ME630 Assignment 4

Aman Parekh (180073)

November 20, 2021

$$\boxed{\frac{\partial \omega}{\partial t} + u\frac{\partial \omega}{\partial x} + v\frac{\partial \omega}{\partial y} = \frac{1}{Re}\left\{\frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2}\right\}}$$

After converting the Navier Stokes equation from primitive variables (u, v, p) to Ψ and ω, we arrive at the above equation, which is the **Vorticity Transport Equation**. This is discretized and solved in a time-marching manner using Runge-Kutta 3 Method.

In each step, u and v also have to be updated and this is done using Ψ. Ψ at each timestep is calculated by solving the following elliptic equation using Guass-Siedel:

$$\boxed{\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} = -\omega}$$

<u>DISCRETIZATION STENCILS</u>

1. **RK-3:**

$$\frac{\partial \omega}{\partial t} = -u_{i,j}\frac{\omega_{i+1,j} - \omega_{i-1,j}}{2\Delta x} - v_{i,j}\frac{\omega_{i,j+1} - \omega_{i,j-1}}{2\Delta y} + \frac{1}{Re}\left\{\frac{\omega_{i+1,j} - 2\omega_{i,j} + \omega_{i-1,j}}{\Delta x^2} + \frac{\omega_{i,j+1} - 2\omega_{i,j} + \omega_{i,j-1}}{\Delta y^2}\right\}$$

2. **Guass-Siedel Formulation for Elliptic Solve (Uniform Grid):**

$$\Psi_{i,j} = \frac{1}{4}\left\{\Psi_{i+1,j} + \Psi_{i-1,j} + \Psi_{i,j+1} + \Psi_{i,j-1} + \omega_{i,j}\Delta x^2\right\}$$

3. **Calculating of u and v using central difference:**

$$u_{i,j} = \frac{\Psi_{i,j+1} - \Psi_{i,j-1}}{2\Delta y}$$

$$v_{i,j} = -\frac{\Psi_{i+1,j} - \Psi_{i-1,j}}{2\Delta x}$$

The code is attached with the submission and the following command can be used to run it :

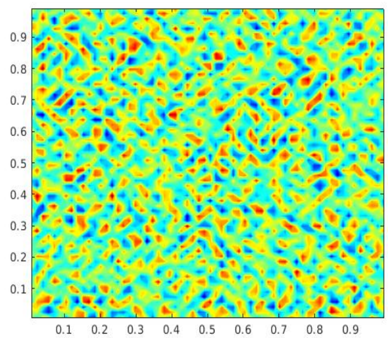
```
>> f95 -o nssolver main.f90 functions.f90 gs.f90 rk3.f90 meshgen.f90 updatebc.f90
>> ./nssolver
```

The video for the submission is attached. For the 64x64 grid, Re100 was run, at a timestep of 1e-4. This took around 40 minutes to give 1000 timesteps. The video file *re100_64.avi* shows the simulation.

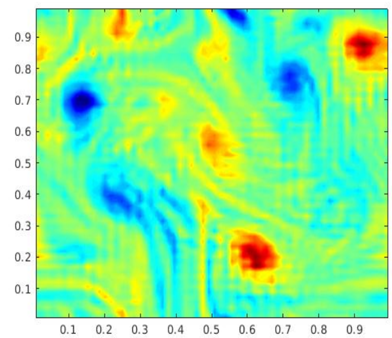
Further increasing the resolution or the Reynolds Number required the timestep to be reduced substantially, otherwise the code was diverging. This would result in very long simulation times.

(Note: I was able to run Re1000 for 81×81 for 430 timesteps at 1e-4 before it diverged. Please also find the video attached as *re1000_81.avi*. However, was not able to repeat it due to long computation times.)

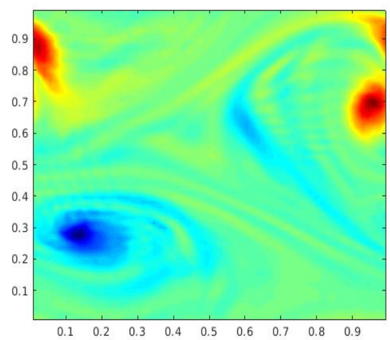
IMAGES OF SIMULATION for Re 100, for 64×64 Grid, dt = 1e-4



(a) At timestep=1



(b) At timestep=500



(c) At timestep=1000

OBSERVATIONS FROM THE SIMULATION:

1. Turbulence, in 3D, is a breakdown phenomena, where large vortices breakdown to form smaller vortices. However, in 2D, this phenomena is reversed. We can see in the video that, starting with small vortices, as we proceed in time, they join to form larger and larger vortices.
2. It was observed that as the number of time iterations increased, the guass-siedel subroutine required more iterations to converge.
3. The Vorticity Transport Equation is interpreted in the following manner:

$$\frac{\partial \omega}{\partial t} + \underbrace{u\frac{\partial \omega}{\partial x} + v\frac{\partial \omega}{\partial y}}_{\text{Convection Term as consists of } u \text{ and } v \text{ variables}} = \underbrace{\frac{1}{Re}\left\{\frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2}\right\}}_{\text{Dissipation Term as consists of viscosity represented by Re}}$$

4. On increasing the Reynolds Number, the larger vortex structures start forming at a faster rate, as expected.