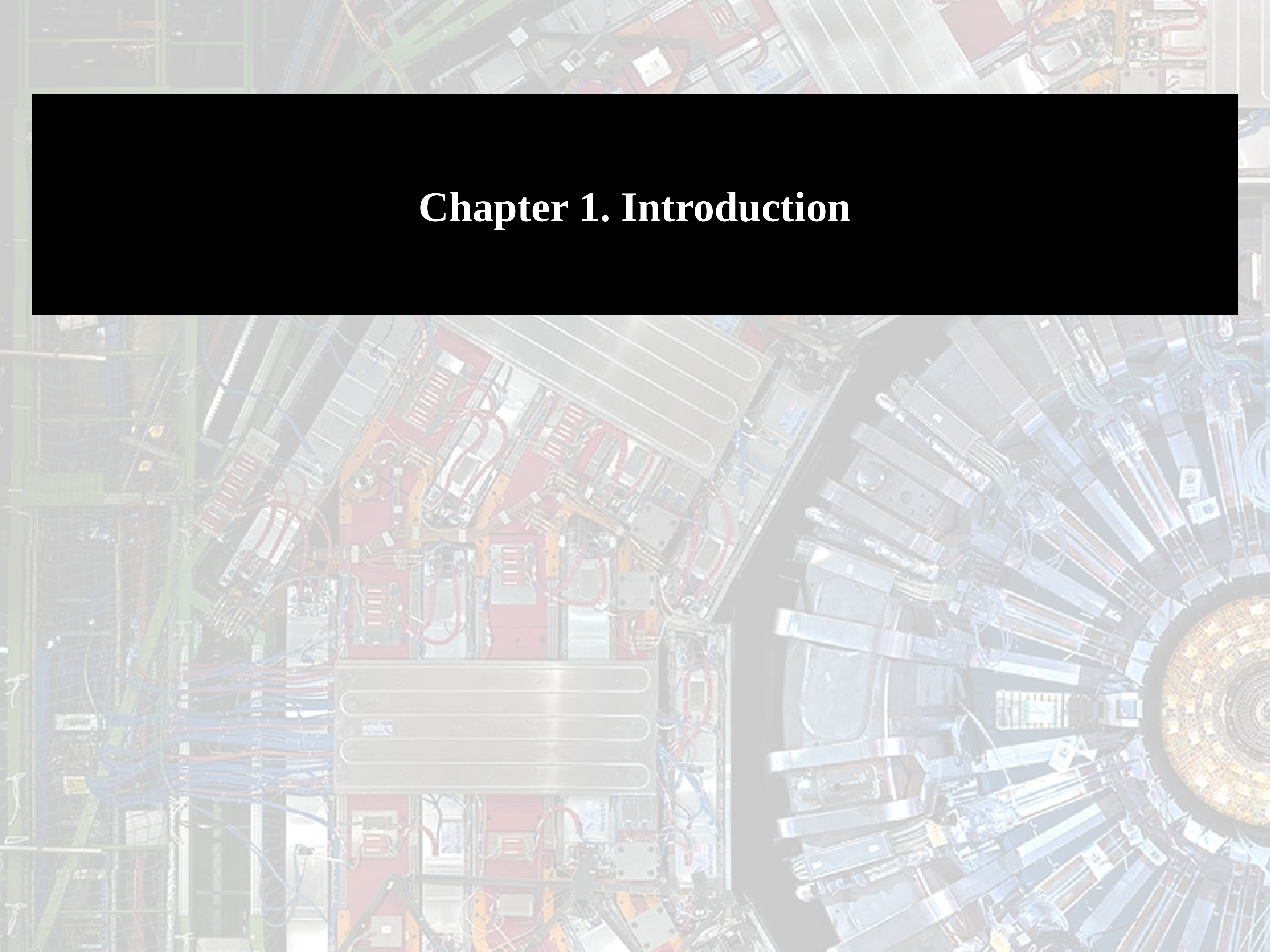
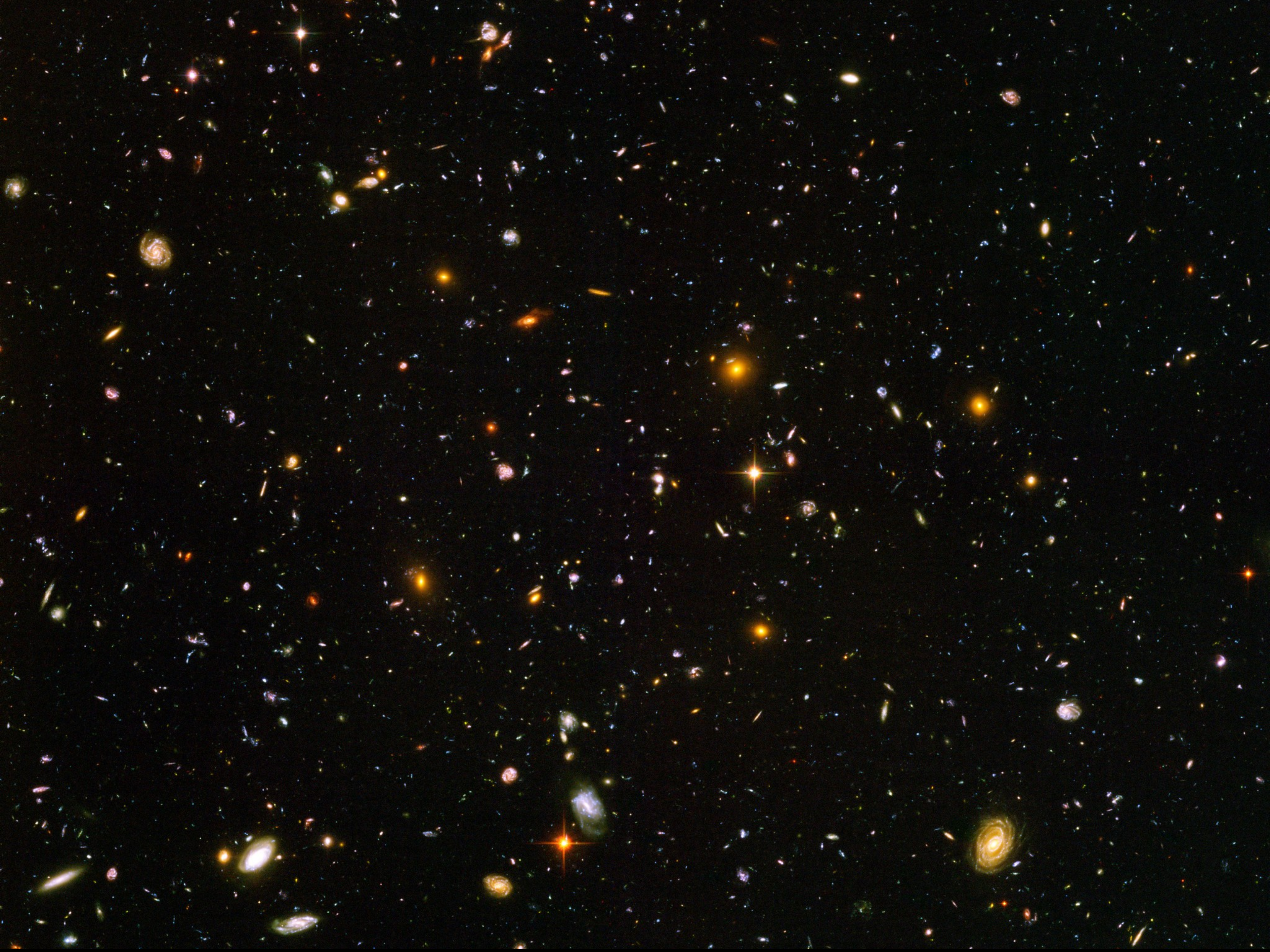
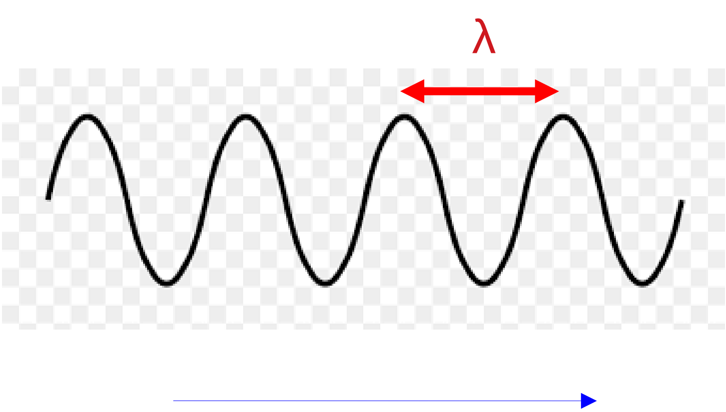
Chapter 1. Introduction

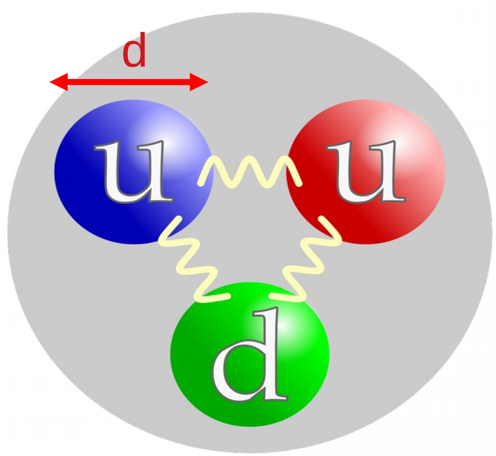




What is Particle Physics?

- Subject that studies the fundamental components of Nature and their interactions
- Nowadays our knowledge about fundamental particles is condensed in the **Standard Model**
- Experimental foundation:
  - Collision experiments at higher and higher energies to reveal the features of the particles
  - Need higher energies to explore smaller and smaller spatial scales
- Theoretical foundation:
  - Quantum Field Theory built on top of Quantum Mechanics and Special Relativity
- The history of Particle Physics is the history of achieving larger and larger energies



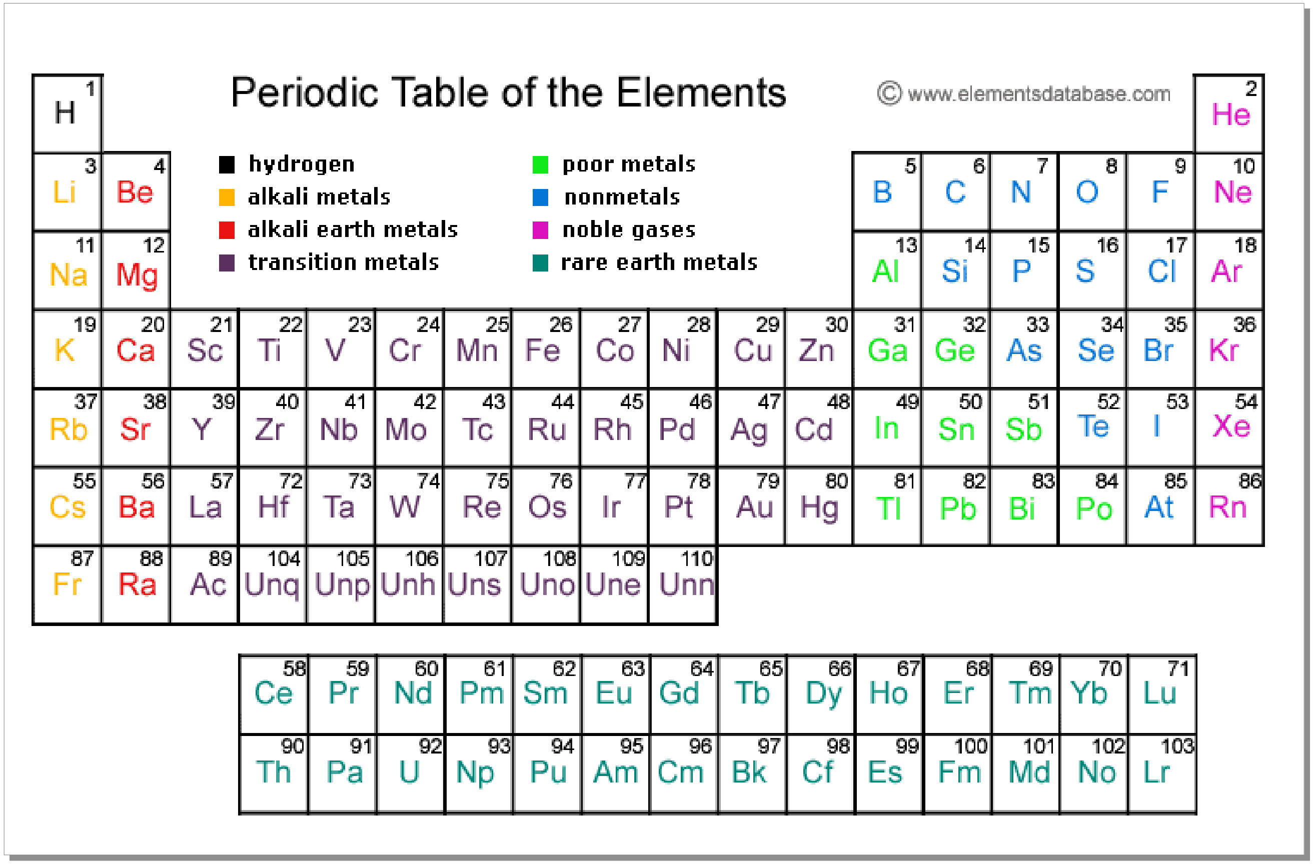


Debroglie:
Wave-Particle duality

Wave properties:
To resolve an object $\lambda \sim d$

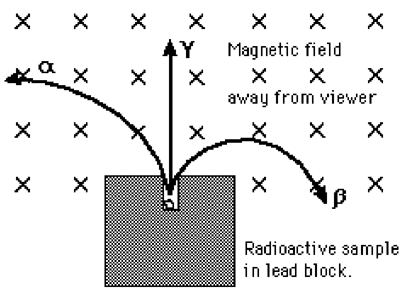
History of Particle Physics: beginnings XIX century

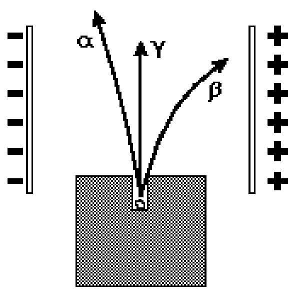
- Dalton postulates that all substances might be composed of single units or atoms
  - As a consecuence of his observations in chemical reaction experiments
- In the late 1800 the atoms were being classified according to the properties
  - This consolidates with the development of the periodic table by Mendeleev



History of Particle Physics: decade of 1890

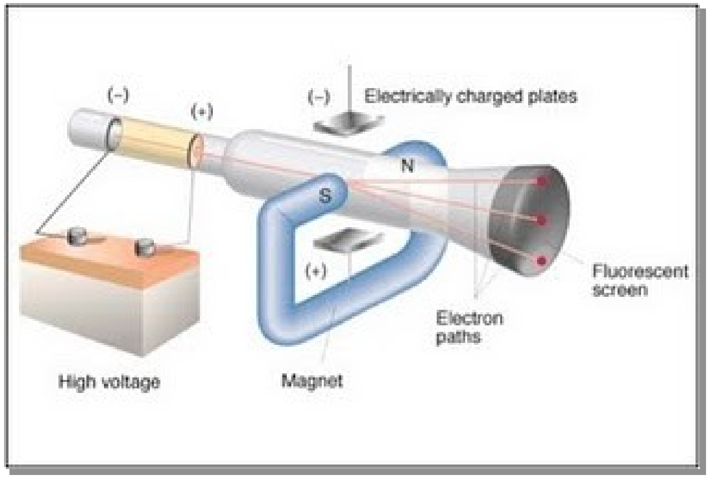
- New unstable elements being investigated by M. and P. Curie, H. Bequerel and E. Rutherford
  - Radioactivity: spontaneous emission of particles from some species of atoms
- At this moment it was determined that there were 3 different kinds of radiation:
  - Radiation α → 2 electric charge and about 4 times proton mass
  - Radiation β → -1 electric charge and about 1/1800 the proton mass
  - Radiation γ → electrically neutral





History of Particle Physics: discovery of the electron

- For a number of years physicist have generated “cathode rays”
  - By simply heating filaments inside gas-filled tubes and applying an electric field
- In 1987 J.J. Thompson attempted to measure the ratio of charge/mass of cathode rays
  - Put a cathode ray into a known electric or magnetic field
  - Measure the cathode ray’s deflection
  - If they are composed of discrete charges → deflection compatible with Lorentz force
- Thompson found that this ratio was ~ 1000 larger than for any known ion
  - He concluded this was a new kind of particle and named it “electron”

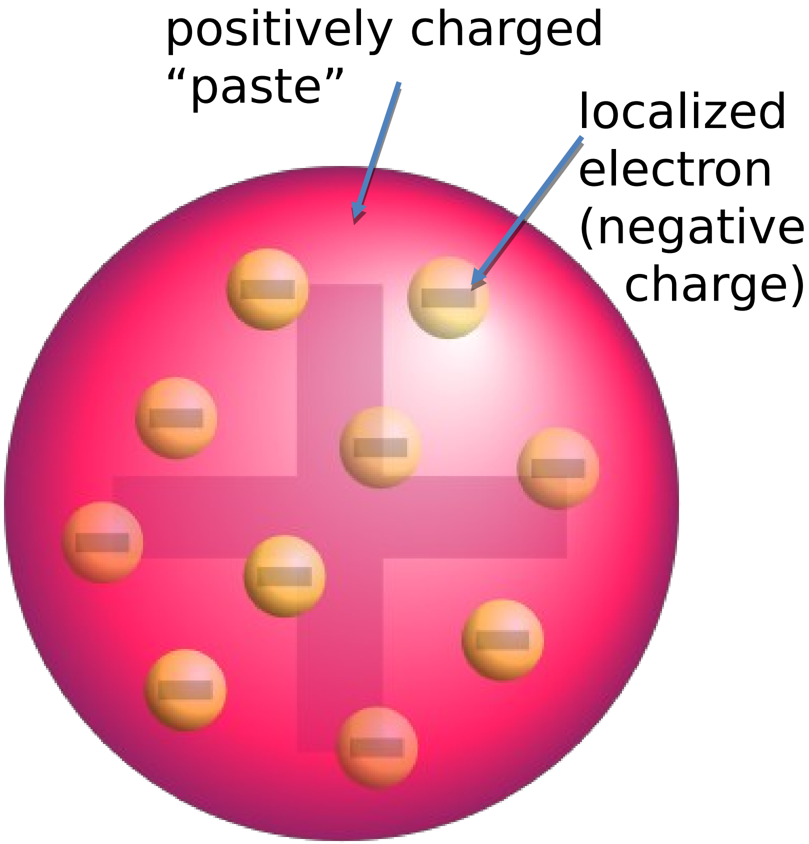


$$\vec{F} = q\left(\vec{E} + \frac{\vec{V}}{c} \times \vec{B}\right)$$

$m_e = 0.511 MeV/c^2$

History of Particle Physics: Thompson’s atom model

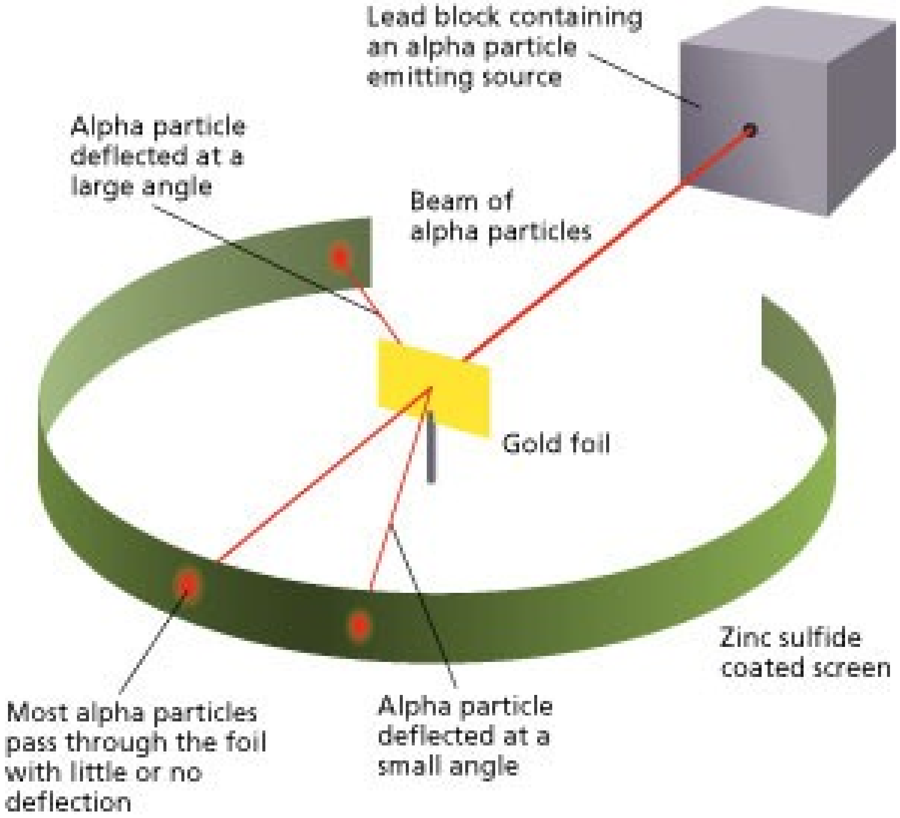
- Thompson correctly believed that electrons were fundamental components of the atoms
- Since atoms were electrically neutral he postulated that point-like electrons should be somehow embedded in a “gel” of positive charge to yield a global neutral charge
- This is the origin of the famous “plum-pudding” atom model

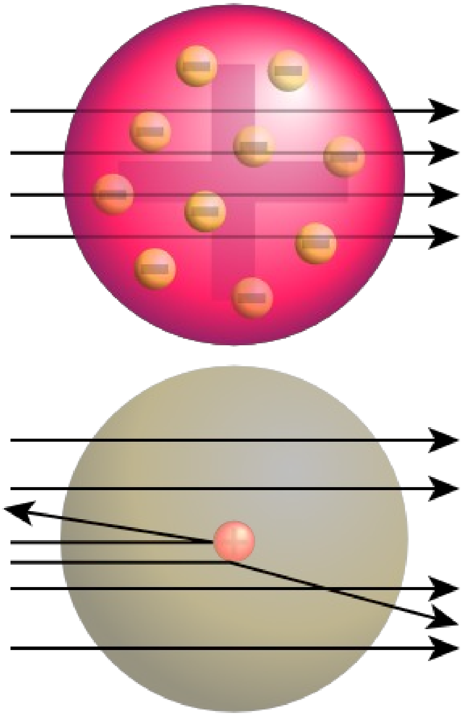


Thomson’s plum-pudding model of the atom.

History of Particle Physics: Rutherford’s experiment

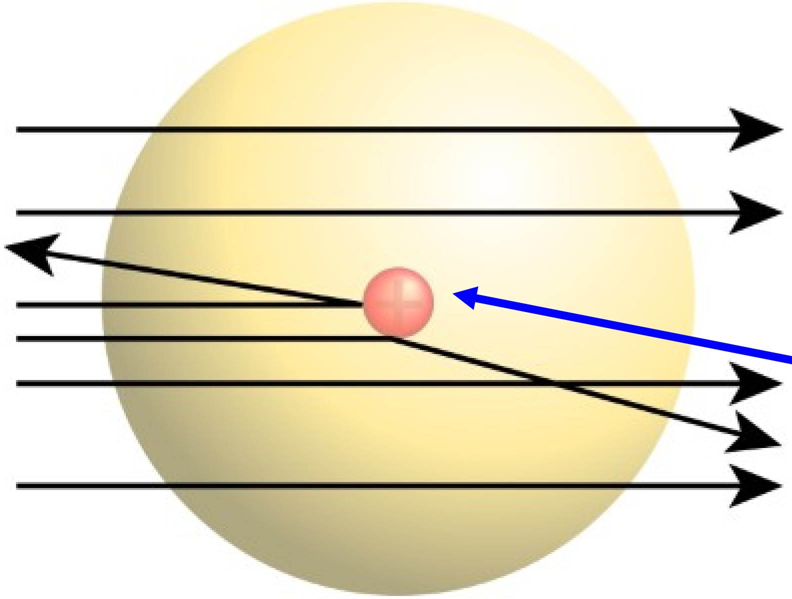
- In the first years of the XX century E. Rutherford wanted to test Thompson’s model
- With this purpose he proposed a setup to make a “scattering” experiment:
  - Bombing a very thin gold foil with α particles and observing the deflection
  - If Thomson’s model was correct the α particles shold not reflect much

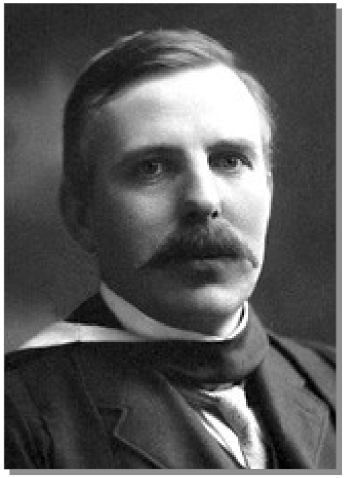




History of Particle Physics: Rutherford’s experiment

- Rutherford observed that a few particles were having a huge deflection (> 90º)
- He wondered what was the origin of such a strong force able to invert the sense of the particles
- The only possibility was some kind of electric force made by another very close-by particle
- He concluded that the positive charge of the atom should be concentrated in small region
- This is the discovery of the nucleous



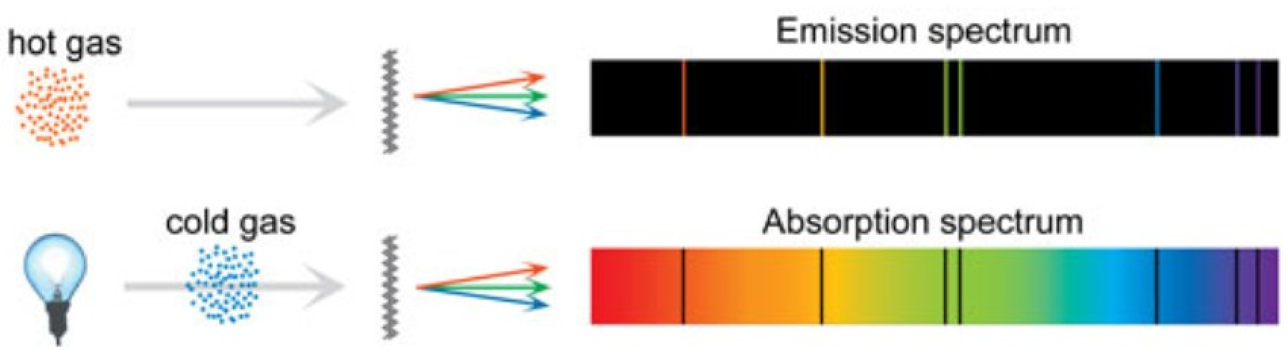


The distance d should be very small to produce such a large F

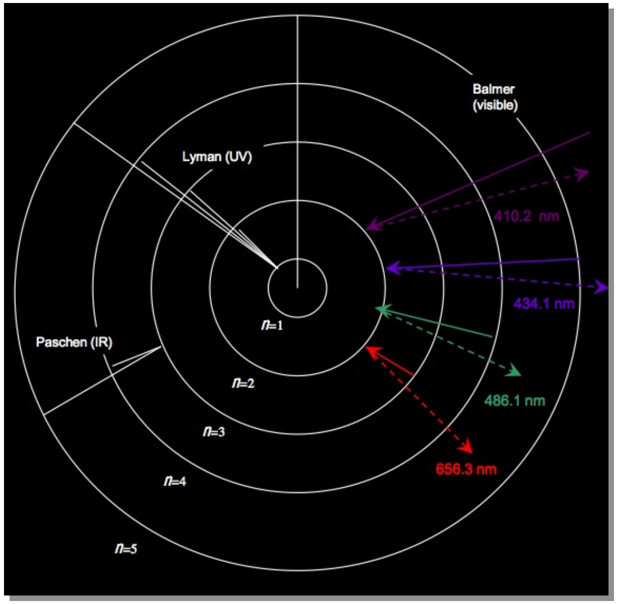
$$F \propto \frac{q_1 q_2}{d^2}$$

History of Particle Physics: Bohr's atom

- In 1914 Bohr proposed a "planetary" model of the atom motivated by the atomic spectrum

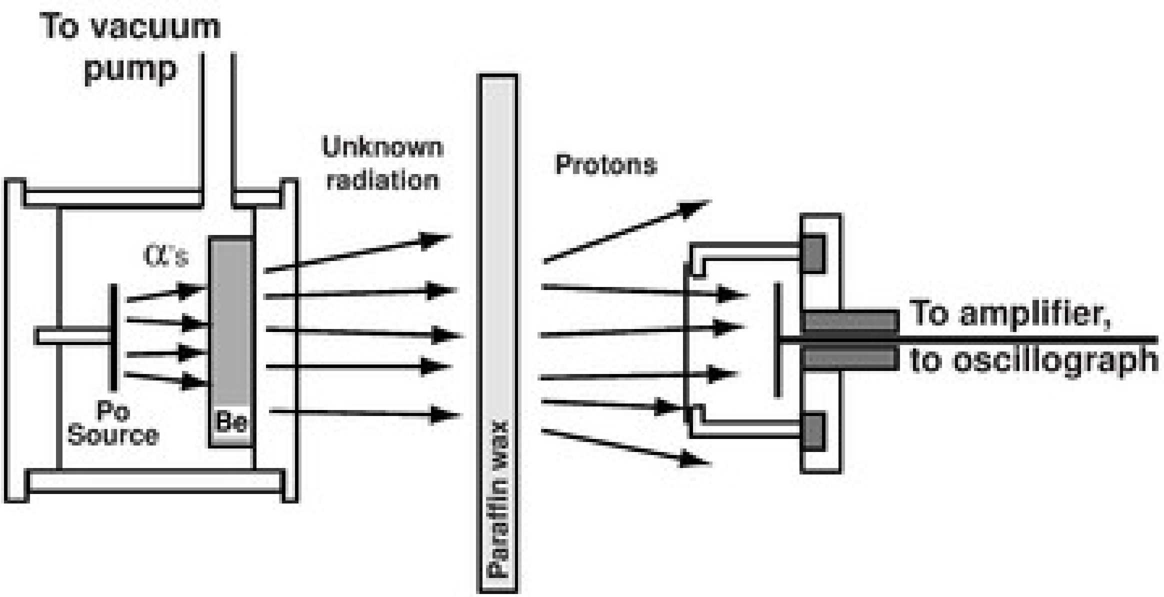


- In this model electrons where occupying discrete orbits (quantized angular momentum)
  - Therefore the energies were also quantized
- When a photon is absorbed one electron can jump from one orbit to another higher orbit
- The electron in a higher orbit can jump back to an empty lower orbit emitting a phton



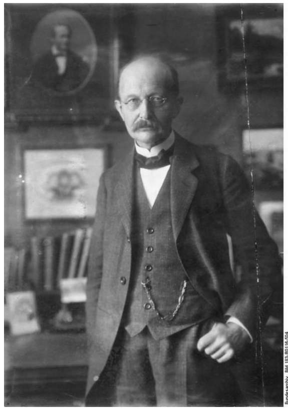
History of Particle Physics: discovery of the neutron

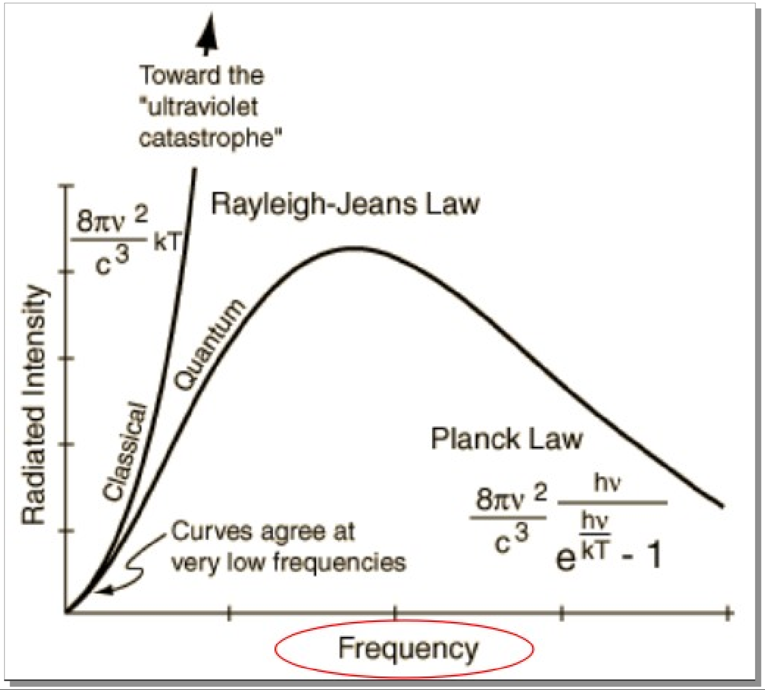
- According to Bohr's model the atom was composed of electrons and protons
  - However the mass of most of the atoms was not consistent with this assumption
- To account for the atom mass, nuclei should contain neutral particles with mass ~ proton
- In 1932, Chadwick proposed an experiment to identify this kind of radiation: the neutrons



History of Particle Physics: quantum mechanics

- In the first decades of the XX century the foundations of quantum mechanics are established
- The first step was the explanation of the radiation of the black body by Max Planck
  - Light is emitted in “quanta” of discrete energy → the photon
- In this period De Broglie postulates the wave-particle duality
- Schrödinger and Heisenberg establish the foundations of quantum mechanics
- A new vision of the atom (Schrödinger’s hydrogen atom) is established

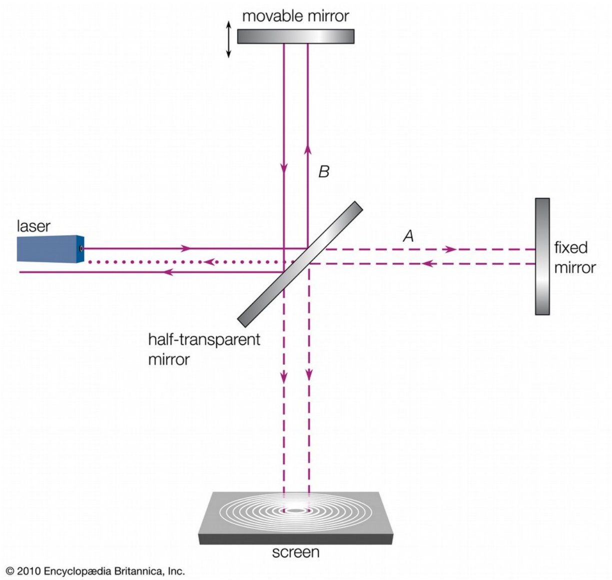


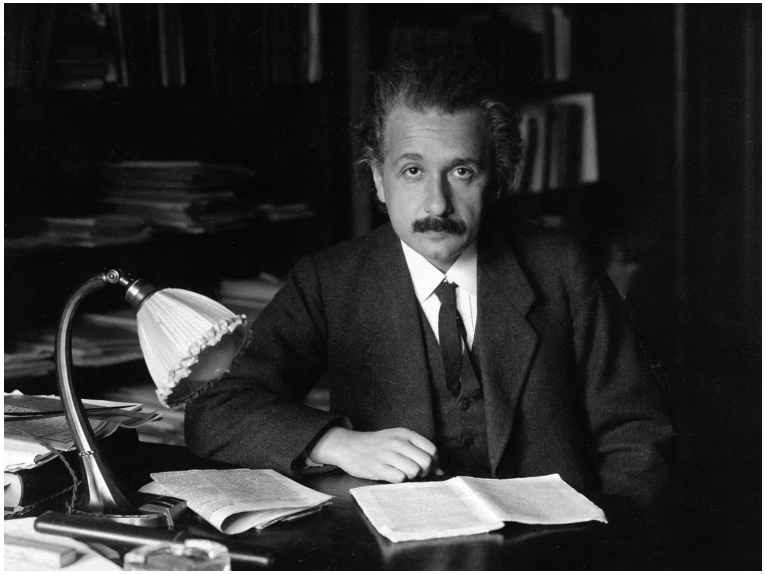


$$i\hbar\frac{\partial\psi}{\partial t}+\frac{\hbar^2}{2m}\nabla^2\psi-V\psi=0$$

History of Particle Physics: special relativity

- In the first decades of the XX century Einstein revolutionized the concepts of space and time
- Galilean transformations worked for classical mechanics but not for Maxwell's equations
  - A new set of transformations was needed → the Lorentz transformations
- Einstein interpreted these transformations creating a new entity, the space-time
- Inertial systems are those related by Lorentz transformations
  - The speed of light is the same in all the systems
  - This was confirmed by the experiment of Michelson-Morley



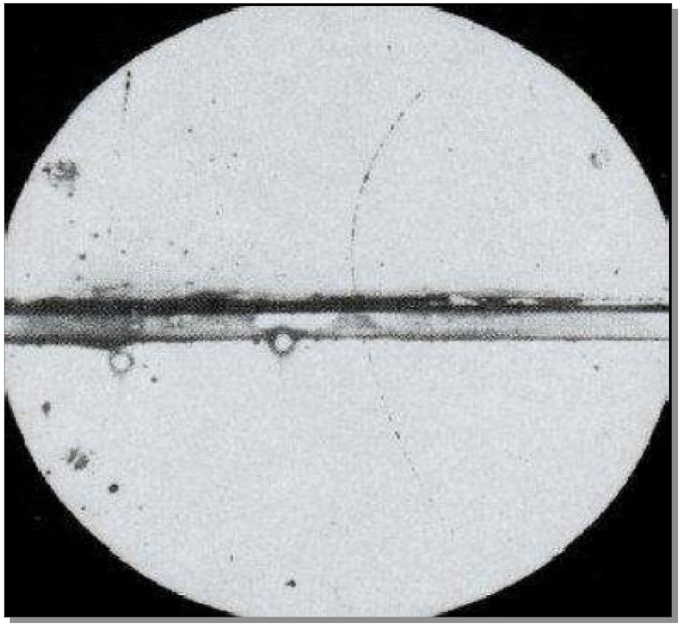


History of Particle Physics: antimatter

- In 1927 Paul Dirac was trying to combine quantum mechanics with special relativity
  - Obtain a quantum-mechanics wave equation starting from $E^2 = p^2 c^2 + m^2 c^4$
- This lead to a problem since this relation yields to negative energy solutions

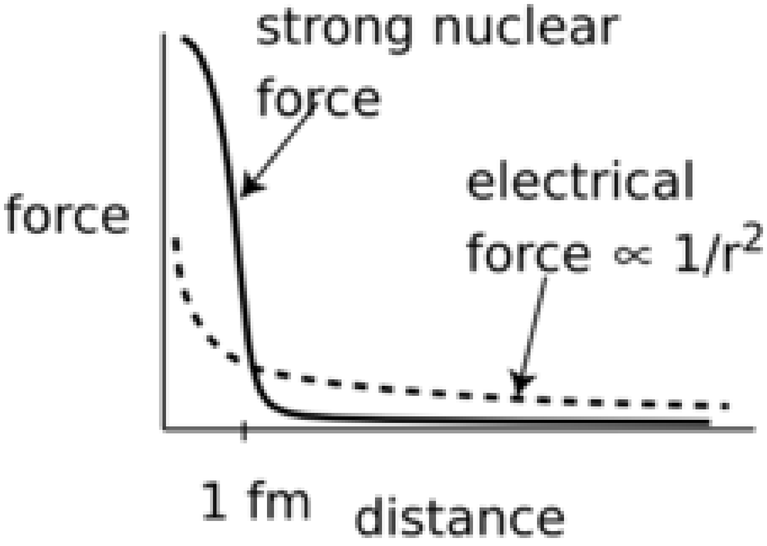
$$E = +\sqrt{p^2 c^2 + m^2 c^4} \qquad E = -\sqrt{p^2 c^2 + m^2 c^4}$$

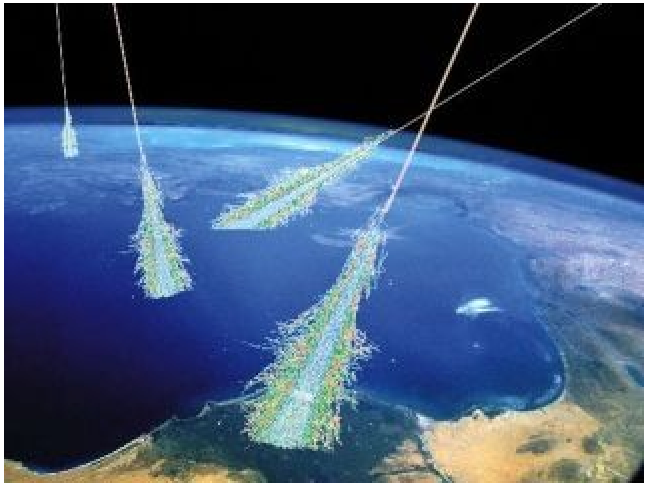
- Dirac interpreted the positive solutions as ordinary particles and the negative as antimatter
- In 1932 Anderson observed for the first time the anti-electron confirming Dirac's theory



History of Particle Physics: the particle zoo

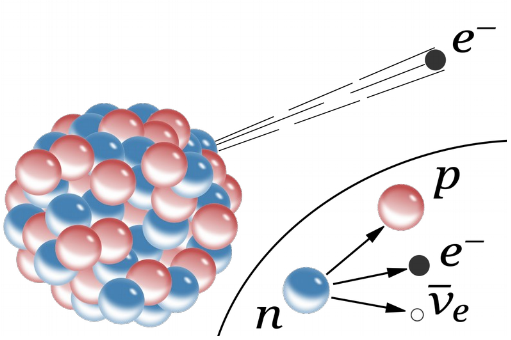
- The new ideas of quantum mechanics were rapidly applied to related phenomena
- The fact that the nucleus is not breaking apart due to electric repulsion suggested a new force
  - Yukawa was able to describe a working potential for this force
  - Same as photons were driving the EM force, other particle should drive this strong force
- In 1936 physicist studying cosmic ray radiation found a candidate for this particle
  - By 1937 they found this particle was not related to the strong force → it was the muon
  - In 1947 the study of cosmic radiation lead to another particle: the pion





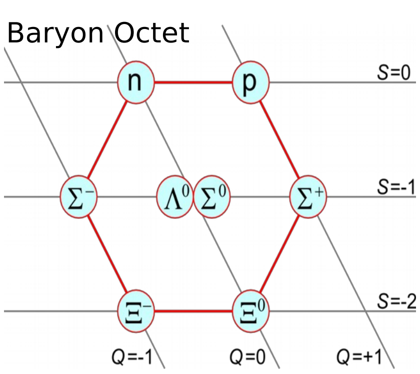
History of Particle Physics: neutrinos

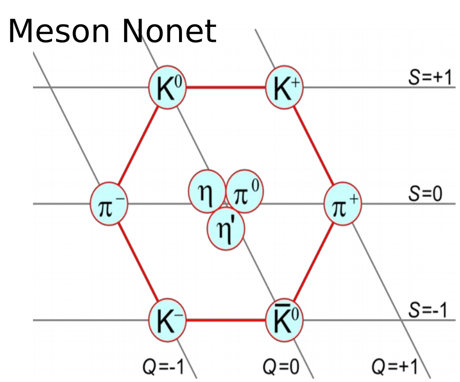
- The study of radioactive decays (beta decays) suggested a possible non-conservation of energy
- Pauli interpreted this phenomenom by postulating a new non-detectable additional particle
- In 1932 Fermi included this idea in his theory of nuclear decays
- In 1950, Cowan and Raines managed to design an experiment to detect neutrinos (indirectly)
  - The idea was to use an inverted beta decay $\bar{\nu} + p \longrightarrow n + e^{+}$

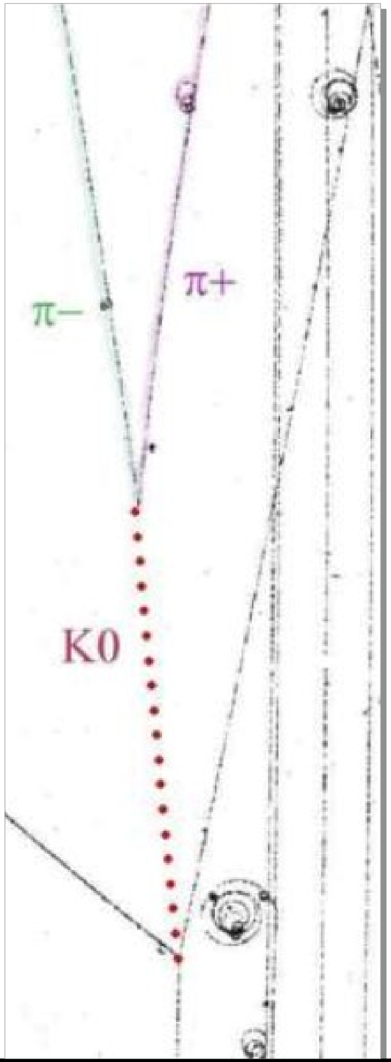


History of Particle Physics: mesons and barions

- In the next years experiments in different accelerators found new mesons and barions
- In 1947 several observations were made of a neutral particle decaying into 2 pions
  - This was the neutral kaon (K^0)
- In 1949 observations of a charged particle decaying into 3 pions
  - This was the charged kaon ($K^{+/-}$)
- In the next years many others: the phi, eta, omega, etc
- In 1950 first evidences of barionic particles such as the Lambda
- Gell-Mann realized that geometrical pattern emerged when organizing the new mesons and barions in terms of the strangeness and charge

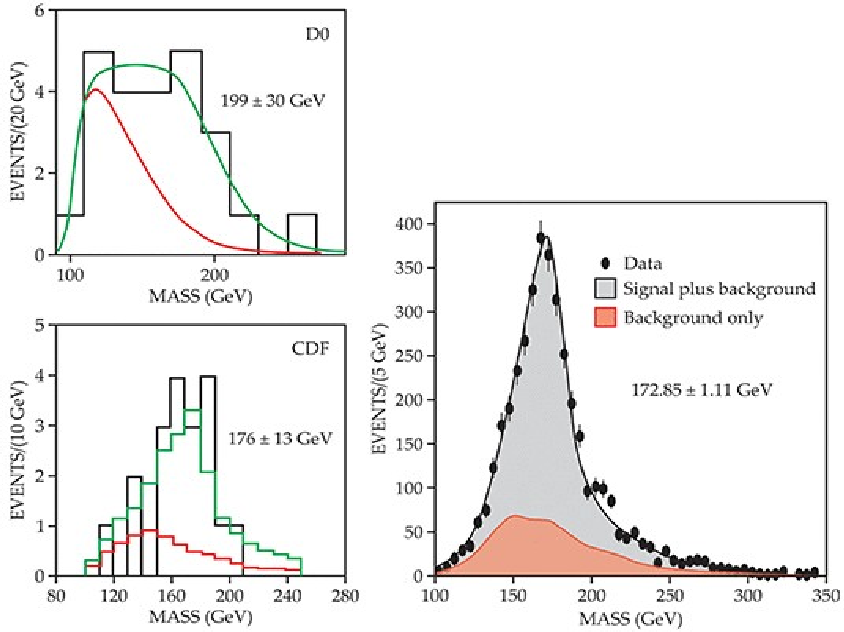


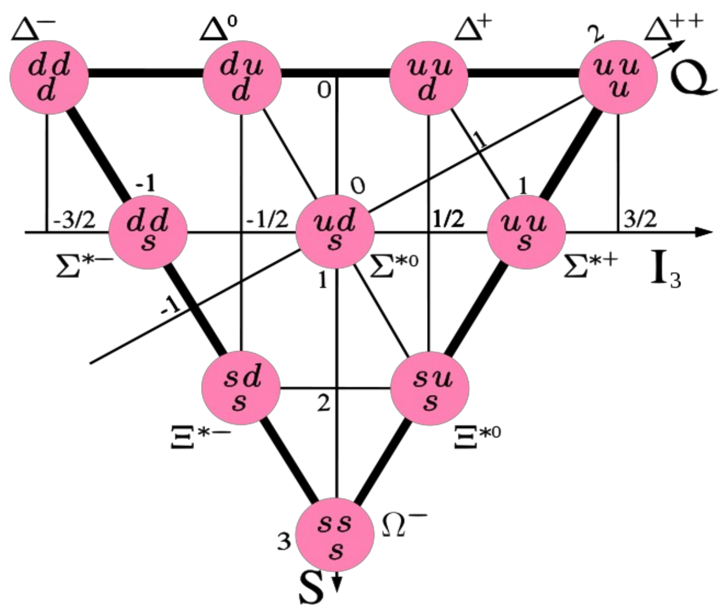




History of Particle Physics: the quark model

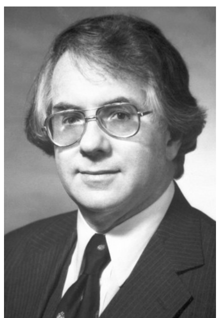
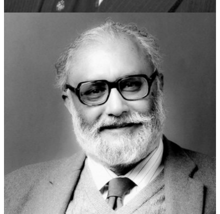
- In 1964 Gell-Mann and Zweig proposed an explanation for the observed patterns
- All hadrons are composed of more fundalmental particles: quarks
- The quarks came in three types “up”, “down” and “strange” and they had fractional charge
- Barions are composed by three quarks while mesons only by two quarks
  - However quarks cannot be found isolated in nature (confinement)
- In the following years the charm, bottom and finally in 1995 the top quark were discovered

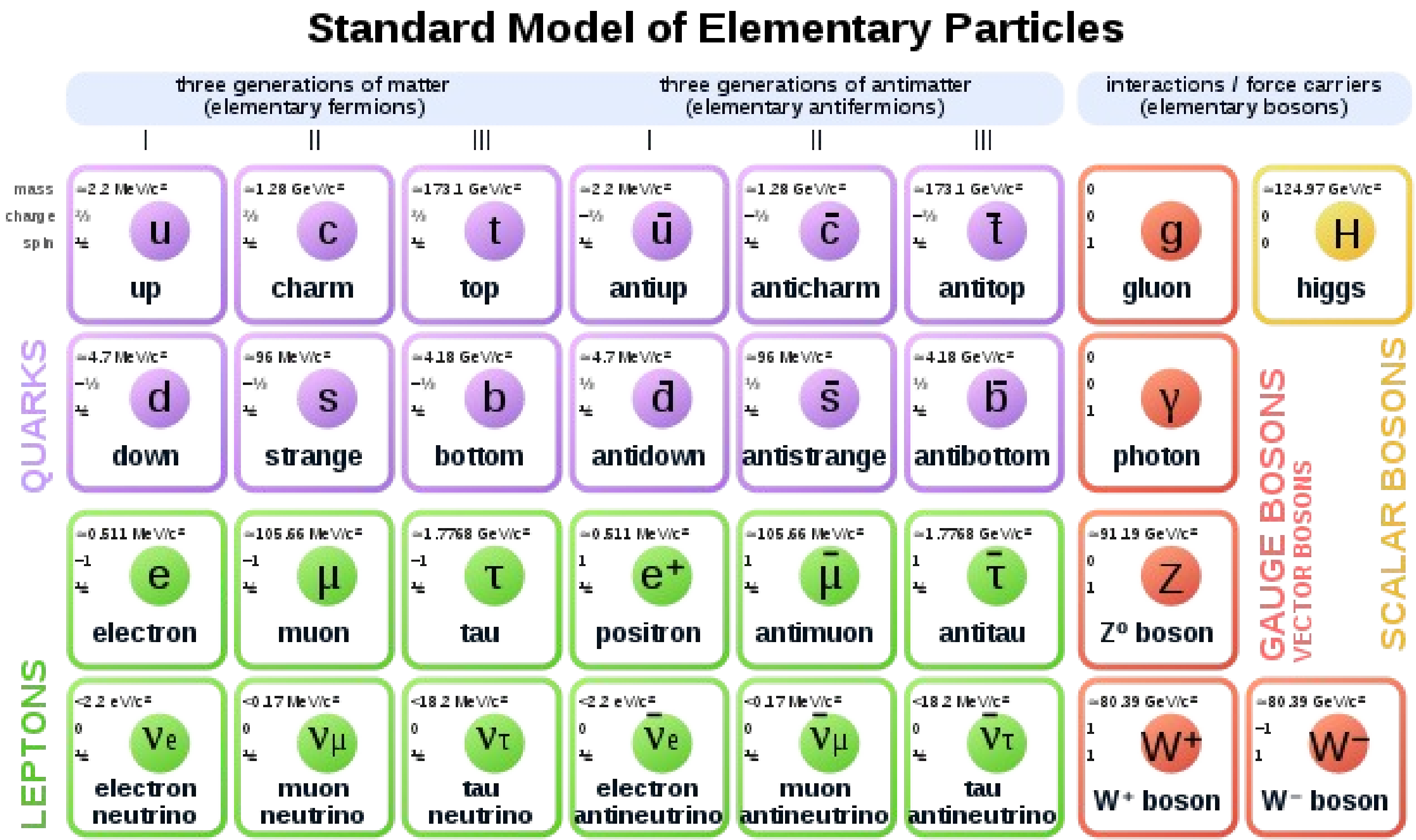




History of Particle Physics: the Standard Model

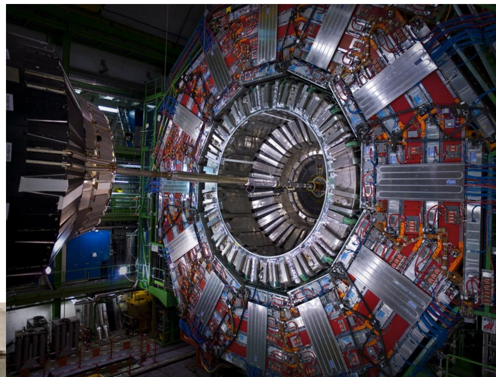
- In the late sixties Glashow, Salam and Weinberg developed a model of the electrowek force
- It included the recently proposed Higgs mechanism for which massive bosons acquire mass
  - The W and Z massive bosons were disvered in 1983 at CERN
- The strong interaction was introduced and the term Standard Model was coined in 1975

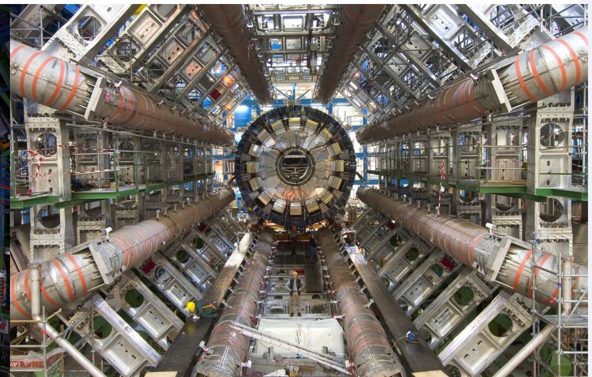



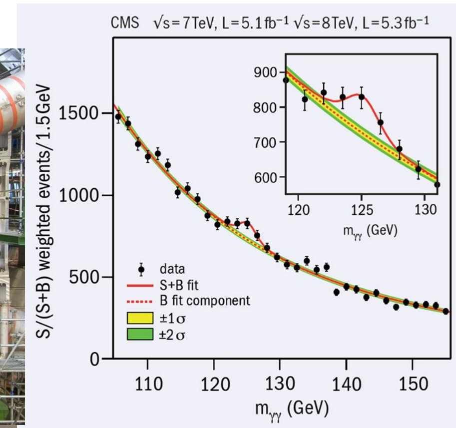



History of Particle Physics: the Higgs boson

- In 2012 the Higgs boson was discovered at the Large Hadron Collider at CERN







Particle Physics: Natural units

- The International System of Units is a natural choice for everyday objects
- However is not very useful for Particle Physics where the scales are very very different
- In Particle Physics we use the Natural Units:
  - From Quantum Mechanics – the unit of action: $\hbar = 6.6 \times 10^{-25}$ GeV s
  - From relativity – the speed of light: $c = 3 \times 10^{8}$ cm/s
  - From Particle Physics – unit of energy: GeV (1 GeV ~ proton mass)
- Simplified in the equations by setting $\hbar = c = 1$
- In this context the main units become :

Energy	GeV	Energy	GeV
Momentum	GeV/c	Momentum	GeV
Mass	GeV/c^2	Mass	GeV
Time	$(\text{GeV}/\hbar)^{-1}$	Time	GeV^{-1}
Length	$(\text{GeV}/\hbar c)^{-1}$	Length	GeV^{-1}
Area	$(\text{GeV}/\hbar c)^{-2}$	Area	GeV^{-2}

Theory of Special Relativity (I) (reminder)

- Galilean transformations of velocity worked very well for classical mechanics
  - The dynamics of the system were the same regardless the inertial system of reference
- However this statement was not true for the Maxwell’s equations
  - A new set of transformations was needed in order to achieve this (Lorentz transformations)

Galilean transformations

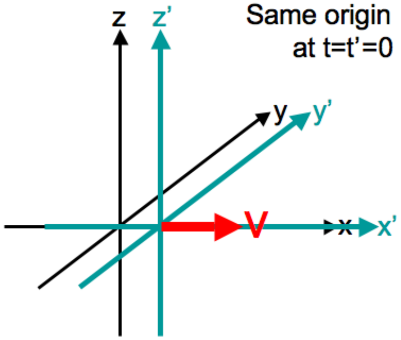
$$\begin{aligned} x' &= x - vt \\ y' &= y \\ z' &= z \\ t' &= t. \end{aligned}$$

Lorentz transformations

$$\begin{aligned} ct' &= \gamma\left(ct - \beta x\right) \\ x' &= \gamma\left(x - \beta ct\right) \\ y' &= y \\ z' &= z. \end{aligned}$$

$$\gamma = \left(\sqrt{1-\frac{v^2}{c^2}}\right)^{-1}$$

$$\beta = \frac{v}{c}$$



- A consecuence of this transformations is that the speed of light is the same in every system
  - And this fact has non-intuitive implications in our conception of space and time
- Einstein understood this by exploring first the concept of simultaneousty
  - Are events simultaneous in one system of reference simultaneous in other systems?
- Einstein established that space and time were concepts attached to a same entity: space-time

Theory of Special Relativity (II) (reminder)

- Einstein's special relativity establishes that:
  - Events in the Universe are encoded by a 4-vector: ($x^0 = ct$, $x^1 = x$, $x^2 = y$, $x^3 = z$)
  - The coordinates between intertial systems of reference are give by Lorentz Transformations
  - For two events x and y the product: $x^0y^0-x^1y^1-x^2y^2-x^3y^3$ is invariant under transformations
- In modern notation, Lorentz transformations are expressed as a tensor

$$\begin{bmatrix} x'^0 \\ x'^1 \\ x'^2 \\ x'^3 \end{bmatrix} = \begin{bmatrix} \Lambda^0{}_0 & \Lambda^0{}_1 & \Lambda^0{}_2 & \Lambda^0{}_3 \\ \Lambda^1{}_0 & \Lambda^1{}_1 & \Lambda^1{}_2 & \Lambda^1{}_3 \\ \Lambda^2{}_0 & \Lambda^2{}_1 & \Lambda^2{}_2 & \Lambda^2{}_3 \\ \Lambda^3{}_0 & \Lambda^3{}_1 & \Lambda^3{}_2 & \Lambda^3{}_3 \end{bmatrix} \begin{bmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{bmatrix} \qquad g_{\mu\nu} = \begin{pmatrix} +1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$$

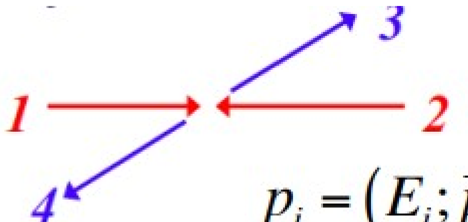
- Define the contravariant $x^\mu = (x^0, x^1, x^2, x^3)$ and covariant $x_\mu = (x_0, -x_1, -x_2, x_3)$ 4-vectors
  - They are related through the so-called metric tensor: $x^\mu = g^{\mu\nu} x_\nu$
  - Lorentz transformations can be expressed as: $x'^\mu = \eta^{\mu\nu} x_\nu$
- This concepts can also be applied to differential operators:

$$\partial_\mu = \frac{\partial}{\partial x^\mu} = \left(\frac{\partial}{\partial t};\frac{\partial}{\partial x},\frac{\partial}{\partial y},\frac{\partial}{\partial z}\right) = \left(\partial^0;\nabla\right) \qquad \partial^\mu = \frac{\partial}{\partial x_\mu} = \left(\partial^0;-\nabla\right)$$

Theory of Special Relativity (III) (reminder)

- From the point of view of the dynamics, Einstein's special relativity establishes that:
  - Energy and momentum in different systems transform as well with Lorentz transformations
  - The energy of an object moving with speed v is given by $E = \gamma mc^2$
- Energy and momentum can also be represented by co-(contra) variante 4-vectors
  - This is the so called 4-momentum: $p^\mu = (p^0 = E, p^1 = p_x, p^2 = p_y, p^3 = p_z)$
- The energy-moment relation is given by $E^2 = p^2 + m^2$ (natural units)
- It's important to notice that for a particle with a given momentum:

$$p^\mu p_\mu = E^2 - p^2 = m^2$$
$$x^\mu p_\mu = Et - \vec{p}.\vec{r}$$



$$p_i = \left(E_i ; \vec{p}_i\right)$$
$$p_i^2 = m_i^2$$

- In general the notation is abbreviated in such a way that:
  - $P^\mu P_\mu$ is simply denoted as P^2
  - $P^\mu Q_\mu$ is simply denoted as PQ

Mandelstam's invariants

$$\boxed{s = (p_1 + p_2)^2, \quad t = (p_1 - p_3)^2, \quad u = (p_1 - p_4)^2}$$

Quantum Mechanics (I) (reminder)

- Physical states (particles) are represented by wave functions Ψ(x,t)
  - This wave functions assign a number (or numbers) to every point of space-time
- We often use the “bra”-”ket” notation for wave functions $\Psi \rightarrow |\Psi>$
- Wave functions belong, mathematically, to a structure called Hilbert space
  - To keep it short: it is a sort of vectorial space with some additional properties
  - An inner product is defined assigning a complex number to every two wave functions
  - The product has several good properties

$$\langle \psi | \Phi \rangle = a \; with \; a \in \mathbb{C}$$

$$\langle \psi | \Phi \rangle = \left(\langle \Phi | \psi \rangle \right)^c$$

C = conjugate

$$\langle \psi | \Phi \rangle = a \; with \; a \in \mathbb{C}$$

$$\langle \psi | \psi \rangle \geqslant 0$$

- The product in Quantum Mechanics is defined as:

$$\langle \psi | \Phi \rangle = \int \psi^c \, \Phi \, dx^3$$

Quantum Mechanics (II) (reminder)

- Wave functions are interpreted in terms of probabilities
- The probability of finding a particle in the state Ψ in the region of space C is given by:

$$Prob\left(x \in C\right) = \int_C \psi^c \psi \, dx^3$$

- Therefore the term $\Psi^c \Psi$ can be associated with a density of probability
- In consequence, wave functions must be normalized in such a way that:

$$Prob\left(x \in \mathbb{R}^3\right) = \int_\infty \psi^c \psi \, dx^3 = 1$$

- Physical states are often described in terms of an orthonormal basis of the Hilbert space

$$\psi = \sum_i a_i \Phi_i \, with \, a_i \in \mathbb{C} \qquad\qquad \langle \Phi_i | \Phi_j \rangle = \delta_{ij}$$

- The contribution of the element of the basis i is given by the product:

$$\langle \Phi_i | \psi \rangle = \left\langle \Phi_i \middle| \sum_j a_j \Phi_j \right\rangle = \sum_j a_j \langle \Phi_i | \Phi_j \rangle = \sum_j a_j \delta_{ij} = a_i$$

Quantum Mechanics (III) (reminder)

- Physical observables in Quantum Mechanics are represented by (anti)linear, unitary operators
- An operator is an object that transforms a state of the Hilbert space into another

$$Prob(x \in C) = \int_C \psi^c \psi \, dx^3$$

- Using the properties of the wave functions we can work with operators in products:

$$\langle \Phi | U \Phi \rangle = (\langle U \Phi | \Phi \rangle)^c = \langle U^h \Phi | \Phi \rangle = \langle \Phi | U \Phi \rangle$$ h = hermitian

- If the operator represents a physical state their transformed fields should be physical

$$1 = \langle \Phi | \Phi \rangle = \langle U \psi | U \psi \rangle = \langle \psi | U^h U \psi \rangle = 1 \Rightarrow U^h U = I$$

- When an observable is measured in the lab an eigenvalue of the operator is always found

$$A \psi_j = a_j \psi_j$$

- If a state is expressed in terms of the eigenstates of an operator, the probability of measuring the eigenvalue a_j is given by:

$$\Phi = \sum_i p_i \psi_i \Rightarrow prob(finding\ a_j) = (|p_j|)^2 = (|\langle \psi_j | \Phi \rangle|)^2$$

Lagrangian Formalism (I) (reminder)

- Classical mechanics can be formulated using the Lagrangian Formalism
- The Lagrangian is a function of the position and velocity of the particles
  - It is defined as L = T – V where T is the kinetic energy and V the potential energy
- The action is the integral of the Lagrangian with respect to time

$$S=\int_{-\infty}^{\infty} L(x, v)\, dt$$

- Physical trajectories for x and v are those for which S in a stationary state
  - This condition can be applied by differentiating and equaling to 0

$$\delta S=0$$

- Notice that S is not a function but a functional: it assigns a number to every x(t) and v(t)
  - The physical x(t) and v(t) functions are those satisfying that condition
  - The equations of motion of a dynamical system can be derived from here

Lagrangian Formalism (II) (reminder)

- Lagrangian formalism is valid for Quantum Field Theory although:
  - Position and velocity are replaced by the fields and their derivatives
- A new quantity known as the "Lagrangian Density" is defined as:

$$L=\int L_d\left(\Phi,\partial_\mu\Phi\right)dx^3$$

- With this new quantity the action can be expressed with the following form:

$$S=\int L_d\left(\Phi,\partial_\mu\Phi\right)dx^4$$

- It is possible to differentiate the action (assuming good properties on the fields functions)

$$\delta S=\int \frac{\partial L_d}{\partial\Phi}\delta\Phi+\frac{\partial L_d}{\partial\left(\partial_\mu\Phi\right)}\delta\left(\partial_\mu\Phi\right)dx^4=\int \frac{\partial L_d}{\partial\Phi}\delta\Phi+\frac{\partial L_d}{\partial\left(\partial_\mu\Phi\right)}\partial_\mu\left(\delta\Phi\right)dx^4$$

- Applying the rule of the derivative of the product:

$$\partial_\mu\left(\frac{\partial L_d}{\partial\left(\partial_\mu\Phi\right)}\delta\Phi\right)=\partial_\mu\left(\frac{\partial L_d}{\partial\left(\partial_\mu\Phi\right)}\right)\delta\Phi+\frac{\partial L_d}{\partial\left(\partial_\mu\Phi\right)}\partial_\mu\delta\Phi$$

Lagrangian Formalism (III) (reminder)

- Replacing this in the previous equation we can write:

$$\delta S=\int \frac{\partial L_d}{\partial \Phi}\delta\Phi+\left[\partial_\mu\left(\frac{\partial L_d}{\partial(\partial_\mu\Phi)}\delta\Phi\right)-\partial_\mu\left(\frac{\partial L_d}{\partial(\partial_\mu\Phi)}\right)\delta\Phi\right]dx^4$$

- That can be expressed as:

$$\delta S=\int\left[\frac{\partial L_d}{\partial\Phi}-\partial_\mu\left(\frac{\partial L_d}{\partial(\partial_\mu\Phi)}\right)\right]\delta\Phi\, dx^4+\int \partial_\mu\left(\frac{\partial L_d}{\partial(\partial_\mu\Phi)}\delta\Phi\right)dx^4$$

- The second term is the volume integral of a divergence through all the space time
  - Applying the divergence theorem: equals the integral of the vector in the surface of infinite
  - If the fields vanished there (they should to satisfy unitarity) the integral is 0

- This means that the physical fields should make the following relation:

$$\delta S=\int\left[\frac{\partial L_d}{\partial\Phi}-\partial_\mu\left(\frac{\partial L_d}{\partial(\partial_\mu\Phi)}\right)\right]\delta\Phi\, dx^4=0\Rightarrow \boxed{\frac{\partial L_d}{\partial\Phi}-\partial_\mu\left(\frac{\partial L_d}{\partial(\partial_\mu\Phi)}\right)=0}$$

Euler-Lagrange equations for quantum fields

Noether's theorem

- Emmy Noether made one of the most fundamental theorems of Physics
- In this theorem Noether established a connection between symmetries and conservation laws
- Let's assume a differential transformation that converts one field into another in such a way:

$$\Phi \rightarrow \Phi' = T\,\Phi \qquad \qquad \delta\Phi = \Phi' - \Phi$$

- If this transformation leaves the Lagrangian unchanged then the transformation is a symmetry
  - If the Lagrangian is unchanged it means that under this transformation $\delta L_d = 0$
- Let's see the implications of this by making the derivatives of the Lagrangians

$$\delta L_d = \left[\frac{\partial L_d}{\partial \Phi} - \partial_\mu \left(\frac{\partial L_d}{\partial(\partial_\mu \Phi)} \right) \right] \delta\Phi + \partial_\mu \left(\frac{\partial L_d}{\partial(\partial_\mu \Phi)} \delta\Phi \right) = 0$$

- The first term vanishes because of the Eular-Lagrange equation, so:

$$\partial_\mu \left(\frac{\partial L_d}{\partial(\partial_\mu \Phi)} \delta\Phi \right) = \partial_\mu f^\mu = 0 \Rightarrow \frac{\partial f^0}{\partial t} = \nabla \vec{f}$$

- Integrating to all the space and applying the divergence theorem we have

$$\frac{\partial \int f^0 dx^3}{\partial t} = \int \nabla \vec{f}\, dx^3 = 0 \Rightarrow Q = \int f^0 dx^3 = cste$$

Gauge theories (I)

- Group of theories in which the interactions are determined by a principle of invariance
  - A transformation of the fields that should leave the Lagrange invariant is considered
  - The transformed Lagrangian is calculated and observed not to be invariant
  - A new field (the interacting field) is added in order to achieve this invariance
- Let's consider the example of electromagnetism
  - As we will see the Lagrangian associated to a fermion Ψ is given by:

$$L_d = \bar{\psi}\left(i\gamma^\mu \partial_\mu - m\right)\psi$$

- Let's assume a local phase transformation in between the fields, such as:

$$\psi \rightarrow \psi' = e^{iq\alpha(x)}\psi \qquad \bar{\psi} \rightarrow \bar{\psi}' = e^{-iq\alpha(x)}\bar{\psi}$$

- If we try to estimate how the Lagrangian changes we obtain

$$L'_d = \bar{\psi}'\left(i\gamma^\mu \partial_\mu - m\right)\psi' = \bar{\psi}'\left(i\gamma^\mu \partial_\mu - m\right)\psi'$$

$$\bar{\psi} e^{-iq\alpha(x)}\left(i\gamma^\mu \partial_\mu - m\right)e^{iq\alpha(x)}\psi + \bar{\psi} e^{-iq\alpha(x)}\left(iq\,\partial_\mu \gamma^\mu \alpha(x)\right)e^{iq\alpha(x)}\psi$$

$$\bar{\psi}\left(i\gamma^\mu \partial_\mu + iq\gamma^\mu \partial_\mu \alpha(x) - m\right)\psi$$

Gauge theories (II)

- Let's introduce a new field A_μ in such a way that:

$$L_d = \bar{\psi}\left(i\gamma^\mu \partial_\mu - iq\gamma^\mu A_\mu - m\right)\psi$$

- Applying the transformations we obtain the same as before plus aditional terms:

$$L'_d = \bar{\psi}\left(i\gamma^\mu \partial_\mu + iq\gamma^\mu \partial_\mu \alpha(x) - m\right)\psi - \bar{\psi}\left(iq\gamma^\mu A'_\mu\right)\psi$$

- Grouping the terms:

$$L'_d = \bar{\psi}\left(i\gamma^\mu \partial_\mu + iq\gamma^\mu\left(\partial_\mu \alpha(x) - A'_\mu\right) - m\right)\psi$$

- If the introduced field transforms according to this transformation:

$$A'_\mu = A_\mu + \partial_\mu \alpha(x)$$

- The transformed Lagrangian is now exactly the same as the original Lagrangian

$$L'_d = \bar{\psi}\left(i\gamma^\mu \partial_\mu - iq\gamma^\mu A_\mu - m\right)\psi = L_d$$

- A_μ is the field of the photon and describes the electromagnetic interactions