ME685 HW9

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1. The Psuedo Code for the Tri-Diagonal Matrix Solver used in solving the system of Finite Difference Equations is as follows:

Algorithm 1: Tri-Diagonal Matrix Algorithm

The arrays a, b, c, d are passed to the TDMA
Performing Forward Elimination
for i = *1,n* **do**
    $b_i = b_i - \dfrac{c_{i-1} \times a_i}{b_{i-1}}$ ▷ Updating b array
    $d_i = d_i - \dfrac{d_{i-1} \times a_i}{b_{i-1}}$ ▷ Updating d array
end
Performing Backward Substitution
$\theta_n = \dfrac{d_n}{b_n}$
for i = *n-1,1* **do**
    $\theta_i = d_i - \dfrac{c_i \times \theta_{i+1}}{b_i}$ ▷ Calculating θ
end

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2. The code is attached with the submission.

3. Let the solution be given by $\theta = Ae^{mx} + Be^{-mx}$. Applying the Boundary Conditions:

$$\theta(x = 0) = 1$$

$$\implies \boxed{A + B = 1}$$

$$\frac{d\theta}{dx}(x = 1) = 0$$

$$\implies Bme^{m} - Cme^{-m} = 0$$

$$\implies \boxed{Be^{m} - Ce^{-m} = 0}$$

Solving the two boxed equations gives us:

$$A = \frac{e^{-m}}{e^{m} + e^{-m}}$$

$$B = \frac{e^{m}}{e^{m} + e^{-m}}$$

$$\therefore \theta_a = \frac{e^{-m+mx} + e^{m-mx}}{e^{m} + e^{-m}}$$

$$\implies \boxed{\theta_a = \frac{cosh(m - mx)}{cosh(m)}}$$

At the ends, $\theta_a(0) = 1$ and $\theta_a(1) \approx 0$. This is will be used as Boundary Conditions in the Numerical Solution.

The 2^{nd}-order central difference discretization will lead to:

$$\frac{\theta_{i-1} - 2\theta_i + \theta_{i+1}}{(\Delta x)^2} = m^2 \theta_i$$

$$\implies \theta_{i-1} + (-2 - m^2(\Delta x)^2)\theta_i + \theta_{i+1} = 0$$

The above equation is valid for nodes $i = 2, 3, ...n$. For $i = 1, n$, we know the solution. This system of equations is solved using the Tri-Diagonal Matrix Solver.

4. The following is the plot of the Analytical and Numerical Solution.

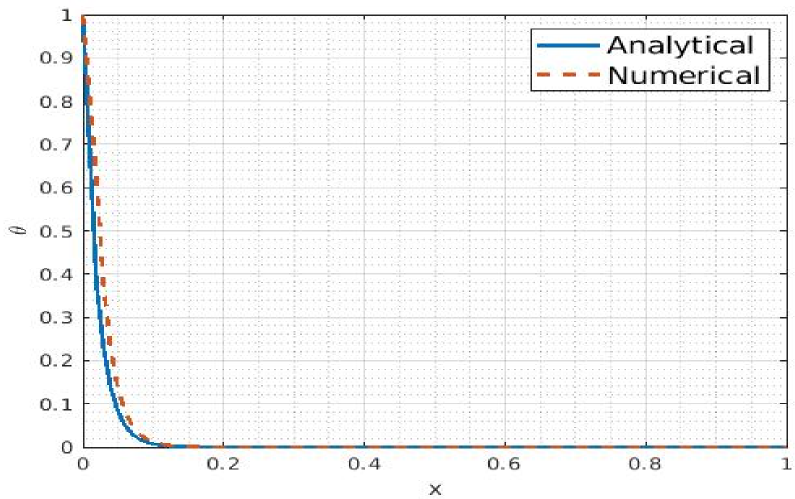

Figure 1: Solution of BVP