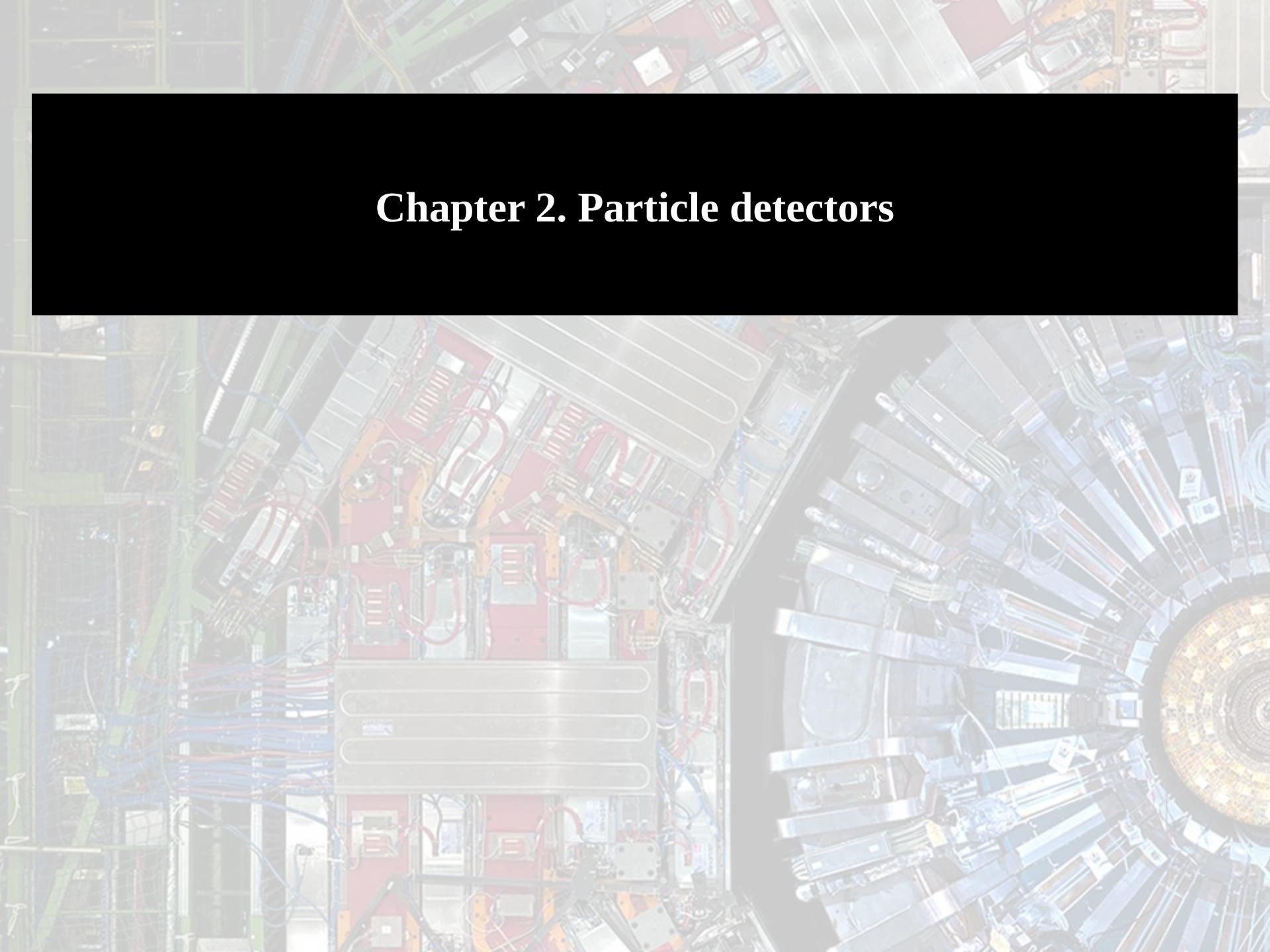
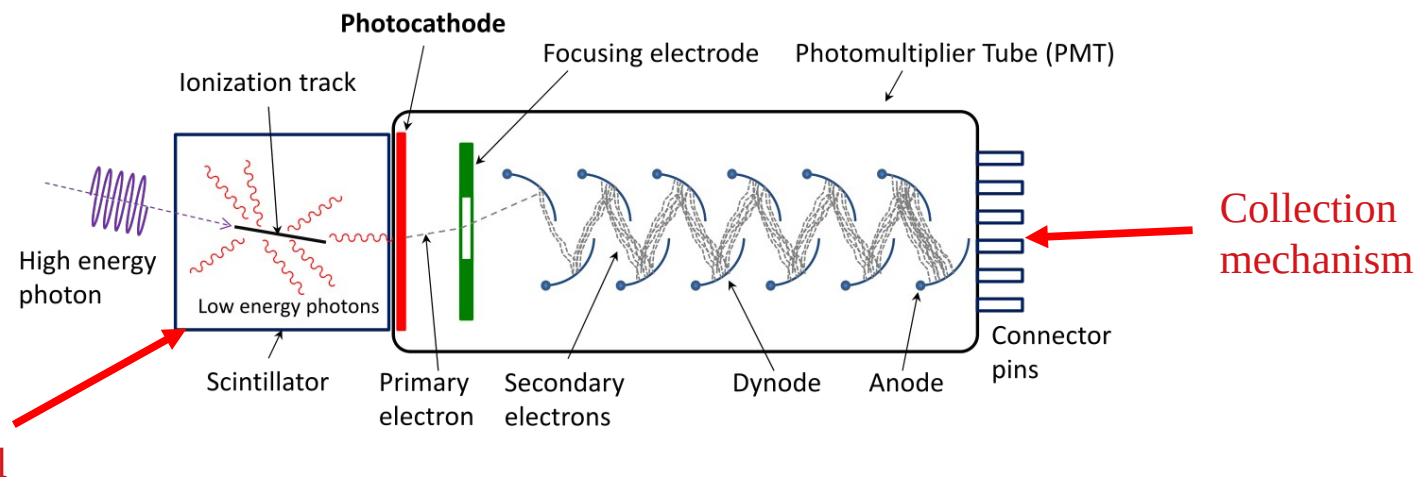


Chapter 2. Particle detectors



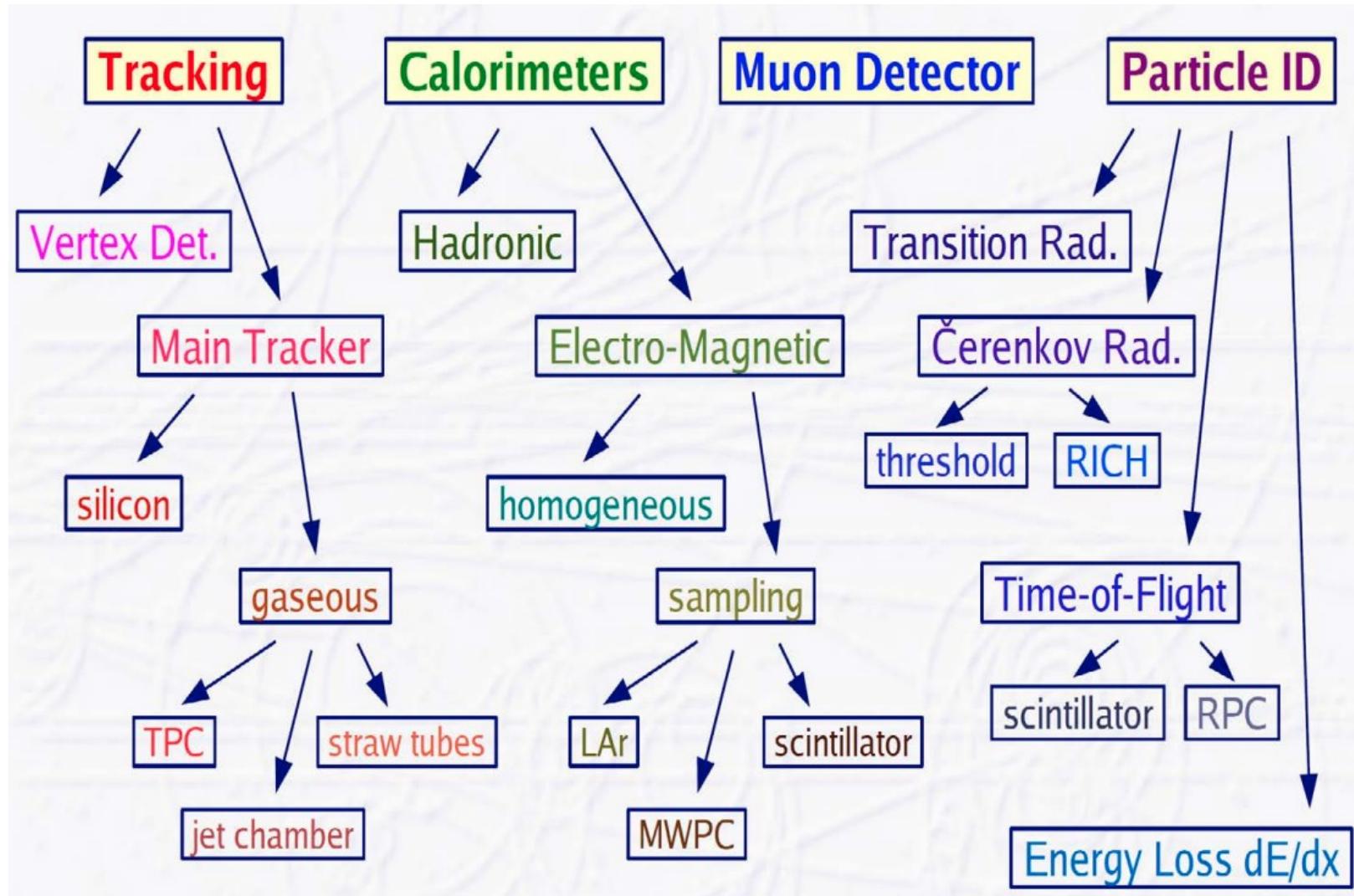
Introduction

- Particle Physics experiments and applications require some kind of device able to:
 - Identify when/if a particle is passing through it
 - Measure one or several properties of the particle (energy, momentum, charge, spin, etc.)
 - Possibly identify the type of particle
- Devices with these functionalities are called “**Particle Detectors**”
- Most particle detectors are logically composed by two parts:
 - **Active material:** volume where the particle interacts with the detector producing “signal”
 - **Collection mechanism:** system able to detect and possibly reconstruct the signal



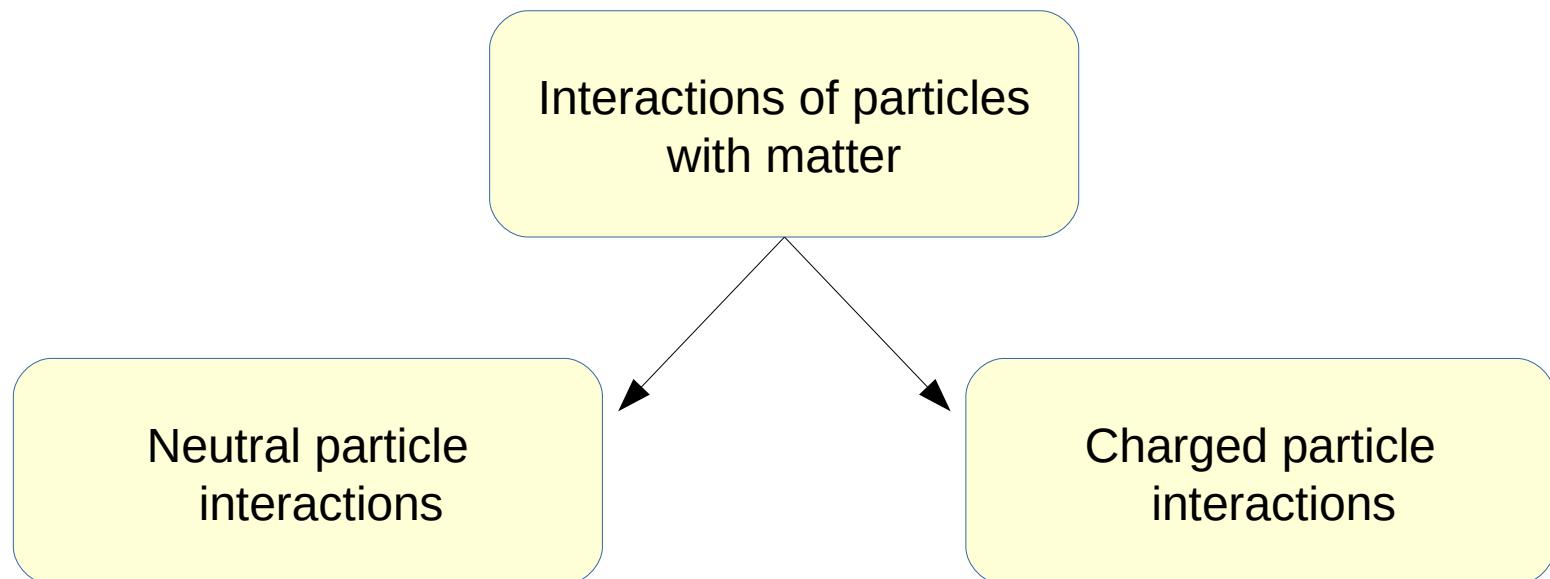
The detector zoo

- There is a large variety of detectors according to the type of particle and the technology used



Active material: interactions with matter

- Particles interact with the electrons and/or nuclei of the medium they cross
- This interaction can be electromagnetic, weak or strong depending on the particle
- The effects of this interaction can be used to detect the particles
- To understand how detectors work we need to understand how particles interact with matter
- Detectors should have an active material in which this interaction takes place



Neutral particle interactions

- Neutral particles such as photons, neutrons, K^0 , and others suffer different interactions

Photons

Photoelectric effect
Compton effect
Pair production

Moderate/Low energy neutrons

Scattering
Absorption
Fission

High Energy neutral hadrons (n , K^0)

Nuclear interactions

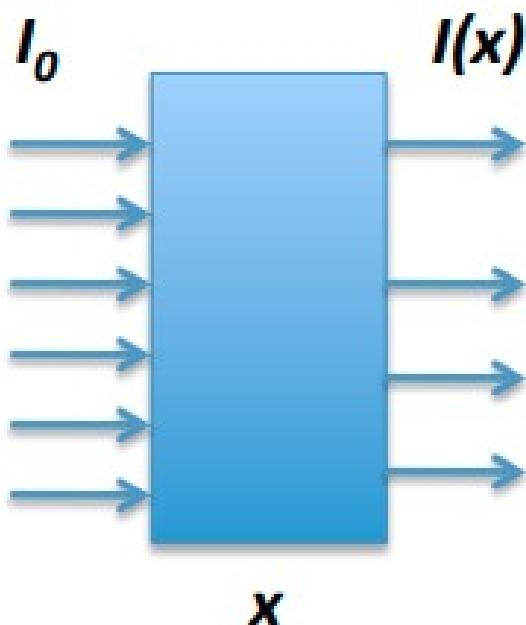
Neutrinos

Weak interactions

Not covered in this lecture

Photon interactions

- Since photons do not have charge they are always indirectly detected:
 - In their interactions they produce electrons/positrons that interact with matter
- Photons may be absorbed (photoelectric effect or e^+e^- creation) or scattered (Compton effect)
- An attenuation law is introduced based on the fact that the interaction probability is constant



$$I(x) = I_0 e^{-\mu x}$$

μ absorption coefficient
 N atoms/m³
 A masse molaire
 N_A nombre Avogadro
 ρ density
 σ Photon cross section
 λ Mean free path or absorption length

$$\mu = N \sigma = \frac{N_A}{A} \quad \rho \sigma = \frac{1}{\lambda}$$

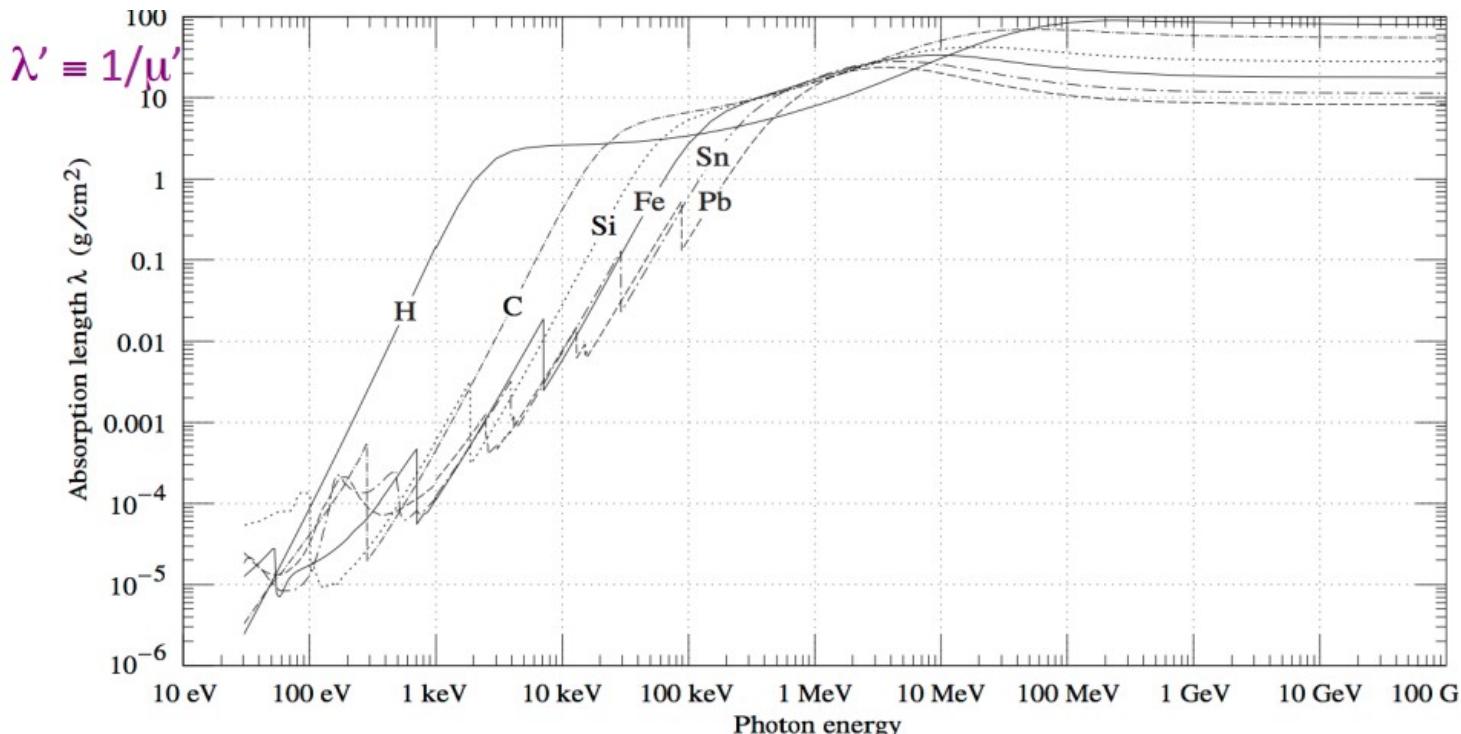
Photon absorption length

- Frequently instead of working with the distance we use the distance times density
 - This quantity is more sensitive to the real amount of matter in the crossed distance

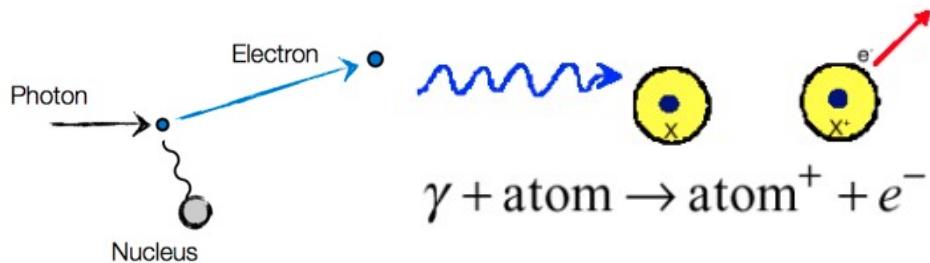
$$x' = x \rho$$

$$I(x) = I_0 e^{-\mu x} \longrightarrow \mu' = \mu/\rho \longrightarrow I(x) = I_0 e^{-\mu' x'}$$

$$\lambda' = 1/\mu'$$



Photoelectric effect



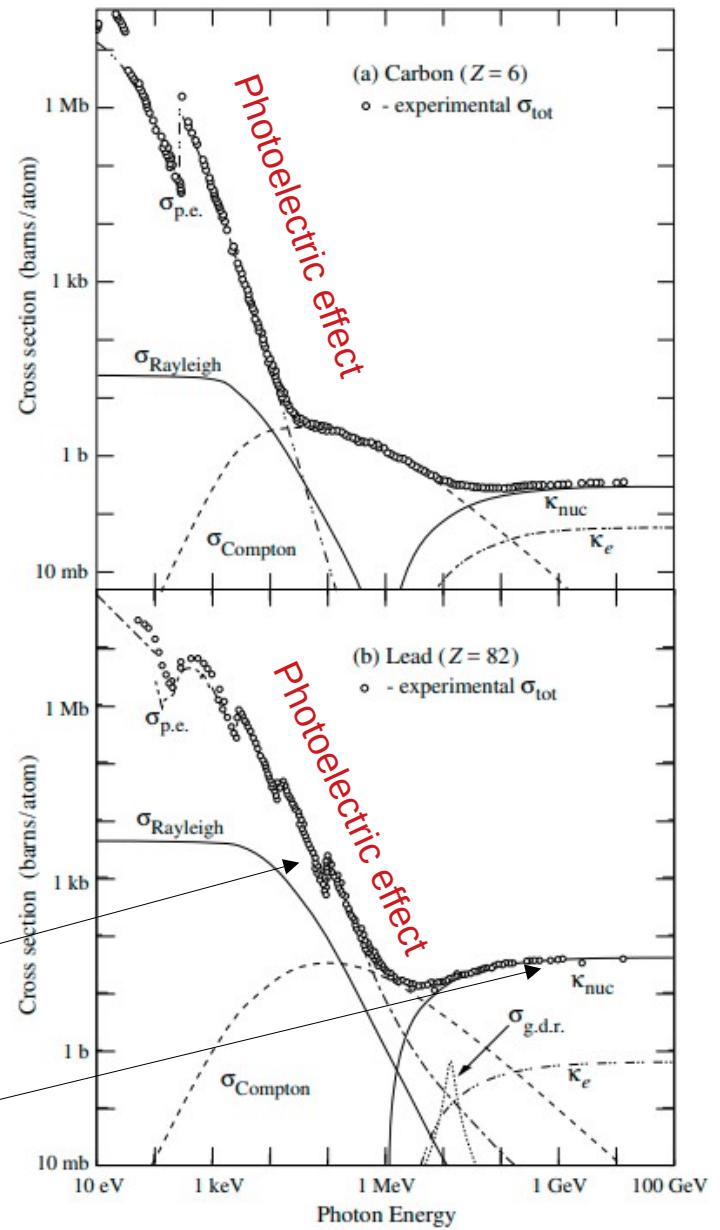
- The energy of the photon is transferred to the electron
- The photon is absorbed and the electron is extracted
 - The atom gets ionized
- The final energy of the electron depends on the binding energy (the electron energy level in the atom)

$$E_e = h\nu - E_{e\,binding}$$

- Different trends at low and high energies

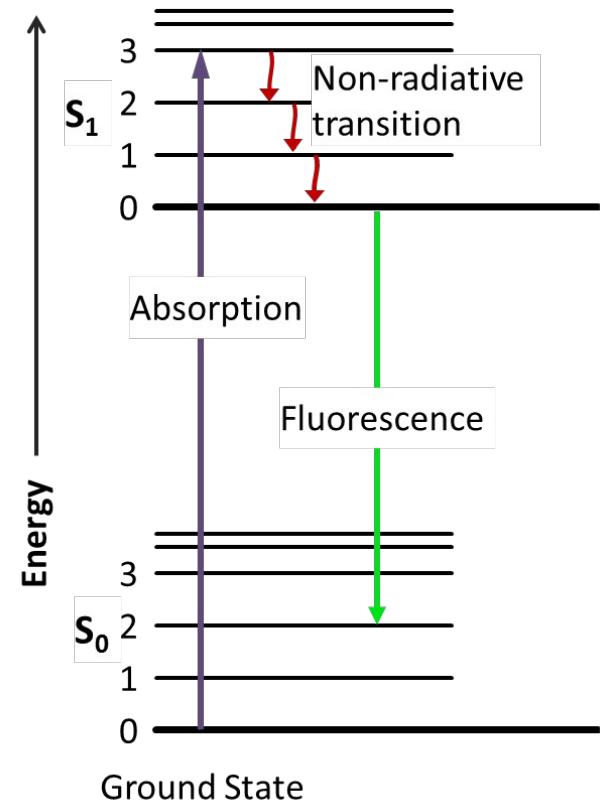
$$\sigma \sim Z^5 / E_\gamma^{3.5}$$

$$\sigma \sim Z^5 / E_\gamma$$



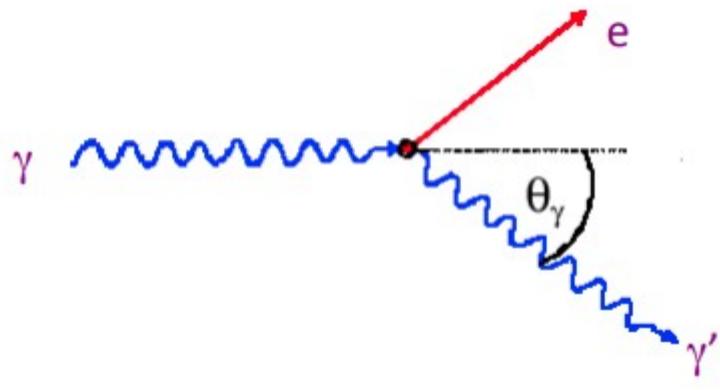
(After) Photoelectric effect: fluorescence

- After the emission of a photoelectron the ionized atom (molecule) is an excited state
- The ionized atom (molecule) returns to the ground state emitting photons
- This is an important effect in a kind of active materials called: scintillators



Compton scattering

- Scattering of light on free electrons (if $E_{\text{photon}} \gg E_{\text{binding}}$ a binded electron is considered free)



$$E_k e = E_\gamma - E_{\gamma'} = E_\gamma \frac{(1-\cos \theta_\gamma) (E_\gamma / m_e c^2)}{1 + (E_\gamma / m_e c^2) (1-\cos \theta_\gamma)}$$

$$E_{\gamma'} = \frac{E_\gamma}{1 + (E_\gamma / m_e c^2) (1-\cos \theta_\gamma)}$$

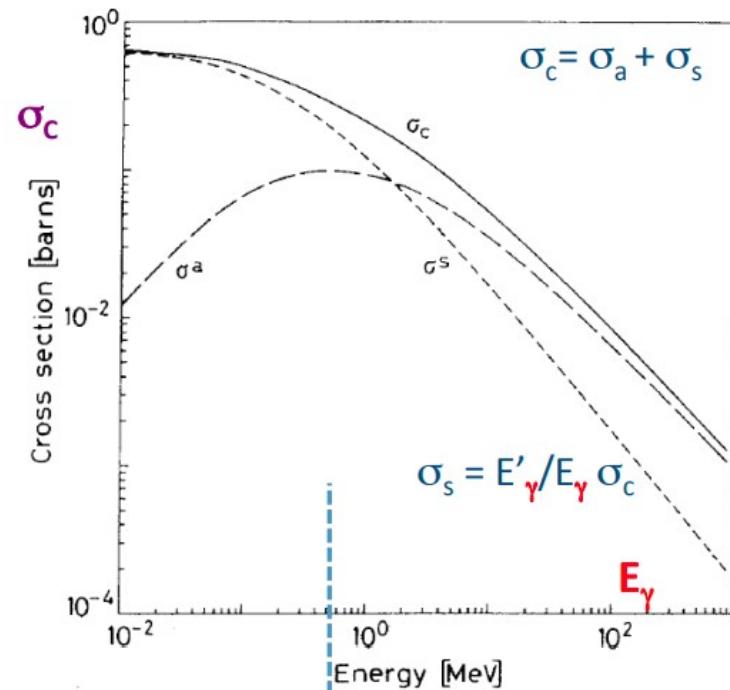
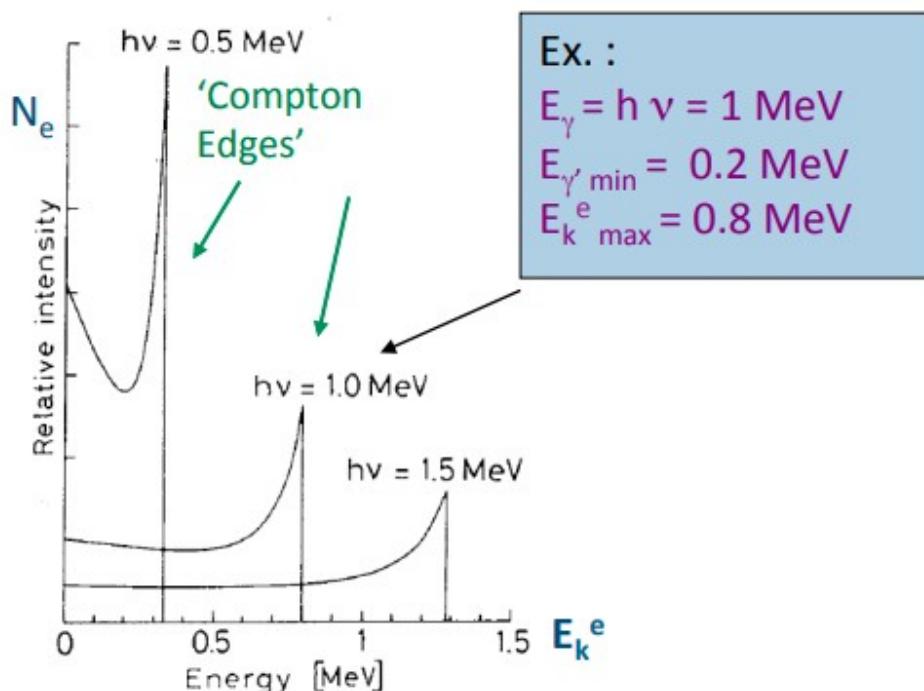
- When the angle is 0 the initial photon gives all its energy to the outgoing photon
- When the angle is backwards the outgoing electron has maximum energy
 - And the outgoing photon minimum energy but it cannot get completely absorbed

Compton edges and cross section

- The photon can never give all its energy to the electron producing the so called “edges”
- The cross section is given by the Klein-Nishina formula

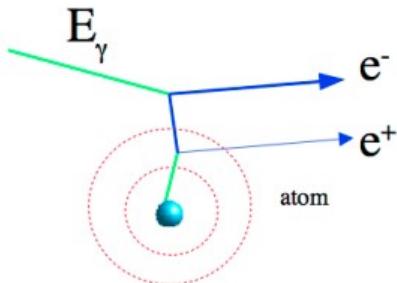
$$\epsilon = \frac{E_\gamma}{m_e}$$

$$\sigma_c^e = 2\pi r_e^2 \left(\frac{1+\epsilon}{\epsilon^2} \right) \left\{ \frac{2(1+\epsilon)}{1+2\epsilon} - \frac{1}{\epsilon} \ln(1+2\epsilon) \right\} + \frac{1}{2\epsilon} \ln(1+2\epsilon) - \frac{1+3\epsilon}{(1+2\epsilon)^2} \quad (\text{per electron})$$

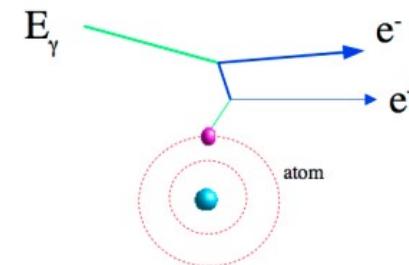


Pair production of electron-positron

- The photon interacts with the field of a nucleus or electron and makes a pair e^+e^-
 - This process cannot take place in the vacuum because of energy-momentum conservation



Pair production in the field of a nucleus



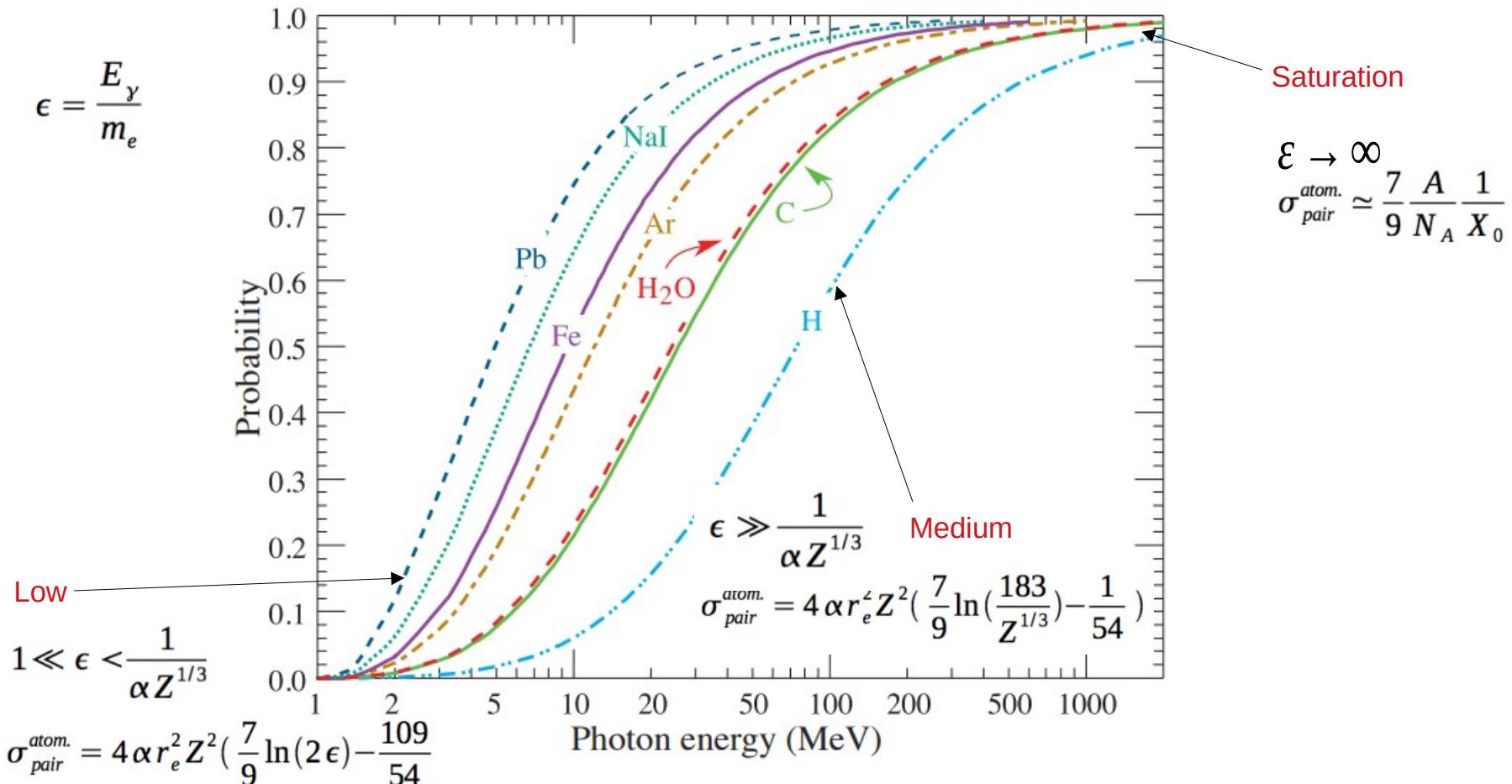
Pair production in the field of an electron

- It is also frequently known as “photon conversion”
- This effect is a “threshold effect”: the photon energy has to be enough to generate 2 electrons

$$E_\gamma > 2 m_e c^2 \left(1 + m_e/m_x\right) \quad m_x = \begin{cases} m_N \\ m_e \end{cases}$$

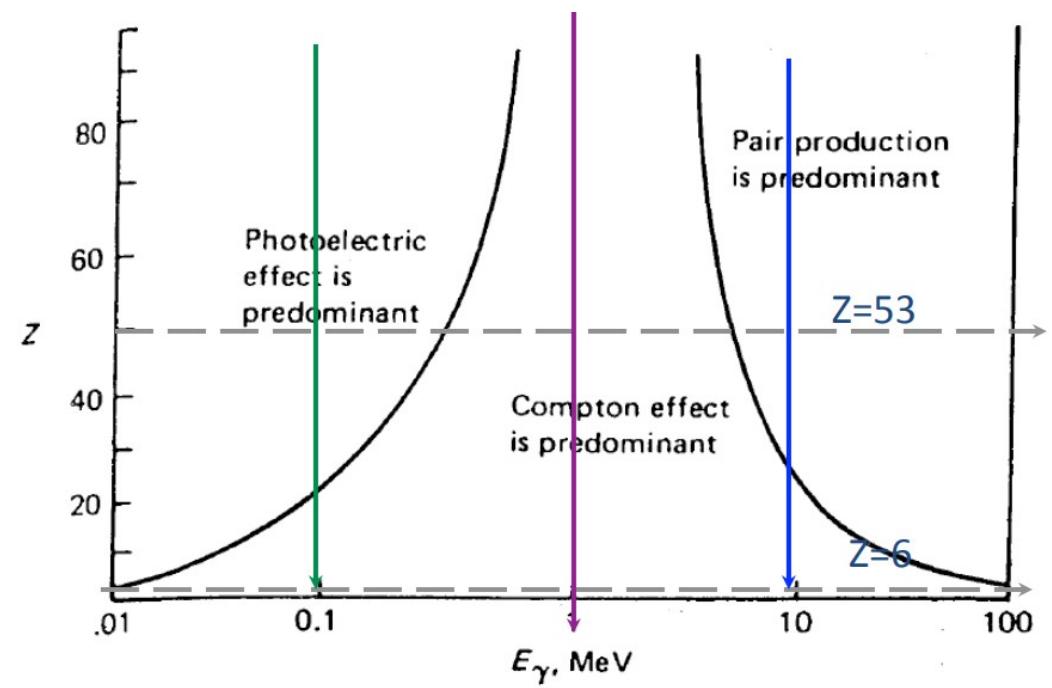
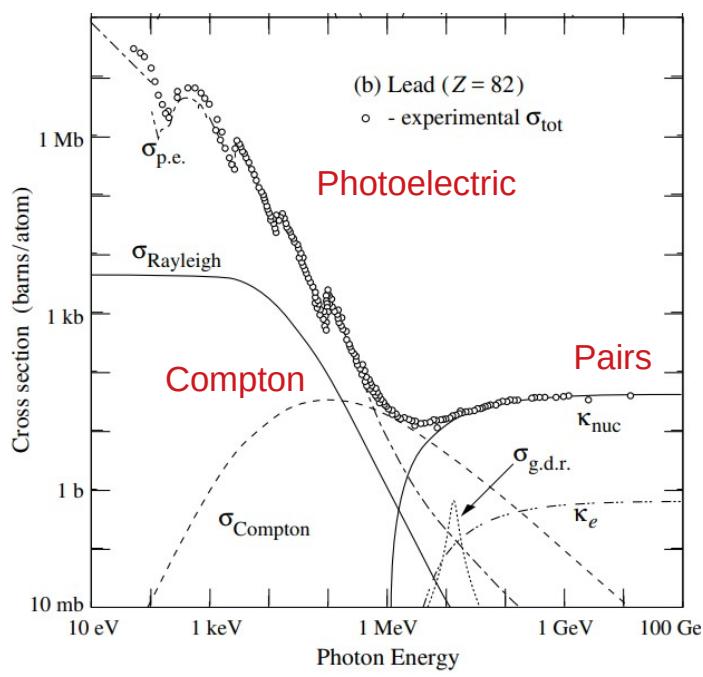
Pair production of electron-positron

- The cross section of pair production rises once the threshold has been reached
- Three different regions: “low”, “medium” and “high” (saturation) energy regimes



Photon total cross section

- Photoelectric effect dominates at low, Compton at medium and pair production at high E
- Compton dominates for low Z materials (photoelectric and pair production increase with Z)
- Other processes not treated here: Rayleigh scattering, photo nuclear interactions



Hadron collisions and interaction lengths

- The total cross section for very high energy hadrons is expressed as: $\sigma_T = \sigma_{elastic} + \sigma_{inelastic}$
- The inelastic part of the total cross-section is susceptible to induce hadron showers
- Two mean lengths are introduced to deal with this kind of interactions
 - Nuclear collision length: $\lambda_T = \frac{A}{N_A \sigma_T} \text{ g cm}^{-2}$
 - Mean free path of a particle before undergoing a nuclear interaction
 - Nuclear interaction length: $\lambda_I = \frac{A}{N_A \sigma_{inelastic}} \text{ g cm}^{-2}$
 - Mean free path of a particle before undergoing an inelastic nuclear interaction

6. ATOMIC AND NUCLEAR PROPERTIES OF MATERIALS

Table 6.1. Abridged from pdg.lbl.gov/AtomicNuclearProperties by D. E. Groom (2007). Quantities in parentheses are for NTP (20°C and 1 atm), and square brackets indicate quantities evaluated at STP. Boiling points are at 1 atm. Refractive indices n are evaluated at the sodium D line blend (589.2 nm); values $\gg 1$ in brackets are for $(n - 1) \times 10^6$ (gases).

Material	Z	A	$\langle Z/A \rangle$	Nucl.coll. length λ_T $\{\text{g cm}^{-2}\}$	Nucl.inter. length λ_I $\{\text{g cm}^{-2}\}$	Rad.len. X_0 $\{\text{g cm}^{-2}\}$	$dE/dx _{\min}$ $\{\text{MeV g}^{-1}\text{cm}^2\}$	Density $\{\text{g cm}^{-3}\}$ $\{(g\ell^{-1})\}$	Melting point (K)	Boiling point (K)	Refract. index (@ Na D)
H ₂	1	1.00794(7)	0.99212	42.8	52.0	63.04	(4.103)	0.071(0.084)	13.81	20.28	1.11[132.]
D ₂	1	2.01410177803(8)	0.49650	51.3	71.8	125.97	(2.053)	0.169(0.168)	18.7	23.65	1.11[138.]
He	2	4.002602(2)	0.49967	51.8	71.0	94.32	(1.937)	0.125(0.166)		4.220	1.02[35.0]
Li	3	6.941(2)	0.43221	52.2	71.3	82.78	1.639	0.534	453.6	1615.	
Be	4	9.012182(3)	0.44384	55.3	77.8	65.19	1.595	1.848	1560.	2744.	
C diamond	6	12.0107(8)	0.49955	59.2	85.8	42.70	1.725	3.520			2.42
C graphite	6	12.0107(8)	0.49955	59.2	85.8	42.70	1.742	2.210			

Neutron interactions

- Neutrons interact via the “strong interaction” with nuclei (short range force $\sim 10^{-13}$ cm)
- Classification of neutrons:

- Cold or ultracold neutrons $E_n < 0.025$ eV
- Thermal or slow neutrons $E_n \sim 0.025$ eV
- Intermediate neutrons $E_n \sim 0.025$ eV – 0.1 MeV
- Fast neutrons $E_n \sim 0.1$ MeV – 10-20 MeV
- High energy neutrons $E_n > 20$ MeV



Low-moderate energy neutrons

Scattering

Absorption

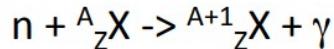
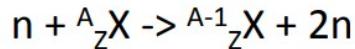
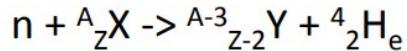
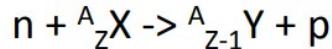
Fission

High energy neutrons

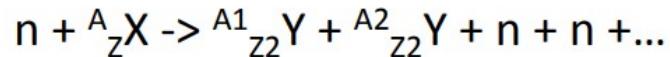
Hadron shower production

Low-intermediate energy Neutron interactions

- Scattering with nuclei (important for moderation): $n + {}^A_z X \rightarrow {}^A_z X^{(*)} + n$
 - The neutron changes energy and the nucleus gets in an excited state
- Absorption and nuclear reactions (the neutron is absorbed and a nuclear reaction takes place)
 - Low A-Z transitions (typically emission of a proton, alpha particle, or neutrons)

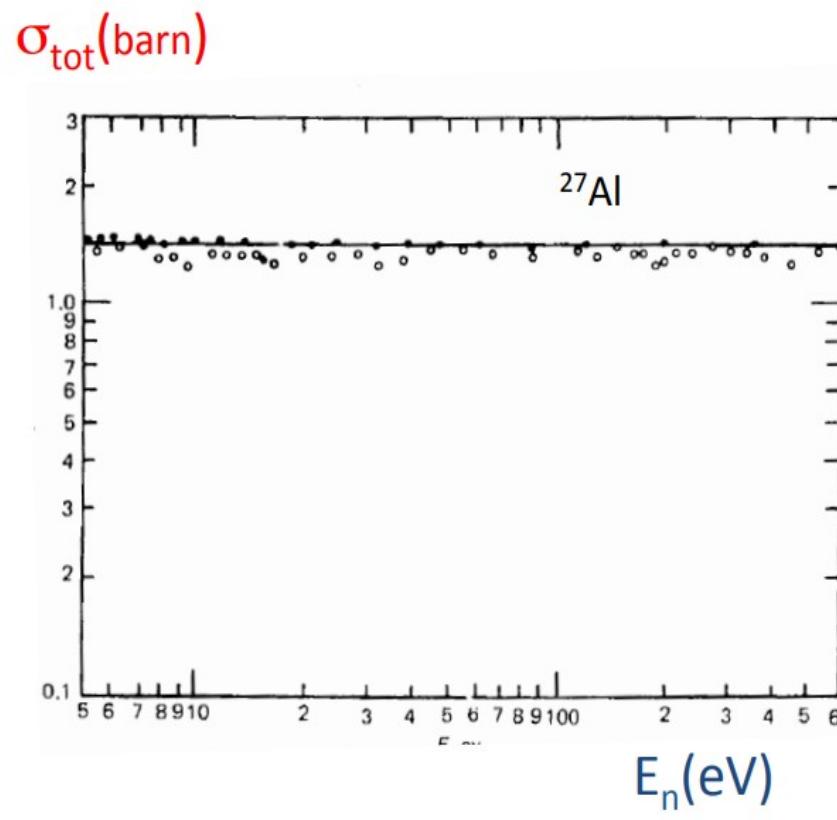
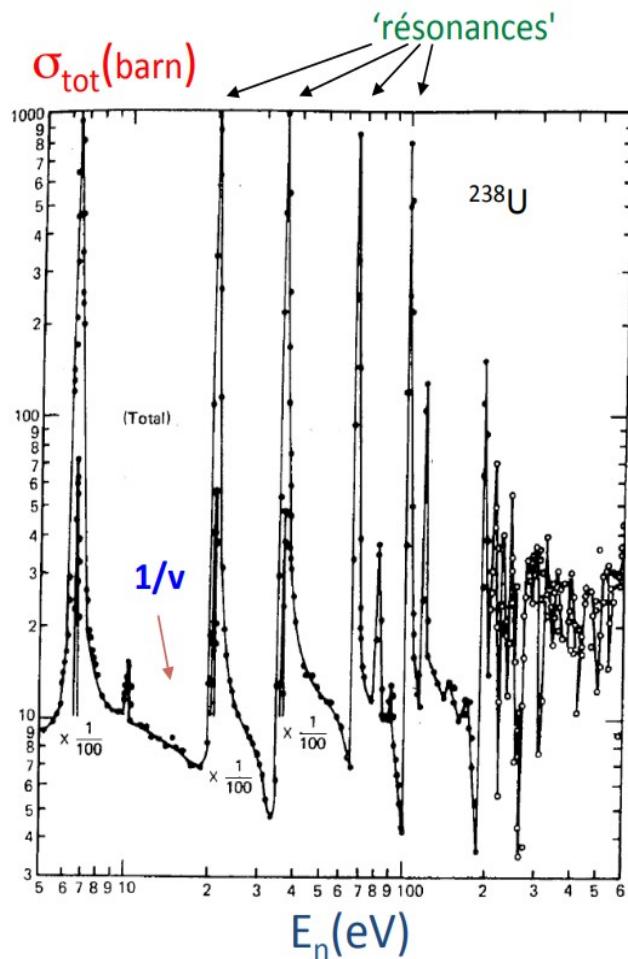


- Fission process: the nucleus is splitted in two and more neutrons are produced
 - High A-Z transitions
 - Importan for chain reactions in nuclear reactors



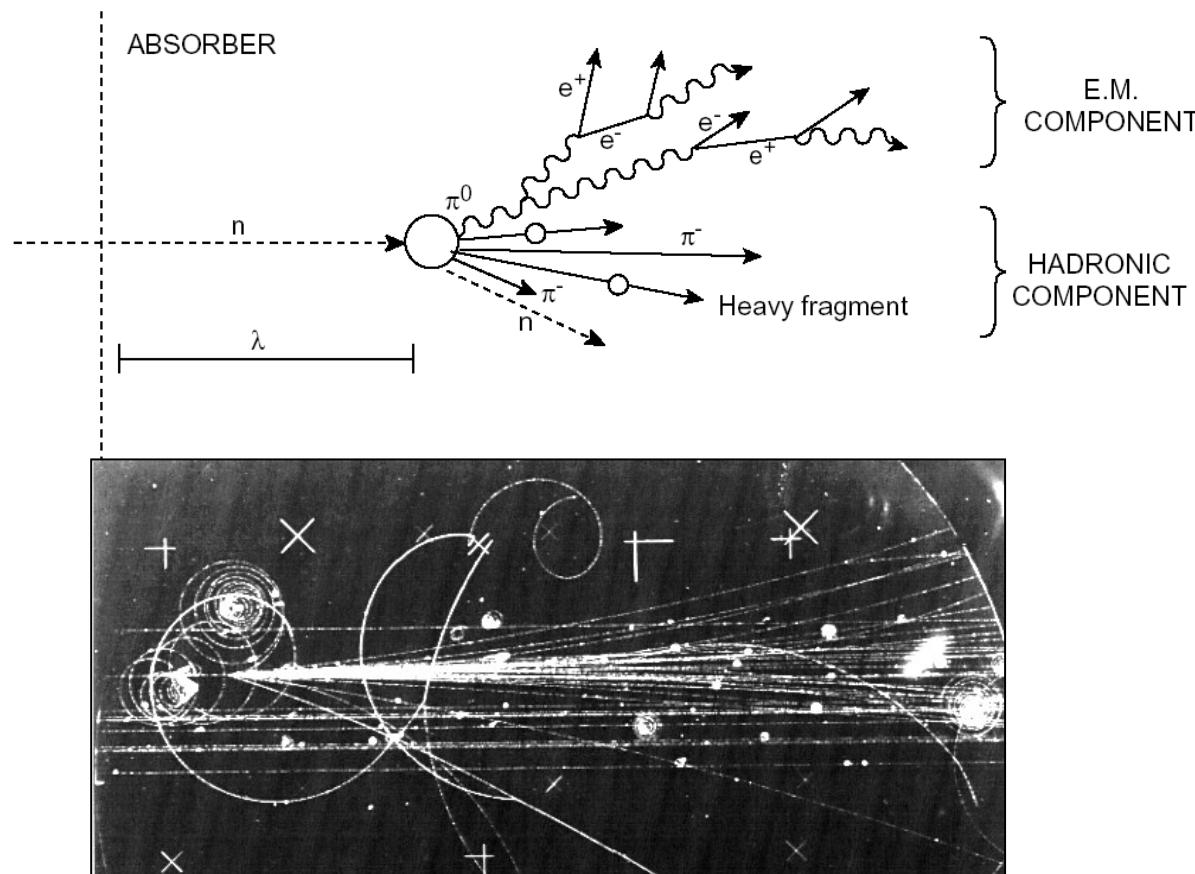
Low-intermediate energy Neutron cross sections

- The cross-section goes like $\sim 1/v_n$ with v_n the velocity of the neutron
- On top of this effect there are the absorption resonances characteristic of every nucleus



Hadron showers

- The incident hadron produces a chain of reactions through inelastic interactions
 - Producing pions, low energy photons, low energy neutrons, neutrinos, photons
- Produces a shower of particles with two different components: electromagnetic and hadronic



Charged particle interactions

- Charged particles suffer a wide variety of different interactions

Ionization

Bremsstrahlung

Multiple scattering

Cherenkov

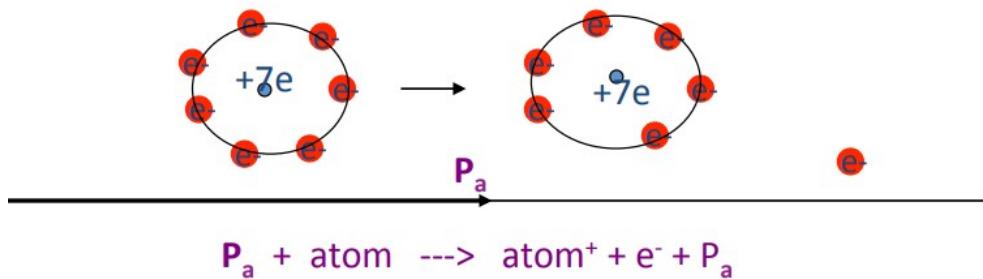
Transition Radiation

Nuclear Interactions

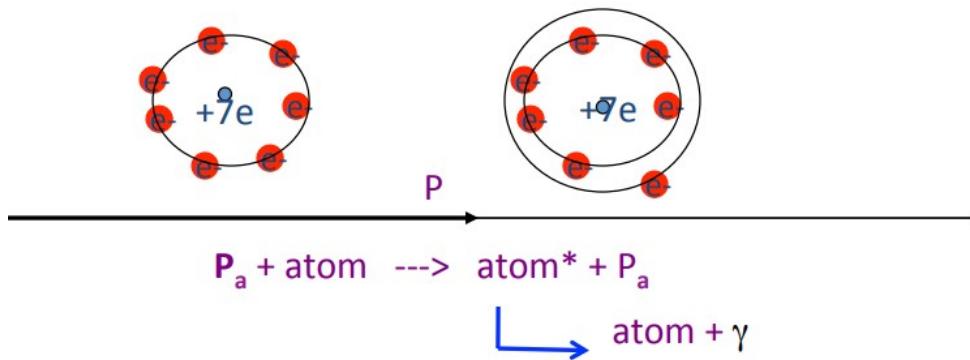
Inelastic collisions with atom electrons (Ionization)

- The charged particle interacts, giving energy, with the electron of an atom
 - This process is dominant for heavy charged particles $M_{pa} \gg m_e$

Ionization



Excitation



- Inelastic collisions on the nucleus are much less frequent (because the mass is much higher)
- The particle loses a bit of energy and its direction remains mostly unchanged

Inelastic collisions with atom electrons (Ionization)

- The average energy loss of a particle due to ionization is given by the Bethe & Bloch Formula
- This formula is valid for particles with a mass much higher than the electrons $m_p \gg m_e$

$$-\frac{dE}{dx} = 2\pi N_a r_e^2 m_e c^2 \rho \frac{Z}{A} \frac{z^2}{\beta^2} \left[\ln \left(\frac{2m_e \gamma^2 v^2 W_{\max}}{I^2} \right) - 2\beta^2 - \delta - 2 \frac{C}{Z} \right]$$

r_e : classical electron
radius = 2.817×10^{-13} cm

m_e : electron mass

N_a : Avogadro's
number = 6.022×10^{23} mol⁻¹

I : mean excitation potential

Z : atomic number of absorbing
material

A : atomic weight of absorbing material

ρ : density of absorbing material

z : charge of incident particle in
units of e

β = v/c of the incident particle

γ = $1/\sqrt{1 - \beta^2}$

δ : density correction

C : shell correction

W_{\max} : maximum energy transfer in a
single collision.

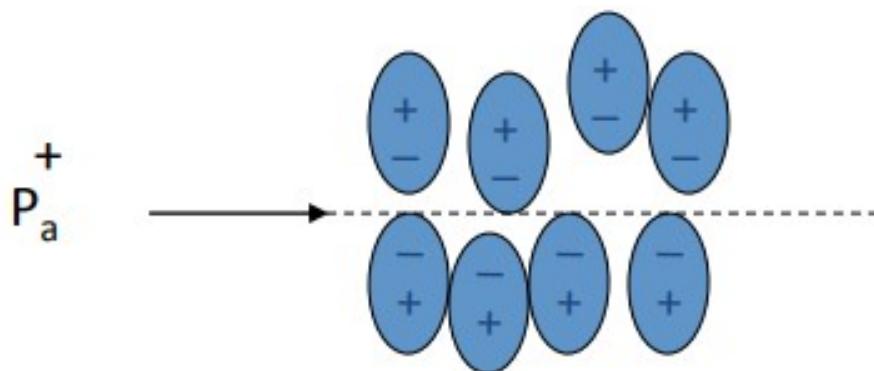
- The maximum energy transfer takes place in head-on or knock-on collisions

$$W_{\max} = E_a^{\max} - m_e = \frac{2m_e \beta^2 \gamma^2}{(E_{CM}/M)^2}$$

Shell and Density effect corrections

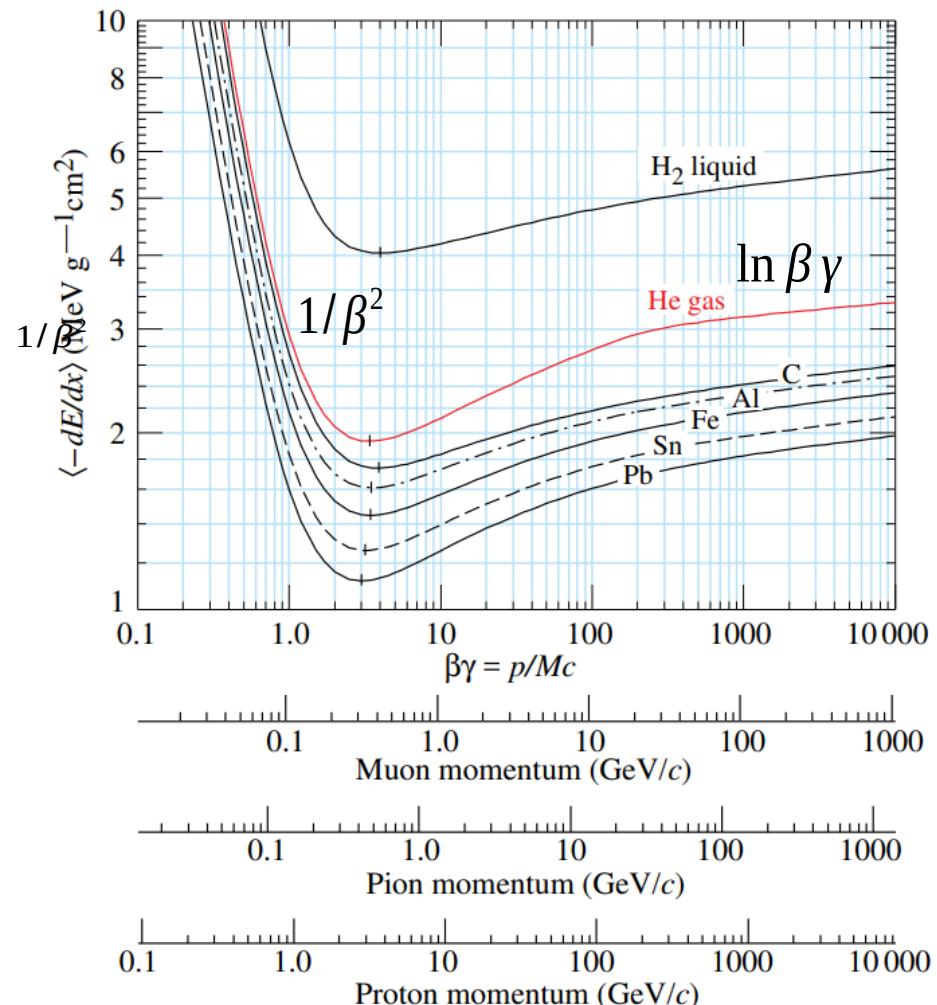
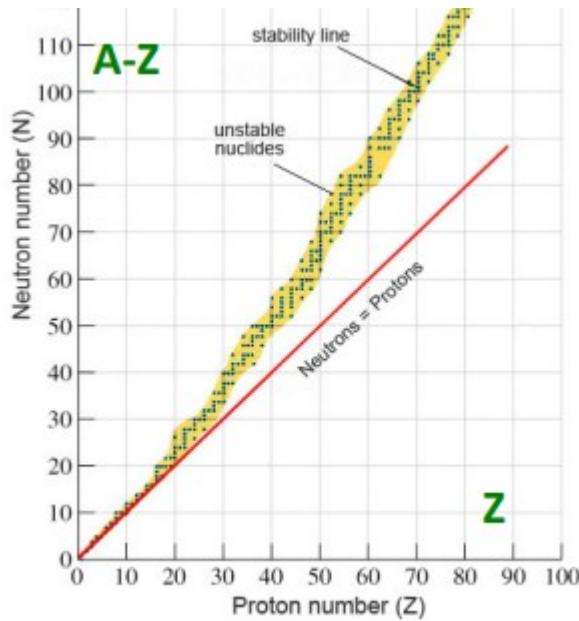
- The C term accounts for the fact that the electron is not at rest in the atom
 - It is a small correction only relevant at low energies
 - If the binding energy of the electron is taken into account the energy loss is reduced
- The “ δ ” term or “density effect” → accounts for the polarization induced by the particle
 - The atoms get polarized due to the charge of the particle
 - The energy loss is reduced because of the shielding of the electric field
 - It depends on the particle speed and on the matter density
 - This process is relevant at high energy

$$-\frac{dE}{dx} = 2\pi N_a r_e^2 m_e c^2 \rho \frac{Z}{A} \frac{z^2}{\beta^2} \left[\ln \left(\frac{2m_e \gamma^2 v^2 W_{\max}}{I^2} \right) - 2\beta^2 \delta - 2 \frac{C}{Z} \right]$$



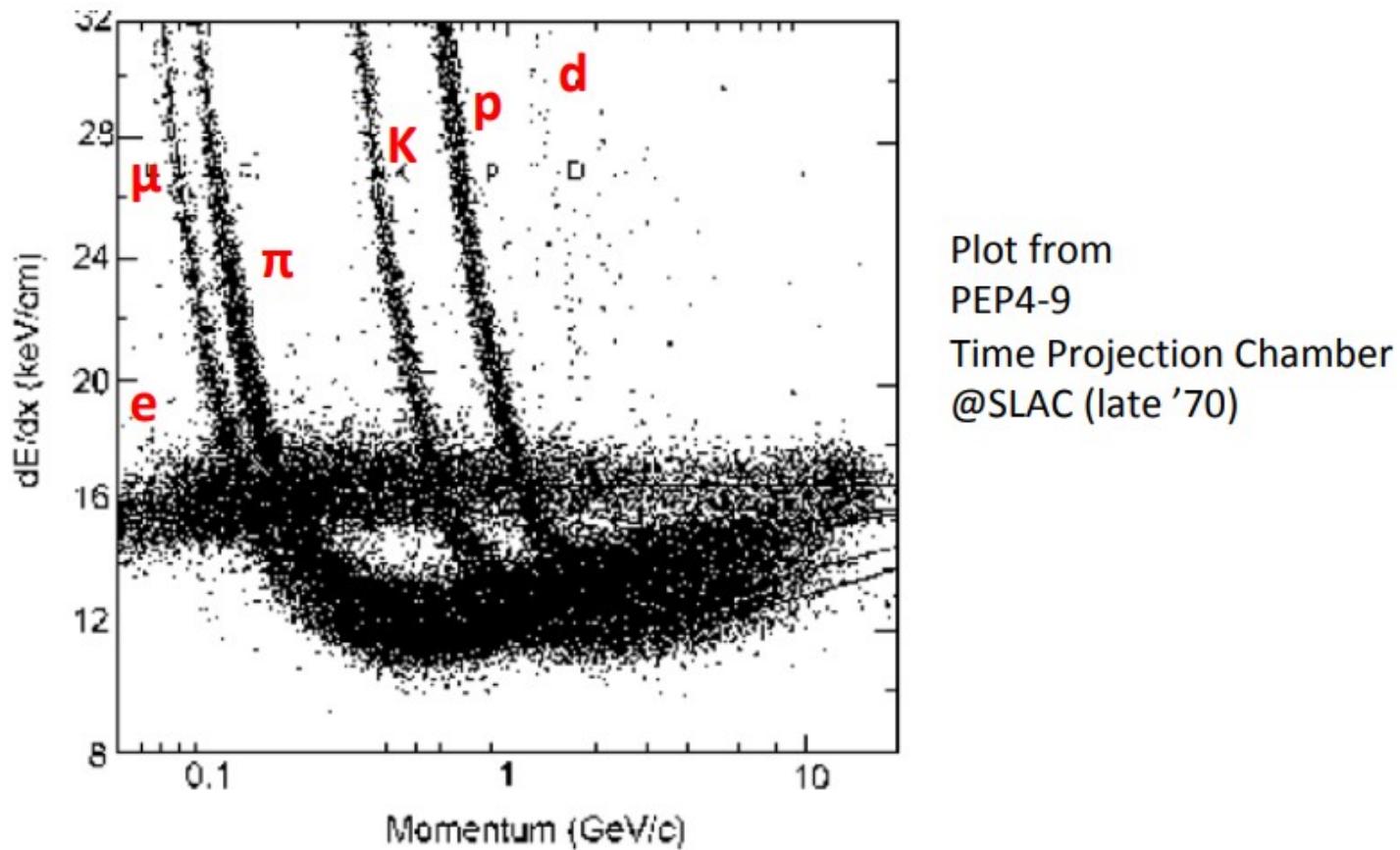
Stopping power or mean specific energy loss

- The energy loss is usually given in the form of the stopping power $-dE/(\rho dx)$
- The dependency on the material is almost removed since $Z/A \sim 0.5$
- At low energy the term $1/\beta^2$ dominates
- At high energy \rightarrow logarithmic term
- There is a minimum of ionization
 - MIP \rightarrow Minimum Ionizing Particle



Use of dE/dx for particle identification

- The energy loss can be used to identify particles if the momentum is measured independently
- Indeed with the momentum known the Bethe Bloch formula can be used to measure β
- With the momentum and β known the mass of the particle can be extracted



Bremsstrahlung. Mean radiative energy loss

- An accelerated or decelerated charged particle emits electromagnetic radiation
- This effect takes place when the particle is accelerated in the field of a nucleous
- It is particularly relevant for electrons and positrons due to their small mass

$$-\left(\frac{dE}{dx}\right)_{\text{brem}} = N \int_{v_0=E_0/h}^{\infty} h\nu \frac{d\sigma}{dv} dv = NE_0 \phi(Z^2)$$

For electrons

If $E_0 \gg m_e c^2$ et $E_0 \ll 137 m_e c^2 / Z^{1/3}$

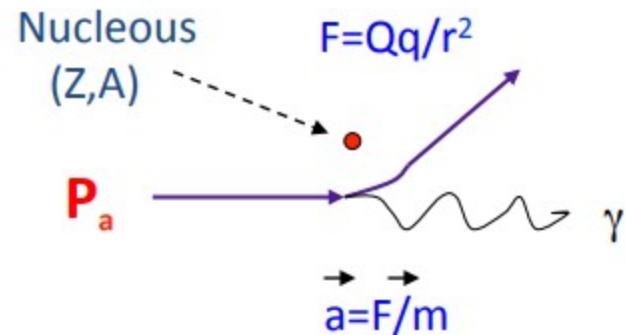
$$\phi(Z^2) = 4\alpha Z^2 r_e^2 \ln(2E_0/m_e c^2 - 1/3 - f(Z))$$

If $E_0 \gg 137 m_e c^2 / Z^{1/3}$

$$\phi(Z^2) = 4\alpha Z^2 r_e^2 \ln(183 Z^{-1/3} - 1/18 - f(Z))$$

For other particles

$$\frac{dE}{dx}_{\text{brem}}(z, m) = \left(\frac{m_e}{m}\right)^2 z^2 \frac{dE}{dx}_{\text{brem}}(e^-)$$



N = atoms/cm³ (N = ρ N_A/A)

Z = atomic number

E_0 = Initial energy of particle P_a

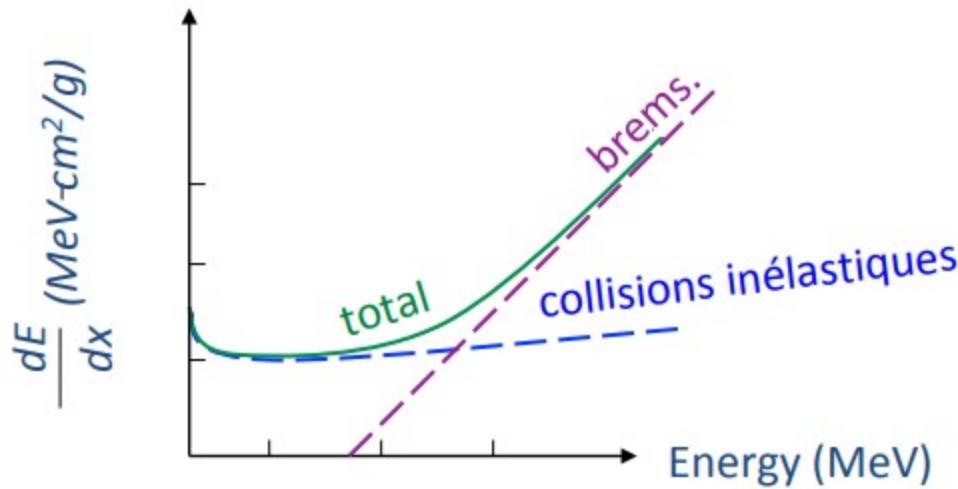
$v_0 = E_0/h$

$h\nu$ = energy of emitted γ

$\frac{d\sigma}{dv}$ = Differential cross section of the bremsstrahlung process

Comparison of Bremsstrahlung vs. ionization

- The average energy loss due to ionization increases with the log of energy and linear with Z
- The average energy loss due to Brem increases linearly with energy and linear with Z^2
- The energy in which the two stopping powers equal each other is called the critical energy



For e^\pm in :

Pb	$E_c = 9.5$ MeV
Cu	$E_c = 24.8$ MeV
Fe	$E_c = 27.4$ MeV
Al	$E_c = 51$ Mev

For liquid and solids: $E_c \sim 610 \text{ MeV}/(Z+1.24)$

For gas $E_c \sim 710 \text{ MeV}/(Z + 0.92)$

- For other particles the critical energy scales according to the square of the mass ratio

$$E_c(Pa) \propto (m_{Pa}/m_e)^2 E_c(e)$$

Radiation length X_0

- › For high energy particles most of the energy loss comes from Bremsstrahlung radiation
- › The radiation length is defined as the length in which a particle has radiated $1/e$ of its energy

$$-\frac{dE}{dx} = NE_0 \Phi \quad X_0 = \frac{1}{N \Phi}$$

- › A specific radiation length is also defined to reduce the dependency with the material

$$X' = X_0 \rho = \frac{\rho}{N \Phi}$$

- › The specific radiation length can be estimated for pure elements approximately as:

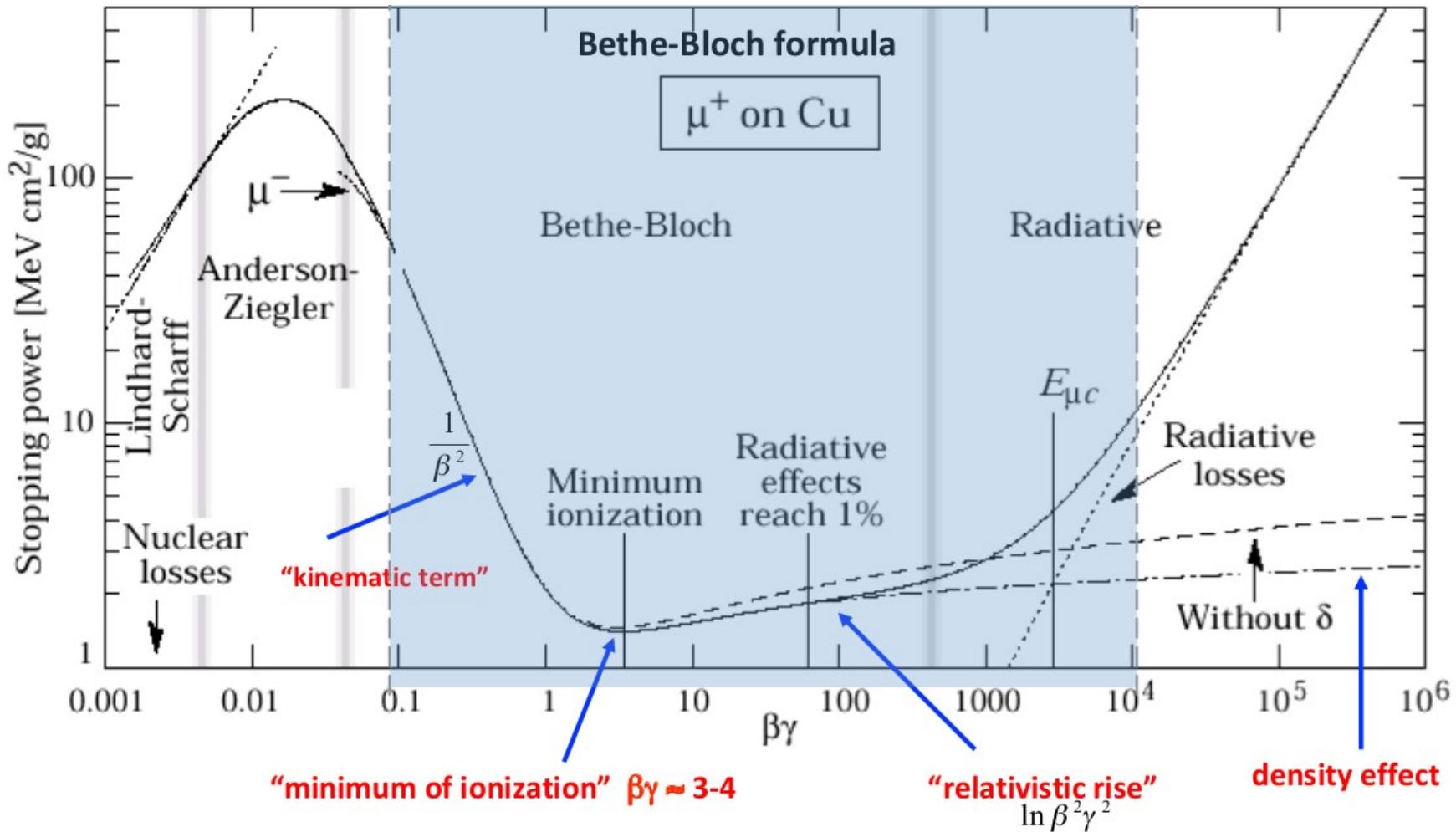
$$X_0 = \frac{716.4 \text{ g cm}^{-2} A}{Z(Z+1) \ln(287/\sqrt{Z})}$$

- › For a material composed of many different elements:

$$\frac{1}{X_0} = \sum_i w_i \frac{1}{X_{0i}}$$

Where w_i is the fraction in mass of the element

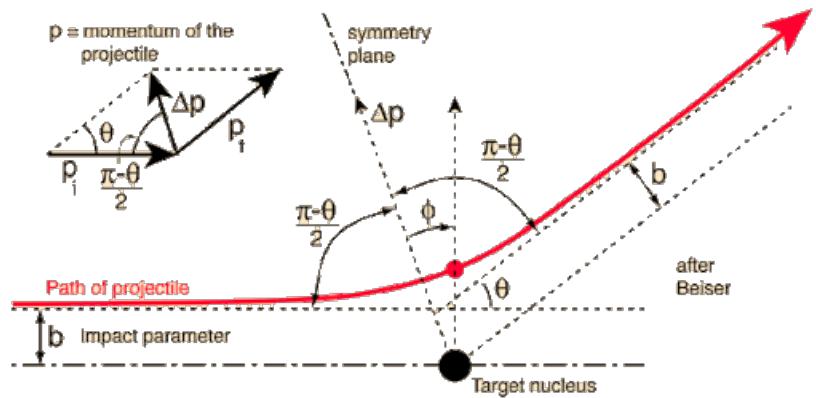
Total energy loss per unit length



Elastic scattering with nuclei

- A charged particle traversing a medium is deflected many times mainly by small angles
- Each deflection is due mainly to the Coulombian interaction of the particle and a nucleous
- Energy transferred to the nucleous is small and neglected
- A single collision is described by Rutherford's formula (ignores spin and screening effects)

$$\frac{d\sigma}{d\Omega} = 4zZr_e^2 \left(\frac{m_e c}{\beta p} \right)^2 \frac{1}{\sin^4 \theta/2}$$



- In multiple scattering typically more than 20 collisions take place during the trajectory
 - The particle follows a zig-zag trajectory of statistical nature
 - A global deflection angle given by Moliere's scattering

Multiple scattering

- The multiple scattering produces a global angular deflection and an orthogonal displacement

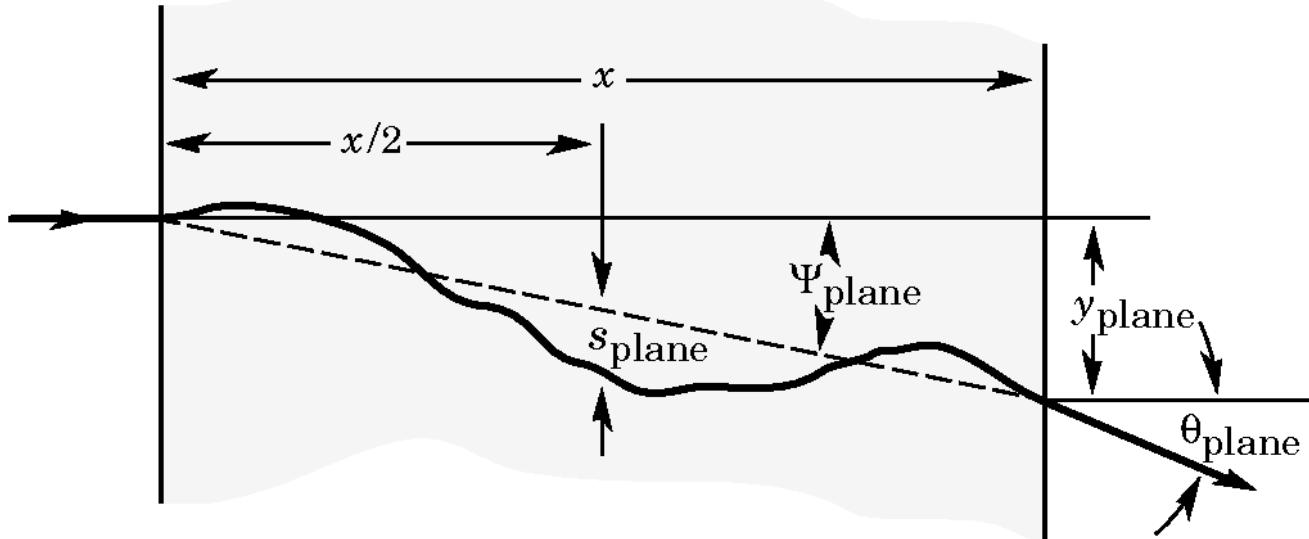


Figure 27.9: Quantities used to describe multiple Coulomb scattering.

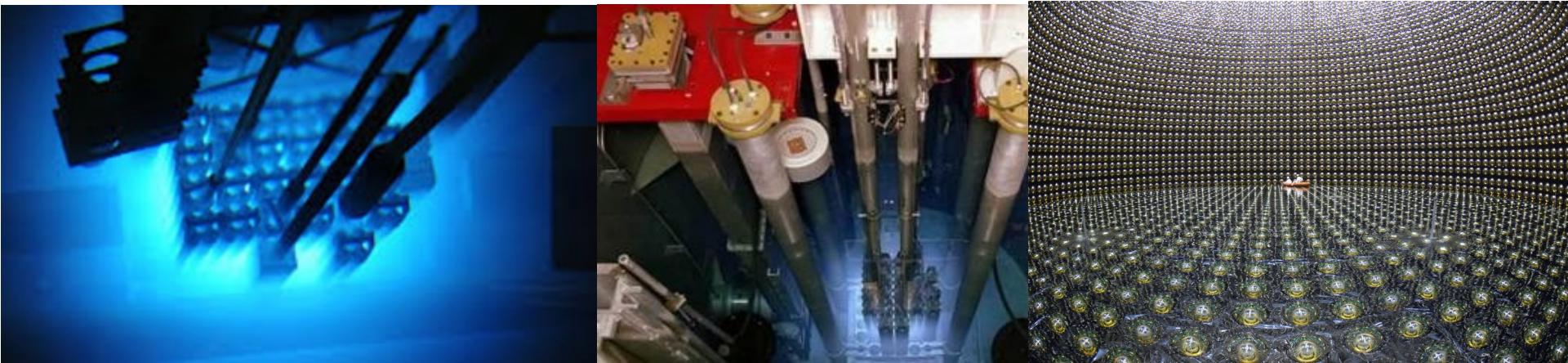
- For small deflection angles the distributions of deflections are approximately Gaussian

$$\sigma(\theta_{\text{plane}}) = \frac{13.6 \text{ MeV}}{\beta p} Z \sqrt{\frac{x}{X_0}} \left(1 + 0.038 \frac{t}{X_0} \right)$$

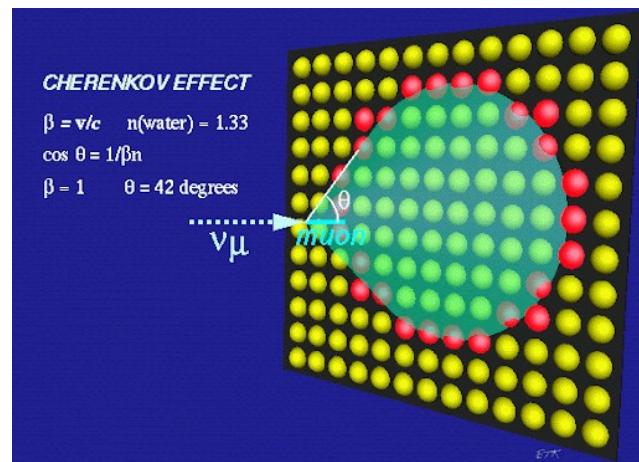
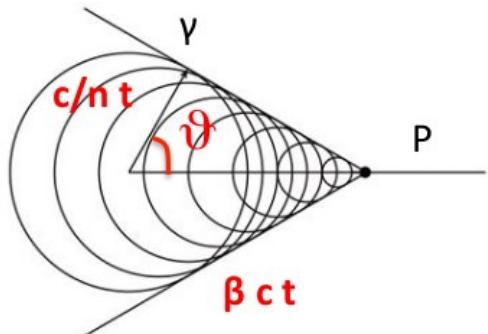
$$\sigma(y_{\text{plane}}) = \frac{x \sigma(\theta_{\text{plane}})}{\sqrt{3}}$$

Cherenkov light emission

- › Radiation emitted by a charged particle when it travels faster than light inside a medium



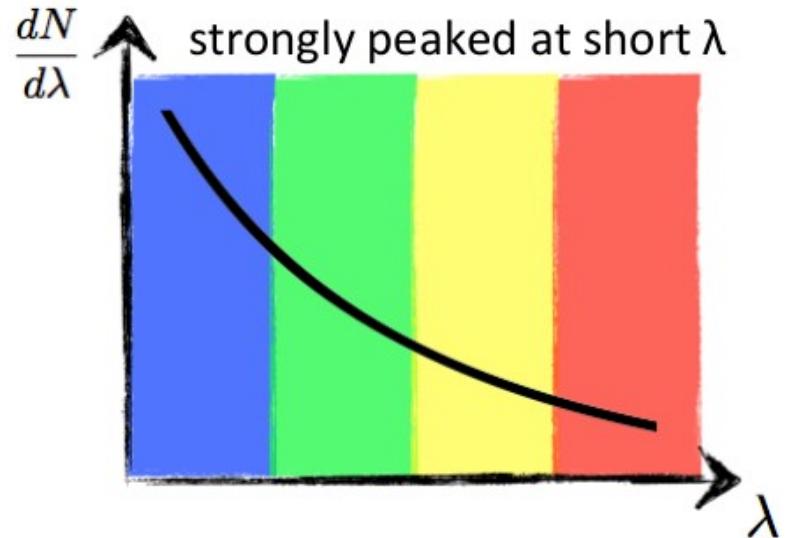
- › The particle electric field polarizes the medium while going through it
 - If $v > c/n$ the light is emitted coherently in a cone behind the particle



Cherenkov light emission

- The number of photons emitted per unit of length path and unit of wave length is

$$\frac{dN}{dx d\lambda} = 2\pi \alpha \frac{1}{\lambda^2} \left(1 - \frac{1}{\beta^2 n^2(\lambda)}\right) z^2$$



- The number of photons per unit path length

$$\frac{dN}{dx} = 2\pi \alpha z^2 \int_{\beta n > 1} \left(1 - \frac{1}{\beta^2 n^2}\right) \frac{d\lambda}{\lambda^2}$$

- Assuming n is constant over the wavelength region detected

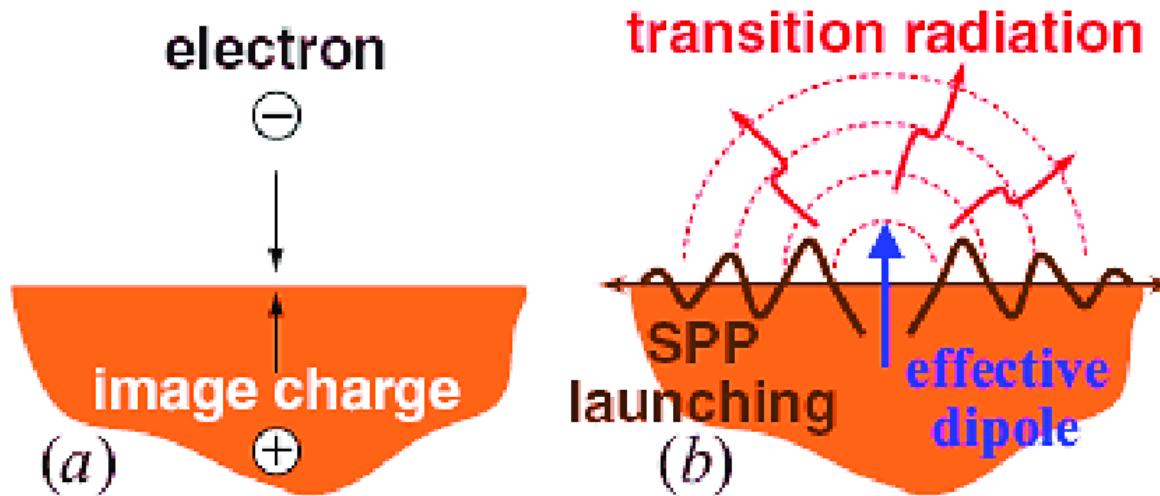
$$\lambda = 350-500\text{nm}$$

$$\frac{dN}{dx} = 2\pi \alpha \sin^2 \theta \left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right) z^2 \longrightarrow \frac{dN}{dx} = 390 \sin^2 \theta \text{ photons/cm}$$

- Loss of energy due to Cherenkov radiation is very small compared to ionization and much weaker than scintillation output so it's usually neglected

Transition radiation

- When a relativistic charged particle crosses a boundary between two materials with different dielectric properties, radiation is emitted, mostly in the form of X-rays
- This effect appears because the electric field generated by the particle is different at both sides



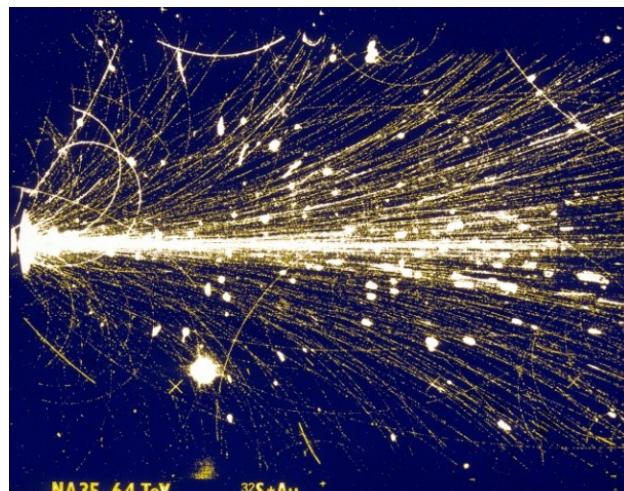
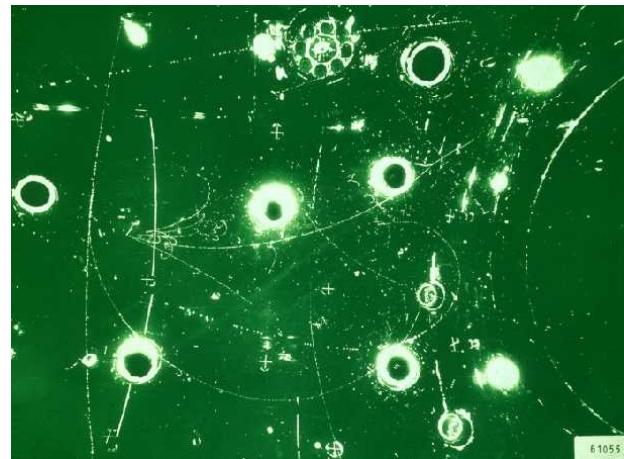
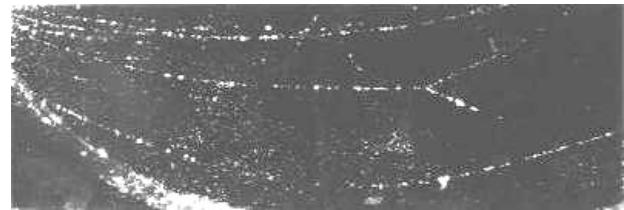
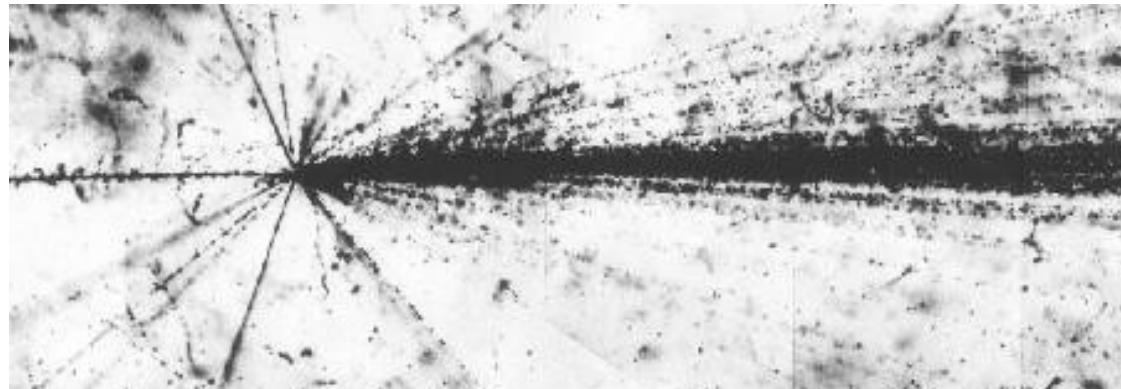
- The radiation is emitted in a cone with $\cos(\theta) = 1/\gamma$
- The probability of radiation per transition surface is low $\sim 1/2 \alpha$ (fine structure constant)

Particle detectors: the ideal detector

- Our ideal particle detector should have the following good properties
 - Good particle identification
 - Precise measurement of the energy and/or the momentum
 - Precise measurement of the trajectory of the particle
 - Coverage of the full solid angle (hermeticity)
- But from a practical and experimental point of view it should also:
 - Be able to take data at a high rate of particles
 - Cope with high particle densities (many particles at the same time)
 - Survive high radiation doses (so they can be installed in colliders, for instance)
 - Survive for a few years of operation (~10 años) without (little) intervention
 - Cheap

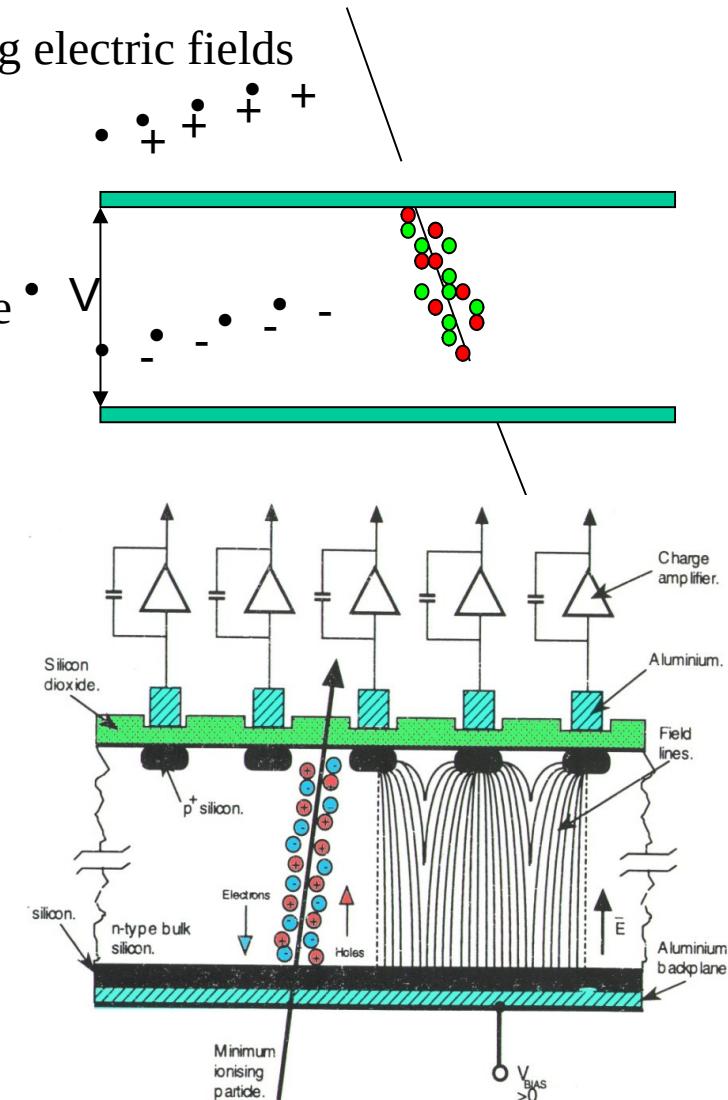
Basic detection techniques: Photographic techniques

- Charged particles ionize the atoms along their trajectory
- Ions can act as seeds for the following processes:
 - Condensation in super saturated gases (Wilson chamber)
 - Bubble formation in super heated liquids
 - Electrical discharge or plasma formation
- Ions can also be made visible chemically
 - Photographic emulsion targets



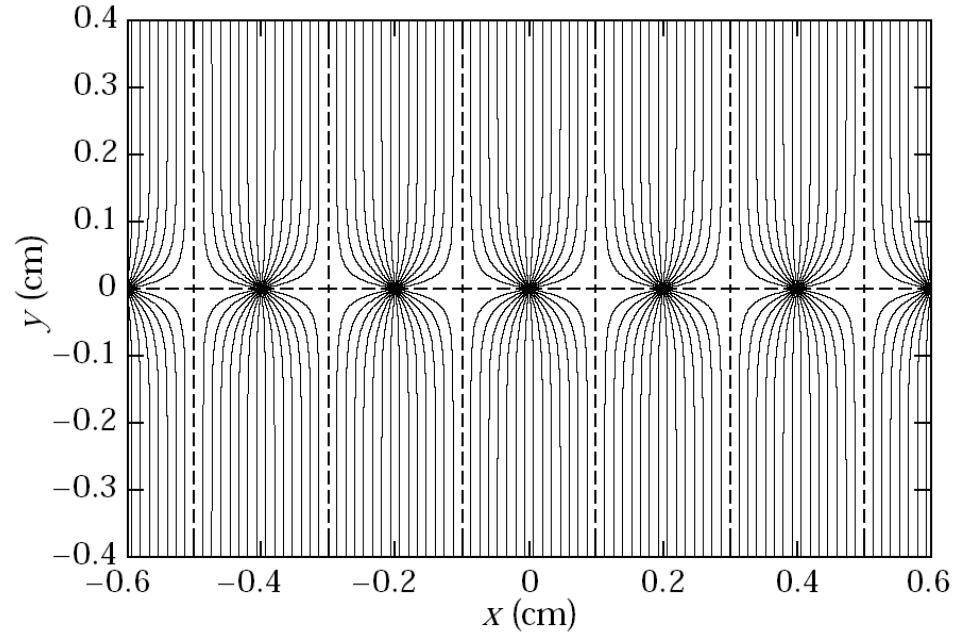
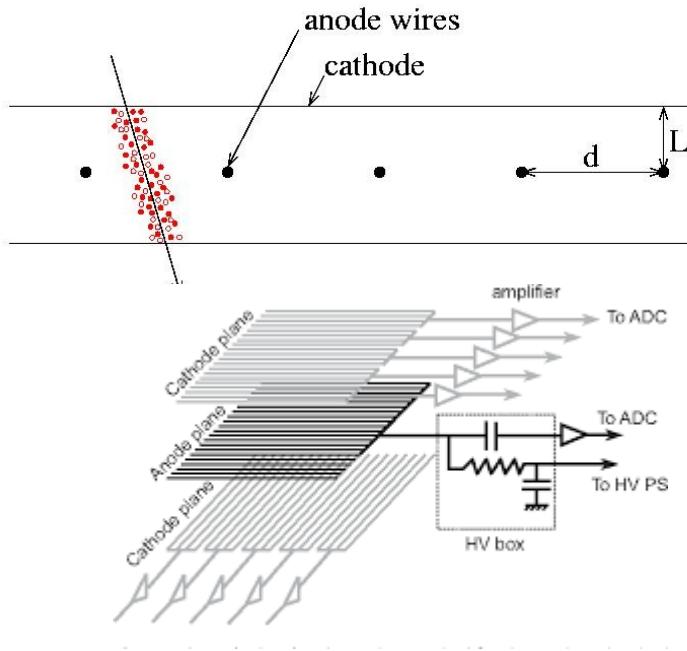
Basic detection techniques: Electrical

- The charge produced by the ionization can be collected electrically
 - The charge generated is conducted and collected using electric fields
- The requirements are the following:
 - The active material should be not conducting
 - There should be a mechanism to transport the charge
- One family of these detectors are the gas/liquid based:
 - Insulating gas or liquid between anode and cathode
 - Sometimes with very low conductivity solids
 - Proportional chambers, drift tube chambers, etc
- Another family are silicon based detectors:
 - Active material is made of semiconductor
 - Usually a reversely polarized p-n junction
 - CCDs, strip detectors, LGADs, etc



An example: Multiwire Proportional Chamber

- Electrons/ions drift along field lines (ions drift to the cathode and electrons to the wire)



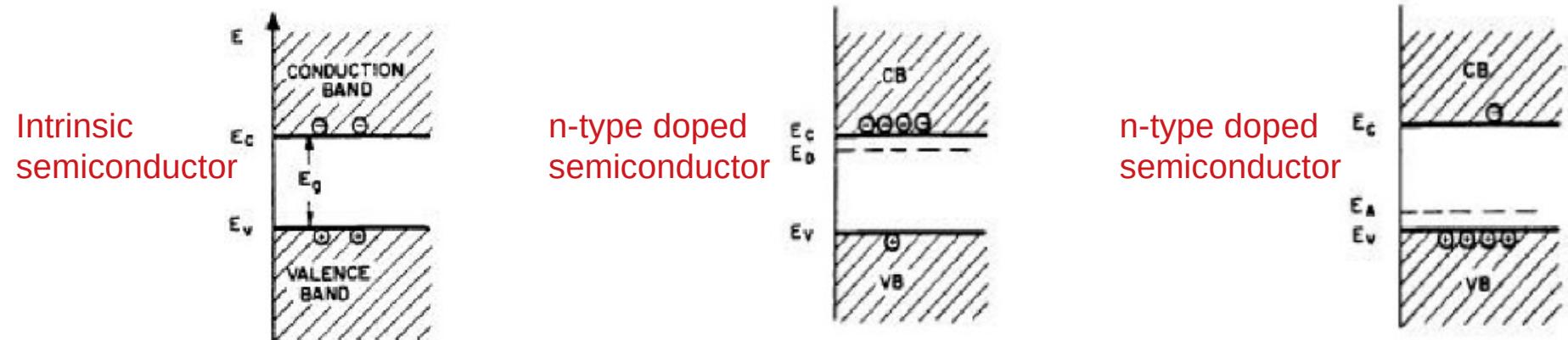
- These devices are sensitive to only one coordinate → usually stacked with 90° rotations
- If all the charge is collected in one wire the resolution is determined by the wire spacing
- Charge sharing among wires → better resolution

$$\sigma_x = \frac{d}{\sqrt{12}}$$

A graph showing the resolution σ_x as a function of the wire spacing d . The x-axis is labeled d and has a double-headed arrow indicating its range. The y-axis is labeled σ_x . A yellow-shaded bell-shaped curve represents the resolution distribution, which is centered at zero and has a width that decreases as d increases. The formula $\sigma_x = d/\sqrt{12}$ is written next to the curve.

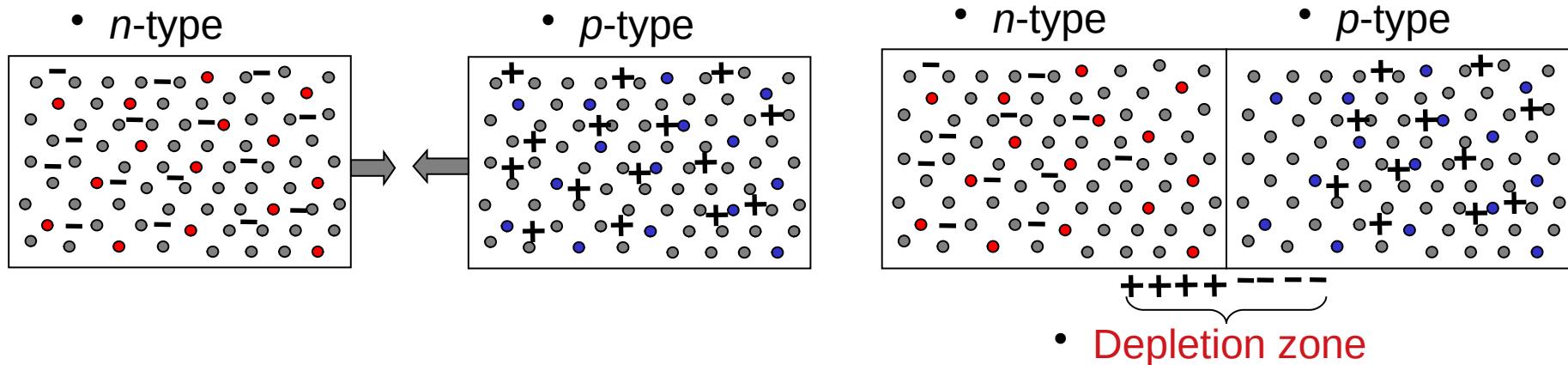
An example: Silicon p-n diodes (I)

- Semiconductor materials have a very particular energy band structure:
 - The valence and the conduction bands are relatively close together
 - The conduction band is empty but thermal excitations can easily populate it
- A particle crossing the semiconductor ionizes moving states from valence to conduction
 - On average a MIP produces about 10000 electron/hole pairs every 100 microns
- The problem is that in the conduction band there are other states promoted thermically
 - We can get rid of those by using the properties of “n-type” and “p-type” doping
 - A semiconductor doped with donor atoms (n-type) easily donate electrons to CB
 - A semiconductor doped with acceptor atoms (p-type) easily accept electrons from the VB

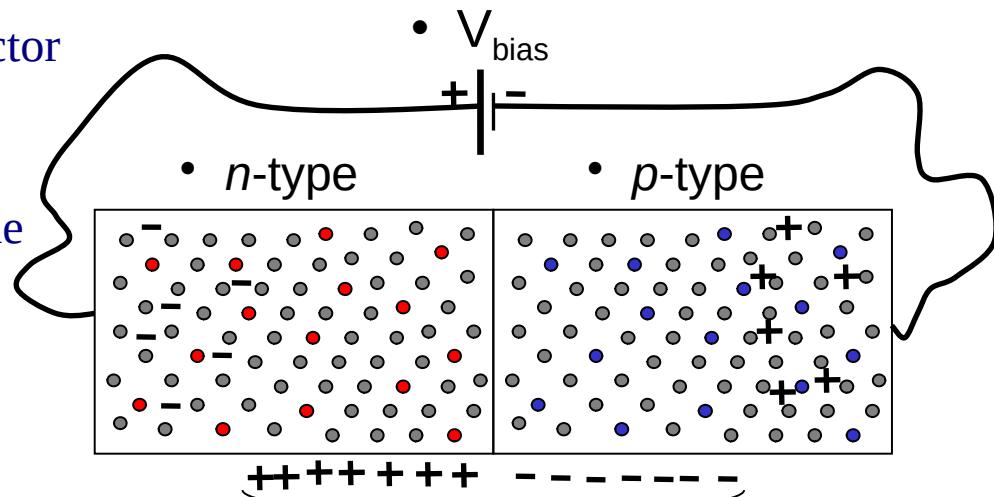


An example: Silicon p-n diodes (II)

- In a p-n junction there is a diffusion process of electrons and holes creating an electric field

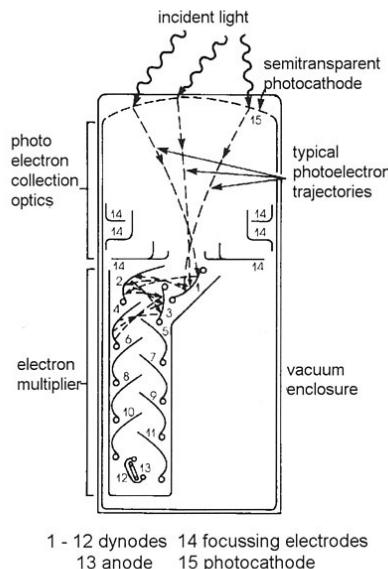


- The electric field removes any free charge carriers creating the so called “depletion zone”
- The size of the depletion zone can be increased by adding a reverse bias voltage
- The depletion zone acts as a particle detector
- The carriers created by a crossing particle will produce a detectable current due to the electric field



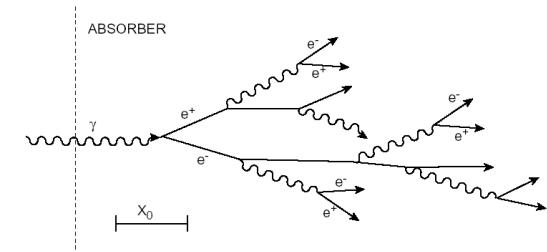
Basic detection techniques: photo-detection

- Charged particles can produce light through several mechanisms
 - Bremsstrahlung (combined with pair production) → scintillators
 - Cherenkov light
 - Transition radiation
- The light produced is collected with photomultipliers
 - Photo-multiplier tubes
 - Semi-conductor based (photo-diodes)



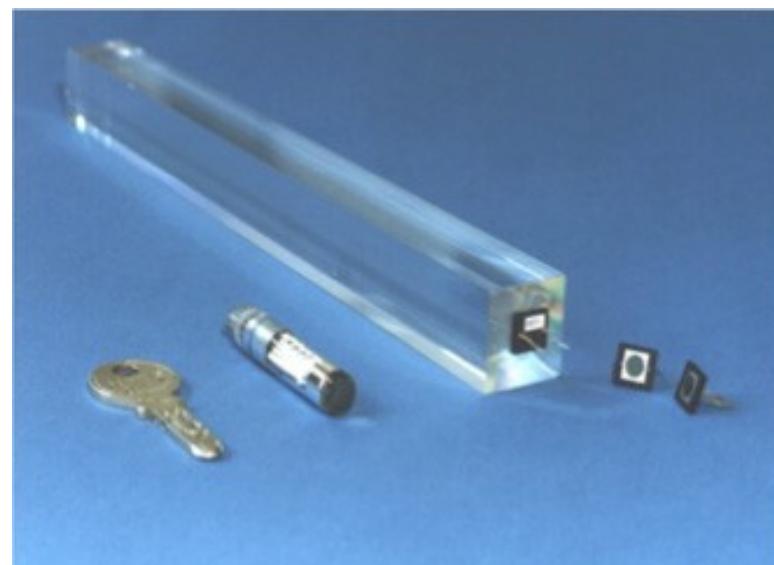
An example: scintillator crystals

- Charged particles crossing dense materials produce photons via Bremsstrahlung
- High energy photons crossing dense materials create e^-e^+ pairs
- In dense crystal scintillators this process occurs in a sort of chain reaction
 - Producing a so-called electromagnetic shower
 - It stops when the energy of the photons is $< 2m_e$
- The crystals conduct the light towards the photomultiplier
- In modern detectors photomultiplier are based on avalanche photodiodes technology



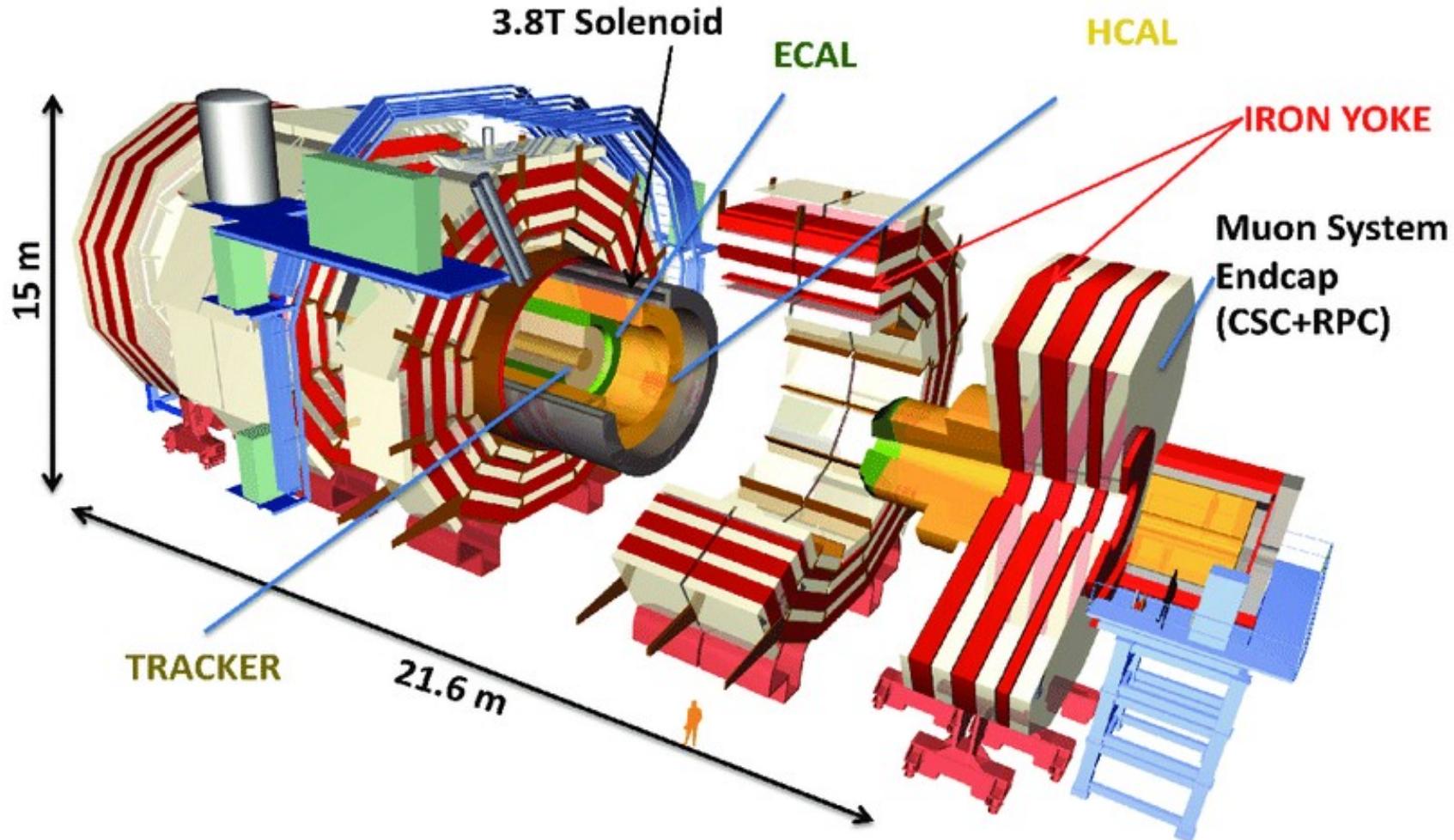
- **CMS ECAL: 80,000 PbWO₄ crystals**
- **Avalanche photodiodes measure scintillation light**

$$\frac{\sigma(E)}{E} \approx \frac{2.7\%}{\sqrt{E}} \oplus \frac{0.2}{E} \oplus 0.55\%$$

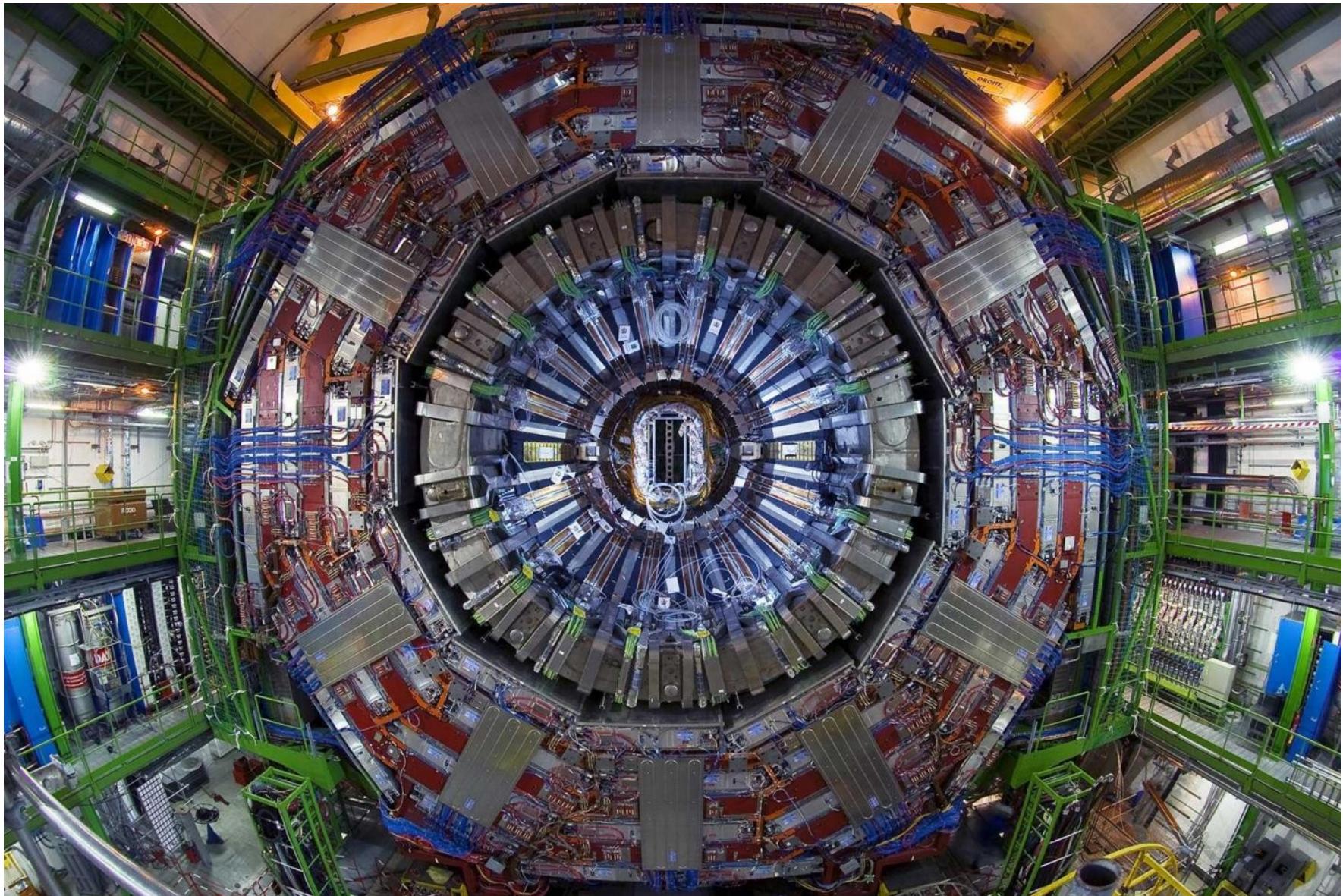


Large Particle Detectors (I)

- Modern detectors use several technologies to detect different kinds of particles

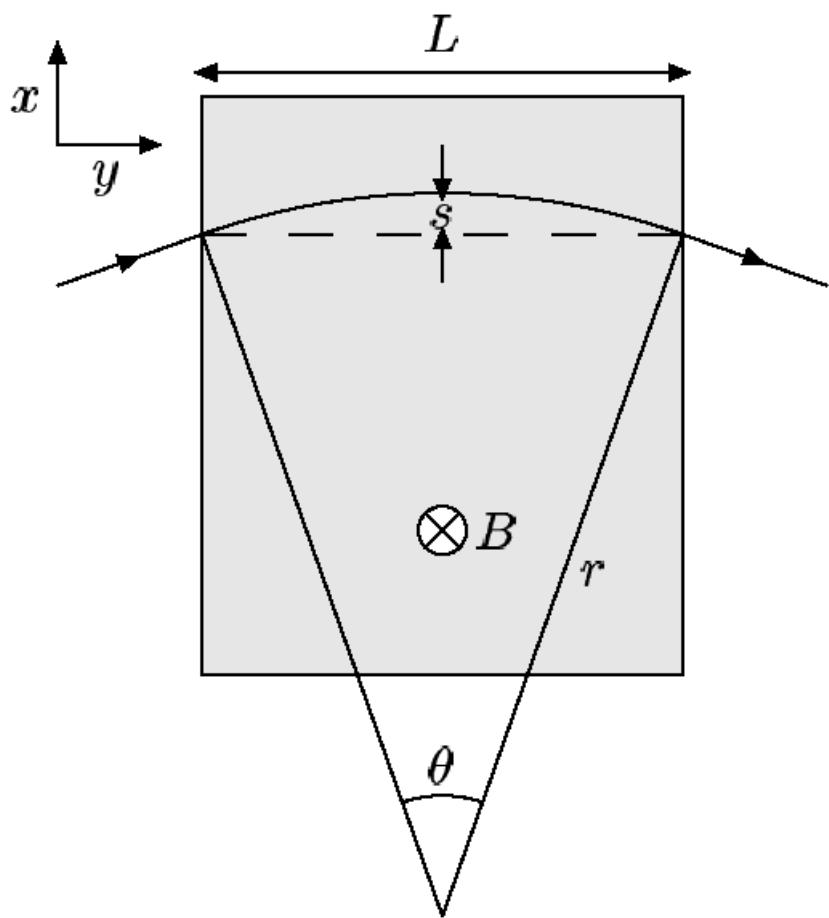


Large Particle Detectors (II)



Large Particle Detectors: momentum measurement

- The momentum of charged particles in large detectors is measured using magnetic fields
 - Charge particles bend in a magnetic field → the more momentum the less bending



Magnetic force: $\vec{F} = q(\vec{v} \times \vec{B})$ or $|F| = qv_{\perp}B$

Centrifugal force: $|F| = \frac{mv_{\perp}^2}{r}$

Radius: $r = \frac{mv_{\perp}}{qB} = \frac{P_{\perp}}{qB}$,

$$r = \frac{P_{\perp}}{0.3B} \quad (P_{\perp} \text{ in } GeV/c \text{ and } B \text{ in Tesla})$$

Bending angle: $\frac{\theta}{2} \approx \sin \frac{\theta}{2} = \left(\frac{L/2}{r} \right) \Rightarrow$

$$\theta \approx \frac{0.3BL}{P_{\perp}}$$

Sagitta:

$$s = r - r \cos \frac{\theta}{2} = r \left(1 - \cos \frac{\theta}{2} \right) \approx r \frac{\theta^2}{8} \approx \frac{0.3}{8} \frac{BL^2}{P_{\perp}}$$

Large Particle Detectors: particle identification

- Information from different sub-detectors can be mixed to identify the particles

