

Artificial astrocyte networks

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Goal

- Use models and proof methods from artificial neural networks to try and set an **upper bound** on astrocyte computation.
- ANNs \leftrightarrow AANs?

ANNs

What are ANNs?

- By example, using vision.
- Sparse networks.

Visual digit recognition.

0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2
3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3
4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4
5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5
6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6
7 7 7 7 7 7 7 7 7 7 7 7 7 7 7 7
8 8 8 8 8 8 8 8 8 8 8 8 8 8 8 8
9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9

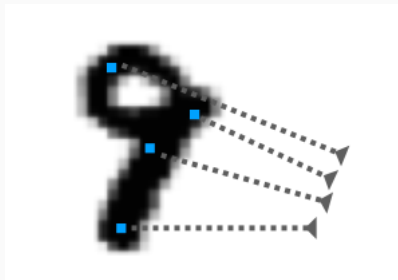
= 1,2,3,4,5,6,...

What are ANNs?



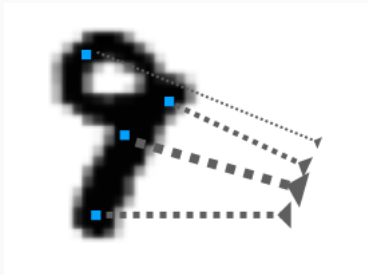
= 9?

What are ANNs?



$$\sum x_{ij} = 9$$

What are ANNs?



$$\sum w_{ij} x_{ij} = 9$$

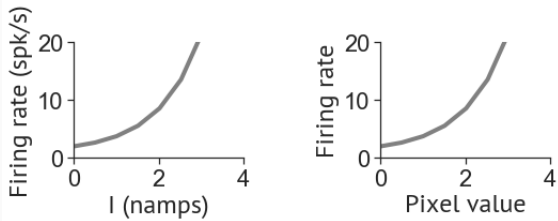
What are ANNs?

$$\sum w_{ij}x_{ij} + b_i = 9$$

What are ANNs?

$$\phi(\sum w_{ij}x_{ij} + b_i) = 9$$

What are ANNs?



FI-curve (left) $\approx \phi$ nonlinearity (right)

What are ANNs?



A deep ANN network. Each box is a *layer*: $\phi(\sum w_{ij}x_{ij} + b_i)$

AANs

Basic astrocyte limits.

- No synapses
- No axons
- No spikes

Basic astrocyte limits.

- No spatial \sum
- No w_i

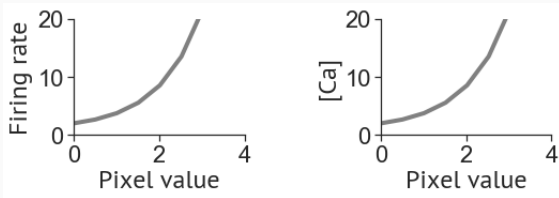
Basic astrocyte properties

- $[Ca^{2+}]$ dynamics
- $[Ca^{2+}]$ dependent gliotransmission

Recovering $\{\phi, w_i, \Sigma\}$?

- Let's make three assumptions that let us form a functional analogy between neurons and astrocytes.

Assumption 1 (ϕ).

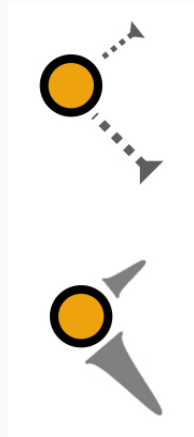


Firing rate (left) \leftrightarrow $[Ca^{2+}]$ dynamics (right)

.

Assumption 2 (w_i).

- **Directional** $[Ca^{2+}]$ -dependent gliotransmission.



Assumption 3 (Σ).

- **Directional** $[Ca^{2+}]$ waves, driven by gliotransmission.
 - Axons send spikes across a spatial distances with fidelity.
 - A $[Ca^{2+}]$ wave can travel a spatial distance with fidelity.

In theory.

Computational calcium waves.

- Let's treat neurons only as a source of input; glia do all the computation!
- Let's study a forward moving $[Ca^{2+}]$ wave.
- With Assumptions 1-3...
- **Prove:** it is a universal function approximator.

A universal function approximator?

$$F(x) - f(x)$$

A universal function approximator?

$$F(x) - f(x)$$

$F(x)$: any target function (whose domain is bounded)

A universal function approximator?

$$F(x) - f(x)$$

$F(x)$: visual recognition system in cat (target)

A universal function approximator?

$$F(x) - f(x)$$

$F(x)$: visual recognition system in cat (*target*)

$f(x)$: an ANN (*approximator*)

A universal function approximator?

$$|F(x) - f(x)| < \epsilon$$

$F(x)$: visual recognition system in cat (*target*)

$f(x)$: an ANN (*approximator*)

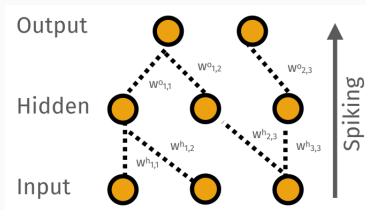
ϵ : the max error boundary

A universal function approximator?

$$|F(x) - f(x)| < \epsilon$$

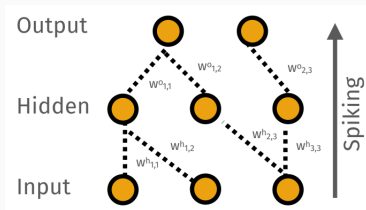
It is possible the target F and learned function f can be made arbitrarily close to ϵ .

Proof sketch.

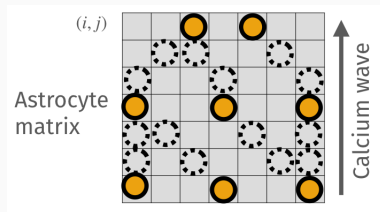


- Bölcskei *et al* (2019) F can be approximated by f in $M \ll N$ connections.
- $f: \phi(\sum w_{ij}x_{ij} + b_i)$

Proof sketch.



- Bölcskei *et al* (2019) F can be approximated by f in $M \ll N$ connections.
- $f: \phi(\sum^M w_{ij} x_{ij} + b_i)$

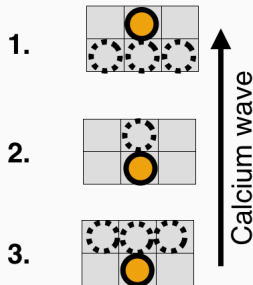


- Approximation of the universal approximator
- $w^o_{1,1} = \sum w^o_{i,j}$
- Astrocyte Sudoku

In practice.

Three astrocyte layers.

- In PyTorch I implemented three kinds of astrocyte layers.
- Using only nearest neighbor connections.
 1. Gather
 2. Slide
 3. Spread



An astrocyte network.

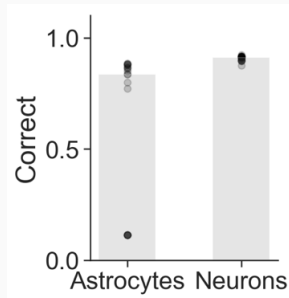


- Two equivariant neural (*left*) and astrocyte networks (*right*).
- For astrocyte computation **width requires depth**.

- Task: MNIST digits
- Use: *{slide, spread, gather}* in pytorch
 1. VAE, $N = (784, 20)$
 2. Perceptron, $N = (20, 30, 10)$
- Loss: Cross-entropy
- Optimizer: ADAM

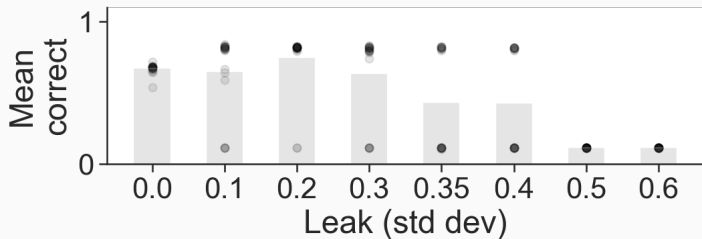
Results.

0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1 1 1 1 1 1 1 1 1 1 1 1 1 1
2 2 2 2 2 2 2 2 2 2 2 2 2 2
3 3 3 3 3 3 3 3 3 3 3 3 3 3
4 4 4 4 4 4 4 4 4 4 4 4 4 4
5 5 5 5 5 5 5 5 5 5 5 5 5 5
6 6 6 6 6 6 6 6 6 6 6 6 6 6
7 7 7 7 7 7 7 7 7 7 7 7 7 7
8 8 8 8 8 8 8 8 8 8 8 8 8 8
9 9 9 9 9 9 9 9 9 9 9 9 9 9



Test set performance (N=20 epochs)

Results.



Effect of gliotransmitter diffusion on model performance

Conclusions.

Conclusions.

- AANs \rightarrow universal function approximator.
- An **upper bound** for the performance of real astrocytes?
- AANs can solve hard vision problems.

Future work.

- Upper bound \rightarrow biological upper bound (help)
- Recurrent waves (in collaboration)
- Better motivated tasks (help)

Code github.com/CoAxLab/glia_playing_atari

Talk github.com/parenthetical-e/glia-talk-sfn-2019

Thank you!