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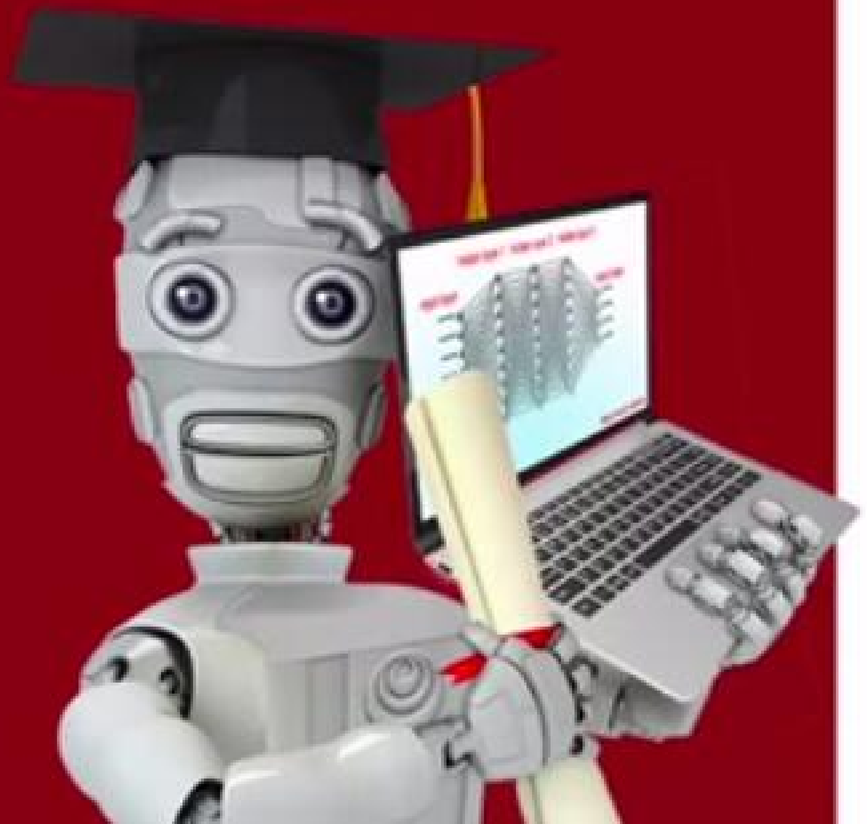
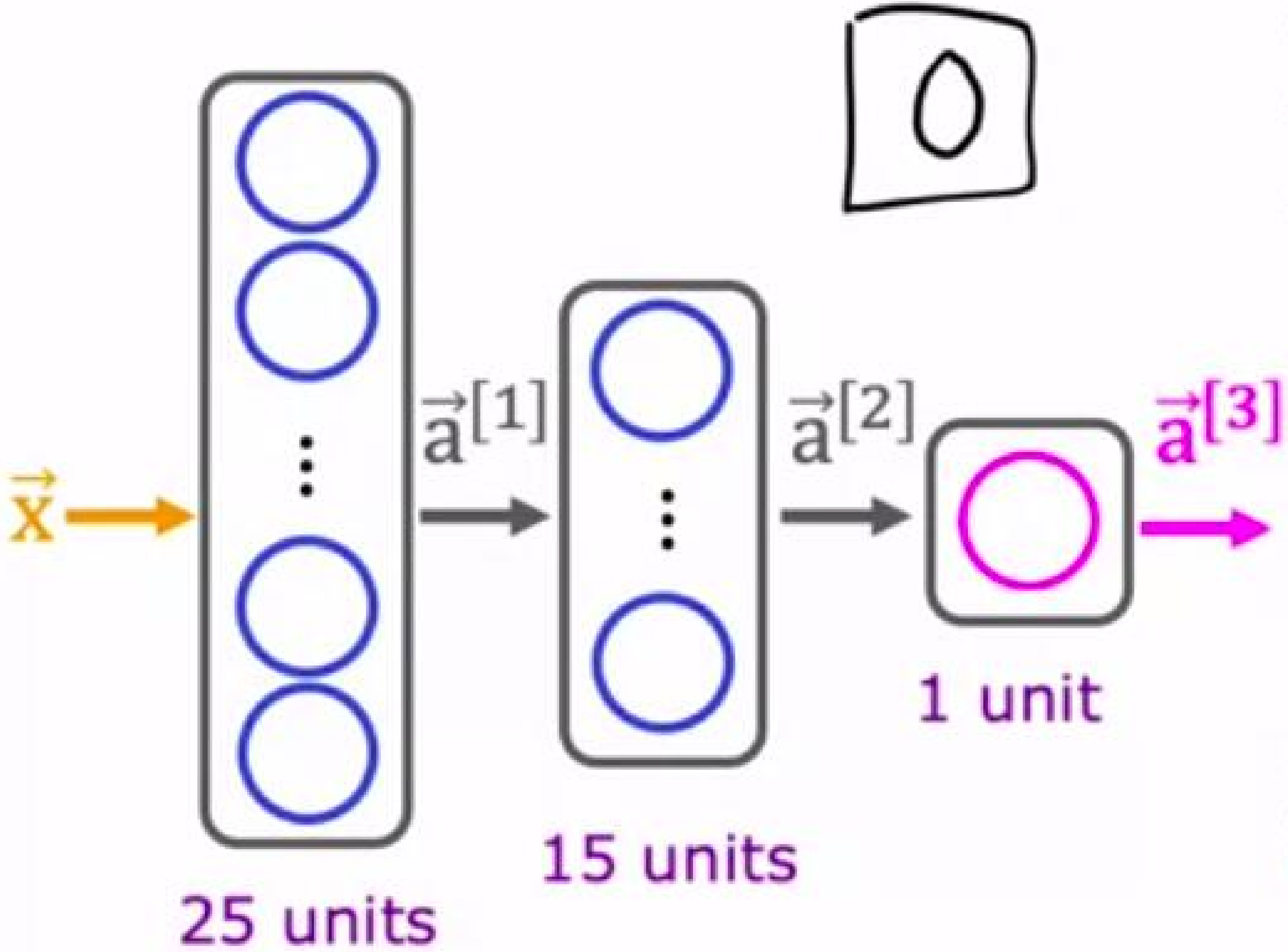


Neural Network Training

TensorFlow implementation

Train a Neural Network in TensorFlow



Given set of (*x*,*y*) examples

How to build and train this in code?

```
import tensorflow as tf
from tensorflow.keras import Sequential
from tensorflow.keras.layers import Dense
   model = Sequential([
     Dense(units=25, activation='sigmoid’),
     Dense(units=15, activation='sigmoid’),
     Dense(units=1, activation='sigmoid’),
                              ])
```

①

```
from tensorflow.keras.losses import
BinaryCrossentropy
   model.compile(loss=BinaryCrossentropy())
```

②

```
   model.fit(X,Y,epochs=100)
```

③

epochs: number of steps
in gradient descent

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Neural Network Training

Training Details

Model Training Steps

Tensor Flow

① specify how to compute output given input x and parameters w,b (define model)

$f_{\vec{w},b}(\vec{x}) = ?$

② specify loss and cost

$L(f_{\vec{w},b}(\vec{x}), y)$ 1 example

$$J(\vec{w},b) = \frac{1}{m}\sum_{i=1}^{m} L(f_{\vec{w},b}(\vec{x}^{(i)}), y^{(i)})$$

③ Train on data to minimize $J(\vec{w},b)$

logistic regression

```
z = np.dot(w,x)+ b

f_x = 1/(1+np.exp(-z))
```

logistic loss

```
loss = -y * np.log(f_x)
   -(1-y) * np.log(1-f_x)
```

```
w = w - alpha * dj_dw
b = b - alpha * dj_db
```

neural network

```
model = Sequential([
        Dense(...)
        Dense(...)
        Dense(...)
                    ])
```

binary cross entropy

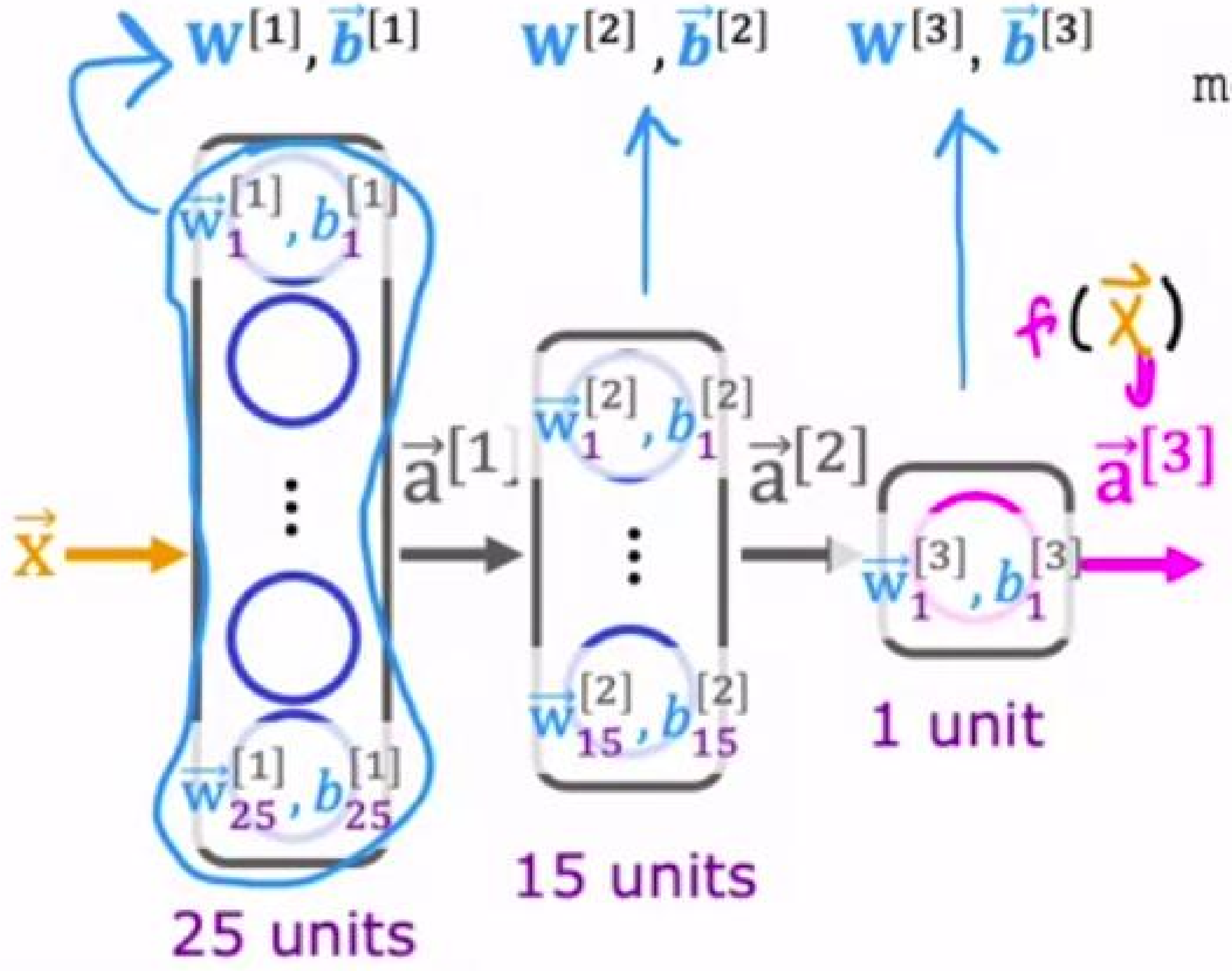
```
model.compile(
loss=BinaryCrossentropy())
```

```
model.fit(X,y,epochs=100)
```

1. Create the model

define the model

$f(\vec{x}) = ?$



```
import tensorflow as tf
from tensorflow.keras import Sequential
from tensorflow.keras.layers import Dense

model = Sequential([
  Dense(units=25, activation='sigmoid’),
  Dense(units=15, activation='sigmoid’),
  Dense(units=1, activation='sigmoid’),
                    ])
```

2. Loss and cost functions

handwritten digit
classification problem

binary classification

$$J(\mathbf{W},\mathbf{B}) = \frac{1}{m}\sum_{i=1}^{m} L\left(f\left(\vec{\mathrm{x}}^{(i)}\right), y^{(i)}\right)$$

$\mathbf{W}^{[1]}, \mathbf{W}^{[2]}, \mathbf{W}^{[3]}$ $\vec{b}^{[1]}, \vec{b}^{[2]}, \vec{b}^{[3]}$

$f_{W,B}(\vec{x})$

$$L(f(\vec{\mathrm{x}}), y) = -y\log\left(f(\vec{\mathrm{x}})\right) - (1-y)\log\left(1 - f(\vec{\mathrm{x}})\right)$$

compare prediction vs. target

logistic loss

also known as binary cross entropy

```
model.compile(loss= BinaryCrossentropy())
```

```
from tensorflow.keras.losses import
  BinaryCrossentropy
```

K Keras

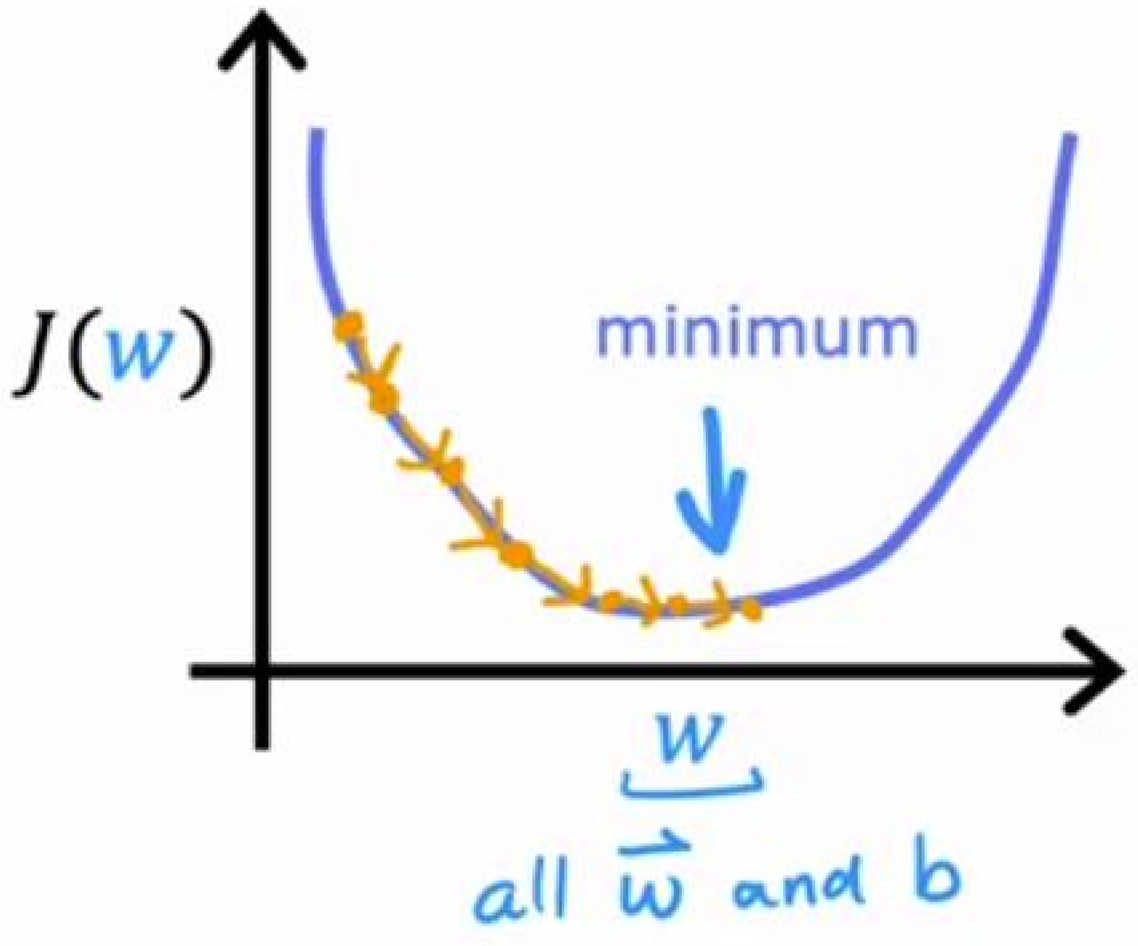
regression
(predicting numbers
and not categories)

mean squared error

```
from tensorflow.keras.losses import
  MeanSquaredError
```

```
model.compile(loss= MeanSquaredError())
```

3. Gradient descent



```
repeat {
    $w_j^{[l]} = w_j^{[l]} - \alpha \frac{\partial}{\partial w_j} J(\vec{\mathrm{w}}, b)$
    $b_j^{[l]} = b_j^{[l]} - \alpha \frac{\partial}{\partial bj} J(\vec{\mathrm{w}}, b)$
}
```

Compute derivatives
for gradient descent
using "backpropagation"

```
model.fit(X,y,epochs=100)
```

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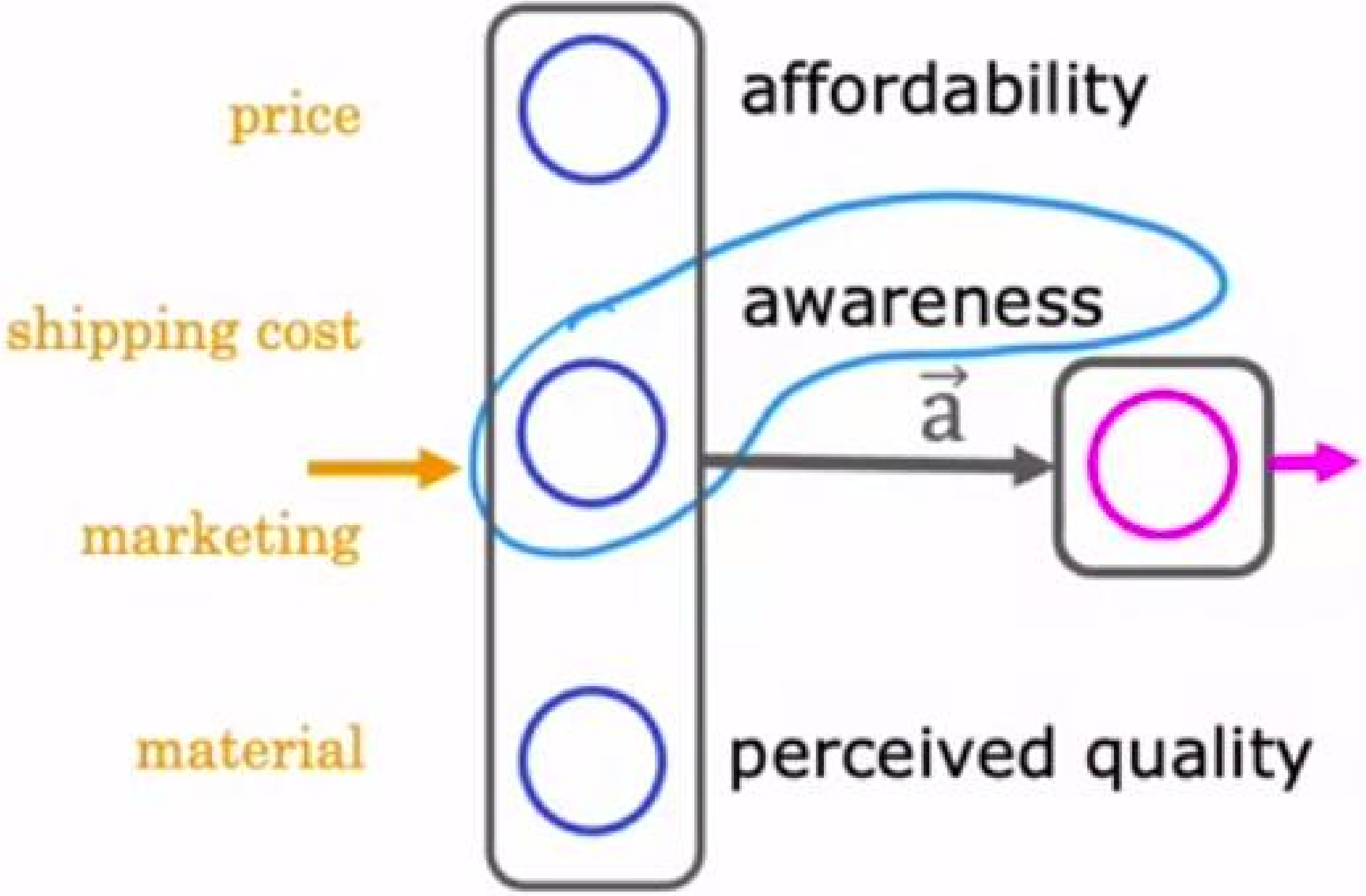
Activation Functions

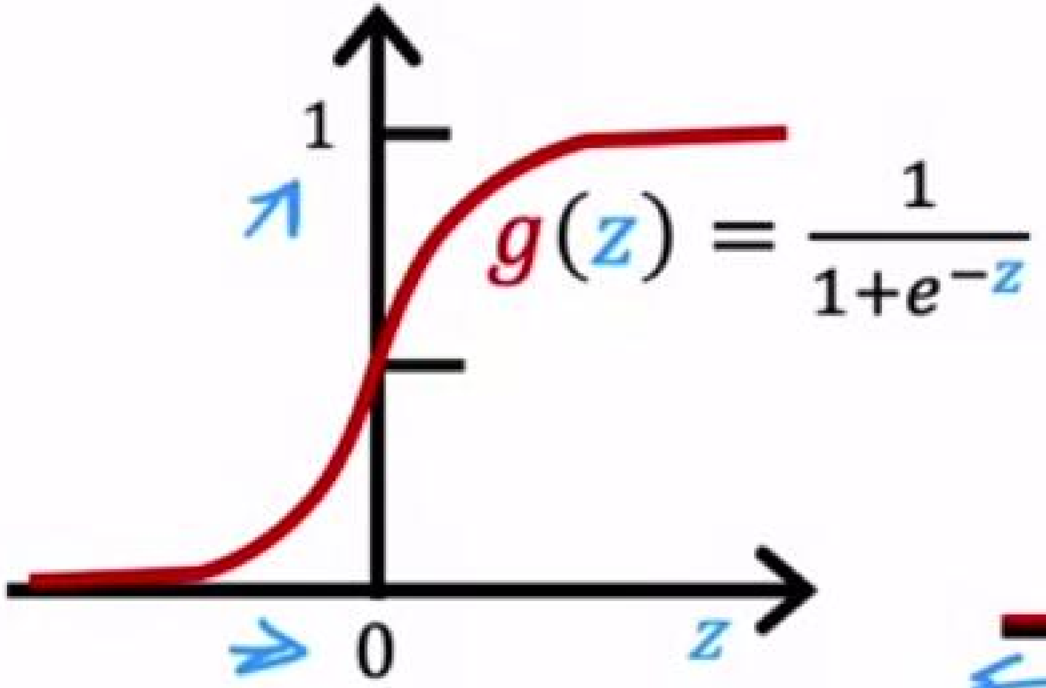
Alternatives to the sigmoid activation

Demand Prediction Example



$$a_2^{[1]} = g(\overbrace{\vec{w}_2^{[1]} \cdot \vec{x} + b_2^{[1]}}^{z})$$

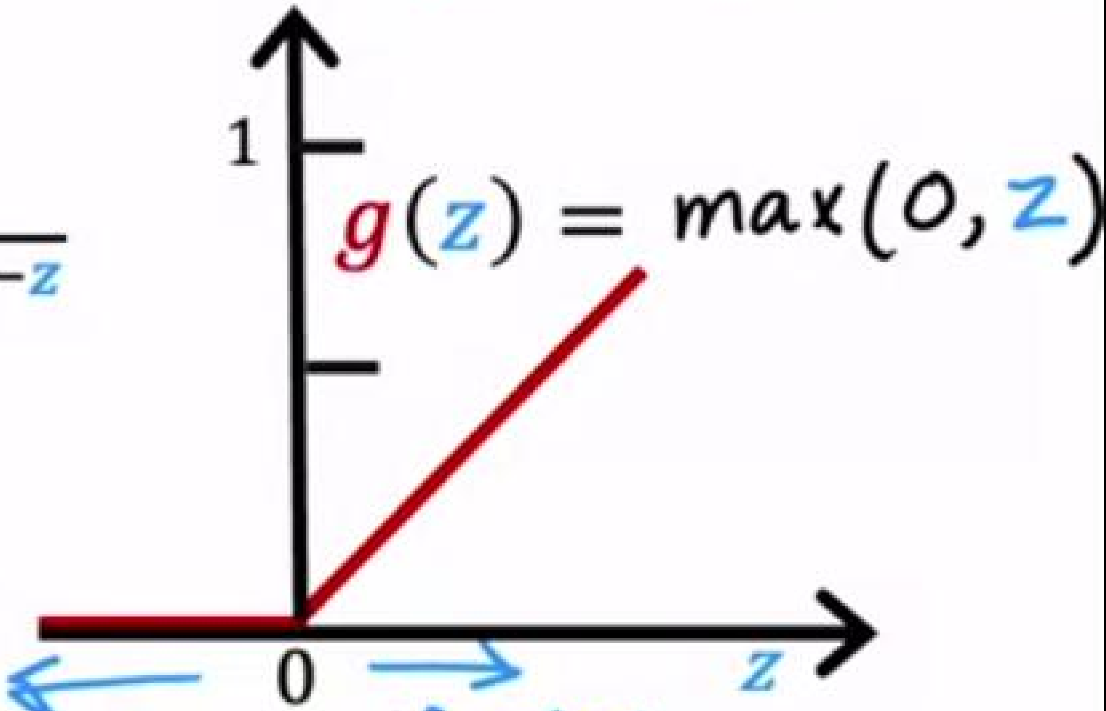
Sigmoid



$$g(z) = \frac{1}{1+e^{-z}}$$

$$0 < g(z) < 1$$

ReLU Rectified Linear Unit



$$g(z) = max(0, z)$$

if $z < 0$
$g(z)$ is 0

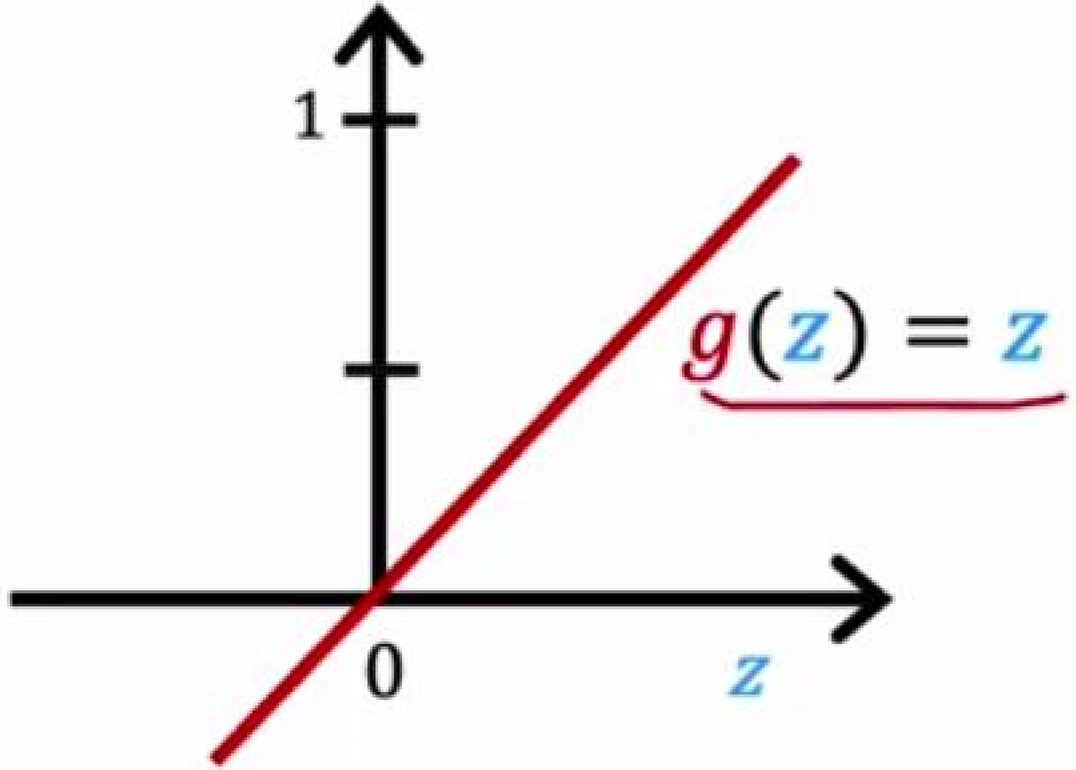
if $z \geq 0$
$g(z)$ is z

Examples of Activation Functions

$$a_2^{[1]} = g(\overbrace{\vec{w}_2^{[1]} \cdot \vec{x} + b_2^{[1]}}^{z})$$

"No activation function"

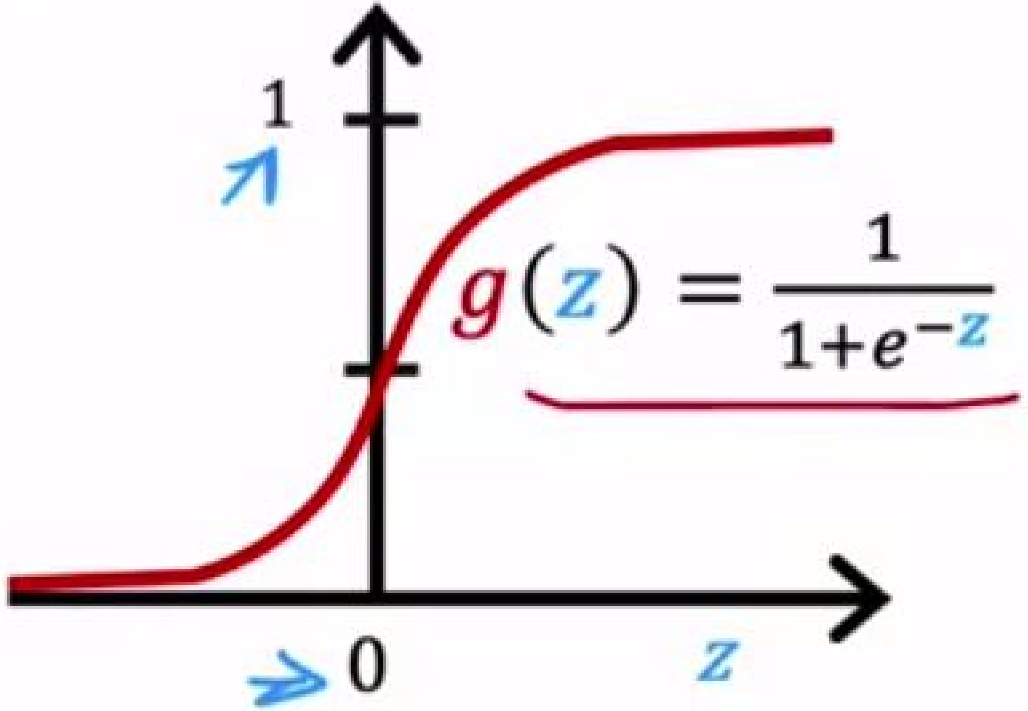
Linear activation function



$g(z) = z$

$a = g(z) = \underbrace{\vec{w} \cdot \vec{x} + b}_{z}$

Sigmoid

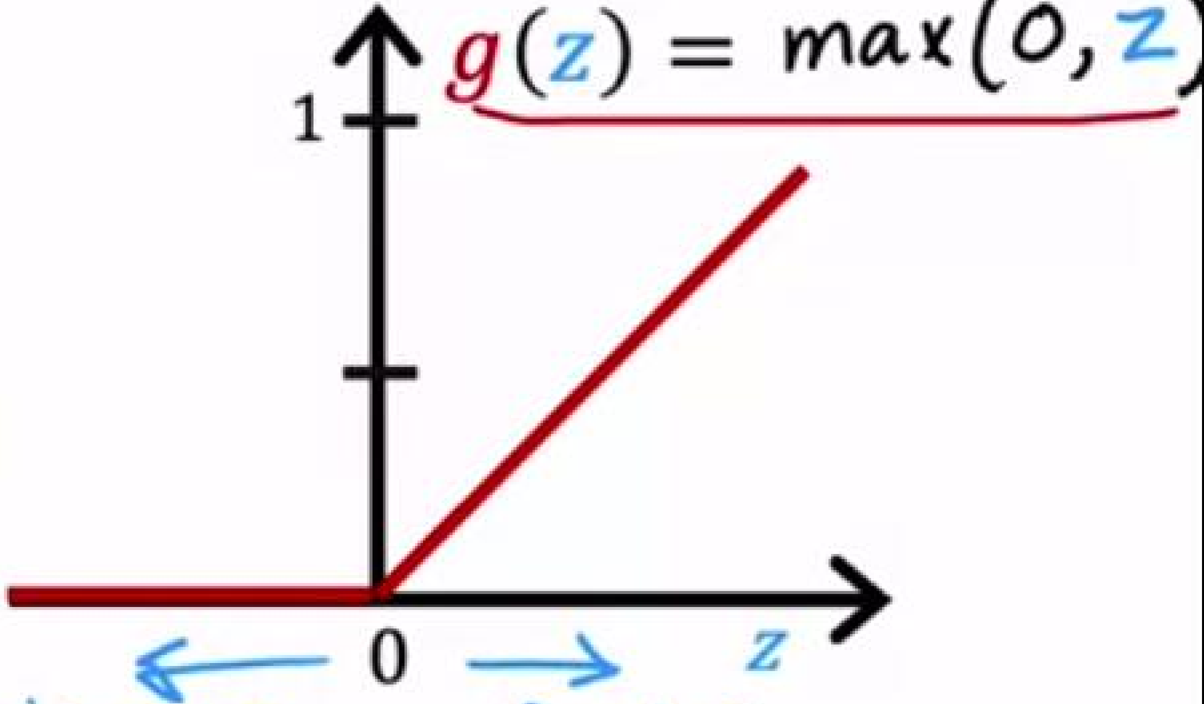


$g(z) = \frac{1}{1+e^{-z}}$

$0 < g(z) < 1$

Later: softmax activation

ReLU Rectified Linear Unit



$g(z) = \max(0, z)$

if $z < 0$ $g(z)$ is 0

if $z \geq 0$ $g(z)$ is z

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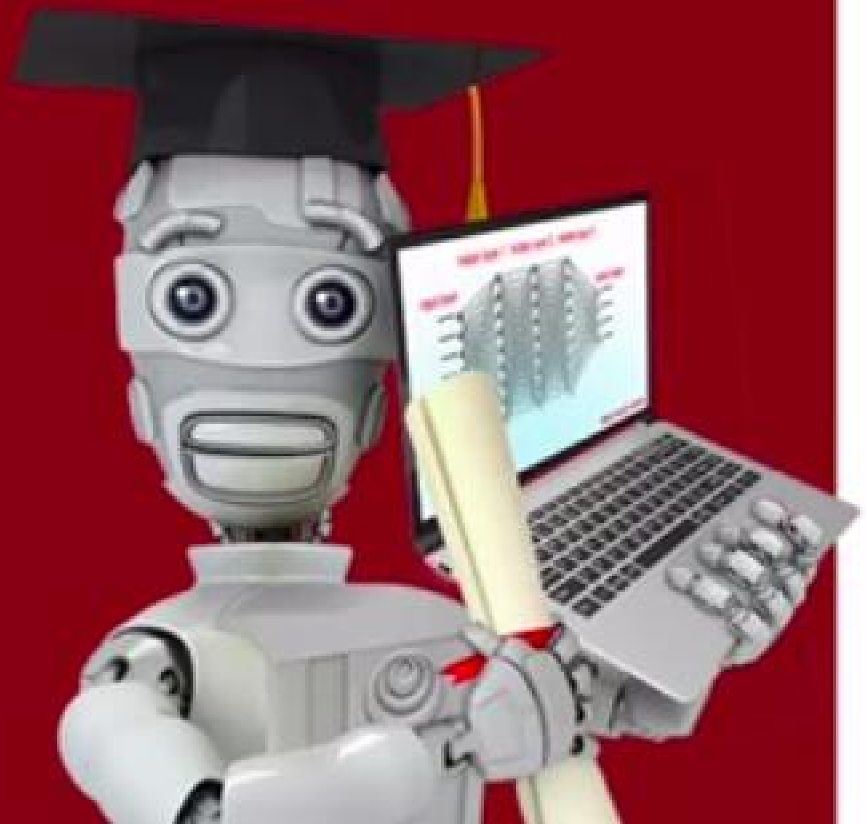
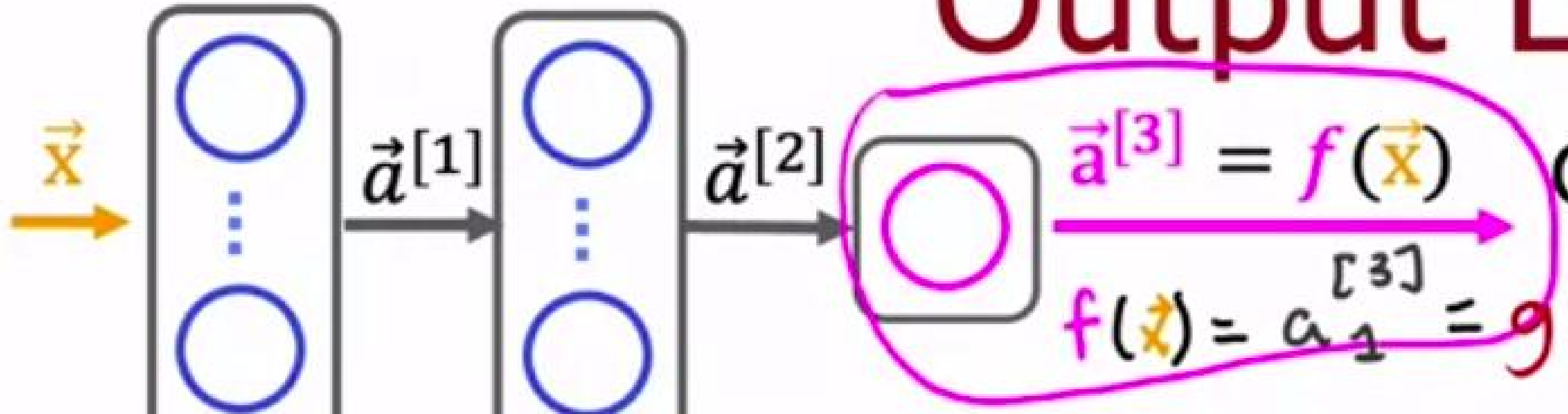


Activation Functions

Choosing activation functions

Output Layer



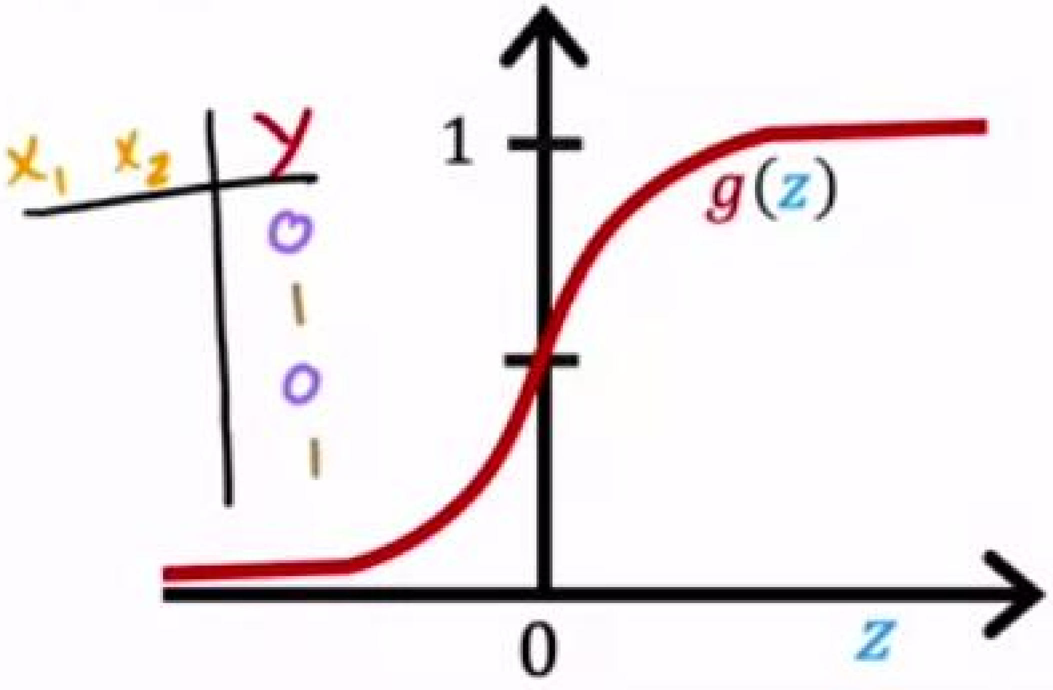
Choosing $g(z)$ for output layer?

$f(\vec{x}) = a_1^{[3]} = g(z_1^{[3]})$

Binary classification

Sigmoid

y=0/1

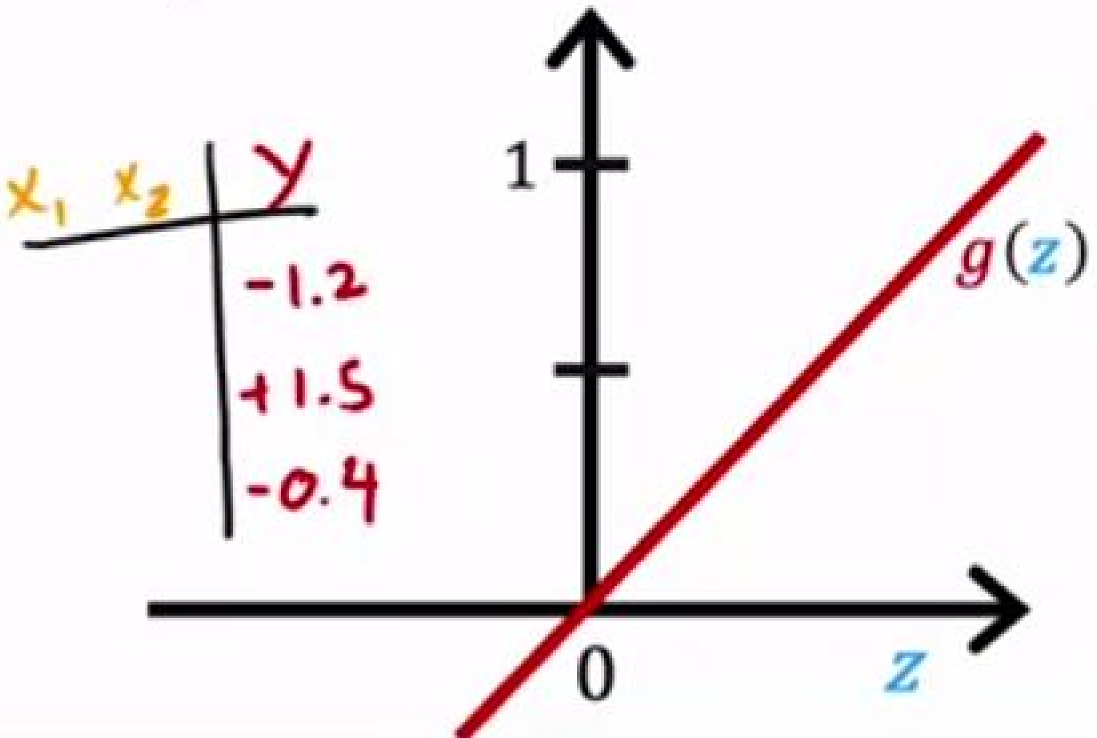


x_1	x_2	y
		0
		1
		0
		1

Regression

Linear activation function

y = +/-

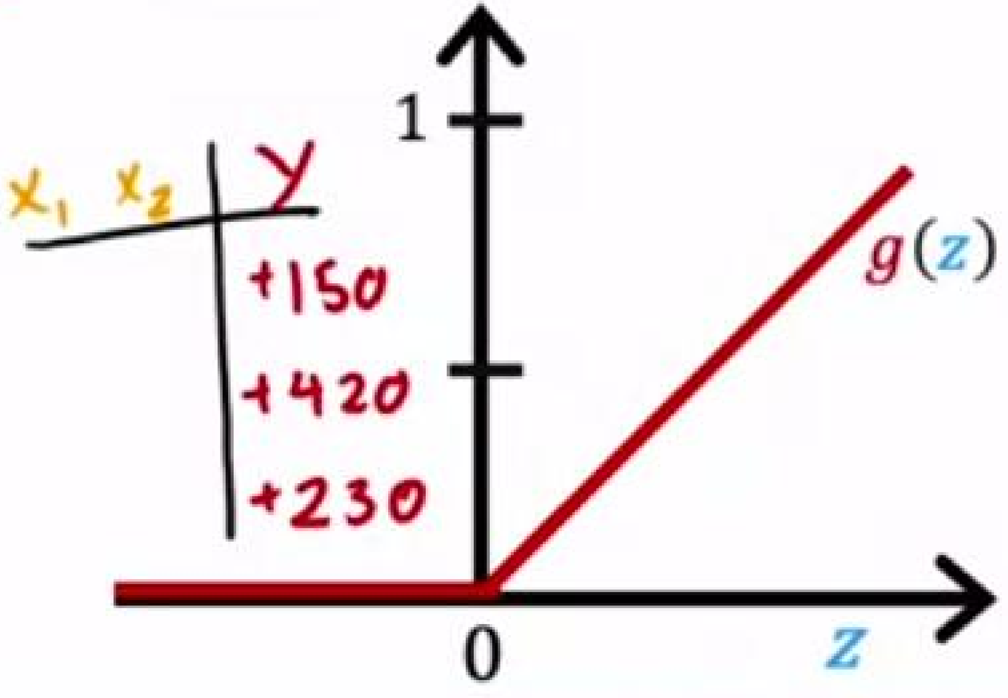


x_1	x_2	y
		-1.2
		+1.5
		-0.4

Regression

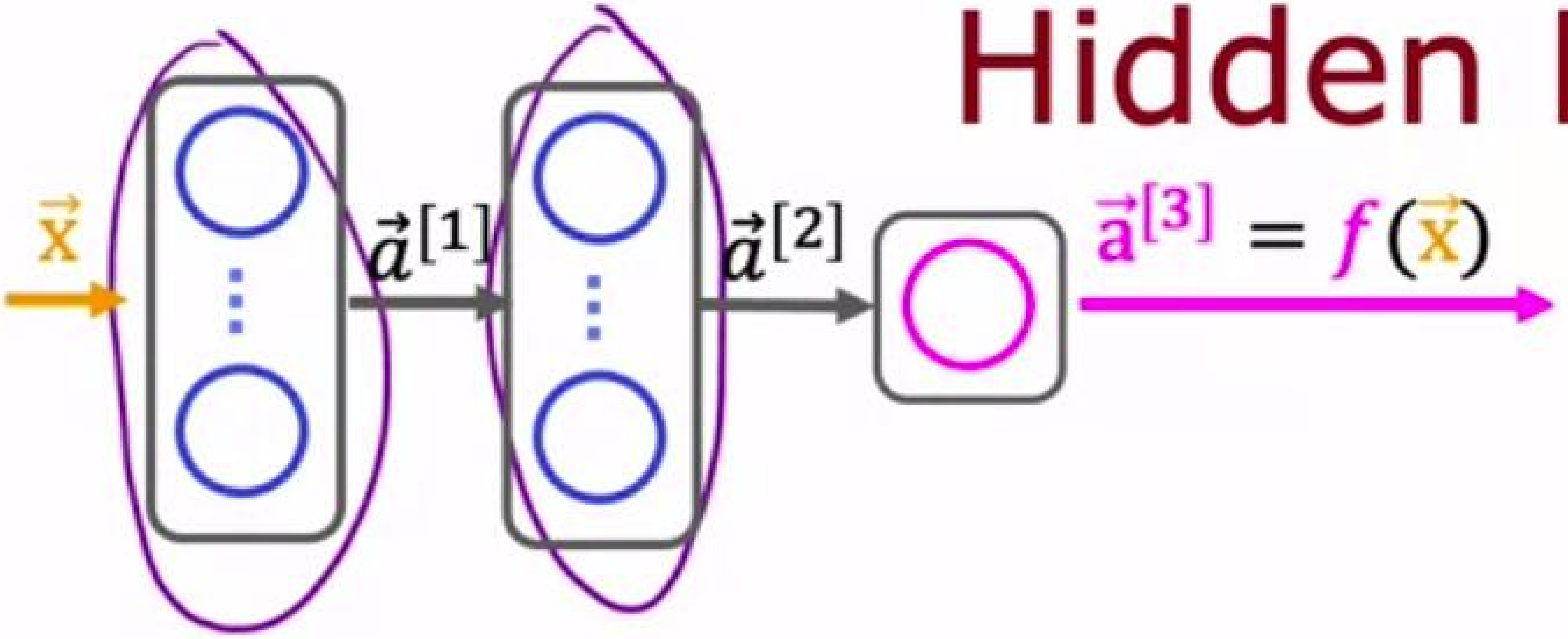
ReLU

y = 0 or +



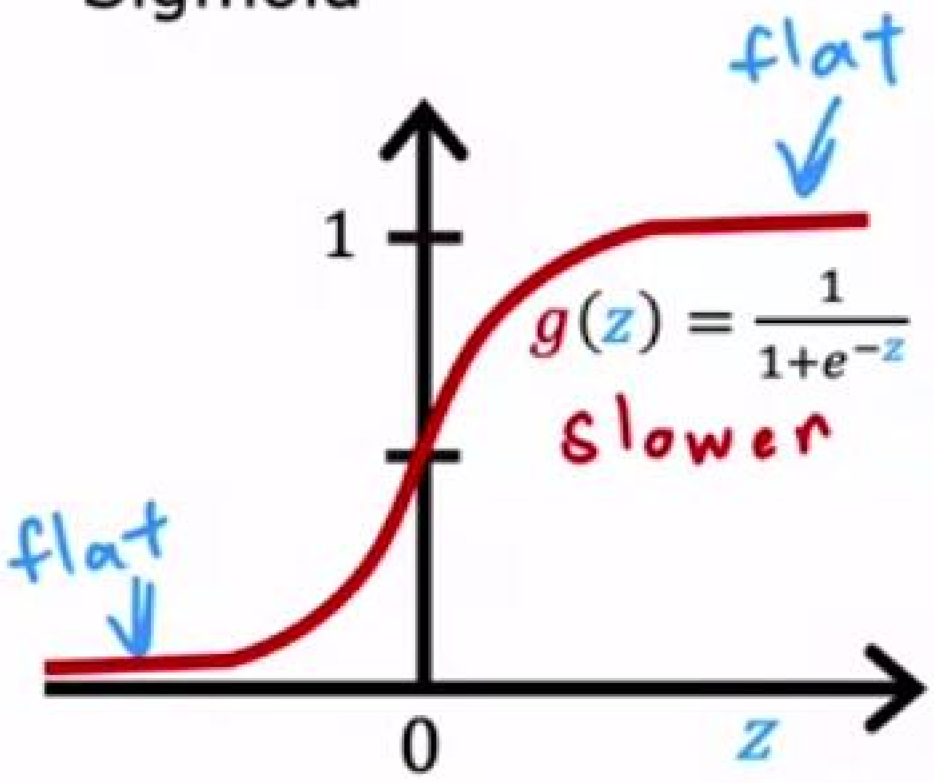
x_1	x_2	y
		+150
		+420
		+230

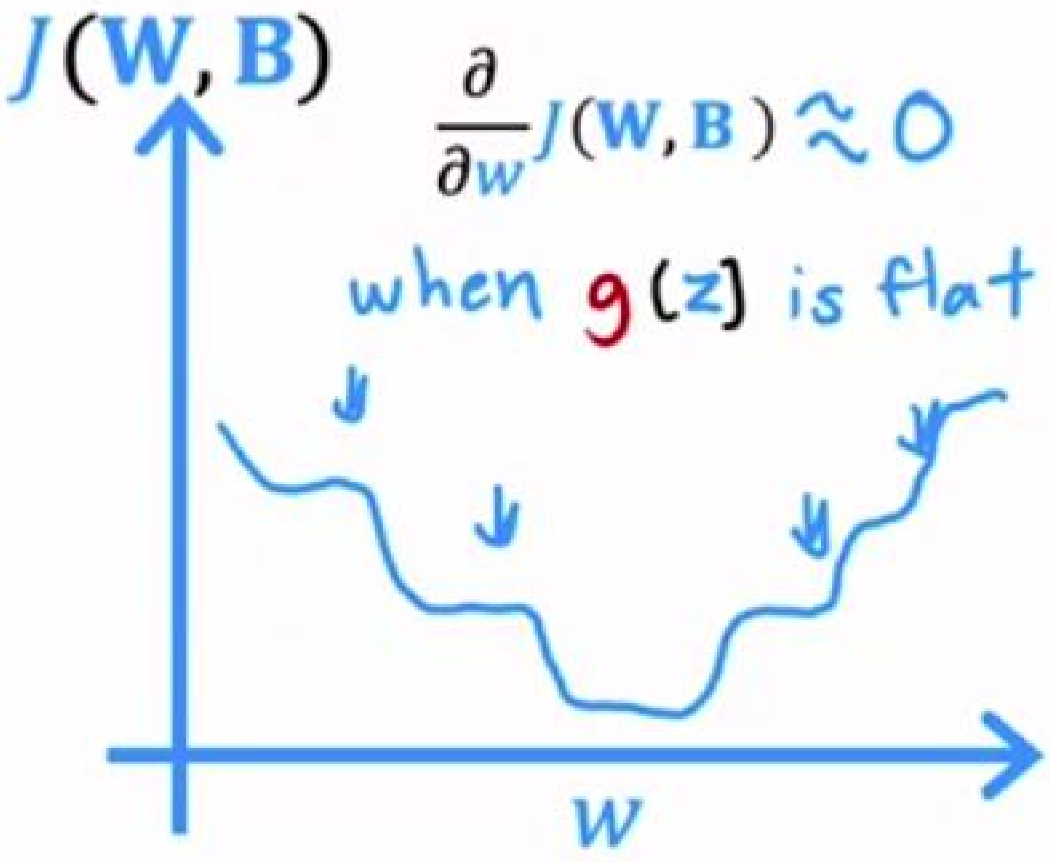
Hidden Layer



Choosing $g(z)$ for hidden layer

Sigmoid

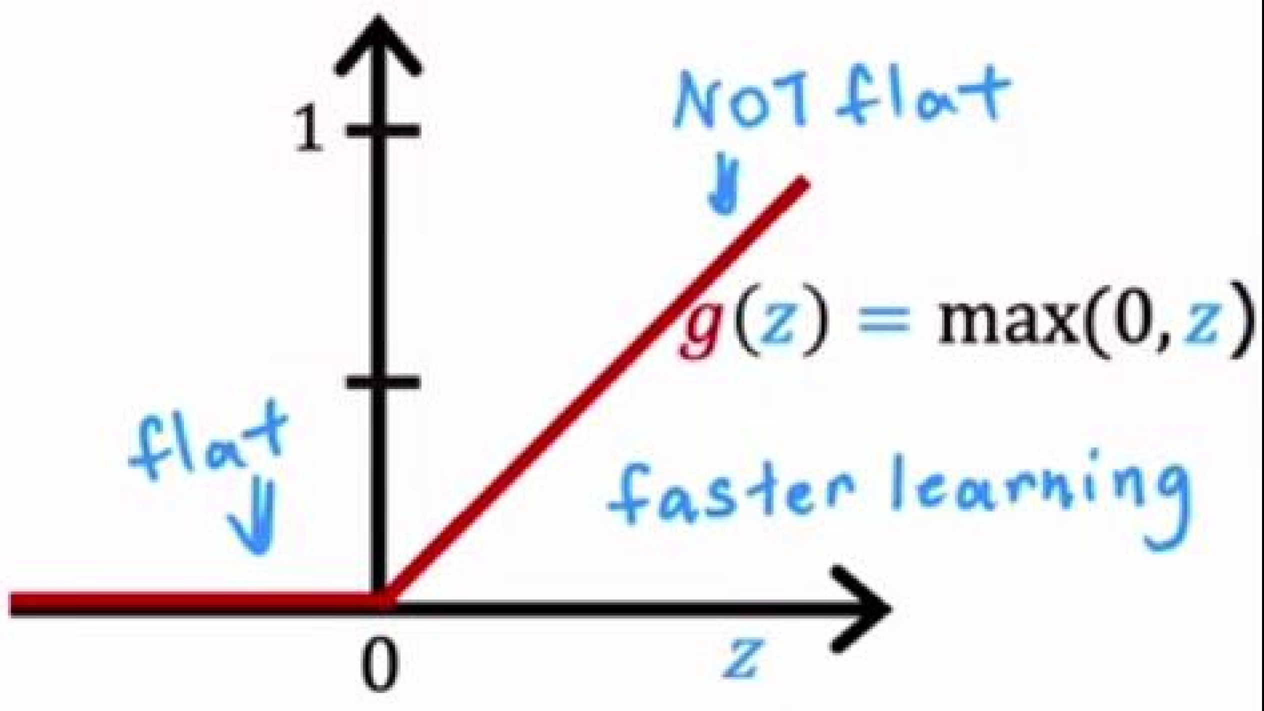




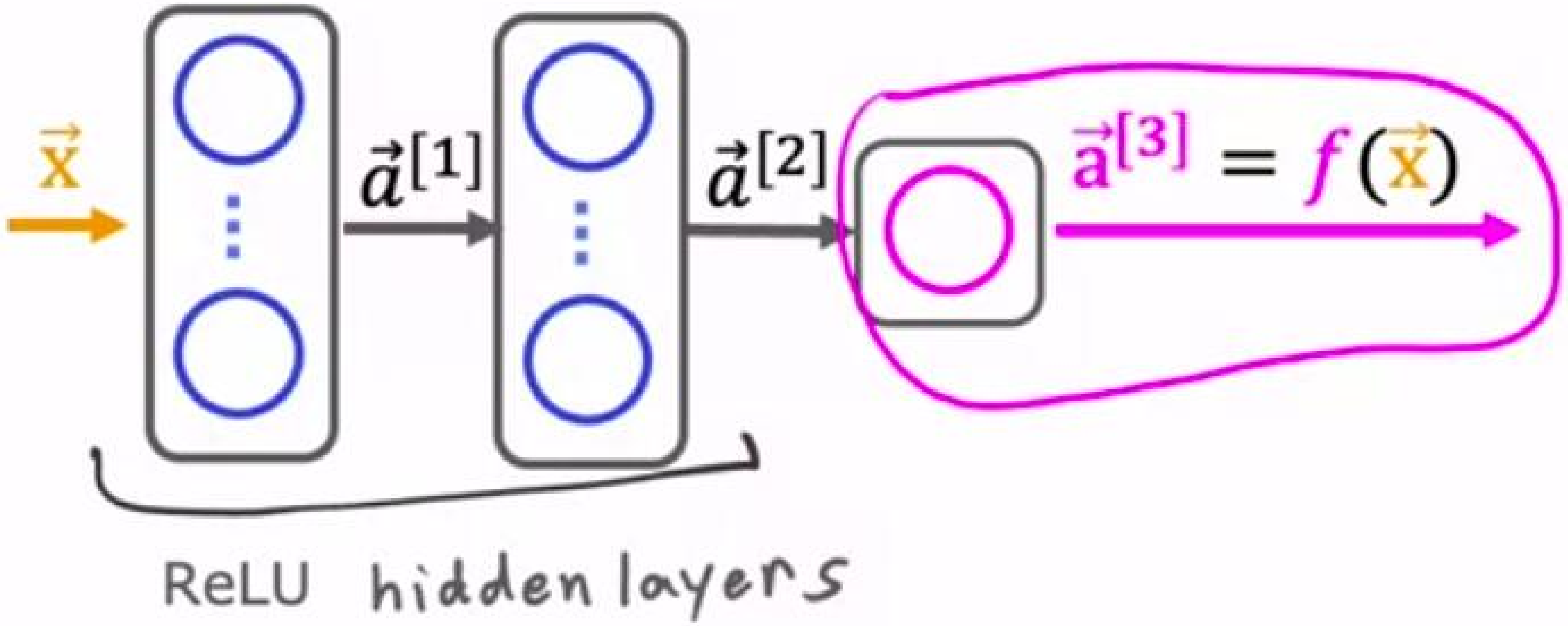
most common choice

ReLU

faster



Choosing Activation Summary



binary classification

activation='sigmoid'

regression y negative/positive

activation='linear'

regression $y \geq 0$

activation='relu'

```
from tf.keras.layers import Dense
model = Sequential([
  Dense(units=25, activation='relu'),   layer1
  Dense(units=15, activation='relu'),   layer2
  Dense(units=1,  activation='sigmoid') layer3
])
```

or 'linear'

or 'relu

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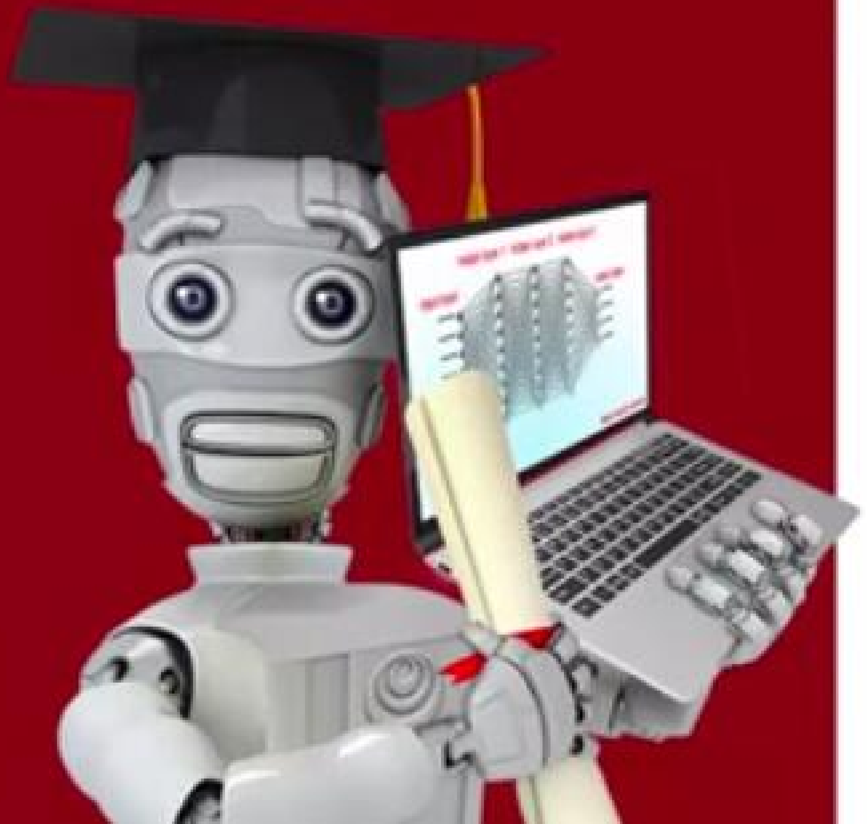
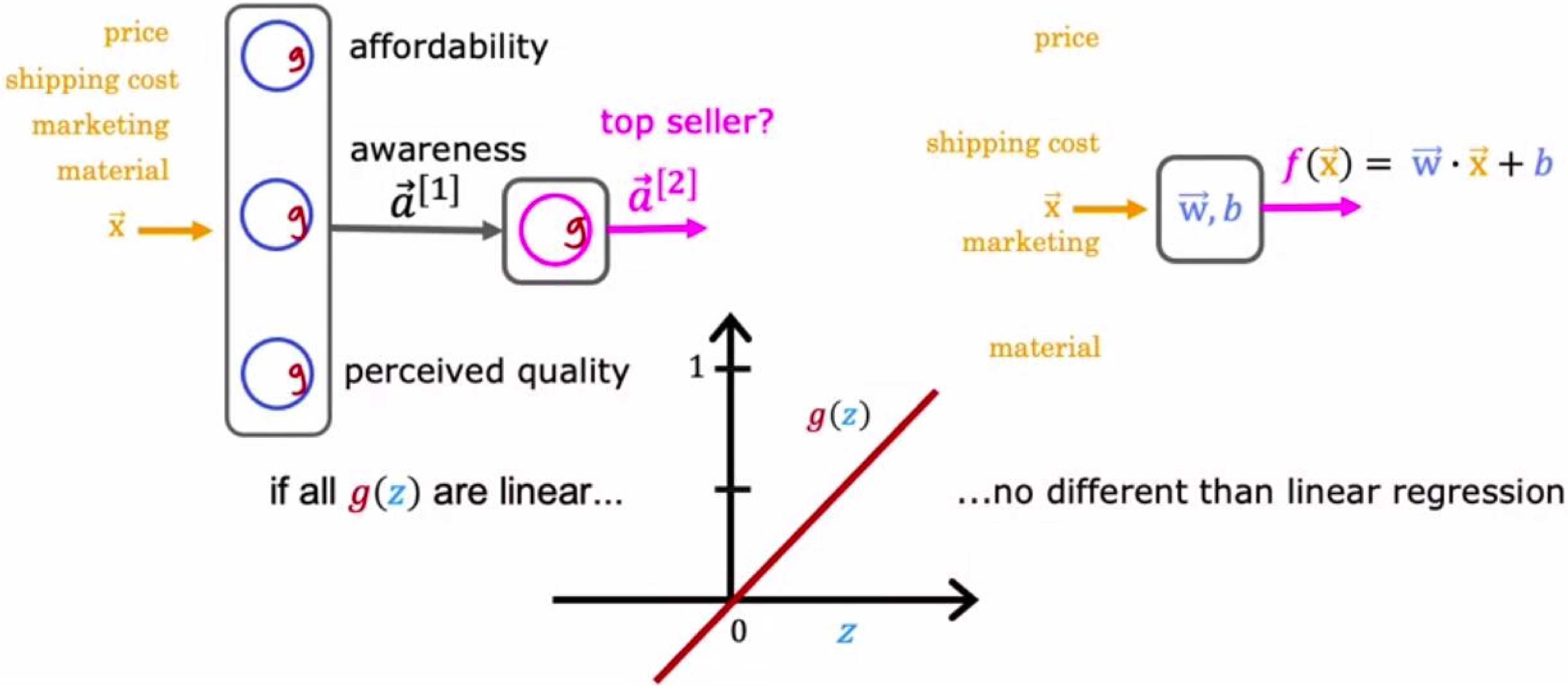


Activation Functions

Why do we need activation functions?

Why do we need activation functions?



price
shipping cost
marketing
material
$\vec{x}$

affordability
awareness
perceived quality

$\vec{a}^{[1]}$

top seller?
$\vec{a}^{[2]}$

if all $g(z)$ are linear...

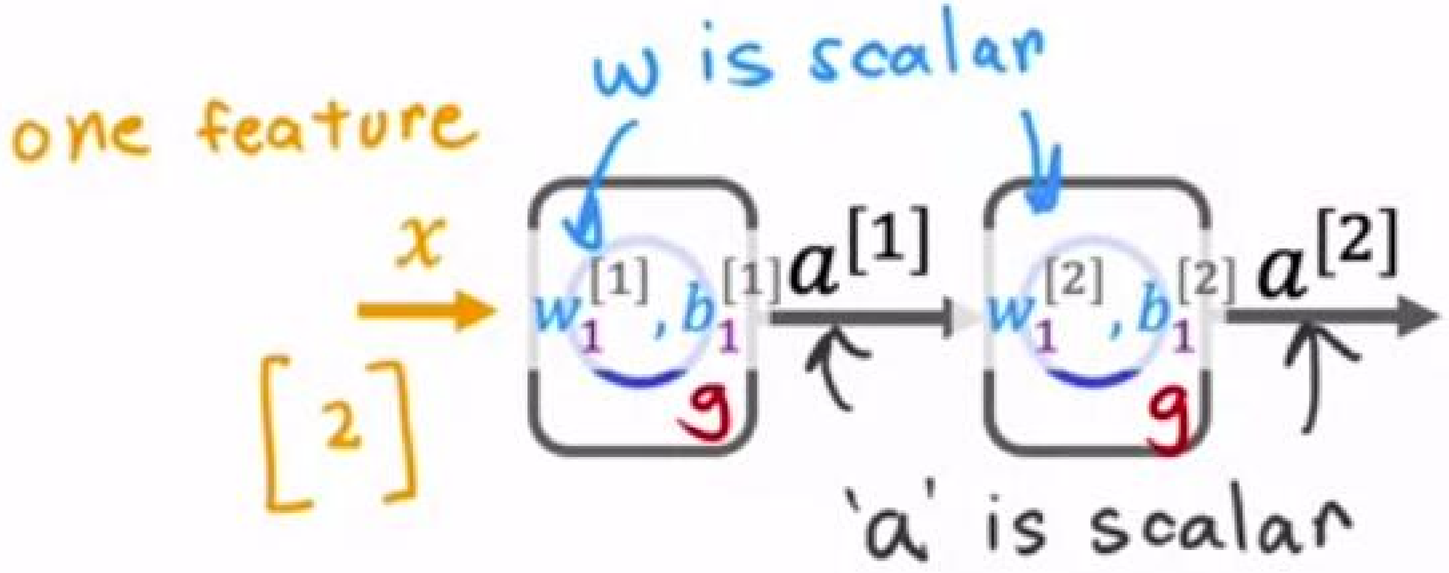
price
shipping cost
$\vec{x}$
marketing
material

$\vec{w}, b$

$f(\vec{x}) = \vec{w} \cdot \vec{x} + b$

...no different than linear regression

Linear Example



$$g(z) = z$$

$$a^{[1]} = \underbrace{w_1^{[1]} x + b_1^{[1]}}_{\downarrow}$$

$$a^{[2]} = w_1^{[2]} a^{[1]} + b_1^{[2]}$$

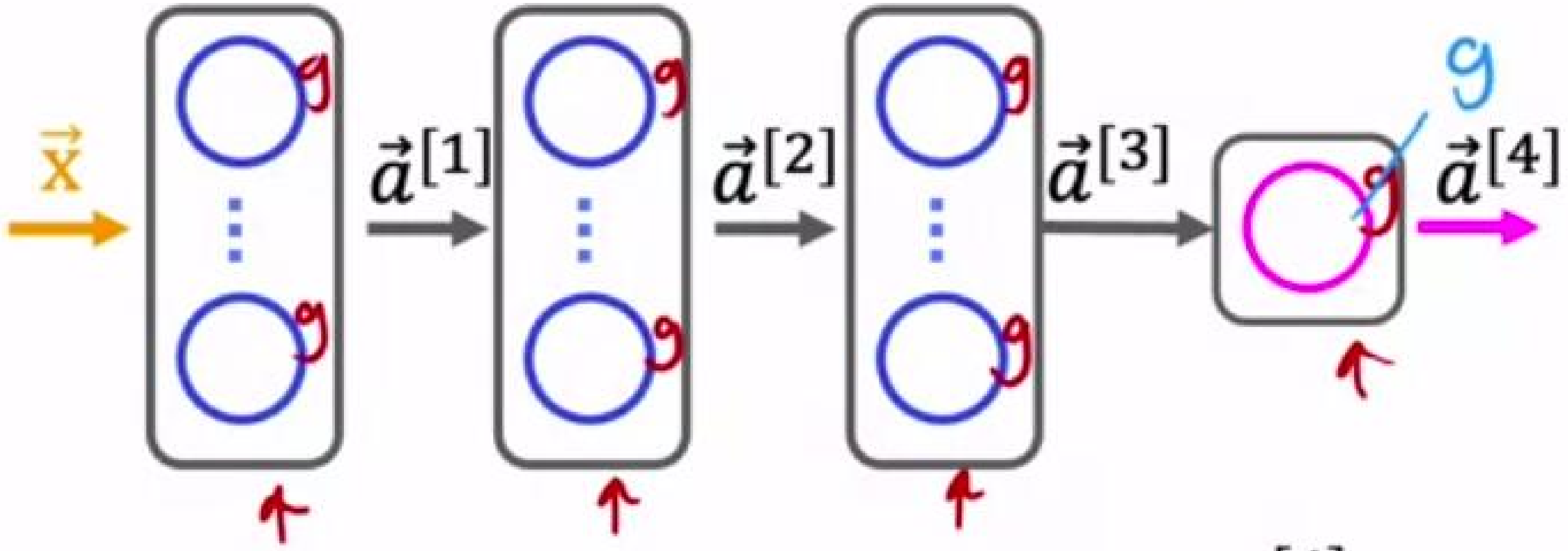
$$= w_1^{[2]} (w_1^{[1]} x + b_1^{[1]}) + b_1^{[2]}$$

$$\vec{a}^{[2]} = \underbrace{(\vec{w}_1^{[2]} \vec{w}_1^{[1]})}_{w} x + \underbrace{w_1^{[2]} b_1^{[1]} + b_1^{[2]}}_{b}$$

$$\vec{a}^{[2]} = w x + b$$

$f(x) = wx + b$ linear regression

Example



$g(z) = z$

$$\vec{a}^{[4]} = \vec{w}_1^{[4]} \cdot \vec{a}^{[3]} + b_1^{[4]}$$

all linear (including output)
↳ equivalent to linear regression

$$\vec{a}^{[4]} = \frac{1}{1+e^{-(\vec{w}_1^{[4]} \cdot \vec{a}^{[3]} + b_1^{[4]})}}$$

output activation is sigmoid
(hidden layers still linear)
↳ equivalent to logistic regression

Don't use linear activations in hidden layers

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Multiclass Classification

Multiclass

MNIST example

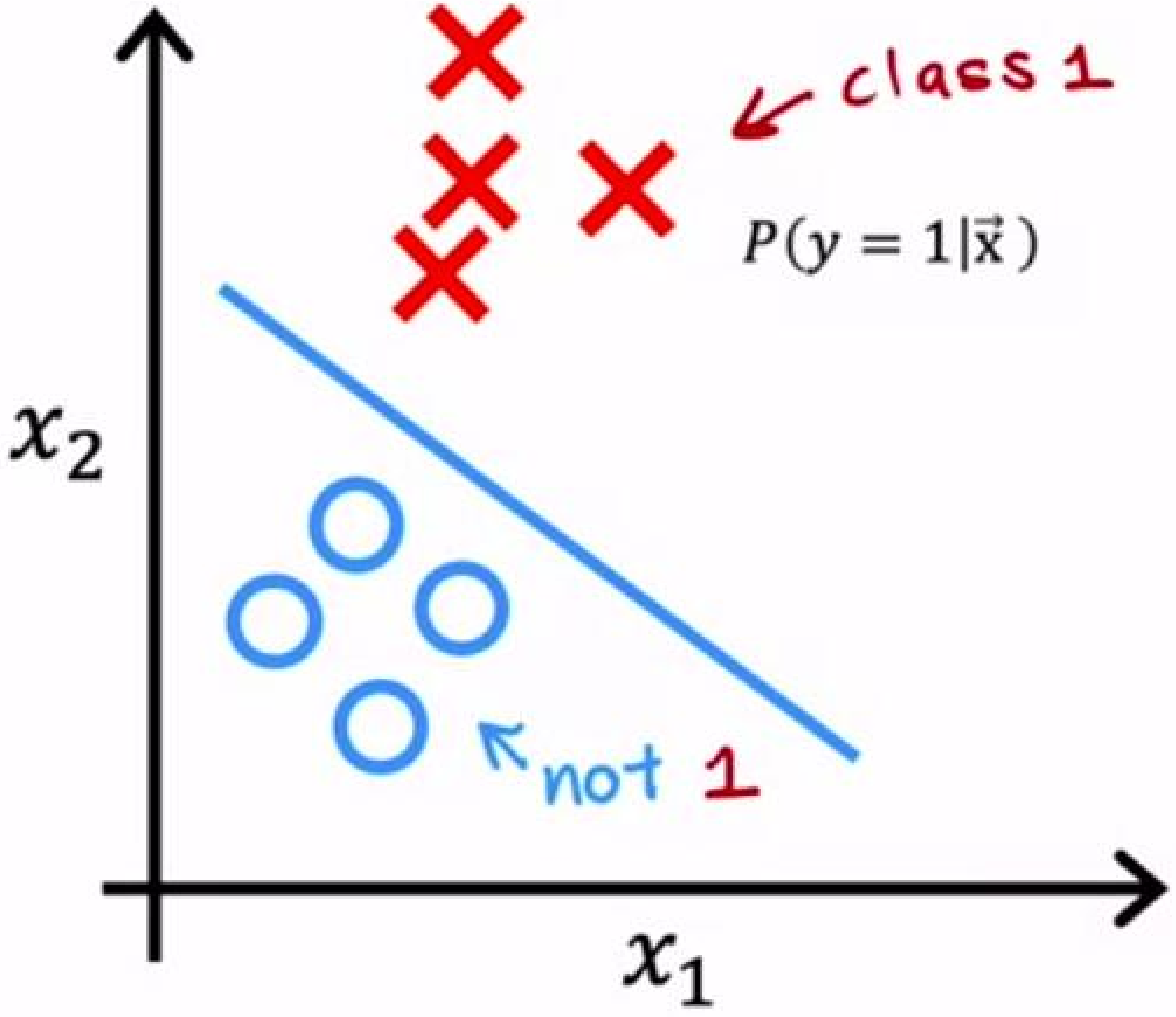
$y = 0$ 1 2 3 4 5 6 7 8 9

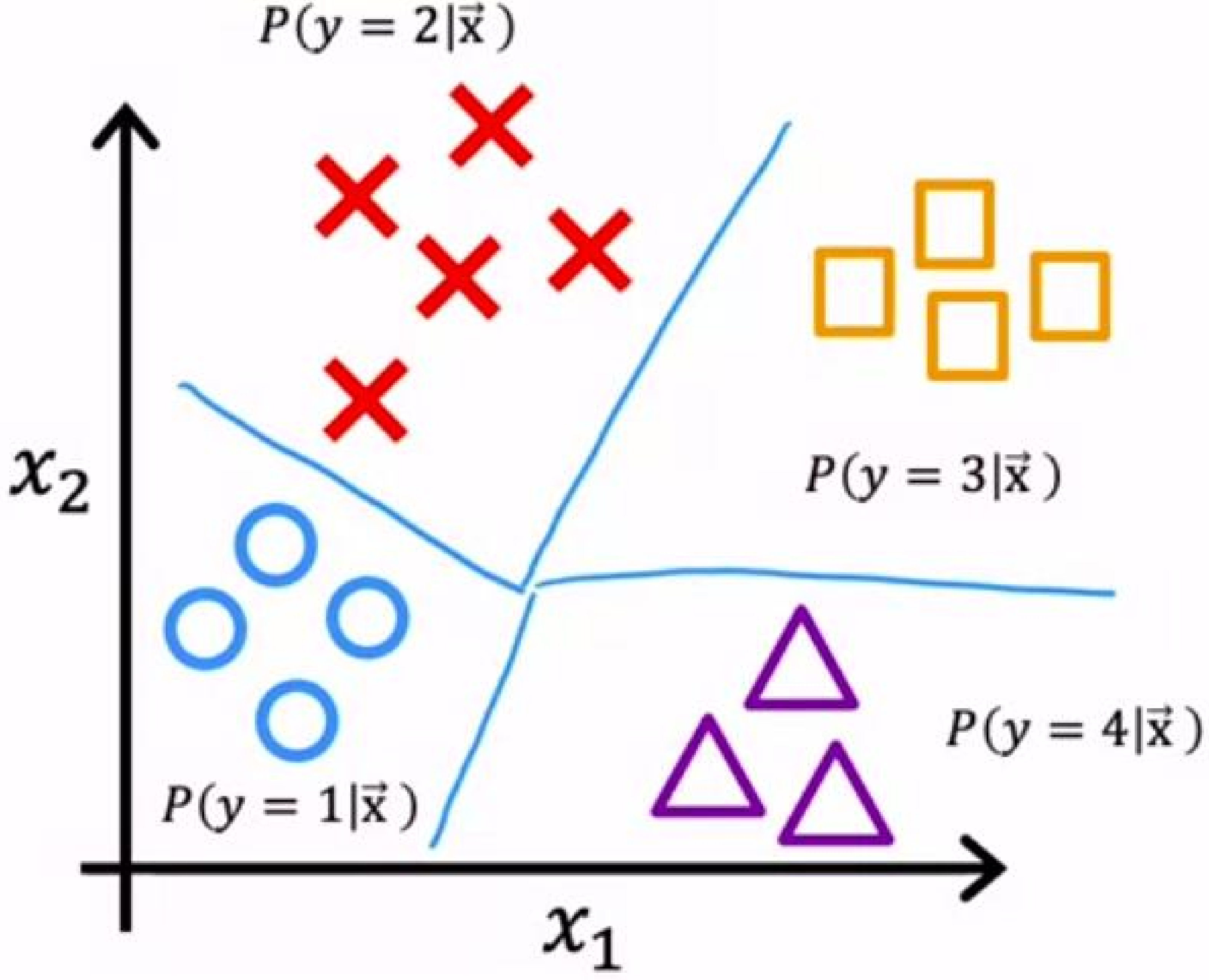
$y = 7$

multiclass classification problem:

target y can take on more than two possible values

Multiclass classification example





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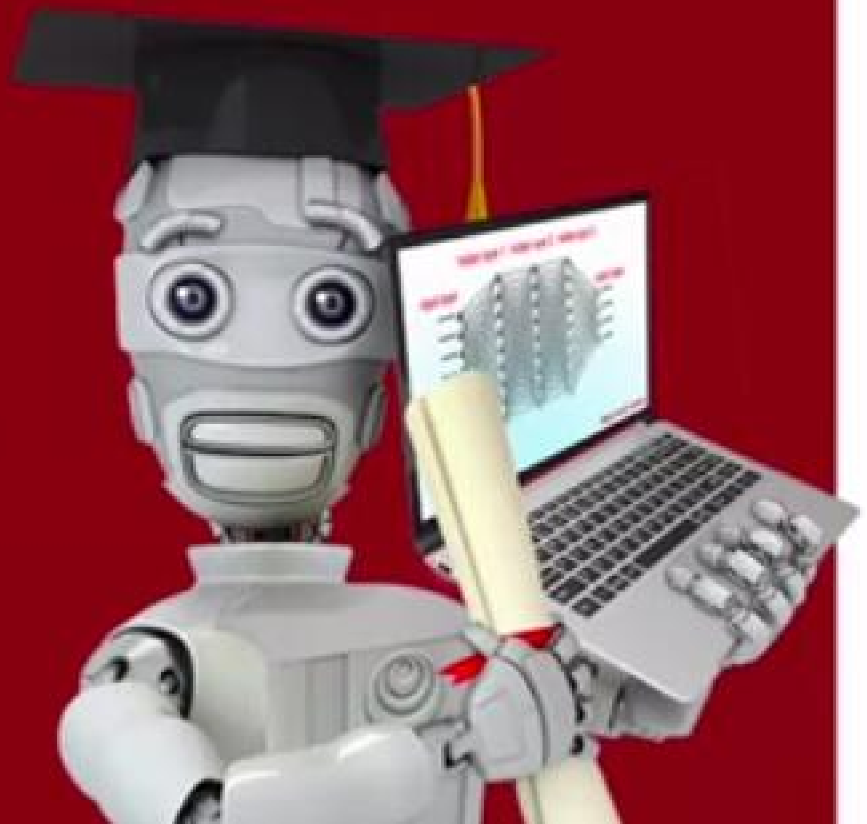


Multiclass Classification

Softmax

Logistic regression (2 possible output values)

$z = \vec{w} \cdot \vec{x} + b$

0.71

× $a_1 = g(z) = \frac{1}{1+e^{-z}} = P(y = 1|\vec{x})$

○ $a_2 = 1 - a_1 = P(y = 0|\vec{x})$

0.29

Softmax regression (N possible outputs) $y = 1, 2, 3, \ldots, N$

$z_j = \vec{w}_j \cdot \vec{x} + b_j \quad j = 1, \ldots, N$

parameters $w_1, w_2, \ldots, w_N$
$b_1, b_2, \ldots, b_N$

$$a_j = \frac{e^{z_j}}{\sum_{k=1}^{N} e^{z_k}} = P(y = j|\vec{x})$$

note: $a_1 + a_2 + \ldots + a_N = 1$

Softmax regression (4 possible outputs) $y = 1, 2, 3, 4$

× $z_1 = \vec{w}_1 \cdot \vec{x} + b_1$

$$a_1 = \frac{e^{z_1}}{e^{z_1} + e^{z_2} + e^{z_3} + e^{z_4}}$$

× ○ □ △

$= P(y = 1|\vec{x})$ 0.30

○ $z_2 = \vec{w}_2 \cdot \vec{x} + b_2$

$$a_2 = \frac{e^{z_2}}{e^{z_1} + e^{z_2} + e^{z_3} + e^{z_4}}$$

$= P(y = 2|\vec{x})$ 0.20

□ $z_3 = \vec{w}_3 \cdot \vec{x} + b_3$

$$a_3 = \frac{e^{z_3}}{e^{z_1} + e^{z_2} + e^{z_3} + e^{z_4}}$$

$= P(y = 3|\vec{x})$ 0.15

△ $z_4 = \vec{w}_4 \cdot \vec{x} + b_4$

$$a_4 = \frac{e^{z_4}}{e^{z_1} + e^{z_2} + e^{z_3} + e^{z_4}}$$

$= P(y = 4|\vec{x})$ 0.35

Cost

Logistic regression

$$z = \vec{\mathrm{w}} \cdot \vec{\mathrm{x}} + b$$

$$a_1 = g(z) = \frac{1}{\underbrace{1 + e^{-z}}} \quad = P(y = 1|\vec{\mathrm{x}})$$

$$a_2 = \underbrace{1 - a_1} \quad = P(y = 0|\vec{\mathrm{x}})$$

$$loss = -y \underbrace{\log a_1}_{\text{if } y=1} - (1 - y) \underbrace{\log\overbrace{(1 - a_1)}^{a_2}}_{\text{if } y=0}$$

$$J(\vec{\mathrm{w}}, b) = \text{average loss}$$

Softmax regression

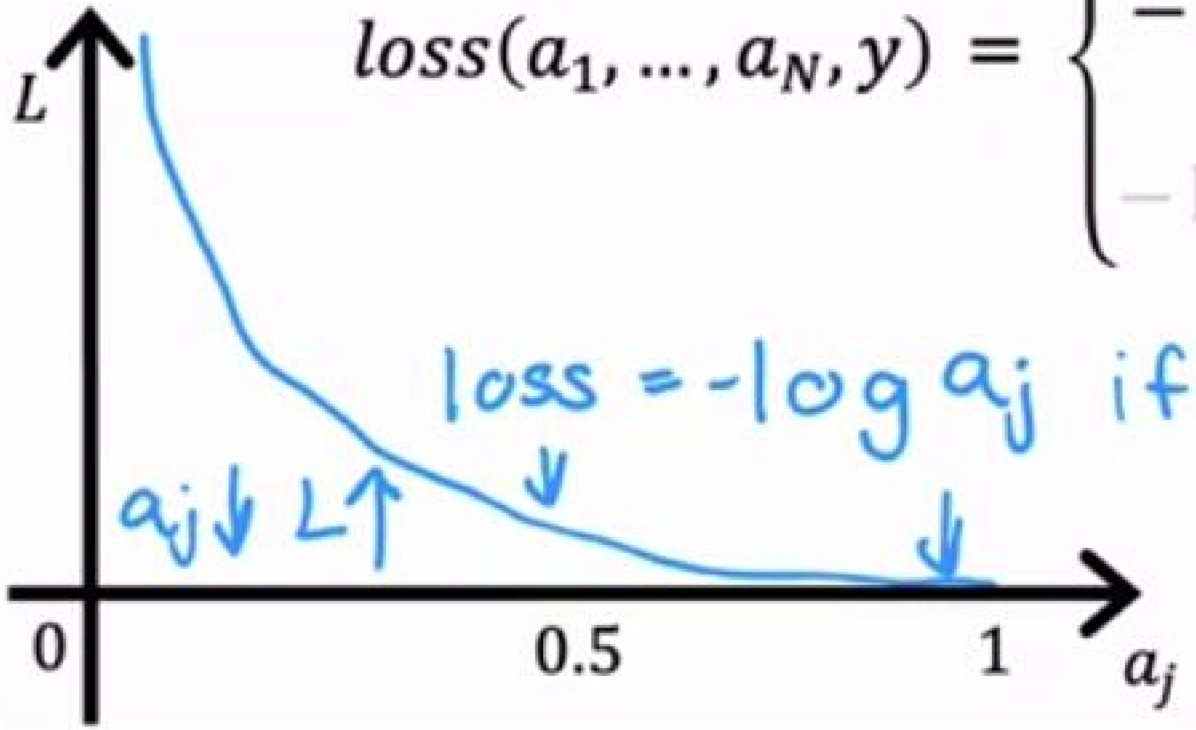
$$a_1 = \frac{e^{z_1}}{e^{z_1} + e^{z_2} + \cdots + e^{z_N}} \quad = P(y = 1|\vec{\mathrm{x}})$$

$$\vdots$$

$$a_N = \frac{e^{z_N}}{e^{z_1} + e^{z_2} + \cdots + e^{z_N}} \quad = P(y = N|\vec{\mathrm{x}})$$

Crossentropy loss

$$loss(a_1, \ldots, a_N, y) = \begin{cases} -\log a_1 & \text{if } y = 1 \\ -\log a_2 & \text{if } y = 2 \\ \vdots & \\ -\log a_N & \text{if } y = N \end{cases}$$



$loss = -\log a_j$ if $y = j$

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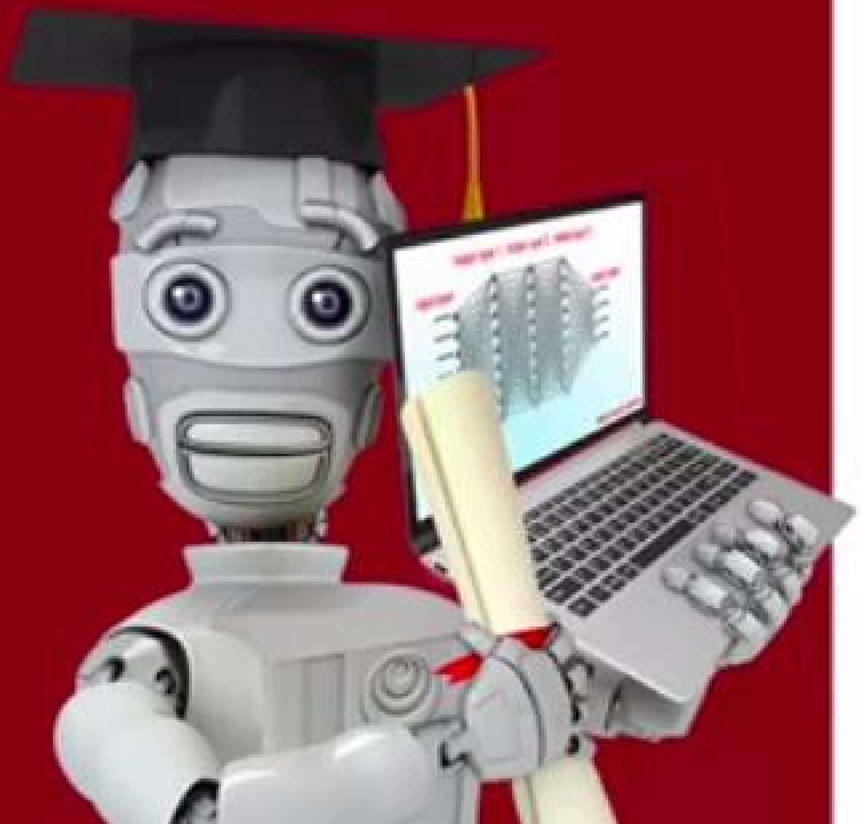
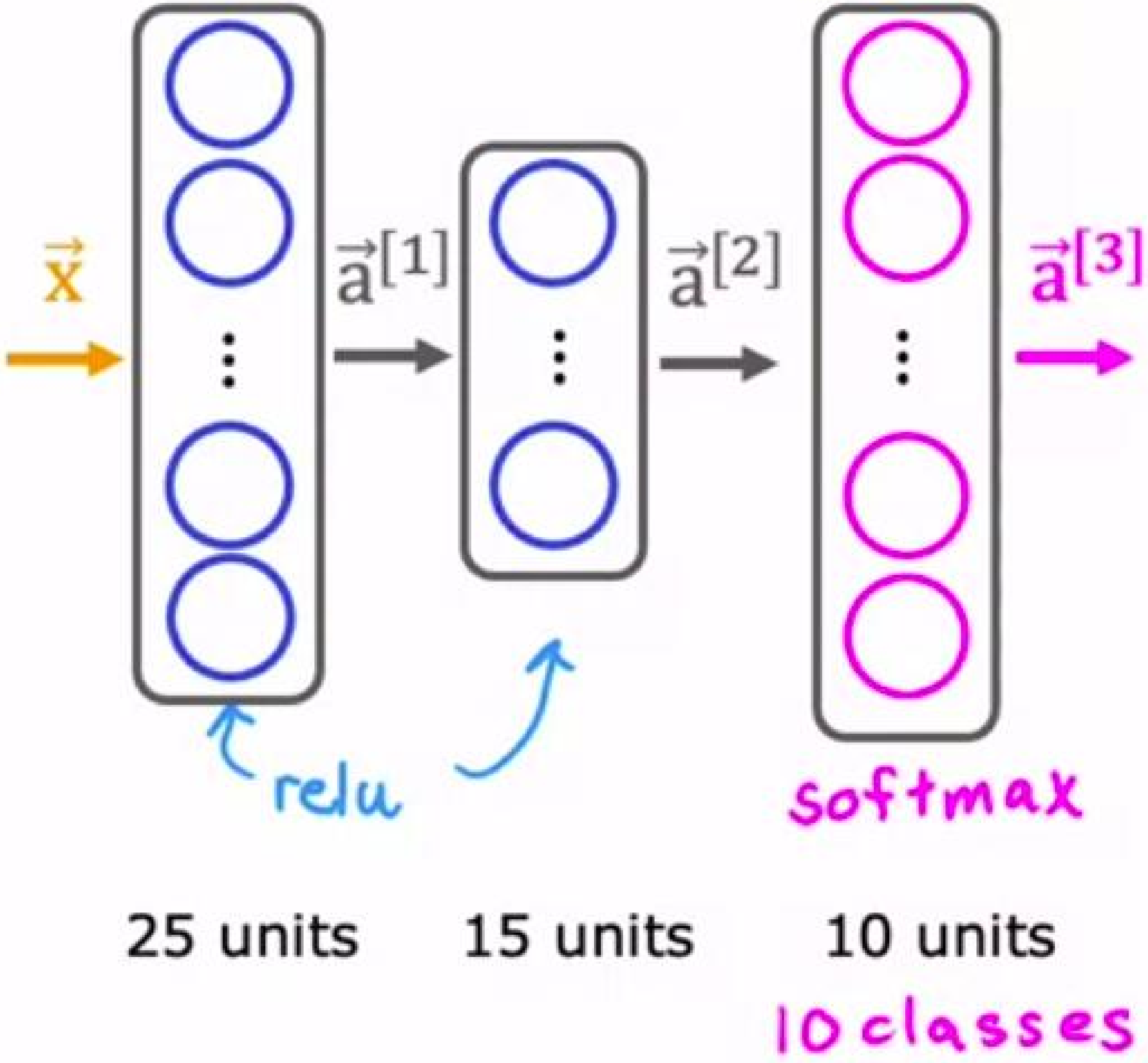


Multiclass Classification

Neural Network with Softmax output

Neural Network with Softmax output



$$z_1^{[3]} = \vec{w}_1^{[3]} \cdot \vec{a}^{[2]} + b_1^{[3]} \qquad a_1^{[3]} = \frac{e^{z_1^{[3]}}}{e^{z_1^{[3]}} + \cdots + e^{z_{10}^{[3]}}}$$

$$= P(y = 1|\vec{x})$$

$$\vdots$$

$$z_{10}^{[3]} = \vec{w}_{10}^{[3]} \cdot \vec{a}^{[2]} + b_{10}^{[3]} \qquad a_{10}^{[3]} = \frac{e^{z_{10}^{[3]}}}{e^{z_1^{[3]}} + \cdots + e^{z_{10}^{[3]}}}$$

$$= P(y = 10|\vec{x})$$

logistic regression

$$a_1^{[3]} = g\left(z_1^{[3]}\right) \qquad a_2^{[3]} = g\left(z_2^{[3]}\right)$$

softmax

$$\vec{a}^{[3]} = \left(a_1^{[3]}, \ldots a_{10}^{[3]}\right) = g\left(z_1^{[3]}, \ldots, z_{10}^{[3]}\right)$$

MNIST with softmax

① specify the model

$f_{\vec{w},b}(\vec{x}) =?$

```
import tensorflow as tf
from tensorflow.keras import Sequential
from tensorflow.keras.layers import Dense
model = Sequential([
  Dense(units=25, activation='relu’),
  Dense(units=15, activation='relu’),
  Dense(units=10, activation='softmax’)
                  ])
```

② specify loss and cost

$L(f_{\vec{w},b}(\vec{x}),y)$

```
from tensorflow.keras.losses import
  SparseCategoricalCrossentropy
model.compile(loss= SparseCategoricalCrossentropy() )
```

③ Train on data to minimize $J(\vec{w},b)$

```
model.fit(X,Y,epochs=100)
```

Note: better (recommended) version later.

Don't use the version shown here!

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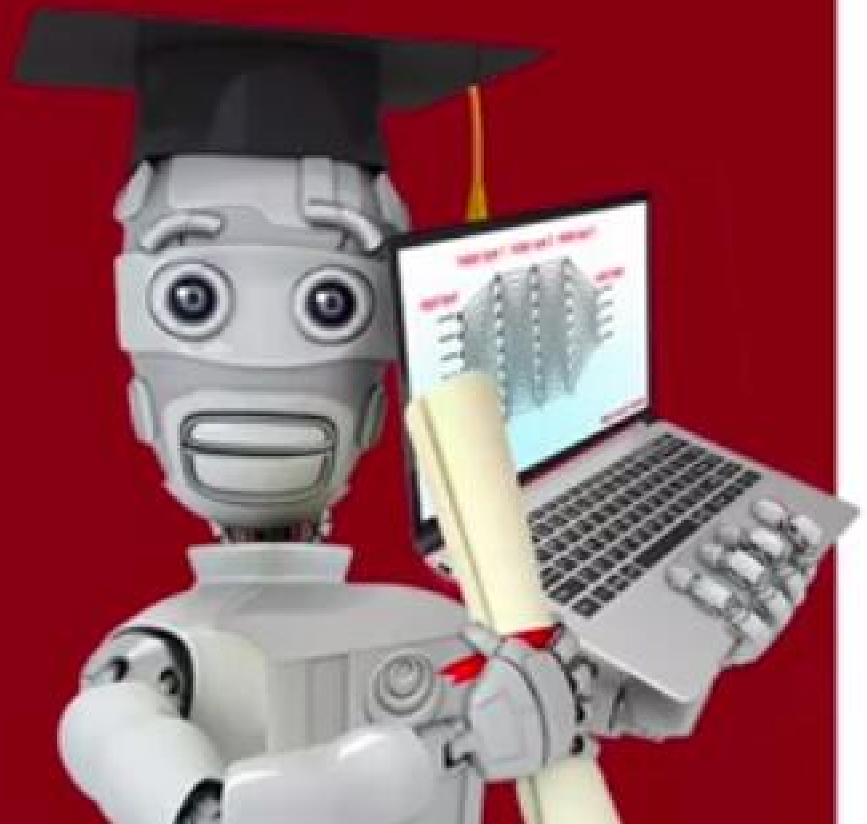


Multiclass Classification

Improved implementation of softmax

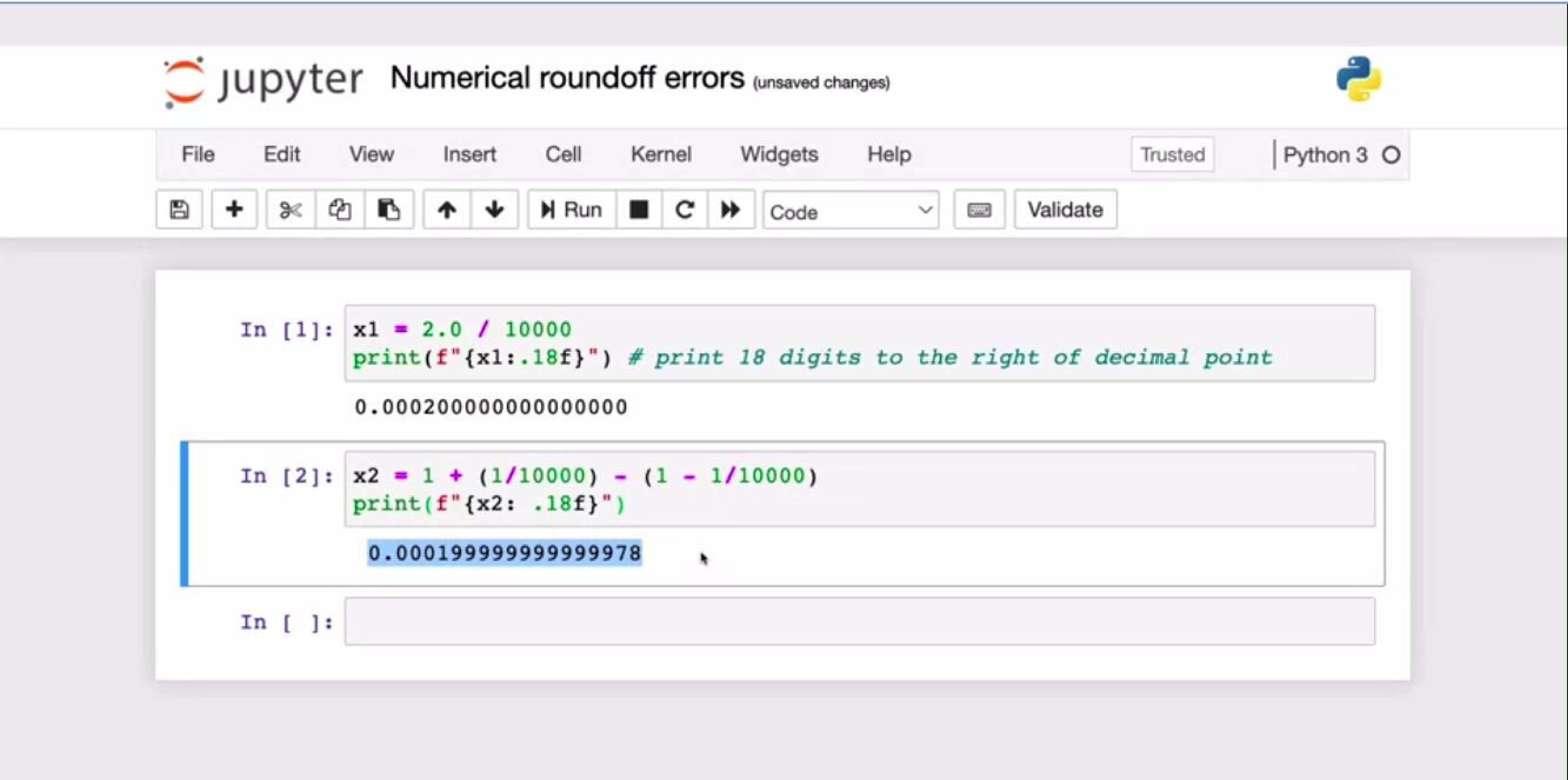
Numerical Roundoff Errors

option 1

$$x = \frac{2}{10,000}$$

option 2

$$x = \underbrace{\left(1 + \frac{1}{10,000}\right)} - \underbrace{\left(1 - \frac{1}{10,000}\right)} =$$



```
In [1]: x1 = 2.0 / 10000
        print(f"{x1:.18f}") # print 18 digits to the right of decimal point

        0.000200000000000000

In [2]: x2 = 1 + (1/10000) - (1 - 1/10000)
        print(f"{x2: .18f}")

         0.000199999999999978

In [ ]:
```

Numerical Roundoff Errors

More numerically accurate implementation of logistic loss:

$1 + \frac{1}{10{,}000}$ $\quad$ $1 - \frac{1}{10{,}000}$

Logistic regression:

$$a = g(z) = \frac{1}{1+e^{-z}}$$

`model = Sequential([`

`    Dense(units=25, activation='relu’),`

`    Dense(units=15, activation='relu’),` 'linear'

`    Dense(units=1, activation='`~~`sigmoid`~~`’)`

`                        ])`

~~`model.compile(loss=BinaryCrossEntropy() )`~~

`model.compile(loss=BinaryCrossEntropy(from_logits=True) )`

Original loss

$$loss = -y\log(a) - (1-y)\log(1-a)$$

More accurate loss (in code)

$$loss = -y\log\left(\frac{1}{1+e^{-z}}\right) - (1-y)\log(1 - \frac{1}{1+e^{-z}})$$

logit: z

More numerically accurate implementation of softmax

Softmax regression

$$(a_{1,}\dots,a_{10}) = g(z_1,\dots,z_{10})$$

$$\text{Loss} = L(\vec{a},y) = \begin{cases} -\log a_1 & \text{if } y = 1 \\ \vdots & \\ -\log a_{10} & \text{if } y = 10 \end{cases}$$

`model = Sequential([`

`  Dense(units=25, activation='relu'),`

`  Dense(units=15, activation='relu'),`

`  Dense(units=10, activation='`~~`softmax`~~`')`

`  ])`

'linear'

~~`model.compile(loss=SparseCategoricalCrossEntropy()`~~` )`

More Accurate

$$L(\vec{a},y) = \begin{cases} -\log \dfrac{e^{z_1}}{e^{z_1}+\dots+e^{z_{10}}} & \text{if } y = 1 \\ \vdots & \\ -\log \dfrac{e^{z_{10}}}{e^{z_1}+\dots+e^{z_{10}}} & \text{if } y = 10 \end{cases}$$

```
model.compile(loss=SparseCategoricalCrossEntropy(from_logits=True))
```

MNIST (more numerically accurate)

model

```
import tensorflow as tf
from tensorflow.keras import Sequential
from tensorflow.keras.layers import Dense
model = Sequential([
  Dense(units=25, activation='relu'),
  Dense(units=15, activation='relu'),
  Dense(units=10, activation='linear') ])
```

loss

```
from tensorflow.keras.losses import
  SparseCategoricalCrossentropy
model.compile(...,loss=SparseCategoricalCrossentropy(from_logits=True) )
```

fit

```
model.fit(X,Y,epochs=100)
```

predict

```
logits = model(X)
f_x = tf.nn.softmax(logits)
```

← not $a_1 \ldots a_{10}$

logistic regression (more numerically accurate)

model

```
model = Sequential([
  Dense(units=25, activation='sigmoid’),
  Dense(units=15, activation='sigmoid’),
  Dense(units=1, activation='linear’)
                  ])
from tensorflow.keras.losses import
  BinaryCrossentropy
```

loss

```
model.compile(..., BinaryCrossentropy(from_logits=True))
```

```
model.fit(X,Y,epochs=100)
```

fit

```
logit = model(X)
```

z

predict

```
f_x = tf.nn.sigmoid(logit)
```

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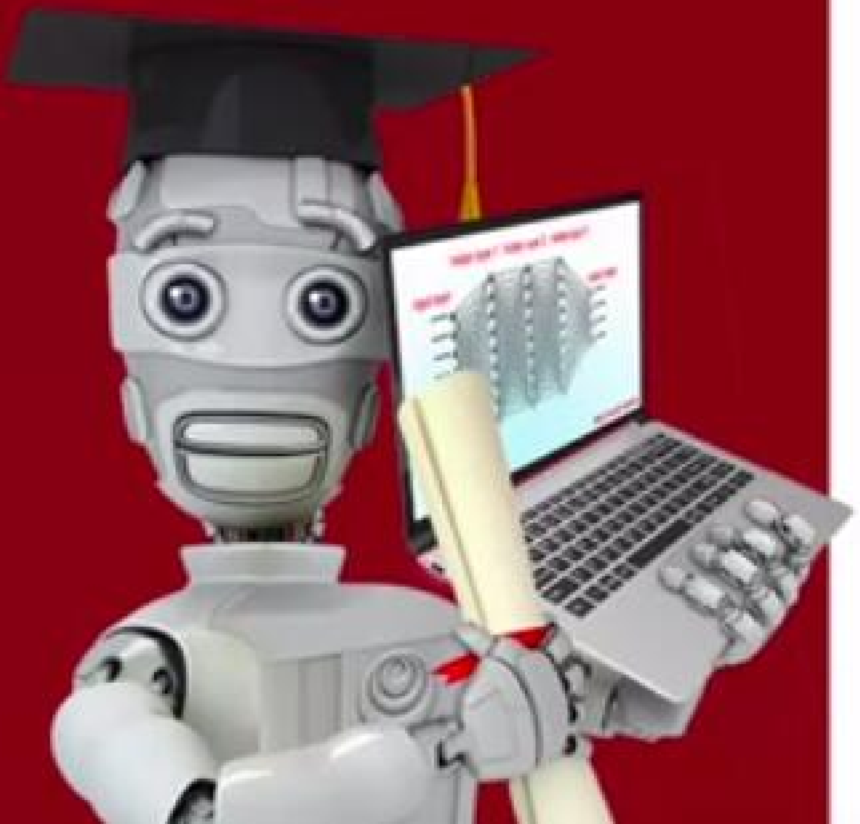


Multi-label Classification

Classification with multiple outputs (Optional)

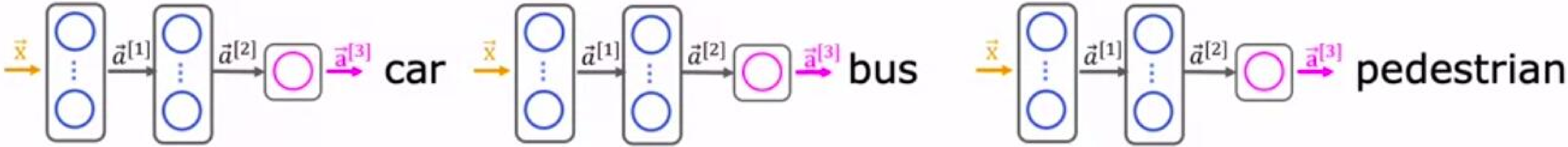
Multi-label Classification

$\vec{x}$

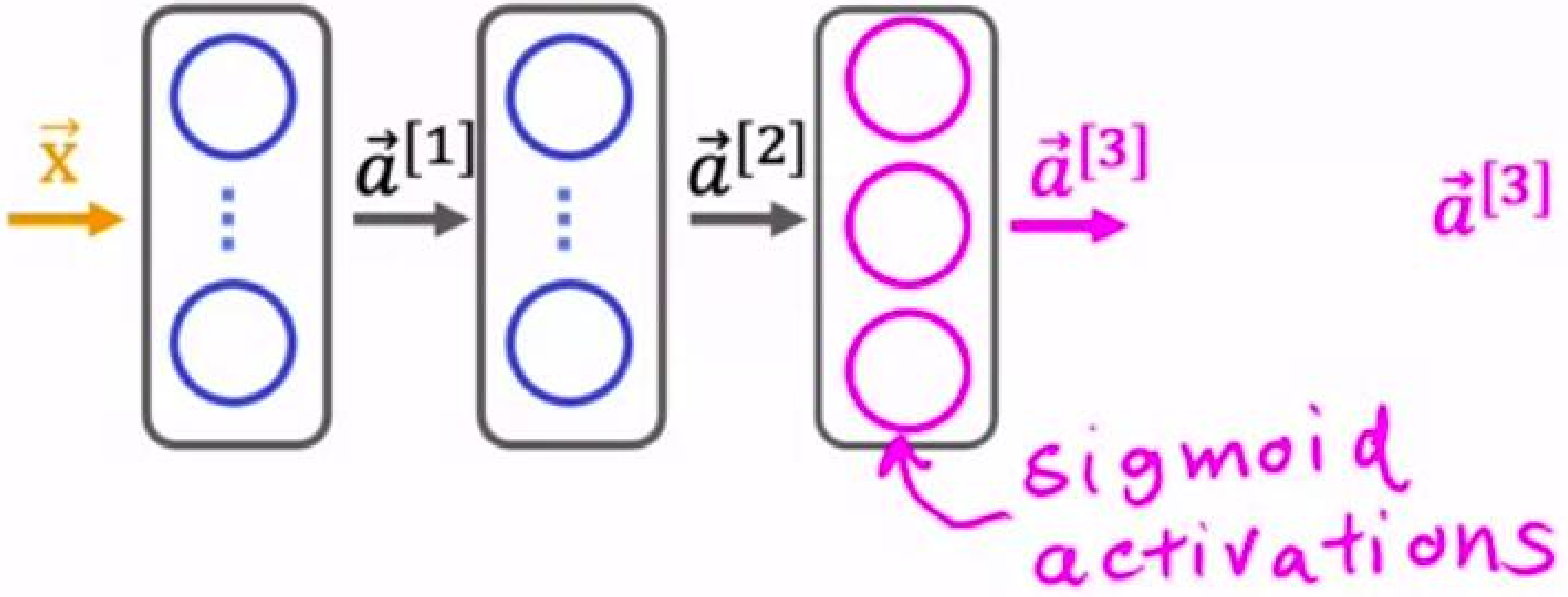
Is there a car?	yes	no	yes
Is there a bus?	no	no	yes
Is there a pedestrian	yes	yes	no

$$y=\begin{bmatrix}1\\0\\1\end{bmatrix} \qquad y=\begin{bmatrix}0\\0\\1\end{bmatrix} \qquad y=\begin{bmatrix}1\\1\\0\end{bmatrix}$$

Multi-label Classification



Alternatively, train one neural network with three outputs



$$\vec{a}^{[3]} = \begin{array}{cl} \begin{bmatrix} a_1^{[3]} \\ a_2^{[3]} \\ a_3^{[3]} \end{bmatrix} & \begin{array}{l} \text{car} \\ \text{bus} \\ \text{pedestrian} \end{array} \end{array}$$

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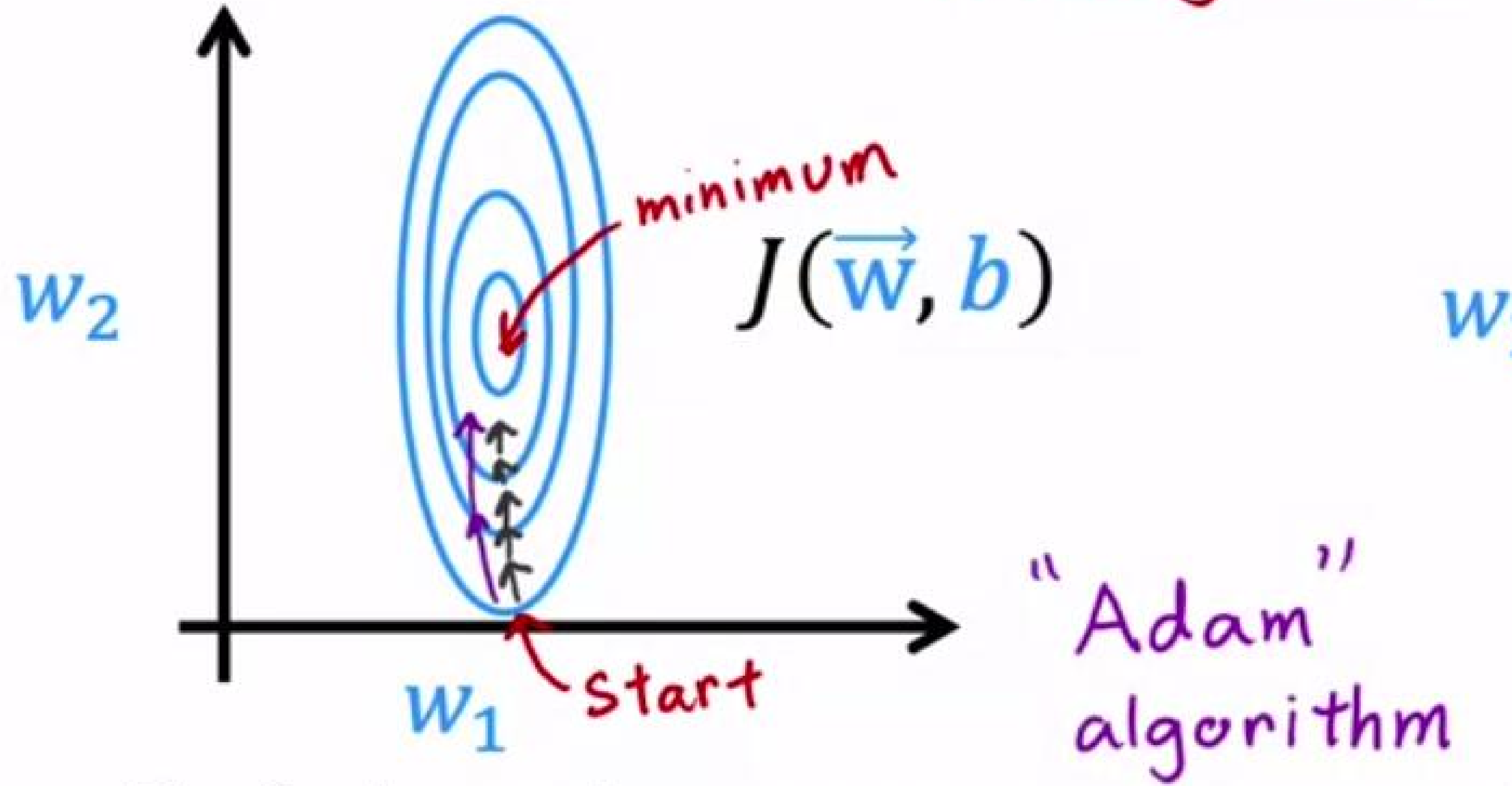
Additional Neural Network Concepts

Advanced Optimization

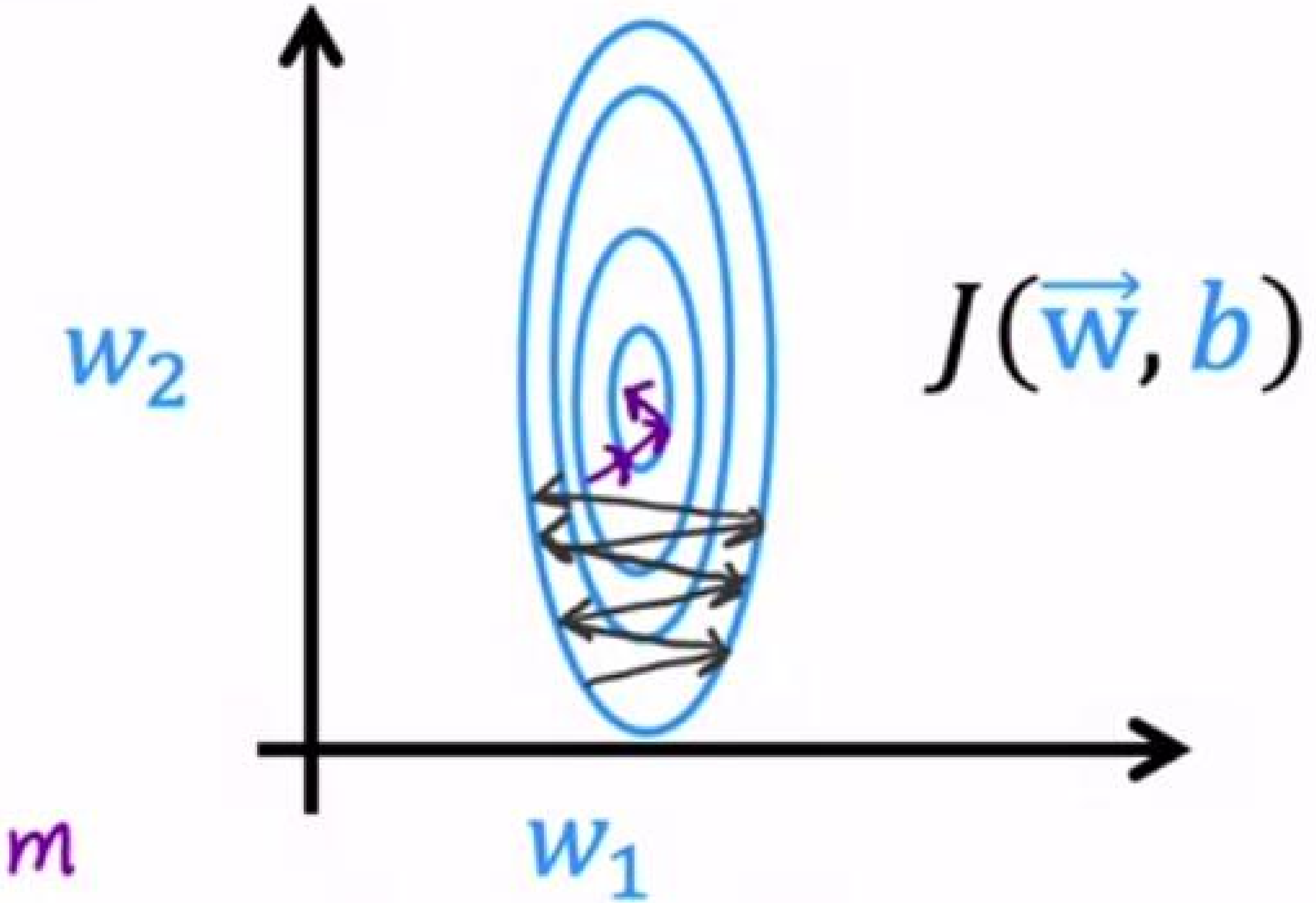
Gradient Descent

$$w_j = w_j - \alpha \frac{\partial}{\partial w_j} J(\vec{\mathrm{w}}, b)$$

learning rate



Go faster – increase α



Go slower – decrease α

Adam Algorithm Intuition

Adam: Adaptive Moment estimation

not just one α

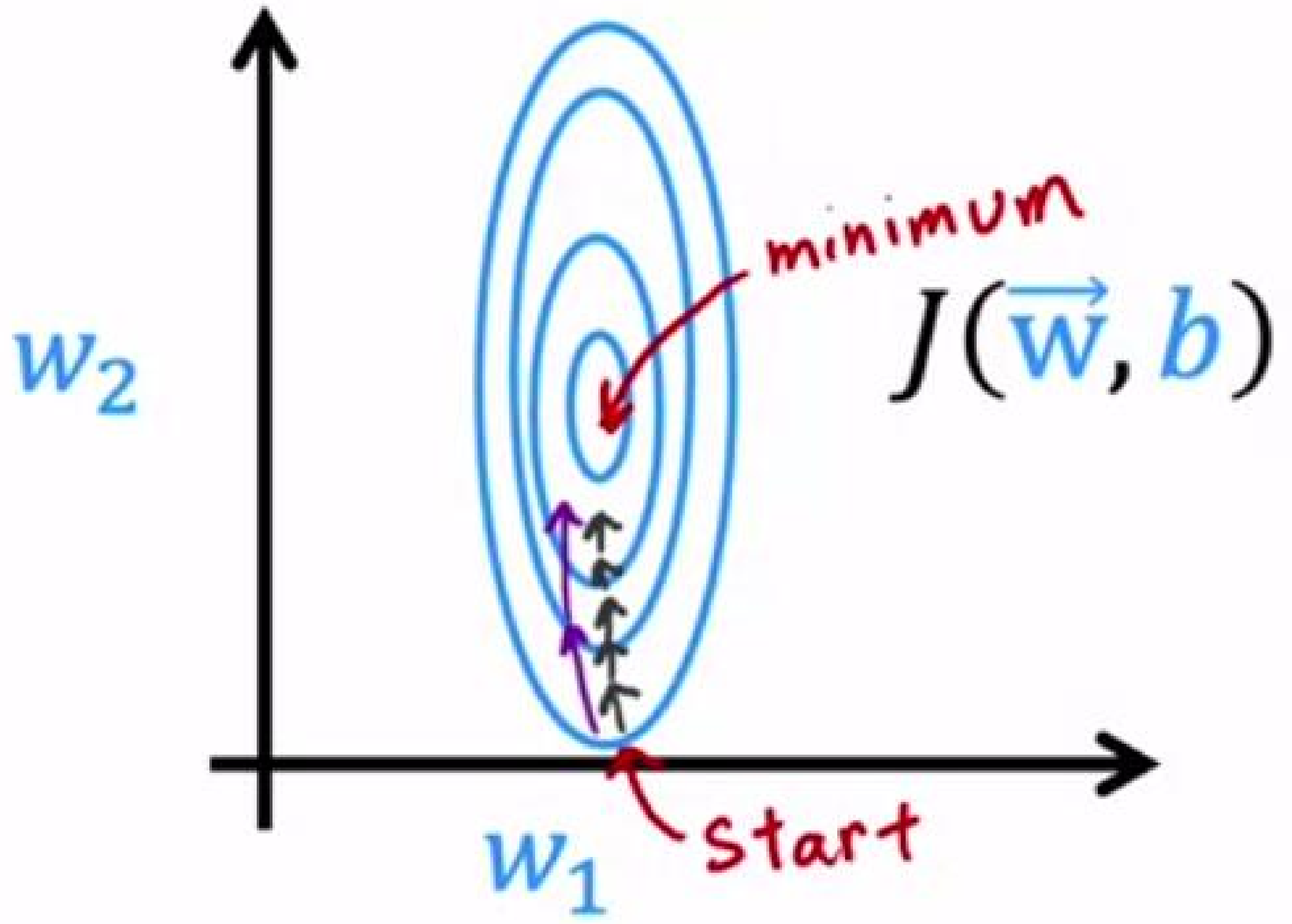
$$w_1 = w_1 - \underbrace{\alpha_1}\frac{\partial}{\partial w_1}J(\vec{\mathrm{w}}, b)$$

$$\vdots$$

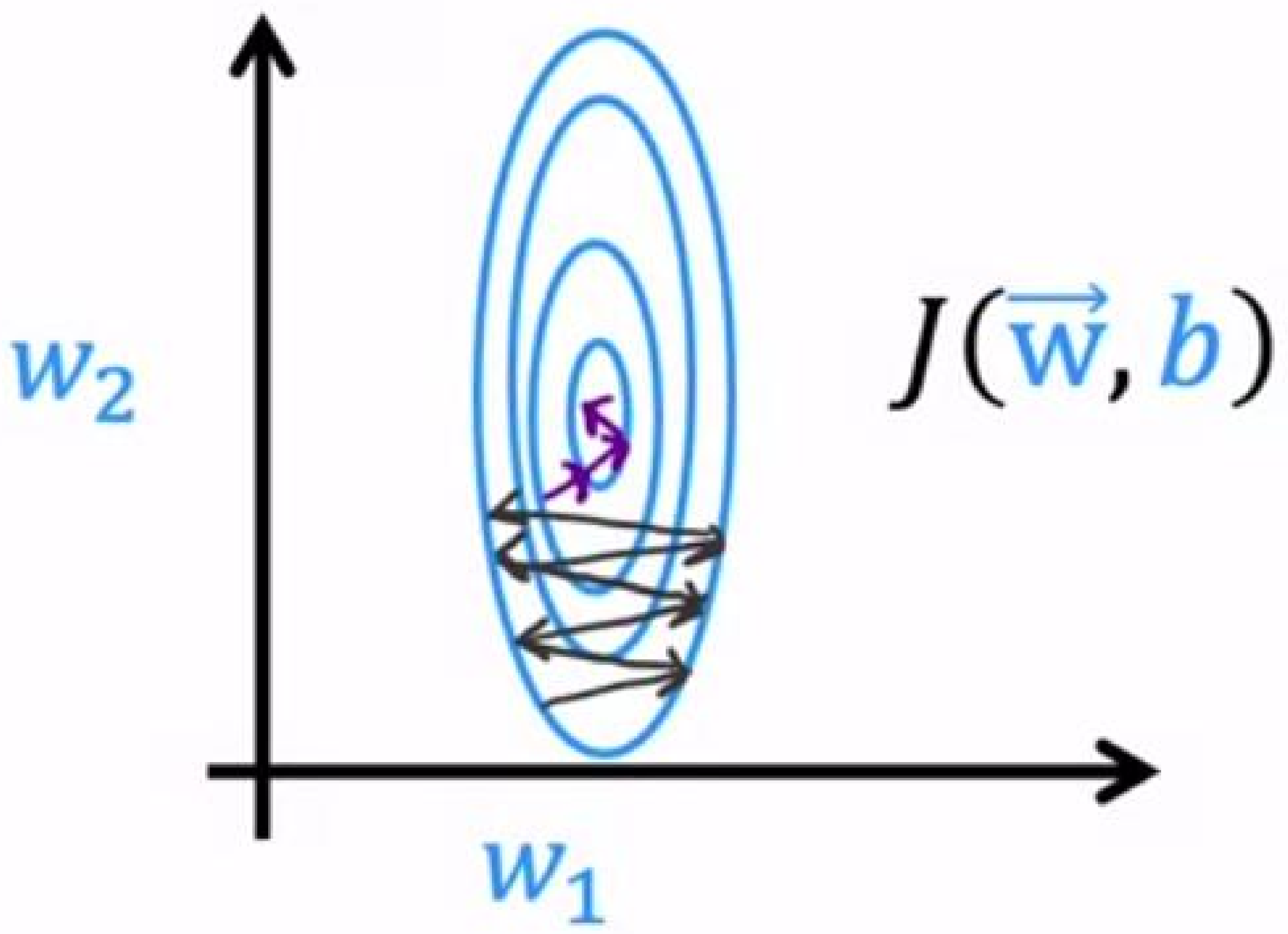
$$w_{10} = w_{10} - \underbrace{\alpha_{10}}\frac{\partial}{\partial w_{10}}J(\vec{\mathrm{w}}, b)$$

$$b = b - \underbrace{\alpha_{11}}\frac{\partial}{\partial b}J(\vec{\mathrm{w}}, b)$$

Adam Algorithm Intuition



If w_j (or b) keeps moving in same direction, $\underbrace{\text{increase}}$ α_j.



If w_j (or b) keeps oscillating, $\underbrace{\text{reduce}}$ α_j.

MNIST Adam

model

```
model = Sequential([
         tf.keras.layers.Dense(units=25, activation='sigmoid’),
         tf.keras.layers.Dense(units=15, activation='sigmoid’),
         tf.keras.layers.Dense(units=10, activation='linear')
])
```

compile

$\alpha = 10^{-3} = 0.001$

```
model.compile(optimizer=tf.keras.optimizers.Adam(learning_rate=1e-3),
  loss=tf.keras.losses.SparseCategoricalCrossentropy(from_logits=True))
```

fit

```
model.fit(X,Y,epochs=100)
```

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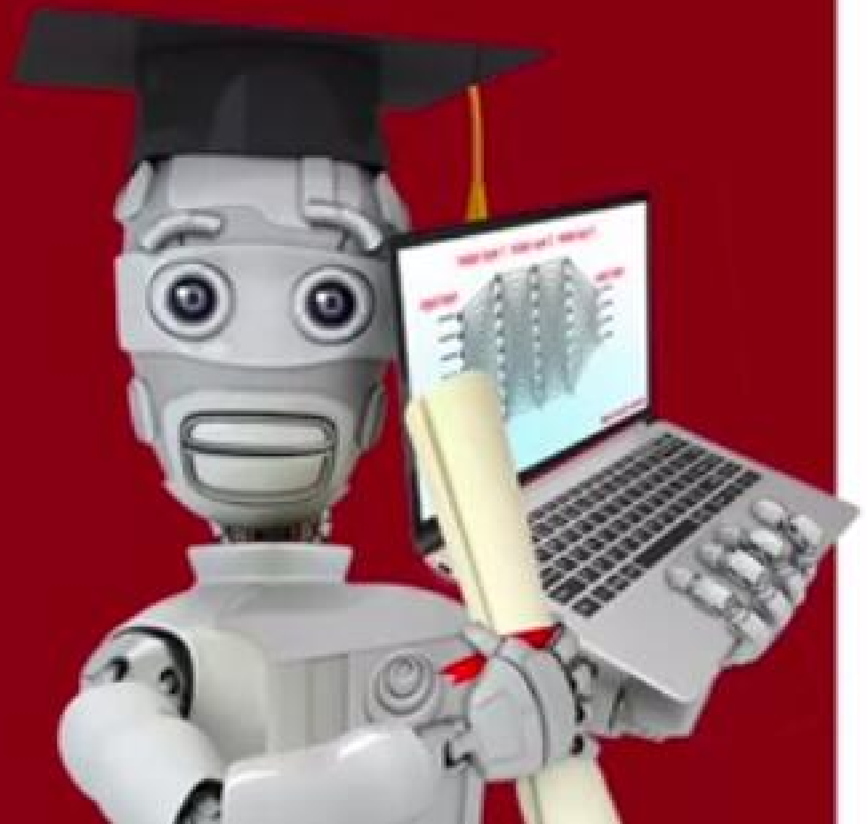
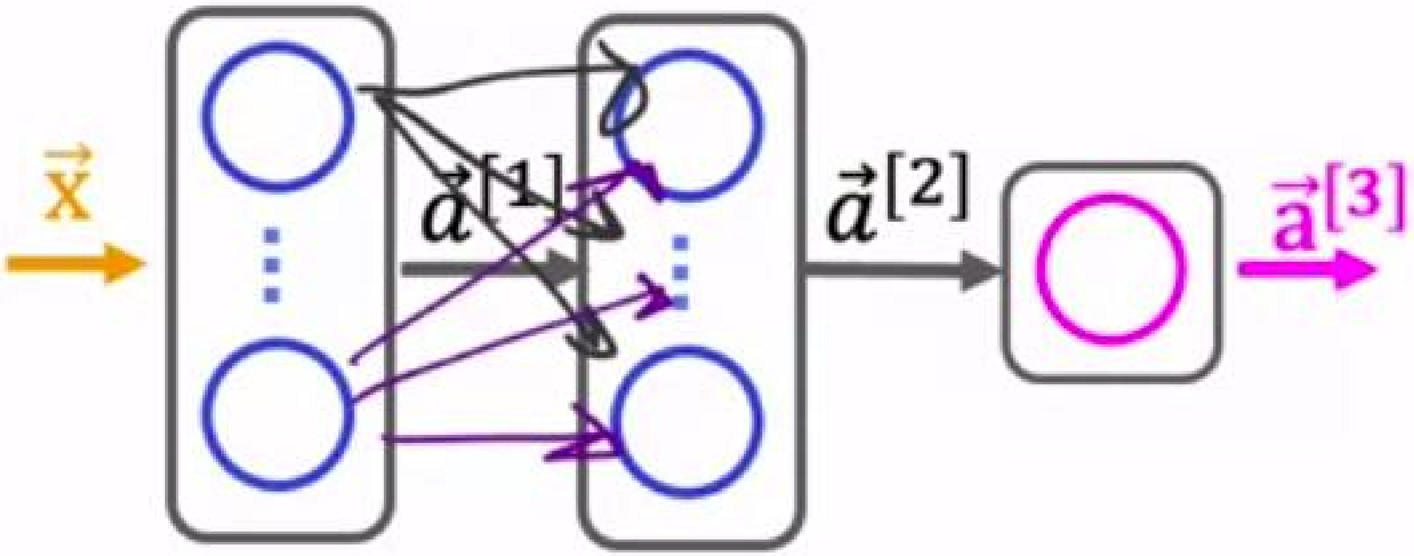


Additional Neural Network Concepts

Additional Layer Types

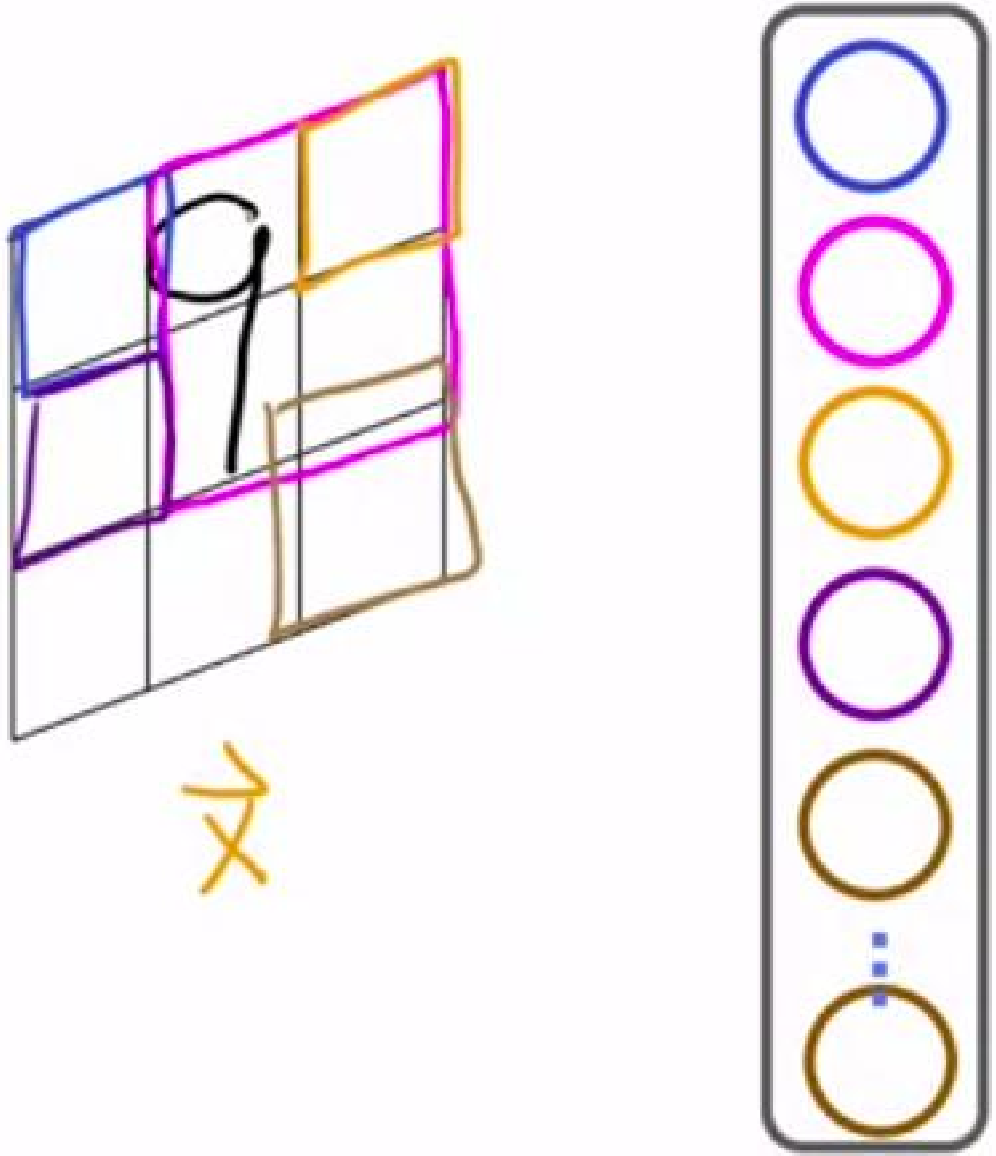
Dense Layer



Each neuron output is a function of
all the activation outputs of the previous layer.

$$\vec{a}_1^{[2]} = g\left(\vec{w}_1^{[2]} \cdot \vec{a}^{[1]} + b_1^{[2]}\right)$$

Convolutional Layer

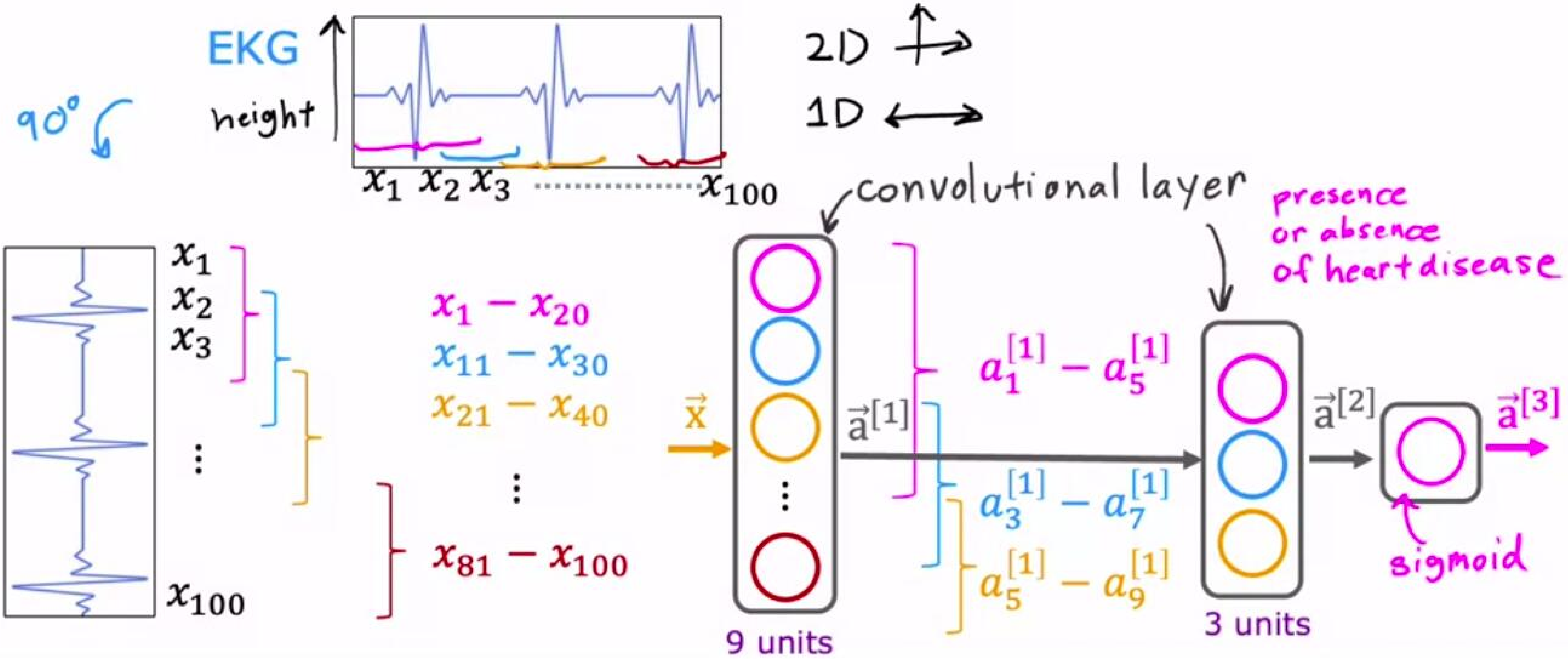


Each Neuron only looks at part of the previous layer's inputs.

Why?

- Faster computation
- Need less training data (less prone to overfitting)

Convolutional Neural Network



EKG

height

90°

2D

1D

x_1 x_2 x_3 .... x_{100}

x_1

x_2

x_3

⋮

x_{100}

$x_1 - x_{20}$

$x_{11} - x_{30}$

$x_{21} - x_{40}$

⋮

$x_{81} - x_{100}$

$\vec{x}$

convolutional layer

9 units

$\vec{a}^{[1]}$

$a_1^{[1]} - a_5^{[1]}$

$a_3^{[1]} - a_7^{[1]}$

$a_5^{[1]} - a_9^{[1]}$

convolutional layer

presence or absence of heart disease

3 units

$\vec{a}^{[2]}$

$\vec{a}^{[3]}$

sigmoid

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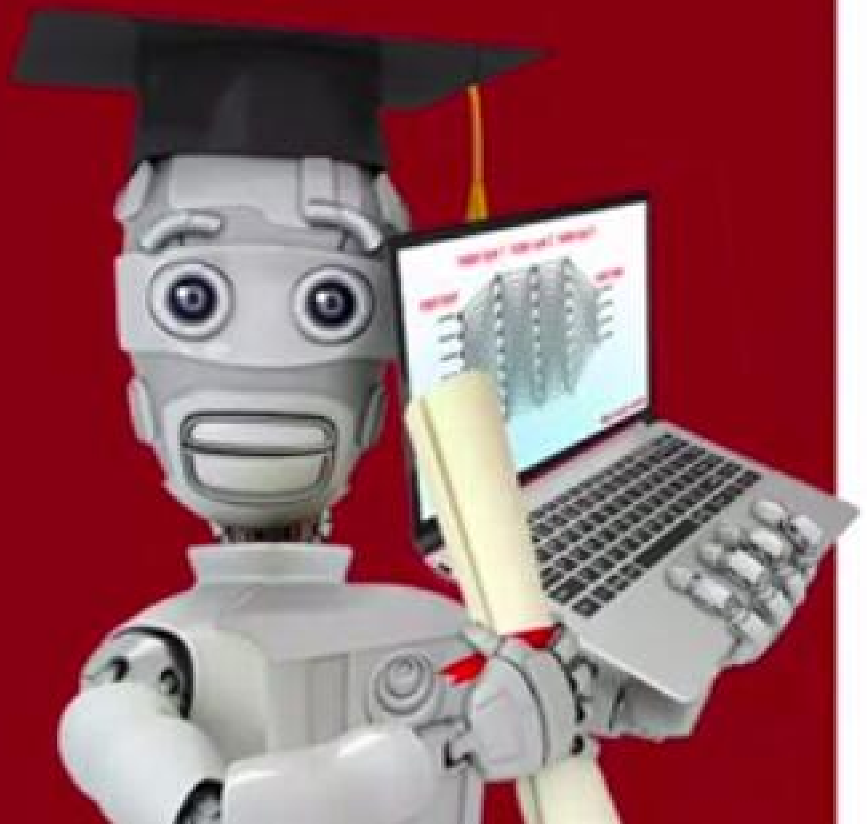


Backprop Intuition (Optional)

What is a derivative?

Derivative Example

Cost function $J(w) = w^2$

Say $w = 3$ $J(w) = 3^2 = 9$

$\varepsilon = 0.002$

If we increase w by a tiny amount $\varepsilon = 0.001$ how does $J(w)$ change?

$w = 3 + \cancel{0.001}$ 0.002

$J(w) = w^2 = \cancel{9.006001}$

$\underbrace{9.012}_{9.012}\underbrace{004}$

If $\underline{w} \uparrow \overset{0.002}{\cancel{0.001}}$ ε ←

$\underline{J(w)} \uparrow 6 \times \overset{0.002}{\cancel{0.001}}$ $6 \times \varepsilon$ $6 \times 0.002 = 0.012$

↖

$\frac{\partial}{\partial w} J(w) = 6$

Informal Definition of Derivative

If $\underline{w \uparrow \varepsilon}$ causes $\underline{J(w)} \uparrow \underset{\uparrow\, 6}{\boxed{k \times \varepsilon}}$ then

$$\frac{\partial}{\partial w} J(w) = k \longrightarrow 6$$

Gradient descent

```
repeat {
    $w_j = w_j - \overset{\text{learning rate}}{\alpha} \frac{\partial}{\partial w_j} J(\vec{w}, b)$
}
```

If derivative is small, then this update step will make a small update to w_j

If the derivative is large, then this update step will make a large update to w_j

More Derivative Examples

$w = 3$ $J(w) = w^2 = 9$ $w \uparrow 0.001$ $J(w) = J(3.001) = 9.006001$ $\frac{\partial}{\partial w} J(w) = 6$

$J(w) \uparrow 6 \times 0.001$

$w = 2$ $J(w) = w^2 = 4$ $w \uparrow 0.001$ $J(w) = J(2.001) = 4.004001$ $\frac{\partial}{\partial w} J(w) = 4$

$J(w) \uparrow 4 \times 0.001$

$J(w) = w^2$

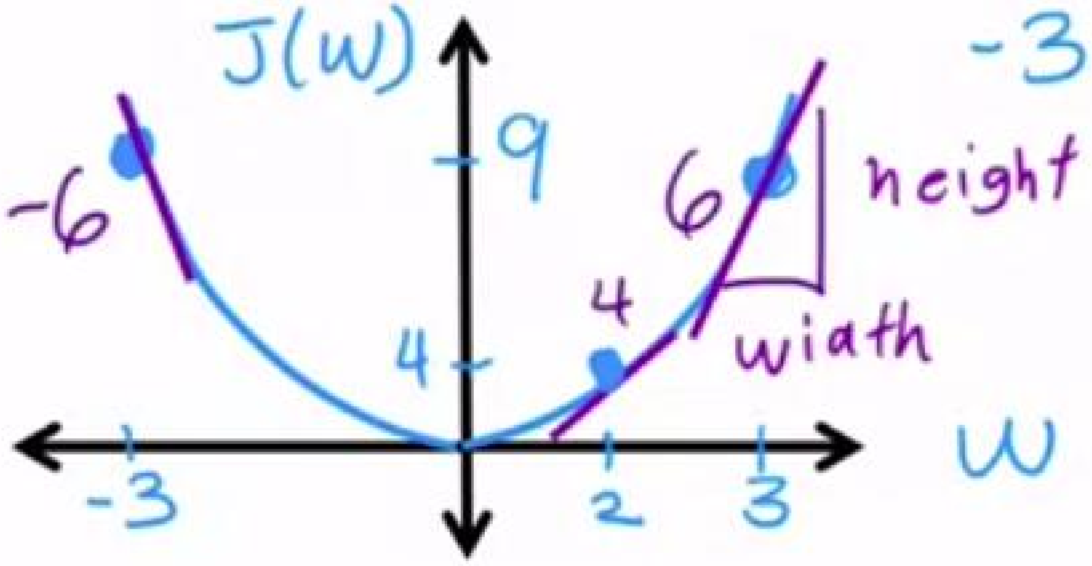
$\downarrow 0.006$

$w = -3$ $J(w) = w^2 = 9$ $w \uparrow 0.001$ $J(w) = J(-2.999) = 8.994001$ $\frac{\partial}{\partial w} J(w) = -6$

$J(w) \downarrow 6 \times 0.001$

$-3 + 0.001$

$J(w) \uparrow -6 \times 0.001$



Calculus

$\frac{\partial}{\partial w} J(w) = 2w$

w	$\partial J(w)/\partial w$
3	$2 \times 3 = 6$
2	$2 \times 2 = 4$
-3	$2 \times -3 = -6$

Even More Derivative Examples

$$w = 2 \left\{ \begin{array}{llll} J(w) = w^2 = 4 & \frac{\partial}{\partial w}J(w) = 2w = 4 & w \uparrow \underbrace{0.001}_{\varepsilon} & \begin{array}{l} J(w) = 4.004001 \\ J(w) \uparrow 4 \times \varepsilon \end{array} \\ J(w) = w^3 = 8 & & & \\ J(w) = w = 2 & & & \\ J(w) = \frac{1}{w} = \frac{1}{2} = 0.5 & & & \end{array}\right.$$

Even More Derivative Examples

$w = 2$

$J(w) = w^2 = 4$	$\frac{\partial}{\partial w}J(w) = 2w = 4$	$w \uparrow \underbrace{0.001}_{\varepsilon}$	$J(w) = 4.004001$ $J(w) \uparrow 4 \times \varepsilon$
$J(w) = w^3 = \boxed{8}$	$\frac{\partial}{\partial w}J(w) = 3w^2 = \boxed{12}$	$w \uparrow \varepsilon$	$J(w) = 8.012006$ $J(w) \uparrow \boxed{12} \times \varepsilon$
$J(w) = w = 2$	$\frac{\partial}{\partial w}J(w) = \boxed{1}$	$w \uparrow \varepsilon$	$J(w) = 2.001$ $J(w) \uparrow \boxed{1} \times \varepsilon$
$J(w) = \frac{1}{w} = \frac{1}{2} = \boxed{0.5}$	$\frac{\partial}{\partial w}J(w) = \frac{-1}{w^2} = \boxed{\frac{-1}{4}}$	$w \uparrow \varepsilon$ $w = \frac{1}{2.001}$	-0.25×0.001 $\underline{0.5 - 0.00025}$ $J(w) = 0.49975$ $J(w) \uparrow \boxed{-\frac{1}{4}} \times \varepsilon$

$\frac{\partial}{\partial w}J(w)$ $\quad$ $w \uparrow \varepsilon$ $\quad$ $J(w) \uparrow \boxed{k} \times \varepsilon$

A note on derivative notation

If $J(w)$ is a function of one variable (w),

d $\quad \frac{d}{dw}J(w)$

If $J(w_1, w_2, \ldots, w_n)$ is a function of more than one variable,

∂ $\quad \frac{\partial}{\partial w_i}J(w_1, w_2, \ldots, w_n)$ $\quad \frac{\partial J}{\partial w_i}$ or $\frac{\partial}{\partial w_i}J$

"partial derivative"

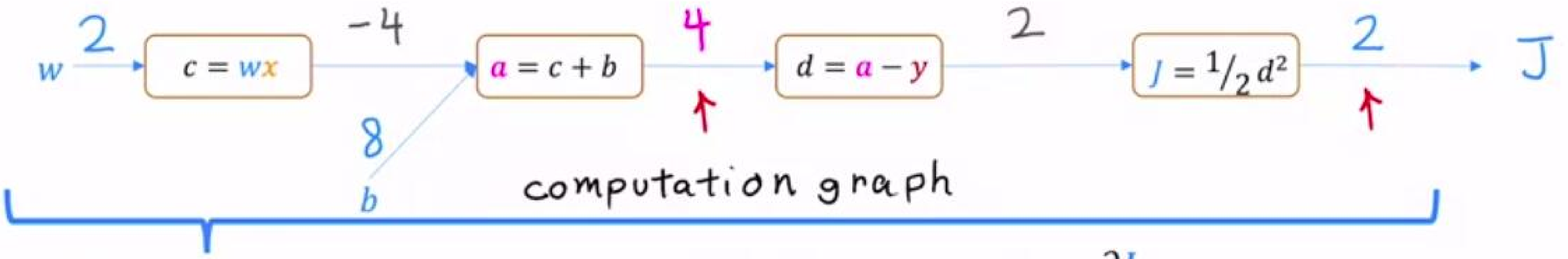
notation used in these courses

Small Neural Network Example

$w=2$ $b=8$ $x=-2$ $y=2$

$a = wx + b$ linear activation $a = g(z) = z$

$$\underbrace{J(w,b)}_{} = \frac{1}{2}\underbrace{(a-y)}_{}^2$$



forward prop

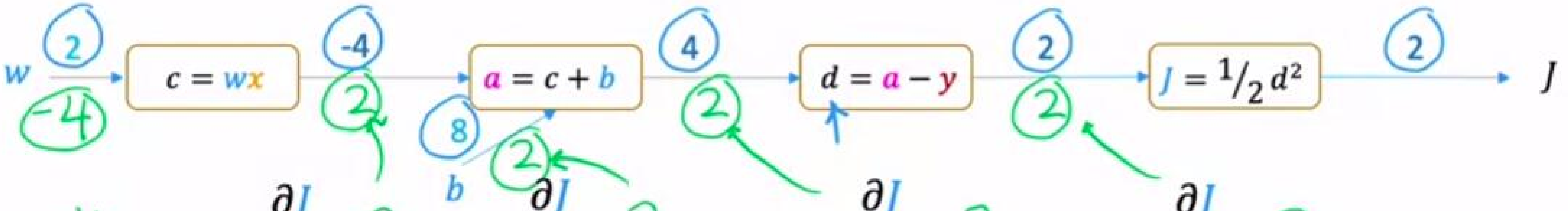
how do we find the derivatives of J? $\frac{\partial J}{\partial w}$

Computing the Derivatives

Forward prop →

← Back prop

$w = 2$ $b = 8$ $x = -2$ $y = 2$ $a = wx + b$ $J = \frac{1}{2}(a - y)^2$



$\frac{\partial J}{\partial w} = -4$ $\frac{\partial J}{\partial c} = 2$ $\frac{\partial J}{\partial b} = 2$ $\frac{\partial J}{\partial a} = 2$ $\frac{\partial J}{\partial d} = 2$

w=2.001 c=-4.002

w ↑ 0.001 c ↓ 2×0.001

c ↑ −2×0.001

J ↑ −4×0.001

$$\frac{\partial J}{\partial w} = \underset{-2}{\frac{\partial c}{\partial w}} \times \underset{2}{\frac{\partial J}{\partial c}}$$

c ↑ 0.001 a ↑ 0.001 J ↑ 0.002

$$\frac{\partial J}{\partial c} = \underset{1}{\frac{\partial a}{\partial c}} \times \underset{2}{\frac{\partial J}{\partial a}}$$

b ↑ 0.001 a ↑ 0.001 J ↑ 0.002

$$\frac{\partial J}{\partial b} = \underset{1}{\frac{\partial a}{\partial b}} \times \underset{2}{\frac{\partial J}{\partial a}}$$

a ↑ 0.001 d ↑ 0.001

a=4.001 d=2.001

J ↑ 0.002

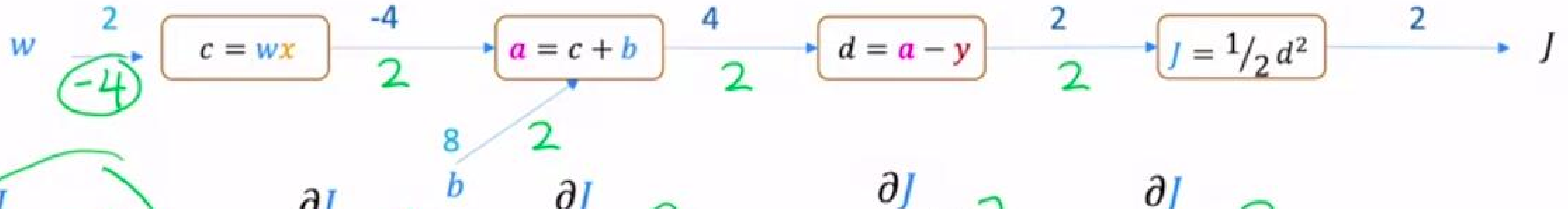
$$\frac{\partial J}{\partial a} = \underset{1}{\frac{\partial d}{\partial a}} \times \underset{2}{\frac{\partial J}{\partial d}}$$

$d \uparrow \underbrace{0.001}_{\varepsilon}$ $J \uparrow \underbrace{0.002}_{2\varepsilon}$

chain rule
(calculus)

Computing the Derivatives

$w = 2$ $b = 8$ $x = -2$ $y = 2$ $a = wx + b$ $J = \frac{1}{2}(a - y)^2$



$\boxed{\frac{\partial J}{\partial w} = -4}$ $\frac{\partial J}{\partial c} = 2$ $\frac{\partial J}{\partial b} = 2$ $\frac{\partial J}{\partial a} = 2$ $\frac{\partial J}{\partial d} = 2$

$$J = \frac{1}{2}\left((wx + b) - y\right)^2 = \frac{1}{2}\left((2 \times -2 + 8) - 2\right)^2 = 2$$

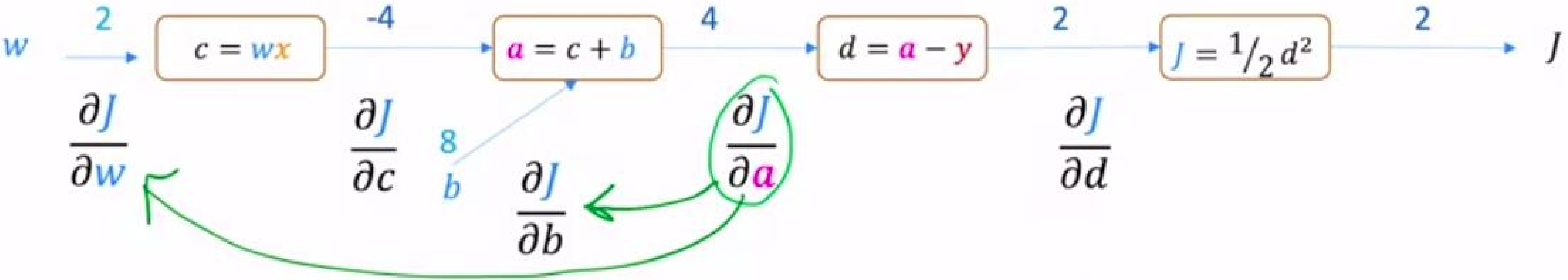
w ↑ 0.001 $J = \frac{1}{2}\left((2.001 \times -2 + 8) - 2\right)^2 = 1.\underbrace{996}002$

$J \downarrow 4 \times 0.001$

$J \uparrow -4 \times 0.001$

$$\frac{\partial J}{\partial w} = -4$$

Backprop is an efficient way to compute derivatives



Compute $\frac{\partial J}{\partial a}$ once and use it to compute both $\frac{\partial J}{\partial w}$ and $\frac{\partial J}{\partial b}$.

If N nodes and P parameters, compute derivatives in roughly $N + P$ steps rather than $N \times P$ steps.

N	P	N + P	N × P
10,000	100,000	1.1×10^5	10^9

Neural Network Example

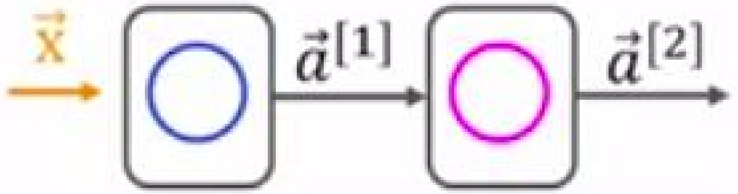
$x = 1 \quad y = 5$

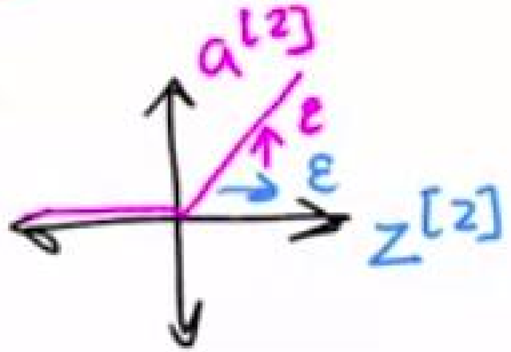
$w^{[1]} = 2, b^{[1]} = 0$

$w^{[2]} = 3, b^{[2]} = 1$

ReLU activation

$g(z) = \max(0, z)$

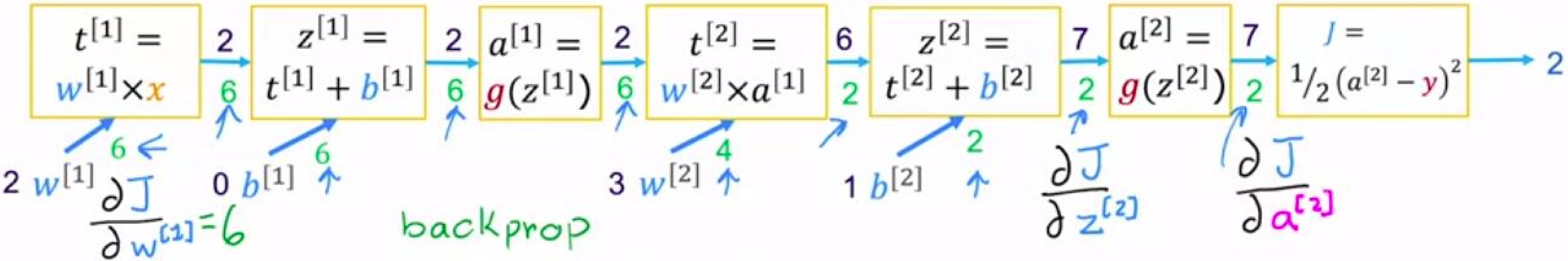




$$a^{[1]} = g(w^{[1]} x + b^{[1]}) = \overbrace{w^{[1]} x + b^{[1]}}^{z^{[1]}} = 2\times 1 + 0 = 2$$

$$a^{[2]} = g(w^{[2]} a^{[1]} + b^{[2]}) = \overbrace{w^{[2]} a^{[1]} + b^{[2]}}^{z^{[2]}} = 3\times 2 + 1 = 7$$

$$J(w, b) = \frac{1}{2}(a^{[2]} - y)^2 = \frac{1}{2}(7 - 5)^2 = 2$$



$t^{[1]} = w^{[1]} \times x$

$z^{[1]} = t^{[1]} + b^{[1]}$

$a^{[1]} = g(z^{[1]})$

$t^{[2]} = w^{[2]} \times a^{[1]}$

$z^{[2]} = t^{[2]} + b^{[2]}$

$a^{[2]} = g(z^{[2]})$

$J = 1/2\,(a^{[2]} - y)^2$

$$\frac{\partial J}{\partial w^{[1]}} = 6$$

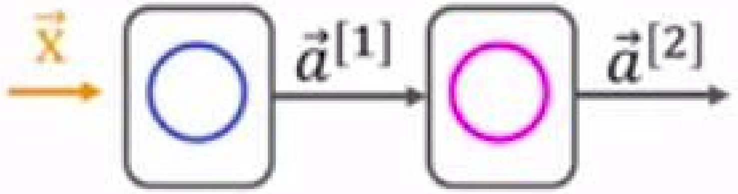
if $w^{[1]} \uparrow \varepsilon$, then $J \uparrow 6 \times \varepsilon$ let's verify this!

Neural Network Example

$x = 1 \quad y = 5$

$w^{[1]} = 2, b^{[1]} = 0$

ReLU activation



$w^{[2]} = 3, b^{[2]} = 1$

$g(z) = \max(0, z)$

$$a^{[1]} = g(w^{[1]}x + b^{[1]}) = w^{[1]}x + b^{[1]} = \overset{2.001}{\cancel{2}} \times 1 + 0 = \overset{2.001}{\cancel{2}}$$

$$a^{[2]} = g(w^{[2]}a^{[1]} + b^{[2]}) = w^{[2]}a^{[1]} + b^{[2]} = 3 \times \overset{2.001}{\cancel{2}} + 1 = \cancel{7} \; 7.003$$

$$J(w, b) = \frac{1}{2}(a^{[2]} - y)^2 = \frac{1}{2}\underbrace{(\overset{7.003}{\cancel{7}} - 5)^2}_{\frac{(2.003)^2}{2}} = \cancel{2} \quad 2\boxed{.006}005$$

$w^{[1]} \uparrow 0.001$

$J \uparrow 6 \times 0.001$

$$\frac{\partial J}{\partial w^{[1]}} = 6$$

$$\frac{\partial J}{\partial w^{[1]}} \quad \frac{\partial J}{\partial b^{[1]}}$$

$$\frac{\partial J}{\partial w^{[2]}} \quad \frac{\partial J}{\partial b^{[2]}}$$

N nodes 

P parameters

$w_1, b_1, w_2, b_2 \ldots$

inefficient way

$N \times P$

efficient way (backprop)

$N + P$