COMPUTER ASSIGNMENT 1

1 QUESTION

1.1 PART A

Equation: $\dot{x} = 1/2x$; $x(0) = 0$

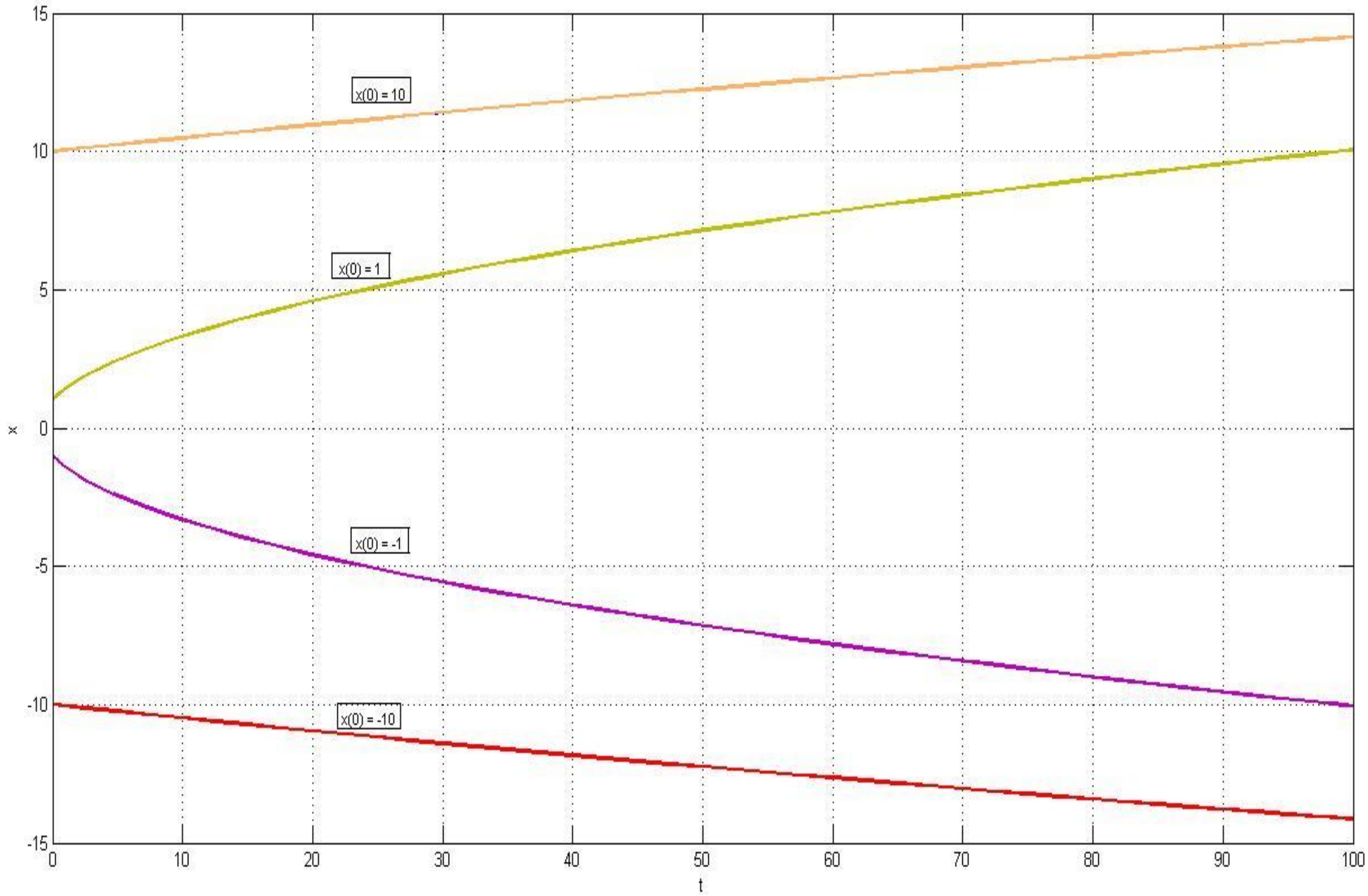

Fig 1: Simulation results for the $\dot{x} = 1/2x$

Simulations carried out for following initial conditions:

1. x(0) = 1
2. x(0) = -1
3. x(0) = 10
4. x(0) = -10

SIMULATION BEHAVIOUR VS PREVIOUS ANALYSIS:

1. Cannot plot the given system of equation for the initial condition *x=0.* When we try to plot for *x=0, ẋ=∞.* So this cannot be plotted in MATLAB and MATLAB throws an error when you try to.

2. Solution obtained solving manually for previous assignment: (x(t) = $\pm\sqrt{t}$).
Graph obtained is similar to when solved mathematically Although, for positive initial conditions, MATLAB plots only the $+\sqrt{(t+C)}$ part and for negative initial conditions only the $-\sqrt{(t+C)}$ part.

3. MATLAB provides only one solution instead of the two, hence is incomplete.

SIMULATION BEHAVIOUR WITH DIFFERENT INITIAL CONDITIONS:

1. Changing the initial condition changes the offset but overall behavior remains same.

2. When $x(0) \neq 0$, solutions have global existence for all initial conditions.

1.2 PART B

Equation: $\dot{x} = 1 + x^2$; $x(0) = 0$

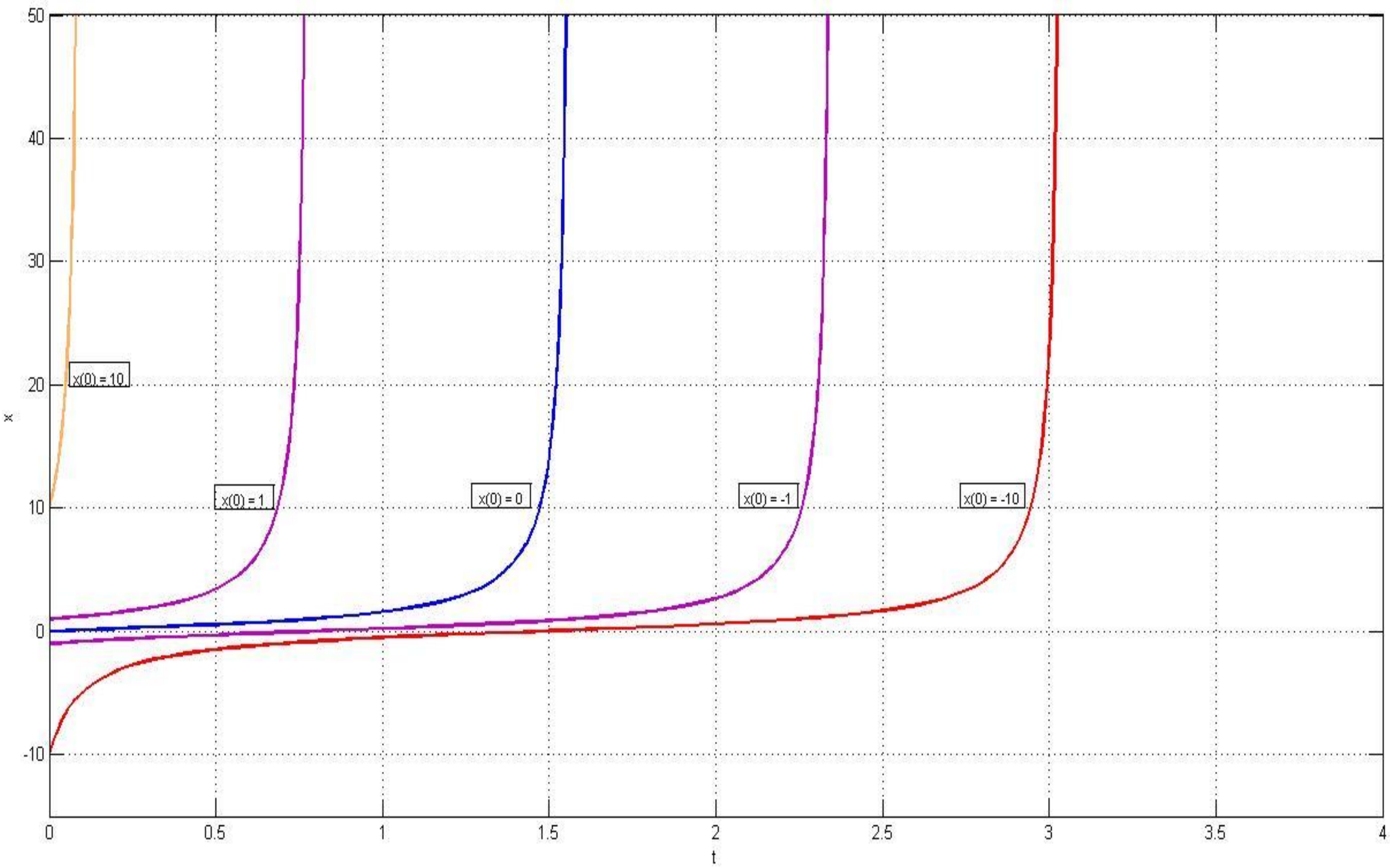

Fig 2: Simulation results for the $\dot{x} = 1 + x^2$

Simulations carried out for following initial conditions:

1. x(0) = 0
2. x(0) = 1
3. x(0) = -1
4. x(0) = 10
5. x(0) = -10

SIMULATION BEHAVIOUR VS PREVIOUS ANALYSIS:

1. Solution obtained solving manually for previous assignment: *x(t) = tan t.*
   When solved and plotted in MATLAB the solutions were same.

2. Similar to the result obtained while solving the DE in assignment 2, the solution is not defined for t $\geq \pi/2$.
   Therefore the solution displays local existence and uniqueness and not global.

SIMULATION BEHAVIOUR WITH DIFFERENT INITIAL CONDITIONS:

1. For different initial conditions, the behavior is similar, but the interval of existence changes.

2. The solution to the nonlinear system is unique.

1.3 PART C

Equation: $\dot{x} = x^{1/3}$; $x(0) = 0$

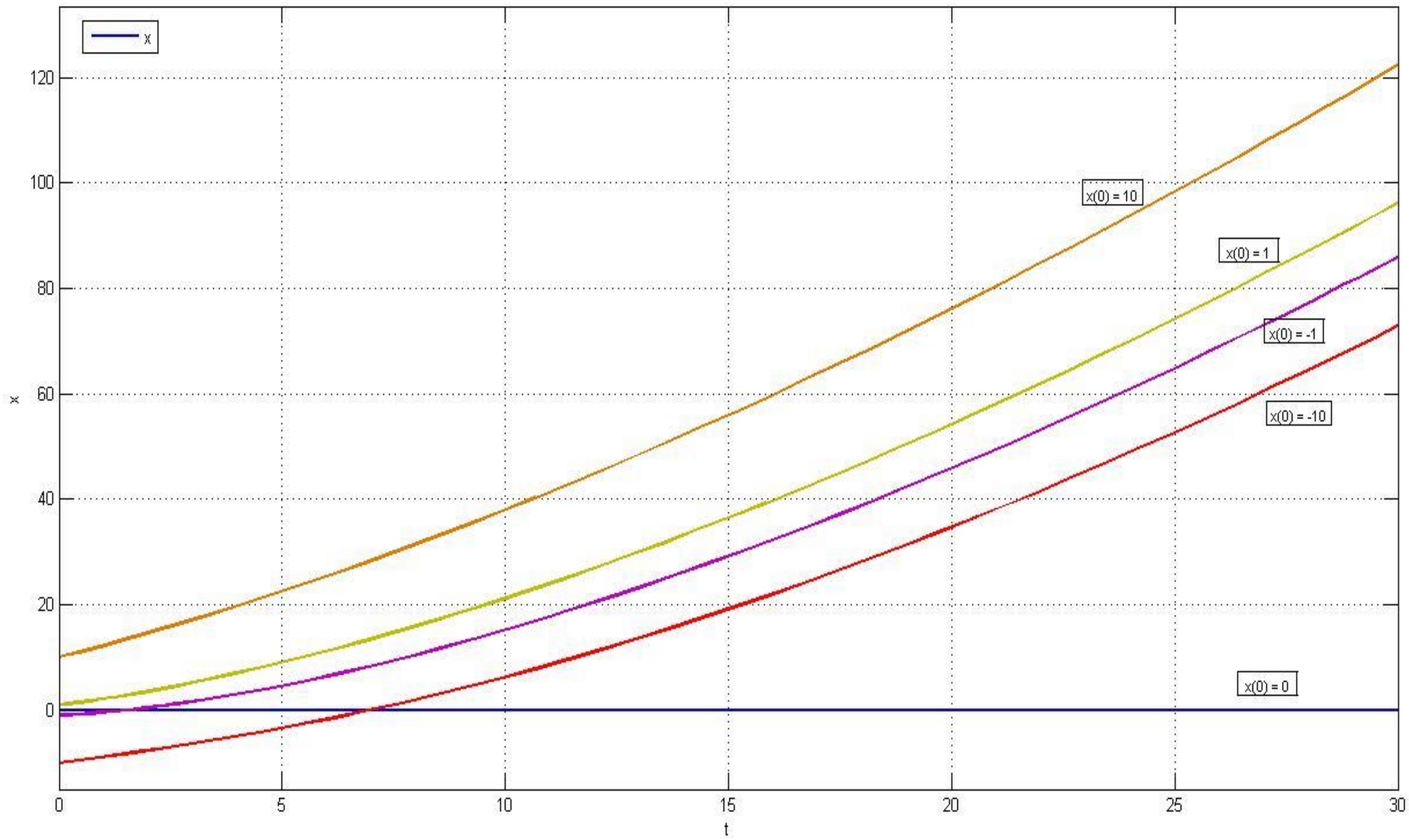

Fig 3: Simulation results for the $\dot{x} = x^{1/3}$

Simulations carried out for following initial conditions:

1. x(0) = 0
2. x(0) = 1
3. x(0) = -1
4. x(0) = 10
5. x(0) = -10

SIMULATION BEHAVIOUR VS PREVIOUS ANALYSIS:

1. Solution obtained solving manually for previous assignment: *x(t) =* $(2t/3)^{3/2}$ *or x(t) = 0.*
   When the system is designed in MATLAB the solution is incomplete since, for x(0) = 0, the tools show the result only as x(t) = 0 line.

2. The solution to the nonlinear system is unique for all non-zero initial conditions. For x(0) = 0, MATLAB gives a unique solution but as per calculations, this result is incomplete.

3. The function is lipschitz as seen from the analytical results from assignment 2.

SIMULATION BEHAVIOUR WITH DIFFERENT INITIAL CONDITIONS:

1. For all non-zero initial conditions, the behavior of the system is similar. Only, the initial offset changes.

1.4 PART D

Equation: $\dot{x} = -2x^2$, $x(0) = -1$

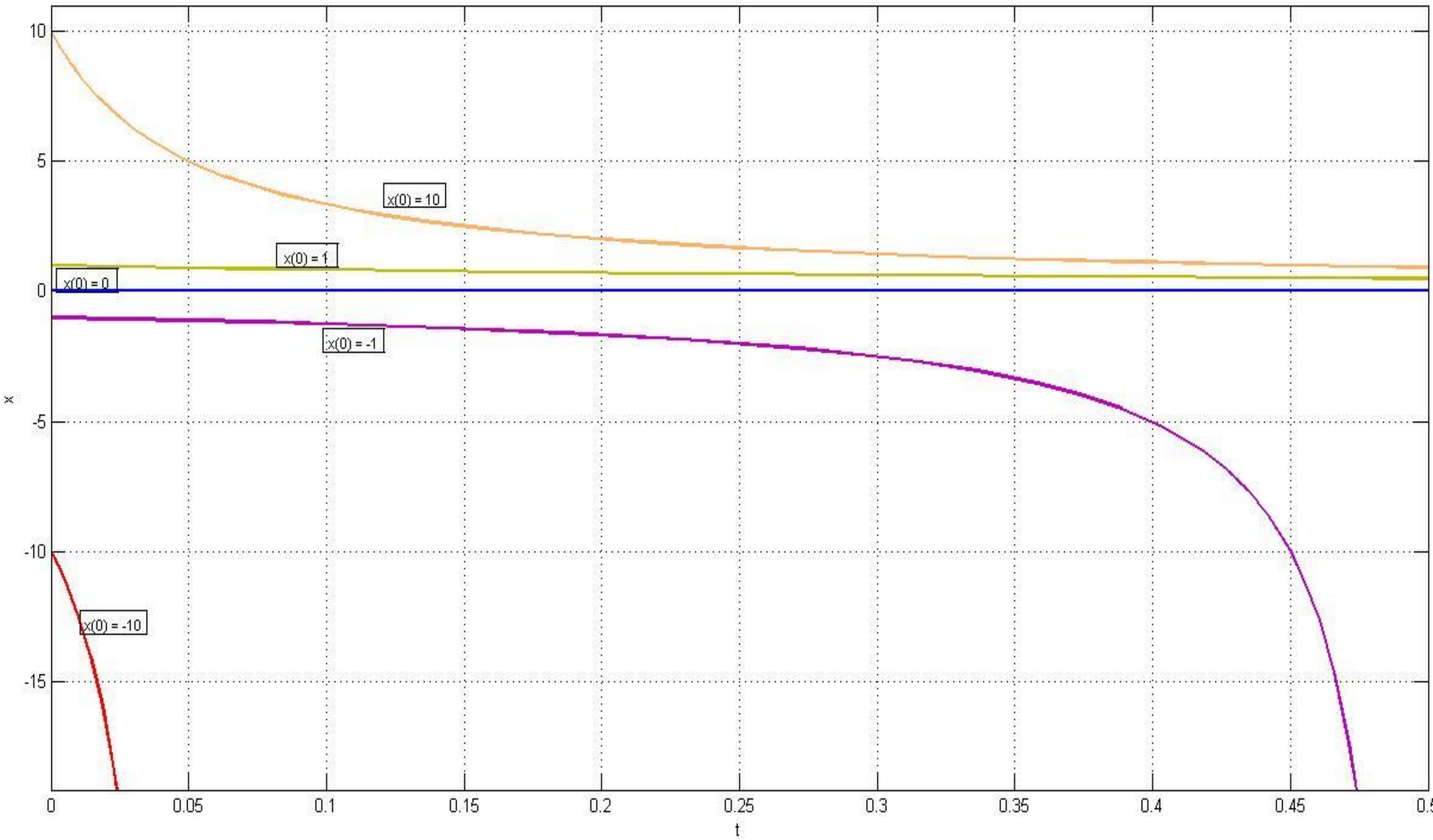

Fig 4: Simulation results for the $\dot{x} = x^{1/3}$

Simulations carried out for following initial conditions:

1. x(0) = 0
2. x(0) = 1
3. x(0) = -1
4. x(0) = 10
5. x(0) = -10

SIMULATION BEHAVIOUR VS PREVIOUS ANALYSIS:

1. For x(0) = 0, the solution obtained is a line x=0.
2. Solution obtained solving manually for previous assignment: *x(t) = x(t)* = $1/(2t-1)$
3. The function is locally lipschitz as seen from the analytical results from assignment 2.

SIMULATION BEHAVIOUR WITH DIFFERENT INITIAL CONDITIONS:

1. Local existence of solution when initial condition is negative, as seen in the graph where x(0) = -10; x(0) = -1. The solution is local and unique.
2. For positive initial conditions, the solution obtained exists globally and is unique.
3. Hence, we can conclude that the solution varies as the initial conditions are varied.

1.5 PART E

$\dot{x}$ = -x if x< 0
$\dot{x}$ = x^2 if x≥ 0 , x(0) = 0

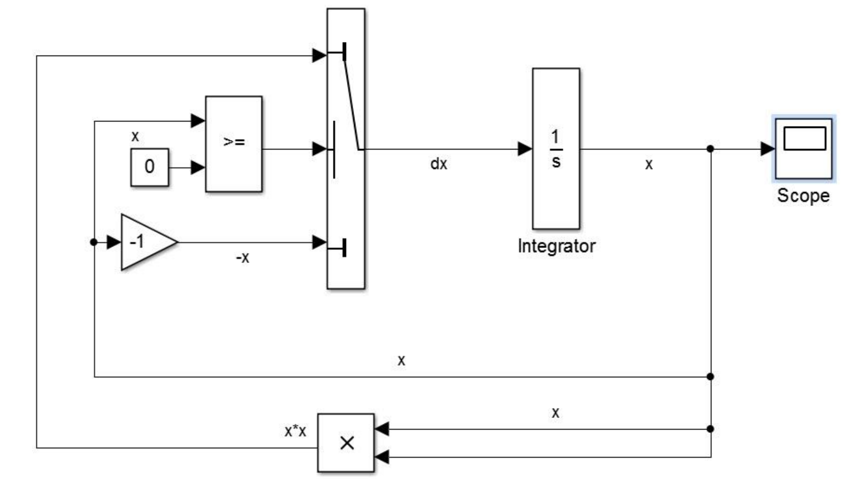

Fig 5: Simulink Model

We Create a Simulink Model to solve this question.

ANALYSIS:

1. As obtained in assignment 2, we get same results in MATLAB model.

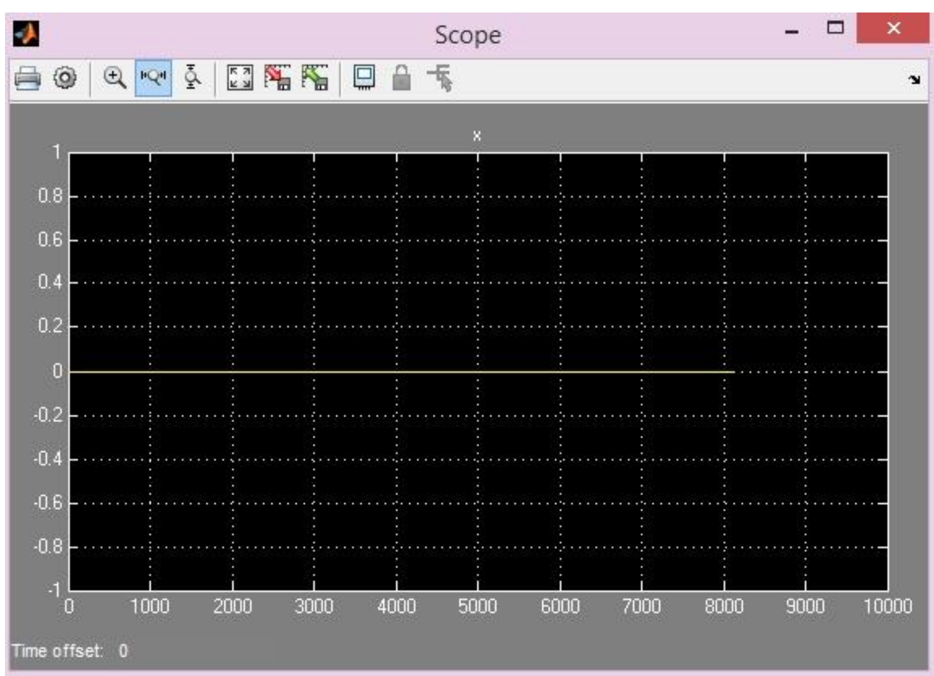

Fig 6: Simulation result

2. Solution has global existence and is unique.
3. For x(0) < 0 we get same result as x(0) = 0. For x(0) > 0 we get an error of singularity.

2 QUESTION

2.1 PART A

Inverse Van der Pol Oscillator

$\dot{x}_1 = -x_2$

$\dot{x}_2 = x_1 - (1 - x_1^2)*x_2$

LYAPUNOV STABILITY ANALYSIS:

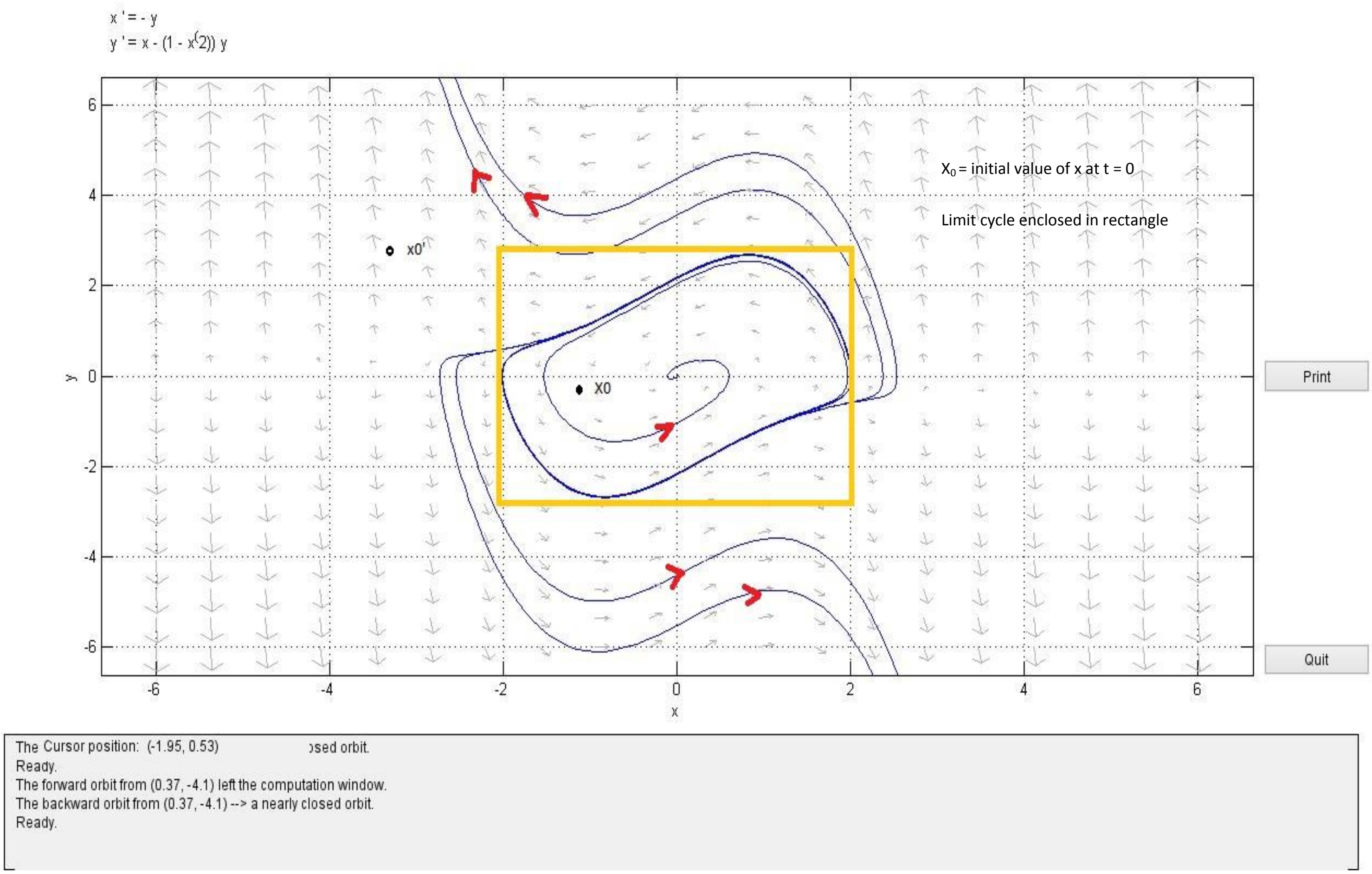

Fig 7: Phase Plane Portrait for Van der Pol Oscillator – Lyapunov Stability Analysis

1. As analyzed in class, the equilibrium point is stable I.S.L.
2. Same result obtained on plotting of the phase plane portrait.
3. For any point within the limit cycle, x_0, the trajectory converges to the equilibrium point.
4. If an initial point is chosen outside the limit cycle, x_0', then its trajectory is repelled from the limit cycle and it goes towards ∞ or $-\infty$.

LAGRANGE STABILITY ANALYSIS:

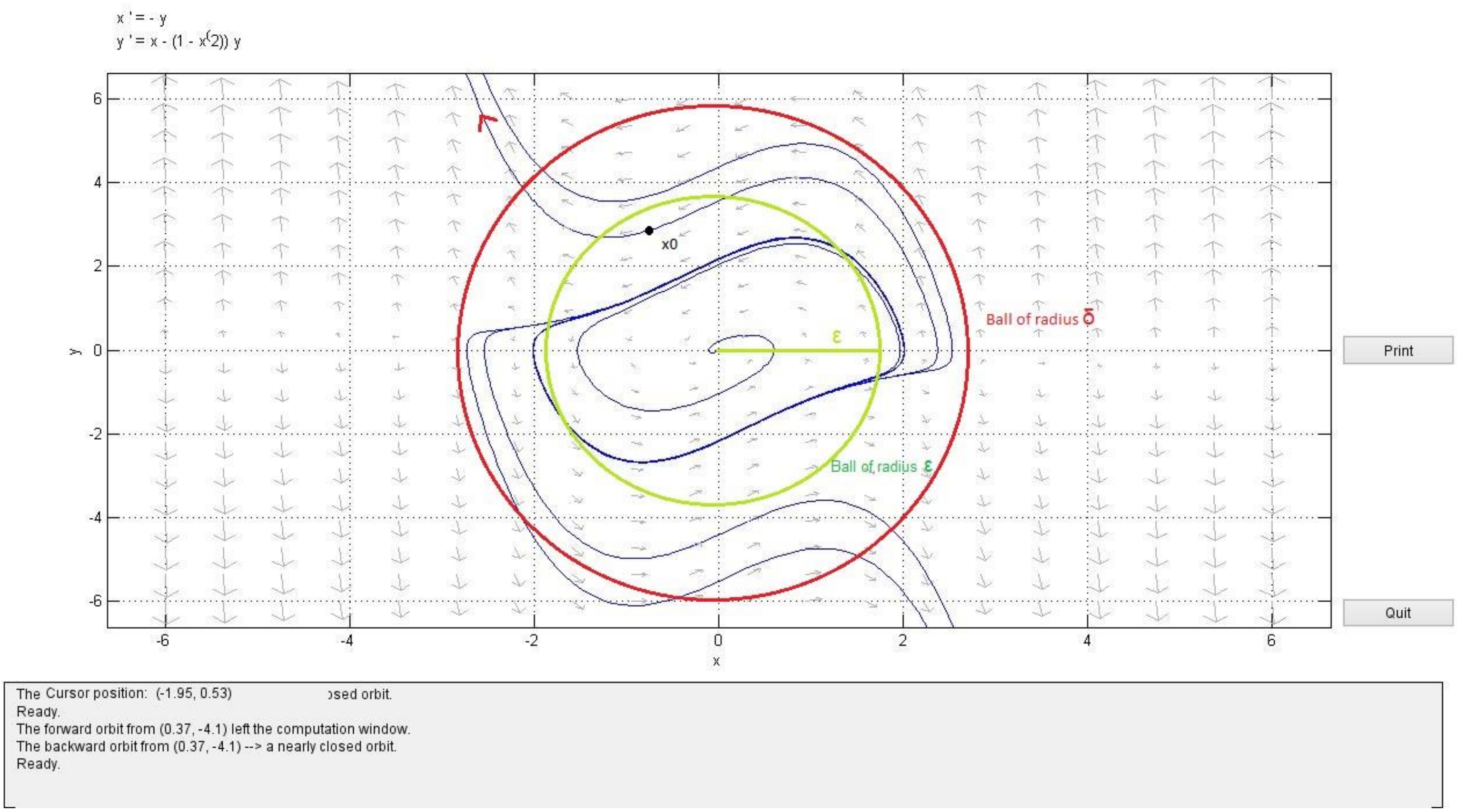

Fig 8: Phase Plane Portrait for Van der Pol Oscillator – Lagrange Stability Analysis

1. As analyzed in class the Van der Pol Oscillator is Lagrange unstable.
2. Same result obtained on plotting of the phase plane portrait.
3. a. Let us choose of ball of radius ε as is shown in the plot.
   b. Then for a system to be Lagrange stable, for any initial condition x0 inside the ball, there exists a ball of size δ, such that the solution stays inside the ball.
   c. As can be seen from the plot, this is not true for bigger values of x0 because if x0 is outside the trajectory, then there will be no ball such that the solution stays inside the ball. So the system is Lagrange Unstable.

2.2 PART B

$$\dot{x}_1 = - x_1 - x_2$$
$$\dot{x}_2 = -x_2 - (x_2^2 - 6)x_2 + x_1$$

LYAPUNOV STABILITY ANALYSIS:

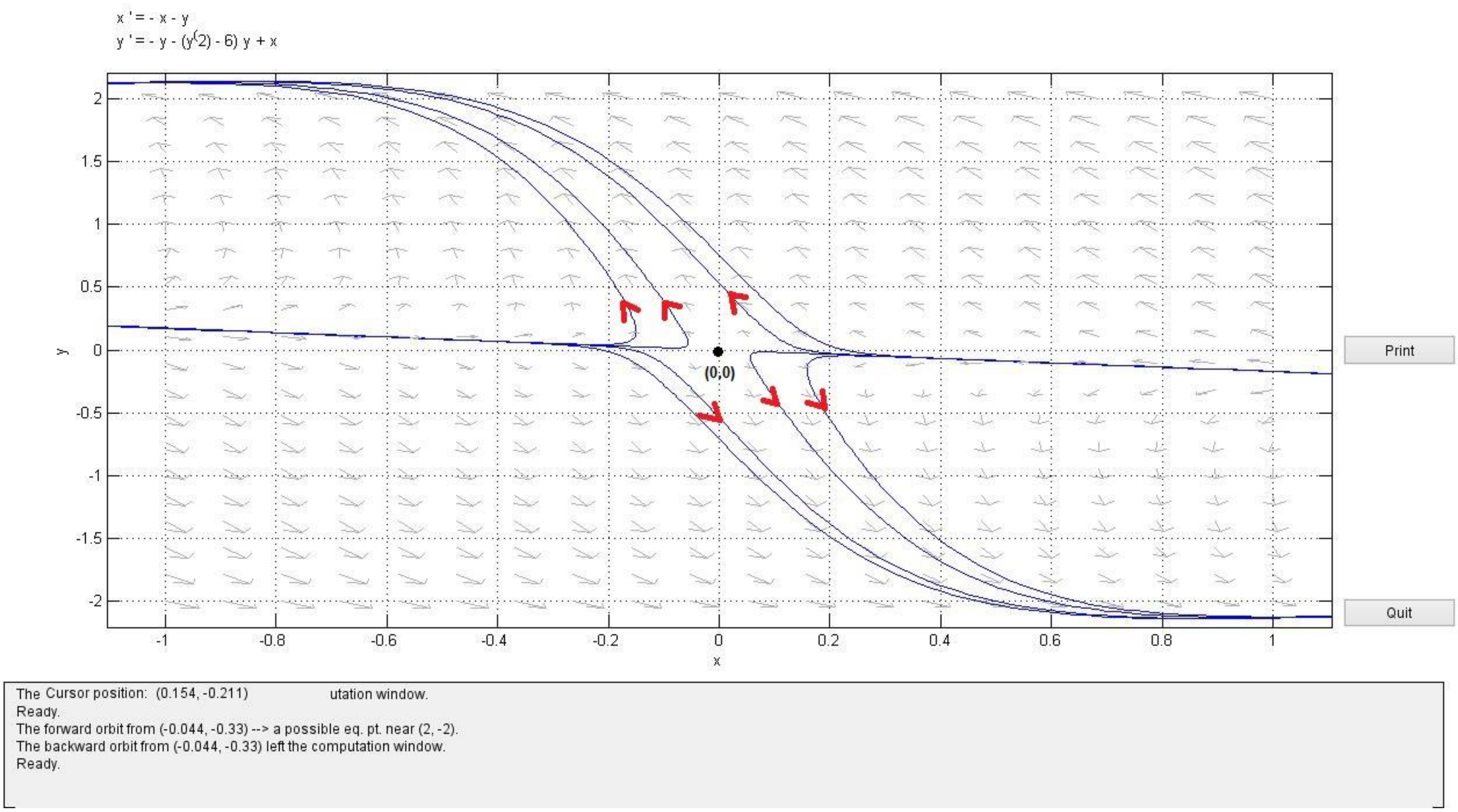

Fig 9: Phase Plane Portrait– Lyapunov Stability Analysis

1. As analyzed in class the system the system is locally unstable I.S.L. since the equilibrium point at origin is a saddle point.
2. Same result obtained on plotting of the phase plane portrait.
3. The above figure shows that all the trajectories diverge away from the origin hence it is a saddle point and locally unstable ISL.

LAGRANGE STABILITY ANALYSIS:

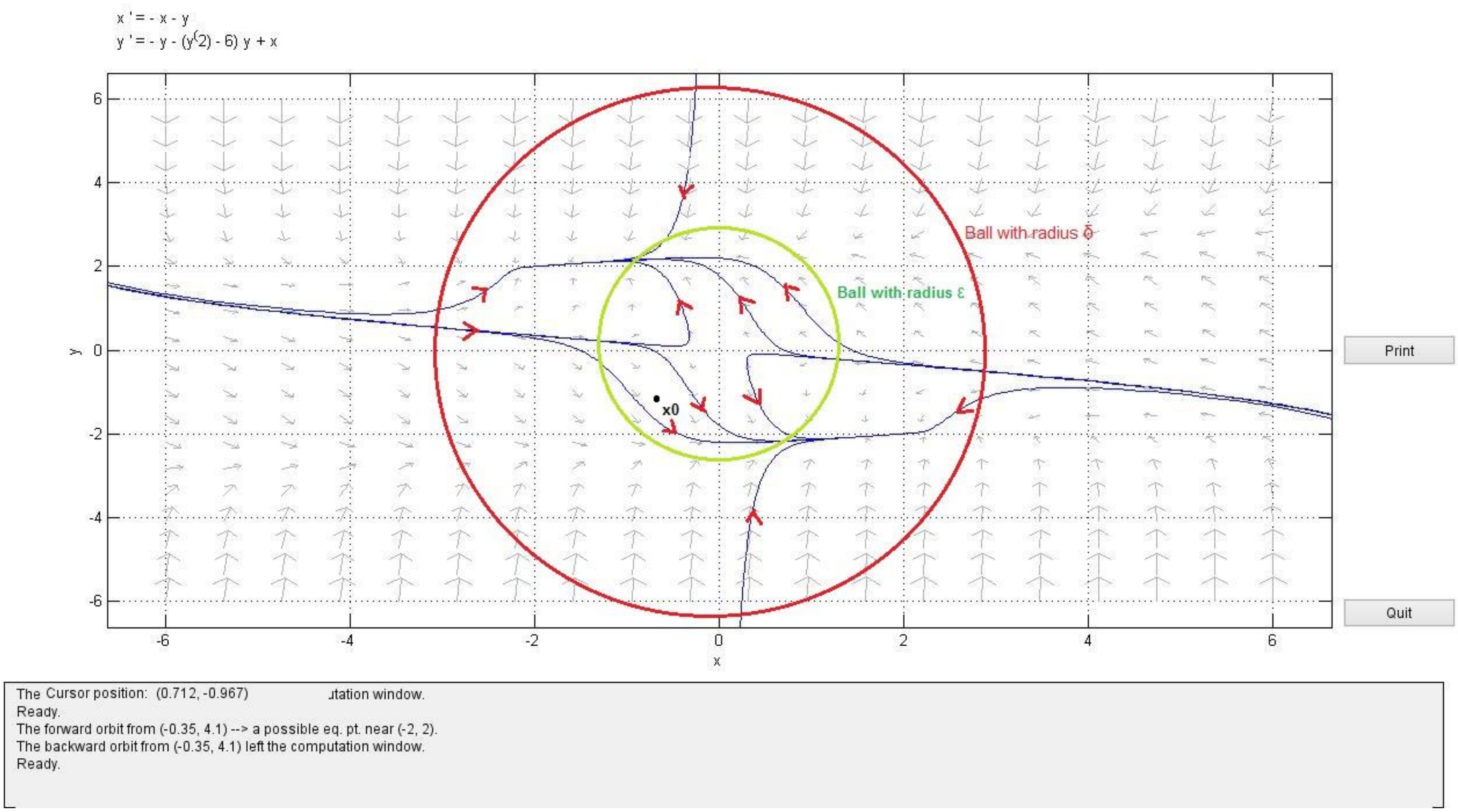

Fig 10: Phase Plane Portrait– Lagrange Stability Analysis

1. As analyzed in class the system is Lagrange stable.
2. Same result obtained on plotting of the phase plane portrait.
3. For any ε, if we pick the initial condition x_0 inside the ball of radius ε, then the trajectory stays inside the ball of radius δ as shown in the figure. Hence, the system is Lagrange stable.

3 BONUS QUESTION

3.1 PART A

Van der Pol Oscillator

$\dot{x}_1 = x_2$
$\dot{x}_2 = -x_1 + (1 - x_1^2)*x_2$

LYAPUNOV STABILITY ANALYSIS:

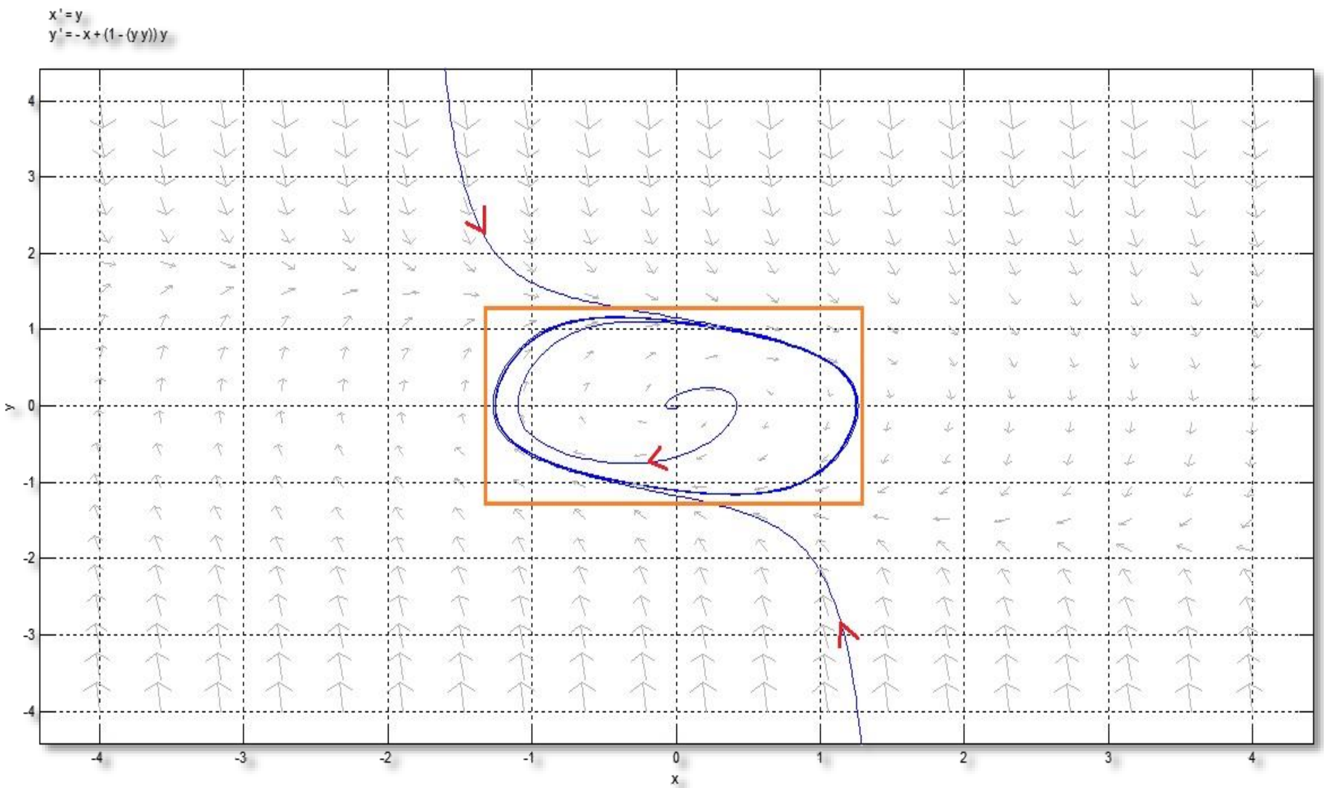

Fig 11: Phase Plane Portrait for Van der Pol Oscillator – Lyapunov Stability Analysis

1. As analyzed in class, the equilibrium point is unstable I.S.L
2. Same result obtained on plotting of the phase plane portrait.
3. if we take any initial condition x0 which lies inside the limit cycle, the trajectory will goes towards the limit cycle.

LAGRANGE STABILITY ANALYSIS:

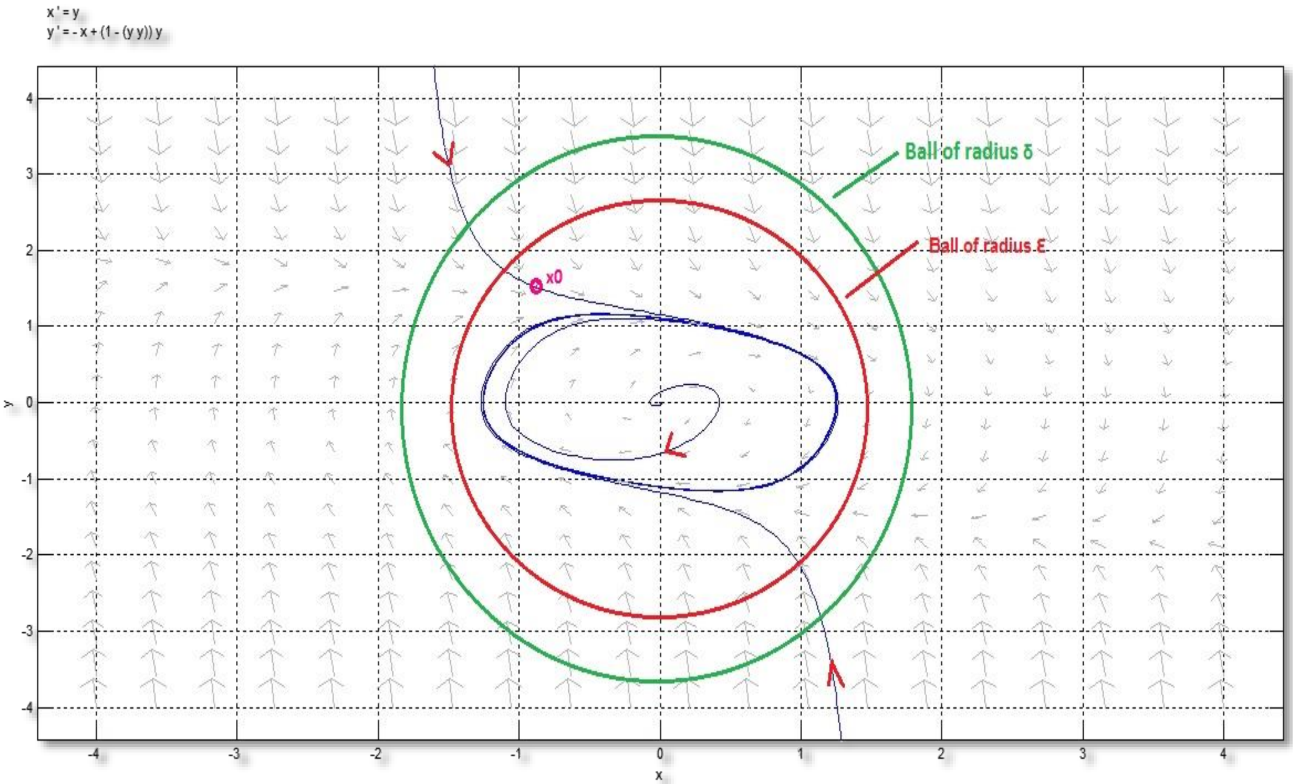

Fig 12: Phase Plane Portrait for Van der Pol Oscillator – Lagrange Stability Analysis

1. As analyzed in class the system is Lagrange stable.
2. Same result obtained on plotting of the phase plane portrait.
3. Let us choose of ball of radius ε as is shown in the figure attached below. Then for a system to be Lagrange stable, for any initial condition x_0 inside the ball, there exists a ball of size δ, such that the solution stays inside the ball. As can be seen from the plot below, this is true for bigger values of x_0 So the system is Lagrange Stable.

3.2 PART B

$\dot{x}_1 = x_2 - (x_1 - (1/3x_1)^3)$
$\dot{x}_2 = -x_1$

LYAPUNOV STABILITY ANALYSIS:

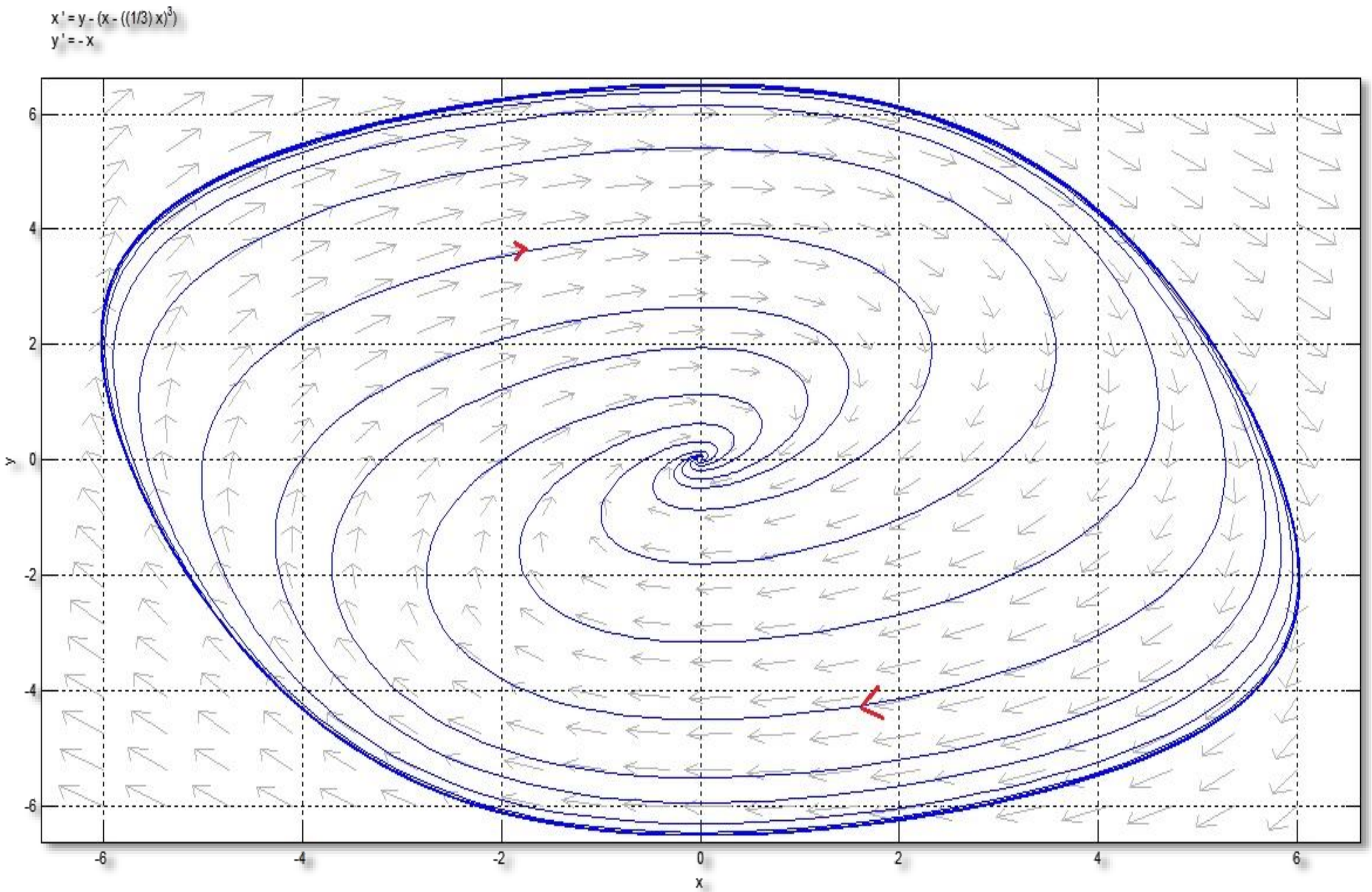

Fig 13: Phase Plane Portrait– Lyapunov Stability Analysis

1. As analyzed in class the equilibrium point is stable ISL.
2. Same result obtained on plotting of the phase plane portrait.
3. For any x0, the trajectory converges to the equilibrium point at the origin.

LAGRANGE STABILITY ANALYSIS:

1. As analyzed in class the system is Lagrange stable.
2. Same result obtained on plotting of the phase plane portrait.
3. Let us choose of ball of radius ε as is shown in the figure attached below. Then for a system to be Lagrange stable, for any initial condition x_0 inside the ball, there exists a ball of size δ, such that the solution stays inside the ball. So the system is Lagrange Stable.