ECE611 - Nonlinear Control Systems

Computer Project #1

1) Consider the following nonlinear differential equations:

a) $\dot{x} = \frac{1}{2x}; \quad x(0) = 0$

b) $\dot{x} = 1 + x^2; \quad x(0) = 0$

c) $\dot{x} = x^{\frac{1}{3}}; \quad x(0) = 0$

d) $\dot{x} = -2x^2; \quad x(0) = -1$

e)

$$\dot{x} = \begin{cases} -x & \text{if } x < 0 \\ x^2 & \text{if } x \geq 0 \end{cases}; \quad x(0) = 0$$

These equations were studied using theory/analysis tools in Homework #2. Study the behavior of these systems numerically by using Simulink to simulate the nonlinear systems. How does the simulation behavior compare to what you found with your previous analysis? Try simulating each system for several different initial conditions. How does this impact the behavior?

2) Consider the following two systems which we analyzed in class. First the Inverse Van der Pol Oscillator, which was shown to be Lyapunov stable but not Lagrange stable:

$$\begin{aligned} \dot{x}_1 &= -x_2 \\ \dot{x}_2 &= x_1 - (1 - x_1^2)x_2 \end{aligned}$$

The we have the following system which was also analyzed in class and shown to be Lagrange stable but not Lyapunov stable

$$\begin{aligned} \dot{x}_1 &= -x_1 - x_2 \\ \dot{x}_2 &= -x_2 - (x_2^2 - 6)x_2 + x_1 \end{aligned}$$

Simulate both of these systems for different initial conditions. Relate the simulation results to the analysis results.

3) BONUS QUESTION Try simulating some of the other examples we have analyzed in class. Consider the properties of these systems (e.g., stability). Can you relate the simulation results to the analysis results?