Variable Key	Variable Description	Variable Category
BUR	Burger	Meal
TAC	Taco	Meal
TS	Texas Sized	Size
NTS	Not Texas Sized	Size

Marginal Probability	NTS Price	TS Price	Probability
BUR	8	10	0.40
TAC	9	11	0.60

P(Meal)

Conditional Matrix	NTS	TS
BUR	0.30	0.70
TAC	0.75	0.25

P(Size | Meal)

Each row in a conditional matrix always sum to 100%. This is a good check.

Joint Matrix	NTS	TS	P(Meal)
BUR	0.12	0.28	0.40
TAC	0.45	0.15	0.60
P(Size)	0.57	0.43	

P(Size & Meal) = P(Size | Meal) * P(Meal)

Each intersection in this joint matrix is calculated using the marginal probabilities and conditional matrix given above.

These marginal probabilities match those given in the problem setup.

These marginal probabilities represent P(Size).

Useful in downstream application of Bayes' Theorem.

Expected Value of a Customer	NTS	TS	
BUR	0.96	2.80	
TAC	4.05	1.65	
			9.46

Customer Volume		139
Expected Total Revenue	$	1,314.94

P(BUR | TS)	P(TS | BUR) * P(BUR) / P(TS)	0.651

Bayes' Theorem used here.

Conditional Matrix	BUR	TAC
NTS	0.211	0.789
TS	0.651	0.349

P(Meal | Size)

P(Meal | Size) = P(Size | Meal) * P(Meal) / P(Size)

Each row in a conditional matrix always sum to 100%. This is a good check.
Note that we could use Bayes' Theorem four times to solve each of the four values in the matrix <u>OR</u> since each row has two values, we could apply Bayes' Theorem to get one value per row and solve for the missing value in each row by calculating --> (100% - solved value).

<u>Issues I'm having understanding the solution to Question 6 in Problem Set #3…</u>

I may not completely understand the wording of the question. However, given what I think the question is asking, my issues are…

1. Why should the customer returning the next day impact the original marginal probabilities of ordering a Burger (40%) or Taco (60%). If this were true, wouldn't this violate the assumption of independence between what any two customers order? Even if it's the same customer, intuitively I would think that subsequent orders by that customer should still be independent. However, let's say that the marginal probabilities did change – as suggested by the solution – then how can the original conditional matrix P(Size | Meal) be used with the revised marginal probabilities for P(Meal)?

2. I think the question is driving at figuring out the probability P(Size = Texas Sized). If this were the case isn't the P(Size = Texas Sized) simply equal to 43% - given the assumption of independent events? Algebraically, however, the solution is calculating this marginal probability by multiplying two conditional probabilities together as follows…

$$65.116\% \times 70\% + (1 - 65.116\%) \times 25\% = 54.302\%$$

$$P(\text{BUR} \mid \text{TS}) \times P(\text{TS} \mid \text{BUR}) + P(\text{TAC} \mid \text{TS}) \times P(\text{TS} \mid \text{TAC})$$

The problem I'm having is that algebraically I don't think this solve for the marginal probability P(Size). If the second conditional matrix (shown above) is derived by the applying Bayes' Theorem to the original marginal probabilities (of both Meal and Size) and the original conditional matrix P(Size | Meal), then how can any of the marginal probabilities change? If they did then wouldn't this invalidate the original conditional matrix and lead to a circularity that would make it impossible to apply Bayes' Theorem?