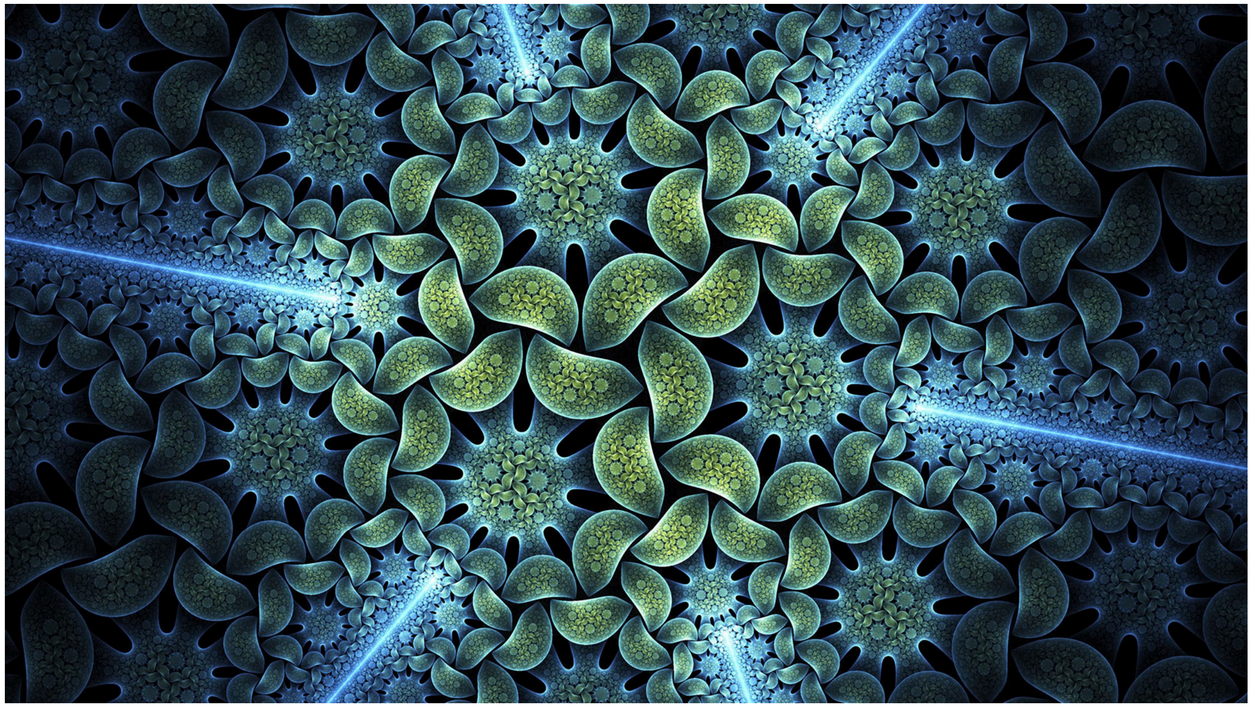
Thinking Recursively

CS 106B

Programming Abstractions
Fall 2016
Stanford University
Computer Science Department



Announcements

- Assignment 2 due Saturday
- Remember no LaIR on Friday night
- New texts for random writer

Today's Goal

1. More practice with recursion
2. Be able to write graphical recursive functions

One of the programs on assignment 3 will use graphical recursion

Recursion Definition

Recursion:

A problem solving technique in which problems are solved by reducing them into **smaller problems** ***of the same form***.

Recursion Template

The structure of recursive functions is typically like the following:

```
recursiveFunction:
   if (test for simple case) {
      Compute the solution without recursion
   } else {
      Break the problem into subproblems of the same form
      Call recursiveFunction on each subproblem
      Reassemble the results of the subproblems
   }
}
```

Three Musts of Recursion

1. Your code must have a case for all valid inputs.

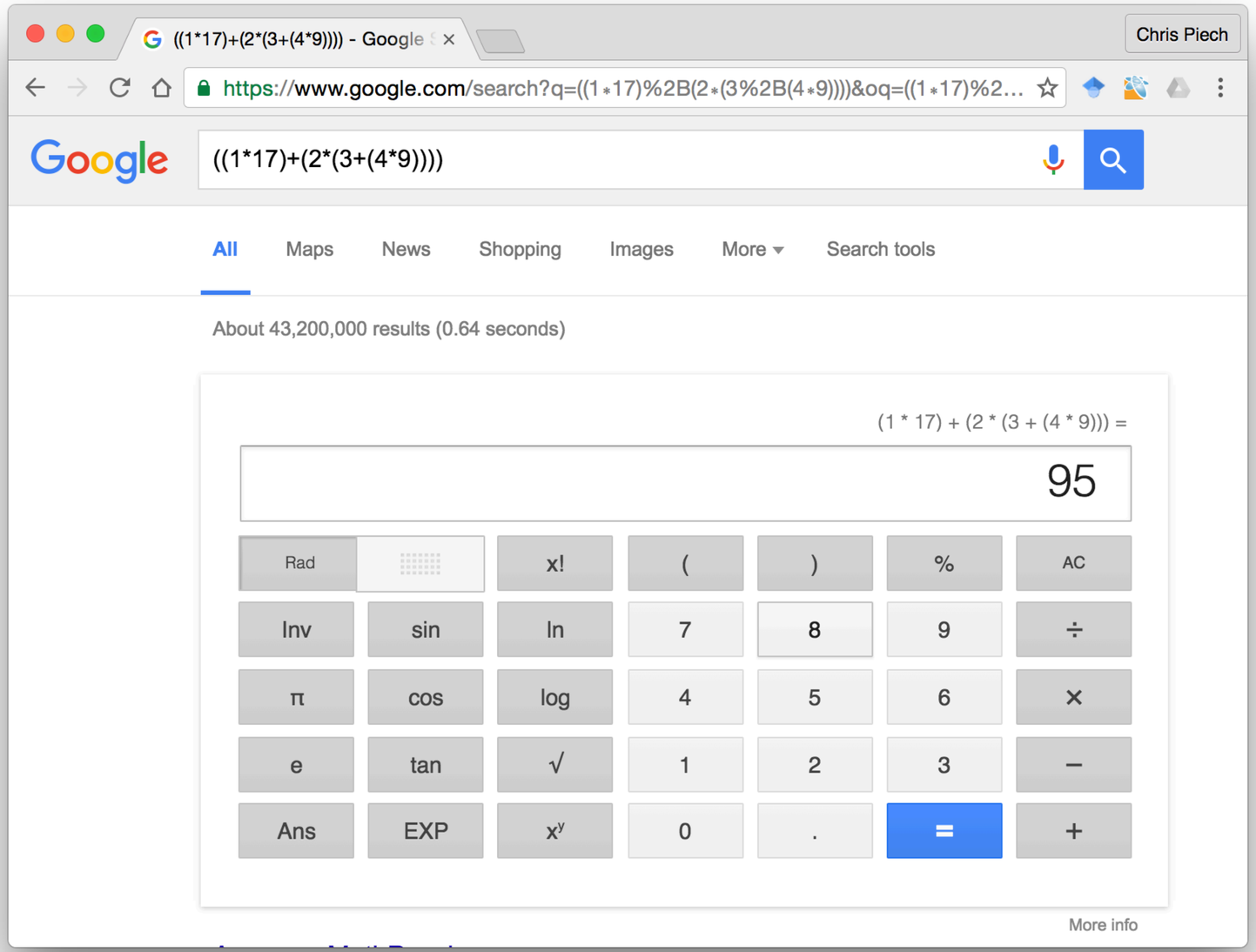
2. You must have a base case (makes no recursive calls).

3. When you make a recursive call it should be to a simpler instance (forward progress towards base case)

Start with Delicious Example

((1+3)*(2*(4+1)))

Google Search I'm Feeling Lucky


((1*17)+(2*(3+(4*9)))) - Google
Chris Piech
https://www.google.com/search?q=((1*17)%2B(2*(3%2B(4*9))))&oq=((1*17)%2...
Google
((1*17)+(2*(3+(4*9))))
All
Maps
News
Shopping
Images
More
Search tools
About 43,200,000 results (0.64 seconds)
(1 * 17) + (2 * (3 + (4 * 9))) =
95
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Ans
EXP
x^y
0
.
=
+
More info

“((1+3)*(2*(4+1)))”

95


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Challenge

Implement a function which evaluates an expression string:

“((1+3)*(2*(4+1)))”

Only * or +

“(7+6)”

“(((4*(1+2))+6)*7)”

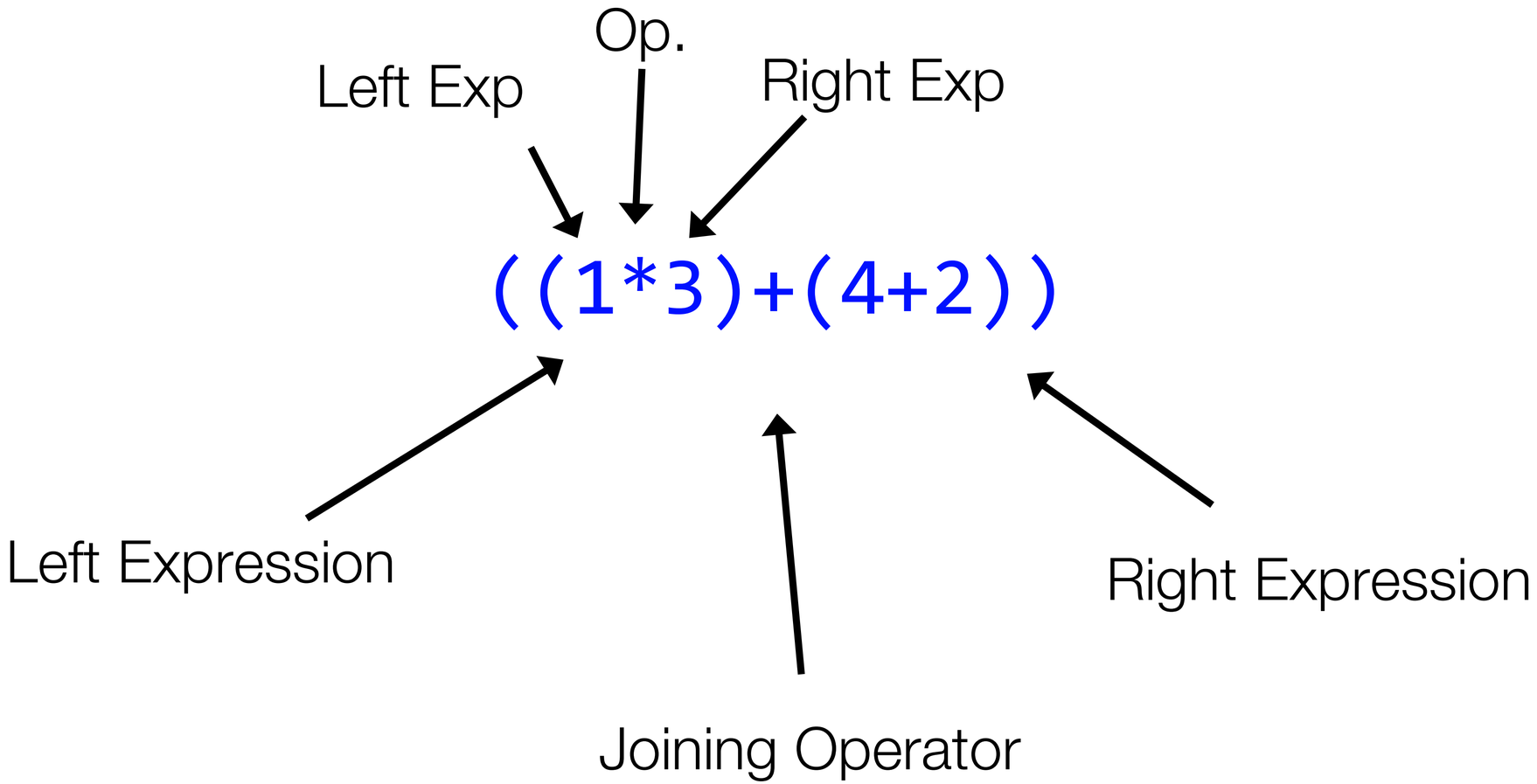
Anatomy of an Expression

An expression is always one of these three things

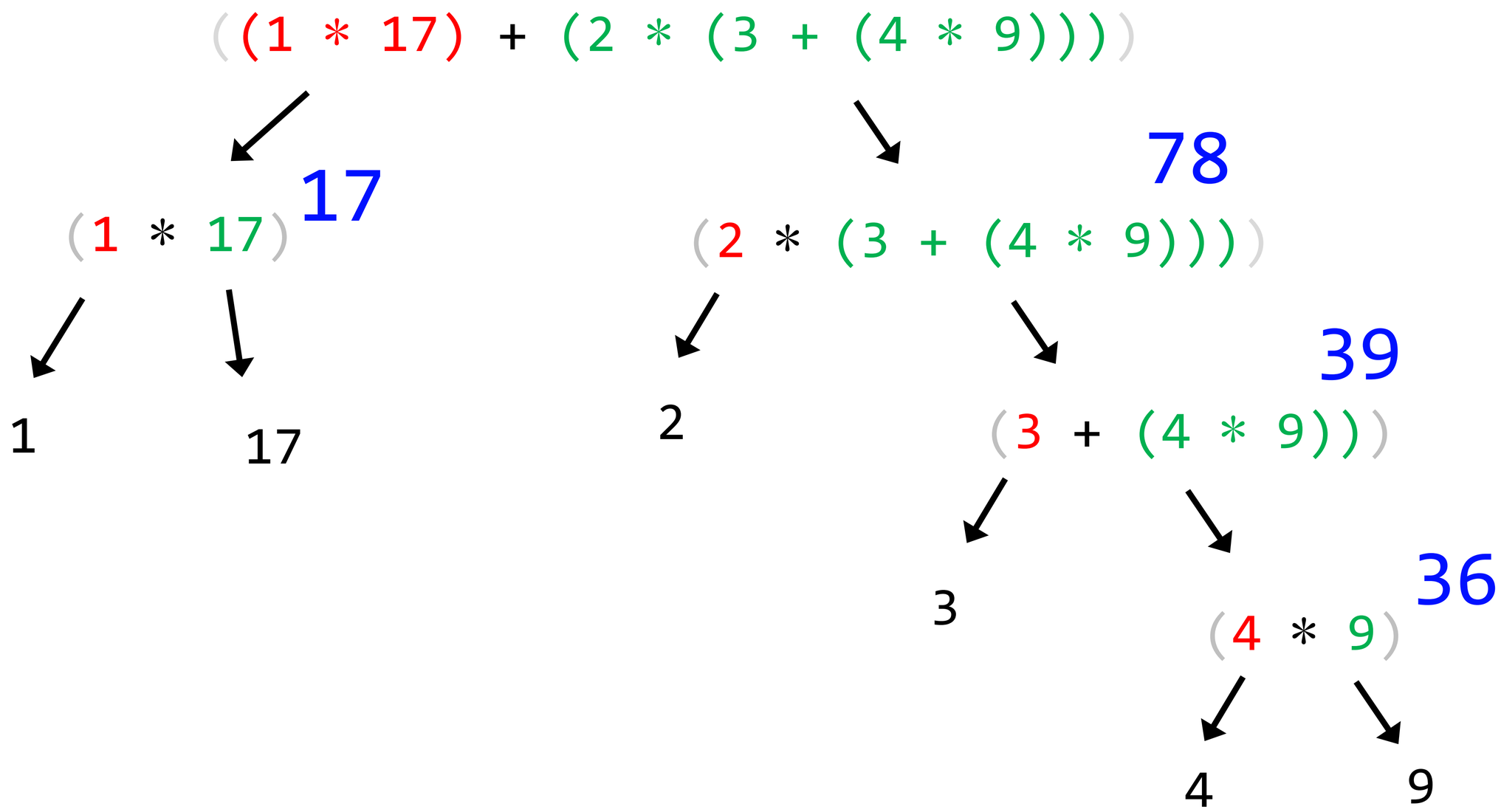
expression

- *number*
- *(expression + expression)*
- *(expression * expression)*

Anatomy of an Expression



How do we evaluate ((1*17)+(2*(3+(4*9))))? 95



It is Recursive?

```
((1*3)+(4+2))
```

The big instance of this problem is:

```
((1*3)+(4+2))
```

The smaller instances are:

```
(1*3)
```
and
```
(4+2)
```

Task

Write this function

```
int evaluate(string exp);
“((1*3)+(4+2))” //returns 11
```

Using these library functions

```
stringIsInteger(exp)
stringToInteger(exp)
```

And these exp helper functions

```
//returns ‘+’
char op = getOperator(exp);
//returns “(1*3)”
string left = getLeftExp(exp);
//returns “(4+2)”
string right = getRightExp(exp);
```

Solution

```
((1*3)+(4+2))
```

`int evaluate(expression):`

- If *expression* is a number, return *expression*
- Otherwise, break up *expression* by its *operator*:
  - *leftResult* = `evaluate(`*leftExpression*`)`
  - *rightResult* = `evaluate(`*rightExpression*`)`
  - `Return` *leftResult operator rightResult*

Solution

```
int evaluate(string exp) {
    if (stringIsInteger(expression)) {
        return stringToInteger(expression);
    } else {
        char op = getOperator(exp);
        string left = getLeftExp(exp);
        string right = getRightExp(exp);
        int leftResult = evaluate(left);
        int rightResult = evaluate(right);

        if (op == '+') {
            return leftResult + rightResult;
        } else if (op == '*') {
            return leftResult * rightResult;
        }
    }
}
```

Solution

```
int evaluate(string exp) {
    if (stringIsInteger(expression)) {
        return stringToInteger(expression);
    } else {
        char op = getOperator(exp);
        string left = getLeftExp(exp);
        string right = getRightExp(exp);
        int leftResult = evaluate(left);
        int rightResult = evaluate(right);

        if (op == '+') {
            return leftResult + rightResult;
        } else if (op == '*') {
            return leftResult * rightResult;
        }
    }
}
```

exp = “((1*3)+((4*5)+2))”

op = ‘+’

left = “(1*3)”

right = “((4*5)+2)”

leftResult = 3

rightResult = 22

Helper Methods

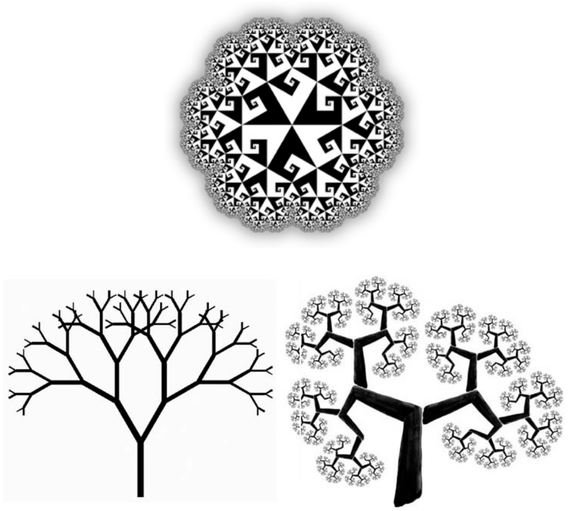
Here is the key function behind the helper methods:

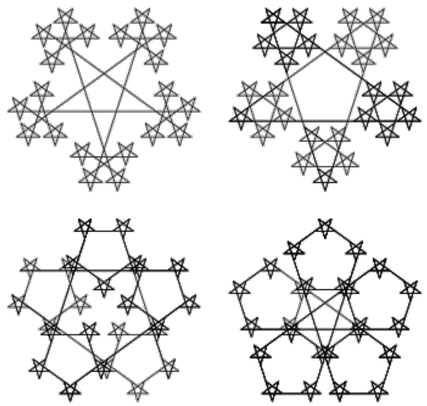
```
int getOppIndex(string exp){
    int parens = 0;
    for (int i = 0; i < exp.length(); i++) {
        char c = exp.charAt(i);
        if (c == '(') { parens++; }
        else if (c == ')') { parens--; }
        if (parens == 0 && (c == '+' || c == '*')) {
            return i;
        }
    }
}
```

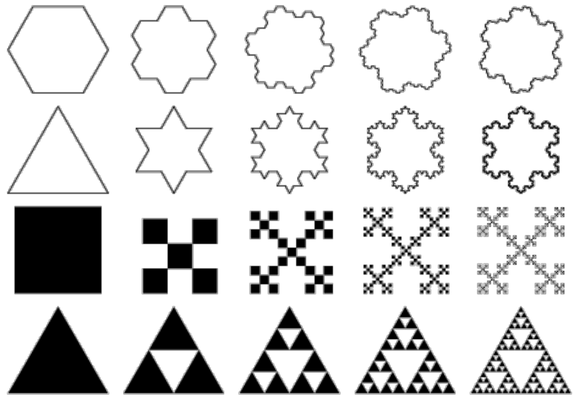
Recursion you can *see*

Fractal

fractal: A recurring graphical pattern. Smaller instances of the same shape or pattern occur within the pattern itself.







Fractals in Nature

Many natural phenomena generate fractal patterns:

1. earthquake fault lines
2. animal color patterns
3. clouds
4. mountain ranges
5. snowflakes
6. crystals
7. DNA
8. ...

Ready?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Let’s go!


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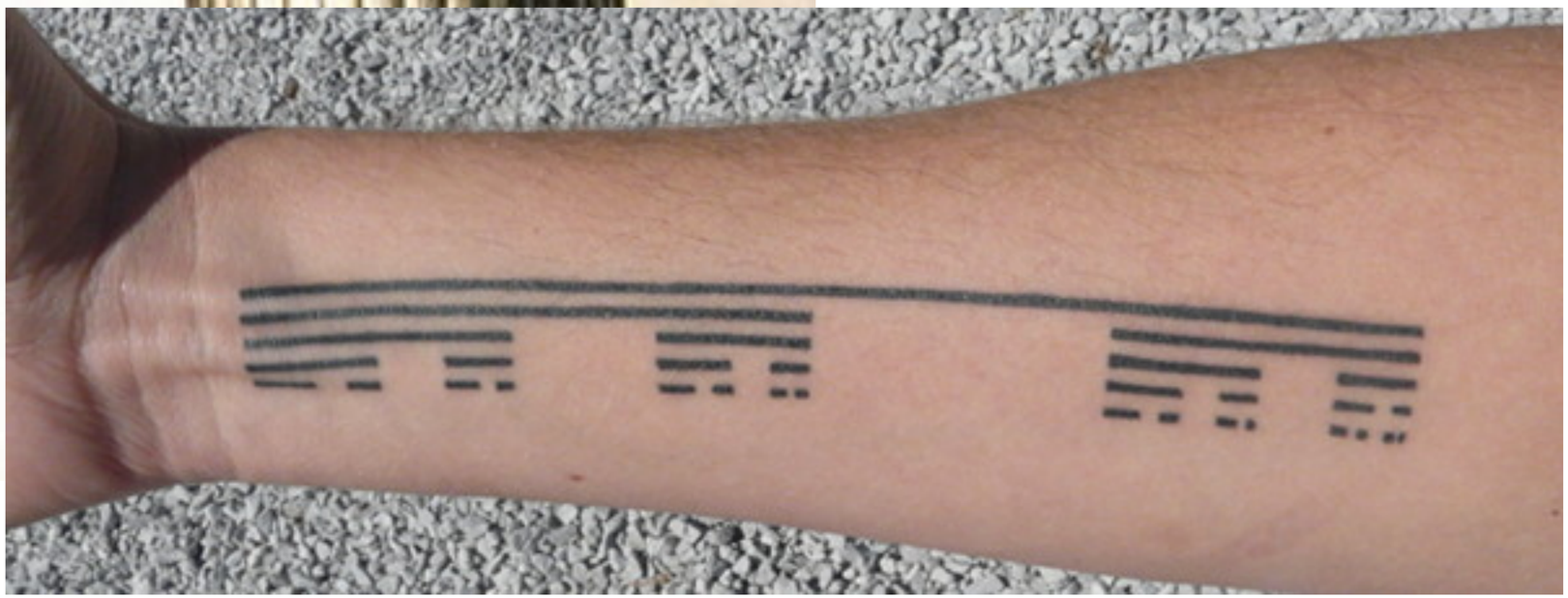
Cantor Fractal



Cantor Fractal

Parts of a Cantor set image…
are Cantor set images

Cantor Fractal

Start

End

Another cantor set

Also a cantor set

Levels of Cantor

6 Levels

Levels of Cantor

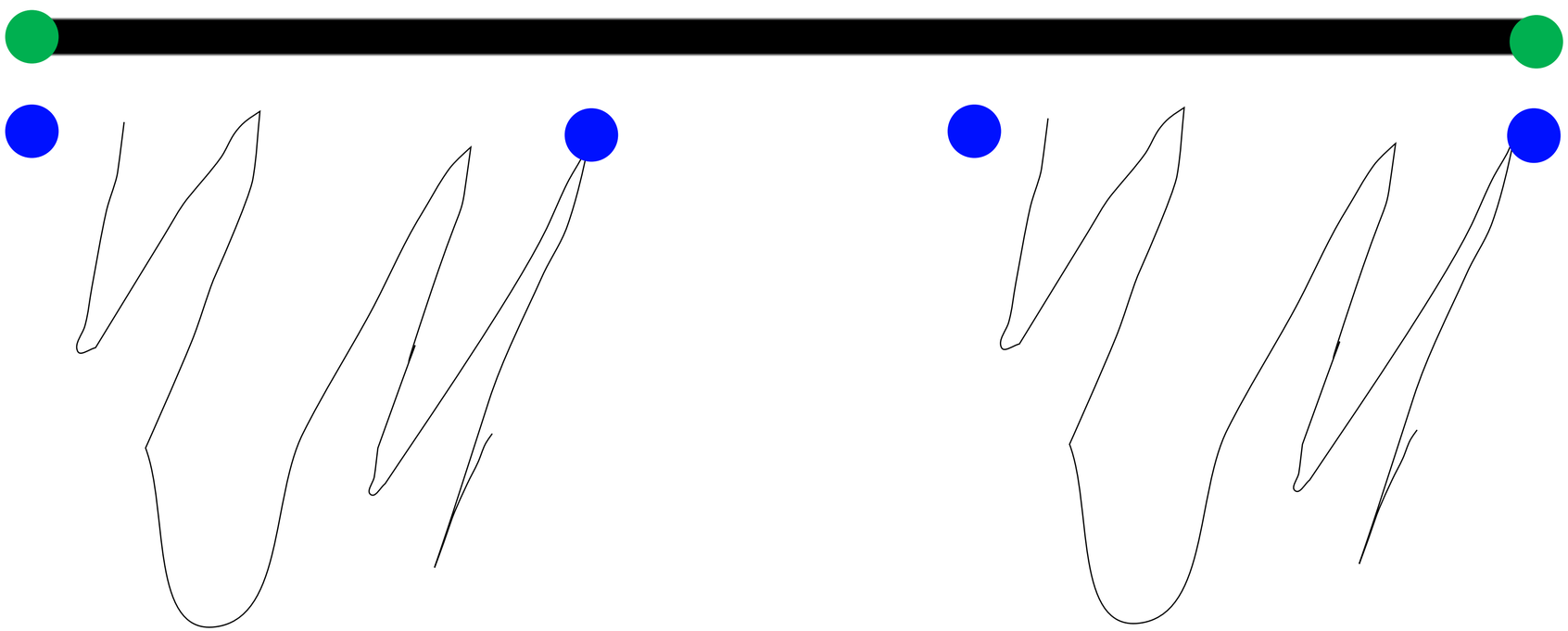
Levels of Cantor

1 Level

How to Draw a Level 1 Cantor?

How to Draw a Level n Cantor?

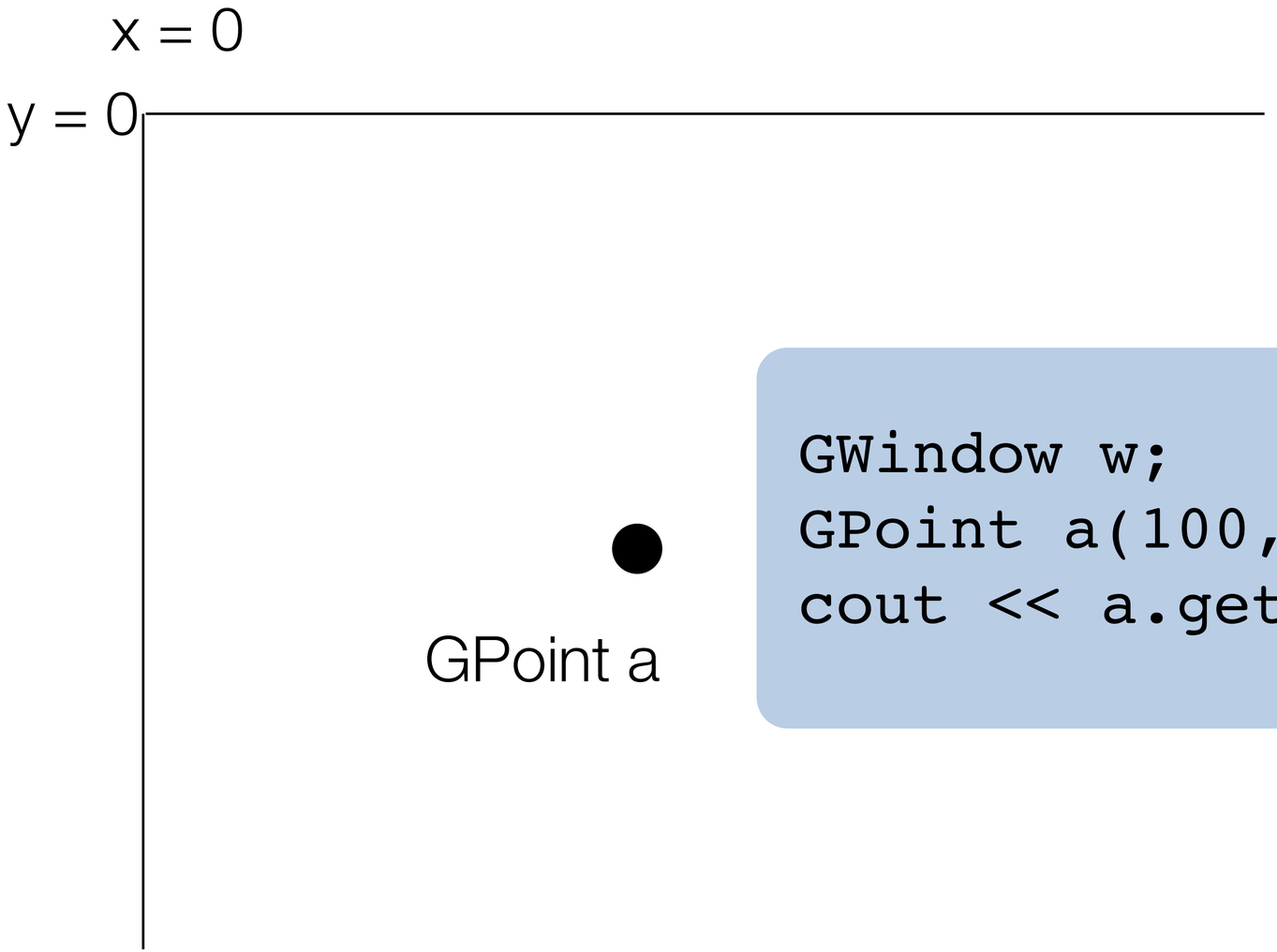
1 Draw a line from start to finish



2 Draw a Cantor of size n-1

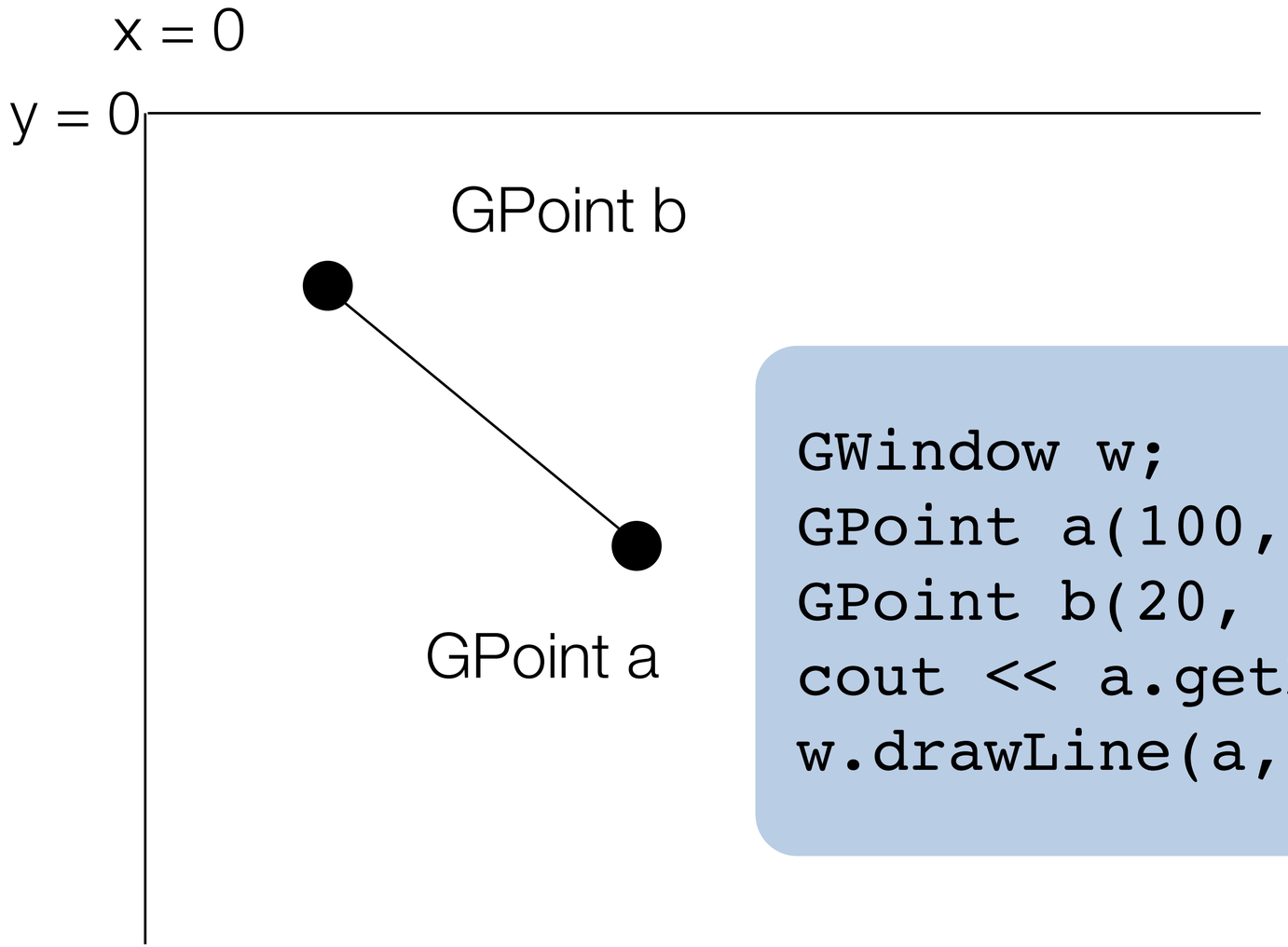
3 Draw a Cantor of size n-1

GPoint



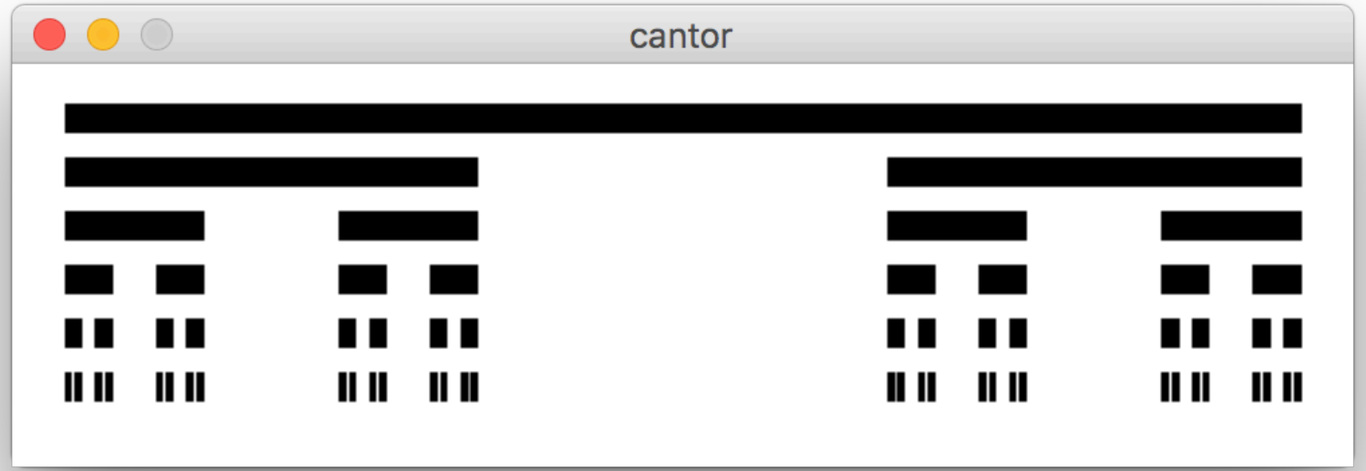

```
GWindow w;
GPoint a(100, 100);
cout << a.getX() << endl;
```

GPoint

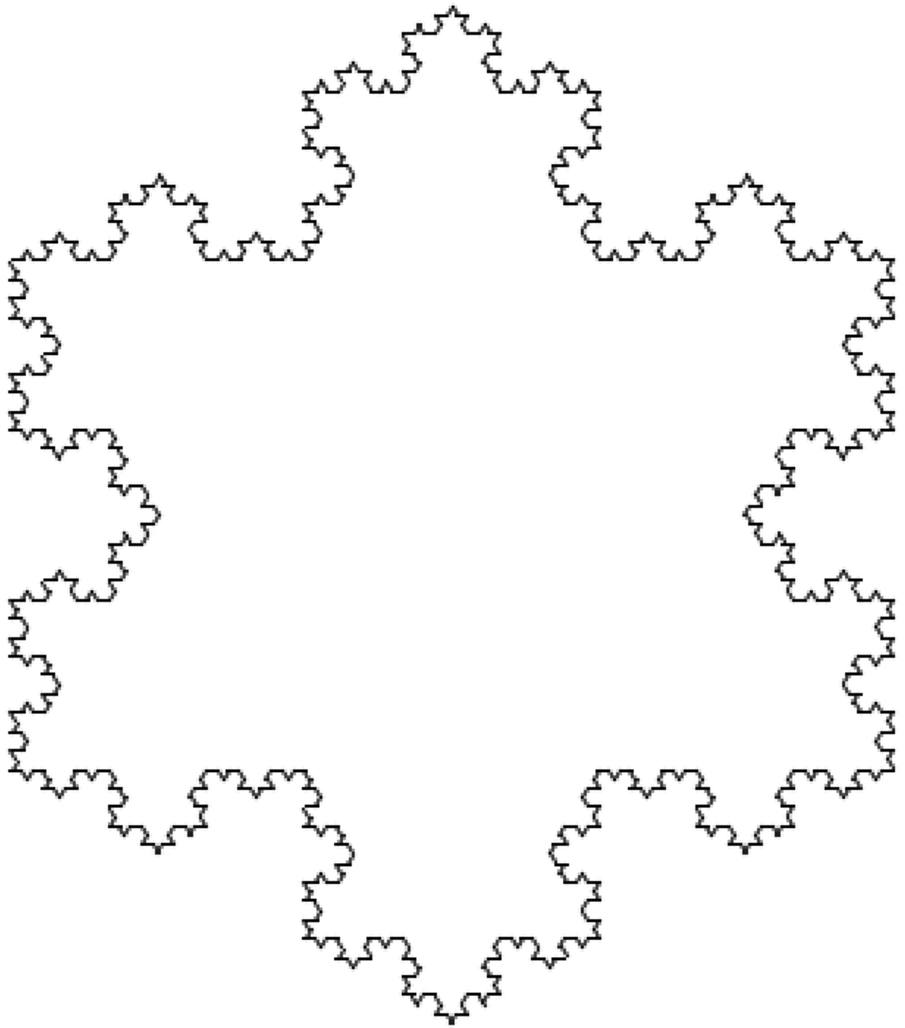


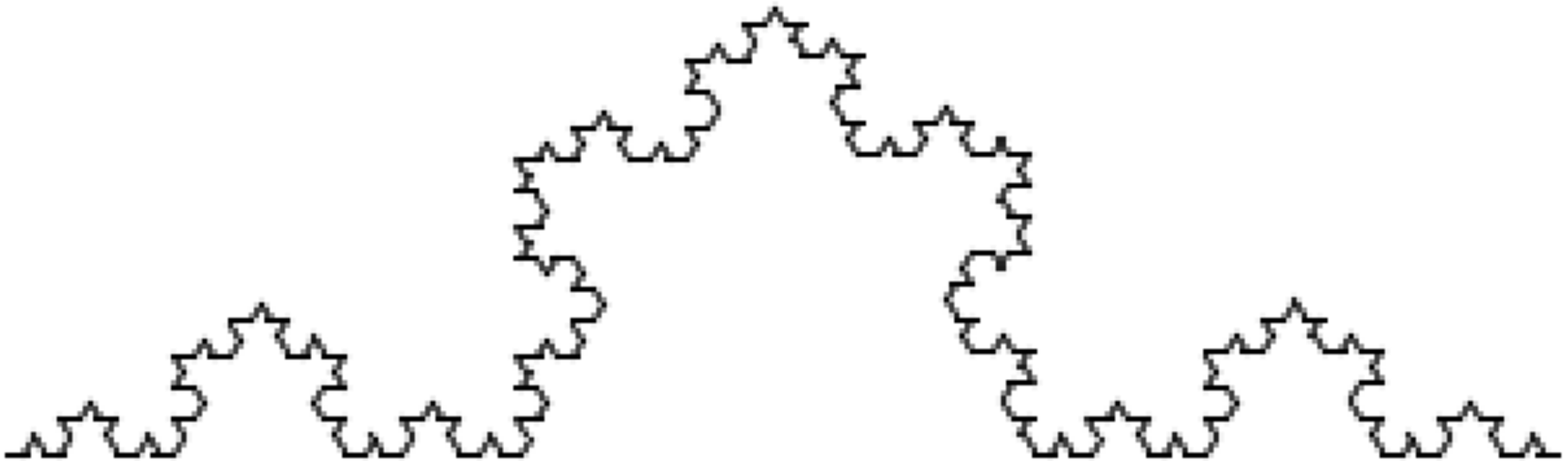

```
GWindow w;
GPoint a(100, 100);
GPoint b(20, 20);
cout << a.getX() << endl;
w.drawLine(a, b);
```

Cantor Fractal



Snowflake Fractal



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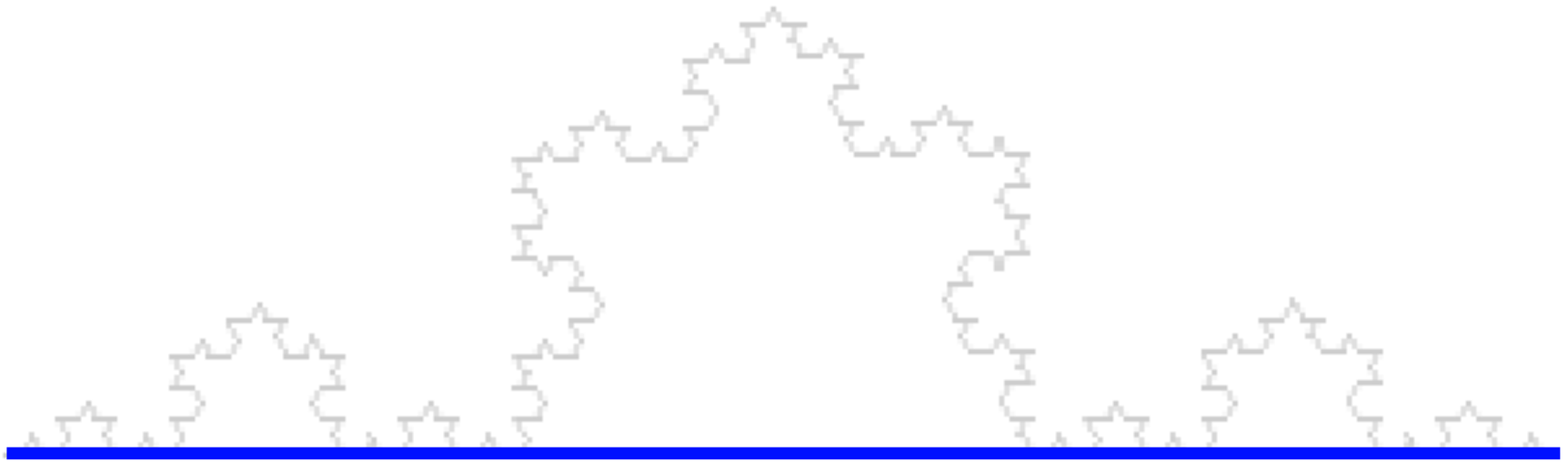
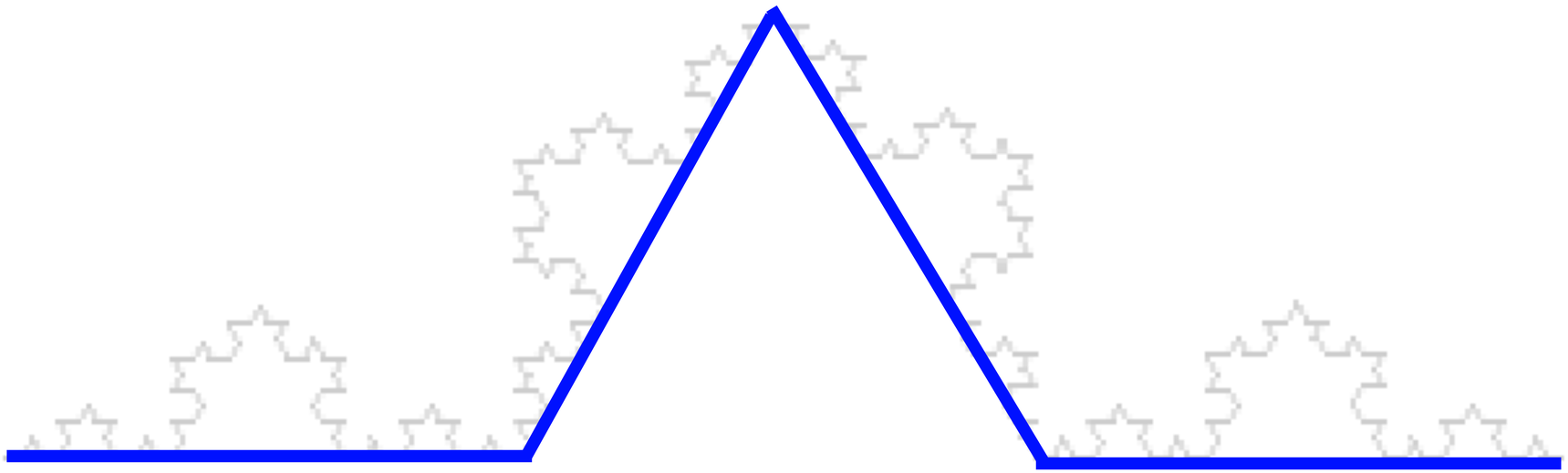
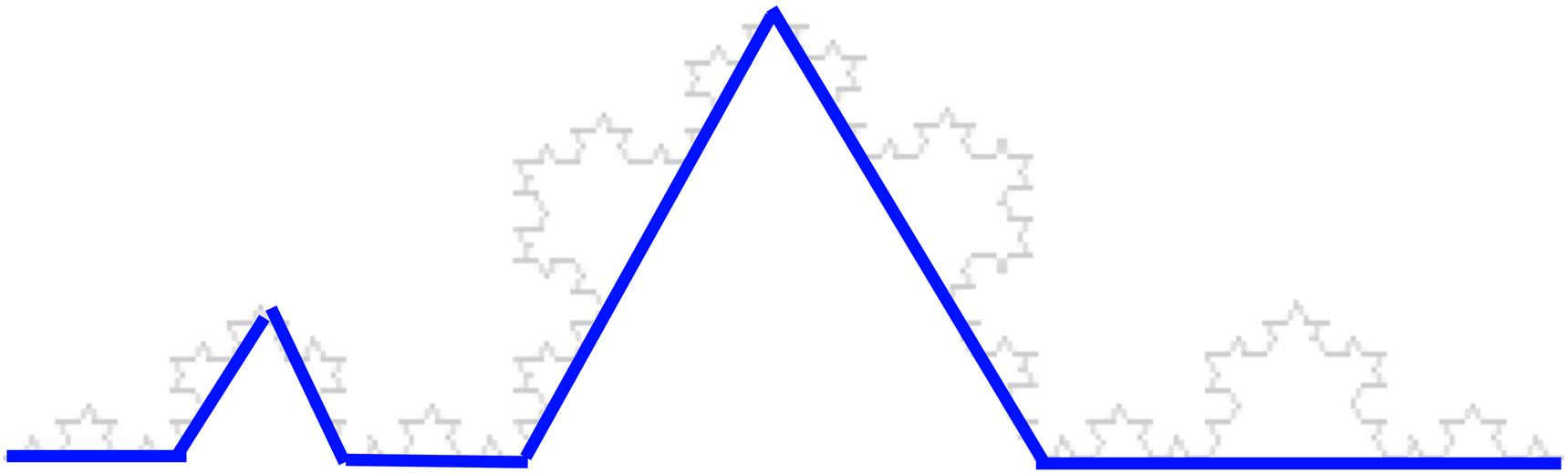
Depth 1 Snowflake Line



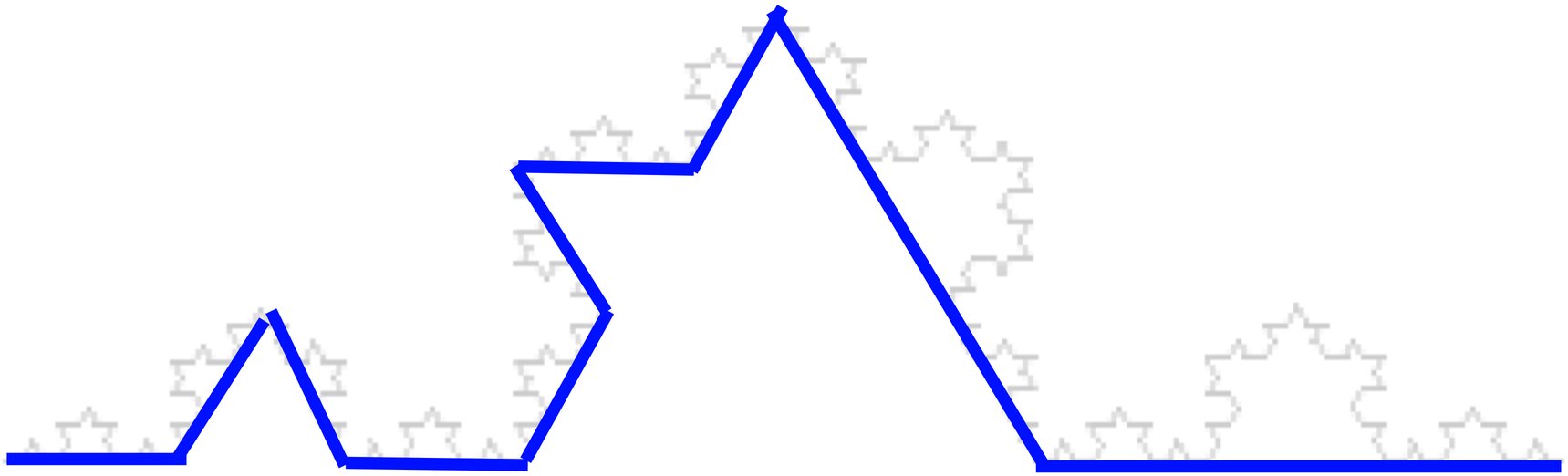
Depth 2 Snowflake Line



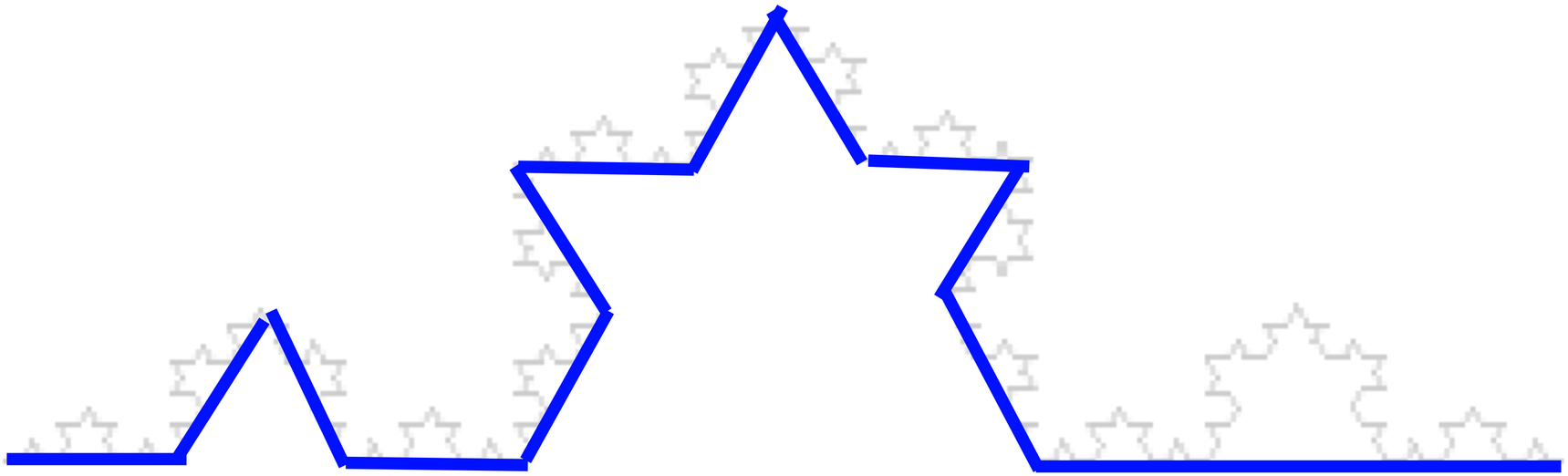
Depth 3 Snowflake Line (in progress)



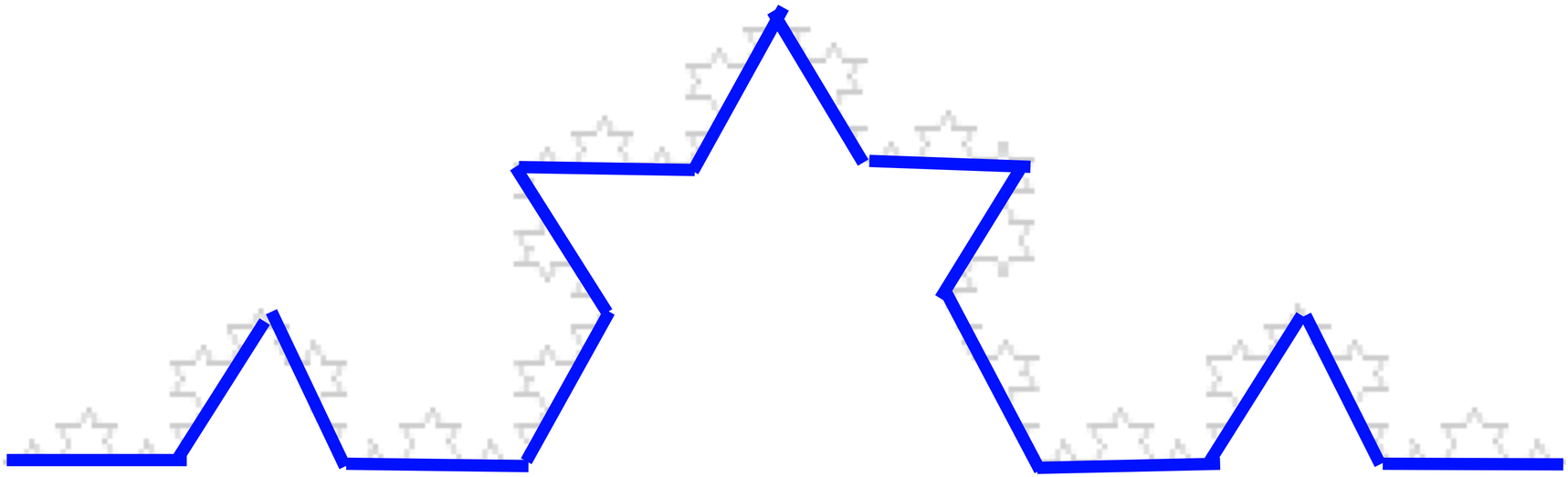
Depth 3 Snowflake Line (in progress)

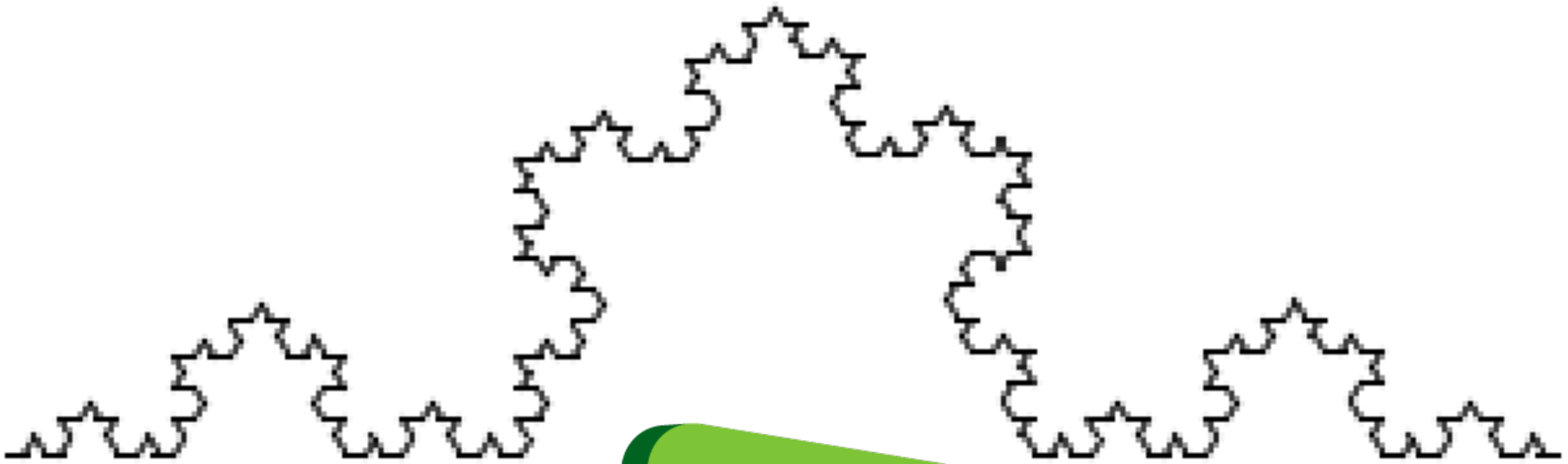


Depth 3 Snowflake Line (in progress)



Depth 3 Snowflake Line (in progress)



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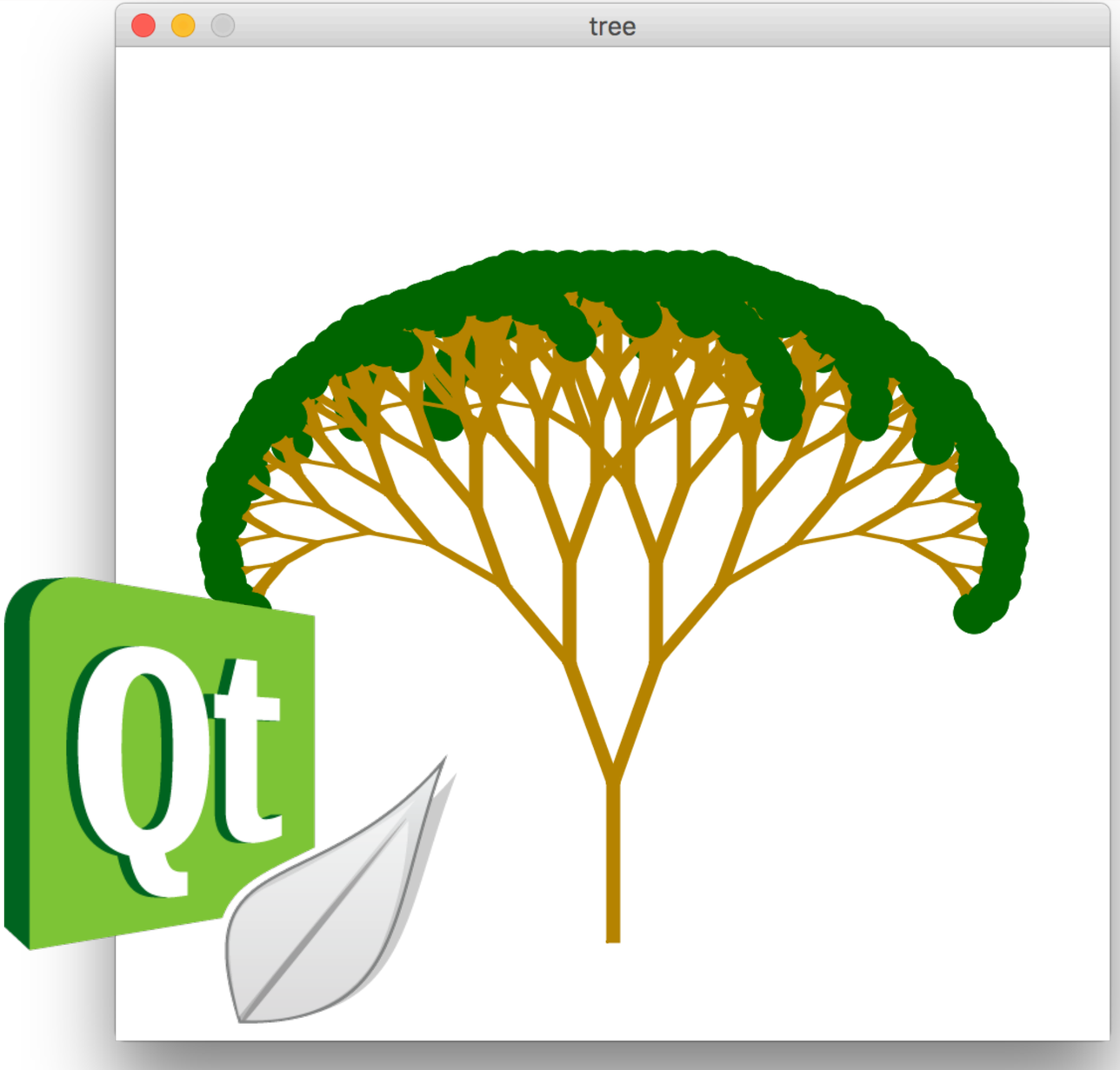
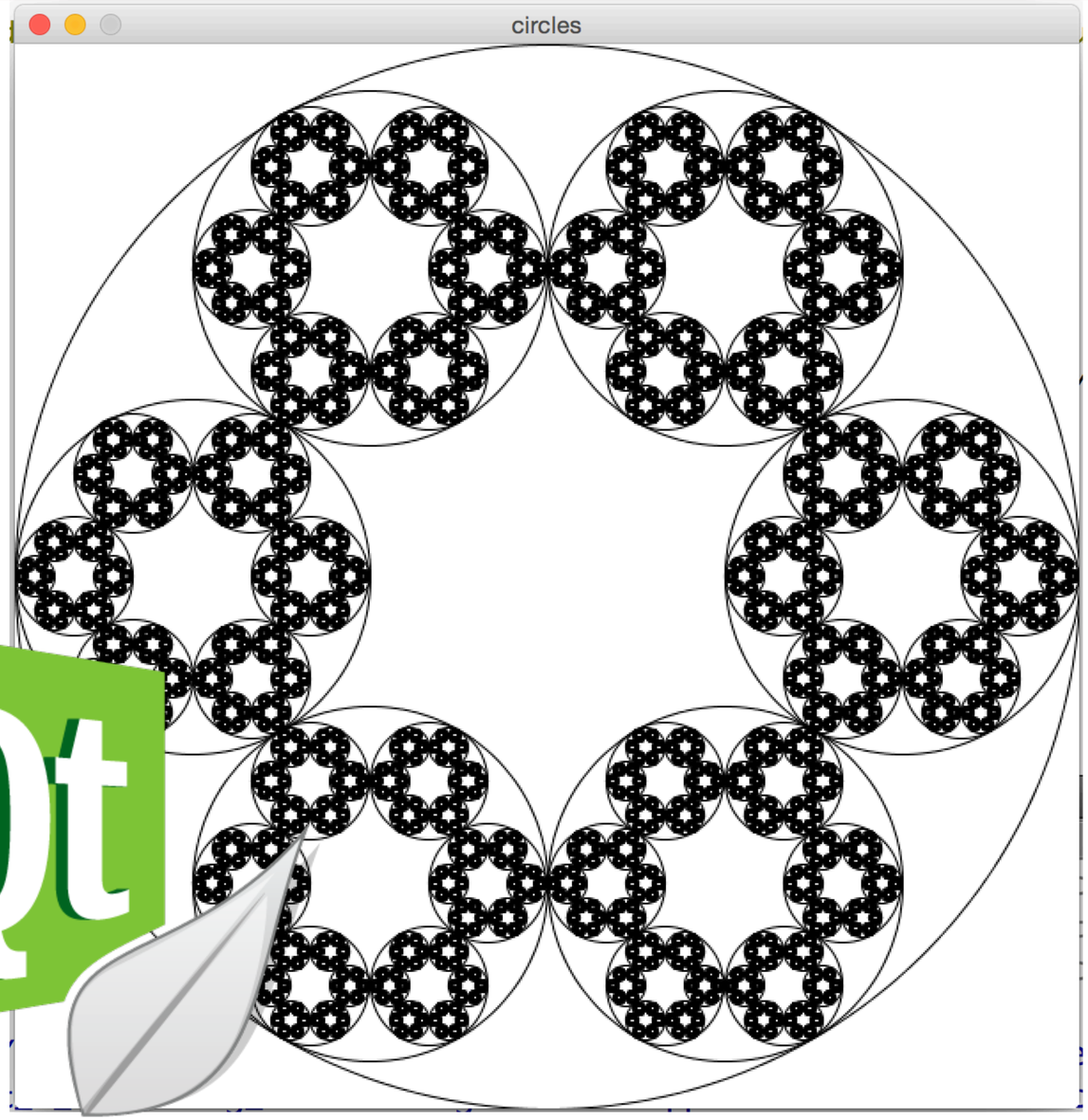
Extra Problems Online

Fractal Tree



Fractal Circles




Recursive
Ray Tracing


ydot

The End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