# ME685 HW10

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$$\frac{\partial \theta}{\tau} = \frac{\partial^2 \theta}{\partial z^2} + \frac{1}{z-1}\frac{\partial \theta}{\partial z} + \frac{m}{z-1}\theta$$

The governing equation given is discretized in the following manner to obey $2^{nd}$ order accuracy:

$$\frac{\theta_i^{n+1} - \theta_i^n}{\Delta t} = \frac{1}{2}\left[\frac{\theta_{i+1}^{n+1} - 2\theta_i^{n+1} + \theta_{i-1}^{n+1}}{\Delta z^2} + \frac{\theta_{i+1}^{n} - 2\theta_i^{n} + \theta_{i-1}^{n}}{\Delta z^2}\right] + \frac{1}{2\times(z_i - 1)}\left[\frac{\theta_{i+1}^{n+1} - \theta_{i-1}^{n+1}}{2\Delta z} + \frac{\theta_{i+1}^{n} - \theta_{i-1}^{n}}{2\Delta z}\right] + \frac{m}{2\times(z_i-1)}\left[\theta_i^{n+1} + \theta_i^n\right]$$

The above is the Crank-Nicolson Algorithm for discretizing the governing equation. This will lead to a system of tri-diagonal matrices which can be solved using TDMA.

The Psuedo Code for the Solver used in the problem is as follows:

**Algorithm 1:** Crank Nicolson Solver

**while** $error \geq 1e-10$ **do**

$u = u_{new}$

*The arrays $a, b, c, d$ are formed in the following manner:*

**for** *$i$ = 2,n-1* **do**

$$a_i = -\frac{\Delta t}{2\Delta x^2} + \frac{\Delta t}{4\Delta z(z(i) - 1}$$ ▷ *Forming a array*

$$b_i = 1 + \frac{\Delta t}{\Delta x^2} - \frac{m\Delta t}{2(z(i) - 1}$$ ▷ *Forming b array*

$$c_i = -\frac{\Delta t}{2\Delta x^2} + \frac{\Delta t}{4\Delta z(z(i) - 1}$$ ▷ *Forming c array*

$$d_i = u_{i-1}\left(\frac{\Delta t}{2\Delta x^2} - \frac{\Delta t}{4\Delta z(z(i) - 1})\right) + u_i\left(1 - \frac{\Delta t}{\Delta x^2} + \frac{m\Delta t}{2(z(i) - 1}\right) + u_{i+1}\left(\frac{\Delta t}{2\Delta x^2} - \frac{\Delta t}{4\Delta z(z(i) - 1}\right)$$ ▷ *Forming d array*

**end**

*The $1^{st}$ and $n^{th}$ elemtns of the array are set.*

*TDMA Solver is Called.*

*error = rms(u,$u_{new}$)*

**end**

The code is attached with the submission.

The following is the plot of the Analytical and Numerical Solution.

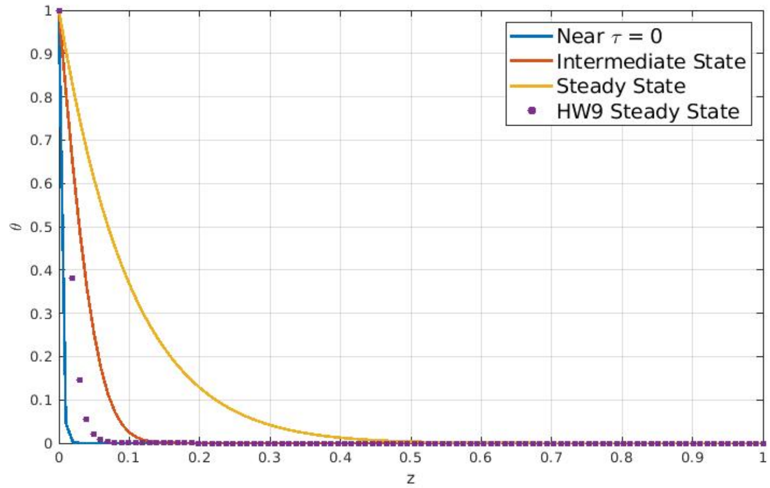


Figure 1: Solution of Transient Equation

The plot clearly shows the evolution of the state of the temperature profile as it converges to steady state. However, the steady state in obtained in HW9 does not match the steady state in the present problem. A possible reason could be due to the difference in governing equations.