for gears 3-4.

Gear computations are largely dependent on the data provided by AGMA. The tables and equations in the *Machine Design* textbook by Robert L. Norton were the main sources of gear and material data required to compute each correction factor. All tables and calculations executed to calculate the bending stresses and strengths and surface contact stresses and strengths for each gear can be found in the Appendix.

To determine if the number of teeth, diametral pitch and face width chosen were suitable to the specific loading scenarios given, the bending safety factors and contact safety factors were calculated by dividing the appropriate strengths by the corresponding stresses. To ensure a sound design, it is imperative that all these safety factors be greater than 1, which is the case in our design as can be seen in the two previous tables.

Weight reduction

Gear weight reduction required the strategic removal of material while maintaining structural integrity. The first step was to increase the diametral pitch, P_d, to decrease the gear's pitch diameter, d_p. As mentioned before, P_d was set to 16 . Furthermore, we want to minimize the tooth face width as it minimizes the width of the entire gear. This, however, can only be done to a certain extent before the gear's safety factor becomes inadequate. The last step taken to minimize the weight of the gears is to implement weight reliefs. This would change the design of the gear from a solid-disk gear to a gear with a rim. The only correction factor this change affected is the AGMA rim thickness factor, K_B. To ensure the gear performance was not affected by the weight reliefs, we opted to keep the backup ratio above 1.2 as this would enable to the rim thickness factor to stay at 1, as mentioned in AGMA figure 12-23. Table 8 below shows the relevant dimensions and ratios chosen in the sizing of the rims.

Table 8: Weight relief dimensions

Weight relief features		
	Gear 2	**Gear 4**
addendum (in)	0.0625	0.0625
dedendum (in)	0.078125	0.078125
tooth thickness (in)	0.140625	0.140625
Rim thickness (in)	1	1
Backup ratio mb	7.111111111	7.111111111
Kb	1	1

These weight reliefs were only added to the bigger gears, 2 and 4, since they were the heaviest and had the most capacity to remove material. The pinion gears, gears 1 and 3 are too small and would not have benefited from weight reliefs as it would have reduced their structural integrity. Weight reliefs that resemble a web were chosen, with no particular attention to the structural design apart from good engineering practices such as adding fillets to avoid stress concentrations. No standards or methods to design such weight relief features were taught, and this was considered outside the scope of this project.

Shafts

To support the gears, shafts are needed. The geometry of the shafts was iterated manually until the desired safety factor was found. In this case, hollow cylindrical shafts were chosen, and thus the geometry is defined by the outer and inner diameters. Choosing hollow shafts was a simple way to reduce the weight of the shafts, while maintaining maximum structural capacity. The shafts are stepped, with a thinner section at each end where the bearings support it.

Loads

The first step was to find the force loads and torque loads acting on the shafts due to the motor, gears, and propeller. Note that there are two shafts 2 on either side of the double-branch gearbox, that are designed to be identical.

The torques in the shafts and tangential loads from the gears were found in the gear section of this report. There are also additional loads on each shaft from the weight of the gears. The tangential loads and gear weights are considered distributed loads across their face width. Likewise, there is a load from the weight of the propeller, considered as a point load since the width of the propeller is not given. There is also an axial load on shaft 3 from the propeller thrust, but this axial load does not induce a bending moment, it is assumed centered on the shaft's axis.

Given the known loads, the reaction forces in the z direction at the bearings can be determined by applying the equations of static equilibrium. To ensure the problem is solvable, the bearings are modeled as simple supports and point loads. With the reaction forces established, shear and moment diagrams are used to determine the bending moments along each shaft. The final bending moments are presented in table 9 below.

Table 9: Bending moments on shafts

Bending Moments on shafts (lb in)	
Shaft 1	
Mmax	14.48938857
Ma	14.48938857
Mm	0
Shaft 2	
Mmax	37906.60005
Ma	37906.60005
Mm	0
Shaft 3	
Mmax	1331.141076
Ma	1331.141076
Mm	0

Since the shafts rotate, the bending is fully-reversed and the mean moment Mm is thus 0, making the alternating moment Ma equal to the max bending moment determined from the diagrams. The FBDs used to derive the loads on shafts 1, 2 and 3 can be found as figures 6 through 8 of the Appendix, along with their shear and moment diagrams. Tables 26 and 27 in the Appendix summarize the torques and bending forces on each shaft.

Stresses and Strength

Each shaft is undergoing bending and torsion loads. The bending loads cause normal stresses and shear stresses in the shaft, and torsional loads cause shear stresses [1]. However, shear stresses due to bending are negligible compared to the normal forces due to bending and the shear forces due to torsional forces. Ignoring these negligible stresses simplifies the shafts' analyses. All of the shafts in a gearbox are undergoing full rotations producing alternating and mean components to the forces. To assess the overall impact of the stresses, the Von Mises equivalent stress can be determined using the following equations:

$$\sigma_a' = \sqrt{\sigma_a^2 - 3\tau_a^2}$$

$$\sigma_m' = \sqrt{\left(\sigma_m + \sigma_{m,axial}\right)^2 - 3\tau_m^2}$$

The shafts are making full rotations and thus undergoing fully reversible loads. This means that the mean normal stresses are zero but non-zero alternating stresses. To ensure the gearbox would be able to withstand the highest loads it might be subject to, the shafts were designed assuming they are subjected to constant torque, taking the maximum value. Note that as mentioned in the project description, the gearbox must be designed to withstand an additional 10% increase in power for future versions of the aircraft, so a total power of 1155HP, which was used in computing the torques on the shafts. These torsional forces provoke non-zero mean shear stresses and zero alternating shear stresses (since it is assumed constant). Bending and torsional stresses can be calculated using the following formulae. The force from the propeller also creates an axial force on shaft 3.

$$\sigma_{axial} = \frac{F}{A}$$

$$\sigma_{bending} = \frac{MD}{2I} \text{where } I = \frac{\pi}{64}\left(D^4 - d'^4\right)$$

$$\tau_{torsion} = \frac{TD}{2J} \text{ where } J = \frac{\pi}{32}\left(D^4 - d'^4\right)$$

D is the outer diameter of the shaft along the bigger step, and d' is the inner diameter (the shafts are hollowed as mentioned previously). The initial safety factor the shafts were designed for is 1.5 for the maximal loads they would be subject to. This safety factor allowed the small changes that needed to be made when implementing the bearings and keys without compromising its structural integrity but also minimize the weight of the shafts. The Modified Goodman Diagram (Figure 3)is used to determine the safety factors of the shafts. Since the Von Mises alternating and mean stresses are constant, their ratio is also constant and case III is used. As can be seen on Figure 4, failure occurs when the CF line is reached. The formula used to calculate the fatigue safety factor is the following:

$$N_f = \frac{S_e S_{ut}}{\sigma'_a S_e + \sigma'_m S_{ut}}$$

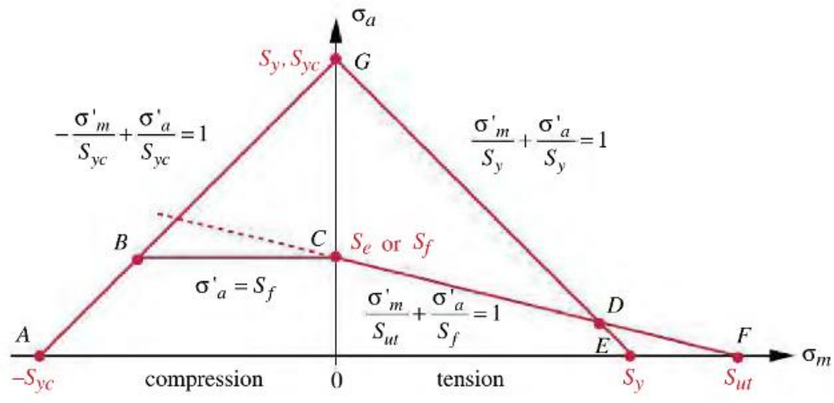


Figure 3: Modified Goodman diagram

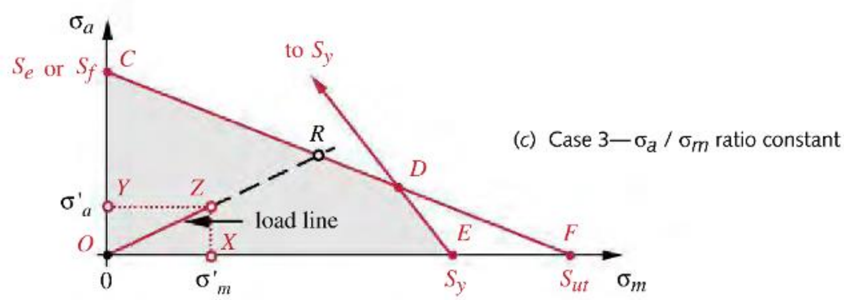


Figure 4: MGD Case III failure

As seen in the fatigue safety factor formula, the endurance limit of the material should be found. The endurance limit represents the maximum stress that can be applied to the beam without causing failure, no matter the number of cycles the beam is subject to, since steel is a knee material. This allows the design to handle any amount of flight time.

$$S_e = C_{load} \cdot C_{size} \cdot C_{surf} \cdot C_{temp} \cdot C_{reliab} \cdot S'_e$$

The correction factors, C_x, are scenario-dependent and are less than or equal to 1. To determine all of the correction factors, some expectations were considered. Gearboxes like this one were assumed to not operate at temperatures higher than 450ºC, so the temperature factors, C_{temp}, is said to be 1, and the reliability is assumed to be 99% so the reliability factor, C_{reliab}, is 0.814. The size correction factor is diameter-dependent and thus underwent many changes with each iteration of the size correction factor.

$$C_{size} = 0.8869D^{-0.097}$$

Failure can occur at two locations on the shafts: the point of maximum moment or at a point with stress concentration. In this case, stress concentrations occur at the gears where there is a keyway, or at the steps on each end. Furthermore, the shafts are subjected to the radial forces from the gears. However, since these are lower than the tangential loads and gear weights, they are neglected: the design will most likely fail first due to the combination of tangential loads and gear weights. At points of stress concentrations, stress concentration factors, K_x, must be applied to account for the higher failure chances in the area. Table 10 below summarizes the K-stress concentration factors.

Table 10: Stress concentration factors

Scenario	Variable	Formula
Static bending, σ	K_t	Norton, Figure 10-16
Alternating bending,σ	K_f	$K_f = 1 + q(K_t - 1)$
Mean bending, σ	K_{fm}	If $K_{fm}\lvert\sigma_{max}\rvert < S_y$ Then $K_{fm} = K_f$ If $K_{fm}\lvert\sigma_{max}\rvert > S_y$ Then $K_{fm} = \frac{S_y - K_f\sigma_a'}{\sigma_m'}$
Static torsion, τ	K_{ts}	Norton, Figure 10-16
Alternating torsion, τ	K_{fs}	$K_{fs} = 1 - q_s(K_{ts} - 1)$
Mean torsion, τ	K_{fms}	If $K_{fms}\lvert\sigma_{max}\rvert < S_y$ Then $K_{fms} = K_{fs}$ If $K_{fsm}\lvert\sigma_{max}\rvert > S_y$ Then $K_{fsm} = \frac{S_y - K_{fs}\sigma_a'}{\sigma_m'}$

Applying the concentration factors to the stress states, results in the following alternating and mean stresses in the shafts:

For all the shafts:

$$\sigma_a' = \sqrt{\left(K_f \frac{MD}{2I}\right)^2 + 3(0)^2} = K_f \frac{MD}{2I}$$

For shafts 1 and 2 (no axial loading):

$$\sigma_m' = \sqrt{(0)^2 + 3\left(K_{fs}\frac{TD}{2J}\right)^2} = \sqrt{3}K_{fs}\frac{TD}{2J}$$

For shaft 3 (with axial loading):

$$\sigma_m' = \sqrt{\left(\frac{4F}{\pi(D^2 - d'^2)}\right)^2 + 3\left(K_{fs}\frac{TD}{2J}\right)^2}$$

Note an increase of 5% was applied to the Von Mises stresses to account for the assumed load factor of 5% for each branch as per the project description.

To find the appropriate diameter of each shaft, an iterative process is used. A safety factor 1.5 was assumed and a diameter was assumed to compute C_{size}, to get the uncorrected endurance strength used in the following:

$$D = \left(\frac{32N_f}{\pi} \left(\frac{\sqrt{\left(K_f M_a\right)^2 + \frac{3}{4}\left(K_{fs} T_a\right)^2}}{S_e} + \frac{\sqrt{\left(K_{fm} M_m\right)^2 + \frac{3}{4}\left(K_{fsm} T_m\right)^2}}{S_{ut}} \right) \right)^{\frac{1}{3}}$$

The assumed and calculated diameter were then iterated until they were equal, resulting in the assumed and calculated safety factor also being equal. The formula above is only usable for a combination of bending and torsional loading with no axial loading. However, this formula was still used for shaft 3 despite the axial load present. To counter this major assumption, an increased safety factor was aimed for by increasing the outer diameter of the shaft.

The shafts' safety factors were originally calculated with the regular stress concentration factors but were then updated to account for the keyways by calculating separate K-stress concentration factors from Norton Figure 12-16. This led to lower effective safety factors, and the shafts' assumed outer diameters were thus increased to reach 1.5 safety factors again. This produced a discrepancy between the assumed outer diameter, and the calculated one, which was to be expected. Table 11 below includes all the shafts' stresses and safety factors, while the detail of each correction factor can be found in table 28 of the appendix.

Table 11: Shaft stresses and safety factors

Shaft Stresses			
	Shaft 1	Shaft 2	Shaft 3
M_a (lb in)	14.48938857	37906.60005	1331.141076
M_m (lb in)	0	0	0
T_a (lb in)	0	0	0
T_m (lb in)	4852.925	14558.775	72793.875
sigma_a (psi)	488.7296055	25809.78429	2662.989095
sigma__m (psi)	0	0	0
Tau_a (psi)	0	0	0
Tau_m (psi)	72783.45654	4281.289496	63923.16864
sigma_m_axial (if hollow) (psi)	0	0	150.6792361
sigma_m_axial (if non-hollow) (psi)	0	0	113.009427
sigma_max' (VM, psi)	184244.27	47342.15144	165582.7764
Von Mises Stresses and Safety Factors (MGD Case 3)			
If not hollow			
sigma_a' (psi)	242.5527595	19127.48981	1313.663547
sigma_m' (psi)	62564.91747	5495.520434	52017.01285
Fatigue safety factor Nf	2.878487766	1.927650768	3.171234866
If hollow			
sigma_a' (psi)	247.4461028	19178.65888	1401.241117
sigma_m' (psi)	63827.12378	5510.221824	58409.59614
Fatigue safety factor Nf	2.821564546	1.922507755	2.708873501
Hollow with keys			
sigma_a' (psi)	304.2666071	24181.4454	1742.86899
sigma_m' (psi)	120175.4935	10990.61301	112779.8399
Fatigue safety factor Nf	1.506884271	1.475078272	1.473953677

Shaft deflection with regards to the bearings was not studied, but the shafts were kept as short as possible with a very small space between the bearings and the gears at each end (0.1 to 0.2in), to minimize the moments and thus the deflection at the bearings.

Keys

Keys and shafts are dependent on each other. The keyways cause abrupt changes in diameter and tight edges in the shaft, creating stress concentrations while the diameter of the shaft influences the torque on the keyway. For simplicity, square keys were chosen, where their width and height are the same. Norton Figure 10-2 is used to determine the nominal key width based on its corresponding shaft diameter. The key length is first set to be equal to the gear face width, as it cannot be less than it. Once the keyway geometry was set, the area for shear and bearing were computed. Taking the torque on each shaft and its radius, the force on the keys was computed and from these the Von Mises alternating and mean stresses were determined. A tapered key was selected for gear 4 due to the axial load of the propeller, that is only in one direction which is applicable to a tapered key. To determine the fatigue safety factor in shear, the calculated Von Mises stresses are applied to case III of the Modified Goodman Diagram.

$$N_f = \frac{S_e S_{ut}}{\sigma_a' S_e + \sigma_m' S_{ut}}$$

The bearing safety factor is calculated:

$$N_b = \frac{S_y}{\sigma_{bearing}} = \frac{S_y}{\frac{F_{max}}{A_{bearing}}} = \frac{S_y}{\frac{F_a + F_m}{A_{bearing}}}$$

Note that all the stresses have an additional 5% accounted for as per the project description. The key lengths were then progressively decreased to have lower safety factors. Indeed, ideally, we would have had the key's safety factors be above 1 but less than that of its associated shaft so that the keys fail first as a safety precaution to protect the more crucial components of the gearbox such as the gears and shafts. It's indeed easier and cheaper to replace a broken key rather than another component. However, the key widths were constraint to the ones on Norton Figure 10-2 and so is the key height, as we opted for square keys. This made it impossible to have safety factors below the shafts', and final key safety factors are about 4 for shear and 2 for bearing. The parameters and safety factors for key 1 are outlined in the table below, while the others can be found in the Results section.

Table 12: Key 1 calculations

Key 1 Calculations			
Strengths and factors		**Loads**	
Material	AISI 1010 Hot Rolled Steel	**Torque Max (lb in)**	4852.925
Sut (psi)	47000	**Torque Min (lb in)**	4852.925
Sy (psi)	26000	**Torque Alternating (lb in)**	0
Se' (psi)	23500	**Torque Mean (lb in)**	4852.925
Se (psi)	12150.28702	**Force Alternating (lb)**	0
C_Surf	0.907394683	**Force Mean (lb)**	9705.85
A	14.4	**Stresses (5% added, psi)**	
b	-0.718	**Tau Alternating**	0
C_Load	0.7	**Tau Mean**	6470.566667
C_reliability (99%)	0.814	**sigma_a'**	0
C_temp	1	**sigma_m'**	11767.71773
C_size	1	**sigma_max (bearing)**	13588.19
Dimensions		**Safety Factors (MGD case 3)**	
Shaft radius (in)	0.5	**Shear Safety Factor**	3.993977513
Key Height (in)	0.25	**Bearing Safety Factor**	1.913426292
Key Width (in)	0.25	**Weight**	
Key Length (in)	6	**Volume (in^3)**	0.375
Area Shear (in^2)	1.5	**Density (lb in^3)**	0.284
Area Bending (in^2)	0.75	**Weight (lb)**	0.1065
d equiv (in)	0.20198105		
A95 (in^2)	0.003125		

The material used for the keys is AISI 1010 hot rolled steel. This material was chosen as it's the one that has the lowest ultimate tensile strength and yield strength on Table A-9. This choice was made with the aim of reducing the safety factors as mentioned above. Likewise, AISI 1010 has a lower Brinell Hardness than AISI 1095 quench and temper at 600ºF used for the shafts to reduce unnecessary wear of the shafts.

Bearings

The appropriate selection of bearings depends on the radial and axial loads they must support, as well as the shaft diameter where they will be installed. Bearing suitability is assessed by comparing its expected lifespan to the total number of load cycles anticipated during operation. Another important factor is whether the bearing's rated speed can match the rotational speed of the shaft.

The L10 value was set as the number of revolutions derived from bearing life and mission parameters (as shown in table 23 of the appendix), divided by 10^6 to get it in millions. From that value, the Basic Dynamic Load Rating C necessary was calculated. P was calculated beforehand from the loads on the shafts and using figure 11-24from SKF.

Shaft 1 undergoes symmetrical loads and is held by the same bearing on each shaft end. Shafts 2 and 3 are asymmetrical and thus could have two different bearings on either side to accommodate the variation in reaction loads on either side. However, it was found that the bearings selected respectively worked for both sides of each shaft. Additionally, the bearings on shaft 3 needed to handle axial force from the propeller. Since many thrust bearings cannot support radial loads, deep groove ball bearings and needle bearings capable of withstanding both axial and radial forces were selected. Bearing comparisons were iteratively performed and visualized in Excel to ensure the suitability of each selection, making sure the catalog C and limiting RPM were in accordance with the values required and the bore diameters were close to those of the shafts. Detailed sample calculations are provided in the appendix, and a final bearings selection table can be found in the results section.

Final Assembly - Results

Gears

As presented previously, below are our final gear ratios of 15:1, with 6:1 for the first stage and 5:2 for the second stage. Our final gear teeth sizing is also outlined, with a chosen diametral pitch of 16.

Table 13: Final Gear ratios

Gear ratios			
Overall Train Ratio	150	10	15
Input Pinion to Driven Gears	30	5	6
Driven Pinions to Output Gear	5	2	2.5
Torque Increase from input to driven gear	6		
driven pinion to output	2.5		

Table 14: Gear Sizing

Gear Sizing								
	Input Pinion	Driven Gear	Driven Pinion	Output Gear	Input side, Total Width (in)	Output side, Total Width (in)	1st Sum of Radiii (in)	2nd Sum of Radii (in)
Number of Teeth N	36	216	72	180				
Pitch Diameter d_p (in), pd=10,	3.6	21.6	7.2	18	46.8	32.4	12.6	12.6
Pitch Diameter d_p (in), pd=12	3	18	6	15	39	27	10.5	10.5
Pitch Diameter d_p (in), pd=16	2.25	13.5	4.5	11.25	29.25	20.25	7.875	7.875

Following the method explained in the Design section, the tables below were used to size the face width of the gears and get safety factors above 1 for both bending and contact. Final face widths are 7.8in and 19.1in.

Table 15: Gear Bending stresses

Gear Calculations												
Bending												
Stresses												
	N (teeth number)	P_d	F (face width, in)	J	K_a	K_m	K_v	K_s	K_b	K_i	sigma_b (psi), 5% extra	W_t (lb)
Gear 1 input pinion	36	16	7.8	0.26	1.12	1.8	1	1	1	1	36020.19249	2156.821
Gear 2 driven gear	216	16	7.8	0.26	1.12	1.8	1	1	1	1	36020.19249	2156.821
Gear 3 driven pinion	72	16	19.1	0.28	1.12	1.8	1	1	1	1	40977.34688	6470.464
Gear 4 output gear	180	16	19.1	0.29	1.12	1.8	1	1	1	1	39564.33492	6470.464

Strength						Safety Factor
	Sfb' (psi)	K_L	K_T	K_r	Sfb (psi)	N_b
Gear 1	45000	0.87399	1	1	39329.37	1.091869961
Gear 2	45000	0.91351	1	1	41108	1.141248827
Gear 3	45000	0.91351	1	1	41108	1.003188482
Gear 4	45000	0.91715	1	1	41271.61	1.043151777

Table 16: Gear contact stresses

Gear Calculations																
Contact																
Stresses																
	N	C_p (psi)	W_t (lb)	F (in)	I	ϕ (pressure angle, rad)	r_p	x_p	P_d	C	d_p	Ca	Cm	Cv	Cs	Cf	sigma_c (psi), 5% extra
Gear 1	36	2300	2156.8	7.8	0.13774	0.34906585	1.125	0	16	16	2.25	1.12	1.8	1	1	1	102423.7249
Gear 2	216	2300	2156.8	7.8	0.13774	0.34906585	6.75	0	16	16	13.5	1.12	1.8	1	1	1	41814.31059
Gear 3	72	2300	6470.5	19.1	0.045913	0.34906585	2.25	0	16	16	4.5	1.12	1.8	1	1	1	138847.3005
Gear 4	180	2300	6470.5	19.1	0.045913	0.34906585	5.625	0	16	16	11.25	1.12	1.8	1	1	1	87814.74332

Strength							Safety Factor
	Sfc' (psi)	C_L	C_T	C_r	C_H	Sfc (psi)	N_c
Gear 1	180000	0.874	1	1	1	157317.4647	1.535947505
Gear 2	180000	0.9135	1	1	1	164432.0097	3.932433835
Gear 3	180000	0.9135	1	1	1	164432.0097	1.184265082
Gear 4	180000	0.9171	1	1	1	165086.4251	1.879939733

When combined with the weight relief features on gears 2 and 4 explained previously, and the inner diameter of the shafts, the final weight of the gears can be computed, amounting to a total of 400lb for the gears 1, 2, 3 and 4 after weight reliefs. It is to be noted that there are 2 additional gears from the second branch, one 2 and one 3, that will be taken into account in the final tallying of the gearbox's weight.

Table 17: Gears' weight

Gear masses											
Material			AISI 41040			Density (lb / in3)			0.284		
	Outer pitch diameter (in)	Inner diameter (in)	Face width (in)	Volume without relief (in3)	Mass without relief (lb)	Rims thickness (in)	Relief ID (in)	Relief OD (in)	Volume of relief (in3)	Mass of relief (lb)	Final mass with relief (lb)
Gear 1	2.25	1	7.8	24.8873043	7.067994422	-	-	-	-	-	7.067994422
Gear 2	13.5	3.3	7.8	1049.769468	298.134529	1	5.3	11.5	638.0951671	181.2190274	116.9155016
Gear 3	4.5	3.3	19.1	140.4103421	39.87653714	-	-	-	-	-	39.87653714
Gear 4	11.25	2.6	19.1	1797.169872	510.3962437	1	4.6	9.25	966.1086597	274.3748593	236.0213844
				Total mass (lb)	855.4753043					Total final mass (lb)	399.8814175

Shafts

The table below shows the final shafts dimensions and their weights.

Table 18: Shaft dimensions

Shaft final geometry and weight			
	Shaft 1	Shaft 2	Shaft 3
D (in)	1	3.3	2.6
d (in)	0.789	2.75	2.166666667
d' (in)	0.375	0.75	1.3
Bigger step length (in)	8	41.4	19.5
(in)	1	2.2	1.3
Total length (in)	10	45.8	22.1
Volume (in3)	7.261039007	380.227744	113.1174068
Material density (lb / in3)	0.284	0.284	0.284
Weight if not hollow (lb)	2.062135078	107.984679	32.12534354
Weight if hollow (lb)	1.748466687	102.238274	23.79453411
Total weight (lb)			
If solid	142.172158	If hollow	127.7812752

These dimensions yield safety factors of 1.5 when hollow and with keyways, as explained previously in table 11. In the table above, D is the outside diameter of the big step, d is that of the small step and d' is the inner diameter for the hollow shafts.

Keys

The tables below summarize the dimensions, weight and safety factors of the keys. As explained previously, the safety factors are about 4 for shear and 2 for bearing.

Table 19: Key Calculations

a. Key 1

Key 1 Calculations			
Strengths and factors		Loads	
Material	AISI 1010 Hot Rolled Steel	Torque Max (lb in)	4852.925
Sut (psi)	47000	Torque Min (lb in)	4852.925
Sy (psi)	26000	Torque Alternating (lb in)	0
Se' (psi)	23500	Torque Mean (lb in)	4852.925
Se (psi)	12150.28702	Force Alternating (lb)	0
C_Surf	0.907394683	Force Mean (lb)	9705.85
A	14.4	Stresses (5% added, psi)	
b	-0.718	Tau Alternating	0
C_Load	0.7	Tau Mean	6470.566667
C_reliability (99%)	0.814	sigma_a'	0
C_temp	1	sigma_m'	11767.71773
C_size	1	sigma_max (bearing)	13588.19
Dimensions		Safety Factors (MGD case 3)	
Shaft radius (in)	0.5	Shear Safety Factor	3.993977513
Key Height (in)	0.25	Bearing Safety Factor	1.913426292
Key Width (in)	0.25	Weight	
Key Length (in)	6	Volume (in^3)	0.375
Area Shear (in^2)	1.5	Density (lb in^3)	0.284
Area Bending (in^2)	0.75	Weight (lb)	0.1065
d equiv (in)	0.20198105		
A95 (in^2)	0.003125		

b. *Key 2*

Key 2 Calculations			
Strengths and factors		**Loads**	
Material	AISI 1010 Hot Rolled Steel	**Torque Max (lb in)**	14558.775
Sut (psi)	47000	**Torque Min (lb in)**	14558.775
Sy (psi)	26000	**Torque Alternating (lb in)**	0
Se' (psi)	23500	**Torque Mean (lb in)**	14558.775
Se (psi)	12150.28702	**Force Alternating (lb)**	0
C_Surf	0.907394683	**Force Mean (lb)**	8823.5
A	14.4	**Stresses (5% added, psi)**	
b	-0.718	**Tau Alternating**	0
C_Load	0.7	**Tau Mean**	5882.333333
C_reliability (99%)	0.814	**sigma_a'**	0
C_temp	1	**sigma_m'**	10697.92521
C_size	1	**sigma_max (bearing)**	12352.9
Dimensions		**Safety Factors (MGD Case 3)**	
Shaft radius (in)	1.65	**Shear Safety Factor**	4.393375264
Key Height (in)	0.75	**Bearing Safety Factor**	2.104768921
Key Width (in)	0.75	**Weight**	
Key Length (in)	2	**Volume (in^3)**	1.125
Area Shear (in^2)	1.5	**Density (lb in^3)**	0.284
Area Bending (in^2)	0.75	**Weight (lb)**	0.3195
d equiv (in)	0.605943151		
A95 (in^2)	0.028125		

c. *Key 3*

Key 3 Calculations			
Strengths and factors		**Loads**	
Material	AISI 1010 Hot Rolled Steel	**Torque Max (lb in)**	14558.775
Sut (psi)	47000	**Torque Min (lb in)**	14558.775
Se' (psi)	23500	**Torque Alternating (lb in)**	0
Se (psi)	12150.28702	**Torque Mean (lb in)**	14558.775
Sy (psi)	26000	**Force Alternating (lb)**	0
C_Surf	0.907394683	**Force Mean (lb)**	8823.5
A	14.4	**Stresses (5% added, psi)**	
b	-0.718	**Tau Alternating**	0
C_Load	0.7	**Tau Mean**	5882.333333
C_reliability	0.814	**sigma_a'**	0
C_temp	1	**sigma_m'**	10697.92521
C_size	1	**sigma_max (bearing)**	12352.9
Dimensions		**Safety factors (MGD case 3)**	
Shaft radius (in)	1.65	**Shear Safety Factor**	4.393375264
Key Height (in)	0.75	**Bearing Safety Factor**	2.104768921
Key Width (in)	0.75	**Weight**	
Key Length (in)	2	**Volume (in^3)**	1.125
Area Shear (in^2)	1.5	**Density (lb in^3)**	0.284
Area Bending (in^2)	0.75	**Weight (lb)**	0.3195
d equiv (in)	0.605943151		
A95 (in^2)	0.028125		

d. *Key 4*

Key 4 Calculations			
Strengths and factors		Loads	
Material	AISI 1010 Hot Rolled Steel	Torque Max (lb in)	72793.875
Sut (psi)	47000	Torque Min (lb in)	72793.875
Sy (psi)	26000	Torque Alternating (lb in)	0
Se' (psi)	23500	Torque Mean (lb in)	72793.875
Se (psi)	12150.28702	Force Alternating (lb)	0
C_Surf	0.907394683	Force Mean (lb)	55995.28846
A	14.4	Stresses (5% added, psi)	
b	-0.718	Tau Alternating	0
C_Load	0.7	Tau Mean	6399.461538
C_reliability	0.814	sigma_a'	0
C_temp	1	sigma_m'	11638.40215
C_size	1	sigma_max (bearing)	13438.86923
Dimensions (tapered)		Safety factors (MGD case 3)	
Radius (Shaft) (in)	1.3	Shear Safety Factor	4.038355041
Key Height, at highest point (in)	0.625	Bearing Safety Factor	1.934686584
Key Height, at lowest point (in)	0.48	Weight	
Key Width (in)	0.625	Volume (in^3)	5.225
Key length, inclined portion (in)	14	Density (lb in^3)	0.284
Key length, straight portion (in)	1	Weight (lb)	1.4839
Area Shear (in^2)	8.75		
Area Bending (in^2)	4.375		
d equiv	0.504952626		
A95	0.01953125		

Bearings

Table 20 below outlines the final bearing selection process, including both the parameters computed from the load cases and the catalog values for the selected bearings.

Table 20: Final bearings selection

Bearing Selection					
	Shaft 1	Shaft 2 - bearing 1	Shaft 2 - bearing 2	Shaft 3 - bearing 1	Shaft 3 - bearing 2
Speed (rpm)	15000	2500	2500	1000	1000
Total revs (from mission requirements)	8606712329	1434452055	1434452055	573780821.9	573780821.9
L_10	8606.712329	1434.452055	1434.452055	573.7808219	573.7808219
Radial Load (lb)	7.067994422	3571.559029	5212.518343	221.856846	164.1645383
Axial Load (lb)	0	0	0	600	600
X	1	1	1	0.56	0.56
Y	0	0	0	1.71	1.71
P Combined Load	7.067994422	3571.559029	5212.518343	1150.239834	1117.932141
C	144.8467073	31612.84948	46137.43086	9558.074536	9289.609367
From catalog					
Bearing Model	SKF 6004	SKF 22314 E	SKF 22314 E	SKF 6311	SKF 6311
Limiting speed (rpm)	24000	4500	4500	8000	8000
Bore (in)	0.787	2.756	2.756	2.165	2.165
Width of bearing (in)	0.472	2.008	2.008	1.142	1.142
Catalog C (lb)	2237	92846	92846	16658	16658
Weight (lb)	0.1446	9.495	9.495	2.949	2.949
Meets Requirements ?	YES	YES	YES	YES	YES

All the selected bearings meet the desired requirements. Note that due to the asymmetry in the reaction forces for shafts 2 and 3, the bearings on each side were evaluated separately. It was still found that similar bearings could be used on both sides. Shaft 1 has symmetric reaction forces so the same bearings can be used on both sides. The bearings used on shaft 3 are able to withstand axial loads, namely the thrust from the propeller.

Assembly

Figure 5 below is a diagram of the final gearbox, including all components, except the keys. The keys are numbered such that they fit in the gear with the same number (key 1 in gear 1). More detailed drawings and CAD models can be found in the Appendix.

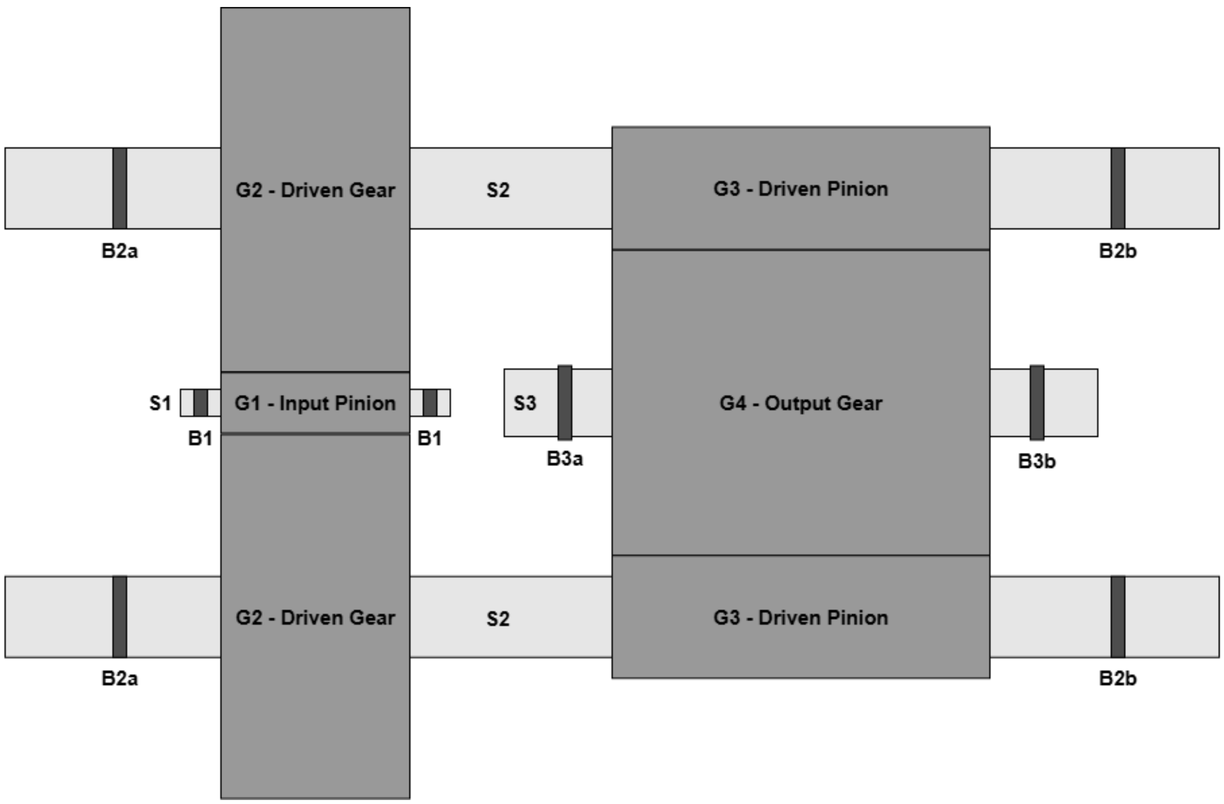


Figure 5: Gearbox assembly

The table below shows the breakdown of the gearbox' weight, which amounts to 831lb in total. Note that the keys are not included in the total. Indeed, the keys' volumes would need to be removed from the gears' and shafts' own volumes to account for the keyways. However, since the keys, gears and shafts all have the same density, the sum in the table below is equivalent to adding the keys' weights but removing their volume from the gears and shafts.

Table 21: Final total weight

Final gearbox weight (lb)			
	QTY	Unit weight (lb)	Total weight (lb)
Gear 1	1	7.067994422	7.067994422
Gear 2	2	116.9155016	233.8310032
Gear 3	2	39.87653714	79.75307429
Gear 4	1	236.0213844	236.0213844
Shaft 1	1	1.748466687	1.748466687
Shaft 2	2	102.2382744	204.4765489
Shaft 3	1	23.79453411	23.79453411
Bearing 1	2	0.1446	0.2892
Bearing 2a	2	9.495	18.99
Bearing 2b	2	9.495	18.99
Bearing 3a	1	2.949	2.949
Bearing 3b	1	2.949	2.949
		Total (lb)	830.8602059

Conclusion

The design process presented in this report successfully fulfills the key objectives set for an auxiliary transmission gearbox intended for geodesic observation aircraft missions. The resulting gearbox design effectively transmits the required power while minimizing weight and ensuring compliance with safety standards by adhering to a structured, iterative approach guided by AGMA standards, combined with detailed Excel modeling and verification through SolidWorks assemblies. The final gearbox weighs 831 lb, achieved through weight-reduction strategies such as gear weight relief features and hollow shafts. All gearbox components, including gears, shafts, keys, and bearings, were meticulously sized and selected, consistently achieving or surpassing FAA/DOT safety factor requirements. Gear safety factors exceeded unity in bending and surface contact stresses, while shaft safety factors were reliably maintained at 1.5. Future enhancements could focus on additional geometric optimizations or advanced material selections to further reduce weight, improve efficiency, and extend component lifetimes.

References

[1] R. L. Norton, *Machine design: An integrated approach*. Prentice Hall, 2011. Available: https://books.google.fr/books?id=wAY-QgAACAAJ

[2] SKF, "Rolling bearings catalog," Oct. 2018. Available: https://cdn.skfmediahub.skf.com/api/public/0901d196802809de/pdf_preview_medium/0901d196802809de_pdf_preview_medium.pdf

Appendix

Additional tables and figures

Table 22: Given values

Givens	
Propeller speed (rpm)	1000
Engine speed (rpm)	15000
Gear life (h)	30000
Bearing life (h)	10000
Take off engine power (hp)	1050
After 10% adjustment	1155
Propeller weight (lb)	150
Propeller max axial load (lb)	600

Table 23: Derived values from givens

Derived from givens	
2(10 LAM + 1 HAM) time (min)	3650
Gear life (mins)	1800000
Bearing life (min)	600000
LAM total # of cycles @ engine output	2591250
HAM total # of cycles @ engine output	266250
2(10 LAM + 1 HAM) # of cycles @ engine output	52357500
Average # of cycles per min @ engine output	14344.52055
Average time per cycle @ engine output (min)	6.9713E-05
Total # of 10-1-10-1 sequences possible for bearing life	164.3835616
Total # of cycles @ engine output for that amount of sequences	8606712329
Total # of 10-1-10-1 sequences possible for gear life	493.1506849
Total # of cycles @ engine output for that amount of sequences	25820136986

Table 24: LAM and HAM loads per phase

a. *LAM*

Load cycles													
LAM 1													
phase	Duration (min)	Timestamp (min)	% of max engine power	actual power (rpm)	% of rotation speed	Actual rotation speed	# of cycles (output from engine)	Engine torque (lb in)	Input pinion torque (lb in)	Driven gear torque (lb in)	Driven pinion torque (lb in)	Output gear torque (lb in)	Propeller torque (lb in)	Axial load from prop (lb)
initial state		0	25	288.75	40	6000	0	1213.23125	1213.23125	3639.69375	3639.69375	18198.46875	18198.46875	150
on tarmac	10	10	25	288.75	40	6000	60000	1213.23125	1213.23125	3639.69375	3639.69375	18198.46875	18198.46875	150
take off	10	20	100	1155	100	15000	150000	4852.925	4852.925	14558.775	14558.775	72793.875	72793.875	600
observation	155	175	80	924	100	15000	2325000	3882.34	3882.34	11647.02	11647.02	58235.1	58235.1	0
landing	5	180	75	866.25	75	11250	56250	3639.69375	3639.69375	10919.08125	10919.08125	54595.40625	54595.40625	450

b. *HAM*

HAM 1														
phase	Duration (min)	Timestamp (min)	% of max engine power	actual power (rpm)	% of rotation speed	Actual rotation speed	# of cycles (output from engine)	Engine torque (lb in)	Input pinion torque (lb in)	Driven gear torque (lb in)	Driven pinion torque (lb in)	Output gear torque (lb in)	Propeller torque (lb in)	Axial load from prop (lb)
Initial state		1800	25	288.75	40	6000	0	1213.23125	1213.23125	3639.69375	3639.69375	18198.46875	18198.46875	150
on tarmac	10	1810	25	288.75	40	6000	60000	1213.23125	1213.23125	3639.69375	3639.69375	18198.46875	18198.46875	150
take off	10	1820	100	1155	100	15000	150000	4852.925	4852.925	14558.775	14558.775	72793.875	72793.875	0
observation	0	1820	0	0	0	0	0	0	0	0	0	0	0	0
landing	5	1825	75	866.25	75	11250	56250	3639.69375	3639.69375	10919.08125	10919.08125	54595.40625	54595.40625	450

c. Max values

	Engine torque (lb in)	Input pinion torque (lb in)	Driven gear torque (lb in)	Driven pinion torque (lb in)	Output gear torque (lb in)	Propeller torque (lb in)	Axial load from prop (lb)
Max values	4852.925	4852.925	14558.775	14558.775	72793.875	72793.875	600

Table 25: Parameters used in gear calculations

Gear calculation parameters	
Max power (hp)	1155
omega_in (rpm)	15000
omega_out (rpm)	1000
Material	AISI 4140
Max input torque T_in max (lb in)	4852.848
Max 2nd level torque T_mid (lb in)	14558.544
total # of cycles for one sequence 10-1-10-1	
Engine output	52357500
Gear 1 input pinion	52357500
Gear 2 driven gear	8726250
Gear 3 driven pinion	8726250
Gear 4 output gear	3490500
Final # of cycles at end of life	
Gear 1 input pinion	5.164E+10
Gear 2 driven gear	4303356164
Gear 3 driven pinion	4303356164
Gear 4 output gear	3442684932

Note that the final cycles at the end of life of the gears are doubled for gears 1 and 4, as their teeth mesh twice per rotation.

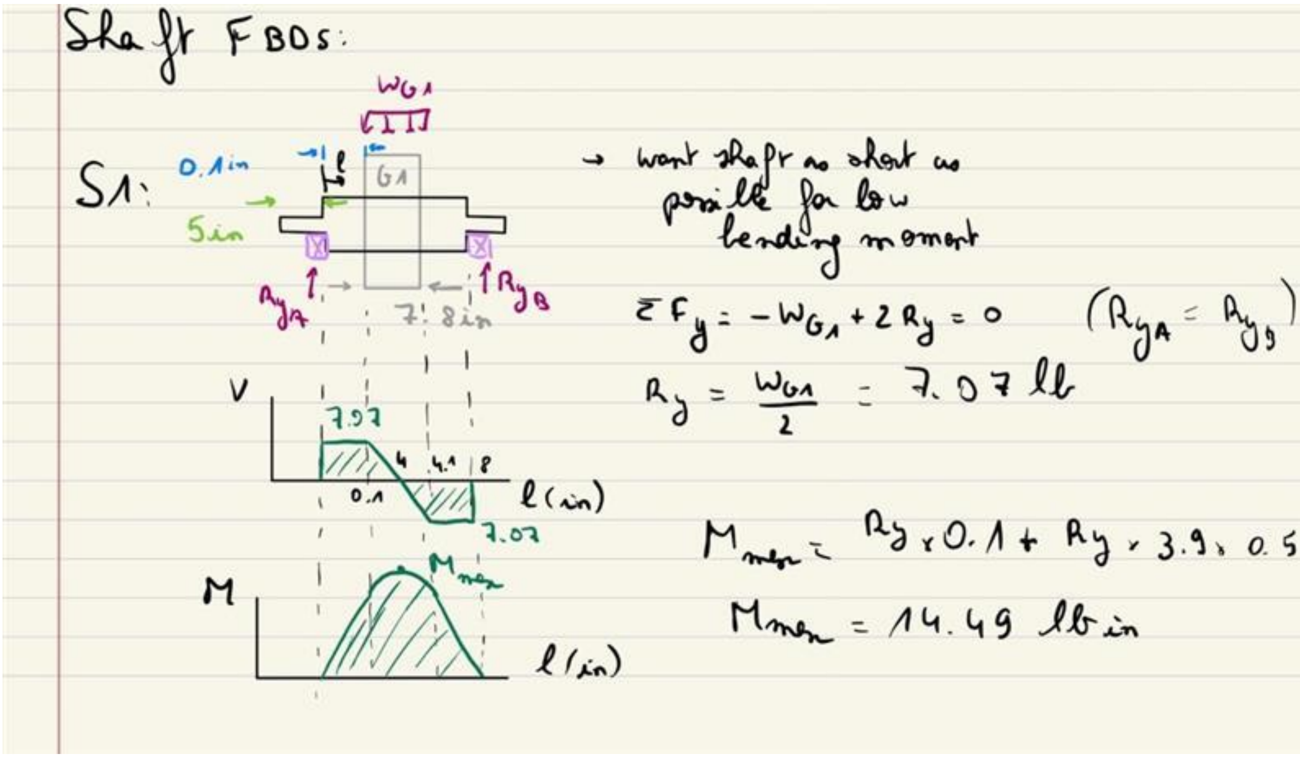
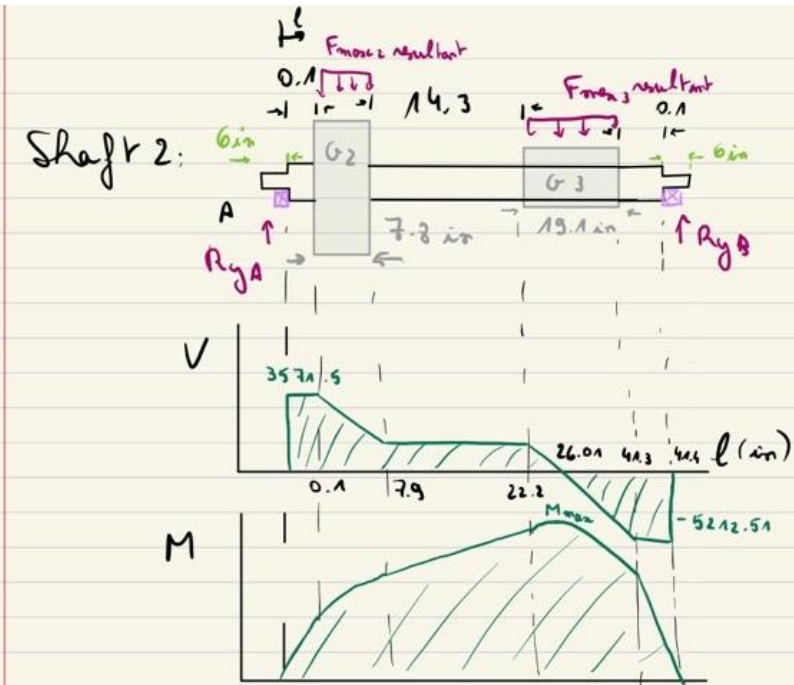


Figure 6: FBD, shear and moment diagram of shaft 1



$$+)\ \Sigma M_A = F_{max\,2} \times \left(\frac{7.8}{2} + 0.1\right) - F_{max\,3} \times \left(\frac{19.1}{2} + 14.3 + 0.1 + 7.8\right)$$

$$+ R_{y_B} \times (53.4 - 6 \times 2) = 0 \Rightarrow R_{y_B} = 5212.52 \text{ lb}$$

$$+\uparrow\ \Sigma F_y = 0 = -F_{max\,2} - F_{max\,3} + R_{y_B} + R_{y_A} \Rightarrow R_{y_A} = 3571.56 \text{ lb}$$

find l for $V = 0$: at $l = 22.2$ in, $V = 1310.85$ lb

at $l = 41.3$, $V = -5245.88$ lb $\rightarrow$ linear curve for uniformly distributed load

$$\frac{-5245.88 - 1310.85}{41.3 - 22.2}\, l' + 1310.85 = 0 \Rightarrow l' = 3.81 \text{ left from } x = 22.2 \text{ in}$$

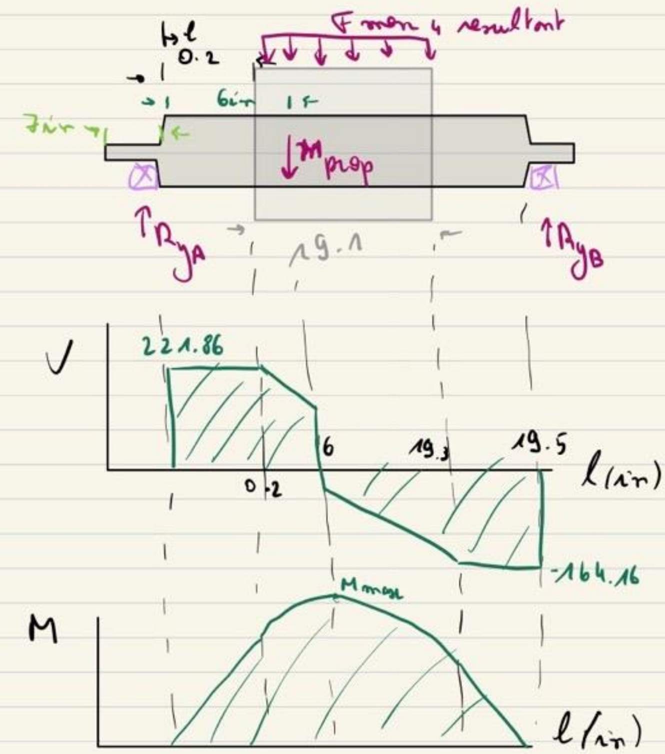
so at $l = 26.01$ in

M_{max} = area under V until l'

$\Rightarrow M_{max} = 37906.6$ lb in

Figure 7: FBD, shear and moment diagram of shaft 2

Shaft 3 :



$$\circlearrowleft + \quad \Sigma M_A = R_{y_B}\,(0.2 \cdot 2 + 19.1) - m_{prop} \cdot 6 - F_{mon\,4}\left(\frac{19.1}{2} + 0.2\right) = 0$$

$$\Rightarrow R_{y_B} = 164.16 \text{ lb}$$

$$+\uparrow \quad \Sigma F_y = 0 = R_{y_A} + R_{y_B} - m_{prop} - F_{mon\,4} \Rightarrow R_{y_A} = 221.86 \text{ lb}$$

$V = 0$ at $l = 6$ in

M_{max} = area under V curve before $l = 6$ in

$$\Rightarrow M_{max} = 1331.14 \text{ lb in}$$

Figure 8: FBD, shear and moment diagram of shaft 3

Table 26: Torques applied on the shafts

Torques on shafts (lb in)	
Shaft 1 (input / gear 1)	
Max torque T_max	4852.925
Min torque T_min	0
Alternating torque T_a	0
Mean torque T_m	4852.925
Shaft 2 (gears 2 and 3)	
Max torque	14558.775
Min torque	0
Alternating torque	0
Mean torque	14558.775
Shaft 3 (gear 4 and propeller)	
Max torque	72793.875
Min torque	0
Alternating torque	0
Mean torque	72793.875

Table 27: Bending forces applied on the shafts

Bending Forces on shafts (lb/in)	
Shaft 1 (weight of gear 1)	
Fmax	0.906153131
Fmin	-0.906153131
Fa	0.906153131
Fm	0
Shaft 2 (torque of mesh 1/2 + weight gear 2)	
Fmax	291.5047224
Fmin	-291.5047224
Fa	291.5047224
Fm	0
Shaft 2 (torque of mesh 2/3 + weight gear 3)	
Fmax	340.8555255
Fmin	-340.8555255
Fa	340.8555255
Fm	0
Shaft 3 (torque of mesh 2/3)	
Fmax	12.35714054
Fmin	-12.35714054
Fa	12.35714054
Fm	0
Resultant of distributed loads (lb)	
Fmax 1	7.067994422
Fmax2	2273.736835
Fmax3	6510.340537
Fmax4	236.0213844

Table 28: Shafts factors and calculations

Shafts calculations			
	SHAFT 1	SHAFT 2	SHAFT 3
Strengths			
Material	AISI 1095 quench and temper @ 600F		
Sut (kpsi)	183	183	183
Sy (kpsi)	118	118	118
Se' (kpsi)	91.5	91.5	91.5
Se (kpsi)	43.94196773	39.13665	28.03658583
Factors			
Cload	1	1	0.7
Csize	0.869	0.773969638	0.792076925
Csurf	0.678913157	0.678913157	0.678913157
Ctemp	1	1	1
Reliability	99%	99%	99%
Creliab	0.814	0.814	0.814
sqrt(a)_t	0.055	0.018	0.055
sqrt(a)_b	0.07	0.024	0.07
qt (Sut+20)	0.810351784	0.960590979	0.87624989
qb (Sut)	0.770499808	0.948135943	0.847641948
Kt (bending)	1.733535404	1.733535404	1.733535404
Kt (in keyway)	2.2	2.2	2.2
Kf (bending)	1.565188888	1.695491282	1.621775379
Kf (in keyway)	1.92459977	2.137763132	2.017170338
Kts (torsion)	1.483614061	1.483614061	1.483614061
Kts (in keyway)	3	3	3
Kfs (torsion)	1.391897517	1.464555304	1.423766767
Kfs (in keyway)	2.620703569	2.921181959	2.752499779
Kfm	1.842675859	1.695491282	1.981309739
Kfm (keyway)	1.878579686	2.137763132	1.971824248
Kfsm	1.391897517	1.464555304	1.423766767
Kfsm (keyway)	1.875881019	2.921181959	1.875370069
Dimensions			
D/d	1.2	1.2	1.2
r/d	0.07	0.07	0.07
d (in)	0.789	2.75	2.166666667
D (calculated, in)	0.7917338	2.9863347	2.0543681
r (in keyway, in)	0.01	0.01	0.01
r (at step, in)	0.05523	0.1925	0.151666667
d' (hollow ID, in)	0.375	0.75	1.3
I (in^4)	0.049087385	5.821376096	2.243175694
I (for hollow shaft)	0.048116663	5.80584454	2.102977214
J (in^4)	0.09817477	11.64275219	4.486351389
J (for hollow shaft)	0.096233326	11.61168908	4.205954427
D (assumed)	1	3.3000000	2.6

Drawings

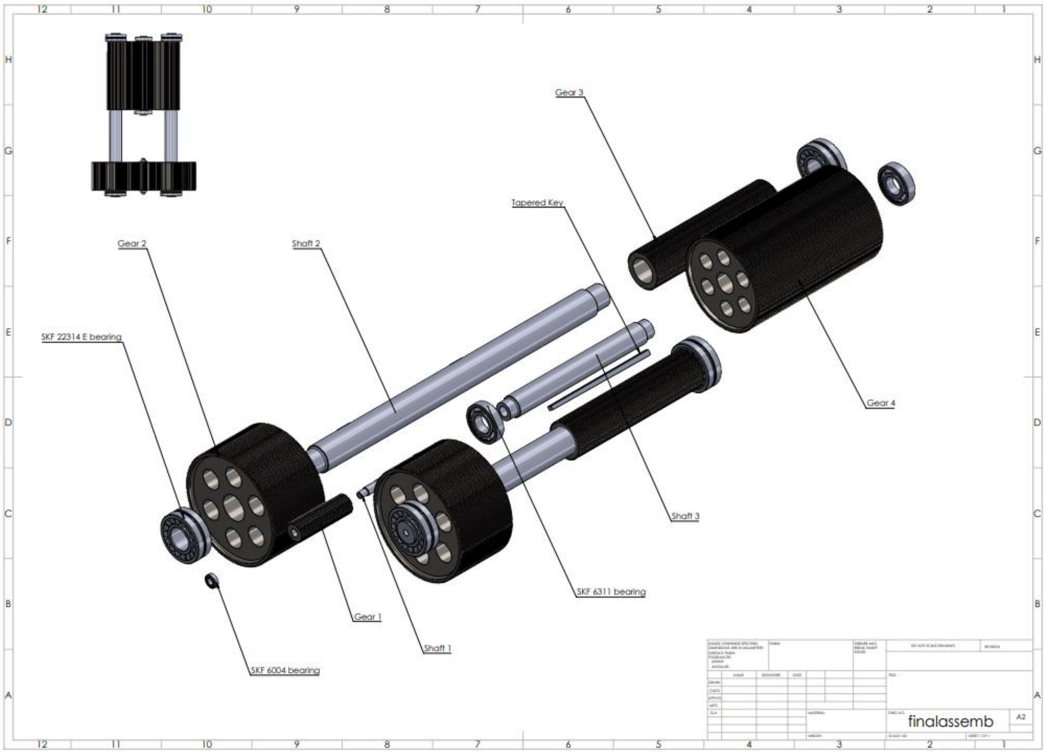


Figure 9: Exploded View Drawing

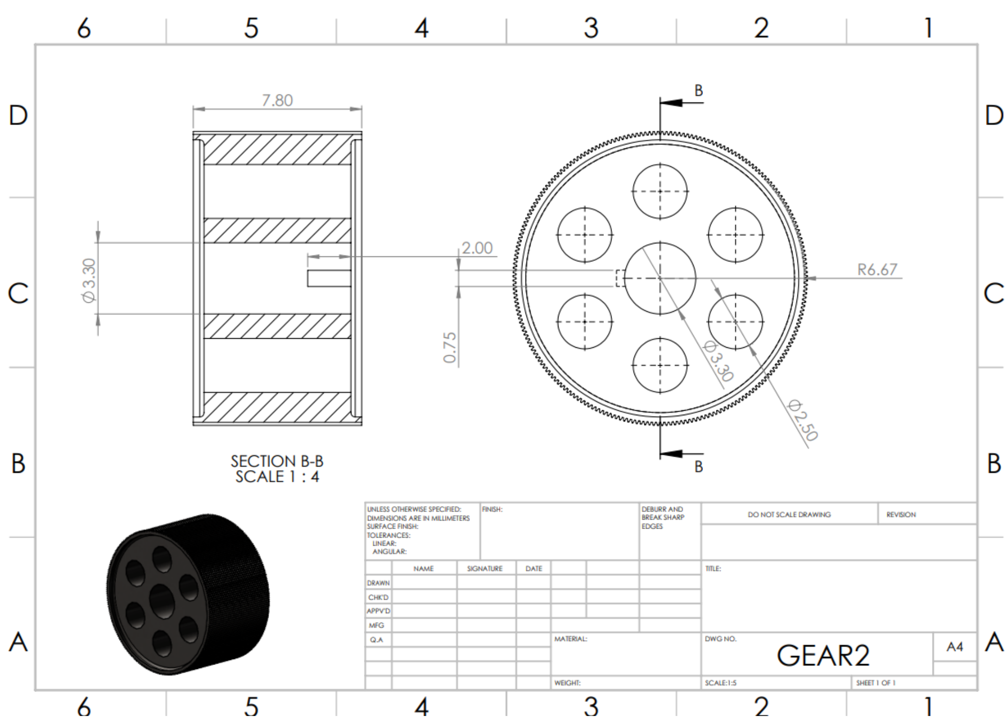


Figure 10: Gear 2 technical drawing

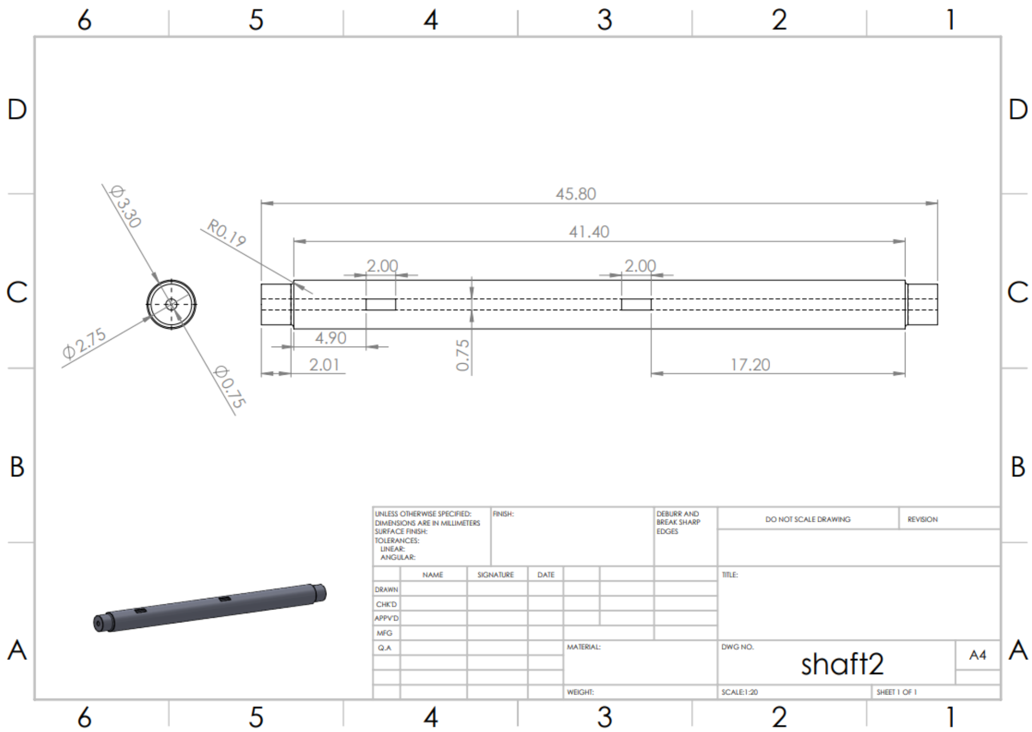


Figure 11: Shaft 2 technical drawing

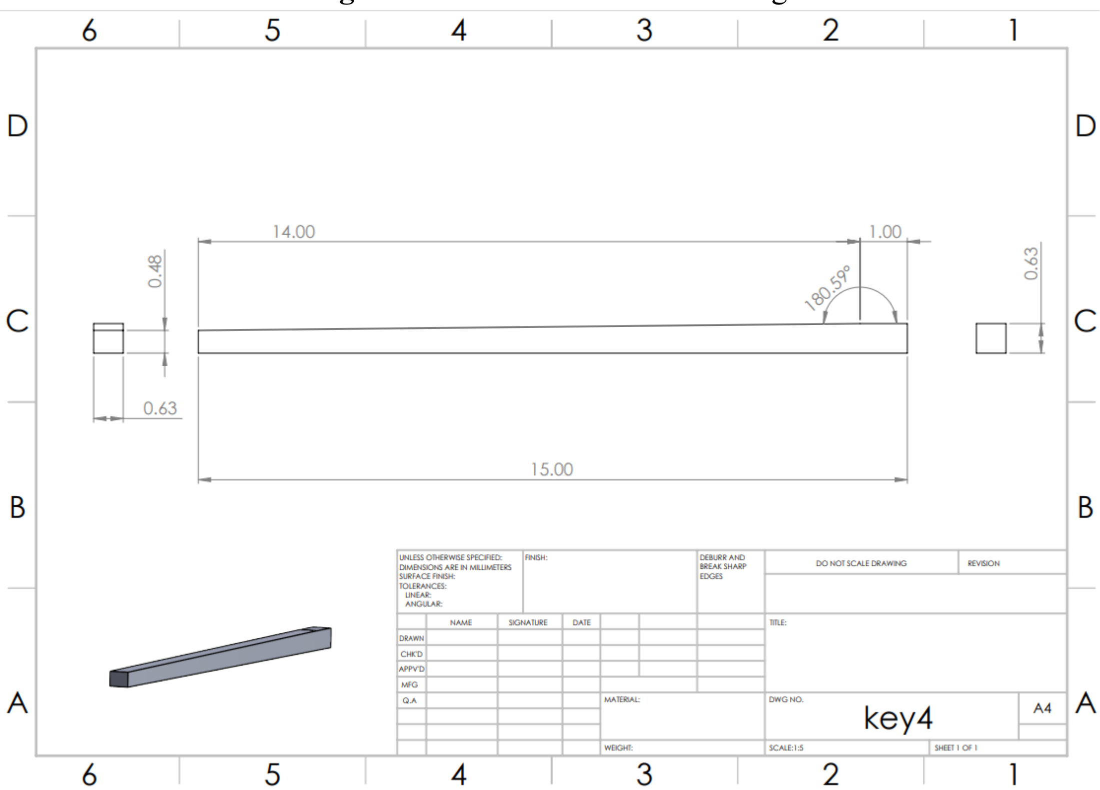


Figure 12: Key 4 technical drawing

Figure 13: Exploded View

Sample calculations

Power transmission and tangential load

The torque on the first gearset is dependent on the power exerted by the motor. The maximum torque is calculated with the input of 1150HP, which takes into account the additional 10% power increase to accommodate potential future changes to the aircraft, and the maximum rotational speed of 15000 rpm.

$$T_{max} = \frac{\max power}{\max rotational\ speed} = \frac{1155 \cdot 63024}{15000} = 4852.8\ lb.in$$

Torque across the second stage of the gearbox is: $T_m = \frac{1}{2} \cdot T_{max} \cdot \frac{N_2}{N_1} = \frac{1}{2} \cdot 4852.8 \cdot \frac{216}{36} = 14558.5\ lb.in$

It is divided by 2 as it is a double-branch gearbox and it is multiplied by the gear ratio as it is the same as the factor of torque increase from the input to the driven gear.

The tangential load, W_t, is: $W_t = 2 \cdot \frac{T_m}{d_p} = 2 \cdot \frac{T_m}{\frac{N}{P_d}}$

The diametral pitch used is of the gear that is on that shaft. Since T_m corresponds to the torque on the second stage of the gearbox (shaft 2), the gears used are gears 2 and 3.

For gears 1 and 2: $W_t = 2 \cdot \frac{T_m}{N/P_d} = 2 \cdot \frac{14558.5}{216/16} = 2156.8\ lb$

Gear calculations

The sample calculations below are for gear 1. The same method was followed for gears 2 to 4.

The geometry factor J is determined by the AGMA table 12-8 because the gears undergo pressure angle of 20º, full teeth meshing and tip loading

Table 12-8 AGMA Bending Geometry Factor J for 20°, Full-Depth Teeth with Tip Loading

	Pinion teeth															
	12		14		17		21		26		35		55		135	
Gear teeth	P	G	P	G	P	G	P	G	P	G	P	G	P	G	P	G
12	U	U														
14	U	U	U	U												
17	U	U	U	U	U	U										
21	U	U	U	U	U	U	0.24	0.24								
26	U	U	U	U	U	U	0.24	0.25	0.25	0.25						
35	U	U	U	U	U	U	0.24	0.26	0.25	0.26	0.26	0.26				
55	U	U	U	U	U	U	0.24	0.28	0.25	0.28	0.26	0.28	0.28	0.28		
135	U	U	U	U	U	U	0.24	0.29	0.25	0.29	0.26	0.29	0.28	0.29	0.29	0.29

Based on this table the geometric factor is 0.26 for gear 1.

The sizing factor K_s is always 1 when no gears are excessively large

The dynamic factor is calculated using:

$$K_V = \left(\frac{A}{A+\sqrt{V_t}}\right)^B \qquad \text{where } A = 50 + 56(1-B) \text{ and } B = \frac{(Q_v - 12)^{\frac{2}{3}}}{4}$$

However, we chose a quality factor Q_v of 12.

The load distribution factor, K_m, is based on AGMA table 12-16

Table 12-16

Load Distribution Factors K_m

Face Width in (mm)	K_m
<2 (50)	1.6
6 (150)	1.7
9 (250)	1.8
≥20 (500)	2.0

Based on this table, since all face diameter of gear 1 is 7.8 inches. For the purposes of this case, the face width is closest to 9 in., and so $K_m = 1.8$.

The rim factor, K_b is calculated by first calculating the backup ratio:

$$m_B = \frac{t_r}{h_t} = \frac{Rim\ thickness}{tooth\ height}$$

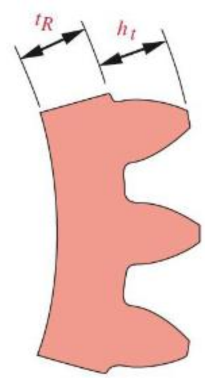


FIGURE 12-23

Parameters for AGMA Rim Thickness Factor K_B

$$K_B = -2m_B + 3.4 \qquad 0.5 \le m_B \le 1.2$$

$$K_B = 1.0 \qquad m_B > 1.2$$

Backup ratios < 0.5 are not recommended. Solid-disk gears will always have $K_B = 1$.

Where the tooth height is calculated by: $h_t = h_{addendum} + h_{dedendum} = \frac{1}{P_d} + \frac{1.25}{P_d}$

Gears 1 and 3 are the smaller gears and are solid disks, so they have a K_b of 1

To save on weight, gears 2 and 3 will have weight reliefs:

$$h_t = \frac{1}{16} + \frac{1.25}{16} = 0.1406\ in$$

We chose a 1in thickness for the inner and outer rim.

So their backup ratio is: $m_B = \frac{t_r}{h_t} = \frac{1}{0.1406} \approx 7.11$

Since $m_b > 1.2$, the rim factor K_b is 1.

The corrected bending stress is then calculated by:

$$\sigma_{ben} = \frac{W_t \cdot P_d \cdot K_a \cdot K_m \cdot K_s \cdot K_B \cdot K_I}{F \cdot I \cdot K_v}$$

For gear 1: $\sigma_{ben} = \frac{2156.8 \cdot 16 \cdot 1.12 \cdot 1.8 \cdot 1 \cdot 1 \cdot 1}{7.8 \cdot 0.26 \cdot 1} \cdot 1.05 = 36020.2\, psi$

An additional 5% is added to the calculated stresses as per the project description.

For the bending strength, the material's bending fatigue strength must be taken into account. The material used for all of the gears is AISI 4140 steel which has an uncorrected bending fatigue strength of 45000 psi based on AGMA table 12-20

Table 12-20 AGMA Bending-Fatigue Strengths S_{fb}' for a Selection of Gear Materials*

Material	AGMA Class	Material Designation	Heat Treatment	Minimum Surface Hardness	Bending-Fatigue Strength psi x 10^3	Bending-Fatigue Strength MPa
Steel	A1–A5		Through hardened	≤ 180 HB	25–33	170–230
			Through hardened	240 HB	31–41	210–280
			Through hardened	300 HB	36–47	250–325
			Through hardened	360 HB	40–52	280–360
			Through hardened	400 HB	42–56	290–390
			Flame or induction hardened	Type A pattern 50–54 HRC	45–55	310–380
			Flame or induction hardened	Type B pattern	22	150
			Carburized and case hardened	55–64 HRC	55–75	380–520
		AISI 4140	Nitrided	84.6 HR15N†	34–45	230–310
		AISI 4340	Nitrided	83.5 HR15N	36–47	250–325
		Nitralloy 135M	Nitrided	90.0 HR15N	38–48	260–330
		Nitralloy N	Nitrided	90.0 HR15N	40–50	280–345
		2.5% Chrome	Nitrided	87.5–90.0 15N	55–65	380–450
Cast iron	20	Class 20	As cast		5	35
	30	Class 30	As cast	175 HB	8	69
	40	Class 40	As cast	200 HB	13	90
Nodular (ductile) iron	A-7-a	60-40-18	Annealed	140 HB	22–33	150–230
	A-7-c	80-55-06	Quenched and tempered	180 HB	22–33	150–230
	A-7-d	100-70-03	Quenched and tempered	230 HB	27–40	180–280
	A-7-e	120-90-02	Quenched and tempered	230 HB	27–40	180–280
Malleable iron (pearlitic)	A-8-c	45007		165 HB	10	70
	A-8-e	50005		180 HB	13	90
	A-8-f	53007		195 HB	16	110
	A-8-i	80002		240 HB	21	145
Bronze	Bronze 2	AGMA 2C	Sand cast	40 ksi min tensile strength	5.7	40
	Al/Br 3	ASTM B-148 78 alloy 954	Heat treated	90 ksi min tensile strength	23.6	160

The life factor, K_L, is calculated based on the number of cycles of each gear.

The number of cycles at the engine output for 2 sets of 10 low-altitude-missions and 1 high-altitude-mission is 52357500 and requires 3650 minutes to complete. Knowing this information and the total life of a gear that is 30000 hours, it can be calculated that the number of revolutions performed by gear 1 is 25820136986. This number needs to be multiplied by 2 as gear 1's teeth meet 2 different gears during 1 revolution due to the gearbox's double-branch design. The overall number cycles gear 1 undergoes is 51640273973.

Using the gear ratio of gear 1 to gear 2, the number of cycles of gear 2 can be calculated:

$$\#\ of\ cycles, gear\ 2 = \frac{216}{36} \cdot 25820136986 = 430335614$$

Gear 2 and gear 3 undergo the same number of cycles as they are mounted onto the same shaft.

$$\#\ of\ cycles, gear\ 3 = 4303356164$$

For gear 4, its number of revolutions is calculated the same way:

$$\#\ of\ cycles, gear\ 4 = 2 \cdot \frac{72}{180} * 4303356164 = 3442684932$$

Similar to gear 1, gear 4 undergoes 2 cycles every revolution as it is meshed with 2 gears.

Based on AGMA graph 12-24, the life factor can be determined.

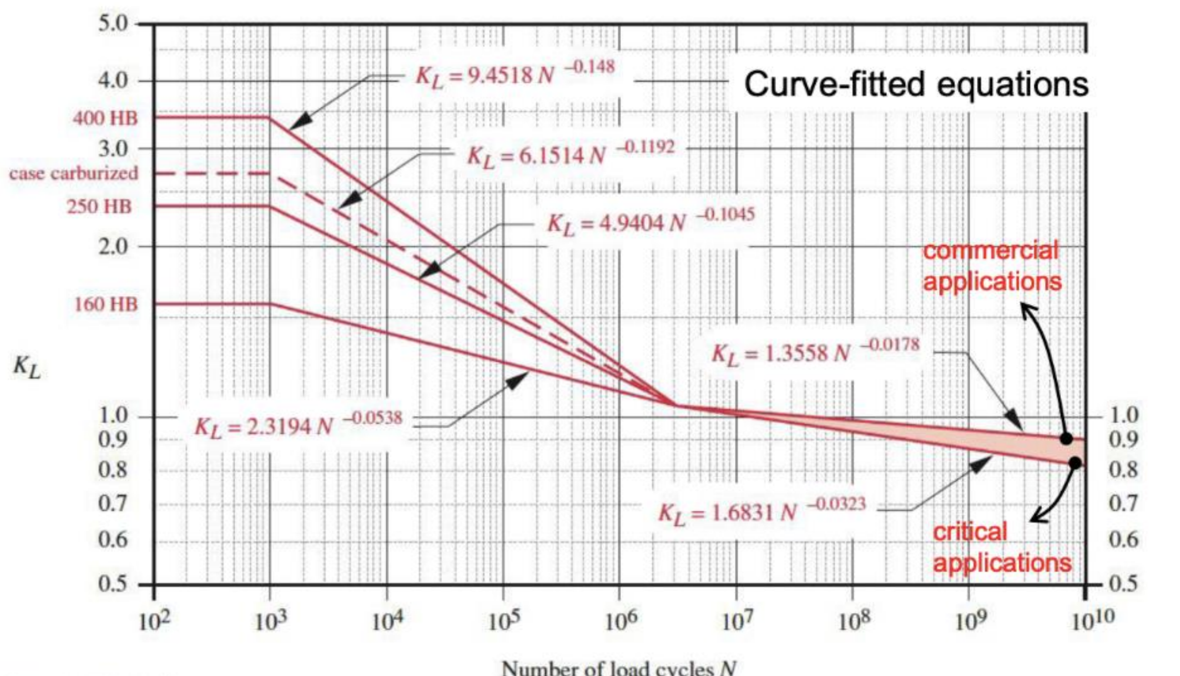


FIGURE 12-24*

AGMA Bending Strength Life Factor K_L

All of the gears undergo more than $1\text{x}10^7$ cycles, so we must chose between the 2 possible lines. Since the gearbox is a commercial application, we opted to use the top line.

$$K_L = 1.3558 \cdot N^{-0.0178}$$

For gear 1, K_L = 0.8739

The temperature factor K_T can be determined with the following:

$$K_T = \frac{460 + T_F}{620}$$

However since the operating temperatures are less than 450ºC, K_T = 1

To find the reliability factor, K_r, the AGMA table 12-19 must be used.

Table 12-19
AGMA Factor K_R

Reliability %	K_R
90	0.85
99	1.00
99.9	1.25
99.99	1.50

Since the reliability was assumed to be 99%, K_r=1

Finally, the corrected bending fatigue strength can be calculated:

$$S_{fb} = \frac{K_L}{K_T \cdot K_r} \cdot S'_{fb}$$

For gear 1: $S_{fb} = \frac{0.8739}{1 \cdot 1} \cdot 45000 = 39329.4\, psi$

The bending safety factors can then be found:

$$N_{ben} = \frac{S_{fb}}{\sigma_{ben}} = \frac{39329.4}{36020.2} = 1.092$$

Similar steps can be taken to find the contact surface safety factor.

The tangential load, W_t, application factor, C_a, load distribution factor, C_m, size factor, C_s, and dynamic factor, C_v are equivalent to their bending stress counterparts:

$C_a = K_a = 1.12$

$$C_m = K_m = 1.8$$

$$C_s = K_s = 1$$

$$C_v = K_v = 1$$

The elastic coefficient, C_p, can be determined using the AGMA table 12-18

Table 12-18 AGMA Elastic Coefficient C_p in Units of $[psi]^{0.5}$ ($[MPa]^{0.5}$)*†

Pinion Material	E_p psi (MPa)	Gear Material: Steel	Malleable Iron	Nodular Iron	Cast Iron	Aluminum Bronze	Tin Bronze
Steel	30E6 (2E5)	2 300 (191)	2 180 (181)	2 160 (179)	2 100 (174)	1 950 (162)	1 900 (158)
Malleable Iron	25E6 (1.7E5)	2 180 (181)	2 090 (174)	2 070 (172)	2 020 (168)	1 900 (158)	1 850 (154)
Nodular Iron	24E6 (1.7E5)	2 160 (179)	2 070 (172)	2 050 (170)	2 000 (166)	1 880 (156)	1 830 (152)
Cast Iron	22E6 (1.5E5)	2 100 (174)	2 020 (168)	2 000 (166)	1 960 (163)	1 850 (154)	1 800 (149)
Aluminum Bronze	17.5E6 (1.2E5)	1 950 (162)	1 900 (158)	1 880 (156)	1 850 (154)	1 750 (145)	1 700 (141)
Tin Bronze	16E6 (1.1E5)	1 900 (158)	1 850 (154)	1 830 (152)	1 800 (149)	1 700 (141)	1 650 (137)

†The values of E_p in this table are approximate and ν = 0.3 was used as an approximation of Poisson's ratio for all materials. If more accurate numbers are available for E_p and ν, they should be used in equation 11.23 to obtain C_p.

For the first mesh set, gear 1 and 2 are both made of steel, so according to the table, $C_p = 2300\ psi^{0.5}$. For the second mesh set, gear 3 and 4 are also both made of steel, so there is also $C_p = 2300\ psi^{0.5}$.

The surface geometry factor, I can be calculated by:

$$I = \frac{\cos(\phi)}{(\frac{1}{\rho_p} \pm \frac{1}{\rho_g}) \cdot d_p}$$

In this formula, all gearsets are external sets, the "+" would be used. However, to simplify calculations, and to avoid having to calculate the radii of curvature of the pinion and gear teeth the following formula can be used:

$$I = \frac{N_g}{N_p + N_g} \cdot \frac{\cos(\phi) \cdot \sin(\phi)}{2}$$

Where ϕ is the pressure angle, in this case it is 20º

For the first gear pair:

$$I = \frac{216}{36 + 216} \cdot \frac{\cos(20º) \cdot \sin(20º)}{2} = 0.1377$$

According to AGMA, the surface finish factor, C_f, does not have a standard yet. For the purposes of this research $C_f = 1$.

The surface contact stress can be calculated by:

$$\sigma_{con} = C_p \sqrt{\frac{W_t}{F \cdot I \cdot d_p} \cdot \frac{C_a \cdot C_m \cdot C_s \cdot C_f}{C_v}}$$

$$\sigma_{con} = 2300 \sqrt{\frac{2156.8}{7.8 \cdot 0.1377 \cdot 2.25} \cdot \frac{1.12 \cdot 1.8 \cdot 1 \cdot 1}{1}} \cdot 1.05 = 102423.7\, psi$$

An additional 5% is added to the stresses.

The general formula to calculate the surface-contact strength is given by:

$$S_{fc} = \frac{C_L \cdot C_H}{C_T \cdot C_R} \cdot S_{fc}'$$

Where:

$C_L = K_L$ = dependent on the number of cycles

$C_T = K_T = 1$

$C_R = K_R = 1$

The hardness ratio factor, C_H accounts for when the pinion teeth work-harden the gear teeth. C_H is only applied to the gears, not the pinion.

It is determined by:

$$C_H = 1 + A(m_G - 1)$$

Where m_G is the gear ratio and A is determined by the harness ratio of the pinion and gear materials.

If $\frac{HB_p}{HB_g} < 1.2$ then A=0

For the first gear set, the pinion and the gear are made of the same material, their ratio of their material's hardness is 1, so A = 0

For the second gear set, the pinion and gear are also made of the same material. There hardness ratio is also 1, so A is also 0.

Since A is 0, based on the formula of C_H, C_H=1 for all the gears.

The uncorrected surface-contact strength for AISI 4140 can be found in the AGMA table 12-21

Table 12-21 AGMA Surface-Fatigue Strengths S_{fc}' for a Selection of Gear Materials*

Material	AGMA Class	Material Designation	Heat Treatment	Minimum Surface Hardness	Surface-Fatigue Strength psi x 10^3	Surface-Fatigue Strength MPa
Steel	A1–A5		Through hardened	≤ 180 HB	85–95	590–660
			Through hardened	240 HB	105–115	720–790
			Through hardened	300 HB	120–135	830–930
			Through hardened	360 HB	145–160	1000–1100
			Through hardened	400 HB	155–170	1100–1200
			Flame or induction hardened	50 HRC	170–190	1200–1300
			Flame or induction hardened	54 HRC	175–195	1200–1300
			Carburized and case hardened	55–64 HRC	180–225	1250–1300
		AISI 4140	Nitrided	84.6 HR15N†	155–180	1100–1250
		AISI 4340	Nitrided	83.5 HR15N	150–175	1050–1200
		Nitralloy 135M	Nitrided	90.0 HR15N	170–195	1170–1350
		Nitralloy N	Nitrided	90.0 HR15N	195–205	1340–1410
		2.5% Chrome	Nitrided	87.5 HR15N	155–172	1100–1200
		2.5% Chrome	Nitrided	90.0 HR15N	192–216	1300–1500
Cast iron	20	Class 20	As cast		50–60	340–410
	30	Class 30	As cast	175 HB	65–70	450–520
	40	Class 40	As cast	200 HB	75–85	520–590
Nodular (ductile) iron	A-7-a	60-40-18	Annealed	140 HB	77–92	530–630
	A-7-c	80-55-06	Quenched and tempered	180 HB	77–92	530–630
	A-7-d	100-70-03	Quenched and tempered	230 HB	92–112	630–770
	A-7-e	120-90-02	Quenched and tempered	230 HB	103–126	710–870
Malleable iron (pearlitic)	A-8-c	45007		165 HB	72	500
	A-8-e	50005		180 HB	78	540
	A-8-f	53007		195 HB	83	570
	A-8-i	80002		240 HB	94	650
Bronze	Bronze 2	AGMA 2C	Sand cast	40 ksi min tensile strength	30	450
	Al/Br 3	ASTM B-148 78 alloy 954	Heat-treated	90 ksi min tensile strength	65	450

† Rockwell 15N scale used for case-hardened materials—see Section 2-4

The corrected surface-contact strength for gear 1 is given by:

$$S_{fc} = \frac{C_L \cdot C_H}{C_T \cdot C_R} \cdot S_{fc}' = \frac{0.8739 \cdot 1}{1 \cdot 1} \cdot 180000 = 157317.5\, psi$$

Finally, the surface-contact safety factor can be calculated for each gear:

$$N_{con} = \frac{S_{fc}}{\sigma_{con}} = \frac{157317.5}{102423.7} = 1.536$$

Shafts

The sample calculations below are for the ideal diameter of shaft 2. The same method can be applied to the other shafts. Recall that this is an iterative process.

The material used for the shafts is AISI 1095 steel, quenched and tempered at 600ºF. This metal has been cold-rolled during manufacturing. This material has an ultimate tensile strength, S_{ut}, of 183 kpsi and a yield strength, S_y, of 118 kpsi according to the AGMA table A-9.

Table A-9 Mechanical Properties for Some Carbon Steels

Data from Various Sources.* Approximate Values. Consult Material Manufacturers for More Accurate Information

SAE / AISI Number	Condition	Tensile Yield Strength (0.2% offset)		Ultimate Tensile Strength		Elongation over 2 in	Brinell Hardness
		kpsi	MPa	kpsi	MPa	%	-HB
1010	hot rolled	26	179	47	324	28	95
	cold rolled	44	303	53	365	20	105
1020	hot rolled	30	207	55	379	25	111
	cold rolled	57	393	68	469	15	131
1030	hot rolled	38	259	68	469	20	137
	normalized @ 1650°F	50	345	75	517	32	149
	cold rolled	64	441	76	524	12	149
	quench & temper @ 1000°F	75	517	97	669	28	255
	quench & temper @ 800°F	84	579	106	731	23	302
	quench & temper @ 400°F	94	648	123	848	17	495
1035	hot rolled	40	276	72	496	18	143
	cold rolled	67	462	80	552	12	163
1040	hot rolled	42	290	76	524	18	149
	normalized @ 1650°F	54	372	86	593	28	170
	cold rolled	71	490	85	586	12	170
	quench & temper @ 1200°F	63	434	92	634	29	192
	quench & temper @ 800°F	80	552	110	758	21	241
	quench & temper @ 400°F	86	593	113	779	19	262
1045	hot rolled	45	310	82	565	16	163
	cold rolled	77	531	91	627	12	179
1050	hot rolled	50	345	90	621	15	179
	normalized @ 1650°F	62	427	108	745	20	217
	cold rolled	84	579	100	689	10	197
	quench & temper @ 1200°F	78	538	104	717	28	235
	quench & temper @ 800°F	115	793	158	1089	13	444
	quench & temper @ 400°F	117	807	163	1124	9	514
1060	hot rolled	54	372	98	676	12	200
	normalized @ 1650°F	61	421	112	772	18	229
	quench & temper @ 1200°F	76	524	116	800	23	229
	quench & temper @ 1000°F	97	669	140	965	17	277
	quench & temper @ 800°F	111	765	156	1076	14	311
1095	hot rolled	66	455	120	827	10	248
	normalized @ 1650°F	72	496	147	1014	9	13
	quench & temper @ 1200°F	80	552	130	896	21	269
	quench & temper @ 800°F	112	772	176	1213	12	363
	quench & temper @ 600°F	118	814	183	1262	10	375

* *SAE Handbook*, Society of Automotive Engineers, Warrendale, Pa.; *Metals Handbook*, American Society for Metals, Materials Park, Ohio.

The initial assumptions include assuming the diameter of the shaft, the relationship between the fillet radius and shaft diameter, and the ratio between the large and small diameters of the step shaft.

$$D = 1\ in$$

$$r/d = 0.07$$

$$D/d = 1.2$$

The initial diameter assumed is 1 in, however this is changed and iterated, until the actual diameter is equal to the assumed diameter. For shaft 1, we got a diameter of 0.792 in, for shaft 2 a diameter of 2.986 in and shaft 3 of diameter 2.054 in.

To calculate the safety factor of the shaft we must calculate the endurance limit:
The uncorrected endurance limit, S_e', is determined by its ultimate tensile strength.

Uncorrected Fatigue Strength Estimates (Section 6.5):

$$\text{steels:} \quad \begin{cases} S_{e'} \cong 0.5 S_{ut} & \text{for } S_{ut} < 200 \text{ kpsi } (1\,400 \text{ MPa}) \\ S_{e'} \cong 100 \text{ kpsi } (700 \text{ MPa}) & \text{for } S_{ut} \geq 200 \text{ kpsi } (1\,400 \text{ MPa}) \end{cases}$$

Since S_{ut} < 200 kpsi, $S_e' = 0.5 \cdot S_{ut} = 0.5 \cdot 183 = 91.5\ kpsi$

To determine the corrected endurance limit, the following correction factors must be applied:
The load factor, $C_{Load} = 1$ since the shafts 1 and 2 only undergo bending and torsion loads and no axial forces. Shaft 3, however, undergoes axial loading due to the propeller thrust and has a $C_{Load} =$ 0.7

Since all of the shaft diameters are between 0.3 in and 10 in and they are hollow, to calculate the size factor we must use:

$$C_{size} = 0.869 \cdot d_{equiv.}^{-0.097}$$

Since the shafts are hollow, the equivalent diameter must be calculated:

$$A_{95} = 0.0766D^2 = 0.0766(3.30)^2 = 0.6789$$

$$d_{equiv.} = \sqrt{\frac{A_{95}}{0.0766}} = \sqrt{\frac{0.6789}{0.0766}} = 3.30$$

$$C_{size} = 0.869 \cdot 3.01^{-0.097} = 0.7740$$

It can be noticed that the actual outer diameter of the shaft and the equivalent diameter are the same. In this case it is to be expected. Area 95 (portion of the cross-sectional) undergoes 95% of the stresses in the shaft, and, for our shafts, that area is not hollow because $d' < 0.95D$.

The surface factor, C_{surf}, is a function of the material's ultimate tensile strength. $C_{surf} = A \cdot S_{ut}^b$

Since the shafts are cold-rolled, A = 2.7 and b = -0.265 based on AGMA table 6-3

Table 6-3 **Coefficients for Surface-Factor Equation 6.7e**

Source: Shigley and Mischke, *Mechanical Engineering Design*, 5th ed., McGraw-Hill, New York, 1989, p. 283 with permission

	For S_{ut} in MPa use		For S_{ut} in kpsi (not psi) use	
Surface Finish	*A*	*b*	*A*	*b*
Ground	1.58	–0.085	1.34	–0.085
Machined or cold-rolled	4.51	–0.265	2.7	–0.265
Hot-rolled	57.7	–0.718	14.4	–0.718
As-forged	272	–0.995	39.9	–0.995

$$C_{surf} = 2.7 \cdot S_{ut}^{-0.265} = 2.7 \cdot 183^{-0.265} = 0.6789$$

Similarly to the gear calculations, we assumed a reliability of 99%, and based on the AGMA table 6-4, $C_{reliab} = 0.814$.

Table 6-4

Reliability Factors for $S_d = 0.08\ \mu$

Reliability %	C_{reliab}
50	1.000
90	0.897
95	0.868
99	0.814
99.9	0.753
99.99	0.702
99.999	0.659
99.9999	0.620

For the temperature correction factor, the operating conditions must be taken into account. Since the gearbox of a gas turbine does not exceed 450ºC (840ºF), $C_{temp} = 1$.

Finally, we can calculate the corrected endurance limit:

$$S_e = C_{Load} \cdot C_{size} \cdot C_{surf} \cdot C_{reliab} \cdot C_{temp} \cdot S'_e = 1 \cdot 0.7740 \cdot 0.6789 \cdot 0.814 \cdot 1 \cdot 91.5 = 39.14\ kpsi$$

To find the safety factor of the shafts we must also consider the Von Mises stresses. To determine these we must consider the forces applied onto the shafts. Below we can observe the shear and moment diagrams of shaft 2. The forces applied onto shaft 2 include the weight of the gears and the tangential forces.

The maximum moment can be observed at 26.1 inches to the right of the left step and is of magnitude 37906.6 lb, as shown on figure 7.

The materials Neuber's constant for steels is found on the AGMA table 6-6

Table 6-6
Neuber's Constant for Steels

S_{ut} (ksi)	$\sqrt{a}$ ($in^{0.5}$)
50	0.130
55	0.118
60	0.108
70	0.093
80	0.080
90	0.070
100	0.062
110	0.055
120	0.049
130	0.044
140	0.039
160	0.031
180	0.024
200	0.018
220	0.013
240	0.009

Based on this table, $\sqrt{a} = 0.024$ for the shaft's bending and $\sqrt{a}_s = 0.018$ for the torsion forces.

Parts of the shaft contains keyways, and other do not.

The stress concentration factor, K_t, outside of the keyway is calculated by the formula:

$$K_t = A(\frac{r}{d})^b$$

As mentioned, we assumed $\frac{r}{d} = 0.07$ and $\frac{D}{d} = 1.2$, and based on the figure C-2 below, since D/d is 1.2, A = 0.97198 and b=-0.21796.

$$K_t = 0.97198 \cdot (0.07)^{-0.21796} = 1.7335$$

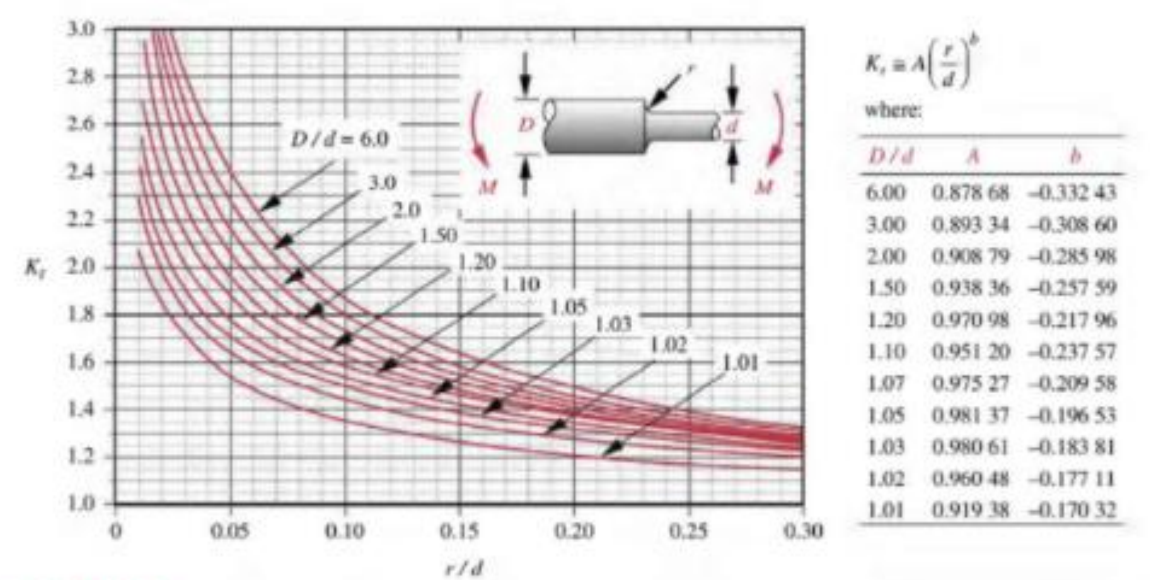


FIGURE C-2

Geometric Stress-Concentration Factor K_t for a Shaft with a Shoulder Fillet in Bending

Similarly for the stress concentration factor for the torsion forces (figure C-3).

$$K_{ts} = 0.83425 \cdot (0.07)^{-0.21649} = 1.4836$$

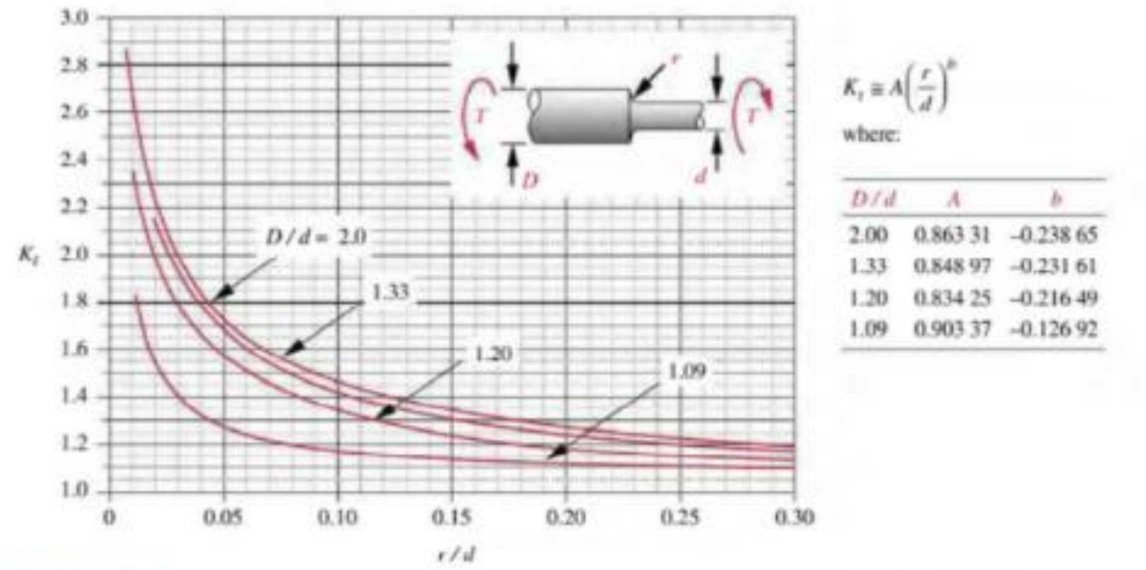


FIGURE C-3

Geometric Stress-Concentration Factor K_t for a Shaft with a Shoulder Fillet in Torsion

For the parts of the shaft subject to higher stress concentrations due to the keyway, the stress concentrations factors can be obtained by the Norton Figure 10-16 below.

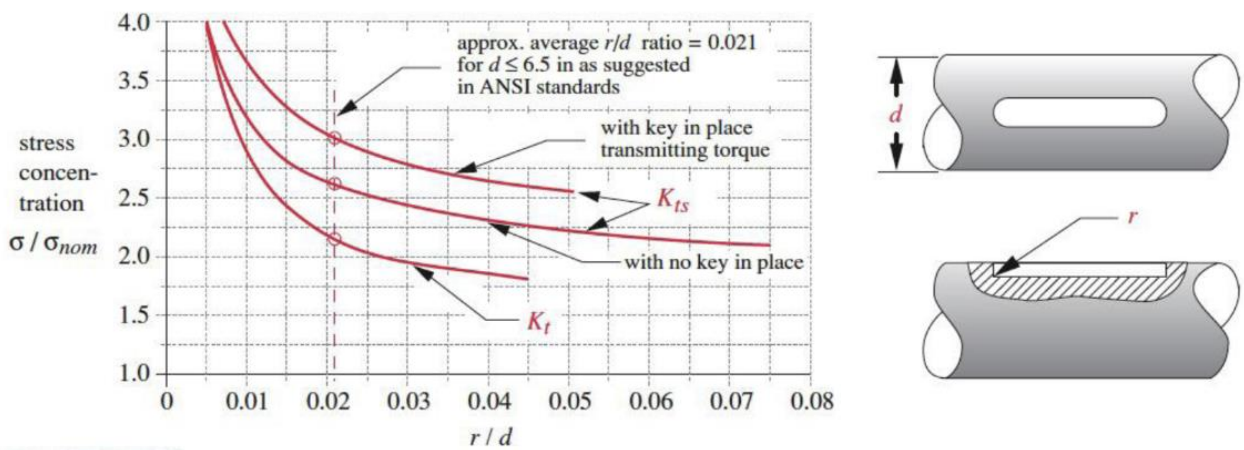


FIGURE 10-16

Stress-Concentration Factors for an End-Milled Keyseat in Bending (K_t) and Torsion (K_{ts}) Source: R. E. Peterson, *Stress Concentration Factors*, 1974, Figures 182 and 183, pp. 266–267, reprinted by permission of John Wiley & Sons, Inc.

$K_t = 2.2$ and $K_{ts} = 3.0$

The notch sensitivity can then be calculated for the bending and torsion loads.

$$q = \frac{K_f - 1}{K_t - 1} \text{ and } q_s = \frac{K_{fs} - 1}{K_{ts} - 1}$$

$$q = \frac{1}{1 + \sqrt{a}/\sqrt{r}} = \frac{1}{1 + 0.024/\sqrt{0.1743}} = 0.9456$$

$$q_s = \frac{1}{1 + \sqrt{a_s}/\sqrt{r}} = \frac{1}{1 + 0.018/\sqrt{0.1743}} = 0.9587$$

The fatigue stress concentration factors are then calculated:

$$K_f = 1 + q(K_t - 1) = 1 + 0.9456(1.7335 - 1) = 1.6937$$

$$K_{fs} = 1 + q_s(K_{ts} - 1) = 1 + 0.9587(1.4836 - 1) = 1.7032$$

Similarly, when considering the kewway the same formula as above is used and gives the following results:

$$K_f = 2.1378$$

$$K_{fs} = 2.9211$$

Since the keyways are the most critical points, we must use their stress concentration values moving forward.

To determine K_{fm} and K_{fsm} we must first determine the maximum nominal stresses. Shaft 2 does not undergo multiaxial stresses, so we do not have to use Von Mises' σ_{max}'.

$$\sigma_{max,nom} = \frac{M_{max} \cdot d}{2 \cdot I} = \frac{M_{max} \cdot d}{2} \cdot \frac{64}{\pi \cdot d^4} = \frac{32 \cdot M_{max}}{\pi \cdot d^3}$$

$M_{max} = M_m + M_a$, however, since the loading is fully reversable, $M_m = 0$ and $M_{max} = M_a$.

$$\sigma_{max,nom} = \frac{32 \cdot M_{max}}{\pi \cdot d^3} = \frac{32 \cdot M_a}{\pi \cdot d^3} = \frac{32 \cdot 37906.6}{\pi \cdot (3.30)^3} = 10744.2$$

If $K_f|\sigma_{max}| < S_y$ then $K_{fm} = K_f$

$$K_f|\sigma_{max}| < S_y$$

$$2.1378 \cdot |10744.2| < 118000$$

$$22968.9 < 118000$$

Which is true, therefore $K_{fm} = K_f = 2.1378$

The same process can be done for the torsional stress concentration factors, and $K_{fs}|\sigma_{max}| < S_y$.

Thus $K_{fsm} = K_{fs} = 2.9211$.

The alternating and mean Von Mises stresses can then be calculated:

$$\sigma_a' = \sqrt{\sigma_a^2 + 3\tau_a^2} = \sqrt{\left(K_f \frac{MD}{2I}\right)^2 + 3(0)^2} = K_f \frac{MD}{2I}$$

$$\sigma_m' = \sqrt{\left(\sigma_m + \sigma_{m_{axial}}\right)^2 + 3\tau_m^2} = \sqrt{(0)^2 + 3\left(K_{fs} \frac{Td}{2J}\right)^2} = \sqrt{3\left(K_{fs} \frac{Td}{2J}\right)^2}$$

Since the shaft is hollow, its second moment of inertia and polar moment of inertia are calculated by:

$$I = \frac{\pi \cdot (D^4 - d^4)}{64} = \frac{\pi \cdot (3.30^4 - 0.75^4)}{64} = 5.806\ in^4$$

$$J = \frac{\pi \cdot (D^4 - d^4)}{32} = \frac{\pi \cdot (3.30^4 - 0.75^4)}{32} = 11.611\ in^4$$

Then the Von Mises stresses are:

$$\sigma_a' = 2.1378 \frac{37906.6 \cdot 3.30}{2 \cdot 5.806} * 1.05 = 24181.4\ psi$$

$$\sigma_m' = \sqrt{3\left(2.9211 \frac{14558.8 \cdot 3.30}{2 \cdot 11.611}\right)^2} \cdot 1.05 = 10990.6\ psi$$

An additional 5% is added to the stresses as per the project description.

According to the Modified Goodman Diagram Case III, the fatigue safety factor is then:

$$N_f = \frac{39136.7 \cdot 183000}{24181.4 \cdot 39136.7 + 10990.6 \cdot 183000} = 1.4751$$

Keys

These sample calculations are for key 1 on the first shaft, however the three other keys follow very similar steps. Key 1's width is 0.25 in, height is 0.25 in and length is 6 in. We opted to make the keys from AISI 1010 hot rolled steel as explained in the design development section. This material has the following properties:

$$S_{ut} = 47000\,psi$$

$$S_e' = 23500\,psi$$

$$S_y = 26000\,psi$$

The endurance limit is calculated the same way at is was for the gears:

$$C_{load} = 0.7\text{, axial forces present}$$

$$C_{reliab} = 0.814\text{, reliability of 99\%}$$

$$C_{temp} = 1\text{, operating temperature < 450ºC}$$

$$C_{size} = 1\text{, equivalent diameter < 0.3 in}$$

$$C_{surf} = 14.4(47kpsi)^{-0.718} = 0.907$$

The corrected endurance limit is: $S_e = 0.7 \cdot 0.814 \cdot 1 \cdot 1 \cdot 0.907 \cdot 23500 = 12150.3psi$

The torques applied on key 1 include:

$$T_a = 0$$

$$T_m = 4852.9\,lb$$

Based on the shaft radius, the applied force can be calculated

$$F_a = \frac{T_a}{r_{shaft}} = \frac{0}{0.5} = 0$$

$$F_m = \frac{T_m}{r_{shaft}} = \frac{4852.9}{0.5} = 9705.9\,lb$$

Based on the key geometry stated above, the shear stresses can be found:

$$\tau_a = \frac{F_a}{A_{shear}} = \frac{0}{0.25 \cdot 6} = 0$$

$$\tau_m = \frac{F_m}{A_{shear}} = \frac{9705.9}{0.25 \cdot 6} = 6470.6\,psi$$

The corresponding Von Mises stresses are found by:

$$\sigma_a' = \sqrt{3 \cdot \tau_a^2} \cdot 1.05 = 0$$

$$\sigma_m' = \sqrt{3 \cdot \tau_m^2} \cdot 1.05 = \sqrt{3 \cdot (6470.6)^2} \cdot 1.05 = 11767.7\,psi$$

An additional 5% was added to the Von Mises stresses as per the project description.

The shear safety factor is calculated using the Modified Goodman Diagram. Again, the Von Mises stresses are constant and so is there ratio, so we must used Case III to calculate the safety factor.

$$N_{shear} = \frac{S_e S_{ut}}{\sigma_a' S_e + \sigma_m' S_{ut}} = \frac{12150.3 \cdot 47000}{0 \cdot 12150.3 + 11767.7 \cdot 47000} = 3.994$$

To find the bearing safety factor, the maximum stress is used.

$$N_{bearing} = \frac{\sigma_{max}}{A_{bending}} \cdot 1.05 = \frac{F_a + F_m}{A_{bending}} \cdot 1.05 = \frac{0 + 9705.9}{0.5 \cdot 0.25 \cdot 6} \cdot 1.05 = 1.913$$

Again, an additional 5% was added to the maximum stress in the key.

Bearings

Similarly to the other sections, the sample calculations will only show the ones for the bearing on shaft 1. Shaft 1 has 2 bearings; however the shaft is symmetrical so the bearing will be the same on each side. Shaft 2 and 3 also each have 2 bearings, but the bearings will undergo different loads as the shafts' loads are not symmetric.

Furthermore, we know the following from the shaft calculations:

$$F_{radial} = 7.068\ lb$$

$$F_{axial} = 0$$

Then, since there is no force in the axial direction:

$$\frac{F_{axial}}{V \cdot F_{radial}} = 0$$

Because the formula above is 0 it is automatically less than e. From Figure 11-24 below we can see that $X = 1$ and $Y = 0$.

Factors V, X, and Y for Radial Bearings

Bearing Type		In Relation to the Load the Inner Ring is: Rotating V	Stationary V	Single Row Bearings 1) $\frac{F_a}{VF_r} > e$: X	Y	Double Row Bearings 2) $\frac{F_a}{VF_r} \le e$: X	Y	$\frac{F_a}{VF_r} > e$: X	Y	e
3) Radial Contact Groove Ball Bearings	4) $\frac{F_a}{C_0}$									
	0.014				2.30				2.30	0.19
	0.028				1.99				1.99	0.22
	0.056				1.71				1.71	0.26
	0.084				1.55				1.55	0.28
	0.11	1	1.2	0.56	1.45	1	0	0.56	1.45	0.30
	0.17				1.31				1.31	0.34
	0.28				1.15				1.15	0.38
	0.42				1.04				1.04	0.42
	0.56				1.00				1.00	0.44
20°				0.43	1.00		1.09	0.70	1.63	0.57
25°				0.41	0.87		0.92	0.67	1.44	0.68
30°		1	1.2	0.39	0.76	1	0.78	0.63	1.24	0.80
35°				0.37	0.66		0.66	0.60	1.07	0.95
40°				0.35	0.57		0.55	0.57	0.93	1.14
Self-Aligning Ball Bearings		1	1	0.40	0.4 cot α	1	0.42 cot α	0.65	0.65 cot α	1.5 tan α
Self-Aligning and Tapered Roller Bearings		1	1.2	0.40	0.4 cot α	1	0.45 cot α	0.67	0.67 cot α	1.5 tan α

1) For single row bearings, when $\frac{F_a}{VF_r} \le e$ use $X = 1$ and $Y = 0$.

For two single row angular contact ball or roller bearings mounted "face-to-face" or "back-to-back" the values of X and Y which apply to double row bearings. For two or more single row bearings mounted "in tandem" use the values of X and Y which apply to single row bearings.

2) Double row bearings are presumed to be symmetrical.

3) Permissible maximum value of $\frac{F_a}{C_0}$ depends on the bearing design.

4) C_0 is the basic static load rating.

5) Units are pounds and inches.

Values of X, Y and e for a load or contact angle other than shown in the table are obtained by linear interpolation.

FIGURE 11-24

V, X, and Y Factors for Radial Bearings *Courtesy of SKF USA Inc.*

Additionally, since the inner ring of the bearing is rotating, $V = 1$.

The equivalent force is then calculated by:

$$P_{eq} = X \cdot V \cdot F_r + Y \cdot F_a = 1 \cdot 1 \cdot 7.068 + 0 = 7.068\ lb$$

From the mission requirements we calculated that the bearing will undergo 8606712329 revolutions at 15000 rpm.

Usually L_{10} is calculated by: $L_{10} = \left(\frac{C}{P_{eq}}\right)^3$ where C is the dynamic load rating of the bearing. The exponent of the L_{10} formula is 3 since we are using ball bearings.

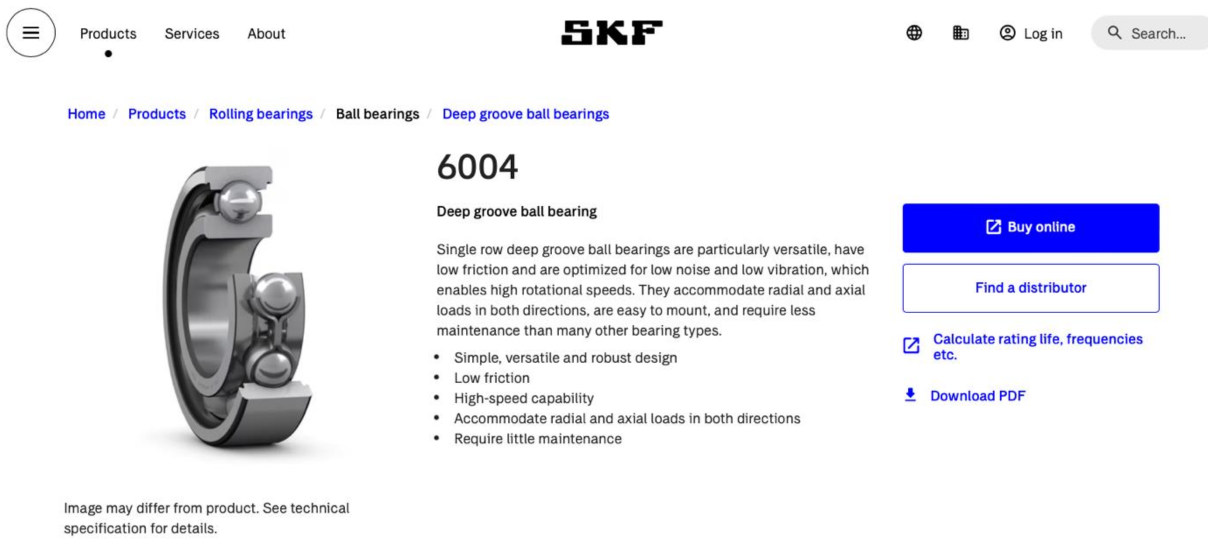
However, since we do not know C, L_{10} can be calculated by:

$$L_{10} = \frac{\#\, of\ cycles}{10^6} = \frac{8606712329}{10^6} = 8606.7$$

The dynamic load rating can then be back calculated:

$$C = (L_{10})^{\frac{1}{3}} \cdot P_{eq} = (8606.7)^{\frac{1}{3}} \cdot 7.068 = 144.8$$

From the SKF catalogue bearing:



Dimensions

Bore diameter	0.787 in
Outside diameter	1.654 in
Width	0.472 in

Performance

Basic dynamic load rating	2 237 lbf
Basic static load rating	1 124 lbf
Reference speed	38 000 r/min
Limiting speed	24 000 r/min
SKF performance class	SKF Explorer

Properties

Filling slots	Without
Number of rows	1
Locating feature, bearing outer ring	None
Bore type	Cylindrical
Cage	Sheet metal
Matched arrangement	No
Radial internal clearance	CN
Material, bearing	Bearing steel
Coating	Without
Sealing	Without
Lubricant	None
Relubrication feature	Without

Logistics

Product net weight	0.1446 lb
eClass code	23-05-08-01
UNSPSC code	31171504

SKF deep groove ball bearing 6004 meets the requirements.

```python
def find_max_teeth():
    max_solution = None
    # We iterate over possible values of N1.
    # The upper bound 500 is arbitrary; our constraints will limit N1.
    for N1 in range(1, 500):
        N2 = 6 * N1
        N3 = 2 * N1
        N4 = 5 * N1

        # Check inequality constraints:
        # (N1/16 + 2*N2/16 <= 30) and (N4/16 + 2*N3/16 <= 30)
        if (N1 / 16 + 2 * N2 / 16 <= 30) and (N4 / 16 + 2 * N3 / 16 <= 30):
            # Check the sum equality: N1 + N2 == N3 + N4
            if N1 + N2 == N3 + N4:
                max_solution = (N1, N2, N3, N4)

    return max_solution

solution = find_max_teeth()
if solution:
    N1, N2, N3, N4 = solution
    print("Optimal teeth numbers (N1, N2, N3, N4):", solution)
else:
    print("No solution found.")
```

Figure 14: Minimum number of teeth code