Setting Hyperparameters

Iterations

- Increase if both train & dev losses are decreasing.
- Stop if dev loss has saturated.
- Decrease if dev loss starts increasing.

Learning Rate

- Increase if train loss is not changing or decreasing fast enough.
- Decrease if train loss keeps fluctuating or starts increasing.

Hidden Nodes

- Increasing it makes harder and longer for the model to train.
- Increase when train loss saturates at a high value.
- Decrease if train loss decreases very slowly or keep fluctuating even after adjusting learning rate.

Sentiment Analysis

classify imdb reviews as POS or NEG.

Project 1:

```
from collections import Counter
positives = Counter()
positives[token] += 1 → used like a dict()
positives.most_common()
```

Project 2:

allocating memory is expensive. It's better to allocate less frequently. `np.zeros((1, vo_s))`

Project 3:

- it's useful to monitor "correct so far" samples as the model trains.
- lowering the learning rate might be useful if the network is not learning anything.

Project 4:

- Weight initialization strategy matters!
- one strategy: `np.random.normal`
- a rule of thumb:
  it's good practice to start weights in range $[-y, y]$ where $y = 1/\sqrt{n}$.
  (n = number of inputs to a given layer)
- noise versus signal in NN;
  value of the inputs of the network, affects weights a lot. If one of them is 18, and the other is 2, the one with value = 18 will dominate highly.
- when we tokenize the data, we should look out for meaningless stuff such as punctuation and common / filler words. Because they are not contributing to the prediction. i.e. noise

Project 5:

- How to improve efficiency of computation?
  i.e., what is wasteful in the network?
  1. long input vector: if many of them are zero, we're doing a useless mat mul.
  2. if we have "1" in the input vector, we should not do mult.

Project 6:

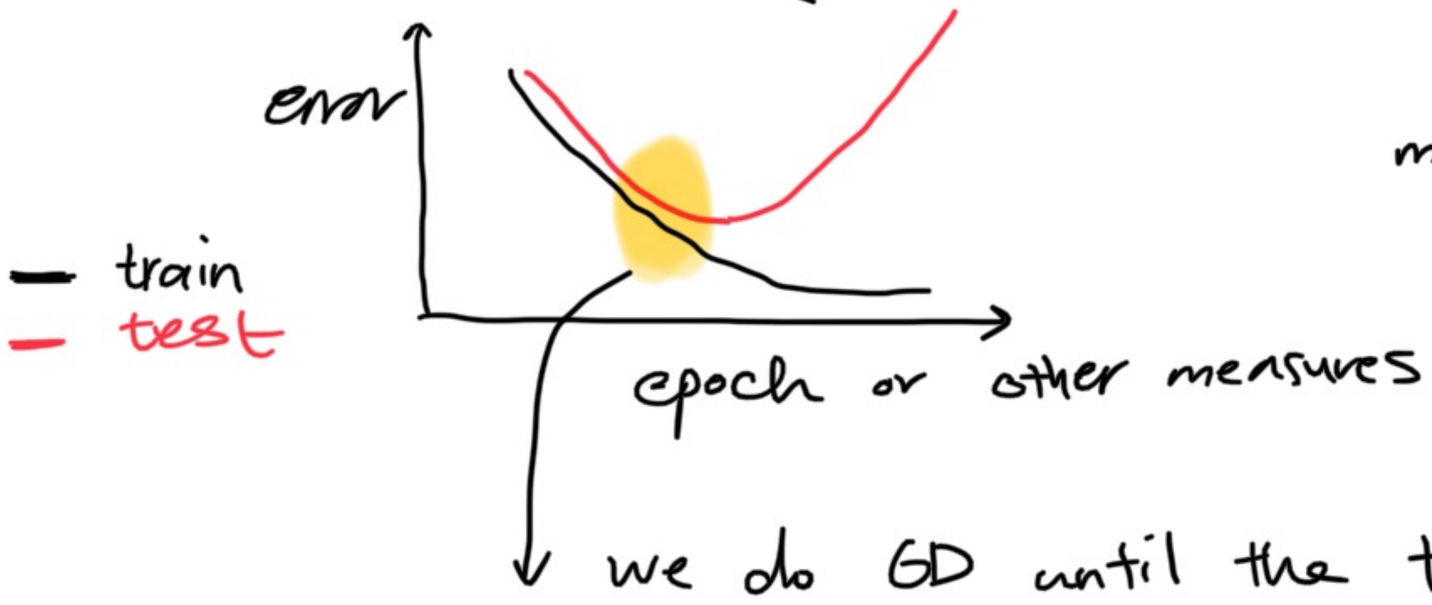
if we strategically reduce vocabulary size, we may improve the accuracy.

Some keywords

underfitting = error due to bias (high bias)

overfitting = error due to variance (high variance)

$$\text{indicator} = \begin{cases} \text{test error : large} \\ \text{train erron : tiny} \end{cases} \longrightarrow \text{overfit}$$



model complexity graph

we do GD until the testing error is decreasing. Once it starts to increase, we stop GD. ⟶ Early Stopping

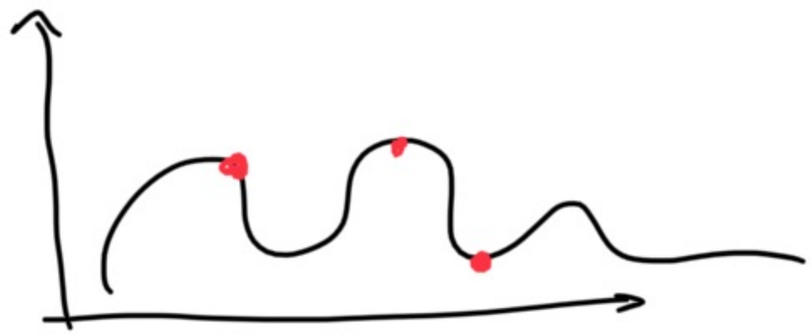
Batch Gradient Descent → for all the data

Stochastic Gradient Descent → make small batches of data

Learning Rate Decay:

rule of thumb: if the model is not working increase the rate.

Random restart to solve local minima



fixing Vanishing Gradient problem:

1. Changing the activation function. tanh, ReLU

Momentum to solve local minima:

β : momentum $\in (0, 1)$

$$step(t) = step(t) + \beta\, step(t-1) + \beta^2\, step(t-2) \dots$$

↳ intuitively it gets us out of local minima.

Dropout

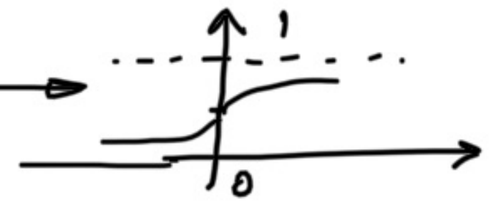
Some
Sometimes, weights in NN are too high that other parts of the model does not train at all.

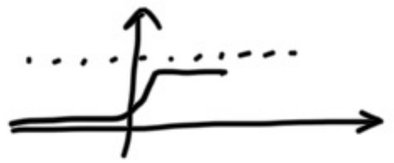
the dropout strategy randomly turns off some nodes during the training

dropout (0.2) → 0.2 = probability that each node will be turned off.

Regularization

example: $\hat{y} = \sigma(wx_1 + wx_2 + b)$ prediction

if $w = 1$ → 

if $w = 10$ → 

too certain

large coefficients

solution

In regularization, we want to penalize large weights.

good for feature selection ← $\text{Error} + \lambda(|w_1| + \cdots + |w_n|)$ l1-regularization

usually good for traing models ← $\text{Error} + \lambda(w_1^2 + \cdots + w_n^2)$ l2-regularization

Perceptron Algorithm

1. start with random weights $w_1, \dots, w_n, b$
2. for every misclassified point $(x_1, \dots, x_n)$:

if prediction == 0:

for $i = 1 \dots n$: update $w_i = w_i + \alpha x_i$;

update b to $b + \alpha$

if prediction == 1:

for $i = 1 \dots n$: update $w_i = w_i - \alpha x_i$;

update b to $b - \alpha$

* Perceptron algorithm has to be generalized for non-linear stuff.

* Error function = Distance : how far are we from the solution?

Gradient Descent:

minimizing the error.

error function must be differetiable

Continuous

So, how do we go from discrete to Continuous?

discrete pred $\xrightarrow[\text{function}]{\text{activation}}$ continuous pred

(σ)

step func $\longrightarrow$ Sigmoid func

$$\frac{1}{1+e^{-x}}$$

$Wx+b$ gives us $\hat{y}$. we have to apply $\sigma()$ to $\hat{y}$.

when the score is zero, probability is 50%.

Multi layer Perceptron

weight matrix

hidden inputs : np.dot (inputs, weights_inp_to_hidden)

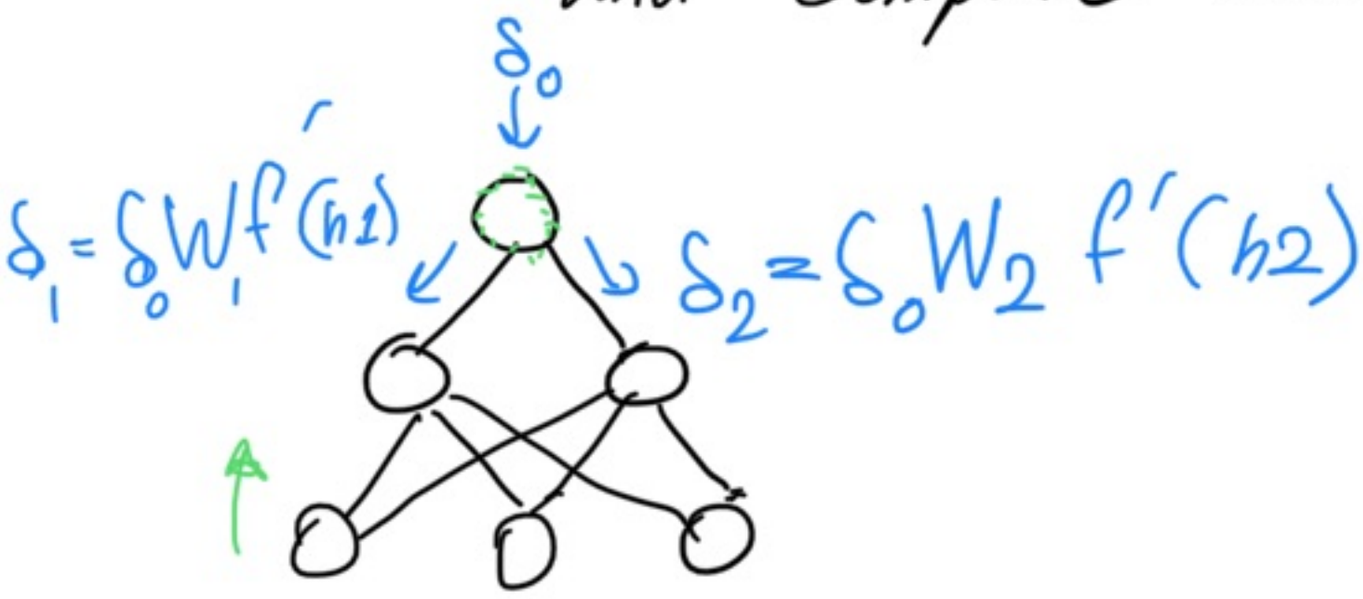
Some numpy tips:

for creating a column vector: data[:, None]

for transposing a matrix: data.T

* Backpropagation

same as forward but we assume the output layer is the input and compute each layer's error.



Squared Errors

$$E = \frac{1}{2} \sum_{\mu} \sum_{j} \left[y_j^{\mu} - \hat{y}_j^{\mu} \right]^2$$

(SSE)

sum of squared errors

μ → data points

j → output units

y_j^{μ} → true value

$\hat{y}_j^{\mu}$ → prediction → depends on w_{ij}'s

Gradient is another word for slope or rate of change.
Gradient descent can get into "local minima".
One way to solve is "momentum".

error term: $\delta = (y - \hat{y}) f'(h)$

↳ activation function

$$w_i = w_i + \eta \delta x_i$$

$$h = \sum w_i x_i$$

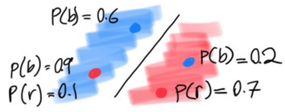
Mean Squared Error (MSE)

$$E = \frac{1}{2m} \sum_{\mu} (y^{\mu} - \hat{y}^{\mu})^2$$

Maximum Likelihood

Pick the model that gives the existing labels the highest probability.
So we try to maximize this probability.

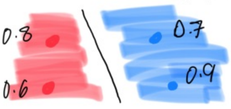
Example:



$\hat{y} = \sigma(Wx+b) = P(\text{blue})$

We assume the points are iid:

$P(\text{all}) = \underbrace{0.1 \times 0.7}_{\text{red dots}} \times \underbrace{0.6 \times 0.2}_{\text{blue dots}} = 0.0084$ ↓ small



$P(\text{all}) = 0.3024$ better than the other model.

Maximum likelihood tries to maximize this.

When dealing with maximum likelihood we prefer sums to products. → $\log() == \ln()$

$\ln(x) < 0$ if $x \in [0,1]$. So we use $-\ln(x)$.

Cross Entropy of good models is smaller.
we can think of it as error

Error Function = Cross Entropy

So we now change the goal to minimizing cross entropy.

Probabilities

	G	R	B
P(gift)	0.8	0.7	0.1
P(no gift)	0.2	0.3	0.9

$0.8 \times 0.7 \times 0.9$

most likely event? choosing the largest prob. in each column.

$$\text{Cross Entropy} = -\sum_{i=1}^{m} y_i \ln(P_i) + (1-y_i)\ln(1-P_i)$$

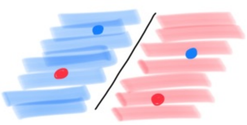
it can tell us how close two vectors are. e.g., $CE[(1,1,0),(0.8,0.7,0.1)] = 0.69$
$CE[(0,0,1),(0.8,0.7,0.1)] = 5.12$ → likely to happen

Multi-class Entropy:

$$CE = -\sum_{i=1}^{n}\sum_{j=1}^{m} y_{ij} \ln(P_{ij})$$

n: number of classes; m: probability of i and j

y_{ij}: 0/1 to indicate which event we are talking about.



if $y=1 \rightarrow P(\text{blue}) = \hat{y} \rightarrow \text{Error} = -\ln\hat{y}$

if $y=0 \rightarrow P(\text{red}) = 1-P(\text{blue}) = 1-\hat{y} \rightarrow \text{Error} = -\ln(1-\hat{y})$

$$\text{Error function: } \frac{-1}{m}\sum_{i=1}^{m}(1-y_i)(\ln(1-\hat{y}_i)) + y_i \ln\hat{y}_i$$

Same as:

binary classification:

$$E(W,b) = \frac{-1}{m}\sum_{i=1}^{m}(1-y_i)\ln(1-\sigma(Wx^i+b)) + y_i \ln(\sigma(Wx^i+b))$$

Multi-class classification:

$$\frac{-1}{m}\sum_{i=1}^{m}\sum_{j=1}^{n} y_{ij} \ln\hat{y}_{ij}$$

Multi-class Classification

We use softmax instead of sigmoid. e.g.,

class1 → score = 2 → $2/(2+1+0)$

class2 → score = 1 → $1/(2+1+0)$

class3 → score = 0 → $0/(2+1+0)$

but it doesn't work for negative scores.

↓

to solve the issue, we make all the score positive.

instead of $2/(2+1+0)$ we have $e^2/(e^2+e^1+e^0)$.

Softmax function ——→

it converts score to probability.

$$\text{probability}(\text{class } i) = \frac{e^{z_i}}{\sum_n e^{z_j}}$$

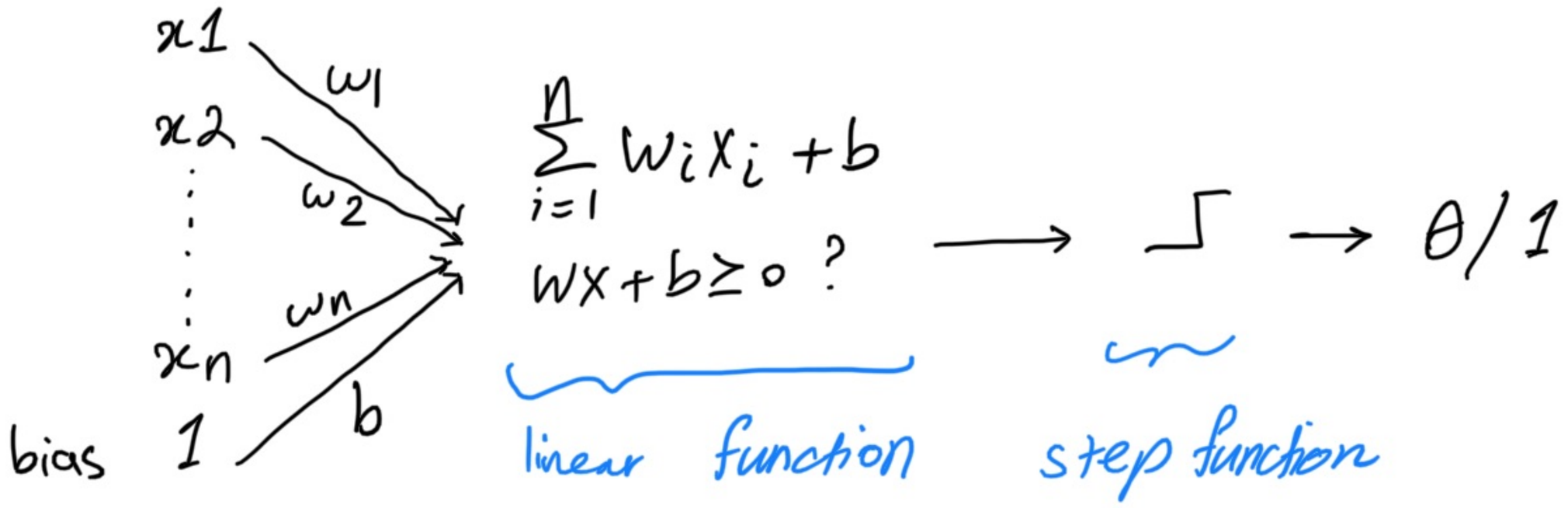
$z_1 \cdots z_N$ are scores.

One-hot encoding:

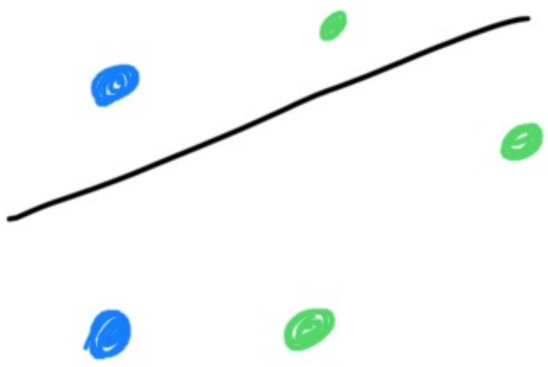
if 3 classes → | c_0 | c_1 | c_2 |
|---|---|---|
| 0 | 0 | 1 |

when label is c2.

Perceptron

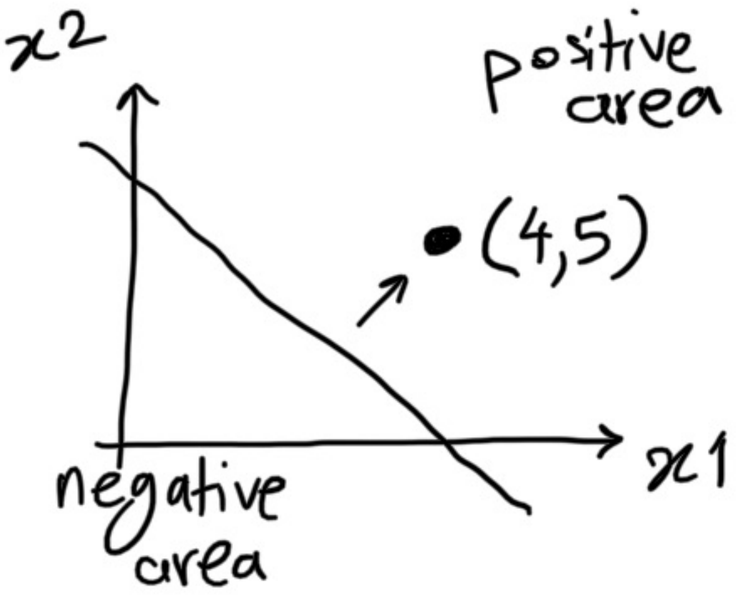


Perceptron Algorithm Goal : split data



→ move this until we get a good split

lines that are misclassified, want the line to come closer.



positive area : $3x_1 + 4x_2 - 10 > 0$

$3x_1 + 4x_2 - 10 < 0$

Line should make small steps towards the line.

But we don't want huge steps.

That's why we use learning rate, e.g., 0.1.

if point was in negative area

point is in pos. area

	3	4	−10	
+	4	5	1	× 0.1
−				

new line : $2.6x_1 + 3.5x_2 - 10.1 = 0$

numpy intro

ndarray → support for math operations

scalars in numpy can be uint8, int8, uint16, ...

example:

- s = np.array(5) → only holds a scalar
- s.shape → () → 0-dim
- x = s+3

- v = np.array([1,2,3])
- v.shape → (3,)
- v[1] → 2, v[1:] → [2,3]

- m = np.array([[1,2,3],[4,5,6],[7,8,9]])
- m.shape → (3,3)

reshaping:

v = np.array([1,2,3,4]) (4,)

X = v.reshape(1,4) (1,4)

element-wise matrix operations:

$$2 + \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 2+1 & 2+2 \\ 2+3 & 2+4 \end{bmatrix}$$

x = np.multiply(some_array, 5) == x = some_array*5

m *= 0 → no matter the dims of m, all of the elements become zero.

m * m → element-wise multiplication

Matrix product:

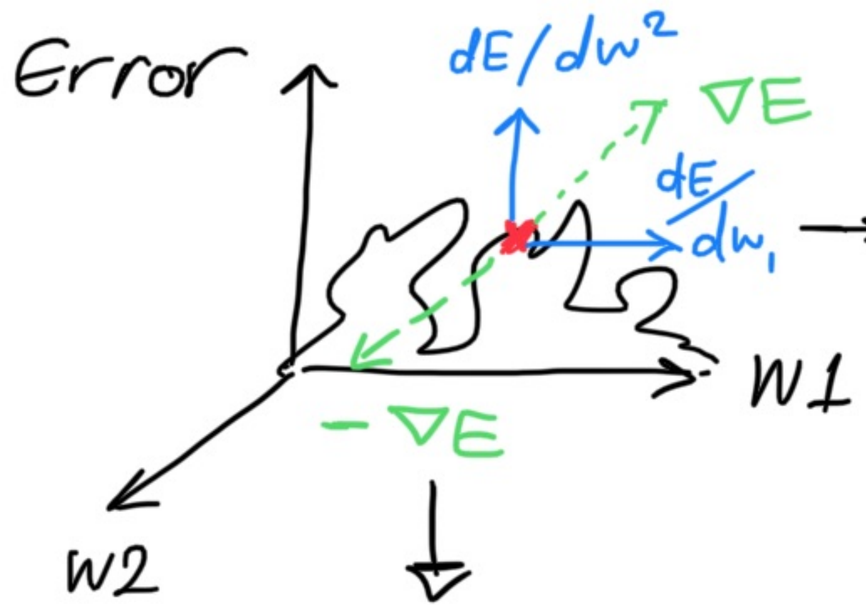
np.matmul(A,B)

$AB \neq BA$

Matrix transpose:

np.transpose(a) or a.T

Gradient Descent



→ minimizing the error means reaching axis w2 as much as we can.

-∇E tells us how to decrease E the most.

In models:

$$\hat{y} = \sigma(Wx+b) \quad \text{Bad}$$

$$\hat{y} = \sigma(w_1x_1 + \cdots + x_nx_n + b)$$

$$\nabla E = \frac{\partial E}{\partial w_1} + \cdots + \frac{\partial E}{\partial w_n} + \frac{\partial E}{\partial b}, \quad \alpha = 0.1 \text{ learning rate}$$

$$W_i' \leftarrow W_i - \alpha \frac{\partial E}{\partial w_i}$$ we want to take small steps.

$$b' \leftarrow b - \alpha \frac{\partial E}{\partial w_i}$$

$$\Longrightarrow \hat{y} = \sigma(W'x + b') \leftarrow \text{better}$$

replacing E

$$\nabla E = -(y-\hat{y})(x_1, \cdots, x_n, 1)$$

↳ gradient at point $(x_1, \cdots, x_n)$ label y

$$\Longrightarrow w_i' \leftarrow w_i + \alpha(y-\hat{y})x_i$$

$$b' \leftarrow b + \alpha(y-\hat{y})$$

Back propagation

do feedforward and get the error
go at the opposite direction.
increase the weights that come from better classification and decrease ones that are worse.

more formally:

$$w_{ij}^{\prime(k)} \leftarrow w_{ij}^{(k)} - \alpha \frac{\partial E}{\partial w_{ij}^{(k)}}$$

Some reminders: chain rule

$A = f(x)$
$B = g \circ f(x)$

$$\frac{\partial B}{\partial x} = \frac{\partial B}{\partial A} \frac{\partial A}{\partial x}$$

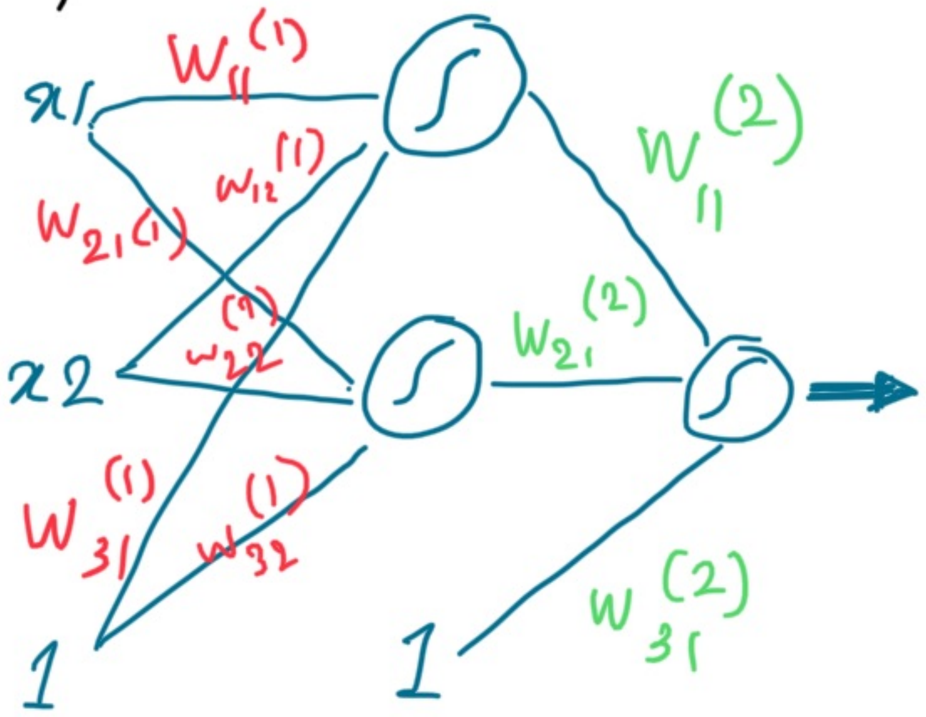
$$\sigma'(x) = \sigma(x)(1-\sigma(x))$$

---

we now have all the ingredients to train neural networks!

Feed forward

The process nn use to turn input into an output.



$$\sigma\begin{pmatrix} w_{11}^{(2)} \\ w_{21}^{(2)} \\ w_{31}^{(2)} \end{pmatrix} \sigma\begin{pmatrix} w_{11}^{(1)} & \cdots & w_{12}^{(1)} \\ \vdots & & \vdots \\ w_{31} & \cdots & w_{32}^{(1)} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ 1 \end{pmatrix} = \hat{y}$$

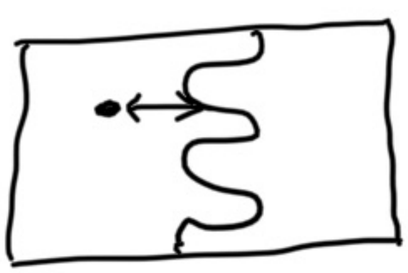
↓ input vector

simply put: $\hat{y} = \sigma \circ W^{(2)} \circ \sigma \circ W^{(1)}(x)$

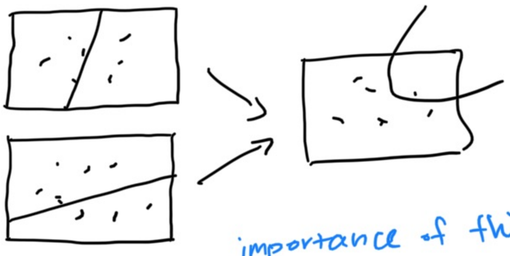
↓ applied to

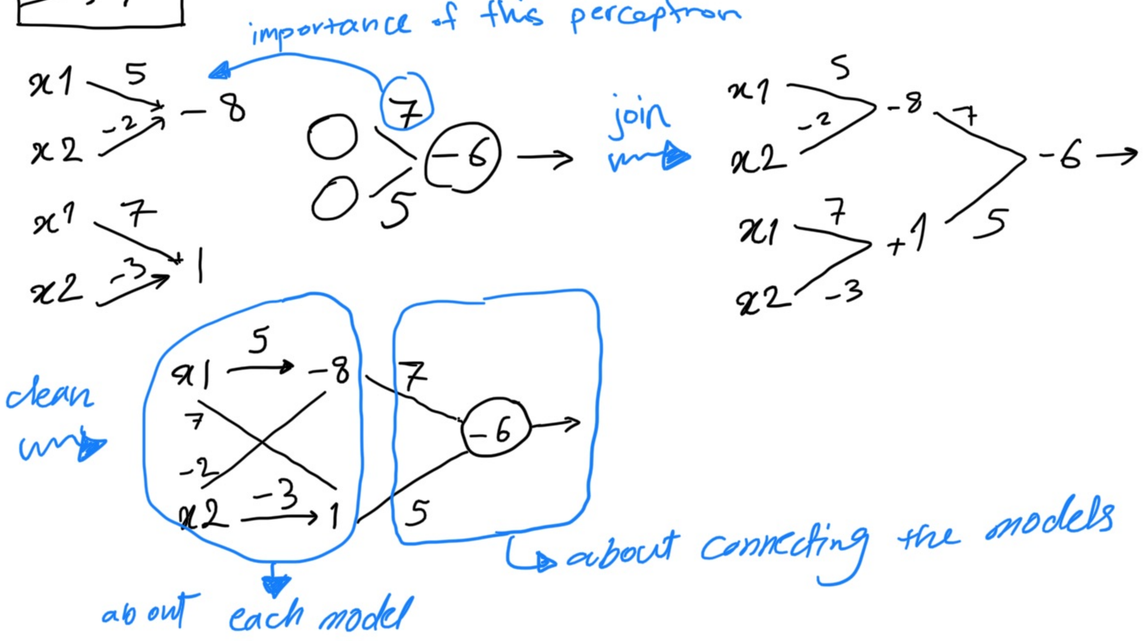
Error function

Similar to single perceptron.

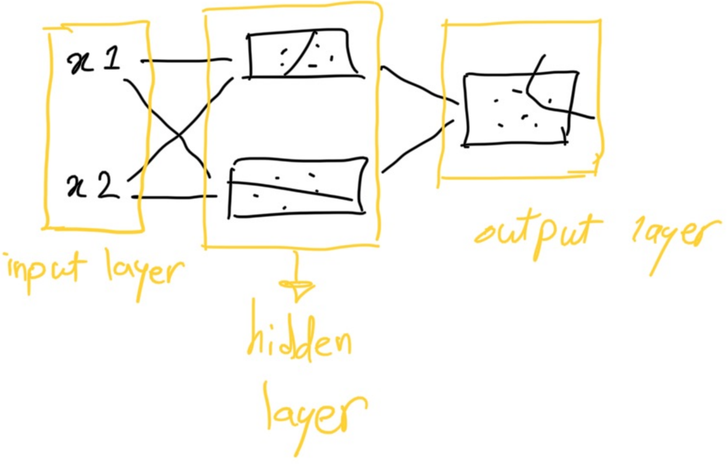


Neural Network Architecture:





Multiple layers:



increasing number of input nodes to n → n-dim data

Deep Neral Networks:

binary classification

Multiclass classification:

class A
class B
class C

option 1 → three biray classification networks
then we can use softmax or max

option 2 → adding more nodes to the output layer!

Gradient Descent Algorithm:

1. start with random weights $w_1, \dots, w_n, b$
2. For every point $(x_1, \dots, x_n)$:

    For $i = 1 \dots n$:

        update $\hat{w}_i \leftarrow w_i - \alpha(\hat{y} - y) x_i$

        update $b' \leftarrow b - \alpha(\hat{y} - y)$
3. Repeat until error is small → #epochs

« Summary »

sigmoid activation function $\sigma(x) = \frac{1}{1 + e^{-x}}$

output (prediction) formula $\hat{y} = \sigma(w_1 x_1 + w_2 x_2 + b)$

Error function $\text{Error}(y, \hat{y}) = -y \log \hat{y} - (1 - y) \log (y - \hat{y})$

Updating weights $w_i \to w_i + \alpha(y - \hat{y}) x_i$

$b \to b + \alpha(y - \hat{y})$

Perceptron v. Gradient Descent

in GDA: change w_i to $w_i + \alpha(y - \hat{y}) x_i$ every time

in PA: only update if misclassified

In fact, they are basically the same!

in GDA: both of <u>come closer</u> & <u>go further away</u>

in PA: only <u>come closer</u>! or do nothing