Minimum Wages, Inequality, and the Informal Sector*

Rafael Machado Parente
International Monetary Fund
rmachadoparente@imf.org

Luiz Brotherhood
Universitat de Barcelona
martinsbrotherhood@ub.edu

Felipe S. Iachan
FGV EPGE
felipe.iachan@fgv.br

September 26, 2025

Abstract

How do minimum wages affect earnings inequality in countries with large informal sectors? We provide reduced-form evidence that the 2000s minimum wage hike in Brazil raised overall inequality by increasing inequality within the informal sector. We develop a model where heterogeneous firms select into informality to investigate when and how raising the minimum wage can increase inequality. We calibrate the model to Brazil and find that, by generating substantial informality, the increase in the minimum wage raised overall inequality by 4.1%. These results highlight how movements into and out of the informal sector modulate the effects of formal labor legislation.

*This paper evolved from Rafael Machado Parente's Ph.D. dissertation. We are indebted to Stephen Redding, Gregor Jarosch and Richard Rogerson for their invaluable guidance and support. We have also benefited from discussions with Mark Aguiar, Christian Moser, Ana Luiza Dutra, Pablo Fajgelbaum, Manuel García-Santana, John Grigsby, Jonathon Hazell, Faizaan Kisat, Nobu Kiyotaki, Ishan Nath, Ezra Oberfield, Lukasz Rachel, Esteban Rossi-Hansberg, Christiane Szerman, João Thereze, and Gianluca Violante. We gratefully acknowledge financial support from Princeton University's International Economics Section, Coordenação de Aperfeiçoamento de Pessoal de Nível Superior - Brasil (CAPES, Finance Code 001), CNPq, the Spanish Government (PID2020-114040RB-I00, PID2021-126549NB-I00, and PID2024-156676NB-I00), and the Generalitat of Catalonia (AGAUR-2020PANDE00036 and 2021-SGR-00862). The views expressed in this paper are our own and do not necessarily reflect those of our employers, including the IMF, its Executive Board, or its Management. All errors are our own.

Earnings inequality fell substantially in Latin America throughout the 2000s. Minimum wage policies were found to be among the most important drivers of these patterns.[1] However, recent studies do not account for the fact that firms and workers can avoid labor legislation by operating informally—a ubiquitous feature in most low-income and developing countries.[2] Hence, when quantifying the consequences of the minimum wage for inequality, it is essential to incorporate this margin of adjustment.[3] What are the effects of the minimum wage on inequality when the informal labor market is taken into account?

This paper proposes answers to this question in three steps. First, using Brazilian survey data on both formal and informal labor markets, we show that inequality in the informal sector did not fall alongside the rapid increase in the minimum wage. Moreover, we provide reduced-form evidence that the minimum wage increased overall inequality because of strong inequality-increasing effects on the informal sector. Second, we develop a stylized model of monopsonistic competition with informality and a minimum wage to investigate under which conditions raising the minimum wage can have the *unintended consequence* of increasing overall inequality. Third, we develop a quantitative model that additionally features heterogeneous workers and skill-biased technology, two other key drivers of informality and inequality. We calibrate the model to Brazil in 1996 and show that, all else equal, the increase in the minimum wage is responsible for a 4.1% increase in the variance of aggregate log earnings. Our framework suggests that the estimated 120% increase in informality costs offsets these unintended consequences. Moreover, improvements in the skill composition of the labor force can complement minimum wage policies in reducing inequality. All in all, these findings suggest that movements into and out of the informal sector modulate the effects of formal labor legislation, such as the minimum wage.

Section 1 establishes stylized facts on informality, inequality, and the minimum

[1] See Lustig et al. (2013) for Latin America; Firpo and Portela (2019), Alvarez et al. (2018), Engbom and Moser (2022) and Haanwinckel (2023) for the case of Brazil.

[2] See Tornarolli et al. (2014) for the case of Latin America.

[3] The intuition that minimum wages can raise overall inequality in segmented labor markets dates back to Lewis (1954) and was formalized in the Harris–Todaro tradition (Cain, 1976; Fields, 2005) as well as in neoclassical settings (Rauch, 1991). Similar predictions also appear in Haanwinckel and Soares (2021), although the impact of minimum wages on inequality is not the focus of their analysis.

wage using Brazilian household survey data from 1996 to 2012. First, we document that informal workers constitute 35% of the labor force, earn lower wages, and are substantially less educated than formal workers. Second, we highlight that while the variance of log earnings in the formal sector fell sharply from 0.65 to 0.33, inequality in the informal sector remained constant at 0.65. Third, we show that the minimum wage became substantially more binding in the formal sector. The share of formal workers at the minimum wage, stable around 7% until 1999, increased sharply to 16% by 2006, stabilizing at that level thereafter.

We then provide reduced-form evidence on the relationship between the minimum wage, inequality, and informality. We leverage both regional- and individual-level variation in exposure to the minimum wage. The regional analysis exploits state-level heterogeneity in the share of formal minimum wage workers in 1999 (Card, 1992). The most exposed states experienced a 26% stronger reduction in the variance of earnings in the formal sector (formal inequality), a 28.2% larger increase in informal inequality, and a 10.3% larger increase in the informal share compared to the least exposed states. Jointly, these led to an 18.3% relative increase in overall inequality in the states where the minimum wage binds the most. Moreover, we show that the effect of the minimum wage on overall inequality varies widely across states: minimum wage increases inequality in the most exposed states but decreases it in less exposed ones. These differences occur because the minimum wage has stronger effects on the informal sector in more exposed states.

While not capturing equilibrium effects at the regional level, the individual-level analysis complements the regional analysis by exploiting more granular variation: rather than comparing average outcomes across states, this approach estimates the impact of the minimum wage on employment *along the wage distribution* (Giupponi et al., 2024). We find labor reallocation around the new minimum wage in both the formal and informal sectors. Importantly, losses in formal jobs are not offset by gains in informal jobs, so the minimum wage raises the informal share. A 1% increase in wages induced by the minimum wage reduces formal employment by 0.29% and increases informal employment by 0.25%. We then construct counterfactual employment distributions to map these job-level effects into aggregate inequality. We find negative effects on formal inequality and positive effects on informal and aggregate inequality. All in all, both regional- and individual-level analyses suggest that the minimum wage in Brazil increased aggregate inequality by increasing both informal inequality and the informal workforce.

Motivated by these findings, Section 2 develops a stylized model where heterogeneous firms compete for labor subject to the minimum wage and can choose to operate informally. Firms trade off minimum wage restrictions when formal versus revenue losses for being informal (e.g., government enforcement or limited access to formal credit markets). In equilibrium, the most productive firms operate formally and, within formal firms, the least productive ones bunch at the minimum wage. We then study how the informal sector shapes the impact of the minimum wage on aggregate inequality. When the minimum wage is low, the economy is predominantly formal and aggregate inequality falls with the minimum wage. As the minimum wage rises, informality expands, and three other forces begin to drive inequality up. First, the continued expansion of the informal sector. Second, the increase in informal inequality, as less productive formal firms become informal. Third, the increased contribution from cross-sector wage dispersion, as mean-wage differences rise and both sectors become economically significant. Together, these forces outweigh the inequality-reducing effects on the formal sector, so the minimum wage increases aggregate inequality.

In the following two sections, we outline the quantitative model and the calibration results. Section 3 extends the stylized model and incorporates worker heterogeneity and skill biases in the production technology. These quantitative features were shown to be important drivers of the informal share of labor and earnings inequality.[4] Moreover, they might interact with the way in which minimum wage increases affect the economy. In Section 4, we calibrate the model to Brazil in 1996. The calibrated framework replicates the observed distribution of wages in the aggregate economy, within each sector, as well as within each skill group.

Section 5 quantifies the effects of the increase in the minimum wage on earnings inequality, holding all other factors constant. We find that the 105% spike in the minimum wage over the 2000s increased overall inequality by 4.1%, despite reducing formal sector wage inequality by 11.5%. This stems from the fact that the minimum wage generated substantial amounts of informality, increasing inequality in this sector, and more than compensating for the inequality-reducing effects in the formal economy.

We then quantify the role of government policies in helping the minimum wage to tackle inequality. First, the estimated 1996-2012 increase in informality costs substantially reduced the informal economy, forcing many employers to cope with

[4] See Haanwinckel and Soares (2021) and Haanwinckel (2023).

the minimum wage and reducing aggregate inequality by 17%. Second, the improvement in the educational composition of the workforce also reduced informality substantially, in line with the literature on the determinants of the informal sector (Haanwinckel and Soares, 2021). Informal firms are more intensive in low-skilled workers. Improvements in the skill composition make this factor of production more scarce, pushing up low skill wages and increasing the associated costs of being informal. These consequently lowered earnings inequality for individuals of all educational levels. These findings highlight that policies to tame the informal sector and improve the quality of the labor force can complement minimum wage policies in reducing overall earnings inequality in countries with a large informal economy.

Section 6 shows that the quantitative results are robust to three key extensions. First, accounting for unemployment does not affect the unintended consequences of the minimum wage, suggesting that the most important margin of adjustment is between the formal and informal sectors rather than between employment and unemployment. Second, following Berger et al. (2025), we allow for a finite number of oligopsonistic firms that compete for labor while choosing whether to operate formally or informally. This extension generates size-dependent markdowns and more reallocation towards larger firms, but the unintended consequences of the minimum wage remain. Third, we recalibrate the model to the regions least and most exposed to the minimum wage in Section 1, and show that the structural framework broadly replicates the regional-level empirical patterns.

Related research. This paper contributes to three strands of literature. First, we relate to the body of work that studies the informal sector in developing economies, summarized in Ulyssea (2020). Within these papers, we contribute to the empirical work on the informal sector (Porta and Shleifer, 2008; La Porta and Shleifer, 2014; Almeida and Carneiro, 2012; Engbom and Moser, 2022) by providing reduced-form evidence suggesting that minimum wages can increase overall inequality when there are substantial effects on the informal sector. Our work is also related to the set of papers that incorporate the informal sector in models of firm heterogeneity (Ulyssea, 2010; Leal Ordóñez, 2014; Meghir et al., 2015; Ulyssea, 2018; Dix-Carneiro et al., 2021). We complement this literature by providing a quantitative model of the minimum wage and the informal sector that delivers realistic wage distributions, and using it to quantify the effects of the minimum wage on the economy.

Second, this paper relates to the literature that studies the effects of the minimum wage on the formal sector.[5] This paper is particularly related to Engbom and Moser (2022) and Haanwinckel (2023), which study the Brazilian context using two different quantitative approaches. We make two main contributions. First, we provide reduced-form evidence that increasing the minimum wage increases overall inequality in highly exposed states relative to the states least exposed. Second, we develop a quantitative model where firms select into the informal sector to quantify the effects of the minimum wage and other mechanisms, such as changes in the relative supply of skills, on inequality and informality.

Third, there is a related literature that studies the effects of the minimum wage in economies with a large informal sector. Jales (2018) develops a density discontinuity design to estimate, in a reduced-form way, the impact of the minimum wage on the joint distribution of employment and wages in Brazil. Jales and Yu (2020) develop a bargaining model featuring compensating differentials and self-selection to microfound the findings in Jales (2018). Derenoncourt et al. (2021) investigate the effects of the minimum wage on racial inequality and the informal sector. Using a reduced-form approach, they show that minimum wage increases are important in explaining the fall in the racial earnings gap in the 2000s, but have little effect on informal labor. Haanwinckel and Soares (2021) develop a quantitative model to study the main drivers of informality. They find that improvements in the education of workers in Brazil were the main force behind the fall in informality, while the minimum wage helped keep the informal share elevated.[6] We contribute to this literature by developing a quantitative model that delivers realistic wage distributions in the aggregate as well as within the formal and informal sectors. This allows us to perform counterfactual exercises and to assess the general equilibrium effects of the minimum wage on inequality within each sector as well as in the aggregate.

The rest of the paper is structured as follows. Section 1 uses Brazilian data to highlight the importance of the informal sector when evaluating the effects of the minimum wage. Section 2 develops a stylized model where the presence of

[5]See Card and Krueger (1993), Lee (1999), Dickens and Manning (2004), Autor et al. (2016), Card et al. (2018), Harasztosi and Lindner (2019), Dustmann et al. (2021), Engbom and Moser (2022), Haanwinckel (2023), Berger et al. (2025), among others.

[6]The last section of Haanwinckel and Soares (2021) discusses the implications of their model for the effect of the minimum wage on relative wages across skills. Our analysis contributes to and extends theirs in two ways. First, we analyze the effects of the minimum wage on both the relative earnings across educational groups and across formality status. Second, and most importantly, we show that there are significant effects of the minimum wage on inequality within the formal and informal sectors.

informality can substantially alter the effects of the minimum wage on inequality and welfare. Section 3 extends the stylized model and introduces other mechanisms that are important in generating the observed changes in inequality and informality in Brazil. Section 4 discusses the calibration and validation. Sections 5 and 6 perform the counterfactual exercises. Section 7 concludes.

1 Empirical motivation

This section uses Brazilian data to examine how the informal sector shapes the effects of the minimum wage on inequality. First, we introduce the data. Second, we present stylized facts on the informal sector, earnings inequality, and the minimum wage in Brazil. We show that a large share of the workforce is informal, that earnings inequality between informal workers, unlike that between formal workers, did not decrease over the 2000s, and that the minimum wage became substantially more binding over time. Third, using two complementary methods that leverage both cross-state and individual-level exposure to the minimum wage, we provide evidence that the minimum wage can increase overall inequality due to its effects on the informal sector.

1.1 Data

The main data sources are the *Pesquisa Nacional por Amostra de Domicílios* (PNAD) from 1996 to 2012 and the Demographic Census from 1991 to 2010.[7] The PNAD is a yearly household survey, whereas the Demographic Census is conducted every 10 years. Both have national coverage and are administered by the *Instituto Brasileiro de Geografia e Estatística* (IBGE). They constitute the primary sources of nationally representative labor market and demographics data in Brazil. Importantly, they complement each other in that the PNAD grants us a yearly time series for each of Brazil's 27 states. In contrast, the Census grants us granular labor market

[7] Panel (a) of Appendix Figure A.1 shows the real value of the minimum wage since 1960. The minimum wage declined between 1960 and 1990, coinciding with the military regime of 1964–1985. It increased starting in 1990 but then fell again between 1992 and 1994 due to the hyperinflation period. Moreover, the PNAD was not conducted in 1994. We choose 1996–2012 for two reasons. First, direct comparability with Engbom and Moser (2022). Second, Panel (b) of Appendix Figure A.1 shows little correlation between the real value of the minimum wage and the share of minimum wage workers outside of the 2000s. However, results are robust to starting the analysis in 1995 or extending it to 2021.

information on 400+ microregions for three years: 1991, 2000, and 2010.[8]

Both sources are particularly well-suited for our analysis as they contain data on both formal and informal work arrangements, which we detail below.[9] We restrict attention to individuals highly attached to the labor force (age between 18-54), and consider only one job per worker (their main job at the reference week).[10] We deflate all nominal variables by the CPI, and express them in terms of 2012. We follow the empirical literature on Brazil and consider monthly gross earnings as the primary measure of earnings. Notably, the minimum wage in Brazil is de facto imposed at the monthly earnings level.[11]

In both sources, households are asked whether they have a signed employment record card (*Carteira de Trabalho Assinada*). When an employer signs an employee's employment record, that labor contract becomes subject to labor legislation such as the minimum wage, unemployment benefits, and others. Throughout this paper, a worker is informal if they do not have a signed employment record. Even though the share of self-employed workers is about as large as that of workers without a signed record, we restrict attention to households engaging in employer-employee working relationships. Because self-employment provides an additional margin for agents to evade labor legislation, the results in this paper can be interpreted as a lower bound on the effects of the minimum wage on compositional changes between the formal and informal sectors.[12]

Minimum wage in Brazil. The minimum wage in Brazil was introduced by Decree-Law No. 2,162 under President Getúlio Vargas in 1940, initially covering only urban areas. It was extended to rural workers in 1963 and unified nationwide in 1984. Today, it is set by the federal government and establishes a nationwide floor

[8] We follow Dix-Carneiro and Kovak (2017) and define a local labor market as a microregion.

[9] Appendix Figures A.2 and A.3 compare the percentiles of the formal and informal earnings distribution in PNAD with those at RAIS and ECINF data sets, respectively, and confirm that PNAD is indeed a unified data set that provides a realistic picture of the earnings distribution in both sectors.

[10] Appendix Figure A.4 shows that less than 5% of formal and informal workers had more than one job according to PNAD data.

[11] Our results are robust to using hourly earnings. However, Appendix Figure A.5 shows a large discrepancy between the distribution of weekly hours worked from PNAD data and the distribution of contractual hours from RAIS data—based on Figure B.24, Panel A of Engbom and Moser (2022). This reflects the fact that PNAD relies on self-reported hours in the reference week of the survey, which may not align with hours in the formal labor contract—the relevant margin for the minimum wage. Hence, we use monthly earnings in our baseline specifications.

[12] Robustness exercises in Appendix A show that the empirical findings are robust to the inclusion of self-employed workers in the definition of informality.

on the monthly nominal earnings of formal workers across all sectors and occupations (Lemos, 2004). The minimum wage is reviewed and adjusted annually, taking into account inflation and economic growth. Since the passage of Lei Complementar No. 103 in July 2000, five states—Rio de Janeiro and Rio Grande do Sul (2001), Paraná (2006), São Paulo (2007), and Santa Catarina (2010)—have introduced state-specific wage floors to different occupations. However, Appendix Table A.1 shows that these floors have had limited bite: more than one-third of formal workers report earnings below their state-occupation-specific floor, while only about 4% declare earnings below the federal minimum. We therefore focus on the federal minimum wage, which remains the binding wage floor for the vast majority of formal workers.[13]

1.2 Stylized facts on the informal sector, inequality, and the minimum wage

This section documents key facts about the informal sector, earnings inequality, and the minimum wage. First, informal workers comprise a large share of the labor force, earn less than formal workers, and are substantially less educated. Second, while formal and overall earnings inequality declined over the sample period, inequality in the informal sector did not. Third, the bite of the minimum wage in the formal sector increased sharply over the 2000s.

Facts on the informal sector. Table 1 reports summary statistics by formality status in 1996 and 2012, highlighting both cross-sectional differences and changes over time. In both years, informal workers represent a large share of the labor force—35% on average in the sample, falling from 39.1% in 1996 to 30.9% in 2012 as the formal sector grew faster. Earnings in the informal sector are, on average, 47% lower than in the formal sector, and formal workers are more educated, with a 20 percentage point higher share having at least a high school diploma. Education levels improved markedly in both sectors: between 1996 and 2012, the share of workers with at least a high school diploma rose from 31.5% to 61.2% in the formal sector and from 14.6% to 38.4% in the informal sector. The informal sector also employs a higher proportion of women and younger workers. Moreover, Appendix Figure A.6 shows that rising education levels were a key driver of falling informality

[13] The PNAD survey is also not designed to be representative by occupation-state-year, so we cannot use it to estimate the share of workers earning their respective wage floors.

Table 1: Summary statistics by formality status

	1996		2012	
	Formal	Informal	Formal	Informal
Employees	18,889.8	11,963.2	35,648.7	15,203.9
Employment share	60.9	39.1	69.1	30.9
Mean earnings	1,387	673	1,388	840
Share with HS	31.5	14.6	61.2	38.4
Age	32.5	31.0	33.7	33.5
Male	63.8	55.2	58.6	50.0

Notes: The first row displays the number of employees (in thousands) in each sector in 1996 and 2012. The second row reports their corresponding shares in total employment. The last four rows calculate the means of the variables in the first column across formal and informal workers in 1996 and 2012. Earnings are deflated by CPI and expressed in 2012 values. *Sources:* 1996/2012 PNAD.

(Haanwinckel and Soares, 2021), highlighting the need to control for changes in skill composition when assessing the economy-wide effects of the minimum wage.

We now provide empirical support for two assumptions underlying our model exercises: abstracting away from industry heterogeneity and the unemployment margin of adjustment. Appendix Table A.2 shows that informality is pervasive across industries, ranging from 17% in Manufacturing to 70% in Domestic Services. Appendix Figure A.7 further shows that the decline in aggregate informality reflects falling informality within industries rather than shifts in the industry mix of employment. Appendix Figure A.8 shows that unemployment remained low (around 7.5%) and stable (between 9% and 6%) over the sample period, especially when compared to the much larger and more volatile informal share. This reflects the high labor-force attachment of workers and suggests that the unemployment margin is less significant than the informal margin—the primary focus of the model exercises in this paper.

Facts on inequality. Figure 1 shows the evolution of the variance of log earnings in the aggregate and by formality status. There was a strong and steady reduction in overall (2.3% per year) and formal (4.5% per year) inequality in log earnings. In contrast, informal-sector inequality remained broadly stable, fluctuating around 0.65 log points. As a result, the gap between formal and informal earnings inequality widened consistently over the 2000s.

We next show that earnings inequality within the formal and informal sectors

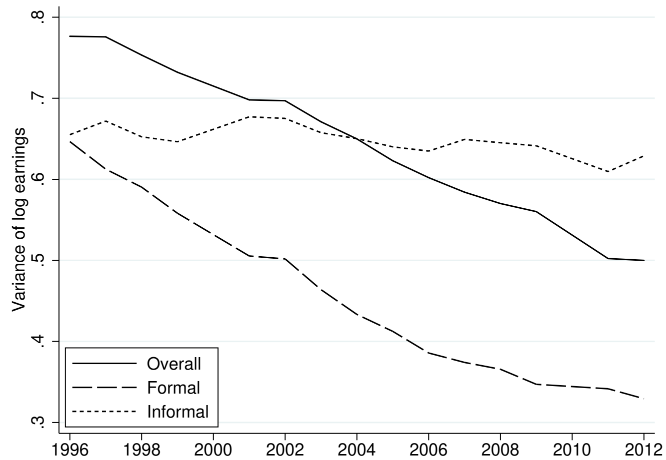


Figure 1: Variance of log earnings, 1996-2012

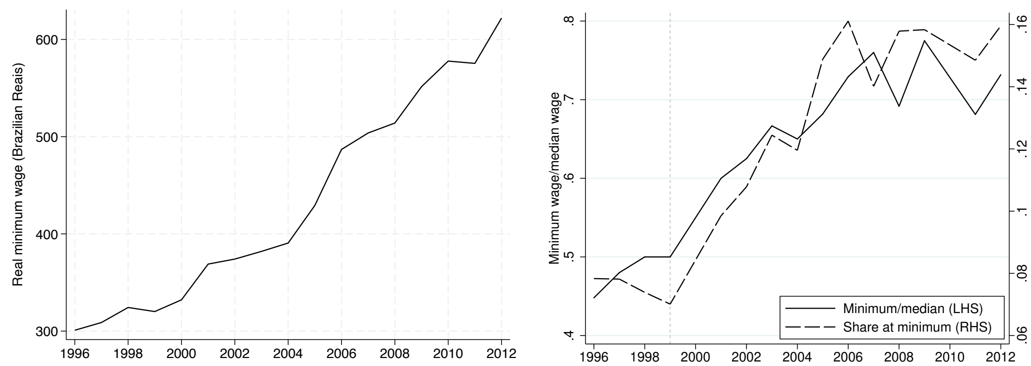
Notes: Variance of overall, formal, and informal log earnings between 1996 and 2012. *Sources:* 1996-2012 PNAD.

accounts for most of the overall inequality. We decompose aggregate inequality into two components: (i) the employment-weighted average of inequality within each sector (the within component) and (ii) the employment-weighted sum of squared differences between each sector's mean earnings and the overall mean (the between component):

$$V_t = \underbrace{\sum_{j \in \{F,I\}} s_t^j V_t^j}_{\text{Within}} + \underbrace{\sum_{j \in \{F,I\}} s_t^j (E_t^j - E_t)^2}_{\text{Between}} \tag{1}$$

Appendix Figure A.9 plots the inequality decomposition over time. Panel (a) shows that within-sector inequality—formal and informal combined—accounts for over 80% of aggregate earnings inequality, and for more than 83% of its decline over the sample period. Panel (b) shows that the formal sector's contribution to the within component fell from 60% in 1996 to 50% in 2006, before rising to 55% in 2012.

Facts on the minimum wage. Figure 2 plots the evolution of the minimum wage and its bite in the economy between 1996 and 2012. Panel (a) shows that the real value of the minimum wage increased steadily from about R\$300 in 1996 to over R\$600 in 2012, with particularly steep increases between 2000-2001 and 2004-2006.



(a) Real minimum wage

(b) Minimum wage bite

Figure 2: The evolution of the minimum wage, 1996-2012

Notes: Left panel shows the real minimum wage (deflated to 2012 prices). In the right panel, the solid line plots the fraction of median earnings represented by the minimum wage (left axis), while the long dashes show the share of formal workers earning exactly the minimum wage (right axis). *Sources:* IPEADATA and 1996-2012 PNAD.

Panel (b) shows two measures of the bite of the minimum wage. On the left y-axis, the minimum wage relative to median earnings rose from 45% in 1996 to 73% in 2012. On the right y-axis, the share of formal sector workers earning the minimum wage increased from 8% to 16% over the same period. Notably, most of this tightening occurred after 1999; before then, the share of minimum wage earners in the formal sector had slightly declined.

The following two sections leverage variation in state- and individual-level exposure to the minimum wage hike in Brazil over the 2000s to estimate the impact of the minimum wage on inequality and informality.

1.3 The effects of the minimum wage: state-level analysis

This section leverages state-level heterogeneity in exposure to the minimum wage to assess its impact on inequality and informality. We find that, relative to states least exposed to the minimum wage, states most exposed experienced decreases in formal inequality, increases in informal inequality, increases in the informal share of labor and unemployment, and, as a consequence, increases in overall inequality among workers.

We rank Brazil's 27 states by their initial exposure to the minimum wage, measured by the share of formal workers in 1999 earning exactly the national wage floor. States are split into nine treatment groups, and we compare the conditional evolution of formal, informal, and overall inequality, as well as the informal share, controlling for key cross-state differences and time-varying factors other than the bite of the minimum wage.[14]

We then implement an event study design that interacts a state's initial exposure to the minimum wage with year fixed effects:

$$y_{sgt} = \alpha + \sum_{h \neq 1} \sum_{k \neq 1999} \beta_{kh} \cdot \mathcal{I}_{g=h} \cdot \mathcal{I}_{t=k} + \delta_s + \delta_t + X'_{st}\Gamma + \varepsilon_{st}, \qquad (2)$$

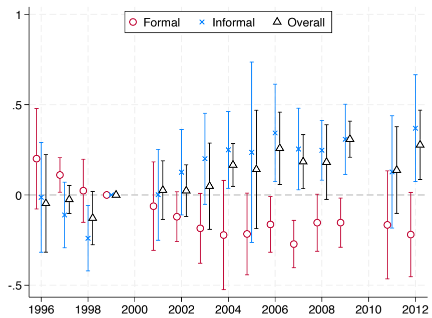
where y_{sgt} denotes the outcome of interest in state $s = 1, ..., 27$, treatment group $g = 1, ..., 9$, and year $t = 1996, ..., 2012$; α is a constant; $\mathcal{I}$ are indicator functions; δ_s and δ_t are state and year fixed effects, respectively; and X_{st} flexibly controls for the age, gender, race, education, and sectoral compositions of the labor force, which might affect inequality, informality, and minimum wage exposure across states.[15] We also control for lagged GDP per capita, and—except when unemployment is the dependent variable—include the unemployment rate as an additional control. We set 1999 and treatment group 1 as the reference categories. The coefficients β_{kh} measure changes in outcomes for group h relative to the three states with the lowest share of formal minimum wage workers in 1999. The identification relies on a parallel trends assumption: absent the post-1999 increase in the minimum wage, differences in outcomes across treatment groups would have evolved similarly over time.

We first discuss the estimates for the most treated states, β_{k9}. Figure 3 plots the annual coefficients for each outcome: Panel (a) shows the log variance of earnings in the formal sector (red circles), informal sector (blue crosses), and aggregate (black triangles); Panel (b) shows the log informal share; and Panel (c) reports the log unemployment rate. The corresponding average post-1999 effects are reported in Appendix Table A.3.

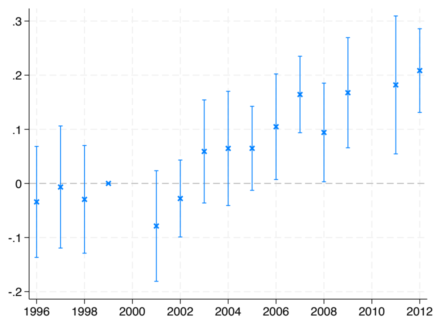
The states most exposed to the minimum wage experienced a 26% larger decline in formal inequality, a 28.2% larger increase in informal inequality, and a 10.3% larger increase in the informal share, resulting in an 18.3% larger increase in overall

[14] Appendix Table A.4 lists the states in each group and Appendix Figure A.10 reports descriptive statistics by treatment group for 1996–1999.

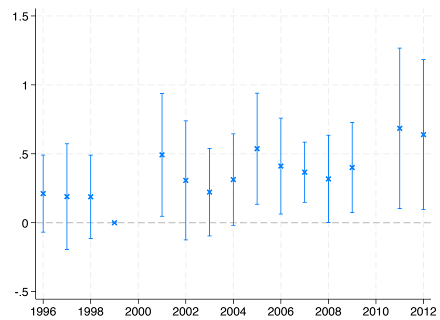
[15] The specification includes quartile fixed effects for each control variable, computed from the distribution across states within each year.



(a) log(Variance)



(b) log(Informal share)



(c) log(Unemployment)

Figure 3: Minimum wage, inequality, and the informal sector (most vs. least binding states)

Notes: This figure plots the ordinary least squares coefficients of Equation (2) for the states in the most treated group, β_{k9} for $k \neq 1999$. Panel (a) shows the results for earnings inequality, Panel (b) for the log of the informal share, and Panel (c) for the log of the unemployment rate. Standard errors are clustered at the state level. *Sources:* 1996-2012 PNAD.

inequality compared with the least exposed states. The minimum wage also raised unemployment by 32% in the most exposed states. These effects are economically significant. Using the labor market composition of the average most-treated state in 1996, we estimate that around 6% of the labor force in these states was pushed from formal into informal jobs, while around 2% became unemployed.[16] Hence, much of the adjustment took the form of reallocation from formal to informal employment rather than exit from the labor force. Lastly, the coefficients for the pre-1999 period support the parallel-trends identification assumption.

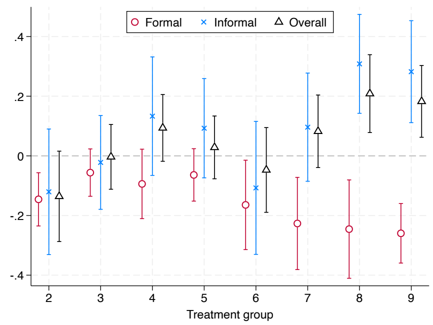
[16]The calculations were $[1-(\text{unemp. rate})]\times(\text{inf. share})\times(\text{inf. share coef.}) = (1-0.071)\times 0.561\times 0.103 = 5.8\%$ for the informal share and $(\text{unemp. rate})\times(\text{unemp. rate coef.}) = 0.071\times 0.322 \approx 2.2\%$ for the unemployment rate.

Figure 4 plots the mean post-1999 effect of the minimum wage (y-axis) by treatment group (x-axis). Panel (a) reports results for overall (black triangles), formal (red circles), and informal (blue crosses) inequality; Panel (b) for the informal share; and Panel (c) for the unemployment rate. Four patterns emerge. First, the minimum wage reduced formal inequality, with effects increasing in exposure. Second, the minimum wage raised informal inequality and the informal share, particularly in the most exposed states (groups 8 and 9). Third, as a result, the effects of the minimum wage on overall inequality change sign as you analyze states that are more or less treated. For example, relative to group 1, group 2 saw a 13.6% larger *decline* in overall inequality, whereas groups 8 and 9 saw a 19% larger *increase*. Fourth, the effects on unemployment are generally small and statistically insignificant, except for groups 7 and 9, which experienced increases in unemployment rates relative to the least treated group.

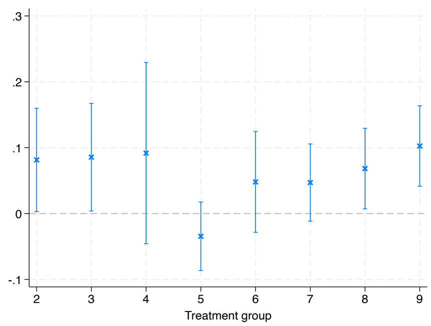
Robustness checks. We implement a battery of robustness checks on the analysis in this section. We complement the PNAD analysis with Census data, which provide much richer cross-sectional variation across 400+ microregions, but only for three years: 1991, 2000, and 2010. Appendix Figure A.11 shows that the most exposed regions experienced declines in formal inequality, increases in informal inequality and the informal share, and rises in overall inequality relative to the least exposed regions.

We then examine the robustness of our results to different regression specifications. First, results are robust to a less saturated specification that linearly controls for the education and sectoral composition of the labor force (Appendix Figure A.12a). Second, weighting regressions by total employment (Figure A.12b) or population (Figure A.12c) leaves the results unchanged, indicating that small outliers are not driving the findings. Third, extending the analysis to 1995-2023 shows that the effects stabilize after 2012, consistent with the slowdown in the real growth of the minimum wage (Figure A.13a). Fourth, splitting states into two groups—above or below the median 1999 share of formal minimum wage workers—yields similar event-study patterns (Figure A.13b).[17] Fifth, replicating the analysis in levels shows a positive and significant relationship between minimum wages and both overall and informal earnings inequality, and a positive but statistically insignificant relationship

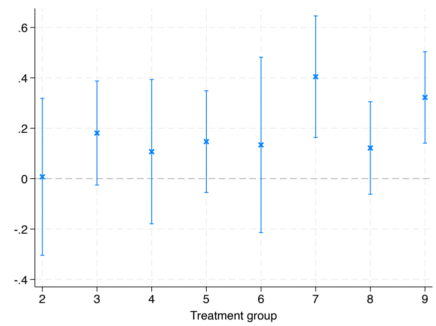
[17] The greater noise in these estimates reinforces the finding in Figure 4 that the minimum wage's effects on the informal sector are concentrated in more exposed states.



(a) log(Variance)



(b) log(Informal share)



(c) log(Unemployment)

Figure 4: Diff-in-diff analysis (post-1999 effects across treatment groups)

Notes: This figure reports, for different treatment groups (x-axis), the β_h coefficients of the OLS regression (y-axis): $y_{sgt} = \alpha + \sum_{h \neq 1} \beta_h \cdot \mathcal{I}_{g=h} \cdot \mathcal{I}_{t>1999} + \delta_s + \delta_t + X'_{st}\Gamma + \varepsilon_{st}$. Panel (a) shows the results for earnings inequality, Panel (b) for the log of the informal share, and Panel (c) for the log of the unemployment rate. The values for the coefficients can be found in Appendix Table A.5. Standard errors in parentheses are clustered at the state level. *Sources:* 1996-2012 PNAD.

with the informal share (Figure A.13c). Sixth, recent work has highlighted potential biases in two-way fixed effects (TWFE) estimators, particularly under staggered treatment.[18] While this concern is less relevant here—as all states faced the same minimum wage—we confirm that our key findings are robust to alternative TWFE estimators (Figure A.14).

We assess the robustness of our results to alternative definitions of informality and earnings. First, our baseline measure of informality excludes self-employed workers, potentially missing an important margin of adjustment away from the formal sector. Appendix Figure A.15 shows that our key findings are robust to

[18]See Borusyak et al. (2024), de Chaisemartin and D'Haultfœuille (2020), and Callaway and Sant'Anna (2020).

including self-employed workers in the informal sector. Second, changes in hours worked could influence the link between the minimum wage and earnings inequality. Appendix Figure A.16 shows that replacing monthly earnings with hourly earnings and adjusting the statutory minimum wage to its full-time equivalent of 44 hours per week do not alter the key findings.

This section uses a difference-in-differences strategy to estimate the relationship between the minimum wage, inequality, and informality. An extensive literature instead studies the link between the minimum wage and earnings inequality using Kaitz regressions.[19] These regressions use the log distance between the minimum wage and median formal-sector earnings (the Kaitz index) as a measure of policy stringency, and relate outcomes to the Kaitz index in a quadratic specification with state and year fixed effects. In Online Appendix A, we follow this approach, describe the specifications and identification assumptions in detail, and show that our main findings hold: the minimum wage is negatively related to formal inequality, positively related to informal inequality and the informal share, and these offsetting forces shape the relationship between the minimum wage and aggregate inequality.

In a recent paper, Derenoncourt et al. (2021) find no evidence that the minimum wage in Brazil displaced workers from the formal to the informal sector. Although seemingly contradictory, our findings highlight the importance of *treatment intensity* in evaluating the impact of the minimum wage on sectoral employment. Figure 4 and Online Appendix B suggest that less-treated states did not experience a relative increase in the informal sector, and that the minimum wage raises informality only in the poorest states, where the minimum wage is the most binding.

1.4 The effects of the minimum wage: individual-level analysis

This section exploits variation in minimum wage exposure across individuals earning similar wages but living in different locations, controlling for regional differences in the cost of living. We find that increases in the minimum wage can increase the informal share of labor, decrease formal inequality, and increase both informal and aggregate earnings inequality.

[19] See Lee (1999), Autor et al. (2016), Engbom and Moser (2022), Saltiel and Urzúa (2022), and Haanwinckel (2023).

1.4.1 Method description

We extend the approach in Giupponi et al. (2024) to take into account the large informal sector in Brazil, and to provide estimates of the impact of the minimum wage on inequality. We leverage yearly state-level data from 1996 to 2012 from the PNAD household survey, and decennial microregion-level data from 2000 and 2010 from the Census. We believe both analyses complement each other. The estimates from yearly PNAD data can be interpreted as short-run effects of yearly minimum wage increases, while estimates from Census data capture long-run effects of the minimum wage increases that occurred over ten years.

We begin by estimating wage premia across regions $\delta_{r(it)}$, conditional on demographic, occupational, and labor market characteristics. Using individual-level data prior to the 2000s minimum wage hike, we regress:

$$\ln w_{it} = \ln \delta_{r(it)} + \theta_t + X'_{it}\beta + \varepsilon_{it}, \tag{3}$$

where w_{it} denotes wages of individual i in year t, $r(it)$ denotes the region, θ_t are year fixed effects, and X_{it} includes age interacted with a female dummy, and indicators for race, education, sector, occupation, hours worked, and formal employment status. Following Giupponi et al. (2024), we rank regions based on $\delta_{r(it)}$ and define the top decile as the control group H (least exposed) and all other regions as the treatment group L (more exposed).

We use a frequency distribution approach to estimate the employment effects of the minimum wage, tracking changes in the employment rate along the wage distribution. Let $\Delta E_{rt}(c)$ denote the change in employment rate (aggregate, formal or informal) up to wage c in treated region $r \in L$ between periods $t-1$ and t. We construct the counterfactual change in $\Delta E_{rt}(c)$ as the (population-weighted) average change in employment rate in control regions $r' \in H$, after adjusting for regional wage differences:[20]

$$\Delta E_{rt}(c)^{\mathrm{CF}} = \frac{\sum_{r' \in H} \Delta E_{r't}\left(\frac{\delta_{r'} c}{\delta_r}\right) N_{r't-1}}{\sum_{r' \in H} N_{r't-1}}. \tag{4}$$

The average treatment effect of the minimum wage on the employment rate up

[20] A worker in treated region r earning wage c would earn $(\delta_{r'}/\delta_r)c$ in control region r'.

to wage c in year t is:

$$\mathrm{DiD}_{E_t(c)} = \frac{1}{\mathrm{ER}_{t-1}} \frac{\sum_{r' \in L} (\Delta E_{r't}(c) - \Delta E_{r't}(c)^{\mathrm{CF}}) N_{r't-1}}{\sum_{r' \in L} N_{r't-1}}, \tag{5}$$

where ER_{t-1} is the aggregate employment rate, constructed as total employment divided by the working-age population. In practice, we estimate Equation (5) for each wage bin k (from k to $k+x$) using the following (population-weighted) regression specification:

$$\frac{\Delta e_{krt} - \Delta e_{krt}^{\mathrm{CF}}}{\mathrm{ER}_{t-1}} = \sum_{f=\underline{F}-x}^{\overline{F}} \alpha_f \mathcal{I}_{k=f} + \nu_{krt} \quad \text{for } r \in L, \tag{6}$$

where $e_{krt} = E_{rt}(k+x) - E_{rt}(k)$ denotes the employment rate in wage bin k, $\mathcal{I}_{k=f}$ is an indicator for whether wage bin k equals the interval $[f, f+x]$ above the new minimum wage at t. We center the wage bin indicators around the post-reform minimum wage to facilitate the visualization of changes in the wage distribution. The coefficients α_f capture the estimated change in employment rate for each bin relative to the new minimum wage, and are expressed as a share of the national employment rate.

We use this method to study the effect of the minimum wage on inequality. To do so, we construct counterfactual employment distributions assuming that the employment rate in the log-wage bin ℓ that would prevail in the absence of a minimum wage increase is:[21]

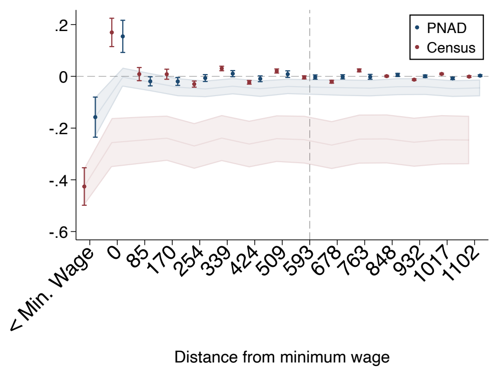
$$e_{\ell rt}^{\mathrm{CF}} = e_{\ell rt-1} + \Delta e_{\ell rt}^{\mathrm{CF}}. \tag{7}$$

We then calculate the counterfactual variance of log-wages in region r as:

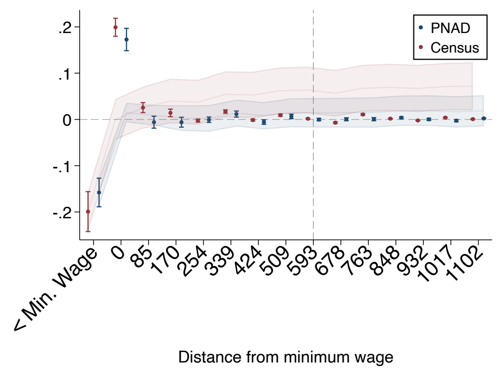
$$V_{rt}^{\mathrm{CF}} = \frac{\sum_\ell e_{\ell rt}^{\mathrm{CF}} \cdot (w_{\ell t} - \mathbb{E}_{rt}^{\mathrm{CF}})^2}{\sum_\ell e_{\ell rt}^{\mathrm{CF}}}, \quad \mathbb{E}_{rt}^{\mathrm{CF}} = \frac{\sum_\ell e_{\ell rt}^{\mathrm{CF}} \cdot w_{\ell t},}{\sum_\ell e_{\ell rt}^{\mathrm{CF}}} \tag{8}$$

where $w_{\ell t}$ is the midpoint of log-wage bin ℓ in t. Similarly to the regression specification in (6), the average treatment effect is obtained as a population-weighted average of $\ln(V_{rt}) - \ln(V_{rt}^{\mathrm{CF}})$.

[21] We restrict $e_{\ell rt}^{\mathrm{CF}} \in [0,1]$. In our implementation, 35% (PNAD) and 30% (Census) of observations have negative counterfactual values and are set to zero. No observation has a counterfactual employment rate exceeding one.



(a) Formal



(b) Informal

Figure 5: The effect of the minimum wage on employment

Notes: The graph reports estimates of the effects of minimum wage increases on the wage distribution. We group the original 50 wage-bin coefficients (α_f) into fifteen broader bins, using linear combinations. Blue dots with confidence intervals denote yearly estimates from the 1996-2012 PNAD, while red dots with confidence intervals come from the 2000-2010 Census. Each circle represents an estimate of employment changes (normalized by the baseline national employment rate) within each wage bin relative to the new minimum wage. The solid lines show the cumulative employment changes up to that point in the distribution. The 95% confidence intervals are based on 100 bootstrap replications, resampling locations with replacement. See Online Appendix C for implementation details. *Sources:* PNAD (1996-2012) and Census (2000, 2010).

1.4.2 Results

We start by discussing the impact of the minimum wage on employment along the wage distribution, as shown in Figure 5. Each circle represents the average change in employment rate for a given wage bin relative to the new minimum wage. For Census data, this is the change between 2000 and 2010; for PNAD data, this is averaged over multiple two-year periods. These changes were estimated using Equation (6). The lines and corresponding shaded areas show the cumulative effect of the minimum wage on the employment rate up to each wage bin.

The first two point estimates show a reallocation of workers from below to around the new minimum wage in both sectors.[22] Figure 5a shows that formal employment below the new minimum fell by 15.8% (PNAD) and 42.6% (Census) of the national pre-treatment formal employment rate, while employment at the minimum rose by 15.4% (PNAD) and 16.9% (Census). A similar pattern emerges in the informal sector: Figure 5b shows a drop in informal jobs below the minimum of 15.8%

[22]The formal sector reallocation mirrors the results in Giupponi et al. (2024).

Table 2: The effect of minimum wages on total employment and own-wage employment elasticities

	Formal		Informal		Overall	
	Est.	SE	Est.	SE	Est.	SE
Panel A. PNAD						
Total employment	-0.045	0.017	0.019	0.017	-0.022	0.014
Employment-wage elasticity	-0.940	7.707	1.014	2.304	-1.738	1.993
Panel B. Census						
Total employment	-0.246	0.048	0.072	0.026	-0.120	0.025
Employment-wage elasticity	-0.290	0.219	0.256	0.120	-0.635	0.117

Notes: This table reports estimates of the effects of minimum wage increases on total employment (normalized by the baseline national employment rate), as well as estimates of the employment elasticity with respect to wage changes induced by minimum wage increases. Total employment effects correspond to the endpoints of the cumulative employment effects in Figure 5. See Online Appendix C for implementation details. *Sources:* PNAD (1996-2012) and Census (2000, 2010).

(PNAD) and 19.9% (Census) of the national pre-treatment informal employment rate, and an increase at the minimum of 17.2% (PNAD) and 20% (Census). This reflects spillovers from the formal to the informal sector and highlights the role of the minimum wage as a benchmark for wage negotiations in Brazil.

Above the new minimum wage, point estimates are closer to zero and the respective lines flatten, suggesting little to no effect on jobs paying more than 600 Reais above the threshold—indicated by the vertical line in each panel. In the Census data, this threshold corresponds to the 88th percentile of the 2000 wage distribution. This result is consistent with Engbom and Moser (2022), who find that minimum wage increases in Brazil do not generate spillover effects beyond the 75th percentile.[23]

Table 2 reports the total employment effects of the minimum wage, aggregating across all wage bins in Figure 5. Three main findings emerge. First, formal job creation above the new minimum wage did not fully offset job losses below it, re-

[23]Following the high-inflation period of the 1980s and early 1990s in Brazil, wages were often set as multiples of the minimum wage in employment contracts, a practice that persisted through the 1990s and possibly into the early 2000s. Although this practice has largely faded, it may still amplify the spillover effects estimated in this and other studies.

sulting in net formal employment loss of 4.5% (PNAD) and 24.6% (Census) relative to pre-treatment national levels. Second, in the informal sector, job creation above the new minimum more than compensated for losses below it, yielding net employment gains of 2.0% (PNAD) and 7.2% (Census). These suggest that the minimum wage increases the informal share in both the short and the long run. Third, in the aggregate, the minimum wage reduced total employment by 2.2% (PNAD) and 12.0% (Census) relative to pre-treatment national levels.

We now turn to own-wage employment elasticities, also reported in Table 2. These measure the percent change in employment for every 1% increase in wages caused by the minimum wage. Using Census data, we estimate a formal (informal) sector elasticity of –0.29 (0.25), meaning that a 1% increase in formal (informal) wages reduces (raises) employment by 0.29% (0.25%).[24] These magnitudes are (in absolute values) close to the average estimate of –0.25 reported by Dube and Lindner (2024) in a review of 72 studies. Putting both sectors together, we estimate an aggregate employment-wage elasticity of –0.63, which Dube and Lindner (2024) classify as a moderate effect. This pooled elasticity does not fall between the sector-specific estimates because the minimum wage had a smaller effect on the wages of affected workers when both sectors are considered jointly, producing a larger employment response for each percentage increase in wages.

We now examine how the minimum wage impacts earnings inequality. Figure 6 presents the impact on formal, informal, and aggregate wage inequality using our extension of the method in Giupponi et al. (2024). In the short run (PNAD data), the minimum wage reduced formal inequality by about 10% and had positive but insignificant effects on overall or informal inequality. In the long run (Census data), however, the minimum wage increased informal and aggregate inequality by 26% and 27%, respectively, while leaving formal inequality virtually unchanged. These results suggest that the minimum wage had negligible-to-negative effects on formal inequality and negligible-to-positive effects on informal and overall inequality, consistent with the findings in the previous section.

Both state- and individual-level evidence suggest that the minimum wage can raise overall earnings inequality when it has a strong, inequality-increasing effect on the informal sector. The analysis in this section compares treated and control regions

[24]We report but do not discuss PNAD wage–employment elasticities for brevity, as they are imprecisely estimated.

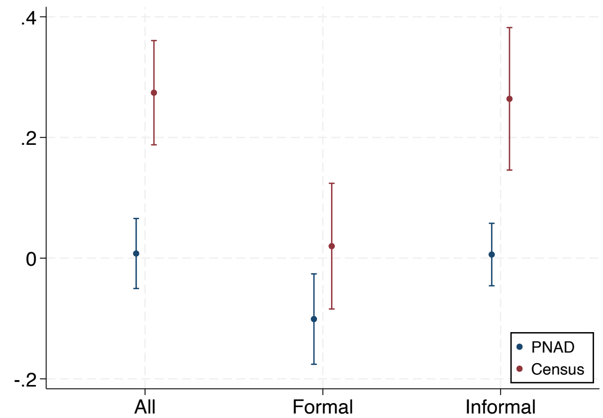


Figure 6: The effect of minimum wages on inequality

Notes: This figure reports estimates of the effects of minimum wage increases on the variance of overall, formal, and informal earnings, $\ln(V_{rt}) - \ln(V_{rt}^{\mathrm{CF}})$, obtained from equations (7) and (8). The 95% confidence intervals are based on 100 bootstrap replications, resampling locations with replacement. See Online Appendix C for implementation details. *Sources:* PNAD (1996-2012) and Census (2000, 2010).

and individuals, abstracting from potential general equilibrium effects of the policy absorbed by the control variables. Such general equilibrium effects are explicitly incorporated in the quantitative model and counterfactual exercises presented in the next sections.

2 Informality and the effects of the minimum wage

This section develops a stylized model to understand the effects of the minimum wage when the informal margin of adjustment is considered, before extending it in several directions in the quantitative analysis.[25] The model consists of ex-ante homogeneous workers and heterogeneous monopsonists that decide to operate formally or informally. Firms operating in the formal sector are subject to the minimum wage. Informal firms can evade the minimum wage but are subject to informality costs.

The model rationalizes the empirical findings from Section 1 on minimum wages, inequality, and informality. For general productivity distributions, if the minimum wage initially reduces aggregate inequality, there is always a range where further increases in the minimum wage have the *unintended consequence* of raising inequal-

[25] All the derivations in the following sections are detailed in Appendix B.

ity. This happens because of adjustments in the informal sector, as higher minimum wages lead to a sizable and unequal informal sector. In the Pareto case, we show that these unintended consequences emerge as soon as informality appears. Moreover, the informal adjustment margin also shapes how inequality evolves within the formal sector.

2.1 Labor supply

There exists a unit measure of ex-ante homogeneous households. Each agent inelastically supplies one unit of labor. Households receive wage offers and must choose, after the realization of firm-specific amenity shocks, which firm to work for (Card et al., 2018). We assume that firm profits and government revenues are rebated to households that consume the final good and do not participate in production.

The utility of an individual depends on their wages and the firm at which they work:

$$V_i(j) = A_i(j)w(j), \tag{9}$$

where $A_i(j)$ is an amenity shock household i gets for working in firm j, and $w(j)$ is their wage. We assume $A_i(j)$ is independently distributed across households and firms, and drawn from a Fréchet distribution with shape parameter η.

The structure of the amenity shocks generates an upward-sloping labor supply curve at the firm level. Moreover, the law of large numbers implies that firm j's labor supply curve equals the probability that household i optimally chooses to work for that firm:

$$l(j) = \Pr_i(j) = \left[\frac{w(j)}{W}\right]^\eta, \tag{10}$$

with $W \equiv \left[\int_{j' \in \Omega} w(j')^\eta dj'\right]^{1/\eta}$ denoting the aggregate wage index and Ω denoting the exogenous set of operating firms.

2.2 Labor demand

There is an exogenous mass of firms that are ex-ante heterogeneous with respect to labor productivity. The productivity distribution follows $z \sim F$ over the support $S = [z_{\min}, z_{\max}]$, with $0 \leq z_{\min} < z_{\max} \leq \infty$. We assume that F is absolutely continuous in $(z_{\min}, z_{\max})$ with density f that is locally bounded and satisfies $\mathbb{E}\big[(\log Z)^2 Z^\eta\big] < \infty$. Firms are perfectly competitive in the goods market,[26] producing homogeneous goods that are perfect substitutes, with the price of each normalized to one. In the labor market, firms compete monopsonistically.

The timing of the firm's problem is as follows. Conditional on productivity, firms decide on their formality status. In doing so, firms trade off minimum wages in the formal sector against the costs of operating in the informal sector. After the formality status is decided, firms maximize profits subject to the labor supply curve (10) and sector-specific constraints. At this stage, the monopsonistic competition assumption implies that larger firms must pay higher wages.

We start by calculating profits, employment, and wages conditional on the formality status. A firm with productivity z operating formally maximizes revenues net of labor costs, subject to the labor supply curve and the minimum wage ($\underline{w}$):

$$\pi^F(z) = \max_{\{l,w\}} \left\{ zl - wl \quad \middle| \quad l = \left(\frac{w}{W}\right)^\eta, \quad w \geq \underline{w} \right\}. \tag{11}$$

Optimal wages and labor of the formal firm are:

$$w^F(z) = \max\left\{\frac{\eta}{\eta+1}z, \underline{w}\right\}, \quad l^F(z) = W^{-\eta}\max\left\{\frac{\eta}{\eta+1}z, \underline{w}\right\}^\eta. \tag{12}$$

When unrestricted by the minimum wage, formal firms set wages as a constant markdown over the marginal product of labor. However, when the productivity of the firm is sufficiently small, the minimum wage becomes binding, and wages and labor no longer vary with firm productivity. Hence, the minimum wage operates as a fixed production cost for low-productivity firms.

By operating informally, a firm avoids the minimum wage but loses a share ρ of revenues. We refer to ρ as the informality cost. This could reflect government detection of informal activity, which could occur with probability ρ, and the penalty is assumed to be a loss of all revenues.[27] The informality cost could also reflect limited access to formal credit markets, which could reduce their productivity. The

[26] Online Appendix D.2 generalizes the results in this section for an environment of monopolistic competition and love for varieties.

[27] An alternative specification is that firms are detected with probability $\tilde{\rho} \in [0,1]$, in which case they lose a fraction $\gamma \leq 1$ of revenues. In this setting, expected revenues are $(1-\tilde{\rho})zl + \tilde{\rho}\gamma zl = [1-\tilde{\rho}(1-\gamma)]zl$. When $\rho = \tilde{\rho}(1-\gamma)$, revenues in this specification are the same as those in the main specification. Hence, changes in ρ reflect both changes in the probability of detection and changes in the share of revenue captured by the government upon inspection.

problem of an informal firm with productivity z is:

$$\pi^I(z) = \max_{\{l,w\}} \left\{ (1-\rho)zl - wl \quad | \quad l = \left(\frac{w}{W}\right)^\eta \right\}, \tag{13}$$

Optimal wages and labor of the informal firm are:

$$w^I(z) = \frac{\eta}{\eta+1}(1-\rho)z, \quad l^I(z) = W^{-\eta}\frac{\eta^\eta}{(\eta+1)^\eta}(1-\rho)^\eta z^\eta \tag{14}$$

Informal firms set wages as a constant markdown over the marginal product of labor. In this case, however, the marginal product of labor is affected by the informality cost, as it scales down productivity. The absence of fixed costs implies positive profits for all firms in the informal sector. That is not the case in the formal sector: firms with productivity below the minimum wage have negative profits. Hence, informality acts as a profitable outside option for firms.

Conditional on productivity, firms choose the formality status that maximizes profits: $\pi(z) = \max\{\pi^F(z), \pi^I(z)\}$. The corresponding labor demand and wages depend upon the formality decision, and are derived in Equations (12) and (14).

2.3 Equilibrium

An equilibrium consists of formality choices, wages, and employment for each firm, together with an aggregate wage index such that firms and households maximize, and the aggregate labor market clears:

$$L^D(W) \equiv \int_{z_{\min}}^{z_{\max}} l(z)f(z)dz = 1 = L^S. \tag{15}$$

The integral aggregates optimal labor demand, $l(z)$, over all firms, weighted by their respective densities, $f(z)$. The last equality arises because aggregate labor supply is inelastic.

We now discuss firm selection into the informal sector. Proposition 1 shows that the solution of the problem of the firm consists of two thresholds, $\underline{z} < \bar{z}$, where firms with $z < \underline{z}$ operate informally, firms with $z \in [\underline{z}, \bar{z}]$ are formal and restricted by the minimum wage, and firms with $z > \bar{z}$ operate formally and unrestricted by the minimum. Importantly, this stylized model environment generates bunching of workers at the minimum wage, a significant feature of the data (Machin et al., 2003).

Proposition 1. *There exist two thresholds linear in the minimum wage,*

$$\underline{z} = \varphi(\eta, \rho) \cdot \underline{w} \qquad \text{and} \quad \bar{z} = \frac{\eta + 1}{\eta} \cdot \underline{w} \tag{16}$$

such that:

1. $\underline{z}$ *solves:* $\frac{\eta^\eta}{(\eta+1)^{\eta+1}}(1-\rho)^{\eta+1}\underline{z}^{\eta+1} - \underline{w}^\eta \underline{z} + \underline{w}^{\eta+1} = 0$;
2. $\underline{w} \leq \underline{z} < \bar{z}$;
3. *Firms with* $z < \underline{z}$ *operate informally, firms with* $z \in [\underline{z}, \bar{z}]$ *are formal but restricted by* $\underline{w}$*, and firms with* $z > \bar{z}$ *are formal and unrestricted by* $\underline{w}$*; and*
4. $\frac{\partial \underline{z}}{\partial \rho} < 0$ *and* $\frac{\partial \underline{z}}{\partial \eta} < 0$.

Proof. See Appendix B for details. □

Why do unproductive firms become informal? Informal firms give up some productivity in exchange for lowering labor costs. When productivity is low, the reduction in labor costs on minimum wage workers more than compensates for those productivity losses. On the other hand, when firms are very productive, the productivity losses are too costly, so firms decide to comply with the minimum. The proposition also shows that larger minimum wages imply larger costs to operate formally, so a smaller share of firms will be productive enough to be formal. At the same time, smaller informality costs compensate firms for being informal, increasing the share of firms that will optimally do so.

Proposition 1 also shows that the aggregate wage index does not change relative profits across sectors. As a consequence, the equilibrium wage index is a markdown over a modified average productivity:

$$W = \frac{\eta}{\eta+1}\left[\int_{z_{\min}}^{\underline{z}} [(1-\rho)z]^\eta f(z)dz + \left[F(\bar{z}) - F(\underline{z})\right]\bar{z}^\eta + \int_{\bar{z}}^{z_{\max}} z^\eta f(z)dz\right]^{\frac{1}{\eta}}, \tag{17}$$

where the first term in brackets represents the average productivity of informal firms, the second represents the minimum wage constraints imposed on unproductive formal firms, and the third represents the average productivity of the unconstrained formal firms. A unique equilibrium exists, as $\lim_{b\to\infty}\int_{z_{\min}}^{b} z^\eta dF(z) < \infty$ and the wage index is finite.

2.4 Inequality, minimum wage, and the informal sector

This section examines the impact of the minimum wage on earnings inequality in the presence of informality. We show that the minimum wage can reduce formal inequality while increasing informal inequality and shifting workers from the formal to the informal sector. These opposing forces can lead to *unintended consequences* of the minimum wage, whereby a policy that intends to reduce inequality may paradoxically increase it by driving workers into the informal sector.

We begin by decomposing the aggregate variance of log earnings as:

$$V = s_I V^I + s_F V^F + s_I s_F (\Delta E)^2, \tag{18}$$

where s_I and $s_F = 1 - s_I$ are informal and formal employment shares, respectively, and $\Delta E \equiv E^I - E^F$ is the mean log-wage gap between sectors. This mirrors Equation (1) and highlights three key drivers of aggregate inequality: informal inequality weighted by the sector share ($s_I V^I$), formal inequality weighted by the sector share ($s_F V^F$), and between-sector wage dispersion.

The response of aggregate inequality to the minimum wage is:

$$\begin{aligned} \frac{\partial V}{\partial \underline{w}} &= s_F \cdot \frac{\partial V^F}{\partial \underline{w}} + s_I \cdot \frac{\partial V^I}{\partial \underline{w}} && \text{[Direct formal/informal effects]} \\ &+ \frac{\partial s_I}{\partial \underline{w}} \cdot \Big[(1 - 2s_I)(\Delta E)^2 + \Delta V \Big] && \text{[Composition effect]} \\ &+ s_I s_F \cdot \frac{\partial (\Delta E)^2}{\partial \underline{w}}, && \text{[Between-sector effect]} \end{aligned} \tag{19}$$

where $\Delta V \equiv V^I - V^F$. Equation (19) decomposes the marginal effect of the minimum wage on the variance of log earnings into three parts. First, the direct within-sector effects. An increase in the minimum wage typically reduces formal inequality while increasing informal inequality, with the former being the focus of existing literature. Second, the composition effects induced by an expanding informal sector. These have ambiguous consequences. For a small informal sector, the contribution of the difference in means is positive, while the contribution of the difference in variances is only positive for a sufficiently unequal informal sector. Third, the changes in mean wages across sectors. The overall effect on aggregate inequality is ambiguous and depends on which component dominates.

Still, when the informal margin of adjustment is considered, the minimum wage can be a limited tool to address inequality. If introducing the minimum wage can reduce aggregate inequality, Proposition 2 shows that continued reliance on this policy will eventually exhaust its effectiveness. At some point, the minimum wage will have the *unintended consequence* of increasing aggregate inequality. This is a general property that relies on a combination of informal sector expansion, a growing contribution from wage differences between formal and informal workers, and an increase in informal variance.

Proposition 2. *If there exists* $\underline{w}_1 > 0$ *such that* $V(\underline{w}_1) < V(\underline{w} = 0)$*, then:*

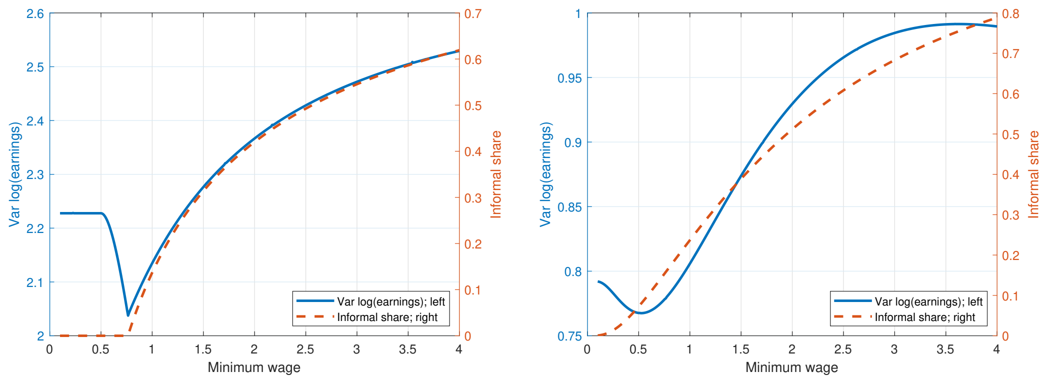
1. *There exists a global inequality minimum* $\underline{w}^*$*; and*
2. *There exists an interval* $(\underline{w}^*, \underline{w}^{**})$ *of unintended consequences, where the minimum wage increases aggregate inequality:* $V'(\underline{w}) > 0, \forall \underline{w} \in (\underline{w}^*, \underline{w}^{**})$.

Proof. See Appendix B for details. □

In what follows, we assume $z \sim \text{Pareto}(\kappa)$, with $\kappa > \eta$. Given the bounded support of this distribution, three regimes arise as a function of the minimum wage. First, when the minimum wage is low, it never binds in the formal sector and there is no informality. Second, the minimum wage binds but there is still no informality. Third, informality emerges, and three types of firms coexist: informal firms, formal firms constrained by the minimum, and unconstrained formal firms. In this regime, the unintended consequences of the minimum wage appear.

Appendix Proposition 5 shows that informality can affect the relationship between the minimum wage and inequality even within the formal sector. Without informality, aggregate inequality equals formal inequality, which strictly declines with the minimum wage as workers bunch at the minimum and the distribution compresses from below. With an informal sector, two patterns emerge. First, informal inequality increases with the minimum wage. Second, formal inequality no longer responds to the minimum. Because the cutoffs $\underline{z}$ and $\bar{z}$ move linearly with the minimum wage, the rate at which formal workers bunch at the minimum is exactly offset by the rate at which formal firms exit to informality. Hence, the share of formal minimum wage workers among all formal workers remains constant, and the minimum wage no longer generates bunching in the formal distribution.[28]

[28] This second property is a feature of the Pareto distribution. For distributions with faster tail decay, such as the Log-Normal in Figure 7, the minimum wage can still lower formal inequality when the informal sector exists. However, as established by Proposition 2, any initial inequality reduction is still followed by a region of unintended consequences.



(a) Pareto distribution (b) Log-Normal distribution

Figure 7: Minimum wages, inequality and the informal share of labor

Notes: Aggregate variance of log earnings and informal employment for different values of the minimum wage. Simulation of the model in Section 2 with Pareto and Log-Normal productivity distributions. For the Pareto, we used location parameter 1 and shape parameter $\kappa = 1.67$. For the Log-Normal, we used a mean of zero and a standard deviation of 0.89. The other parameters values are: $\eta = 1$; $\rho = 0.15$; and $\underline{w} \in [0.1, 4]$. *Sources:* Model simulations.

A key limitation in Proposition 2 is that it does not fully characterize the link between the size of the informal sector and the unintended consequences of the minimum wage. Figure 7 illustrates this relationship for both the Pareto and Log-Normal cases, by plotting aggregate inequality and informality against different values of the minimum wage. The figure shows that unintended consequences emerge soon after informality appears, and they persist until most workers in the economy are informal.

While this stylized framework clarifies the core mechanisms, it also involves strong simplifications. In particular, the sharp wage separation between sectors likely overstates between-sector variance, and worker homogeneity abstracts from skill-based sorting observed in the data. The following sections address these limitations in a richer quantitative framework, which we use to quantify the unintended consequences of the minimum wage.

3 Quantitative extension

This section describes the extended model, used to quantify the general equilibrium effects of the minimum wage. Consistent with empirical evidence for Brazil, the additional features are important in shaping informality, inequality, and the way in which the minimum wage influences the economy. On the household side, we assume that workers differ in their skill levels and that formal wages may be worth more (or less) than informal wages due to the valuation of labor legislation. On the firm side, we introduce decreasing returns to scale, payroll tax rates, and allow for the possibility that the productivity distributions in the formal and informal sectors overlap.

3.1 Labor supply

There is a unit measure of workers who differ in their skill level. In particular, there are H different skill levels, and N_h denotes the fraction of workers with skill level h. We maintain the assumption that profits and government revenues are rebated to households that consume the final good but do not participate in production.

The utility of worker i, of skill h, working at firm j is:

$$V_{ih}(j) = A_i(j) \cdot (1 + \varsigma_h(j)) \cdot w_h(j), \tag{20}$$

where $\varsigma_h(j) = 0$ if firm j is informal and $\varsigma_h(j) = \varsigma_h$ if formal. This formulation allows for a wedge between nominal and perceived wage value due to formal sector benefits (e.g., vacation pay, unemployment insurance). Thus, one dollar of formal earnings may be worth more (or less) than one dollar of informal earnings.

The combination of minimum wages and decreasing returns to scale at the firm level generates a possibility of labor rationing by low productivity formal firms (Berger et al., 2025). Therefore, we introduce a "congestion wedge" $r_h(j) \leq 1$ on firm j, to be determined in equilibrium. It reflects the possibility that a worker could have firm j as her preferred employer and still be rationed out of that firm.[29]

The structure of amenity shocks is the same, so the labor supply curve firm j faces in the market for skill h is:

$$l_h^s(j) = N_h \left[\frac{(1 + \varsigma_h(j)) w_h(j)}{W_h} \right]^\eta, \quad W_h = \left[\int_{j \in \Omega} [(1 + \varsigma_h(j)) r_h(j) w_h(j)]^\eta \, dj \right]^{\frac{1}{\eta}}, \tag{21}$$

[29]See Berger et al. (2025) for a more detailed discussion on labor rationing in monopsonistic models of the minimum wage, and how that affects labor supply decisions of households. Appendix C shows that this individual-level problem is equivalent to the problem of a representative family allocating its members, taking as given firms' rationing constraints.

with W_h being the wage index for skill h, which incorporates the "congestion wedges" to guarantee labor market clearing. Appendix C shows that the welfare of worker with skill h is proportional to their respective wage index W_h.

3.2 Labor demand

There is a fixed mass of firms with heterogeneous labor productivity. We assume that productivity has two components, $z = \nu\theta$, each drawn independently from its respective distribution F_ν and F_θ. Labor markets are segmented by skill, and firms compete monopsonistically in each of them. We assume that firms aggregate labor from different skills linearly and under decreasing returns to scale to produce a single, homogeneous good, sold under perfect competition.[30]

The timing of the problem of the firm is as follows. First, firms draw ν. Conditional on ν, firms decide whether to be formal or informal. Formal firms are subject to the minimum wage and payroll taxes, applied to all workers. Informal firms are subject to informality costs. After they decide on the formality status, firms draw θ (hence, z is realized). Conditional on z, firms maximize profits subject to skill-specific labor supply curves and sector-specific constraints.

As in the last section, we start by discussing the problem of the firm conditional on formality status and labor productivity z. A formal firm has profits:

$$\pi^F(z) = \max_{\{l_h(z), w_h(z)\}_h} \left\{ \sum_h z\xi_h^F(z) l_h(z)^\alpha - (1+\tau) \sum_h w_h(z) l_h(z) \right\} \tag{22}$$

$$\text{s.t.} \quad l_h(z) \leq l_h^s(z) = N_h \left[\frac{(1+\varsigma_h) w_h(z)}{W_h} \right]^\eta, \quad w_h(z) \geq \underline{w} \quad \forall h, \tag{23}$$

where τ is the payroll tax rate, α is a scale parameter, and $\xi_h^F(z)$ represent skill-specific demand shifters of formal firms. These demand shifters capture potential skill biases in the production function, and will be allowed to change over time to capture skill-biased technological change.

Proposition 3 shows that there exists a threshold solution for the formal firm problem, where wages are either the minimum wage or a markdown over the marginal product of labor. Moreover, due to decreasing returns to scale, firms with low productivity can optimally choose to ration hiring (i.e., hire strictly below their

[30] Linearity across skill types follows Engbom and Moser (2022) and Berger et al. (2025), whereas decreasing returns to scale follows Ulyssea (2018) and Berger et al. (2025).

labor-supply curve at $\underline{w}$).

Proposition 3. *For each skill level h, there exist two unique productivity thresholds $\underset{\sim}{z}_h$ and $\overline{z}_h$ such that, for formal firms:*

i) If $z \geq \overline{z}_h$, wages are a constant markdown over the marginal product of labor, and firms hire at their labor supply curve

$$\begin{aligned} w_h^F(z) &= \frac{W_h}{1+\varsigma_h}\left(\frac{(1+\varsigma_h)\alpha\eta z\xi_h^F(z)}{(1+\tau)W_h(\eta+1)N_h^{1-\alpha}}\right)^{\frac{1}{1+\eta(1-\alpha)}}, \\ l_h^F(z) &= N_h\left[\frac{(1+\varsigma_h)w_h(z)}{W_h}\right]^{\eta}. \end{aligned} \tag{24}$$

ii) If $\underset{\sim}{z}_h \leq z \leq \overline{z}_h$, wages equal the minimum wage and firms hire at their labor supply curve

$$w_h^F(z) = \underline{w}, \quad l_h^F(z) = N_h\left[\frac{(1+\varsigma_h)\underline{w}}{W_h}\right]^{\eta}. \tag{25}$$

iii) If $z < \underset{\sim}{z}_h$, wages equal minimum wage and firms ration. That is, they hire strictly less than their labor supply curve at the minimum wage

$$w_h^F(z) = \underline{w}, \quad l_h(z) = \left(\frac{\alpha z\xi_h^F(z)}{(1+\tau)\underline{w}}\right)^{\frac{1}{1-\alpha}} < N_h\left[\frac{(1+\varsigma_h)\underline{w}}{W_h}\right]^{\eta}. \tag{26}$$

Proof. See Appendix B for details. □

Informal firms are not subject to minimum wages or payroll taxes. However, they still lose a fraction ρ of their revenue—the informality costs. An informal firm with productivity z has profits:

$$\pi^I(z) = \max_{\{l_h(z),w_h(z)\}_h}\left\{\sum_h (1-\rho)z\xi_h^I(z)l_h(z)^{\alpha} - \sum_h w_h(z)l_h(z)\right\} \tag{27}$$

$$\text{s.t.} \quad l_h(z) \leq l_h^s(z) = N_h\left[\frac{w_h(z)}{W_h}\right]^{\eta} \quad \forall h. \tag{28}$$

Proposition 4 details the closed-form solution for the informal firm's problem. Like unconstrained formal firms, informal firms set wages as a constant markdown over marginal labor productivity and will never hire less than what is allowed by their labor supply curve.

Proposition 4. *Optimal wages and labor of informal firms for each skill $h = 1, ..., H$:*

$$w_h^I(z) = W_h \left(\frac{\alpha\eta(1-\rho) z \xi_h^I(z)}{W_h(\eta+1) N_h^{1-\alpha}} \right)^{\frac{1}{1+\eta(1-\alpha)}}, \quad l_h^I(z) = N_h \left[\frac{w_h(z)}{W_h} \right]^{\eta} \tag{29}$$

Proof. See Appendix B for details. □

3.3 Formality decision

Given the signal ν, firms choose their formality status before observing θ. Firms choose to be formal if their expected formal profits $\Pi^F(\nu) = \int \pi^F(\nu\theta) dF_\theta(\theta)$ exceed their expected informal profits $\Pi^I(\nu) = \int \pi^I(\nu\theta) dF_\theta(\theta)$. This two-stage process generates an overlap in the productivity distribution of firms in the formal and informal sectors, leading to an overlap in the wage distributions in the two sectors, a predominant feature in the data.

3.4 Equilibrium

An equilibrium consists of formality choices, wages, and employment levels for each firm, together with wage indices for each skill level, such that:

1. Firms optimally choose their formality status. Given this choice, wage indices W_h, and perceived labor supply curves, firms maximize profits (Propositions 3 and 4).

2. Labor markets clear for each skill level $h = 1, \ldots, H$.

Online Appendix D.3 calculates the market-clearing condition in the goods markets. Appendix C derives the labor supply functions, the formulas for the equilibrium wage indices that take labor rationing into account, and the connection between these wage indices and worker welfare from worker-side primitives, as in Berger et al. (2025). Appendix D details the algorithm used to solve for the equilibrium numerically.

4 Calibration and validation

This section calibrates the quantitative model to Brazilian data from 1996. We classify parameters into two groups. Externally calibrated parameters are obtained

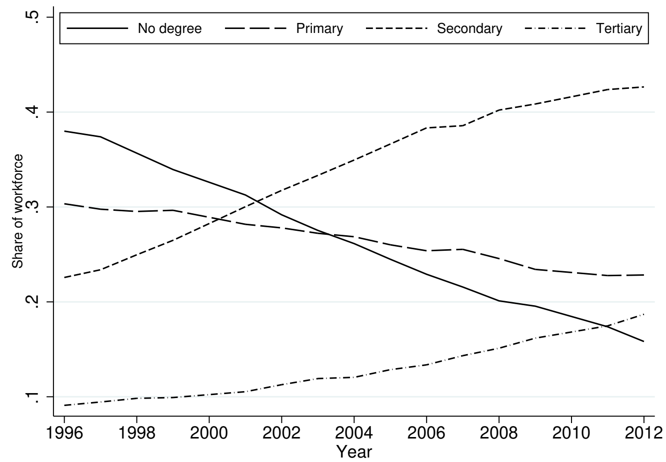


Figure 8: Skill composition of Brazilian labor force, 1996-2012

Notes: Share of labor force that belongs to each education group. *Sources:* 1996-2012 PNAD.

directly from official statistics or recent literature. Internally calibrated parameters are chosen to minimize the distance between model and data moments. We then externally validate the model and show that it delivers realistic earnings distributions for the overall economy, within each sector, and within each skill group.

4.1 External calibration

The externally calibrated parameters are: (i) H, the number of skill groups; (ii) N_h, the skill composition of the workforce; (iii) η, the elasticity of the labor supply curve; (iv) τ, the payroll tax rate; and (v) ς_h, workers' valuation of formal wages.

We map skill groups to education categories in the data. Specifically, we construct four education groups ($H = 4$): workers with no degree (4 years of schooling or less), a primary degree (5–8 years), a secondary degree (9–11 years), or a tertiary degree (12 or more years). Figure 8 plots the share of each group from 1996 to 2012, which maps directly into N_h. Educational attainment rose substantially over this period, as the share of workers without a degree (N_1) declined from 38% in 1996 to 16% in 2012.

The definition of skills above captures substantial heterogeneity in the earnings distribution. For example, Appendix Figure A.17 plots the distribution of log earnings relative to the minimum wage for 1996 and 2012 across different skill groups.

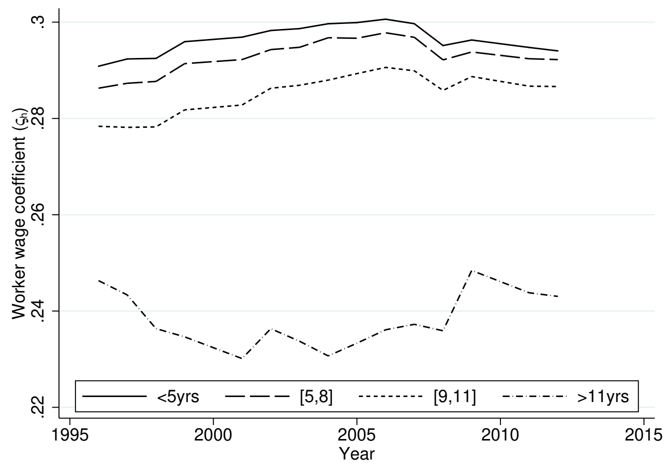


Figure 9: Workers' valuation of wages (ς_h), 1996-2012

Notes: Valuation of gross wages, estimated from labor legislation using the methodology in Souza et al. (2012). *Sources:* 1996-2012 PNAD and Brazilian labor legislation.

Even though there is substantial overlap, workers with tertiary degree earn, on average, four times more than non-degree workers, a pattern that the calibration of the model will capture.

The firm-level labor supply elasticity is set to $\eta = 1$, following Felix (2022). This value implies that a firm must raise wages by 1 percent to expand employment by 1 percent when facing labor supply constraints. It also corresponds to a markdown of 2, meaning workers capture only half of the marginal value they produce: for each additional dollar of value generated, they receive 50 cents ($\eta/(\eta + 1) = 0.5$).

We assume that the formal firm has a total labor cost of $1 + \tau$ times its gross wage bill. This takes into consideration the fact that a firm must pay vacation stipends, social security contributions, severance payments, and other transfers to its workers. Online Appendix E adapts the method from Souza et al. (2012) and estimates $\tau = 71.4\%$.

Lastly, Figure 9 shows the values of ς_h, estimated following Souza et al. (2012) and detailed in Online Appendix E. Two patterns stand out. First, $\varsigma_h > 0$ for all h, indicating that benefits such as vacation pay and unemployment insurance more than offset income taxes and mandatory social security contributions. Second, the nominal–real wage gap is 30% for no-degree workers and 24% for tertiary-educated workers, reflecting the progressivity of Brazil's social security and income tax systems.

4.2 Internal calibration

We start by specifying the functional forms for the productivity distributions, F_ν and F_θ, and the skill-specific demand shifters, $\xi_h^j(z)$ for $j \in \{F, I\}$.

The first productivity component, ν, is drawn from a Log-Normal distribution where the underlying Normal has mean zero and standard deviation σ. The second productivity component, θ, is drawn from a Pareto distribution with shape parameter κ^j that depends on the firm's formality status, $j \in \{F, I\}$. This yields a Pareto Log-Normal distribution of firm productivity in each sector, first introduced in Colombi (1990) and subsequently used in the literature (Rothschild and Scheuer, 2016; Ulyssea, 2018).

We incorporate skill bias in the technology by assuming that skill-specific demand shifters depend on firms' productivity and formality status. We follow Burstein and Vogel (2017) and assume:

$$\xi_h^j(z) = \frac{z^{\phi_h^j}}{\sum_k z^{\phi_k^j}}, \quad \sum_h \phi_h^j = 0, \quad h \in \{1, 2, 3, 4\}, \quad j \in \{F, I\}. \tag{30}$$

When $\phi_h^j > 0$, more productive firms are more intensive in skill h.

The internally calibrated parameters are: (i) α, the degree of decreasing returns to scale; (ii) σ, the standard deviation of the Log-Normal productivity component; (iii) κ^F and κ^I, the shape parameters of the Pareto productivity component; (iv) $\underline{w}$, the real value of the minimum wage; (v) ρ, the informality cost; and (vi) ϕ_h^j, the demand shifter parameters. Since the demand shifters sum up to zero, we have a total of 12 free parameters.

The targeted moments are: (i) the mean earnings ratio across skills (3 moments); (ii) the mean earnings ratio between formal to informal workers (1 moment); (iii) the variance of log earnings by skill (4 moments); (iv) the variance of formal and informal log earnings (2 moments); (v) the fraction of formal workers at the minimum wage by skill (4 moments); (vi) the ratio between the minimum wage and the mean formal wage (1 moment); (vii) the informal shares by skill (4 moments); and (viii) the labor share of GDP (1 moment). These result in a total of 20 linearly independent moments.

We calibrate the model using a simulated method of moments via indirect infer-

Table 3: Parameters of the model (1996)

External calibration			Internal calibration		
Parameter	Description	Value	Parameter	Description	Value
H	# of skills	4	α	Returns to scale	0.85
N_h	Skill supply	Figure 8	σ	ν std. dev.	0.89
η	LS elasticity	1	κ^F	θ^F shape	1.69
ς_h	Earnings tax	Figure 9	κ^I	θ^I shape	1.65
τ	Payroll tax	71.4%	ϕ_h^F, ϕ_h^I	Demand shifters	Figure 10
			$\underline{w}$	Minimum wage	0.10
			ρ	Informality cost	0.15

Notes: This table reports the values of the model parameters used in the simulations. Externally calibrated parameters are set using direct data counterparts or estimates from the literature, while internally calibrated parameters are chosen to ensure that the model replicates selected moments in the data. *Sources:* 1996 PNAD and model simulations.

ence, assigning greater weight to moments central to our analysis, such as the size of the informal sector.[31] Table 3 reports the resulting parameter values, and Table 4 compares the targeted moments in the model to the data. The model matches most targets closely. Remaining discrepancies are concentrated in earnings inequality and the informal share by skill level: in both cases, the model reproduces the skill gradient but slightly overstates the inequality gradient and understates the informality gradient.

Following Ottonello and Winberry (2020), Appendix Figures A.18 and A.19 report, respectively, the elasticity of the objective function to each parameter and the sensitivity matrix of Andrews et al. (2017). Two findings emerge. First, moments respond to parameters in intuitive ways: for example, increasing ρ reduces the equilibrium size of the informal sector. Second, the absence of linearly dependent columns in the sensitivity matrix (i.e., the sensitivity of parameter estimates to misspecified moments) confirms that all parameters are identified in the data. Finally, Appendix Figure A.20 shows that small deviations from the calibrated parameters raise the value of the objective function, indicating that the calibration is at a local minimum.

The calibration yields $\alpha = 0.85$. This value lies between the estimate of 0.94 in

[31] Appendix D details the calibration procedure.

Table 4: Model and data moments

Moment	Data	Model	Moment	Data	Model
Variance of log earnings			(Formal) Fraction at $\underline{w}$		
Overall	0.78	0.73	Overall	0.08	0.06
Formal	0.65	0.61	No degree	0.13	0.10
Informal	0.66	0.62	Primary	0.08	0.06
No degree	0.54	0.45	Secondary	0.05	0.03
Primary	0.54	0.58	Tertiary	0.01	0.00
Secondary	0.64	0.75			
Tertiary	0.91	1.28	(Formal) $\frac{\text{Min. wage}}{\text{Mean wage}}$	0.22	0.24
Mean earnings			Informal share		
Formal/Informal	2.06	2.01	Overall	0.39	0.38
Primary/No degree	1.39	1.28	No degree	0.52	0.41
Secondary/Primary	1.46	1.38	Primary	0.37	0.38
Tertiary/Secondary	2.49	2.53	Secondary	0.26	0.36
			Tertiary	0.22	0.31

Notes: This table reports the model and data moments targeted in the calibration. *Sources:* 1996 PNAD and model simulations.

Berger et al. (2025) using a similar framework in the context of the United States, and the estimate of 0.6 in Ulyssea (2018), which models the informal sector in Brazil but abstracts away from monopsony power. The lower value of α relative to estimates for the United States is consistent with the smaller average firm size in Brazil (Eslava et al., 2024). In fact, although not directly targeted, our baseline model estimates that 61% of firms in Brazil are informal, broadly consistent with the 68% reported in Ulyssea (2018).

Regarding the productivity distributions, the calibration yields $\sigma = 0.89$, $\kappa^F = 1.69$, and $\kappa^I = 1.65$. While we allow the Pareto shape parameter κ to differ between the formal and informal sectors, the estimated values are similar in magnitude. Allowing for sector-specific κ is nonetheless important to obtain a good fit in the 2012 calibration (Appendix E), which we use to back out counterfactual 2012 values for parameters not directly observed in the data—such as the informality cost ρ—that feed into the joint counterfactual exercises presented in the next sections.

The calibration yields values of 0.10 for the minimum wage and 0.15 for the

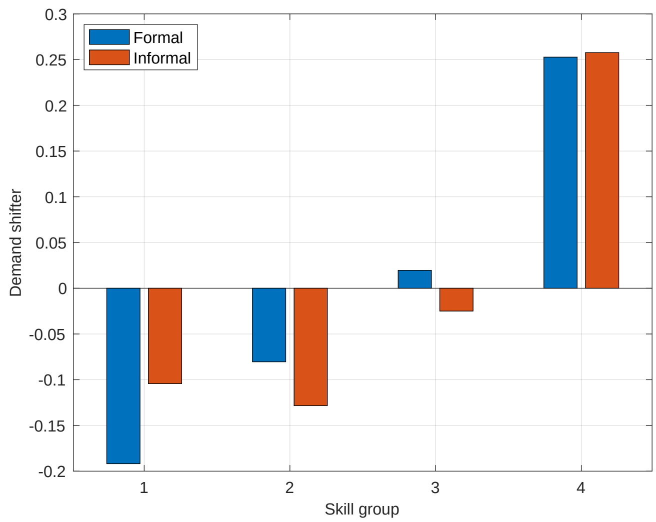
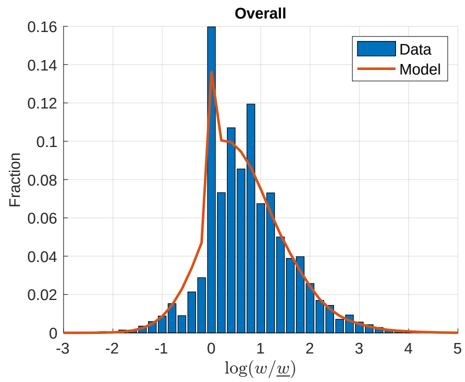


Figure 10: Calibrated demand shifter parameters (ϕ_h)

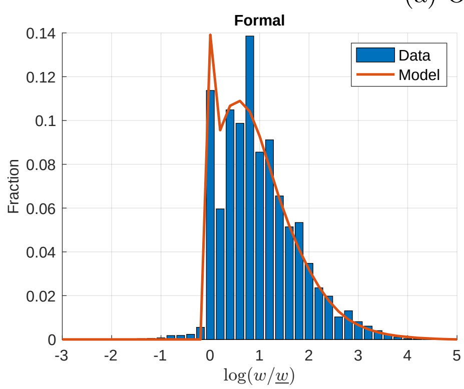
Sources: 1996 PNAD and model simulations.

informality cost. To interpret the informality cost ρ—together with the shape parameters κ^F and κ^I—we compare effective productivity in the formal sector (z) with that in the informal sector ($(1-\rho)z$). The mean of the formal effective productivity distribution, measured before the formality decision, is 15% higher than that of the informal distribution. Naturally, selection of firms into formality amplifies productivity differences: the average formal firm is 4.5 times more productive than the average informal firm in equilibrium. This productivity gap, consistent with observed differences in technology adoption and market access, helps explain the positive formal-informal wage premium replicated by the model in the internal calibration.

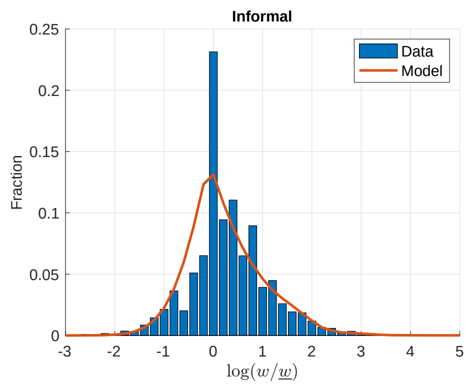
Lastly, Figure 10 plots the estimated demand shifters by skill for the formal and informal firms. In both sectors, more productive firms allocate a larger share of employment to higher-skill labor while reducing their reliance on lower-skill labor. The skill bias is stronger in the formal sector, where firms have a larger positive bias toward the highest skill group. This is consistent with formal firms' greater propensity to adopt advanced technologies and management practices—often to comply with regulatory standards or integrate into supply chains—that are complementary to skilled labor, thereby raising the marginal productivity of high-skill workers (Amaral and Quintin, 2006; Bloom et al., 2013; Ulyssea, 2018).



(a) Overall



(b) Formal



(c) Informal

Figure 11: Log earnings histogram

Notes: Histograms of log earnings relative to the minimum wage. Blue bars represent the data, and red lines represent the model. *Sources:* 1996 PNAD and model simulations.

4.3 External validation

We now show that the calibrated model generates realistic earnings distributions. This gives us confidence in using the model to study quantitatively the impact of the minimum wage on inequality at the aggregate level. We do so by comparing data and model-generated histograms of log earnings relative to the minimum wage. Figure 11 displays histograms for the aggregate, formal, and informal distributions of earnings in 1996. Panel (a), which looks at the aggregate distribution, shows that the model economy generates similar moments other than the mean and the variance. Moreover, the figure suggests that the Pareto Log-Normal assumption for the productivity distribution, which ultimately shapes the wage distribution in the model, provides a good approximation for the lower and upper tails of earnings.

The bottom-most plots compare the earnings distribution within the formal and informal sectors in the model and data. The model accurately captures the bunching at the minimum wage in the formal sector and is again capable of generating realistic earnings distributions. The same is true for the informal sector, although there is less bunching at the minimum wage in the model than in the data, a phenomenon highlighted in Derenoncourt et al. (2021). Lastly, Appendix Figure A.21 analyzes the within-skill earnings distribution and shows that, yet again, the model economy reproduces realistic earnings distributions along this dimension.

5 Main counterfactuals

This section evaluates the role of the minimum wage in shaping the aggregate distribution of earnings and the informal share of labor. We simulate a 105% increase in the real value of the minimum wage—mirroring the observed rise from 1996 to 2012—holding all other parameters fixed at their 1996 values. We then assess how changes in the informality cost and skill composition can complement the minimum wage in tackling earnings inequality. We calibrate the model to Brazil in 2012 to back out counterfactual values of the informality cost (see Appendix E for details) and use the 2012 skill distribution directly from the data.

5.1 Minimum wages, inequality, and the informal sector

We start by analyzing the impact of the minimum wage on the formal sector. The second column of Table 5 shows that the minimum wage increase accounts for 32% of the observed decline in formal earnings inequality, consistent with the findings in Engbom and Moser (2022). Moreover, Figure 12 shows that the minimum wage increase has spillover effects up to the 50th percentile of the formal wage distribution. All else equal, raising the minimum wage increases the p10p90 ratio by 19.7%, the p25p90 ratio by 7.3%, and the p50p90 ratio by around 1%, with little effects beyond the median. These spillovers take place due to imperfect labor market competition and heterogeneous firm exposure to the minimum wage.[32]

At the same time, the increase in the minimum wage had sizeable effects on the informal sector. As formal employment became substantially more costly, the share of informal workers rose from 38% to 78%, and the variance of informal earnings

[32] See also Engbom and Moser (2022) and Berger et al. (2025), who document similar spillover patterns along the formal wage distribution in Brazil.

Table 5: The effects of the minimum wage

	1996	$\underline{w}$	2012
V(log earnings)			
Overall	0.73	0.76	0.54
Formal	0.61	0.54	0.39
Informal	0.62	0.70	0.59
Fraction at $\underline{w}$	0.06	0.12	0.18
Informal share	0.38	0.78	0.34
Wage indices (W_h)			
No degree	0.15	0.17	0.23
Primary	0.17	0.18	0.23
Secondary	0.20	0.21	0.33
Tertiary	0.30	0.33	0.39

Notes: The first and last columns of the table report moments of the model calibrated to 1996 and 2012 data, respectively, while the middle column presents the counterfactual exercise in which the 1996 economy's minimum wage is raised to its 2012 level. Appendix E details the calibration of the model for 2012. *Sources:* Model simulations.

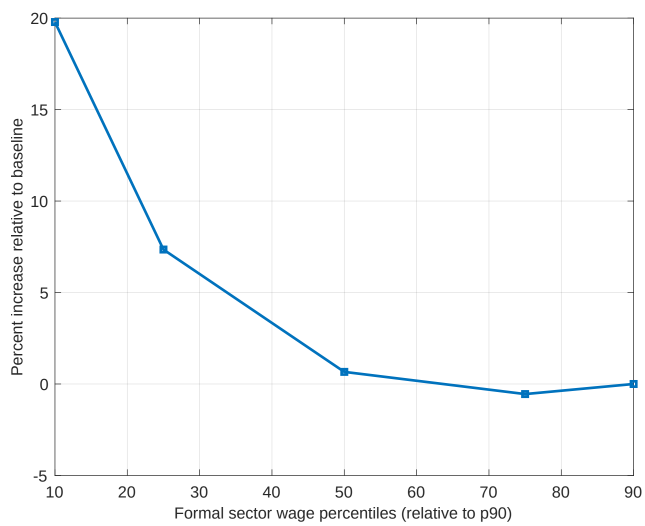


Figure 12: The effects of the minimum wage along the formal earnings distribution

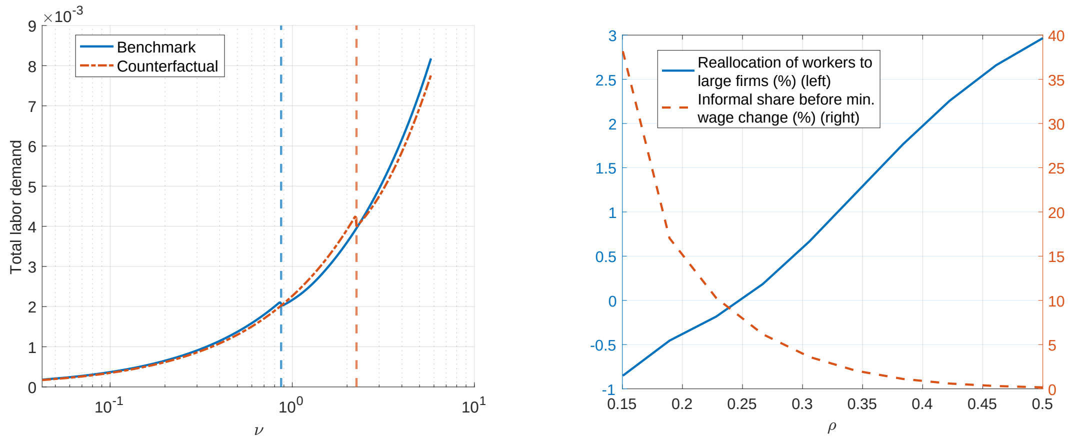
Notes: Percent increase in different percentile ratios, relative to the 1996 baseline. All percentile ratios are relative to the 90th percentile. *Sources:* Model simulations.

increased from 0.62 to 0.70. Two considerations help reconcile this sharp rise in informality. First, informality was already widespread in 1996: four of Brazil's 27 states had informal shares above 60%, with the highest exceeding 75% (Appendix Figure A.22). Second, the simulated 105% real increase in the minimum wage ranks among the largest in the literature. For comparison, recent studies report increases of 24% in Morocco (Paul-Delvaux, 2024), 17% in the UK (Giupponi et al., 2024), and 60% in Hungary (Harasztosi and Lindner, 2019). Hence, absent improvements in technology, informality cost, or skill composition—all key to reducing informality in Brazil (Haanwinckel and Soares, 2021)—–it is plausible that the national informality rate could have exceeded 70% by 2012.

Taken together, our results suggest that Brazil's minimum wage hike had the *unintended consequence* of increasing overall earnings inequality. The rise in informality and informal inequality more than offsets the inequality-reducing effect in the formal sector, leading to a 4.1% increase in aggregate earnings inequality. At the same time, by raising wages in the formal sector, the minimum wage intensifies competition for labor, pushing up wages economy-wide and improving ex-ante welfare for all workers. Welfare gains are most significant at the bottom and top of the skill distribution, reflecting relative bunching at the new minimum wage and heterogeneous demand for skills in the formal and informal production functions.

We now examine how the minimum wage reallocates workers across firms. Panel (a) of Figure 13 shows average firm size by productivity ν. As expected, more productive firms employ more workers. However, formal firms near the formality threshold employ fewer workers than similarly productive informal firms due to payroll tax burdens. When the minimum wage rises, the formality threshold shifts rightward, pushing marginal firms into informality. These newly informal firms regain market power and expand, drawing workers away from both larger formal firms and smaller informal ones.

This pattern contrasts with Engbom and Moser (2022), where the minimum wage reallocates labor from smaller to larger firms. To unpack this difference, Panel (b) of Figure 13 plots changes in employment at large firms—those that remain formal in the baseline counterfactual—as a function of ρ. Under our baseline $\rho = 0.15$, the number of workers at large firms falls. As ρ increases—and informality becomes less attractive—reallocation toward large firms becomes stronger. This indicates that the reallocation margin highlighted by Engbom and Moser (2022) is present



(a) Workers across ν

(b) Reallocation to large firms for different informality levels

Figure 13: Reallocation of workers following a minimum wage increase

Notes: Panel (13a) shows the proportion of the economy's workforce allocated to firms according to their ν productivity component before (blue) and after (red) the minimum wage increase. Vertical lines show the formality thresholds. The blue line in Panel (13b) shows the change (after vs. before the policy) in the share of workers allocated to large firms—defined as those that remain formal in the baseline minimum wage counterfactual—across models with different informality costs (ρ). The red line plots the informal labor share before the minimum wage increase. *Sources:* Model simulations.

but quantitatively limited unless informality costs are large.[33] In short, informality serves as a fallback that allows low-productivity firms to remain competitive by not complying with the minimum wage. These firms absorb labor that would otherwise shift to more productive formal firms, making the rise in informality a natural and quantitatively significant response to minimum wages in low informality cost environments.

5.2 The role of informality cost and skill supply

Over the 2000s, Brazil experienced government efforts to curb informality (Corseuil et al., 2012) and sharp improvements in education levels (Table 1). Motivated by these trends, Table 6 studies how changes in the informality cost and skill composition complement the minimum wage in reducing earnings inequality. The third and fourth columns report the joint effects of changing the minimum wage alongside in-

[33] Appendix F shows that this result holds in an extended version of the model that incorporates unemployment and is closer to the framework in Engbom and Moser (2022).

Table 6: Joint counterfactuals: minimum wages, informality costs and skill composition

	1996	Counterfactuals: Only $\underline{w}$	Counterfactuals: $\underline{w}$ and ρ	Counterfactuals: $\underline{w}$ and N_h
V(log earnings)	0.73	0.76	0.62	0.80
By formality status				
Formal	0.61	0.54	0.36	0.58
Informal	0.62	0.70	0.52	0.71
By skill				
No degree	0.45	0.54	0.39	0.46
Primary	0.58	0.59	0.50	0.54
Secondary	0.75	0.76	0.65	0.70
Tertiary	1.28	1.28	1.18	1.23
Variance decomposition				
Within skill	0.63	0.67	0.55	0.73
Between skills	0.09	0.08	0.07	0.08
Fraction at $\underline{w}$	0.06	0.12	0.49	0.23
Informal share	0.38	0.78	0.17	0.56

Notes: This table shows the counterfactual effects of a joint change in the minimum wage and the informality cost, and the minimum wage with the skill composition. The 1996 and $\underline{w}$ columns replicate the results in Table 5. The "$\underline{w}$ and ρ" column evaluates the effects of changes in both the minimum wage and the estimated increase in the informality cost. The "$\underline{w}$ and N_h" analyzes the joint effects of the minimum wage increase and the improvement in the skill composition. *Sources:* Model simulations.

formality costs and skill composition, respectively. The sharp rise in the calibrated ρ—from 0.15 in 1996 to 0.34 in 2012—reduces the informal sector to 17% of the workforce while increasing the bunching of formal workers at the minimum wage. The smaller informal sector exhibits lower inequality, while the expanded formal sector shows reduced dispersion due to increased bunching. As a result, aggregate inequality declines from 0.76 to 0.62. These findings underscore the importance of aligning formalization policies with minimum wage adjustments to avoid unintended increases on inequality.

The last column of Table 6 shows that the shift in the skill composition towards a more educated workforce helped curb the spike in informality in response to the minimum wage (Haanwinckel and Soares, 2021). Low productivity firms select themselves into the informal sector (Ulyssea, 2018). These firms are more intensive in low-skilled workers. As this factor of production becomes more scarce, its wages go up, and operating informally becomes relatively more expensive. As a result, the size of the informal sector decreases from 78% in the "Only $\underline{w}$" scenario to 56% in the "$\underline{w}$ and N_h" scenario. At the same time, the share of formal workers bunched at the minimum wage increases from 12% to 23%. Consequently, earnings inequality *within* each skill group goes down, with no-degree workers experiencing the most substantial reduction as they are more exposed to the minimum wage.

However, aggregate earnings inequality increases from 0.76 to 0.80 when we increase the minimum wage and improve the skill composition. As discussed above, this does not reflect higher inequality within skill groups. Table 6 also shows that this does not reflect substantial changes in inequality *between* different skill groups, which stays constant at 0.08. Rather, the increase in earnings inequality reflects a compositional shift toward higher skill groups that *inherently display more wage dispersion*, even before the rise in the minimum wage, as shown by the first column of Table 6.

Hence, education policies complement minimum wage reforms by enhancing their coverage, taming informality, and compressing earnings within skill groups. Even though aggregate inequality may rise, this reflects a mechanical shift toward higher-skilled groups that exhibit structurally greater wage dispersion—a feature outside the scope of our analysis.

6 Extended counterfactuals

This section extends our counterfactual analysis along three dimensions: unemployment, heterogeneous markdowns, and heterogeneous regional exposure.

6.1 Unemployment

A standard neoclassical prediction is that minimum wages could lead to higher unemployment. We now test whether accounting for this adjustment margin alters our main finding, namely, the unintended consequences of the minimum wage driven by shifts between formal and informal sectors. To do so, Appendix F extends the quantitative model to include unemployment, following Caliendo et al. (2019). Table F.1 reports the calibration of the model parameters, including the one that governs unemployment benefits, which is set to match the unemployment rate in 1996. Table F.2 shows that the extended model continues to replicate key moments in the data.

We then conduct a similar set of counterfactuals under two alternative assumptions about unemployment benefits. First, we hold benefits constant across scenarios. Second, we index unemployment benefits to the minimum wage—an empirically relevant case in Brazil, where benefits are legally tied to the minimum wage.

Appendix Table F.3 summarizes the key findings. In both counterfactuals, the minimum wage has the unintended consequence of increasing overall earnings inequality: the expansion of informality and greater dispersion within the informal sector more than offset the inequality-reducing effects within the formal economy. When unemployment benefits are held constant, aggregate inequality rises by 1.3%; when benefits are indexed to the minimum wage, aggregate inequality increases by 2.7%.

At the same time, the first counterfactual shows that a higher minimum wage reduces unemployment. As labor market competition increases and firms raise wages, the relative value of unemployment benefits declines, lowering the unemployment rate. However, when unemployment benefits are indexed to the minimum, firms no longer provide a sufficient wage premium over benefits, and the unemployment rate rises by 3 pp in the counterfactual equilibrium—consistent with the reduced-form estimates in Section 1.

Overall, these findings suggest that the unintended consequences of the minimum wage are robust to the inclusion of unemployment in the quantitative model. The key main margin of adjustment remains between the formal and informal sectors, not between employment and unemployment. This reflects the relatively low value of unemployment benefits compared to the minimum wage and their respective positions along the earnings distribution.

6.2 Heterogeneous markdowns

Firms of different sizes may exert varying degrees of market power in setting wages. This heterogeneity can shape how labor reallocates across formal and informal firms in response to the minimum wage. We now test whether size-dependent market power alters our main results on the unintended consequences of the minimum wage. To do so, we modify the benchmark model to allow for a discrete number of firms competing for workers while choosing their formality status. This approach builds on the framework of Berger et al. (2025), adding an informal sector to capture the reallocation channel central to our analysis. See Appendix G for more details.

Appendix Table G.1 presents results for two scenarios. First, we confirm that with a large number of firms, the model converges to the case with monopsonistic competition. Second, we evaluate the impact of the minimum wage in an economy with fewer firms and oligopsonistic competition.

Columns (3) and (4) of Appendix Table G.1 report results for the first scenario. With $250,000$ firms, the wage-setting decision of each firm has a negligible effect on the aggregate wage index. Firms operating along their labor supply curves set a markdown of 2 ($(\eta + 1)/\eta$). However, the minimum wage introduces variation in markdowns: firms paying the minimum wage face binding constraints and operate with markdowns between 1 and $(\eta + 1)/\eta$. As a result, even in this near-monopsonistic setting, there is markdown dispersion across firms.

Columns (5) and (6) present results for a setting with fewer firms. Comparing the equilibria before the minimum wage hike in Columns (3) and (5), the share of informal employment falls as the number of firms declines. With fewer competitors, firms have more market power, making the productivity loss from informality (governed by ρ) more costly.[34] A smaller number of firms also reduces overall wage inequality, both in the aggregate and within each sector, as fewer firms employ the same number of workers, and wages are identical for workers of the same skill in each firm. Importantly, in this setting, the minimum wage still has unintended consequences, leading to higher aggregate and informal inequality, more informality, and lower formal inequality.

Lastly, we examine how size-dependent market power shapes worker reallocation in response to the minimum wage. Appendix Figure G.2 plots the share of workers shifting from large to smaller firms across economies with different numbers of firms. Large firms are defined as those that remain formal after the minimum wage increase. When there are fewer competitors, large firms exercise greater market power and lose fewer workers as the minimum wage raises labor market competition. Still, the main adjustment margin is unchanged: most displaced workers ultimately move to the informal sector, even in economies with as few as 100 firms.

[34] Appendix Figure G.1 shows that, holding prices fixed at their equilibrium values under $M = 250,000$, a reduction in the number of firms decreases both the share of informal firms and informal workers.

6.3 Diff-in-diff analysis

This section shows that our structural model can replicate the reduced-form results from Section 1. To do so, we recalibrate the model separately for the most exposed region (treatment group 9) and the least exposed region (treatment group 1). In both cases, we hold the minimum wage fixed at the national level of 0.1, but let the mean of the productivity distribution of ν to differ across regions to capture variation in average productivity. Appendix Tables A.6 and A.7 report the calibrated parameters and the fit between model and data moments.

We then apply the 105% increase in the minimum wage to both economies and replicate the DID analysis by comparing changes in key outcomes across regions over time. Appendix Table A.8 reports the evolution of inequality and informality before and after the reform for each treatment group. Three findings emerge. First, the least exposed states experience more informality but lower inequality within sectors and in the aggregate. Second, in the most exposed states, the rise in informality and informal inequality more than offsets the decline in formal inequality, leading to unintended consequences. Third, the model replicates the signs of the reduced-form DID estimates for aggregate, formal, and informal inequality: most exposed regions see a larger increase in aggregate and informal inequality, and a sharper decline in formal inequality, relative to least exposed regions.

However, the model does not replicate the sign of the reduced-form estimates for the informal share: it predicts a smaller increase in informality for more exposed states relative to less exposed ones. This discrepancy arises because the informal share, bounded between zero and one, responds concavely to increases in the minimum wage. Since more exposed states begin with higher informality, their relative increase is mechanically smaller. Appendix Figure A.23 illustrates this pattern: the informal share rises convexly at low minimum wage levels but turns concave as the minimum wage increases. Because the effect of the minimum wage is stronger in more exposed states, their curves flatten earlier. There are two ways to reconcile the model with the reduced-form evidence. First, starting from a lower minimum wage, both groups would have similar informality levels, and a wage hike would produce a larger rise in informality in more exposed states, consistent with the data. Second, applying only a marginal increase to the 1996 minimum wage—as opposed to the full 105% hike—also generates relative increases in the informal share in the least exposed states.

These results validate our quantitative findings, since none of the cross-state differences over time were targeted in the calibration of the two regions. The model correctly predicts that a higher minimum wage leads to a larger increase in aggregate inequality in more exposed states, driven by a more substantial rise in inequality within the informal sector.

7 Conclusion

There is a long-standing literature suggesting that minimum wages are an important tool for reducing earnings disparity. This paper examines how the presence of the informal sector shapes this effect. We find that, in the Brazilian context, the spike in the minimum wage over the 2000s increased overall inequality, highlighting the *unintended consequences* of the minimum wage. That is, policies that aim at reducing inequality might increase it due to strong informal margins of adjustment.

We reach this conclusion in three steps. Our empirical work provides reduced-form evidence that the minimum wage increases inequality in the informal sector, and that this offsets the inequality-reducing effects the minimum wage has in the formal sector. We then devise a theoretical model and derive analytical results showing that there is scope for higher minimum wages to increase aggregate inequality. In the last step, we build a quantitative framework to study the role of changes in the minimum wage, informality costs, and the skill composition on the Brazilian economy. We show that the minimum wage increase, albeit responsible for a strong reduction in formal inequality, is also responsible for a rise of 4.1% in aggregate inequality, due to a substantial informal margin of adjustment.

This paper opens important avenues for further research. First, it provides a tool for addressing the discussion about federal-level minimum wages in countries where local labor markets differ substantially in informality levels. Second, as gig and platform-allocated work expand in developed countries (e.g., ride-sharing, food delivery, freelance services), the question of how minimum wage policies affect workers who can easily move between formal employment and alternative work arrangements becomes increasingly relevant. All in all, our findings suggest that movements into and out of the informal sector modulate the effects of formal labor legislation in developing countries.

References

Adão, Rodrigo, "Worker Heterogeneity, Wage Inequality, and International Trade: Theory and Evidence from Brazil," Working Paper 11 2016.

Almeida, Rita and Pedro Carneiro, "Enforcement of Labor Regulation and Informality," *American Economic Journal: Applied Economics*, 7 2012, *4* (3), 64–89.

Alvarez, Jorge, Felipe Benguria, Niklas Engbom, and Christian Moser, "Firms and the Decline in Earnings Inequality in Brazil," *American Economic Journal: Macroeconomics*, 1 2018, *10* (1), 149–89.

Amaral, Pedro S. and Erwan Quintin, "A competitive model of the informal sector," *Journal of Monetary Economics*, 2006, *53* (7), 1541–1553.

Andrews, Isaiah, Matthew Gentzkow, and Jesse M. Shapiro, "Measuring the Sensitivity of Parameter Estimates to Estimation Moments*," *The Quarterly Journal of Economics*, 06 2017, *132* (4), 1553–1592.

Autor, David H., Alan Manning, and Christopher L. Smith, "The Contribution of the Minimum Wage to US Wage Inequality over Three Decades: A Reassessment," *American Economic Journal: Applied Economics*, 1 2016, *8* (1), 58–99.

Berger, David, Kyle Herkenhoff, and Simon Mongey, "Labor Market Power," *American Economic Review*, April 2022, *112* (4), 1147–93.

_ , _ , and _ , "Minimum Wages, Efficiency, and Welfare," *Econometrica*, 2025, *93* (1), 265–301.

Bloom, Nicholas, Benn Eifert, Aprajit Mahajan, David McKenzie, and John Roberts, "Does Management Matter? Evidence from India *," *The Quarterly Journal of Economics*, 2013, *128* (1), 1–51.

Borusyak, Kirill, Xavier Jaravel, and Jann Spiess, "Revisiting Event-Study Designs: Robust and Efficient Estimation," *The Review of Economic Studies*, 02 2024, *91* (6), 3253–3285.

Burstein, Ariel and Jonathan Vogel, "International Trade, Technology, and the Skill Premium," *Journal of Political Economy*, 2017, *125* (5), 1356–1412.

Cain, Glen G., "The Challenge of Segmented Labor Market Theories to Orthodox Theory: A Survey," *Journal of Economic Literature*, 1976, *14* (4), 1215–1257.

Caliendo, Lorenzo, Maximiliano Dvorkin, and Fernando Parro, "Trade and Labor Market Dynamics: General Equilibrium Analysis of the China Trade Shock," *Econometrica*, 2019, *87* (3), 741–835.

Callaway, Brantly and Pedro H.C. Sant'Anna, "Difference-in-Differences with multiple time periods," *Journal of Econometrics*, 2020.

Card, David, "Using Regional Variation in Wages to Measure the Effects of the Federal Minimum Wage," *ILR Review*, 1992, *46* (1), 22–37.

_ , Ana Rute Cardoso, Joerg Heining, and Patrick Kline, "Firms and Labor Market Inequality: Evidence and Some Theory," *Journal of Labor Economics*, 2018, *36* (S1), S13–S70.

_ and Alan B Krueger, "Minimum Wages and Employment: A Case Study of the Fast Food Industry in New Jersey and Pennsylvania," Working Paper 4509, National Bureau of Economic Research 10 1993.

Colombi, Roberto, "A New Model Of Income Distribution: The Pareto-Lognormal Distribution," in Camilo Dagum and Michele Zenga, eds., *Income and Wealth Distribution, Inequality and Poverty*, 1990, pp. 18–32.

Corseuil, Carlos Henrique L., Rita Almeida, and Pedro Carneiro, "Inspeção do Trabalho e Evolução do Emprego Formal no Brasil," Discussion Papers 1688, Instituto de Pesquisa Econômica Aplicada - IPEA January 2012.

Corseuil, Carlos Henrique, Miguel Foguel, and Marcos Hecksher, "Efeitos dos pisos salariais estaduais sobre o mercado de trabalho: uma nova abordagem empírica," *Economia Aplicada*, 2015, *19* (1), 131–169.

Costa, Francisco, Jason Garred, and João Paulo Pessoa, "Winners and losers from a commodities-for-manufactures trade boom," *Journal of International Economics*, 2016, *102*, 50–69.

de Chaisemartin, Clément and Xavier D'Haultfœuille, "Two-Way Fixed Effects Estimators with Heterogeneous Treatment Effects," *American Economic Review*, 09 2020, *110* (9), 2964–96.

Derenoncourt, Ellora, François Gérard, Lorenzo Lagos, and Claire Montialoux, "Racial Inequality, Minimum Wage Spillovers, and the Informal Sector," Working Paper 5 2021.

Desmet, Klaus, Dávid Krisztián Nagy, and Esteban Rossi-Hansberg, "The Geography of Development," *Journal of Political Economy*, 2018, *126* (3), 903–983.

Dickens, Richard and Alan Manning, "Spikes and Spill-overs: The Impact of the National Minimum Wage on the Wage Distribution in a Low-Wage Sector," *The Economic Journal*, 2004, *114* (494), C95–C101.

Dix-Carneiro, Rafael and Brian K. Kovak, "Trade Liberalization and Regional Dynamics," *American Economic Review*, 10 2017, *107* (10), 2908–46.

_ , Pinelopi K Goldberg, Costas Meghir, and Gabriel Ulyssea, "Trade and Informality in the Presence of Labor Market Frictions and Regulations," Working Paper 28391, National Bureau of Economic Research 01 2021.

Dube, Arindrajit and Attila Lindner, "Chapter 4 - Minimum wages in the 21st century," in Christian Dustmann and Thomas Lemieux, eds., *Handbook of Labor Economics*, Vol. 5, Elsevier, 2024, pp. 261–383.

Dustmann, Christian, Attila Lindner, Uta Schönberg, Matthias Umkehrer, and Philipp vom Berge, "Reallocation Effects of the Minimum Wage," *The Quarterly Journal of Economics*, 08 2021, *137* (1), 267–328.

Engbom, Niklas and Christian Moser, "Earnings Inequality and the Minimum Wage: Evidence from Brazil," *American Economic Review*, 2022, *112* (12), 3803–3847.

Eslava, Marcela, Marcela Meléndez, Gabriel Ulyssea, Nicolás Urdaneta, and Ignacio Flores, "Firms and Inequality in Latin America," IDB Publications (Working Papers) 13480, Inter-American Development Bank April 2024.

Felix, Mayara, "Trade, Labor Market Concentration, and Wages," October 2022. Working Paper.

Fields, Gary S., "A guide to multisector labor market models," Policy Research Working Paper Series 32547, The World Bank April 2005.

Firpo, Sergio and Alysson Portela, "Decline in Wage Inequality in Brazil: A Survey," Policy Research Working Paper 9096, World Bank, Washington, DC 2019.

Giupponi, Giulia, Robert Joyce, Attila Lindner, Tom Waters, Tom Wernham, and Xiaowei Xu, "The Employment and Distributional Impacts of Nationwide Minimum Wage Changes," *Journal of Labor Economics*, 2024, *42* (S1), 293–333.

Gonzaga, Gustavo, William F. Maloney, and Alejandra Mizala, "Labor Turnover and Labor Legislation in Brazil [with Comments]," *Economía*, 2003, *4* (1), 165–222.

Haanwinckel, Daniel, "Supply, Demand, Institutions, and Firms: A Theory of Labor Market Sorting and the Wage Distribution," Working Paper 31318, National Bureau of Economic Research June 2023.

— **and Rodrigo R. Soares**, "Workforce Composition, Productivity, and Labour Regulations in a Compensating Differentials Theory of Informality," *The Review of Economic Studies*, 2021, *88* (6), 2970–3010.

Harasztosi, Peter and Attila Lindner, "Who Pays for the Minimum Wage?," *American Economic Review*, 8 2019, *109* (8), 2693–2727.

Jales, Hugo, "Estimating the effects of the minimum wage in a developing country: A density discontinuity design approach," *Journal of Applied Econometrics*, 2018, *33* (1), 29–51.

— **and Zhengfei Yu**, "Labor Market Policies in a Roy-Rosen Bargaining Economy," GLO Discussion Paper Series 577, Global Labor Organization (GLO) 6 2020.

Leal Ordóñez, Julio César, "Tax collection, the informal sector, and productivity," *Review of Economic Dynamics*, 2014, *17* (2), 262 – 286.

Lee, David S., "Wage Inequality in the United States during the 1980s: Rising Dispersion or Falling Minimum Wage?," *The Quarterly Journal of Economics*, 1999, *114* (3), 977–1023.

Lemos, Sara, "The Effects of the Minimum Wage on Wages, Employment and Prices," IZA Discussion Papers 1135, Institute for the Study of Labor (IZA), Bonn 5 2004.

Lewis, W. Arthur, "Economic Development with Unlimited Supplies of Labour," *The Manchester School*, 1954, *22* (2), 139–191.

Lustig, Nora, Luis F. Lopez-Calva, and Eduardo Ortiz-Juarez, "Declining Inequality in Latin America in the 2000s: The Cases of Argentina, Brazil, and Mexico," *World Development*, 2013, *44*, 129–141.

Machin, Stephen, Alan Manning, and Lupin Rahman, "Where the Minimum Wage Bites Hard: Introduction of Minimum Wages to a Low Wage Sector," *Journal of the European Economic Association*, 2003, *1* (1), 154–180.

Meghir, Costas, Renata Narita, and Jean-Marc Robin, "Wages and Informality in Developing Countries," *American Economic Review*, 4 2015, *105* (4).

Ottonello, Pablo and Thomas Winberry, "Financial Heterogeneity and the Investment Channel of Monetary Policy," *Econometrica*, 2020, *88* (6), 2473–2502.

Paul-Delvaux, Louise, "Minimum Wage, Informality, and Gender Gaps: Evidence from Morocco," Working Paper 3 2024.

Porta, Rafael La and Andrei Shleifer, "The Unofficial Economy and Economic Development," *Brookings Papers on Economic Activity*, 2008, *2008*, 275–352.

Porta, Rafael La and Andrei Shleifer, "Informality and Development," *Journal of Economic Perspectives*, 9 2014, *28* (3), 109–26.

Presidência da República, "Lei nº 7.998, de 11 de janeiro de 1990," `https://www.planalto.gov.br/ccivil_03/leis/l7998.htm` jan 1990.

Rauch, James E., "Modelling the informal sector formally," *Journal of Development Economics*, 1991, *35* (1), 33 – 47.

Rothschild, Casey and Florian Scheuer, "Optimal Taxation with Rent-Seeking," *The Review of Economic Studies*, 03 2016, *83* (3), 1225–1262.

Saltiel, Fernando and Sergio Urzúa, "Does an Increasing Minimum Wage Reduce Formal Sector Employment? Evidence from Brazil," *Economic Development and Cultural Change*, 2022, *70* (4), 1403–1437.

Souza, André Portela, Sérgio P. Firpo, Vladimir P. Ponczek, Eduardo Zylberstajn, and Felipe Garcia Ribeiro, "Custo do Trabalho no Brasil: Proposta de uma nova metodologia de mensura cão," FGV/EESP 5 2012.

Tornarolli, Leopoldo, Diego Battistón, Leonardo Gasparini, and Pablo Gluzmann, "Exploring Trends in Labor Informality in Latin America, 1990-2010," CEDLAS, Working Papers 0159, CEDLAS, Universidad Nacional de La Plata 05 2014.

Ulyssea, Gabriel, "Regulation of entry, labor market institutions and the informal sector," *Journal of Development Economics*, 1 2010, *91* (1), 87–99.

_ , "Firms, Informality, and Development: Theory and Evidence from Brazil," *American Economic Review*, 8 2018, *108* (8), 2015–47.

_ , "Informality: Causes and Consequences for Development," *Annual Review of Economics*, 2020, *12* (1), 525–546.

Minimum Wages, Inequality, and the Informal Sector

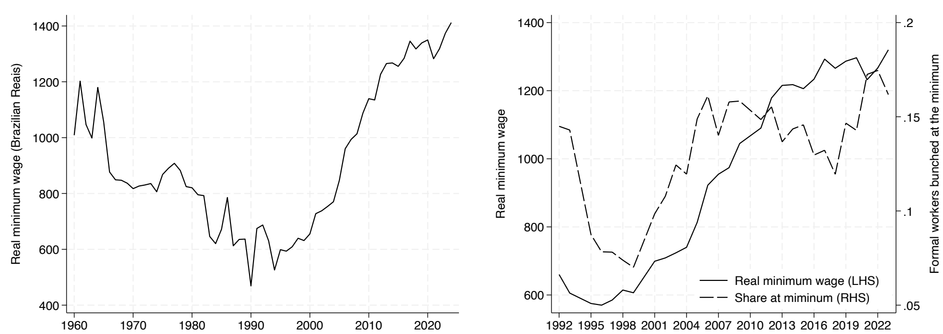
Machado Parente, Brotherhood, and Iachan

Appendix

Appendix A presents additional figures and tables. Appendix B contains all the calculations and proofs used in the main text. Appendix C shows that the model in Section 3 is isomorphic to the problem of a representative family assigning workers under rationing constraints. Appendix D details the computation of the quantitative model. Appendix E details the calibration for the 2012 economy. Appendix F adds the unemployment margin to the quantitative model. Lastly, Appendix G details the model under oligopsonistic competition and size-dependent markdowns.

A Additional tables and figures

This section contains additional figures and tables referenced in the main text.

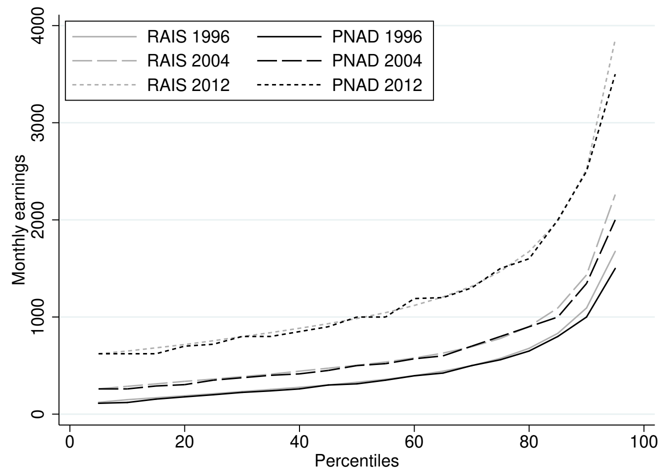


(a) Real minimum wage, 1960-2023 (b) Min. wage and its bite, 1992-2023

Figure A.1: Real minimum wage and minimum wage bite

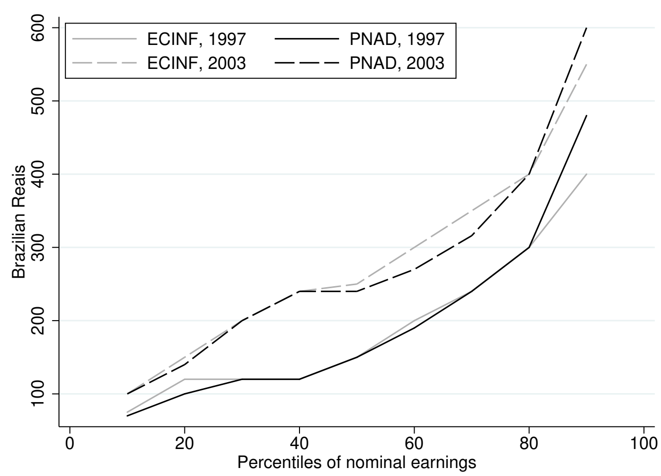
Sources: IPEADATA and 1992-2023 PNAD.

Figure A.2: Comparison between RAIS and PNAD data sets, 1996-2012



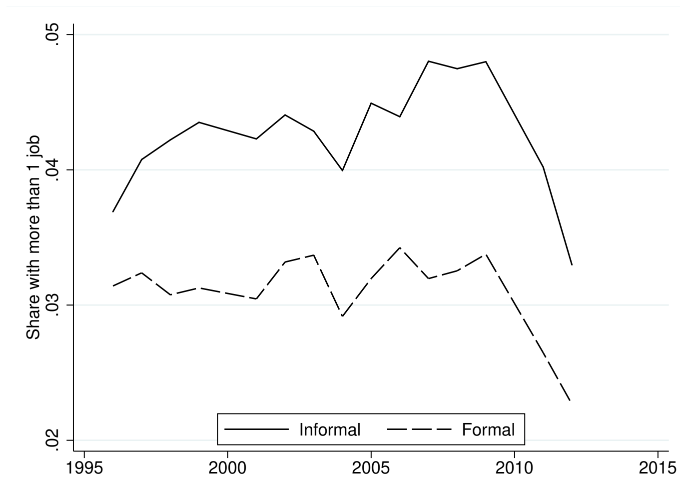
Notes: Comparison between formal earnings distributions in PNAD (black) and RAIS (grey) across different years (line patterns). *Sources:* 1996-2012 PNAD and RAIS.

Figure A.3: Comparison between ECINF and PNAD data sets, 1997 and 2003



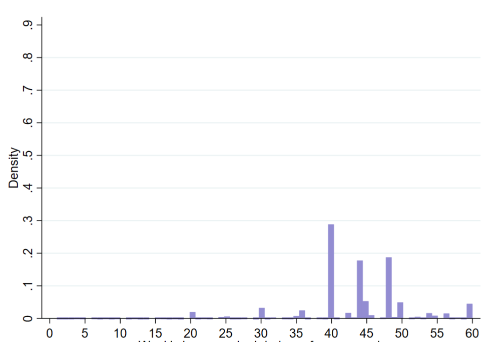
Notes: Comparison between informal earnings distributions in PNAD (black) and ECINF (grey) across different years (line patterns). *Sources:* 1997 and 2003 PNAD and ECINF.

Figure A.4: Share of formal/informal workers with more than one job, 1996-2012

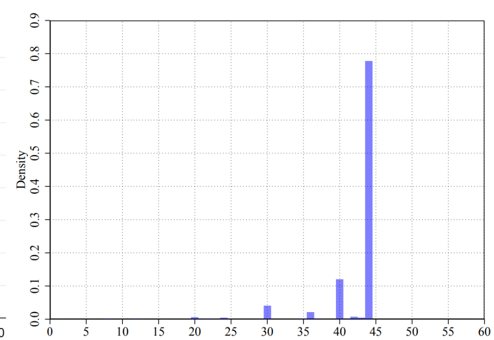


Notes: Share of workers in the formal and informal sectors with more than one job in the reference week. *Sources:* 1997-2012 PNAD.

Figure A.5: Weekly and Contractual Hours, 1996



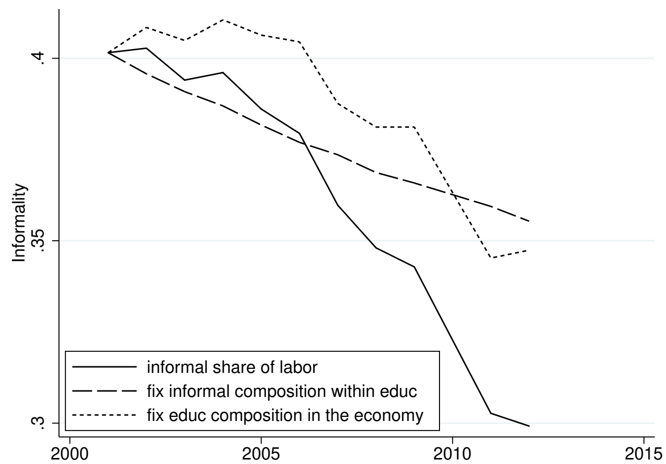
(a) Weekly hours (PNAD)



(b) Contractual hours (RAIS)

Notes: Panel (a) plots the histogram of weekly hours using PNAD data from 1996. The blue bars sum up to 1. Panel (b) reports Figure B.24, Panel A of Engbom and Moser (2022). For comparability, we capped the PNAD histogram at 60 hours. *Sources:* 1996 PNAD and Engbom and Moser (2022).

Figure A.6: Informality decomposition across education groups, 1996-2012



Notes: This figure performs a decomposition of the informal share of labor across different education groups: $(L_t^I/L_t) = \sum_e (L_{et}/L_t) \cdot (L_{et}^I/L_{et})$ where e denotes education groups, t time and superscript I denotes informal employment. The solid line shows the observed movement in the informal share of labor. The long dashes plot a counterfactual curve that fixes the share of informality within education groups (L_{et}^I/L_{et}) in its initial value. The short dashes plot a counterfactual curve that fixes the educational composition of the labor force (L_{et}/L_t) in its 1996 value. *Sources:* 1996-2012 PNAD.

Table A.1: Compliance by type of minimum wage regulation

	N. of workers	Share
All	1,735,859	1.00
Wages > state-occupation min. w.	1,081,356	0.62
Wages = state-occupation min. w.	41,849	0.02
Wages < state-occupation min. w.	612,655	0.35
Wages > national min. w.	1,215,514	0.7
Wages = national min. w.	453,521	0.26
Wages < national min. w.	66,824	0.04

Notes: This table presents compliance with the national minimum wage and subnational wage floors that vary by state and occupation, using data on formal workers in states and occupations where these floors were in effect. The fourth row of the table shows that 35% of workers in occupations subject to an occupation-specific minimum wage were earning less than that floor. In contrast, the seventh row indicates that only 4% of formal workers earned below the national minimum wage. *Sources:* 2010 Census and state/occupation-level wage floor data provided by Marcos Hecksher (Corseuil et al., 2015).

Table A.2: Informal share in different industries

	Share informal	Share of total employment
Manufacturing	16.5	18.1
Other activities	16.5	9.8
Transport, storage, and communic.	20.1	5.8
Commerce and repair	24.5	18.2
Undefined	30.4	0.0
Education, health, and social serv.	32.8	9.5
Restaurant and accommodation	38.8	5.6
Construction	43.5	6.5
Other services	46.4	3.5
Public admin	55.2	3.5
Agriculture	61.6	7.8
Domestic services	69.4	11.7

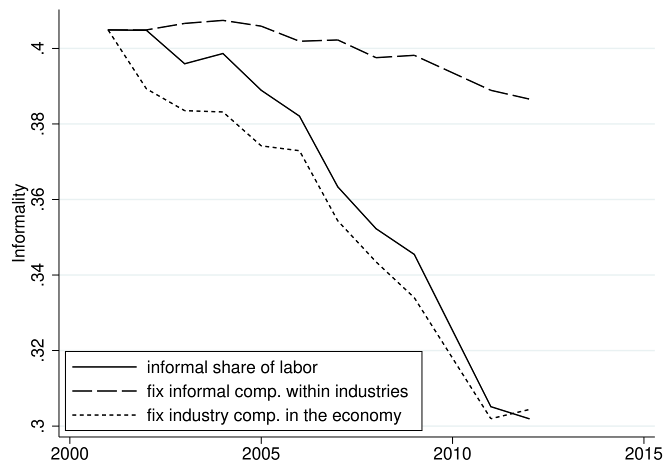
Notes: Table restricts data to 2001-2012 period, as industry definitions are consistent across surveys. The second column shows the share of employment that is informal in each industry. The third column shows the size of each industry in terms of total employment. Sample weights are used. *Sources:* 2001/2012 PNAD.

Table A.3: Minimum wage, inequality, and the informal sector (mean effects post-1999)

	$\log(V^{\text{All}})$	$\log(V^F)$	$\log(V^I)$	log(Inf. share)	log(Unemp.)
β_9	0.183	-0.260	0.282	0.103	0.322
	(0.059)***	(0.049)***	(0.083)***	(0.030)***	(0.088)***
State FE	✓	✓	✓	✓	✓
Year FE	✓	✓	✓	✓	✓
Observations	405	405	405	405	405
R^2	0.859	0.900	0.665	0.971	0.799

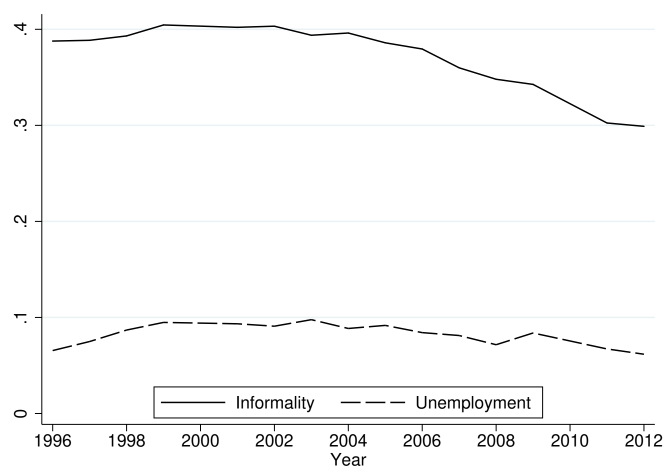
Notes: This table displays the coefficients of the OLS regression: $y_{sgt} = \alpha + \sum_{h \neq 1} \beta_h \cdot \mathcal{I}_{g=h} \cdot \mathcal{I}_{t>1999} + \delta_s + \delta_t + X'_{st}\Gamma + \varepsilon_{st}$. The coefficients β_2-β_8 can be found in Figure 4 or in Appendix Table A.5. Standard errors in parentheses are clustered at the state level. ***p<1%, **p<5%, *p<10%. *Sources:* 1996-2012 PNAD.

Figure A.7: Informality decomposition across industries, 1996-2012



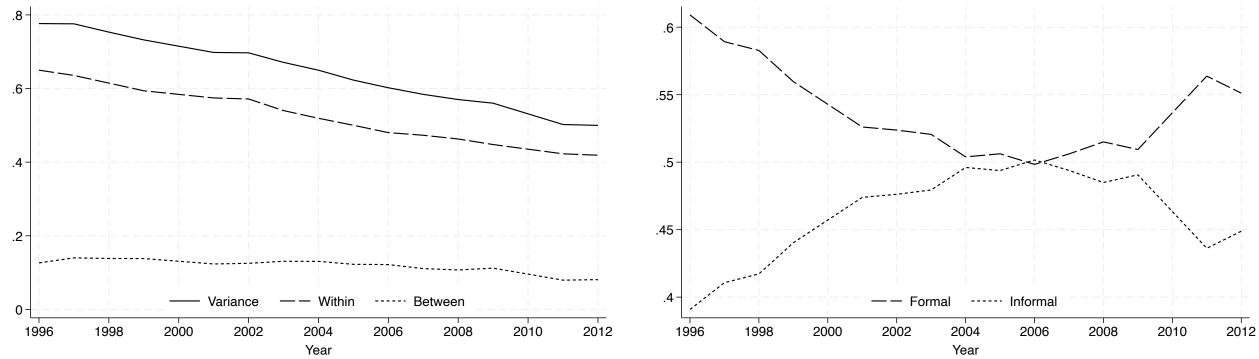
Notes: This figure performs a decomposition of the informal share of labor across different industries: $(L_t^I/L_t) = \sum_j (L_{jt}/L_t) \cdot (L_{jt}^I/L_{jt})$ where j denotes industry, t time, and superscript I denotes informal employment. The solid line shows the observed movement in the informal share of labor. The long dashes plot a counterfactual curve that fixes the share of informality within industries (L_{jt}^I/L_{jt}) in its initial value. The short dashes plot a counterfactual curve that fixes the industry composition of the labor force (L_{jt}/L_t) in its 1996 value. *Sources:* 1996-2012 PNAD.

Figure A.8: Informality and unemployment, 1996-2012



Notes: Solid line shows the fraction of informal workers. Long dashes show the share of unemployed workers. *Sources:* 1996-2012 PNAD.

Figure A.9: Decomposition of overall variance of log earnings



(a) Overall decomposition

(b) Within-group decomposition by formality

Notes: Panel (a) decomposes overall variance in log earnings into within and between terms, following Equation (1). Panel (b) shows the share of the within component accounted for by formal and informal workers. *Sources:* 1996-2012 PNAD.

Table A.4: Brazilian states and respective treatment groups

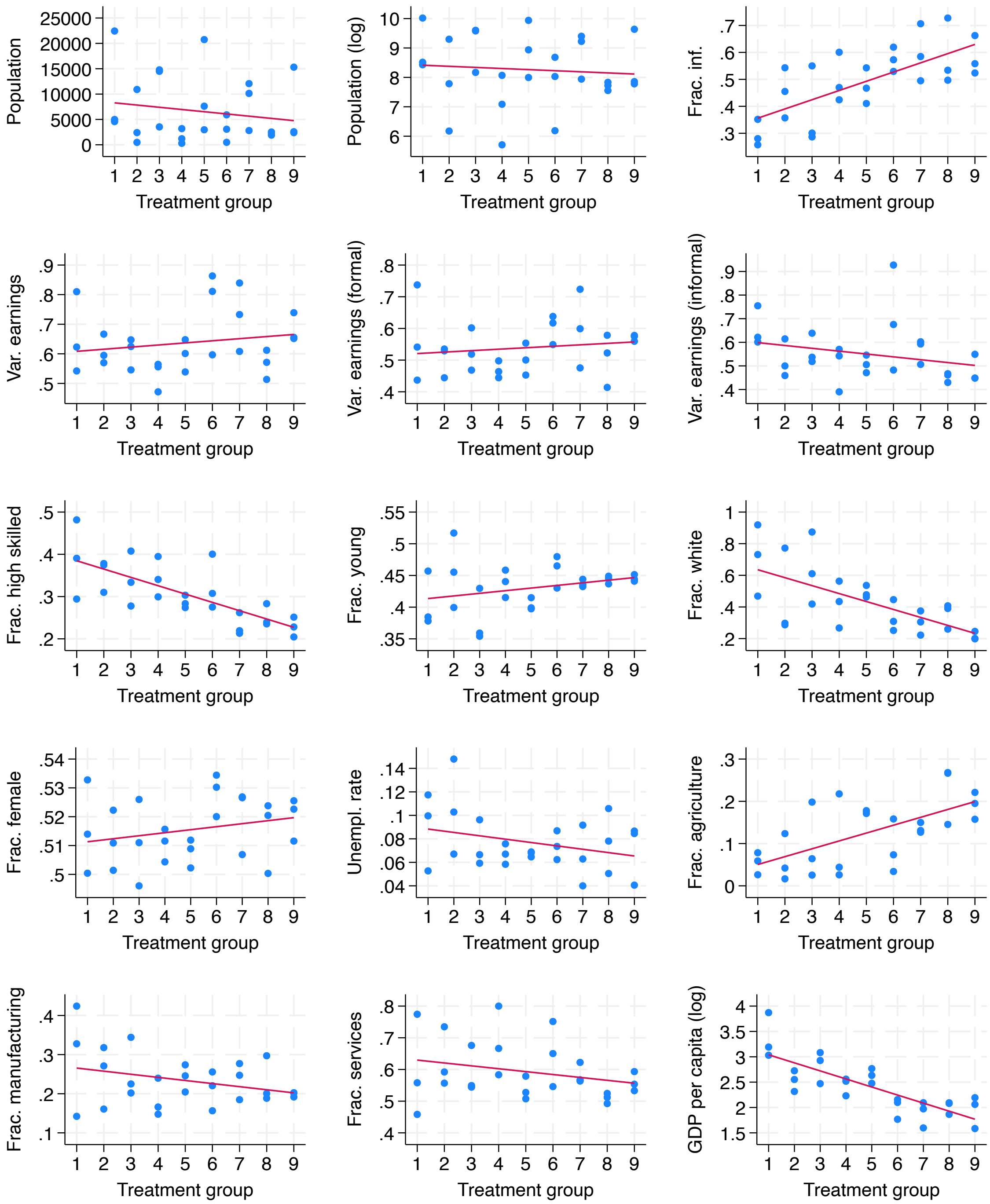
Group	State	Group	State
1	São Paulo	6	Pará
1	Santa Catarina	6	Paraíba
1	Distrito Federal	6	Acre
2	Amapá	7	Maranhão
2	Paraná	7	Pernambuco
2	Amazonas	7	Ceará
3	Mato Grosso	8	Alagoas
3	Rio de Janeiro	8	Tocantins
3	Rio Grande do Sul	8	Rio Grande do Norte
4	Rondônia	9	Bahia
4	Mato Grosso do Sul	9	Sergipe
4	Roraima	9	Piauí
5	Goiás		
5	Espírito Santo		
5	Minas Gerais		

Table A.5: Diff-in-diff results (complete table)

	$\log(V^{\text{All}})$	$\log(V^F)$	$\log(V^I)$	log(Inf. share)	log(Unemp.)
β_2	-0.136	-0.146	-0.120	0.081	0.007
	(0.074)*	(0.043)***	(0.102)	(0.038)**	(0.152)
β_3	-0.003	-0.056	-0.022	0.086	0.181
	(0.053)	(0.039)	(0.076)	(0.040)**	(0.100)*
β_4	0.094	-0.094	0.133	0.092	0.107
	(0.054)*	(0.057)	(0.097)	(0.067)	(0.139)
β_5	0.028	-0.064	0.093	-0.035	0.147
	(0.051)	(0.043)	(0.081)	(0.025)	(0.098)
β_6	-0.047	-0.164	-0.107	0.048	0.134
	(0.069)	(0.073)**	(0.108)	(0.037)	(0.169)
β_7	0.082	-0.227	0.096	0.047	0.405
	(0.059)	(0.075)***	(0.088)	(0.028)	(0.117)***
β_8	0.209	-0.246	0.308	0.068	0.122
	(0.063)***	(0.080)***	(0.081)***	(0.030)**	(0.089)
β_9	0.183	-0.260	0.282	0.103	0.322
	(0.059)***	(0.049)***	(0.083)***	(0.030)***	(0.088)***
State FE	✓	✓	✓	✓	✓
Year FE	✓	✓	✓	✓	✓
Observations	405	405	405	405	405
R^2	0.859	0.900	0.665	0.971	0.799

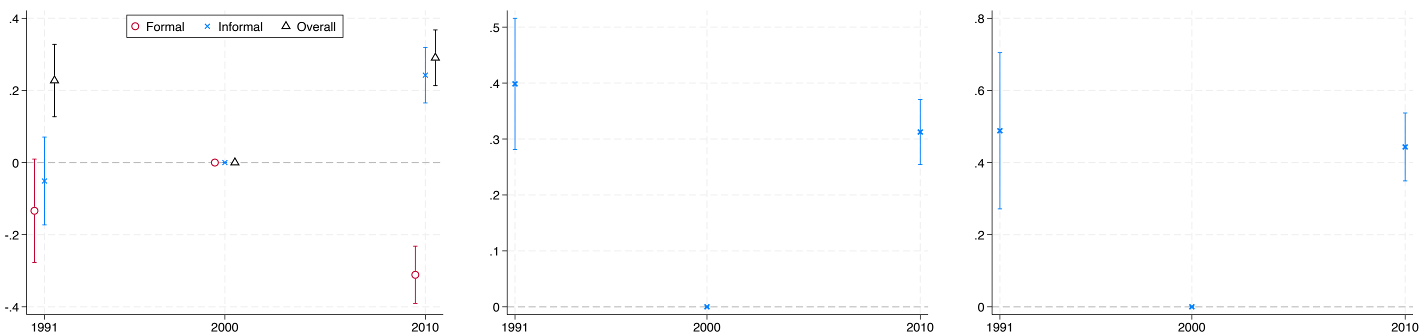
Notes: This table displays the coefficients of the OLS regression: $y_{sgt} = \alpha + \sum_{h \neq 1} \beta_h \cdot \mathcal{I}_{g=h} \cdot \mathcal{I}_{t>1999} + \delta_s + \delta_t + X'_{st}\Gamma + \varepsilon_{st}$. Standard errors in parentheses are clustered at the state level. ***p<1%, **p<5%, *p<10%. *Sources:* 1996/2012 PNAD.

Figure A.10: Descriptive statistics by treatment group



Notes: This figure displays various state-level statistics by treatment intensity. Each circle in the subplots represents the average value of the corresponding variable for a given state over the period 1996–1999. *Sources:* 1996-1999 PNAD.

Figure A.11: Event-study analysis: Census data

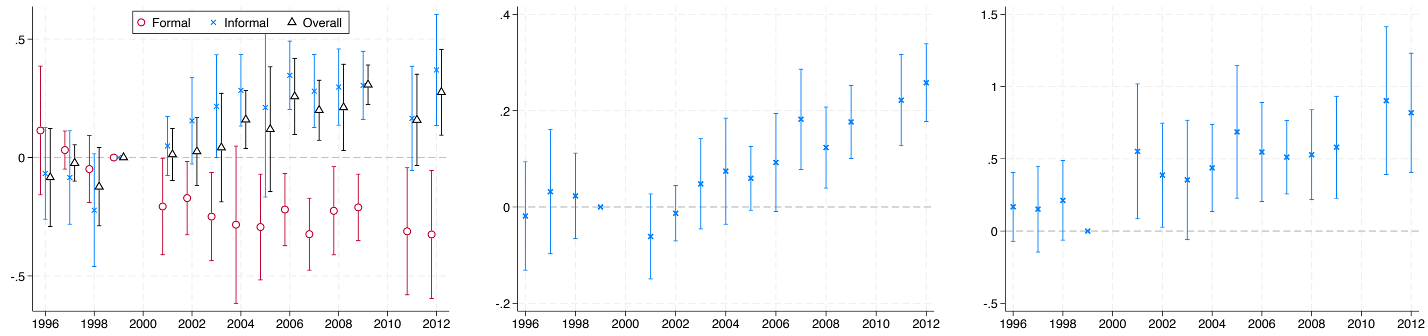


(a) log(Variance) (b) log(Informal share) (c) log(Unemployment)

Notes: This figure plots the ordinary least squares coefficients of Equation (2) for regions in the most treated group, β_{k9} for $k \neq 1999$. The estimates use Census data at the microregion level. *Sources:* 1991/2000/2010 Census.

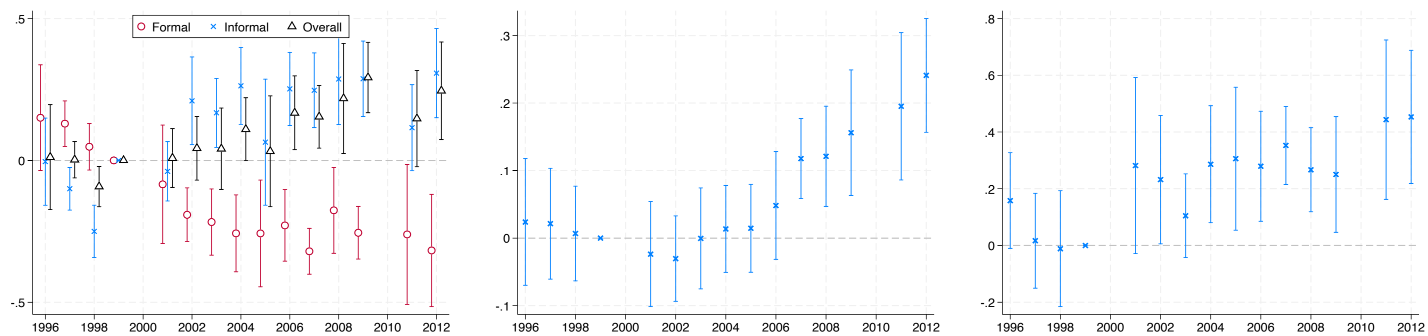
Figure A.12: Event-study analysis: linear controls and weighted regressions

(a) Linear controls



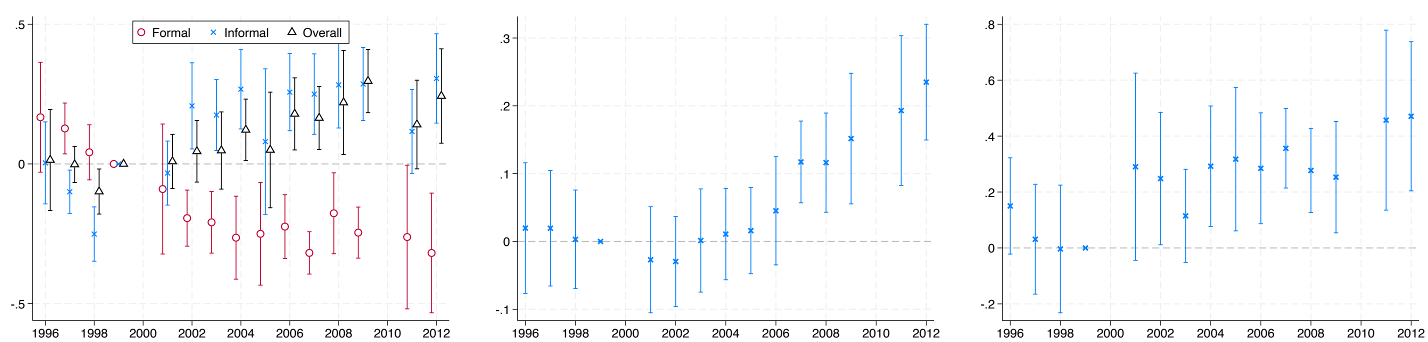
log(Variance) log(Informal share) log(Unemployment)

(b) Employment weights



log(Variance) log(Informal share) log(Unemployment)

(c) Population weights

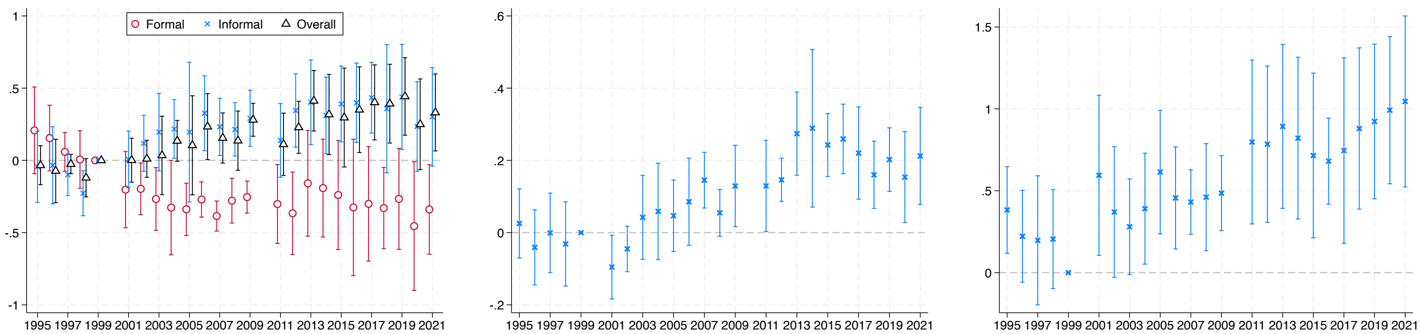


log(Variance) log(Informal share) log(Unemployment)

Notes: This figure plots the ordinary least squares coefficients of Equation (2) for states in the most treated group, β_{k9} for $k \neq 1999$. *Sources:* 1996-2012 PNAD.

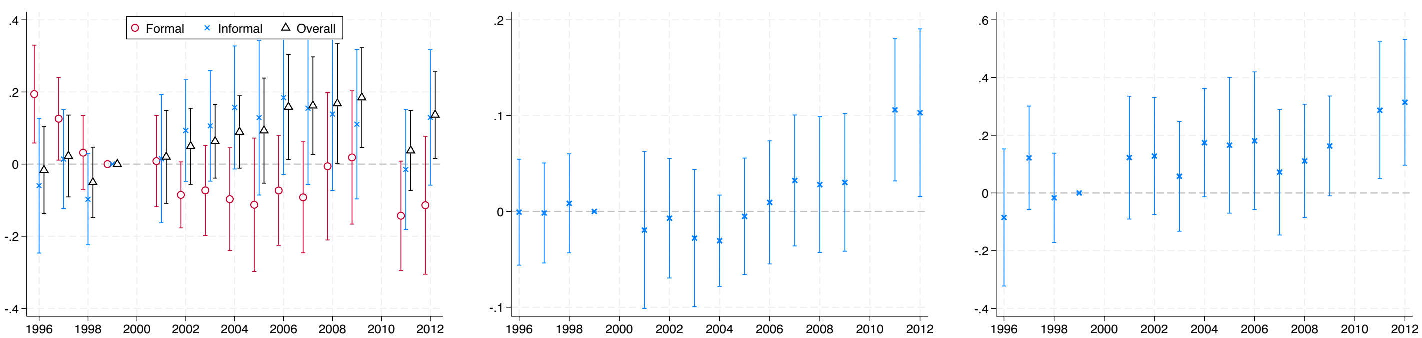
Figure A.13: Event-study analysis: extended years, two treatment groups, and dependent variables in levels

(a) Years: 1995-2021



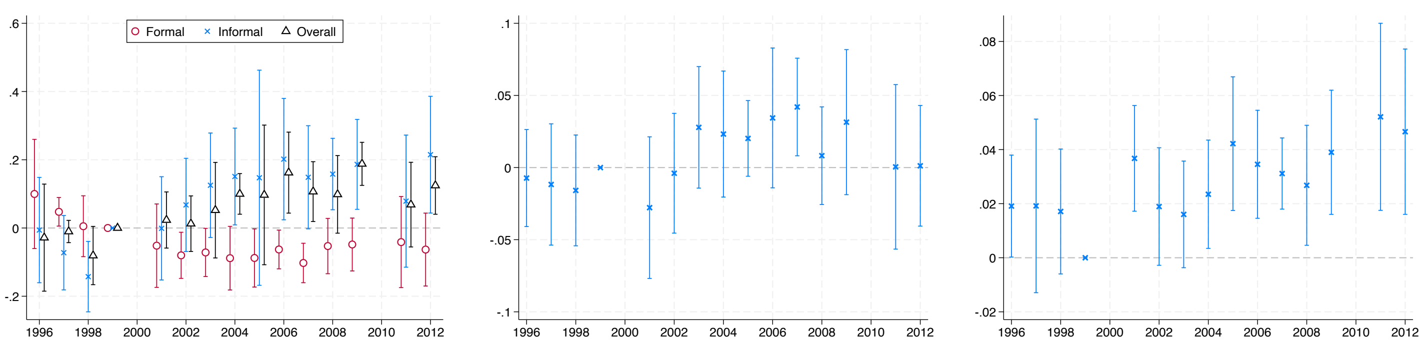
log(Variance) (Informal share) log(Unemployment)

(b) Two groups: above/below median



log(Variance) (Informal share) log(Unemployment)

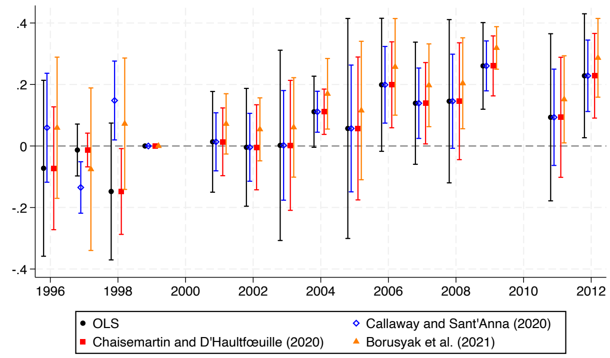
(c) Dependent variables in levels



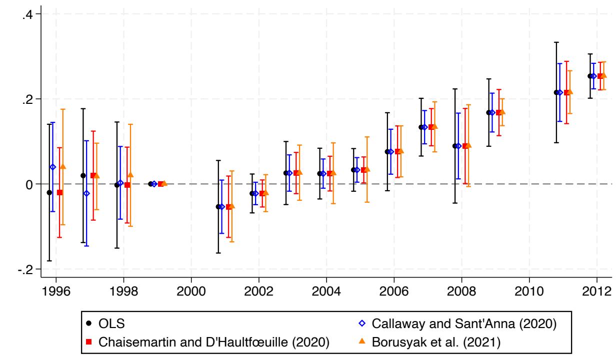
Variance Informal share Unemployment

Notes: This figure plots the ordinary least squares coefficients of Equation (2) for states in the most treated group, β_{k9} for $k \neq 1999$.. *Sources:* 1996-2021 PNAD.

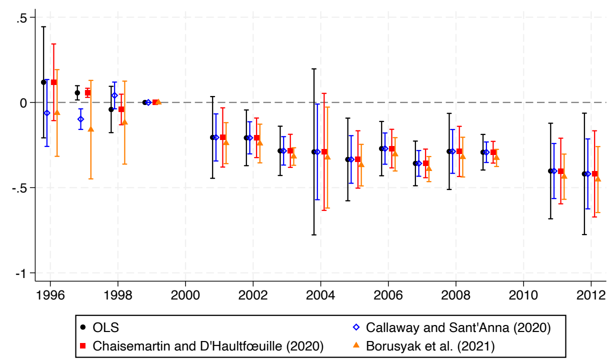
Figure A.14: Robustness to different TWFE estimators



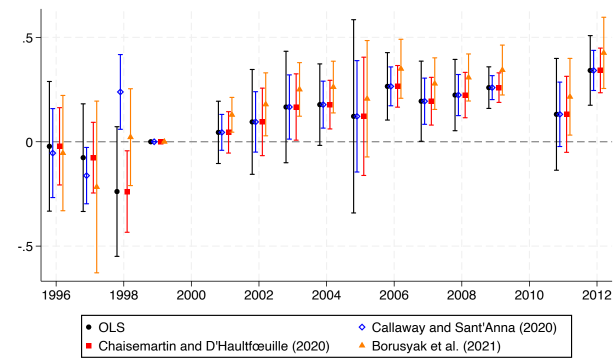
(a) log(Variance)



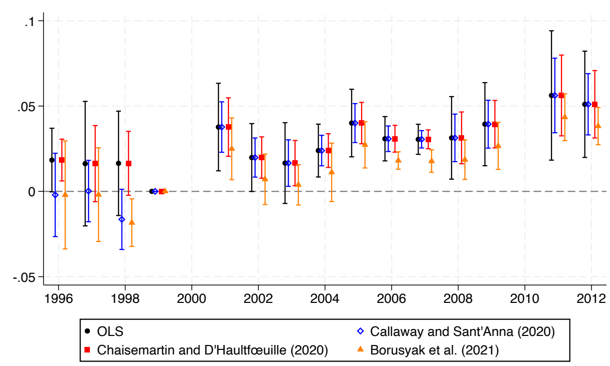
(b) log(Informal share)



(c) log(Formal variance)



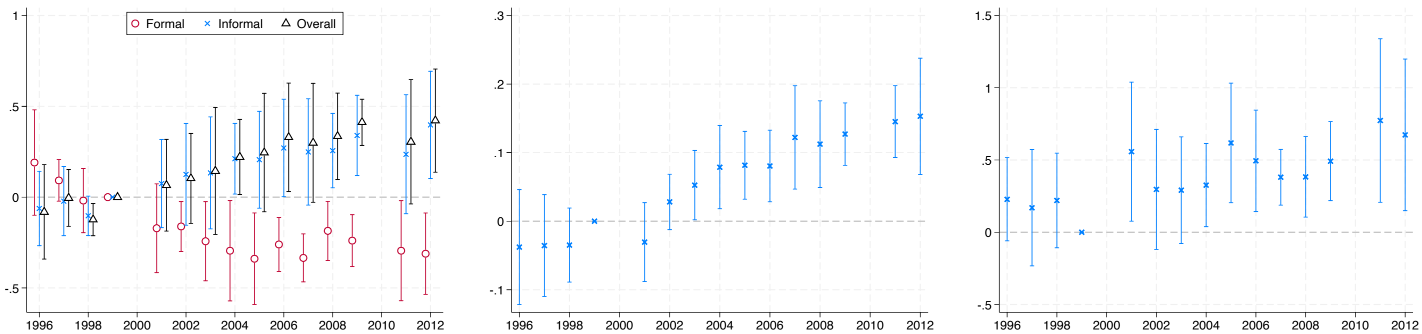
(d) log(Informal variance)



(e) log(Unemployment)

Notes: This figure plots different estimators of the two-way fixed effect model in Equation (2) for the states in the most treated group, β_{k9} for $k \neq 1999$. Standard errors are clustered at the state level. *Sources:* 1996-2012 PNAD.

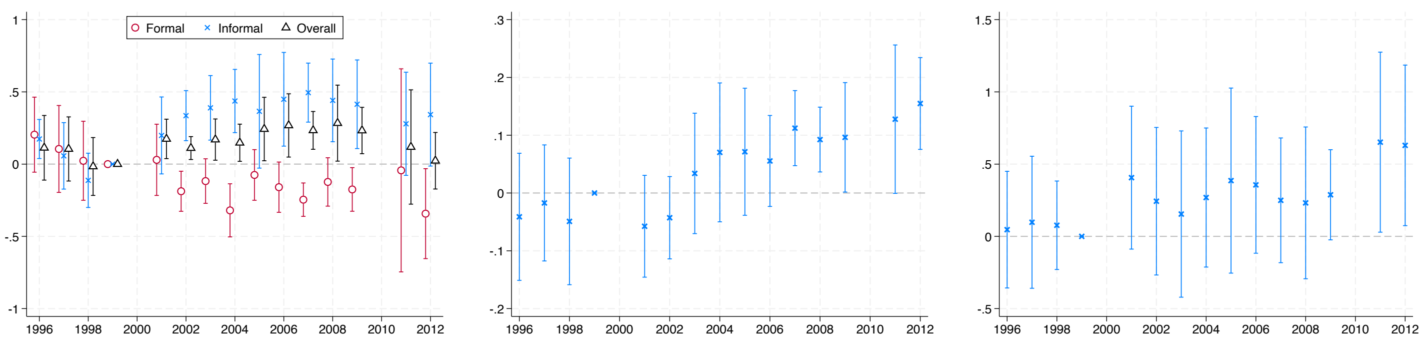
Figure A.15: Event-study analysis: including self-employed



(a) log(Variance) (b) log(Informal share) (c) log(Unemployment)

Notes: This figure plots the ordinary least squares coefficients of Equation (2) for states in the most treated group, β_{k9} for $k \neq 1999$. Self-employed workers are considered informal. *Sources:* 1996-2023 PNAD.

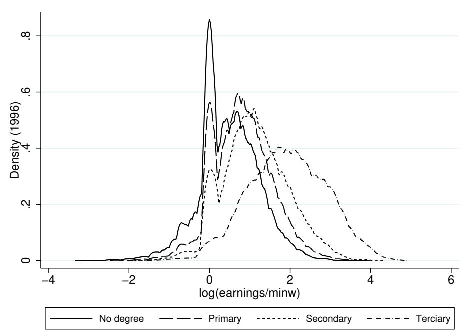
Figure A.16: Event-study analysis: hourly earnings



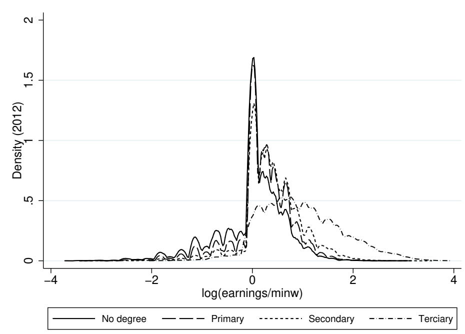
(a) log(Variance) (b) log(Informal share) (c) log(Unemployment)

Notes: This figure plots the ordinary least squares coefficients of Equation (2) for states in the most treated group, β_{k9} for $k \neq 1999$. This exercise uses hourly earnings instead of monthly earnings. *Sources:* 1996-2023 PNAD.

Figure A.17: Earnings distribution relative to the minimum wage



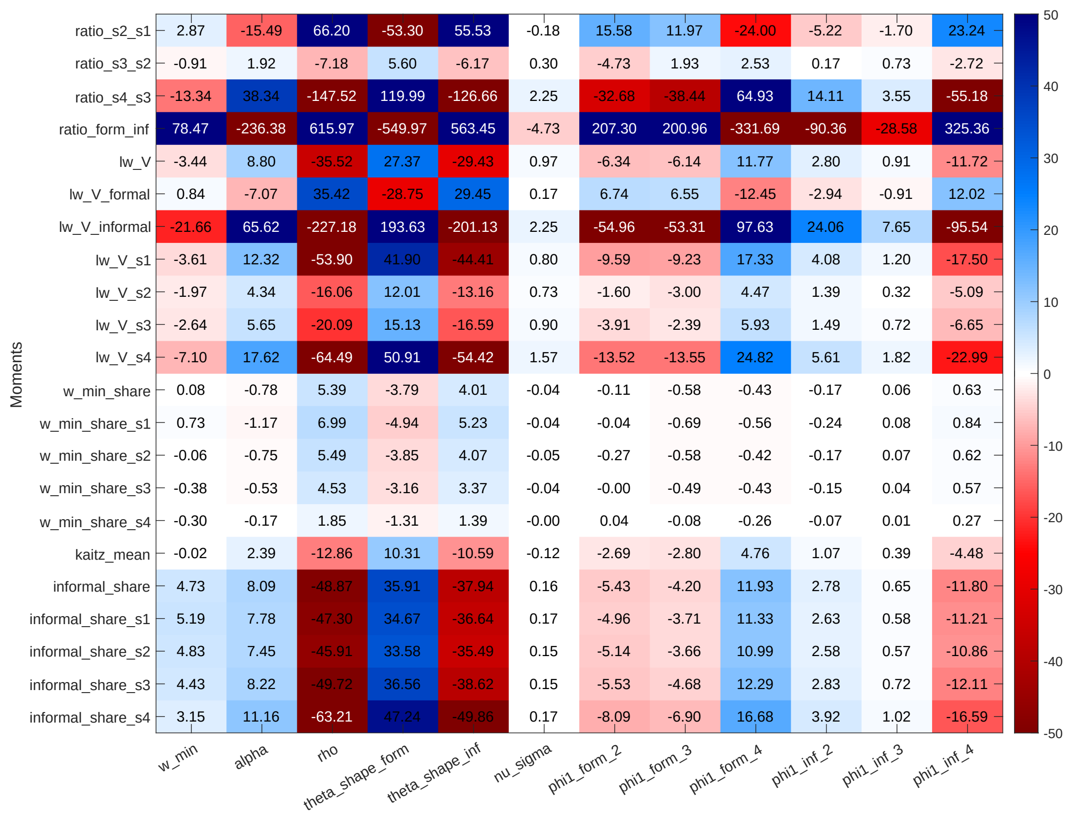
(a) 1996



(b) 2012

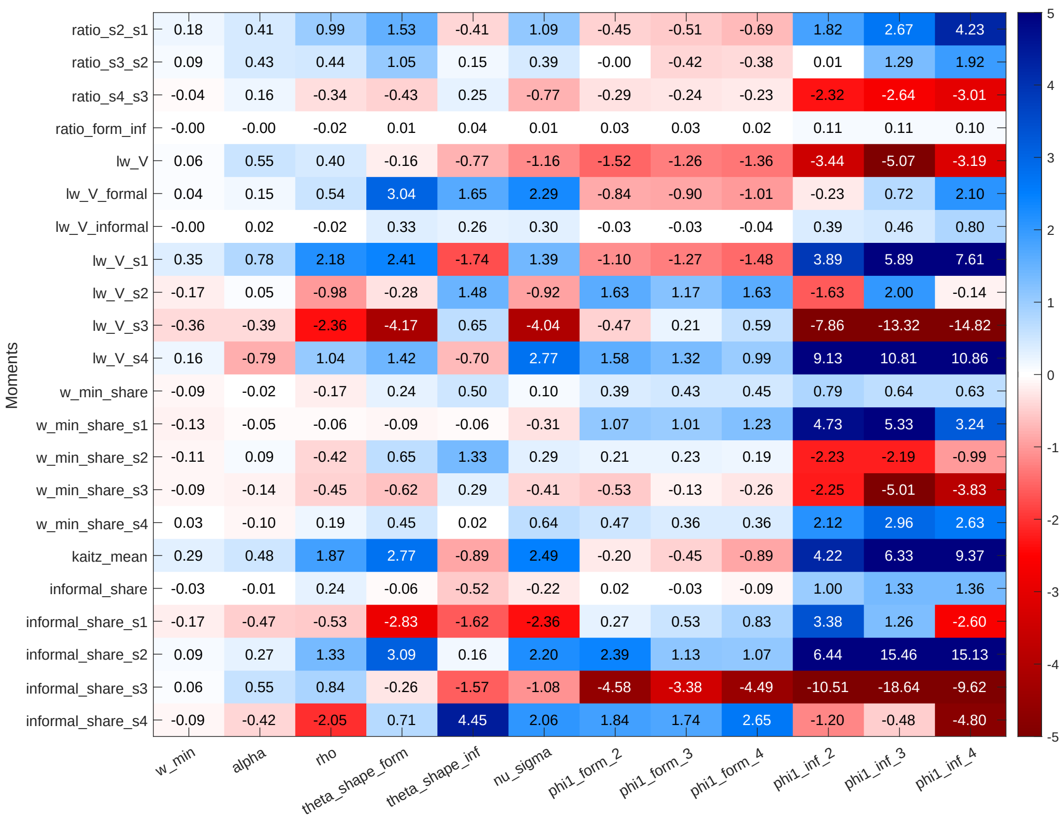
Notes: This figure presents the empirical distribution of the ratio of earnings to the minimum wage, expressed in logarithms, in 1996 and 2012 across different education groups. *Sources:* 1996/2012 PNAD.

Figure A.18: Sensitivity of moments to parameters



Notes: This figure illustrates how each targeted moment in the calibration responds to small changes in the parameters. Each number represents the numerical derivative calculated from the calibrated model, using a 0.001-unit perturbation to the left and right of each parameter, centered at its calibrated value. *Sources:* Model simulations.

Figure A.19: Sensitivity of parameters to moments (Andrews et al., 2017)



Notes: This figure illustrates how each parameter responds to a small perturbation in the moments. The sensitivity matrix was calculated following Andrews et al. (2017). *Sources:* Model simulations.

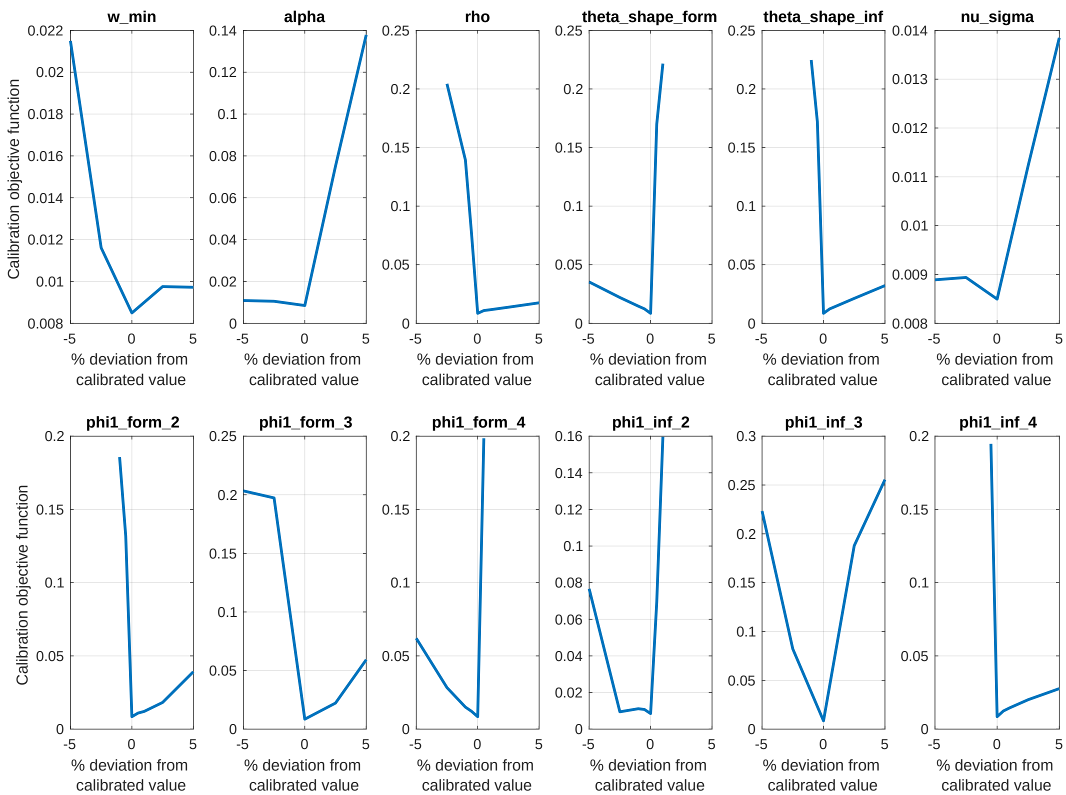
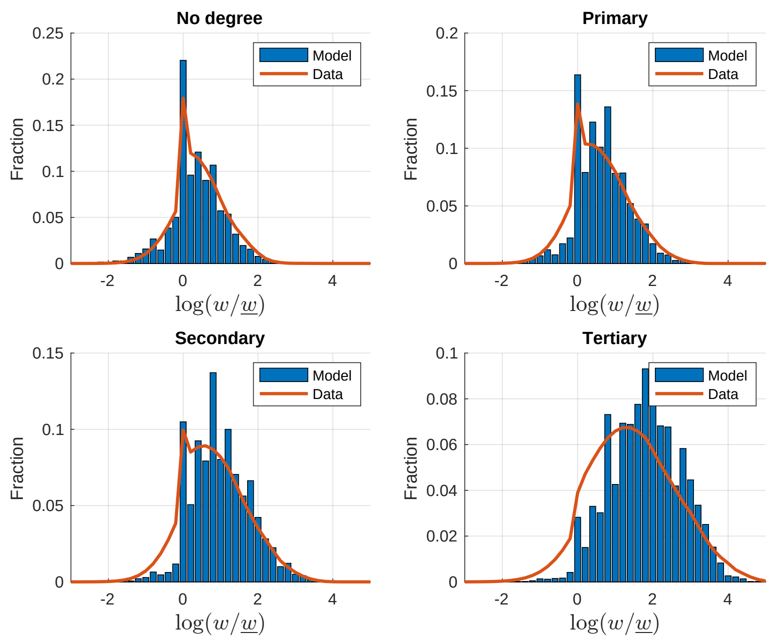


Figure A.20: Local sensitivity of the calibration objective function

Notes: Each plot shows the effect of a marginal change in a parameter value on the calibration objective function. The parameters are centered around their calibrated values. In some simulations that alter parameters dependent on a firm's formality status, only formal or only informal firms remain. In such cases, the objective function is not plotted, as it takes on a very large value to penalize the calibration algorithm. *Sources:* Model simulations.

Figure A.21: Log earnings histogram by skills, 1996



Notes: Histograms of log earnings relative to the minimum wage by education groups. Blue bars represent the data, and red lines represent the model. *Sources:* 1996 PNAD and model simulations.

Table A.6: Parameters of the models calibrated to the least and most exposed states

Parameter	Least exposed states	Most exposed states
α	0.65	0.60
ρ	0.08	0.02
ν shifter	0.31	-0.28
σ	0.88	1.11
κ^F	1.60	1.51
κ^I	1.62	1.63
ϕ_1^F	-0.15	-0.29
ϕ_2^F	-0.08	-0.17
ϕ_3^F	0.02	0.17
ϕ_4^F	0.22	0.29
ϕ_1^I	-0.12	0.03
ϕ_2^I	-0.05	-0.07
ϕ_3^I	-0.05	-0.32
ϕ_4^I	0.22	0.37
N_1	0.33	0.45
N_2	0.31	0.25
N_3	0.24	0.24
N_4	0.12	0.06

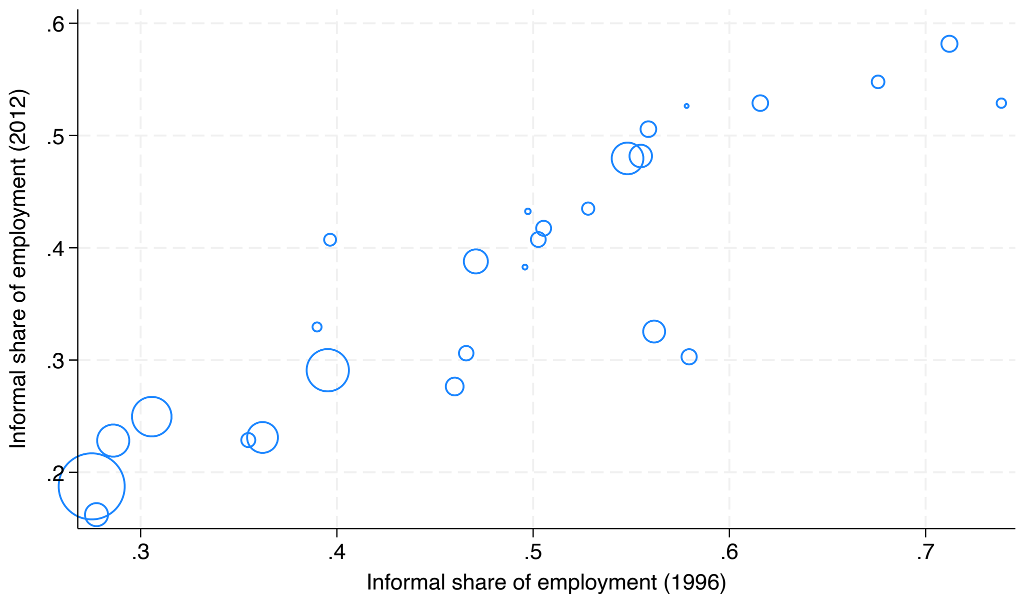
Notes: This table shows the parameters obtained through internal calibration for the two groups of states, least and most exposed to the minimum wage, as defined in Section 1. *Sources:* Model simulations.

Table A.7: Data and model moments for the calibration to least and most exposed states

	Least exposed states		Most exposed states	
	Data	Model	Data	Model
1. *Variance of log-earnings*:				
Overall	0.63	0.63	0.70	0.68
Formal	0.57	0.57	0.64	0.62
Informal	0.59	0.57	0.49	0.51
No degree	0.43	0.37	0.44	0.37
Primary	0.45	0.46	0.47	0.40
Secondary	0.50	0.57	0.69	0.74
Tertiary	0.85	0.93	1.03	1.10
2. *Mean earnings*:				
Formal/Informal	1.72	1.66	2.35	2.31
Primary/No degree	1.23	1.22	1.33	1.36
Secondary/Primary	1.32	1.35	1.88	1.90
Tertiary/Secondary	2.37	2.39	2.78	2.78
3. *Minimum wage*:				
(Formal) $\frac{\text{Min wage}}{\text{Mean wage}}$	0.17	0.23	0.32	0.31
(Formal) Fraction at $\underline{w}$				
Overall	0.02	0.03	0.22	0.20
No degree	0.04	0.06	0.32	0.40
Primary	0.02	0.04	0.23	0.11
Secondary	0.01	0.01	0.13	0.03
Tertiary	0.00	0.00	0.04	0.00
4. *Informal share*:				
Overall	0.28	0.26	0.56	0.52
No degree	0.36	0.28	0.71	0.56
Primary	0.28	0.26	0.48	0.55
Secondary	0.21	0.24	0.32	0.46
Tertiary	0.19	0.21	0.27	0.42

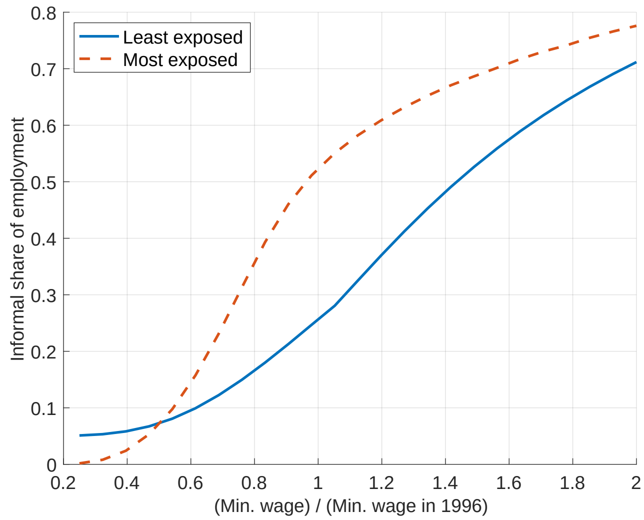
Notes: This table reports the model and data moments targeted in the calibration for the least and most exposed states. *Sources:* 1996 PNAD and model simulations.

Figure A.22: Informal share of employment in 1996 and 2012



Notes: Each circle in this scatterplot represents a state, with the vertical axis showing the informal employment share in 2012 and the horizontal axis showing the share in 1996. Circle sizes are proportional to employment in 1996. *Sources:* 1996/2012 PNAD.

Figure A.23: Informal employment share as a function of the minimum wage in the least and most exposed states



Notes: This figure shows the relationship between the share of informal employment and the minimum wage in the models calibrated to the least and most exposed states. *Sources:* Model simulations.

Table A.8: Differences-in-differences estimates in the structural model

	Overall variance			Formal variance		
	Before	After	Diff.	Before	After	Diff.
Least exposed	0.63	0.59	-0.04	0.57	0.50	-0.07
Most exposed	0.68	0.68	0.01	0.62	0.48	-0.14
Diff-in-diff			0.04			-0.06
	Informal variance			**Informal share**		
	Before	After	Diff.	Before	After	Diff.
Least exposed	0.57	0.54	-0.03	0.26	0.73	0.47
Most exposed	0.51	0.60	0.08	0.52	0.78	0.26
Diff-in-diff			0.12			-0.21

Notes: This table replicates the differences-in-differences estimation from Section 1.3 using simulated data from the structural model calibrated to the least and most exposed states. The "Before" and "After" columns show the levels of each statistic, while "Diff." reports the difference calculated as "After" minus "Before." The "Diff-in-diff" row presents the difference-in-differences estimates. *Sources:* Model simulations.

B Theory appendix

This section details the proofs for all propositions in the main text. When a complete proof is excessively long, we provide the main arguments and lemmas and refer the reader to the supplementary material

B.1 Proof of Proposition 1

Proof of Proposition 1. Let $c := \frac{\eta}{\eta+1}$ and $\kappa := \frac{(1-\rho)^{\eta+1}\eta^{\eta}}{(\eta+1)^{\eta+1}}$. Define $\bar{z} := \frac{1}{c}\,\underline{w} = \frac{\eta+1}{\eta}\,\underline{w}$.

(i) Unconstrained formal region ($z > \bar{z}$). When $z > \bar{z}$, the minimum wage is slack and the formal optimum is $w_F^*(z) = cz > \underline{w}$. Up to a common positive factor, unconstrained formal profit is

$$\pi^F_{\text{uncon}}(z) \;\propto\; (1-c)c^{\eta} z^{\eta+1} = \frac{\eta^{\eta}}{(\eta+1)^{\eta+1}}\, z^{\eta+1},$$

while the (optimized) informal profit is $\pi^I_{\max}(z) \propto \kappa z^{\eta+1}$. Since $\frac{\eta^{\eta}}{(\eta+1)^{\eta+1}} > \kappa$ (because $1 > (1-\rho)$), formality strictly dominates informality for all $z > \bar{z}$.

(ii) Wage-constrained range and the cutoff $\underline{z}$. For $z \le \bar{z}$, the formal wage is forced to $\underline{w}$, giving $\pi^F_{\max}(z) \propto (z-\underline{w})\,\underline{w}^{\eta}$, whereas $\pi^I_{\max}(z) \propto \kappa z^{\eta+1}$. Set $z = \varphi\,\underline{w}$ and consider

$$\tilde{h}(\varphi) := \frac{\pi^F_{\max}(\varphi\,\underline{w}) - \pi^I_{\max}(\varphi\,\underline{w})}{\underline{w}^{\eta+1}} = \varphi - 1 - \kappa\,\varphi^{\eta+1}.$$

Note that $\tilde{h}(1) = -\kappa < 0$ and, with $\varphi_0 := \frac{1}{c} = \frac{\eta+1}{\eta}$,

$$\tilde{h}'(\varphi) = 1 - (\eta+1)\kappa\,\varphi^{\eta} > 0 \quad \text{for all } \varphi \le \varphi_0, \qquad \tilde{h}(\varphi_0) = \frac{1-(1-\rho)^{\eta+1}}{\eta} > 0.$$

Hence there exists a unique root $\varphi(\eta,\rho) \in (1,\varphi_0)$ with $\tilde{h}(\varphi(\eta,\rho)) = 0$. Define $\underline{z} := \varphi(\eta,\rho)\,\underline{w}$. Then $\underline{w} < \underline{z} < \bar{z}$ for $\rho \in (0,1)$, $\underline{z} = \bar{z}$ when $\rho = 0$ and $\underline{z} = \underline{w}$ when $\rho = 1$, establishing item 2.

Polynomial characterization (item 1). Multiplying $\tilde{h}(\varphi) = 0$ by $\underline{w}^{\eta+1}$ and using $\varphi = z/\underline{w}$ yields

$$\kappa\,\underline{z}^{\eta+1} - \underline{w}^{\eta}\,\underline{z} + \underline{w}^{\eta+1} = 0,$$

i.e.

$$\frac{\eta^\eta}{(\eta+1)^{\eta+1}}(1-\rho)^{\eta+1}\,\underline{z}^{\eta+1} - \underline{w}^\eta\,\underline{z} + \underline{w}^{\eta+1} = 0.$$

(iii) Regime choice (item 3). Because $\tilde{h}$ is increasing and crosses zero at $\varphi(\eta,\rho)$, we have $\pi^I_{\max} > \pi^F_{\max}$ when $z < \underline{z}$, while $\pi^F_{\max} \geq \pi^I_{\max}$ when $\underline{z} \leq z \leq \bar{z}$. Together with the result for $z > \bar{z}$ from part (i), this gives the three-region partition.

(iv) Comparative statics of $\underline{z}$ (item 4). Let $F(\varphi;\eta,\rho) := \varphi - 1 - \kappa(\eta,\rho)\varphi^{\eta+1}$. At the root $\varphi = \varphi(\eta,\rho)$,

$$\partial_\varphi F = 1 - (\eta+1)\kappa\,\varphi^\eta > 0$$

(by the monotonicity shown above), so the implicit function theorem applies.

With respect to ρ. Since $\partial_\rho\kappa = -(\eta+1)\frac{\eta^\eta}{(\eta+1)^{\eta+1}}(1-\rho)^\eta < 0$, we have $\partial_\rho F = -(\partial_\rho\kappa)\varphi^{\eta+1} > 0$, hence

$$\frac{\partial\varphi}{\partial\rho} = -\frac{\partial_\rho F}{\partial_\varphi F} < 0 \quad\Rightarrow\quad \frac{\partial\underline{z}}{\partial\rho} = \underline{w}\,\frac{\partial\varphi}{\partial\rho} < 0.$$

With respect to η. Note $\partial_\eta \ln\kappa = \ln\Big(\frac{\eta}{\eta+1}(1-\rho)\Big) < 0$, so $\partial_\eta\kappa = \kappa\,\ln\Big(\frac{\eta}{\eta+1}(1-\rho)\Big) < 0$. Moreover,

$$\partial_\eta F = -\varphi^{\eta+1}\Big(\partial_\eta\kappa + \kappa\ln\varphi\Big) = -\kappa\,\varphi^{\eta+1}\,\ln\Big(\varphi\frac{\eta}{\eta+1}(1-\rho)\Big).$$

Because $\varphi < \varphi_0 = \frac{\eta+1}{\eta}$ and $(1-\rho) \leq 1$, the logarithm is negative for $\rho < 1$, so $\partial_\eta F > 0$ and therefore

$$\frac{\partial\varphi}{\partial\eta} = -\frac{\partial_\eta F}{\partial_\varphi F} < 0 \quad\Rightarrow\quad \frac{\partial\underline{z}}{\partial\eta} = \underline{w}\,\frac{\partial\varphi}{\partial\eta} < 0.$$

(At the boundary $\rho = 1$ one has $\kappa = 0$ and $\underline{z} = \underline{w}$; the strict inequalities hold for $\rho < 1$.)

Finally, linearity in $\underline{w}$ is immediate from $\bar{z} = \frac{\eta+1}{\eta}\underline{w}$ and $\underline{z} = \varphi(\eta,\rho)\underline{w}$. □

B.2 Proof of Proposition 2

Proof. Let us write $y = \log z$ and f_Y for its density. Now, given the cutoffs, $\underline{z} = \varphi(\rho, \eta)\, \underline{w}$ and $\overline{z} = \frac{\eta+1}{\eta} \underline{w}$., let us move to log-space and define $\underline{y}(\underline{w}) = \log \varphi(\rho, \eta) + \log \underline{w}$ and $\overline{y}(\underline{w}) = \log \frac{\eta+1}{\eta} + \log \underline{w}$.

Log wages are given by

$$\log w(y) = \begin{cases} A_I + y & , \text{if } y < \underline{y}(\underline{w}), \\ \log \underline{w} = A_F + \overline{y} & , \text{if } y \in \left[\underline{y}(\underline{w}), \overline{y}(\underline{w})\right], \\ A_F + y & , \text{if } y > \overline{y}(\underline{w}), \end{cases}$$

where $A_I = \log \frac{\eta(1-\rho)}{\eta+1}$ and $A_F = \log \frac{\eta}{\eta+1}$.

The employment density is $g(y) = \left(\frac{e^{\log w(y)}}{W}\right)^\eta f_Y(y) \propto e^{\eta \log w(y)} f_Y(y)$ and market-clearing means that

$$W^\eta = \int_{-\infty}^{\infty} e^{\eta \log w(y)} f_Y(y)\, dy.$$

Let us define the auxiliary functions, for $k = 0, 1, 2$,

$$I_k(\underline{w}) = \int_{-\infty}^{\underline{y}(\underline{w})} (A_I + y)^k e^{\eta(A_I + y)} f_Y(y)\, dy \tag{31}$$

$$+ \int_{\underline{y}(\underline{w})}^{\overline{y}(\underline{w})} (\log \underline{w})^k \underline{w}^\eta f_Y(y)\, dy \tag{32}$$

$$+ \int_{\overline{y}(\underline{w})}^{\infty} (A_F + y)^k e^{\eta(A_F + y)} f_Y(y)\, dy. \tag{33}$$

Then, the worker-weighted mean and variance of log wages are

$$\mathbb{E}[\log w | \underline{w}] = \frac{I_1(\underline{w})}{I_0(\underline{w})} \qquad \text{and} \qquad V(\log w | \underline{w}) = \frac{I_2(\underline{w})}{I_0(\underline{w})} - \left[\frac{I_1(\underline{w})}{I_0(\underline{w})}\right]^2. \tag{34}$$

A similar reasoning allows us to evaluate left and right tails with

$$I_k^L(\underline{w}) = \int_{-\infty}^{\underline{y}(\underline{w})} (A_I + y)^k e^{\eta(A_I + y)} f_Y(y)\, dy$$

and

$$I_k^R(\underline{w}) = \int_{\overline{y}(\underline{w})}^{\infty} (A_F + y)^k e^{\eta(A_F + y)} f_Y(y)\, dy,$$

so that

$$\mathbb{E}\left[\log w | y < \underline{y}\left(\underline{w}\right)\right] = \frac{I_1^L\left(\underline{w}\right)}{I_0^L\left(\underline{w}\right)} \qquad \text{and} \qquad V\left(\log w | \underline{w}\right) = \frac{I_2^L\left(\underline{w}\right)}{I_0^L\left(\underline{w}\right)} - \left[\frac{I_1^L\left(\underline{w}\right)}{I_0^L\left(\underline{w}\right)}\right]^2,$$

gives us the mean and variance of log wages for informal firms, and analogously for the right-tail expressions give us the conditional mean and variance for the unconstrained formal firms.

Laissez-faire ($\underline{w} = 0$), the "all formal" situation: Under laissez-faire, $I_k\left(\underline{w}\right) = \int_{-\infty}^{\infty} \left(A_F + y\right)^k e^{\eta(A_F+y)} f_Y\left(y\right) dy = e^{\eta A_F} \int_{-\infty}^{\infty} \left(A_F + y\right)^k e^{\eta y} f_Y\left(y\right) dy$, so all moments coincide with moments derived under $f_\eta\left(y\right) = \frac{e^{\eta y}}{\mathbb{E}[e^{\eta y}]} f_Y\left(y\right)$: a tilted distribution.

Let $\mathbb{E}_\eta\left[y\right]$ and $Var_\eta\left[y\right]$ denote the expectation and variance of log-wages under this tilted distribution. Notice that a bounded $\mathbb{E}_\eta\left[y^2\right] = \frac{\mathbb{E}\left[y^2 e^{\eta y}\right]}{\mathbb{E}[e^{\eta y}]} < \infty$ guarantees the existence of the variance (and lower moments), so

$$V\left(\log w | \underline{w} = 0\right) = Var_\eta\left[y\right].$$

$\underline{w} \to \infty$, the "all informal" limit: As $\underline{w} \to \infty$, $\underline{z} \to \infty$ (and so does $\underline{y}$). In the limit, the term in line 32 vanishes, all firms are informal and $\lim_{\underline{w}\to\infty} I_k\left(\underline{w}\right) = \int_{-\infty}^{\infty} \left(A_I + y\right)^k e^{\eta(A_I+y)} f_Y\left(y\right) dy = e^{\eta A_I} \int_{-\infty}^{\infty} \left(A_I + y\right)^k e^{\eta y} f_Y\left(y\right) dy$.

We are again dealing with the same tilted distribution as in $\underline{w} = 0$. As a consequence

$$\lim_{\underline{w}\to\infty} V\left(\log w | \underline{w}\right) = Var_\eta\left[y\right].$$

So, laissez-faire and the all-informal limit share the same log-wage variance, $Var_\eta\left[y\right]$.

Continuity: Set $g_{I,k}(y) := (A_I + y)^k e^{\eta y} f_Y(y)$ and $g_{F,k}(y) := (A_F + y)^k e^{\eta y} f_Y(y)$.

The assumption $\mathbb{E}[y^2 e^{\eta y}] < \infty$ implies $g_{I,k}, g_{F,k} \in L^1(\mathbb{R})$ for $k = 0, 1, 2$. Hence the primitives $G_{I,k}(t) = \int_{-\infty}^{t} g_{I,k}(y)\, dy$ and $G_{F,k}(t) = \int_{-\infty}^{t} g_{F,k}(y)\, dy$ are continuous. Since $\underline{y}(\cdot)$ and $\overline{y}(\cdot)$ are continuous in $\underline{w}$, the maps $\underline{w} \mapsto \int_{-\infty}^{\underline{y}(\underline{w})} g_{I,k}(y)\, dy$ and $\underline{w} \mapsto \int_{\overline{y}(\underline{w})}^{\infty} g_{F,k}(y)\, dy$ are continuous. The middle term is continuous as a product of $(\log \underline{w})^k \underline{w}^\eta$ and $F(\overline{y}(\underline{w})) - F(\underline{y}(\underline{w}))$. Therefore each $I_k(\underline{w})$ is continuous, $I_0(\underline{w}) > 0$, and $V(\log w \mid \underline{w}) = I_2/I_0 - (I_1/I_0)^2$ is continuous on $(0, \infty)$.

Global minimum exists and is interior: Consider there is $\underline{w}_1 > 0$ with $V(w_1) < Var_\eta[y]$. Take $\epsilon \in (0, Var_\eta[y] - V(\underline{w}_1))$. Then, by existence of the limits, there exists $w_L < \underline{w}_1$ such that $|V(w) - Var_\eta[y]| < \epsilon$ for all $w \in (0, w_L)$ and, analogously, there exist $w_R > \underline{w}_1$ such that $|V(w) - Var_\eta[y]| < \epsilon$ for all $w \in (w_R, +\infty)$. By Weierstrass, V attains a global minimum on the compact interval $[w_L, w_R]$, moreover

$$V(w_L), V(w_R) > Var_\eta[y] - \epsilon > V(w_1),$$

so any global minimizer lies strictly inside (w_L, w_R). Let $\underline{w}^* := \sup \arg\min_{w \geq 0} V(w)$ (the rightmost minimizer) to absorb possible flatness or multiplicity.

Increasing range to the right of $\underline{w}^*$: Given the continuous density f_Y and Leibniz applied to 34, $V(\underline{w})$ is C^1 for all $\underline{w} > 0$ such that $\underline{z}(\underline{w}), \overline{z}(\underline{w}) \in (z_{\min}, z_{\max})$.

Since $\underline{w}^*$ is a global minimizer, we have $V(\underline{w}) \geq V(\underline{w}^*)$ for all $\underline{w} \geq \underline{w}^*$. Suppose, for contradiction, that $V'(\underline{w}) \leq 0$ for all $\underline{w} > \underline{w}^*$. Then V is weakly decreasing on $[\underline{w}^*, \infty)$, which implies $V(\underline{w}) \leq V(\underline{w}^*)$ for all $\underline{w} \geq \underline{w}^*$. This contradicts the fact that $\lim_{\underline{w}\to\infty} V(\underline{w}) = Var_\eta[y] > V(\underline{w}^*)$.

Hence V' must be strictly positive at some point $w_0 > \underline{w}^*$. By continuity of V' in the interior of the support, there exists an open interval $(\underline{w}^*, \underline{w}^{**}) \ni w_0$ such that $V'(w) > 0$ for all $w \in (\underline{w}^*, \underline{w}^{**})$. In this interval, V is strictly increasing. □

B.3 Analysis of the Pareto case

Proposition 5 (Pareto case: three equilibrium regimes and the sign of $V'(\underline{w})$)**.** *Assume $Z \sim \mathrm{Pareto}(\nu)$ on $[z_{\min}, \infty)$ with $\nu > \eta$ and $0 < \rho < 1$. Let $c := \frac{\eta}{\eta+1}$, $\bar{z}(\underline{w}) = (\eta+1)\underline{w}/\eta$, and $\underline{z}(\underline{w}) = \varphi(\eta, \rho)\,\underline{w}$. Define the critical minimum wage levels $\underline{w}_0 := c\, z_{\min}$ (first bite of minimum wage on the formal sector) and $\underline{w}_1 := z_{\min}/\varphi(\eta,\rho)$ (informality onset).*

Then $\underline{w}_0 < \underline{w}_1$, and there are three regions of interest:

1. ***Non-binding minimum wage:*** *$0 \leq \underline{w} \leq \underline{w}_0$. All formal and unconstrained; $V(\underline{w}) \equiv Var_\eta[\log Z]$, $V'(\underline{w}) = 0$.*

2. ***Binding, without informality:*** *$\underline{w}_0 < \underline{w} < \underline{w}_1$. $s^I = 0$ and $V'(\underline{w}) = \frac{dV^F}{d\underline{w}} < 0$.*

3. ***Binding minimum wage and informality present:*** *$\underline{w} > \underline{w}_1$: V^F is constant in $\underline{w}$ and $dV^F/d\underline{w} = 0$; $s^I(\underline{w})$ is strictly increasing; $V^I(\underline{w})$ is strictly*

increasing.

Unintended consequences: Additionally, the right derivative at the onset of informality ($\underline{w} \geq \underline{w}_1$) is

$$\frac{dV}{d\underline{w}}\bigg|_{\underline{w}_1^+} = \frac{ds^I}{d\underline{w}}\bigg|_{\underline{w}_1} \Big(\underbrace{V^I(\underline{w}_1^+) - V^F}_{=-V^F} + (E^I - E^F)^2\Big) > 0,$$

ensuring that variance is strictly increasing in a right neighborhood of $\underline{w}_1$.

Proof. Let $Z \sim Pareto\,(\nu)$ and $Y = \log Z \sim F_Y$ with density f_Y. Let $y_{\min} :=$ $\inf supp(Y)$ and define $\alpha := \nu - \eta > 0$.

Write the employment kernel $e^{\eta \log w(y)} f_Y(y)$. As before, let the cutoffs in log space be

$$\underline{y}(w) = \log \varphi(\rho, \eta) + \log \underline{w}, \qquad \bar{y}(w) = \log\Big(\frac{\eta+1}{\eta}\Big) + \log \underline{w},$$

and $A_I = \log \frac{\eta(1-\rho)}{\eta+1}$ and $A_F = \log \frac{\eta}{\eta+1}$ denote the log-wage intercepts.

Under Pareto, $f_Y(y) \propto e^{-\nu y}\mathbf{1}\{y \geq y_{\min}\}$, hence

$$e^{\eta \log w(y)} f_Y(y) \propto \begin{cases} e^{-\alpha y}, & y < y(w), \\ w^\eta e^{-\nu y}, & y \in [y(w), \bar{y}(w)], \\ e^{-\alpha y}, & y > \bar{y}(w). \end{cases}$$

and $\bar{y}(w) - \underline{y}(w) = \log\big(\frac{\eta+1}{\eta}\big) - \log \varphi(\rho, \eta)$ is constant. Once again, define $k \in \{0, 1, 2\}$ the auxiliary integrals

$$I_k(\underline{w}) = \underbrace{\int_{y_{\min}}^{\underline{y}(\underline{w})} (A_I + y)^k e^{\eta(A_I + y)} f_Y(y)\, dy}_{I_k^L(\underline{w})} + \underbrace{\int_{\underline{y}(\underline{w})}^{\bar{y}(\underline{w})} (\log(\underline{w}))^k \underline{w}^\eta f_Y(y)\, dy}_{\text{bunching at } \underline{w}} \tag{35}$$

$$+ \underbrace{\int_{\bar{y}(\underline{w})}^{\infty} (A_F + y)^k e^{\eta(A_F + y)} f_Y(y)\, dy}_{I_k^R(\underline{w})} \tag{36}$$

and the corresponding means/variances:

$$\mathbb{E}[\log w \mid \underline{w}] = \frac{I_1}{I_0}, \qquad V(\log w \mid \underline{w}) = \frac{I_2}{I_0} - \Big(\frac{I_1}{I_0}\Big)^2,$$

$$E^I = \frac{I_1^L}{I_0^L}, \quad V_I = \frac{I_2^L}{I_0^L} - \left(\frac{I_1^L}{I_0^L}\right)^2, \qquad E^F = \frac{I_1 - I_1^L}{I_0 - I_0^L}, \quad V^F = \frac{I_2 - I_2^L}{I_0 - I_0^L} - \left(\frac{I_1 - I_1^L}{I_0 - I_0^L}\right)^2.$$

There are three regimes to consider:

1) $y_{\min} \geq \bar{y}(w) > \underline{y}(w)$**:** The minimum wage does not bind, and all firms are unconstrained. Then,

$$I_k\left(\underline{w}\right) = \int_{y_{\min}}^{\infty} (A_F + y)^k e^{\eta(A_F+y)} e^{-\nu y} dy$$

and

$$V\left(\underline{w}\right) = \frac{I_2}{I_0} - \left(\frac{I_1}{I_0}\right)^2 = \frac{1}{(\nu - \eta)^2},$$

so that variance is constant and $V'\left(\underline{w}\right) = 0$ in this regime.

2) $\bar{y}(w) > y_{\min} \geq \underline{y}(w)$ **:** Constrained and unconstrained formal firms coexist; informal firms are absent.

$$I_k(\underline{w}) = \int_{y_{\min}}^{\bar{y}(\underline{w})} (\log \underline{w})^k \underline{w}^\eta e^{-\nu y}\, dy + \int_{\bar{y}(\underline{w})}^{\infty} (A_F + y)^k e^{\eta(A_F+y)} e^{-\nu y}\, dy.$$

We can resort to a variance decomposition

$$V^F\left(\underline{w}\right) = p_{U|F} V^{UF}\left(\underline{w}\right) + p_{U|F}\left(1 - p_{U|F}\right)\Delta^2,$$

where $p_{U|F}$ denotes the share of unconstrained workers among formal workers, its complement $\left(1 - p_{U|F}\right)$ denote the share of formal workers at the minimum wage and Δ denote the difference between (worker-weighted) mean log-wages of these two groups. Let

$$CF_k\left(\underline{w}\right) = \int_{y_{\min}}^{\bar{y}(\underline{w})} (\log \underline{w})^k \underline{w}^\eta e^{-\nu y}\, dy$$

so that $p_{U|F} = \frac{I_0^R(\underline{w})}{CF_0(\underline{w}) + I_0^R(\underline{w})}$ and notice that $\Delta_F = \mathbb{E}_\eta\left[\left(A_F + y - \log \underline{w}\right) | y \geq \overline{y}\left(\underline{w}\right)\right] = \mathbb{E}_\eta\left[\left(y - \overline{y}\left(\underline{w}\right)\right) | y \geq \overline{y}\left(\underline{w}\right)\right] = \frac{1}{\nu - \eta}$ and

$$V^{UF}\left(\underline{w}\right) = Var_\eta\left(\underline{w}\right) = \frac{I_2^L}{I_0^L} - \left(\frac{I_1^L}{I_0^L}\right)^2 = \frac{1}{(\nu - \eta)^2}.$$

The variance decomposition then becomes

$$V^F(\underline{w}) = \left(\frac{1}{\nu-\eta}\right)^2 \left[p_{U|F} + p_{U|F}\left(1-p_{U|F}\right)\right] = \left(\frac{1}{\nu-\eta}\right)^2 \left[2p_{U|F} - p_{U|F}^2\right],$$

so

$$\frac{dV^F}{d\underline{w}} \propto \left[1-p_{U|F}\right] p_{U|F}^{'} < 0,$$

as the proportion of workers at unconstrained firms is decreasing in the minimum wage, i.e., $p_{U|F}^{'} < 0$.

In this region the informal share is zero, so $V(\underline{w}) = V^F(\underline{w})$ and derivatives coincide.

3) $\bar{y}(w) > \underline{y}(w) \geq y_{\min}$: Constrained, unconstrained, and informal firms coexist. Now, it is useful to define formal sector auxiliary functions with

$$I_k^F(\underline{w}) = \underbrace{\int_{\underline{y}(\underline{w})}^{\bar{y}(\underline{w})} (\log \underline{w})^k \underline{w}^\eta e^{-\nu y}\, dy}_{\text{bunching}} + \underbrace{\int_{\bar{y}(\underline{w})}^{\infty} (A_F+y)^k e^{\eta(A_F+y)} e^{-\nu y}\, dy}_{I_k^R(\underline{w})} = I_k(\underline{w}) - I_k^L(\underline{w}).$$

We now define the constrained-firm minimum wage contribution as

$$CF_k(\underline{w}) = (\log \underline{w})^k \underline{w}^\eta \int_{\underline{y}(\underline{w})}^{\bar{y}(\underline{w})} e^{-\nu y}\, dy,$$

noting that both extrema now depend on the minimum wage. They also have a fixed distance $\log\left(\frac{\eta+1}{\eta}\right) - \log\varphi(\rho,\eta) > 0$ in log-space. As in the previous region, the same variance decomposition steps establish that $V^F(\underline{w}) = \left(\frac{1}{\nu-\eta}\right)^2 \left[2p_{U|F} - p_{U|F}^2\right]$ and $\frac{dV^F}{d\underline{w}} \propto \left[1-p_{U|F}\right] p_{U|F}^{'}$, but in this region

$$p_{U|F} = \frac{I_0^R(\underline{w})}{CF_0(\underline{w}) + I_0^R(\underline{w})} = \frac{\int_{\bar{y}(\underline{w})}^{\infty} e^{\eta A_F + (\eta-\nu)y}\, dy}{\underline{w}^\eta \int_{\underline{y}(\underline{w})}^{\bar{y}(\underline{w})} e^{-\nu y}\, dy + \int_{\bar{y}(\underline{w})}^{\infty} e^{\eta A_F + (\eta-\nu)y}\, dy}.$$

As

$$I_0^R(\underline{w}) = \int_{\bar{y}(\underline{w})}^{\infty} e^{\eta A_F + (\eta-\nu)y}\, dy = \frac{e^{\eta A_F}}{\nu-\eta} \left(\frac{\eta+1}{\eta}\right)^{\eta-\nu} \underline{w}^{\eta-\nu}$$

and

$$CF_0\left(\underline{w}\right) = \underline{w}^{\eta} \int_{\underline{y}(\underline{w})}^{\bar{y}(\underline{w})} e^{-\nu y}\, dy = \frac{1}{\nu}\left[\varphi\left(\eta,\rho\right)^{-\nu} - \left(\frac{\eta+1}{\eta}\right)^{-\nu}\right]\underline{w}^{\eta-\nu},$$

both terms share the $\underline{w}^{\eta-\nu}$ factor that cancels out in $p_{U|F}$. So, in this region, where informality is present, the share $p_{U|F}$ does not depend on the minimum wage and

$$p'_{U|F} = 0 \implies \frac{dV^F}{d\underline{w}} \propto \left[1 - p_{U|F}\right] p'_{U|F} = 0.$$

For the informal sector, we have that employment weighted density is $f_\eta\left(y\right) \propto e^{-(\nu-\eta)y}$ is exponential and truncated at $\underline{y}\left(\underline{w}\right)$ and $y_{\min}$. As such, let $\delta = \underline{y}\left(\underline{w}\right) - y_{min}$,

$$V^I\left(\underline{w}\right) = Var_\eta\left(y|y \leq \underline{y}\right) = \frac{1}{(\nu-\eta)^2} - \frac{\delta^2\, e^{-(\nu-\eta)\delta}}{\left(1 - e^{-(\nu-\eta)\delta}\right)^2}$$

and

$$\frac{dV^I\left(\underline{w}\right)}{d\underline{w}} = \frac{2\delta^2\, e^{-(\nu-\eta)\delta}}{\left(1 - e^{-(\nu-\eta)\delta}\right)^3}\left[1 - e^{-(\nu-\eta)\delta} - (\nu-\eta)\delta\, e^{-(\nu-\eta)\delta}\right]\frac{d\underline{y}}{d\underline{w}} > 0,$$

as the fraction that leads, the term in brackets, and $\frac{d\underline{y}}{d\underline{w}}$ are all positive.

Last, we show that $s^{I'}(\underline{w}) > 0$: Because $\bar{y} - \underline{y}$ is constant, the block masses can be written

$$I_0^L(\underline{w}) = L_0 - L_1\, \underline{w}^{-(\nu-\eta)}, \qquad CF_0(\underline{w}) = M_1\, \underline{w}^{-(\nu-\eta)}, \qquad I_0^R(\underline{w}) = R_1\, \underline{w}^{-(\nu-\eta)},$$

for constants $L_0, L_1, M_1, R_1 > 0$ independent of $\underline{w}$. Let $t := \underline{w}^{-(\nu-\eta)}$. Then,

$$s^I(\underline{w}) = \frac{I_0^L}{I_0^L + CF_0 + I_0^R} = \frac{L_0 - L_1 t}{L_0 + (M_1 + R_1 - L_1)t} =: S(t),$$

so

$$S'(t) = -\frac{L_0(M_1 + R_1)}{\left[L_0 + (M_1 + R_1 - L_1)t\right]^2} < 0, \qquad \frac{dt}{d\underline{w}} = -(\nu-\eta)\underline{w}^{-(\nu-\eta+1)} < 0.$$

Hence $\dfrac{ds^I}{d\underline{w}} = S'(t)\,\dfrac{dt}{d\underline{w}} > 0.$

Increasing variance region: In the absence of informality, aggregate variance is either flat (when the minimum does not bind) or decreasing (in region 2). We now use the variance decomposition

$$V = s^F V^F + s^I V^I + s^F s^I \left(E^I - E^F\right)^2$$

in a right neighborhood of $\underline{y}(\underline{w}) = y_{\min}$, where informality starts to emerge. Differentiating (from the right), we obtain

$$V^{'} = s^F \left(V^F\right)^{'} + s^I \left(V^I\right)^{'} + \left(s^I\right)^{'} \left[\left(V^I - V^F\right) + \left(1 - 2s^I\right)\left(E^I - E^F\right)^2\right] + s^F s^I \frac{\partial \left(E^I - E^F\right)^2}{\partial \underline{w}}.$$

Evaluating at $\underline{w} \to \underline{w}_1^+$ (from the right, inside regime 3, with $\underline{w}_1 := \varphi\left(\eta, \rho\right)^{-1} z_{\min}$), we have $s^I = 0$, $\left(V^F\right)^{'} = 0$, and $V^I = 0$, so

$$V^{'} = \left(s^I\right)^{'} \left[-V^F + \left(E^I - E^F\right)^2\right].$$

Now, using $\underline{y}\left(\underline{w}\right) = y_{\min}$,

$$E^I = A_I + y_{\min} = A_I + \underline{y}$$

and

$$\begin{aligned} E^F &= \ln \underline{w} + p_{U|F} \mathbb{E}_\eta \left[\left(y - \overline{y}\right)|y \geq \overline{y}\right] \\ &= \underline{y} - \ln \varphi(\rho, \eta) + p_{U|F} \frac{1}{\nu - \eta}, \end{aligned}$$

where we used $\underline{y} = \ln \varphi(\rho, \eta) + \ln \underline{w}$. So, at the regime transition (from the right), we have

$$E^I - E^F = A_I + \ln \varphi(\rho, \eta) - p_{U|F} \frac{1}{\nu - \eta}.$$

Also, at that limit from the right,

$$\begin{aligned} V^F &= p_{U|F} V_{U|F} + p_{U|F}\left(1 - p_{U|F}\right) \mathbb{E}_\eta \left[\left(y - \overline{y}\right)|y \geq \overline{y}\right]^2 \\ &= p_{U|F} \left(\frac{1}{\nu - \eta}\right)^2 + p_{U|F}\left(1 - p_{U|F}\right)\left(\frac{1}{\nu - \eta}\right)^2 \\ &= p_{U|F}\left(2 - p_{U|F}\right)\left(\frac{1}{\nu - \eta}\right)^2, \end{aligned}$$

where

$$p_{U|F}=\frac{\nu\left(\frac{\eta+1}{\eta}\right)^{-\nu}}{\nu\left(\frac{\eta+1}{\eta}\right)^{-\nu}+(\nu-\eta)\left[\varphi\left(\eta,\rho\right)^{-\nu}-\left(\frac{\eta+1}{\eta}\right)^{-\nu}\right]}.$$

Hence at the limit of interest $V^{'}=\left(s^{I}\right)^{'}\mathcal{B}$, where $\mathcal{B}\equiv -V^{F}+(E^{I}-E^{F})^{2}=\left(C_{\Delta}-\frac{p_{U|F}}{\alpha}\right)^{2}-\frac{p_{U|F}(2-p_{U|F})}{\alpha^{2}}$, and $C_{\Delta}\equiv\log\left(\frac{\eta}{\eta+1}\right)+\log(1-\rho)+\log\varphi$.

In Region 3,

$$p_{U|F}=\frac{\nu\, q}{(\nu-\eta)\big(\varphi^{-\nu}-q\big)+\nu\, q},\qquad q\equiv\Big(\frac{\eta+1}{\eta}\Big)^{-\nu}.$$

So, we can write

$$\mathcal{B}=\frac{\left(\alpha C_{\Delta}-p_{U|F}\right)^{2}-p_{U|F}\left(2-p_{U|F}\right)}{\alpha^{2}}.$$

It suffices then to show that $\left(\alpha C_{\Delta}-p_{U|F}\right)^{2}-p_{U|F}\left(2-p_{U|F}\right)>0$ for ensuring $\mathcal{B}>0$. To streamline notation, let $\zeta=\frac{\varphi(\eta,\rho)^{-\nu}}{q}$, so that

$$\zeta=1+\frac{\nu}{\alpha}\left(\frac{1}{p}-1\right),$$

$u:=\frac{\zeta}{(1-\rho)^{\nu}}$, and $p:=p_{U|F}\in(0,1)$. Notice that $\alpha C_{\Delta}=-c\log u$.

Define, $c:=\frac{\alpha}{\nu}\in(0,1)$, and

$$\Psi\left(u,p\right)=\left(c\log u+p\right)^{2}-p\left(2-p\right).$$

Notice that

$$\zeta-\frac{1}{p}=\left(\frac{1}{p}-1\right)\left(\frac{1}{c}-1\right)>0\implies\zeta>\frac{1}{p}.$$

Also notice that $u=\frac{\zeta}{(1-\rho)^{\nu}}>\zeta$. As a consequence, $\log u>\log\zeta>0$.

From the monotonicity of Ψ in is first argument, then

$$\Psi\left(u,p\right)>\Psi\left(\zeta,p\right)=\left(c\log\zeta+p\right)^{2}-p\left(2-p\right).$$

Write $\theta:=\frac{1-p}{cp}\geq 0$ and notice that $\zeta=1+\theta$. We now use this fundamental bound (valid for all $\theta > -1$):

$$\log(1+\theta) \geq \frac{\theta}{1+\theta}.$$

Then,

$$\begin{aligned} c\log\zeta = c\log(1+\theta) &\geq c\frac{\theta}{1+\theta} \\ &= c\frac{1-p}{cp+1-p}. \end{aligned}$$

Define $D := cp+1-p = 1-(1-c)\,p \in (0,1]$. Then,

$$c\log\zeta \geq \frac{1-p}{D} \implies c\log\zeta + p \geq \frac{pD+(1-p)}{D} = \frac{1-(1-c)\,p^2}{D}.$$

So, combining the inequalities,

$$\Psi(u,p) > \Psi(\zeta,p) \geq \left(\frac{1-(1-c)\,p^2}{D}\right)^2 - p\,(2-p)\,.$$

Then, define $\beta = 1-c$,

$$\Psi(u,p) > \frac{\left(1-\beta p^2\right)^2 - p\,(2-p)\,[1-\beta p]^2}{D^2}.$$

The numerator above factors as

$$N(p,\beta) := (p-1)\left(2\beta^2p^3 - 2\beta p^2 + p - 1\right).$$

The second term is convex quadratic in β, as $2\beta^2p^3 > 0$. At the endpoint $\beta = 0$, its value is $\left(p-1\right) < 0$. At the endpoint $\beta = 1$, $\left(2p^3-2p^2+p-1\right) = (p-1)\left(2p^2+1\right) < 0$. Hence, this second term is strictly negative for all $\beta \in (0,1)$.

Consequently, $N(p,\beta) > 0$ as it is the product of two strictly negative terms and $\Psi(u,p) > \frac{N(p,\beta)}{D^2} > 0$. □

B.4 Proofs of Results from Section 3

Proof of Proposition 3. Fix a productivity realization z and a skill h. Conditional on the aggregate wage index W_h and skill mass N_h, a formal firm takes $\{W_h, N_h\}$ as given and chooses $\{w_h, l_h\}$ to maximize

$$\pi_h(z) = z\,\xi_h(z)\,l_h^\alpha - (1+\tau)w_h l_h$$

subject to the firm-level labor supply and the minimum wage constraint

$$l_h \;\le\; N_h\Big(\tfrac{(1+\varsigma_h)w_h}{W_h}\Big)^\eta, \qquad w_h \ge \underline{w},$$

where ς_h is the value-of-formality wedge for skill h. This is the per–skill version of the firm optimization problem, where we exploit the separability across skills.

KKT conditions. The Lagrangian for a given h is

$$\mathcal{L} = z\,\xi_h(z) l_h^\alpha - (1+\tau)w_h l_h + \lambda_h\Big[N_h\Big(\tfrac{(1+\varsigma_h)w_h}{W_h}\Big)^\eta - l_h\Big] + \mu_h\,(w_h - \underline{w}),$$

with multipliers $\lambda_h, \mu_h \ge 0$. The first-order and complementary slackness conditions are

$$\partial_{l_h}\mathcal{L}: \quad \alpha z \xi_h(z) l_h^{\alpha-1} - (1+\tau)w_h - \lambda_h = 0, \tag{37}$$

$$\partial_{w_h}\mathcal{L}: \quad -(1+\tau)l_h + \lambda_h N_h \eta\Big(\tfrac{(1+\varsigma_h)}{W_h}\Big)^\eta w_h^{\eta-1} + \mu_h = 0, \tag{38}$$

$$\lambda_h\left[N_h\Big(\tfrac{(1+\varsigma_h)w_h}{W_h}\Big)^\eta - l_h\right] = 0, \quad \mu_h(w_h - \underline{w}) = 0. \tag{39}$$

Because $\alpha \in (0,1)$, profits are strictly concave in l_h and decreasing in w_h, so w_h is optimally set at its minimum feasible level (statutory minimum or the supply-implied bound); fixing w_h reduces the problem to a concave maximization in l_h with convex constraints, so KKT conditions are necessary and sufficient.

Region (i): unconstrained by the minimum wage; hire on the labor-supply curve. Here $\mu_h = 0$ and the labor-supply constraint binds, so

$$l_h = N_h\Big(\tfrac{(1+\varsigma_h)w_h}{W_h}\Big)^\eta, \qquad \lambda_h > 0.$$

From (38) and the binding labor-supply equation, we obtain $\lambda_h = (1+\tau)w_h/\eta$. Substituting into (37) yields

$$\alpha z \xi_h(z) l_h^{\alpha-1} = (1+\tau)w_h\Big(1+\tfrac{1}{\eta}\Big) \quad\Longleftrightarrow\quad (1+\tau)w_h = \frac{\eta}{\eta+1}\,\alpha z \xi_h(z)\, l_h^{\alpha-1}.$$

Thus wages are a constant markdown $\eta/(\eta+1)$ of the marginal product (net of the payroll tax): $w_h = \frac{\eta}{\eta+1}\frac{\mathrm{MPL}_h}{(1+\tau)}$. Using $l_h = N_h((1+\varsigma_h)w_h/W_h)^\eta$ to eliminate l_h and solving for w_h gives

$$w_h^F(z) = \frac{W_h}{1+\varsigma_h}\left[\frac{(1+\varsigma_h)\,\alpha\eta\,z\,\xi_h(z)}{(1+\tau)\,W_h\,(\eta+1)\,N_h^{1-\alpha}}\right]^{\frac{1}{1+\eta(1-\alpha)}}, \qquad l_h^F(z) = N_h\Big(\tfrac{(1+\varsigma_h)w_h^F(z)}{W_h}\Big)^\eta.$$

These coincide with part (i) of Proposition 3.

Region (ii): minimum wage binds; hire on the labor-supply curve. If the expression above implies $w_h^F(z) < \underline{w}$, then the minimum wage is binding at the optimum. Setting $w_h = \underline{w}$ and enforcing the labor-supply constraint with equality yields

$$w_h^F(z) = \underline{w}, \qquad l_h^F(z) = N_h\Big(\tfrac{(1+\varsigma_h)\underline{w}}{W_h}\Big)^\eta,$$

which is part (ii) of Proposition 3.

Region (iii): minimum wage binds; rationing (labor-supply slack). If, at $w_h = \underline{w}$, the unconstrained optimal l_h from (37) is below the supply implied by $\underline{w}$, then the labor-supply constraint is slack ($\lambda_h = 0$) and (37) gives

$$\alpha z\xi_h(z)l_h^{\alpha-1} = (1+\tau)\underline{w} \quad\Longrightarrow\quad l_h^F(z) = \left(\frac{\alpha z\xi_h(z)}{(1+\tau)\underline{w}}\right)^{\frac{1}{1-\alpha}}.$$

Rationing obtains precisely when $l_h^F(z) < N_h((1+\varsigma_h)\underline{w}/W_h)^\eta$, which is part (iii) of Proposition 3, with $w_h^F(z) = \underline{w}$.

Cutoffs and uniqueness. Define $\overline{z}_h$ by the unique solution to $w_h^F(\overline{z}_h) = \underline{w}$ using the strictly increasing mapping in Region (i) (the bracket is increasing in z and the outer power is positive), and define $\underline{z}_h$ by the unique z at which $\Big(\frac{\alpha z\xi_h(z)}{(1+\tau)\underline{w}}\Big)^{\frac{1}{1-\alpha}} = N_h\big(\frac{(1+\varsigma_h)\underline{w}}{W_h}\big)^\eta$. Monotonicity of the left-hand side in z delivers uniqueness. The three regions then correspond to $z \geq \overline{z}_h$ (Region i), $\underline{z}_h \leq z \leq \overline{z}_h$ (Region ii), and $z < \underline{z}_h$ (Region iii), as stated in Proposition 3.

□

Proof of Proposition 4. Informal firms face no minimum wage and no payroll tax, but revenues are realized with probability $1-\rho$. For each skill h they solve

$$\max_{w_h, l_h} \; (1-\rho)\, z\, \xi_h(z) l_h^{\alpha} - w_h l_h \quad \text{s.t.} \quad l_h \le N_h \Big(\tfrac{w_h}{W_h}\Big)^{\eta}.$$

(Note $\varsigma_h = 0$ for informal firms.)

As in the formal case, the solution must have the labor-supply constraint binding: if $\lambda_h = 0$, then $\partial \mathcal{L}/\partial w_h = -l_h < 0$ at any $l_h > 0$, contradicting optimality. With $\lambda_h > 0$, proceed exactly as in Region (i) of Proposition 3, replacing z by $(1-\rho)z$ and setting $\tau = \varsigma_h = 0$. The KKT system delivers the same constant-markdown relation,

$$w_h = \frac{\eta}{\eta+1}\, \alpha(1-\rho) z \xi_h(z)\, l_h^{\alpha-1}, \qquad l_h = N_h \Big(\tfrac{w_h}{W_h}\Big)^{\eta}.$$

Solving as before yields

$$w_h^I(z) = W_h \left[\frac{\alpha\eta(1-\rho)\, z\, \xi_h(z)}{W_h(\eta+1)\, N_h^{1-\alpha}}\right]^{\frac{1}{1+\eta(1-\alpha)}}, \qquad l_h^I(z) = N_h \Big(\tfrac{w_h^I(z)}{W_h}\Big)^{\eta},$$

which establishes Proposition 4.

$\square$

Corollary 1 (Formal cutoffs $\underset{\sim}{z}_h$ and $\overline{z}_h$)**.** *For each skill h, define*

$$S_h \;\equiv\; N_h^{1-\alpha} \left(\frac{(1+\varsigma_h)\, \underline{w}}{W_h}\right)^{\eta(1-\alpha)} \underline{w}.$$

Then the two formal-sector productivity cutoffs are characterized by

$$\underset{\sim}{z}_h\, \xi_h(\underset{\sim}{z}_h) \;=\; \frac{1+\tau}{\alpha}\, S_h, \qquad \overline{z}_h\, \xi_h(\overline{z}_h) \;=\; \frac{(1+\tau)(\eta+1)}{\alpha\,\eta}\, S_h.$$

If $z \mapsto z\,\xi_h(z)$ is strictly increasing on its support, both solutions are unique and satisfy $\underset{\sim}{z}_h < \overline{z}_h$.

Proof. At the rationing/supply boundary, set $w_h = \underline{w}$, equate the unconstrained l_h^* to the supply $l_h = N_h\big(\frac{(1+\varsigma_h)\underline{w}}{W_h}\big)^{\eta}$, and use the formal FOC $\alpha z\, \xi_h(z)\, l_h^{\alpha-1} = (1+\tau)\underline{w}$ to obtain the first identity. At the constrained/unconstrained boundary, combine $\partial_{w_h}\mathcal{L} = 0$ with the binding supply and $w_h = \underline{w}$ to get $\alpha z\, \xi_h(z)\, l_h^{\alpha-1} = (1+\tau)\underline{w}\left(1+\frac{1}{\eta}\right)$, yielding the second identity. Monotonicity of $z \mapsto z\,\xi_h(z)$ gives uniqueness and ordering. $\square$

C Representative family, rationing, and the wage index

This section restates the worker assignment problem subject to firm-level rationing. It shows that it is isomorphic to the decentralized worker problem in Section 3 once we introduce firm-specific rationing wedges. We derive: (i) the optimal assignment rule for a representative family, following the reasoning of Berger et al. (2022), (ii) the mapping from Lagrange multipliers to rationing wedges, (iii) the wage index that appears in Section 3, and (iv) the labor supply curve a firm faces. We then compare this "efficient" rationing (which can condition worker allocations on realized amenity shocks) to an *anonymous ex-ante rationing* scheme. Both deliver the *same* wedges $r(j)$, worker allocations, and wage index; only welfare differs.

Notation. We suppress skill indices h to keep notation light; all statements hold skill-by-skill. Let Ω be the set of operating firms. Worker $i \in [0,1]$ has idiosyncratic amenity draws $\{A_{ij}\}_{j\in\Omega}$, i.i.d. Fréchet with shape $\eta > 0$. Firm j posts a wage $w(j)$ and an effective valuation factor $(1+\varsigma(j))$, so the *effective wage* is

$$x(j) \equiv (1+\varsigma(j))\, w(j).$$

Some firms ration: they commit to hiring strictly fewer workers than the mass of workers who would optimally choose them at the minimum wage. We capture this with a *rationing wedge* $r(j) \in [0,1]$. Throughout, $r(j) = 1$ if j does not ration. For rationing firms, $r(j) \in [0,1)$.

C.1 Representative-family assignment with firm capacity

The representative family assigns workers to firms, respecting that each worker has a single unit of labor and that each firm cannot hire more than its capacity $\bar{l}(j)$. The family maximizes expected utility:

$$\max_{\{p_{ij}\}} \int_0^1 \int_\Omega p_{ij}\, \tilde{U}_{ij}\, dj\, di, \qquad \tilde{U}_{ij} \equiv \log A_{ij} + \log x(j) \tag{40}$$

$$\text{s.t.} \quad \int_\Omega p_{ij}\, dj = 1 \ \ \forall i, \qquad \int_0^1 p_{ij}\, di \le \bar{l}(j) \ \ \forall j, \qquad p_{ij} \ge 0. \tag{41}$$

Let λ_i be the Lagrange multiplier on the "one unit of labor per worker" constraint and $\mu(j) \ge 0$ the multiplier on firm j's capacity. The first-order condition for p_{ij} is

$$\tilde{U}_{ij} - \lambda_i - \mu(j) = 0. \tag{42}$$

Hence the optimal *assignment rule* is:

$$p_{ij} = \begin{cases} 1, & \text{if } j \in \arg\max_{k\in\Omega}\{\tilde{U}_{ik} - \mu(k)\}, \\ 0, & \text{otherwise.} \end{cases} \tag{43}$$

Congestion wedges. Define the congestion/rationing wedge

$$r(j) \;\equiv\; e^{-\mu(j)} \in (0,1]. \tag{44}$$

Then the *adjusted utility* relevant for the assignment is

$$U_{ij} \equiv \tilde{U}_{ij} - \mu(j) = \log A_{ij} + \log\big(x(j)r(j)\big) \quad\Longleftrightarrow\quad e^{U_{ij}} = A_{ij}\,x(j)\,r(j). \tag{45}$$

If firm j does not ration, the capacity constraint is slack, $\mu(j) = 0$ and $r(j) = 1$. If it binds, $\mu(j) > 0$ and $r(j) \in (0,1)$. Equation (45) shows that rationing is observationally equivalent to rescaling the firm's effective wage by $r(j)$. This rescaling is analogous to the shadow wage definition explored in Berger et al. (2022). Notice that the stronger the rationing (larger $\mu(j)$), the lower is $r(j)$, and the lower will be the contribution of firm j to the wage index W.

C.2 Choice probabilities and the wage index with wedges

Under i.i.d. Fréchet(η) amenities, the standard extreme-value algebra implies that, for any *fixed* vector $x(\cdot)$ and wedges $r(\cdot)$,

$$\Pr\{i \text{ allocated to } j\} \;=\; \frac{\big(x(j)\,r(j)\big)^{\eta}}{\displaystyle\int_{k\in\Omega}\big(x(k)\,r(k)\big)^{\eta}\,dk}. \tag{46}$$

It is therefore convenient to define the *wage index with rationing*:

$$W \;\equiv\; \left(\int_{k\in\Omega}\big(x(k)\,r(k)\big)^{\eta}\,dk\right)^{1/\eta}. \tag{47}$$

With this index, equation (46) can be rewritten compactly as

$$\Pr\{i \text{ allocated to } j\} \;=\; \Big(\frac{x(j)\,r(j)}{W}\Big)^{\eta}. \tag{48}$$

Realized employment vs. *perceived* labor supply. Realized employment at firm j is

$$l(j) = \min\Big\{\Big(\frac{x(j)\,r(j)}{W}\Big)^{\eta},\ \bar{l}(j)\Big\}. \tag{49}$$

When the capacity constraint is slack, $\mu(j) = 0$ and $r(j) = 1$, so $l(j) = \big(x(j)/W\big)^{\eta}$. For firm problems, it is convenient to define the *labor supply the firm faces* as the upper envelope

$$l^s(j) \equiv \Big(\frac{x(j)}{W}\Big)^{\eta}, \qquad \text{and hence} \quad l(s) \leq l^s(j) \ \text{ with equality if } j \text{ does not ration.} \tag{50}$$

Note that W applies the wedges of *all* firms through the denominator, which is the only way in which rationing by any firm affects the supply curve any firm faces.

C.3 Mapping to the quantitative model

Restoring skill indices, take $x_h(j) = (1 + \varsigma_h(j))w_h(j)$ and $r_h(j) \in [0, 1]$. Then equations (47) and (50) deliver exactly the objects used in 3:

$$l^s_h(j) = N_h\Big(\frac{(1 + \varsigma_h(j))\,w_h(j)}{W_h}\Big)^{\eta}, \tag{51}$$

$$W_h = \left(\int_{k\in\Omega} \Big[(1 + \varsigma_h(k))\,r_h(k)\,w_h(k)\Big]^{\eta} dk\right)^{1/\eta}. \tag{52}$$

Thus, the representative-family problem reproduces (i) the labor supply curve that each firm perceives and (ii) the wage index with rationing wedges used in the main text. In particular, a firm's *own* wedge does not enter the numerator of its supply curve; wedges matter only through the common index W_h. Restoring skill indices, the index in Equation (52) is exactly the one used in Section 3, Equation (21), once we identify $p_h(j) = r_h(j)^{\eta}$.

C.4 Efficient rationing, anonymous rationing, and welfare

Efficient rationing (planner). In the problem above, rationing is "efficient" in the sense that the planner can condition access on the realized utility shocks A_{ij} that depend on the quality of the employer-employee match. This raises expected utility relative to schemes that ration in an *anonymous* manner.

Anonymous ex-ante rationing. Consider instead the following two-stage random-access implementation: before workers observe $\{A_{ij}\}$, each firm j is independently available to each worker with probability $p(j)$. Define $r(j) := p(j)^{1/\eta}$. We now show that the resulting choice shares and the index W coincide with (9) and (8).

Formally, let $Z_{ij} \in \{0,1\}$ indicate whether j is available to worker i, with $Z_{ij} \sim$ Bernoulli$(p(j))$, independent across j and independent of $\{A_{ij}\}$. Conditional on $Z_i \equiv \{Z_{ik}\}_{k\in\Omega}$, the usual extreme-value algebra yields the standard logit share over the *available* set:

$$\Pr\{i \text{ chooses } j \mid Z_i\} = \frac{Z_{ij}\, x(j)^\eta}{\int_{k\in\Omega} Z_{ik}\, x(k)^\eta\, dk}.$$

Because the set of firms Ω is a continuum, independence across k and integrability of $x(k)^\eta$ allow a law of large numbers:

$$\int_{k\in\Omega} Z_{ik}\, x(k)^\eta\, dk = \int_{k\in\Omega} \mathbb{E}[Z_{ik}]\, x(k)^\eta\, dk = \int_{k\in\Omega} p(k)\, x(k)^\eta\, dk \quad \text{a.s.}$$

In particular, the denominator is almost surely non-random and does not depend on Z_{ij} (firm j has measure zero). Taking expectations of the conditional share then gives the unconditional choice probability:

$$\Pr\{i \text{ chooses } j\} = \frac{\mathbb{E}[Z_{ij}]\, x(j)^\eta}{\int_\Omega p(k)\, x(k)^\eta\, dk} = \frac{p(j)\, x(j)^\eta}{\int_\Omega p(k)\, x(k)^\eta\, dk}.$$

With the identification $r(j) = p(j)^{1/\eta}$ and recalling the wage index in (8),

$$W \;:=\; \left(\int_{k\in\Omega} \big[x(k)\, r(k)\big]^\eta\, dk \right)^{1/\eta},$$

we can rewrite the share compactly as

$$\Pr\{i \text{ chooses } j\} = \left(\frac{x(j)\, r(j)}{W} \right)^\eta,$$

which is exactly Equation (9). Hence the function $r(\cdot)$, the worker distribution across firms, and the wage index W are identical under efficient rationing and this anonymous ex-ante random-access implementation.

Welfare. Under anonymous ex-ante rationing, the ex-ante expected utility of a worker is

$$\mathbb{E}\left[\max_{j\in\Omega \text{ available}} \{\log A_{ij} + \log x(j)\}\right] \;=\; \frac{\gamma}{\eta} + \log W, \tag{53}$$

where γ is Euler's constant and the equality uses the fact that $\log A$ is Gumbel with scale $1/\eta$ when $A \sim$ Fréchet(η). In the representative-family problem, welfare is (weakly) higher because rationing can be targeted on the realized $\{A_{ij}\}$. Crucially for our paper, *both* rationing notions deliver the same $r(\cdot)$ and the same W, so the implications for the wage index and all equilibrium conditions in 3 are unchanged.

C.5 Summary

(i) The representative-family assignment with capacity constraints generates a firm-specific multiplier $\mu(j)$; setting $r(j) = e^{-\mu(j)}$ embeds rationing as an effective wage shifter. (ii) With Fréchet shocks, workers are allocated to firms with probability proportional to $\big(x(j)r(j)\big)^{\eta}$, which delivers the wage index W in (47). (iii) The labor supply curve a firm faces is the usual $x(j)/W$-based curve, equation (50); wedges matter only through W (equivalently W_h with skill indices).

D Quantitative model computation and calibration

This section details the computation of the quantitative model and its calibration.

We discretize the state space of firms. For each productivity component, we construct 501 equal-sized bins covering the range from 0 to 0.995, and define the productivity shock grid as the values of the inverse cumulative distribution function at the midpoint of each bin. This ensures that each grid point has the same probability, and that the discretization spans the distribution over 500 equally spaced percentiles between 0 and 99.5%. Hence, the final grid for firm-level productivities has 250,000 points, consisting of all possible combinations of points in the two grids for ν and θ.

We solve the model computationally using MATLAB. The general equilibrium is obtained by applying the fsolve routine to a function that takes as input an initial guess for the wage indices $W = [W_1, W_2, \ldots, W_H]$ and returns the excess demand in each labor market.[35]

[35] It is straightforward to show that, in the absence of labor rationing by firms and with all labor markets clearing, wages aggregate to W_h according to equation (21). When labor rationing is present, shadow wages must be taken into account to establish that equation (21) remains valid.

To smooth the general equilibrium search and the calibration process, we introduce taste shocks in firms' formality decisions. Specifically, the probability that a firm with productivity ν chooses to be formal is given by

$$\Pr^F(\nu) = \frac{\exp\{\mathbb{E}[V^F(\nu)]/\sigma_{\text{taste}}\}}{\exp\{\mathbb{E}[V^F(\nu)]/\sigma_{\text{taste}}\} + \exp\{\mathbb{E}[V^I(\nu)]/\sigma_{\text{taste}}\}}. \tag{54}$$

We set $\sigma_{\text{taste}} = 0.0001$, ensuring that taste shocks do not alter the model's economic content while making the excess labor demand in each market respond very smoothly to changes in the wage indices.

Our internal calibration yields shape parameter values for θ_F and θ_I between 1 and 2, which would imply finite means but infinite variances for standard Pareto distributions. However, since these distributions are discretized in the numerical implementation, this does not pose an issue, and they can be interpreted as proxies for Pareto distributions with finite moments.

Notes on the calibration. To calibrate the model, we select the parameter vector $\boldsymbol{p} = [p_1, p_2, \dots, p_N] \in \mathcal{P}$ that minimizes the weighted sum of squared deviations between the model-generated moments and their empirical counterparts:

$$\boldsymbol{p}^* = \arg\min_{\boldsymbol{p}\in\mathcal{P}} \sum_{i=1}^{K} w_i \left[m_i^{\text{model}}(\boldsymbol{p}) - m_i^{\text{data}}\right]^2. \tag{55}$$

The weights w_i are chosen to adjust the relative importance assigned to each moment and are determined based on the behavior of the minimization algorithm throughout the calibration iterations. For instance, in the 1996 benchmark calibration, all moments are assigned the same weight, except for informal inequality, which receives a weight five times larger than that of the other moments.

E Calibrating the model to 2012 data

This section describes the calibration strategy for the year 2012, discusses the estimated parameter values, and assesses the model's fit against both targeted and untargeted moments.

The calibration for the year 2012 is similar to that for 1996, with the following differences. First, we use the real increase in the minimum wage from 1996 to 2012 (105%) to compute its 2012 value: $\underline{w}_{2012} = \underline{w}_{1996} \cdot 2.05 = 0.1 \cdot 2.05 = 0.21$. Second, we allow the mean of the underlying Normal distribution driving the ν shock to differ from zero. Since the production function exhibits decreasing returns to scale and the minimum wage is increased exogenously, we must allow total factor productivity to adjust for the model to fit the data.

Table E.1: Parameters of the 2012 calibration

External calibration			Internal calibration			Internal calibration (Demand shifters)	
Parameter	Description	Value	Parameter	Description	Value	Parameter	Value
H	# of skills	4	α	Returns to scale	0.69	ϕ_2^F	-0.12
N_h	Skill supply	Figure 8	ν shifter	ν mean shifter	0.63	ϕ_3^F	0.26
η	LS elasticity	1	σ	ν std. dev.	0.84	ϕ_4^F	0.26
ς_h	Earnings tax	Figure 9	κ^F	θ^F shape	2.18	ϕ_2^I	-0.22
τ	Payroll tax	71.4%	κ^I	θ^I shape	1.71	ϕ_3^I	0.23
$\underline{w}$	Min. wage	0.21	ρ	Informality cost	0.34	ϕ_4^I	0.08

Sources: Model simulations.

Table E.2: Model and data moments, 2012

Moment	Data	Model	Moment	Data	Model
Variance of log earnings			(Formal) Fraction at $\underline{w}$		
Overall	0.50	0.54	Overall	0.16	0.18
Formal	0.33	0.39	No degree	0.29	0.48
Informal	0.63	0.59	Primary	0.19	0.31
No degree	0.45	0.19	Secondary	0.16	0.09
Primary	0.34	0.23	Tertiary	0.06	0.02
Secondary	0.32	0.61			
Tertiary	0.64	0.59	(Formal) $\frac{\text{Min. wage}}{\text{Mean wage}}$	0.45	0.40
Mean earnings			Informal share		
Formal/Informal	1.65	1.56	Overall	0.31	0.34
Primary/No degree	1.21	1.05	No degree	0.49	0.43
Secondary/Primary	2.15	2.08	Primary	0.36	0.36
Tertiary/Secondary	1.19	1.17	Secondary	0.23	0.32
			Tertiary	0.22	0.31

Sources: Model simulations and 2012 PNAD.

F Unemployment

This section adds unemployment to the quantitative model, following Caliendo et al. (2019). After recalibrating the model and performing the same set of counterfactuals, we show that the most important margin of adjustment in response to the minimum wage is still between the formal and informal sectors, not between employment and unemployment, and that the unintended consequences of the minimum wage still hold.

Households now face unemployment utility b on top of firm-specific wage offers. Importantly, households draw a taste shock for being unemployed, which is also Fréchet distributed and independent from other firm-specific shocks. This formulation implies that a share $U_h = N_h \left(\frac{b}{W_h}\right)^\eta$ of households of skill h will opt out of the labor force. The aggregate wage indices in the economy now read:

$$W_h = \left[b^\eta + \int_{j\in\Omega} [(1+\varsigma(j))w(j)r(j)]^\eta dj\right]^{\frac{1}{\eta}} \tag{56}$$

The problem of the firm does not change, so we do not discuss it in this Appendix. An equilibrium in this model consists of an allocation and wage indices that equate aggregate labor supply and demand for each skill. The calibration procedure is identical to the one in the main specification, with the addition of the unemployment utility parameter b, which is endogenously chosen to match aggregate unemployment $U = \sum_h U_h$ in 1996. Table F.1 describes the parameters of this alternative calibration procedure, and Table F.2 compares the aggregate moments in the model with the data. The results of the calibration procedure are similar to those displayed in the model without unemployment.

Table F.3 examines the consequences of increasing the minimum wage while holding all other parameters fixed at their 1996 levels. We consider two scenarios: one in which the unemployment utility parameter remains unchanged in the counterfactual, and another in which it increases proportionally with the minimum wage.[36] Notice that there are still unintended consequences of the minimum wage on overall earnings inequality, and that these appear because of strong informal sector responses.

The effect of the minimum wage on unemployment depends on whether b is

[36] Brazilian law stipulates that unemployment benefits cannot be lower than the minimum wage (Presidência da República, 1990).

Table F.1: Calibration results: model with unemployment

Parameter	Description	Value
b	Unemployment utility	0.04
σ	ν std. dev.	1.21
κ^F	θ^F shape	1.75
κ^I	θ^I shape	1.55
ϕ_2^F	Demand shifters	-0.08
ϕ_3^F	Demand shifters	0.06
ϕ_4^F	Demand shifters	0.23
ϕ_2^I	Demand shifters	-0.12
ϕ_3^I	Demand shifters	-0.04
ϕ_4^I	Demand shifters	0.30
$\underline{w}$	Min. wage	0.09
ρ	Inf. cost	0.21
α	Returns to scale	0.61

Sources: Model simulations.

Table F.2: Data vs. model comparison: model with unemployment

Moment	Data	Model	Moment	Data	Model
Variance of log earnings			(Formal) Fraction at $\underline{w}$		
Overall	0.78	0.74	Overall	0.08	0.06
Formal	0.65	0.56	No degree	0.13	0.11
Informal	0.66	0.63	Primary	0.08	0.05
No degree	0.54	0.41	Secondary	0.05	0.01
Primary	0.54	0.54	Tertiary	0.01	0.00
Secondary	0.64	0.74			
Tertiary	0.91	1.09	(Formal) $\frac{\text{Min. wage}}{\text{Mean wage}}$	0.22	0.25
Mean earnings			Informal share		
Formal/Informal	2.06	2.07	Overall	0.39	0.35
Primary/No degree	1.39	1.35	No degree	0.52	0.39
Secondary/Primary	1.46	1.52	Primary	0.37	0.36
Tertiary/Secondary	2.49	2.43	Secondary	0.26	0.33
			Tertiary	0.22	0.29
Unemployment rate	0.07	0.07			

Sources: 1996 PNAD and model simulations.

Table F.3: The unintended consequences of the minimum wage in a model with unemployment

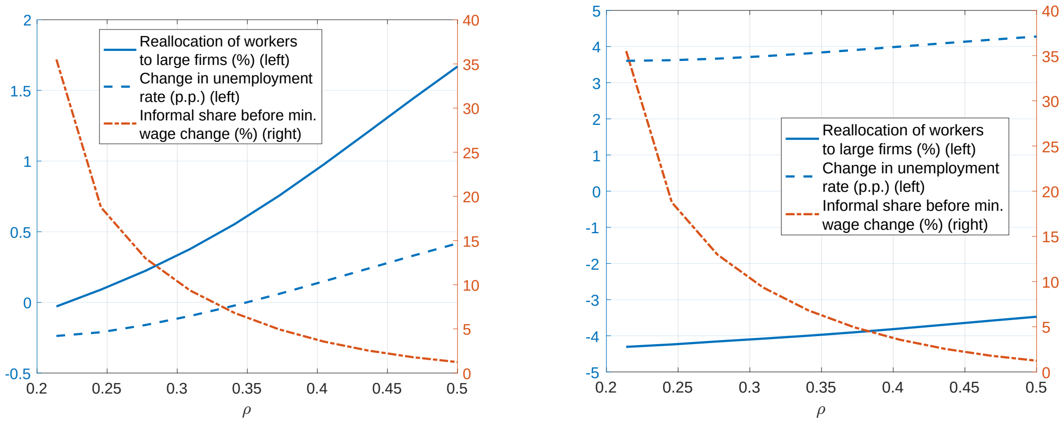
	1996	Min. w. counterfactual: b not indexed to $\underline{w}$	Min. w. counterfactual: b indexed to $\underline{w}$
V(log earnings)			
Overall	0.74	0.75	0.76
Formal	0.56	0.46	0.46
Informal	0.63	0.65	0.70
Fraction at $\underline{w}$	0.06	0.16	0.13
Informal share	0.35	0.68	0.67
Unemployment	0.07	0.06	0.10

Notes: The first column reports model statistics with unemployment calibrated to 1996. The second column shows the counterfactual with a 105% increase in the minimum wage, and the third column shows the exercise with both the minimum wage and the unemployment utility increased by 105%. *Sources:* Model simulations.

a function of $\underline{w}$. On the one hand, if b is not indexed to $\underline{w}$, a higher minimum wage reduces unemployment because overall wages increase, making unemployment a less attractive alternative. On the other hand, if b increases in proportion to $\underline{w}$, a higher minimum wage raises unemployment, since the wage indices W_h grow less than proportionally relative to the utility of being unemployed.

The effect of the minimum wage on informality is large in both cases. Hence, even in a model with unemployment, the predominant margin of adjustment to a higher minimum wage remains the reallocation between the formal and informal sectors. Consequently, regardless of the assumption on unemployment utility, increasing the minimum wage has unintended consequences.

We now discuss how the minimum wage reallocates labor across firms, and how it varies with the size of the informal sector. Figure F.1 uses the model with unemployment to replicate Figure 13 from the no-unemployment setting. When b is not indexed to the minimum wage, the baseline result holds: with the calibrated informality cost $\rho = 0.21$, fewer workers are in large firms after the minimum wage increase. When informality is more costly (higher ρ), the increase shifts workers toward large formal firms. If b rises proportionally with the minimum wage, the higher unemployment utility strongly competes with productive firms, reallocating



(a) b not indexed to $\underline{w}$

(b) b indexed to $\underline{w}$

Figure F.1: Reallocation of workers to large firms in the model with unemployment

Notes: This figure shows the change (after vs. before the policy) in the share of workers allocated to large firms—defined as those that remain formal in the baseline minimum wage counterfactual—across models with different informality costs (ρ). The solid blue line represents worker reallocation to large firms, the dashed blue line shows the change in the unemployment rate (in percentage points) resulting from the policy, and the red line plots the informal employment share before the minimum wage increase. In the left panel, the minimum wage is increased by 105%, while in the right panel, both the minimum wage and the unemployment utility are increased by 105%. *Sources:* Model simulations.

their workers into unemployment.

We then evaluate the extreme case where there is no informality ($\rho = 1$) to connect our unemployment model and findings to Engbom and Moser (2022). In their framework, non-employment captures both unemployment and informal work, with utility levels held constant before and after the minimum wage increase. As a result, their setup resembles our model with unemployment but no informality. Figure F.2 shows that, in this model, small and unproductive firms that cannot afford the higher wage reduce employment, and displaced workers are absorbed by larger and more productive firms. Unemployment increases from 6.8% to 7.7% in this simulation.

In contrast, in our calibrated models, informality functions as a fallback option that allows low-productivity firms to survive by avoiding minimum wage compliance. These firms shift to the informal sector, thereby preventing full reallocation of their workers to more productive formal firms. As a result, the rise in informality is a natural and quantitatively significant response to the minimum wage shock when enforcement is limited.

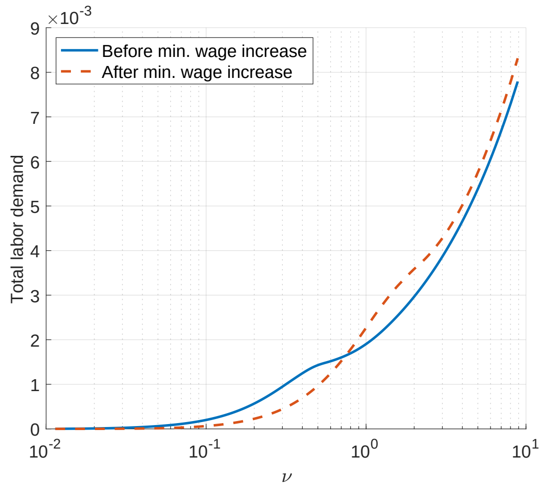


Figure F.2: Distribution of workers across firm productivities (unemployment model without informality)

Notes: This figure shows the total number of workers across firms at each productivity level ν. For these simulations, we start with the unemployment model calibrated to 1996, but set ρ equal to one. *Sources:* Model simulations.

G Oligopsonistically competitive model

This section presents a variation of the quantitative model where a finite number of firms M compete for workers, rather than the continuum of firms assumed in the main model. The key difference is that each firm now takes into account the effect of its own wage setting on the wage index and aggregate labor market conditions.

For simplicity of implementation and for simpler comparison with the case in which there is no oligopsonistic behavior, we make the following assumptions: (i) when firms make their formality decisions, they disregard the impact of this decision on the formality decisions of other firms and the behavior of the wage index in the later, wage-setting stage; (ii) there is no aggregate uncertainty: productivity draws $z = (\nu, \theta)$ are obtained from a permutation of a fixed grid of realizations that approximates the continuous distribution from the main model. [37] Assumptions (i) and (ii) jointly avoid an incomplete information game, aggregate uncertainty from finite shocks, and a learning problem from the formality decisions of other firms.

[37]Fix the finite productivity grid $\mathcal{Z} = (\nu_k, \theta_k)_{k=1}^{K}$. Firms are allocated across this grid according to the same distribution used in the numerical implementation of the benchmark model (uniform; see Appendix D), through a random permutation. Thus, each firm's productivity is idiosyncratic, while a negative correlation ensures that there is no aggregate uncertainty.

Notice that in the wage-setting stage, firms do take into account their impacts on the index, and markdowns will depend on productivity and firm size.

G.1 Labor supply and wage competition

The wage index for skill h now depends on the wages set by all M firms:

$$W_h\left(w_{i,h}\right) = \left(\frac{1}{M}\sum_{i=1}^{M}(1+\chi_{i,h})^{\eta} r_{i,h}^{\eta} w_{i,h}^{\eta}\right)^{\frac{1}{\eta}}, \tag{57}$$

where $\chi_{i,h} = \mathbf{1}_{formal}\chi_h$and $r_{i,h}$ is the rationing wedge at firm i. Each firm i faces the same labor supply curve as before:

$$l_{i,h}^{s}\left(w_{i,h}\right) := \left(\frac{(1+\chi_{i,h})w_{i,h}}{W_h\left(w_{i,h}\right)}\right)^{\eta} N_h.$$

However, unlike in the main model, firm i's wage choice now affects W_h, which determines how many workers all other firms can attract.

G.2 How firm decisions change

When setting wages, each firm now considers how this affects its ability to attract workers. The key change appears in how responsive the labor supply is to wage increases. In the main model, a firm could increase employment by raising wages without affecting other firms. Now, when firm i raises wages, it makes all other firms less attractive to workers.

This leads to a modified elasticity of labor supply:

$$\frac{dl_{i,h}^{s}}{dw_{i,h}} = \eta\frac{l_{i,h}^{s}}{w_{i,h}}\left(1-\frac{l_{i,h}^{s}}{MN_h}\right). \tag{58}$$

The term $\frac{l_{i,h}^{s}}{MN_h}$ represents firm i's share of total employment in skill h. Larger firms face a less responsive labor supply because their wage increases have bigger effects on the overall wage level.

Formality decision. As in the original model, given the signal ν, firms choose their formality status before observing θ. Firms choose to be formal if their expected formal profits $\Pi^F(\nu) = \int \pi^F(\nu\theta)dF_{\theta}(\theta)$ exceed their expected informal profits $\Pi^I(\nu) = \int \pi^I(\nu\theta) dF_\theta(\theta)$.

G.3 Equilibrium

An equilibrium consists of formality choices, wages, and employment levels for each skill level, together with wage indices W_h, such that:

1. Firms choose formality optimally and, given their formality status, wage indices W_h, and perceived labor supply curves, solve their respective optimization problems, accounting for how wage choices affect the wage index through equation (58).

2. Labor markets clear for each skill level $h = 1, \dots, H$.

G.4 Main differences from the baseline model

The oligopsonistic model differs from the main model in three important ways. First, firms are no longer infinitesimal: when one firm raises wages, it takes into account its indirect effect on the wage index and its own labor supply curve. Second, different wage markdowns: Larger firms that are unconstrained by the minimum wage have more market power and discount wages further away from marginal productivity. In the main model, all unconstrained firms had the same markdown. Third, market concentration matters: When there are fewer firms (smaller M), each firm has more market power, which can affect both formality choices, in the first stage, and how the economy responds to minimum wage changes.

Despite these differences, the qualitative results regarding minimum wage effects and informality remain similar to the main model, as shown in the quantitative analysis. See Section 6.2 for a discussion of the simulations obtained from the oligopsonistic competition model (Table G.1 and Figures G.1 and G.2).

Table G.1: Oligopsonistic competition model

	Benchmark model		Oligopsony $M = 250,000$		Oligopsony $M = 100$	
	1996	$\underline{w}$	1996	$\underline{w}$	1996	$\underline{w}$
V(log earnings)						
Overall	0.73	0.76	0.73	0.76	0.49	0.53
Formal	0.61	0.54	0.61	0.54	0.39	0.29
Informal	0.62	0.70	0.62	0.70	0.39	0.51
Fraction at $\underline{w}$	0.06	0.12	0.06	0.12	0.10	0.20
Informal share	0.38	0.78	0.38	0.78	0.35	0.77

Notes: The first two columns replicate Table 5 and report moments from the calibrated model as well as from the counterfactual equilibrium following the minimum wage increase. The third and fourth columns present the oligopsonistic model evaluated with a finite but large number of firms ($M = 500^2$). The last two columns show statistics from the oligopsonistic model with $M = 10^2$ firms. *Sources:* Model simulations.

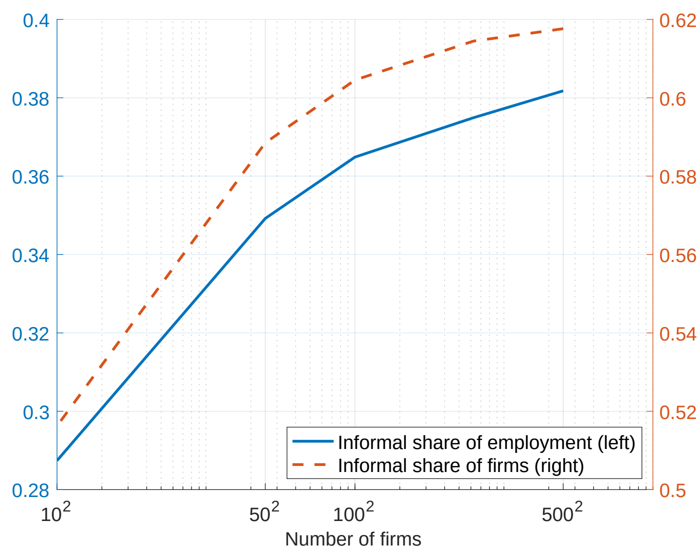


Figure G.1: Informal shares under partial equilibrium

Notes: This figure shows the informal share of employment and firms in partial equilibria for economies with different numbers of firms, where prices are held fixed at their levels from the general equilibrium with $M = 500^2$ firms. *Sources:* Model simulations.

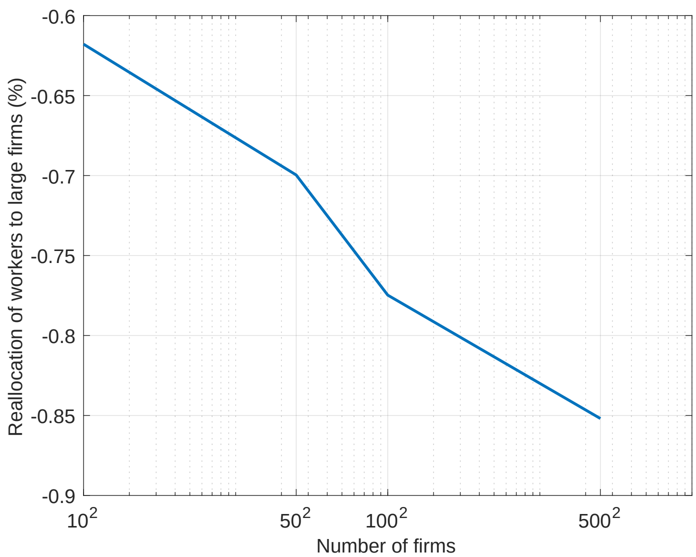


Figure G.2: Reallocation of workers to large firms in the oligopsonistic model

Notes: This figure shows the change (after vs. before the minimum wage increase) in the share of workers employed by large firms—defined as those that remain formal after the minimum wage increase—across oligopsonistic models with different numbers of firms. *Sources:* Model simulations.

Minimum Wages, Inequality, and the Informal Sector

Machado Parente, Brotherhood, and Iachan

Online Appendix not for publication

Online Appendix A performs the Kaitz analysis of the effects of the minimum wage. Online Appendix B reconciles our findings with those in Derenoncourt et al. (2021). Online Appendix C details our adaptation of Giupponi et al. (2024). Online Appendix D contains additional calculations and proofs used in the main text. Lastly, Online Appendix E calculates workers' and firms' formal sector wage valuations.

A Kaitz analysis

This section leverages variation at the state level over time to correlate the increases in the minimum wage with the earnings inequality in the formal sector, informal sector, and in the aggregate. There are three main takeaways: first, the results suggest that inequality in the formal sector falls with the minimum wage. Second, an increase in the minimum wage correlates positively with inequality in the informal sector and the informal share of labor. Third, and as a consequence, the reduced-form relationship between the minimum wage and aggregate inequality is negative, but smaller in magnitude than the relationship with formal sector inequality.

Differently to the estimation strategy in the main text, the analysis in this appendix closely follows the empirical framework and methods in Autor et al. (2016).[38] We use the log-distance between the minimum wage and the median wage in the formal sector (also known as the Kaitz index) as a measure for how restrictive the minimum wage is for state s in year t:

$$\text{Kaitz}_{st} \equiv \log\left(\frac{\underline{w}_t}{w_{st}^{50,F}}\right). \tag{59}$$

We correlate the minimum wage with different measures of earnings inequality (y_{st}) by regressing:

$$y_{st} = \beta_1 \cdot \text{Kaitz}_{st} + \beta_2 \cdot \text{Kaitz}_{st}^2 + \alpha(s,t) + \varepsilon_{st}, \tag{60}$$

[38] See Lee (1999), Haanwinckel (2023), and Engbom and Moser (2022) for papers with similar specifications.

where $\alpha(s,t)$ represents controls at the state and year level. These controls absorb state and national-level changes in the shape of the wage distribution that could influence both the bite of the minimum wage and outcome variables. We also experiment with controlling for the unemployment rate in state s time t as a proxy for heterogeneous shocks to a state's labor market.[39] Our preferred specification follows Engbom and Moser (2022) and includes state fixed effects and state-specific quadratic time trends, even though we display the results for a variety of different controls. The identification assumption is that, conditional on $\alpha(s,t)$, the error term ε_{st} is uncorrelated with the Kaitz index. Identification of β_1 and β_2 comes from movements in the minimum wage that deviate from state-specific quadratic time trends. The marginal coefficient on the minimum wage, the object displayed in the figures that follow, is estimated as: $\rho = \hat{\beta}_1 + 2\hat{\beta}_2\overline{kaitz}$, and we evaluate it at the employment-weighted median Kaitz index.

Table A.1 reports the estimated relationships between the minimum wage and different outcomes, ρ. Each row corresponds to a different inequality measure, and each column corresponds to a specific distribution of earnings. The first column displays the results for the formal earnings distribution, the second column displays the results for the informal earnings distribution, and the last column discusses the results for the aggregate earnings distribution. The last row calculates the relationship between the minimum wage and the informal share of labor. There is a negative and significant relationship between the minimum wage and formal inequality (-0.985***). Importantly, these regression estimates are consistent with other evidence for Brazil (Engbom and Moser (2022) and Haanwinckel (2023)).

The second column estimates the same set of regressions, but focuses on the informal earnings distribution. There is a significant relationship of 0.172** between the minimum wage and the variance of informal earnings. Moreover, the last row in Table A.1 reports that increases in the minimum wage are associated with increases in the informal share of labor (0.162***). Hence, either through the movement of more productive workers from formal to informal jobs (Jales, 2018), or through competition effects in the labor markets (Derenoncourt et al., 2021), the reduced-form evidence suggests that there exists a non-trivial relationship between the minimum wage and the informal sector.

Lastly, the third column looks at the association between the minimum wage and

[39] For instance, Costa et al. (2016) and Adão (2016) study the regional effects of the commodity boom in 2000s Brazil.

Table A.1: Reduced-form evidence on the effects of the minimum wage

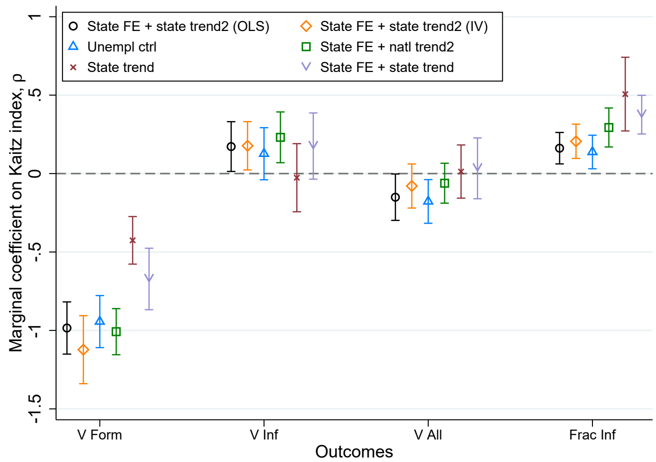
Outcomes	Formal	Informal	Aggregate
log(Variance)	-0.985***	0.172**	-0.151*
	(0.085)	(0.081)	(0.076)
log(Informal share)		0.162***	
		(0.051)	

Notes: Each cell represents a separate regression. Each cell reports the marginal coefficient on the minimum wage ($\rho = \hat{\beta}_1 + 2\hat{\beta}_2\overline{kaitz}$), where the β-coefficients are obtained by regressing (60). All specifications control for state fixed effects and state-specific quadratic time trends. Marginal coefficients are evaluated at median wage. All regressions are employment-weighted and have 405 observations (27 states by 15 years). Standard errors in parentheses are clustered at the state level. *Sources:* 1996/2012 PNAD.

aggregate distribution of earnings. This takes into consideration not only the within-sector associations discussed above, but also how the minimum wage is related to the distance between mean earnings in the formal and informal sectors. The relationship between the minimum wage and aggregate inequality is negative, but less significant and smaller in magnitude than the relationship with formal sector inequality (-0.151* vs. -0.985***, respectively), due to the counteracting forces presented by the informal sector earnings distribution.

We now discuss the robustness of these results. Figure A.1 shows that the results are robust to different specifications of the control variables: controlling for unemployment rate, state-specific linear time trends, no state fixed effects as in Lee (1999), state-specific linear time trends and national quadratic time trends as in Haanwinckel (2023), among others. We use two different strategies to control for the possibly mechanical endogeneity of the Kaitz index, as it might correlate with the residual term, because median wages might affect the dispersion in earnings. First, we redo the analysis with the share of formal workers at the minimum wage as the measure for how binding the minimum is in a given state-year. Second, we follow the 2SLS IV approach from Autor et al. (2016), where the first stage projects the Kaitz index and its square on log minimum wage, its square, and its interaction with the state's overall median earnings throughout the sample period, thus filtering for transitory shocks on median wages. The results are similar and displayed in Table A.2 and Figure A.1, respectively. Lastly, Figure A.2 compares the estimates of the effect of the minimum wage on informal share of labor with those found at Engbom

Figure A.1: Marginal effect of the minimum wage (alternative specifications)



Notes: Plot shows the marginal effect of the minimum wage on different outcomes (x-axis) for different specifications (colors). "State trend" denote state-specific linear time trends, "state trend2" denote state-specific quadratic time trends, and "natl trend2" denote a national quadratic time trend. *Sources:* 1996-2012 PNAD.

Table A.2: Reduced-form evidence on the effects of the minimum wage (share of minw formal workers as main measure)

Outcomes	Formal	Informal	Aggregate
log(Variance)	-1.382***	0.897***	0.730***
	(0.264)	(0.286)	(0.210)
log(Informal share)		0.445**	
		(0.208)	

Notes: Each cell represents a separate regression. Each cell reports the marginal coefficient on the minimum wage on the regression: $y_{st} = \beta \cdot atminw_{st} + \alpha_s + \alpha_t + \varepsilon_{st}$. All regressions are employment-weighted and have 405 observations (27 states by 15 years). Standard errors are clustered at the state level. *Sources:* 1996/2012 PNAD.

and Moser (2022), and shows that if we apply similar sample restrictions, we also obtain a null relationship between the minimum wage and informal share.

Figure A.2: Comparison with Engbom and Moser (2022)

Notes: This figure plots the estimated marginal effect of the minimum wage on the informal share of labor for different specifications. The first row ("Engbom Moser 2021") shows the weakly positive effect of minimum wage on formal share, taken from Engbom and Moser (2022). The second row is a replication attempt of the RAIS data set with the PNAD data set. The third row includes female workers in the sample. The "Self empl" row excludes self-employed workers from the sample. "Both" considers both male and female and excludes SE workers, which corresponds to the main specification in this paper. "Both+IV" uses the 2SLS strategy in Autor et al. (2016). *Sources:* 1996-2012 PNAD.

B Discussion of Derenoncourt et al. (2021)'s findings

This section reconciles our findings with those in Derenoncourt et al. (2021). We confirm that specifications that compare states above and below the median treatment do not estimate informality responses to the minimum wage. We estimate the following regression:

$$y_{st} = \alpha + \beta \cdot \text{Treated}_s \cdot \mathcal{I}_{t>1999} + \delta_s + \delta_t + X'_{st}\Gamma + \varepsilon_{st}, \tag{61}$$

where y denotes the outcome of interest for state s state and year t and Treated_s is an indicator for states that are highly treated (i.e., above median treatment) by the minimum wage in the pre-period average. The specification for controls and fixed effects is the same as that in equation (2).Moreover, we closely follow Derenoncourt et al. (2021) in including self-employed workers in the definition of informality.[40]

Table B.1 shows the difference-in-differences coefficients for two treatment measures. Columns (1) and (3) confirm the findings in Derenoncourt et al. (2021) that, relative to below-median treatment states, above-median treatment states did not experience a larger increase in the share of the informal sector after 1999. Column (1) uses the treatment measure in this paper and Column (2) uses the Kaitz index as an alternative measure. Specifications (2) and (4) highlight that treatment heterogeneity is important when analyzing the informality effects of the minimum wage.

[40]Results in this Appendix are invariant to whether or not we include self-employed individuals in the analysis.

Table B.1: Exploring treatment heterogeneity

	Bunching at $\underline{w}$		Kaitz index	
	(1)	(2)	(3)	(4)
β	0.0269		0.0202	
	(0.0177)		(0.0172)	
β_2		0.0848***		0.110***
		(0.0193)		(0.0161)
β_3		0.0554		0.0785***
		(0.0376)		(0.0221)
β_4		0.0723***		0.0278
		(0.0249)		(0.0228)
β_5		0.0103		0.0167
		(0.0163)		(0.0132)
β_6		0.0766***		0.0204
		(0.0213)		(0.0157)
β_7		0.0470**		0.0776***
		(0.0224)		(0.0186)
β_8		0.0900***		0.108***
		(0.0235)		(0.0188)
β_9		0.104***		0.100***
		(0.0187)		(0.0165)
R-squared	0.976	0.979	0.976	0.981
Observations	405	405	405	405

Notes: The first two columns measure treatment to the minimum wage using the fraction of workers bunched at the minimum before 1999. The latter two columns use the same definition of treatment as Derenoncourt et al. (2021): the Kaitz index. The dependent variable is the log of the informal share in each state. All regressions include self-employed workers in the definition of informality. Standard errors in parentheses clustered at the state level. ***p<1%, **p<5%, *p<10%. *Sources:* 1996-2012 PNAD.

C Details on our adaptation of Giupponi et al. (2024)

This section further details our adaptation of Giupponi et al. (2024).

Outliers. Following Giupponi et al. (2024), we drop the top 1% of wages in each year from the analysis.

Years. In the PNAD exercise, we use the years 1996-1999 to estimate the regional wage premia (equation 3). The effect of the minimum wage is estimated using the years 2001-2012, excluding pairs of years in which the real minimum wage fell (2001-2002) and the year in which no PNAD is available (2010). With Census data, we use the year 2000 to estimate the regional wage premia, and the years 2000 and 2010 to estimate the effect of the minimum wage.

Seeds. In the PNAD exercise, we discard bootstrap iterations (seeds) in which it is not possible to define control groups. In some bootstrap iterations, too few states are selected, preventing the division of regions into ten groups.

Wage bins. To define the wage bins k used to construct Figure 5, we create a linearly spaced grid with 50 points between the wages corresponding to the 1st and 95th percentiles of the 2000 Census wage distribution, where percentiles are calculated after dropping the top 1% of wages.

For the bins of log wages, ℓ, used to compute the observed and counterfactual distributions of log wages, we use linearly spaced grids between the log wages corresponding to the 1st and 99th percentiles of each year's log wage distribution, again calculated after dropping the top 1% of wages.

D Theory appendix

This section details additional proofs and derivations used the main text.

D.1 Fréchet calculations

This section details the Fréchet calculations of the labor supply curve at the firm level, as well as worker welfare, for the stylized model. Results for the quantitative model extend trivially, and are not derived for brevity.

Assume that the utility of household i working at firm j reads:

$$U_i(j) = A_i(j)w(j) \tag{62}$$

where we assume that the amenity shocks $A_i(j)$ are iid and follow a Fréchet distribution with shape η, scale equals to one and location equals to zero:

$$F(A(j)) = e^{-A(j)^{-\eta}}, \quad f(A(j)) = e^{-A(j)^{-\eta}}\eta A(j)^{-\eta-1} \tag{63}$$

The share of households that optimally choose firm j is:[41]

$$l(j) = \int_0^1 Pr\left(U_h(j) \geq U_h(j') \;\; \forall j' \neq j\right) dh \tag{64}$$

$$l(j) = \int_0^1 \int_0^\infty f(A(j)) \prod_{j' \in \Omega \setminus \{j\}} F\left(\frac{w(j)A(j)}{w(j')}\right) dA(j)dh \tag{65}$$

$$l(j) = \int_0^1 \int_0^\infty e^{-A(j)^{-\eta}}\eta A(j)^{-\eta-1} \prod_{j' \in \Omega \setminus \{j\}} e^{-\left(\frac{w(j)A(j)}{w(j')}\right)^{-\eta}} dA(j)dh \tag{66}$$

$$l(j) = \int_0^1 \int_0^\infty \eta A(j)^{-\eta-1} e^{-\int_{j' \in \Omega}\left(\frac{w(j)A(j)}{w(j')}\right)^{-\eta}} dA(j)dh \tag{67}$$

$$l(j) = \int_0^1 \int_0^\infty \eta A(j)^{-\eta-1} e^{-(w(j)A(j))^{-\eta}\int_{j' \in \Omega} w(j')^{\eta}} dA(j)dh \tag{68}$$

[41] In the case of a discrete number of firms $j = 1, ..., J$, the labor share allocated at firm 1 would be:

$$l(1) = \int_0^1 \int_0^\infty \int_0^{\frac{w(1)A(1)}{w(2)}} ... \int_0^{\frac{w(1)A(1)}{w(J)}} f(A(J))...f(A(1))\, dA(J)...dA(2)dA(1)dh$$

which denotes the probability that firm 1 is chosen over all other firms $j = 2, ..., J$ in the economy. The equation for the continuum of firms is an alternative form of expressing the same variable, calculated in Desmet et al. (2018).

$$l(j) = \int_0^1 \int_0^\infty \eta A(j)^{-\eta-1} e^{-\left(\frac{A(j)}{\frac{\left[\int_{j' \in \Omega} w(j')^\eta\right]^{1/\eta}}{w(j)}}\right)^{-\eta}} dA(j) dh \tag{69}$$

Define $s \equiv \frac{\left[\int_{j' \in \Omega} w(j')^\eta\right]^{1/\eta}}{w(j)}$, and manipulate to find:

$$l(j) = s^{-\eta} \int_0^1 \int_0^\infty \frac{\eta}{s} \left[\frac{A(j)}{s}\right]^{-\eta-1} e^{-\left(\frac{A(j)}{s}\right)^{-\eta}} dA(j) dh \tag{70}$$

and use the fact that households are homogeneous (so integral over h is irrelevant), and that $\frac{\eta}{s}\left[\frac{A(j)}{s}\right]^{-\eta-1} e^{-\left(\frac{A(j)}{s}\right)^{-\eta}}$ is the pdf of a Fréchet distribution with shape η and scale s (so it integrates to one) to find:

$$l(j) = \frac{w(j)^\eta}{\int_{j' \in \Omega} w(j')^\eta} \tag{71}$$

We now calculate the expected utility of a household in the model. The probability that the utility of household i being less than u conditional on firm j being its optimal choice is:

$$F_U(u) = Pr(U_i(j) \leq u \,|\, U_i(j) \geq U_i(j') \, \forall j' \neq j) = \frac{Pr(U_i(j) \leq u \,\&\, U_i(j) \geq U_i(j') \, \forall j' \neq j)}{Pr(U_i(j) \geq U_i(j') \, \forall j' \neq j)} \tag{72}$$

$$F_U(u) = \frac{\int_0^{\frac{u}{w(j)}} f(A(j)) \prod_{j' \in \Omega \setminus \{j\}} F\left(\frac{w(j)A(j)}{w(j')}\right) dA(j)}{\int_0^\infty f(A(j)) \prod_{j' \in \Omega \setminus \{j\}} F\left(\frac{w(j)A(j)}{w(j')}\right) dA(j)} \tag{73}$$

$$F_U(u) = \frac{\int_0^{\frac{u}{w(j)}} \eta A(j)^{-\eta-1} e^{-\left(\frac{A(j)}{\frac{\left[\int_{j' \in \Omega} w(j')^\eta\right]^{1/\eta}}{w(j)}}\right)^{-\eta}} dA(j)}{\frac{w(j)^\eta}{\int_{j' \in \Omega} w(j')^\eta}} \tag{74}$$

$$F_U(u) = \int_0^{\frac{u}{w(j)}} \frac{\int_{j' \in \Omega} w(j')^\eta}{w(j)^\eta} \eta A(j)^{-\eta-1} e^{-\left(\frac{A(j)}{\frac{\left[\int_{j' \in \Omega} w(j')^\eta\right]^{1/\eta}}{w(j)}}\right)^{-\eta}} dA(j) \tag{75}$$

$$F_U(u) = \int_0^{\frac{u}{w(j)}} \frac{\eta}{\left[\int_{j'\in\Omega} w(j')^\eta\right]^{1/\eta}} \left[\frac{w(j)A(j)}{\left[\int_{j'\in\Omega} w(j')^\eta\right]^{1/\eta}}\right]^{-\eta-1} e^{-\left(\frac{w(j)A(j)}{\left[\int_{j'\in\Omega} w(j')^\eta\right]^{1/\eta}}\right)^{-\eta}} w(j)dA(j) \tag{76}$$

change variables and call $x = w(j)A(j)$ to find:

$$F_U(u) = \int_0^{u} \frac{\eta}{\left[\int_{j'\in\Omega} w(j')^\eta\right]^{1/\eta}} \left[\frac{x}{\left[\int_{j'\in\Omega} w(j')^\eta\right]^{1/\eta}}\right]^{-\eta-1} e^{-\left(\frac{x}{\left[\int_{j'\in\Omega} w(j')^\eta\right]^{1/\eta}}\right)^{-\eta}} dx \tag{77}$$

so the optimal utility U is a Fréchet random variable with shape η and scale $\left[\int_{j'\in\Omega} w(j')^\eta\right]^{1/\eta}$, which means that its mean is given by:

$$\mathbb{E}[U] = \Gamma\left(\frac{\eta-1}{\eta}\right)\left[\int_{j'\in\Omega} w(j')^\eta\right]^{1/\eta} = \Gamma\left(\frac{\eta-1}{\eta}\right) W \tag{78}$$

D.2 Monopolistic competition

This section considers the case in which firms not only have monopsony power in the labor market but also are monopolistic competitors in the goods market. We assume each firm produces a different variety, which workers demand in a CES fashion. We show that this changes slightly the problem of the firm, but does not alter the threshold characterization of the solution, in which low-productivity firms select into the informal sector. Hence, the qualitative results in Section 2 do not change.

We first analyze the problem of the household. The consumption problem of household i working for firm j, consuming varieties from all other firms k is:

$$V_i(j) = \max_{c(k)} \left\{ A_i(j) \left[\int_{k\in\Omega} c_i(k)^{1-\frac{1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \quad \middle| \quad \int_{k\in\Omega} p(k)c_i(k) = w(j) \right\} \tag{79}$$

where σ is the elasticity of substitution between varieties. The first order conditions of this problem give rise to a downward sloping demand curve for the product of firm k consumed by household i employed at firm j:

$$c_i(k) = \left[\frac{p(k)}{P}\right]^{-\sigma} \frac{w(j)}{P} \tag{80}$$

where $P = \left[\int_{k\in\Omega} p(k)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$ denotes the CES price index such that $PC_i(j) = w(j)$ with $C_i(j) = \left[\int_{k\in\Omega} c_i(k)^{1-\frac{1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$.

The solution for the consumption problem yields the following indirect utility function:

$$V_i(j) = \frac{A_i(j)w(j)}{P} \tag{81}$$

and the employment decision of household i boils down to selecting the employer that offers the highest amenity-adjusted wage:

$$U_i = \max_{j\in\Omega} \left\{ \frac{A_i(j)w(j)}{P} \right\} \tag{82}$$

First, notice that the introduction of monopolistic competition in the goods market does not alter the choice probabilities of workers to different firms, hence the labor supply curve faced by firm j still reads:

$$l(j) = \left(\frac{w(j)}{W} \right)^{\eta} \tag{83}$$

where $W = \left[\int_{j\in\Omega} w(j)^{\eta}\right]^{\frac{1}{\eta}}$ denotes the aggregate wage index. Second, using calculations similar to those in Online Appendix D.1, it is easy to calculate the aggregate demand for products from firm $k \in \Omega$, coming from all households working at all firms $j \in \Omega$:

$$c(k) = \left[\frac{p(k)}{P}\right]^{-\sigma} \frac{W}{P} \tag{84}$$

This result comes from the fact that the aggregate wage index represents the total amount of earnings earned by households after their optimal employment decisions. So far, we have distinguished the monopsonist j from the monopolist k, but from now on, we will look at the labor supply and product demand curves for the same firm.

Consider the problem of the informal firm operating with productivity z:

$$\pi^I(z) = \max_{\{c,p,l,w\}} \left\{ (1-\rho)pc - wl \quad | \quad c = zl, \;\; c = P^{\sigma-1}W \cdot p^{-\sigma}, \;\; l = W^{-\eta} \cdot w^{\eta} \right\} \tag{85}$$

The first constraint is the linear production function, the second constraint represents market power in the goods market, and the last constraint represents the monopsony power.

Substitute the constraints in this problem to find:

$$\pi^I(z) = \max_{\{l\}} \left\{ (1-\rho) P^{\frac{\sigma-1}{\sigma}} W^{\frac{1}{\sigma}} (zl)^{1-\frac{1}{\sigma}} - W l^{1+\frac{1}{\eta}} \right\} \tag{86}$$

and the solution reads:

$$l^I(z) = (W/P)^{-\frac{\sigma-1}{\sigma}\frac{\eta\sigma}{\eta+\sigma}} \left(\frac{\sigma-1}{\sigma} \frac{\eta}{\eta+1} \right)^{\frac{\eta\sigma}{\eta+\sigma}} (1-\rho)^{\frac{\eta\sigma}{\eta+\sigma}} z^{\frac{\sigma-1}{\sigma}\frac{\eta\sigma}{\eta+\sigma}} \tag{87}$$

Importantly, define the adjusted markdown as $\tilde{\eta} \equiv \frac{\sigma-1}{\sigma}\frac{\eta\sigma}{\eta+\sigma}$ and the real wage of workers $\tilde{W} \equiv W/P$ to find:

$$l^I(z) = \tilde{W}^{-\tilde{\eta}} \left(\frac{\tilde{\eta}}{\tilde{\eta}+1} \right)^{\frac{\sigma}{\sigma-1}\tilde{\eta}} (1-\rho)^{\frac{\sigma}{\sigma-1}\tilde{\eta}} z^{\tilde{\eta}} \tag{88}$$

There are two takeaways from the above equation. First, as σ goes to infinity, we have varieties that are perfect substitutes, and we get back the same expression as in the main text. Second, the labor allocation, and consequently profits in the informal sector are a "modified" version of the ones derived in the main text, except that now there is curvature with respect to productivity z that comes from both the elasticity of the labor supply curve, η, and the elasticity of the demand curve, σ.

The intuitions and results for the firms in the formal sector are available upon request.

D.3 Walras' Law

This section calculates the goods' market-clearing. Aggregate demand for goods read:

$$C = \sum_{h=1}^{H} \int_{i \in h} \int_{j \in \Omega} \Pr_{ih}(j) c_{ih}(j) dj di \tag{89}$$

From the household problem $c_{ih}(j) = w_h(j)$ and conditional on h and the firm j the household is working for, all individual i's are symmetric (that is: $\Pr_{ih}(j) = \Pr_h(j)$). This implies:

$$C = \sum_{h=1}^{H} \int_{j \in \Omega} N_h \Pr_h(j) w_h(j) dj \tag{90}$$

From the structure of the problem, in equilibrium, we have that the labor demand for skill h by firm j equals the fraction of households of skill h that are choosing to work in that firm: $l_h(j) = N_h \Pr_h(j)$, hence:

$$C = \int_{j \in \Omega_{form}} \sum_{h=1}^{H} l_h(j) w_h(j) dj + \int_{j \in \Omega_{inf}} \sum_{h=1}^{H} l_h(j) w_h(j) dj \tag{91}$$

where we also inverted the order of the summation and split firms into informal and formal sectors. Each term in turn becomes:

$$\int_{j \in \Omega_{form}} \sum_{h=1}^{H} l_h(j) w_h(j) dj = \int_{j \in \Omega_{form}} q(j) dj - \int_{j \in \Omega_{form}} \pi^F(j) dj \tag{92}$$

$$\int_{j \in \Omega_{inf}} \sum_{h=1}^{H} l_h(j) w_h(j) dj = (1-\rho) \int_{j \in \Omega_{inf}} q(j) dj - \int_{j \in \Omega_{inf}} \pi^I(j) dj \tag{93}$$

This implies that the goods market clearing condition:

$$C + \Pi^F + \Pi^I + \rho Q^I = \int_{j \in \Omega} q(j) dj = Q \tag{94}$$

which states that total production is split into consumption, profits for formal and informal firms, and government collection of revenue due to audited informal units.

E Calculation of workers' and firms' wage valuations

This section details the methodology used to estimate the valuation of formal nominal wages for workers and firms (ς_h and τ). We closely follow the work in Haanwinckel and Soares (2021) and Souza et al. (2012). The main idea is that households and firms value additional payments they receive, or have to incur, because of labor legislation.

We start by estimating the total labor cost of hiring a formal worker at a nominal monthly wage of 100 Brazilian Reais. The results are displayed in Table E.1. First, formal workers are entitled to a 13th salary by the end of the year (A.1). Second, the firm must pay a vacation stipend of 1/3 of the monthly wage (A.2). Third, in the period of 30 days prior to dismissal (Advance notice), formal employees can spend up to 25% of their work time searching for a new job. As discussed in Gonzaga et al. (2003), this advance notification is in practice an additional severance payment, as workers are not expected to put effort into working during that month.

The three items above represent transfers from firms to workers. We now dis-

Table E.1: Calculating τ

Item	Formula	Value
Nominal wage (A)		100
13th salary (A.1)	A/12	8.33
Vacation (A.2)	(A/3)/12	2.78
Advance notice	(A+A.1+A.2)*dismiss prob.	3.33
Raw total (B)		114.44
FGTS contribution (B.1)	8% of B	9.16
FGTS fund (B.2)	B.1*duration	304.33
Severance payment	B.2/2*dismiss prob.	4.56
INSS employer	20% of B	22.89
Other contributions	5.3% of B	6.07
Total with contributions (C)		157.12
Vacation adjustment	C/11	14.28
Total cost (D)		171.40
Payroll tax rate: τ	D/A-1	71.4%

Notes: Calculation of payroll tax rate τ used in the model. The above calculations were made under a dismissal probability of 3% and expected duration of employment of 33 months (Haanwinckel, 2023). *Sources:* Labor legislation.

cuss government taxation, which falls upon the raw total wage (B). In Brazil, formal workers have a severance payment fund (FGTS), where withdrawal can occur at the time of dismissal. Firms must make monthly contributions of 8% of the raw total wage to this fund (B.1). We estimate the total value of the FGTS fund by multiplying the monthly contribution times the average duration of a formal job from Haanwinckel and Soares (2021). Upon firing a worker, firms must incur severance payments of 50% of the value of the FGTS fund, with 40% going directly to the worker and 10% going to the government. Firms must also contribute to the retirement fund of the worker (INSS), as well as other social security contributions, which amount to a total of 25.3% of the raw total wage. Lastly, formal workers have one month of paid vacation per year. Hence, an adjustment factor of 1/11 is needed to represent the fact that the employee is only productive during 11 months in a year. These calculations result in an effective payroll tax rate of 71.4%.

To calculate the valuation of formal nominal wages for workers, we apply the labor legislation to each observation in the PNAD data and average the resulting wedges across the educational groups. This process generates Figure 9. To illustrate the procedure, Table E.2 estimates the wedges in 1996 for four representative levels of earnings, corresponding to mean earnings in each education category in the data. The first three items are the direct transfers from firms to workers in terms of 13th salary, vacation stipends and advance notices. Then come the two largest deductions: worker contributions to the retirement system (INSS deduction) and income taxes. Importantly, these rates depend on the earnings level analyzed, a feature that is taken into account in these calculations. After that comes the valuation of the FGTS fund, severance payments made in the case of dismissal, and disability insurance. Lastly, one must adjust for the fact that workers are entitled to one month of paid vacation. The results show that the benefits accrued from having a formal labor contract more than compensate for the income and social security taxation (the wedges are positive). Moreover, notice that the wedges are larger for less-educated workers, a reflection of the progressiveness of the tax system in Brazil.

Table E.2: Calculating ς_h

Employee items	Formula	No degree	Primary	Secondary	Tertiary
Nominal wage (A)	Mean for each skill group	306	382	534	1323
13th salary (A.1)	A/12	25.5	31.8	44.5	110.3
Vacation (A.2)	(A/3)/12	8.5	10.6	14.8	36.8
Advance notice	(A+A.1+A.2)*dismiss prob.	10.2	12.7	17.8	44.1
Raw total (B)		350.2	437.2	611.1	1514.1
INSS deduction	INSS formula over (B)	-31.5	-39.3	-67.2	-105.3
Income tax	Income tax formula over (B)	0.0	0.0	0.0	-92.1
Valuation of FGTS	50% of firm contribution	14.0	17.5	24.4	60.6
Severance payment	40% FGTS fund*dismiss prob.	11.2	14.0	19.5	48.3
Accident insurance	2% of (B)	7.0	8.7	12.2	30.3
Total		350.9	438.0	600.1	1455.8
Vacation adjustment	Vacation cost paid by firm	43.7	54.6	76.3	189.0
Overall valuation		394.6	492.6	676.4	1644.8
Worker wedge		28.9%	28.9%	26.7%	24.3%

Notes: Calculation of formal worker wedges ς_h used in the model. The above calculations were made under a dismissal probability of 3% and expected duration of employment of 33 months (Haanwinckel, 2023). *Sources:* Labor legislation.