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Neural Networks Intuition

Neurons and the brain

Neural networks

Origins: Algorithms that try to mimic the brain.

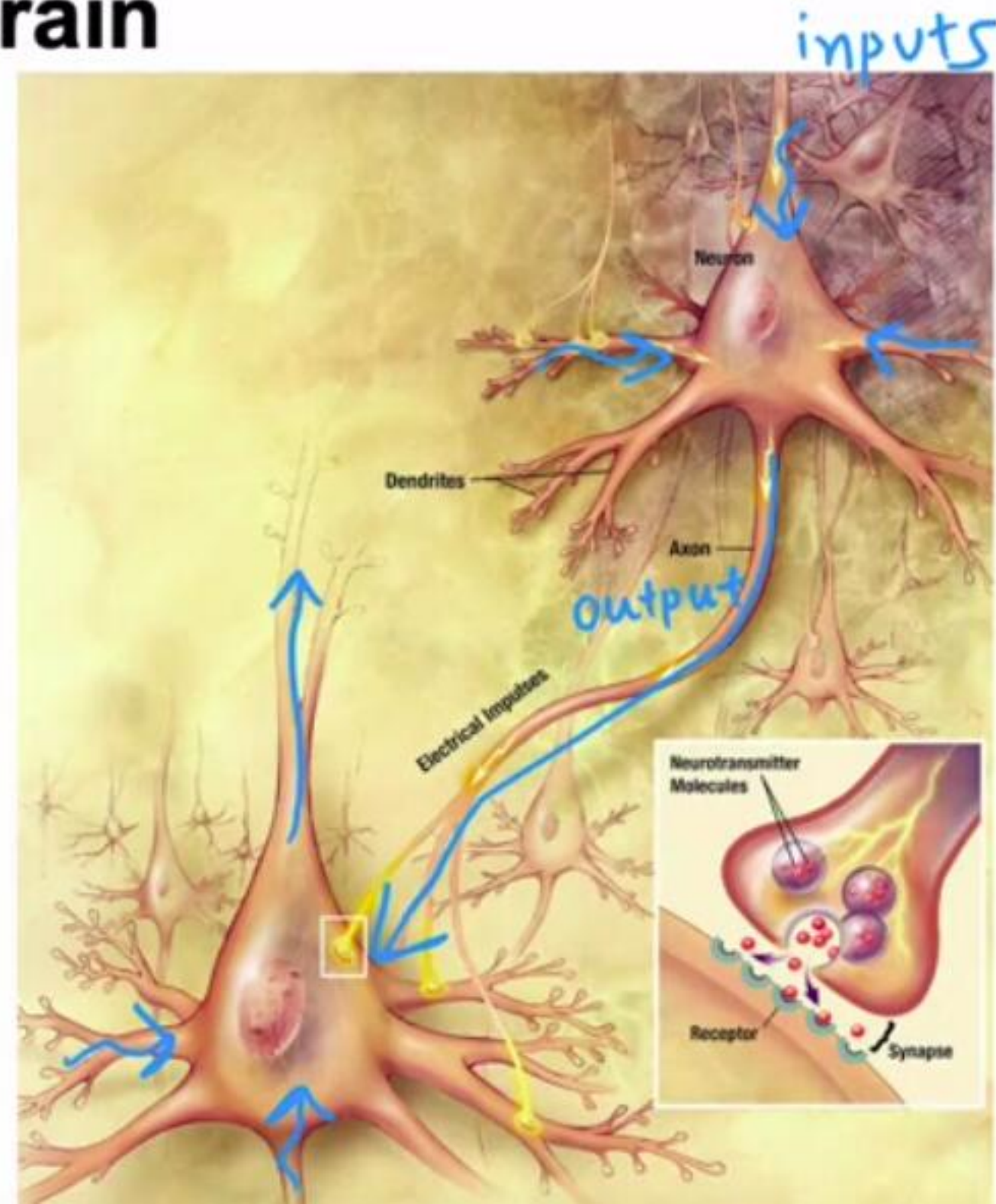


Used in the 1980's and early 1990's.
Fell out of favor in the late 1990's.

Resurgence from around 2005.

speech → images → text (NLP) → ...

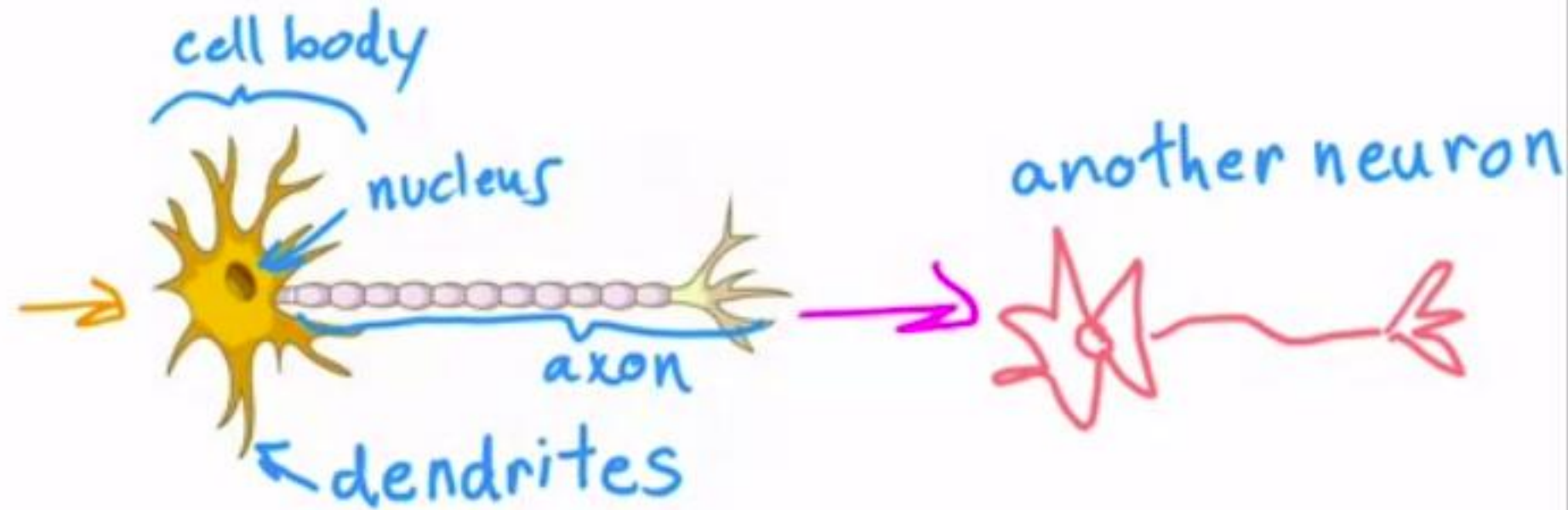
Neurons in the brain



Biological neuron

inputs

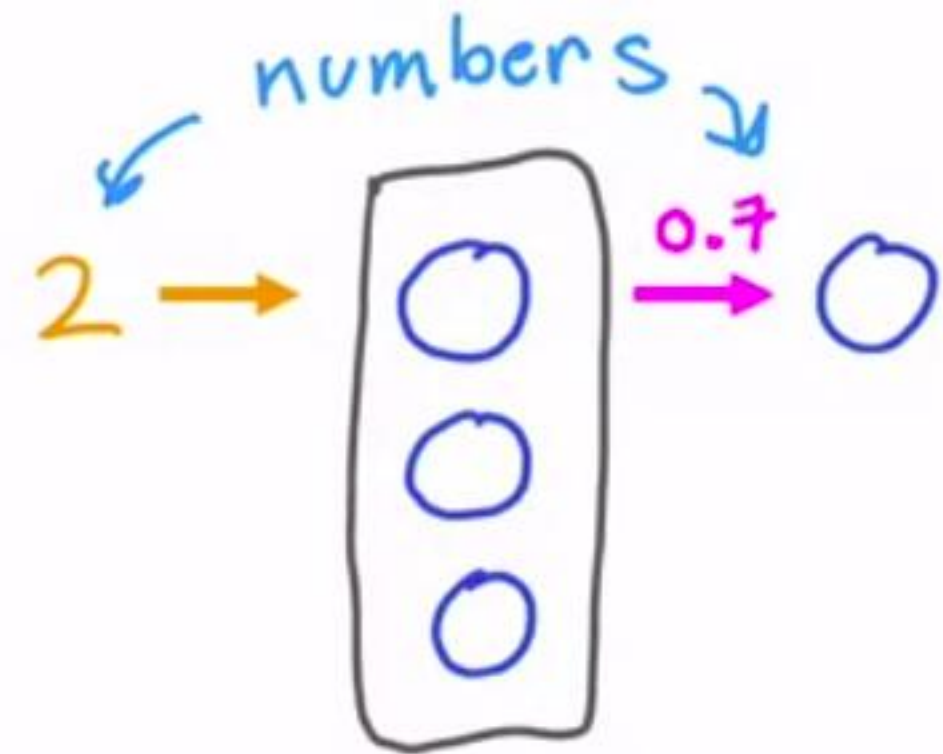
outputs



Simplified mathematical model of a neuron

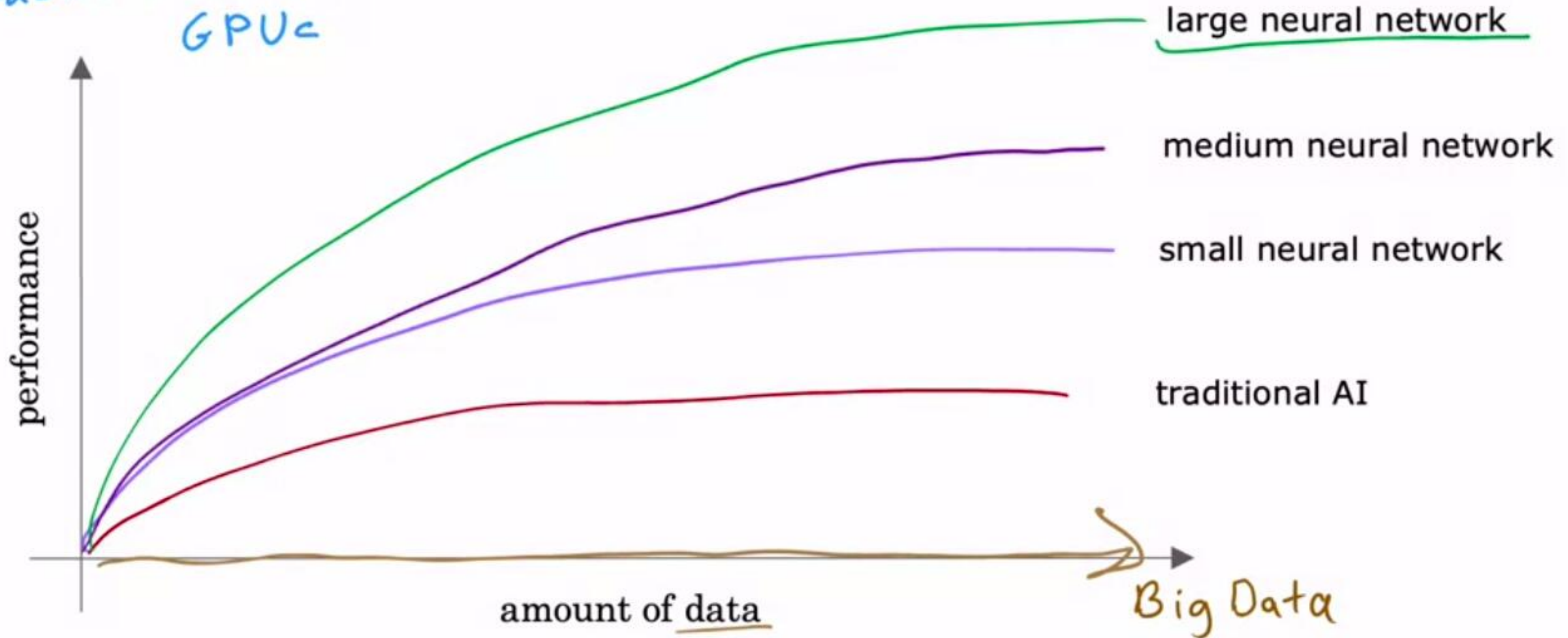
inputs

outputs



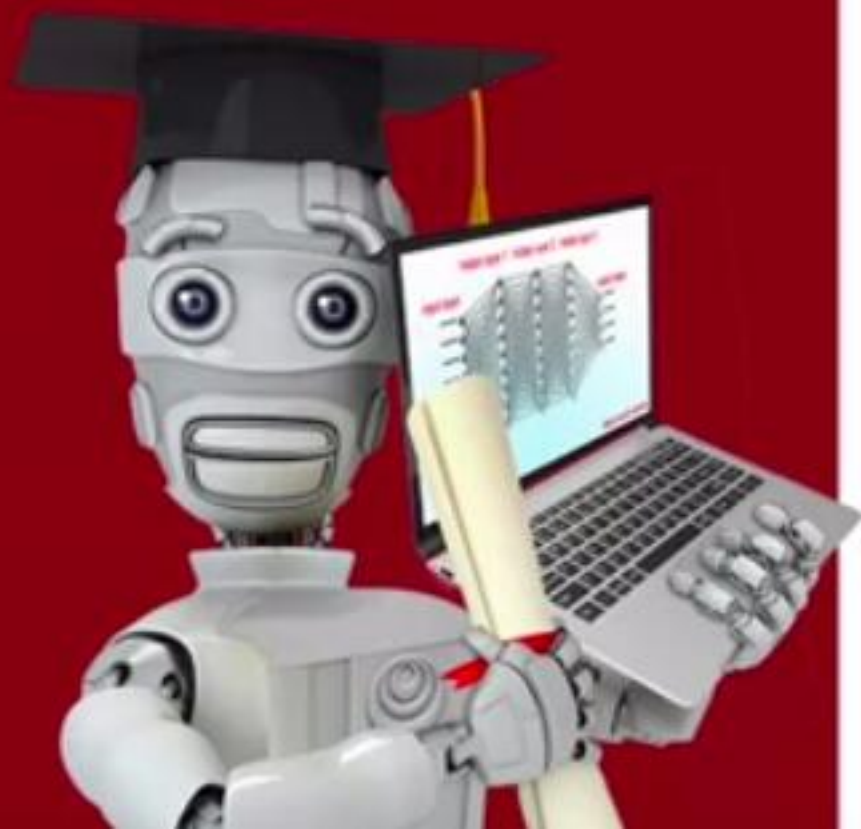
Why Now?

Faster computer processors
GPU



 DeepLearning.AI

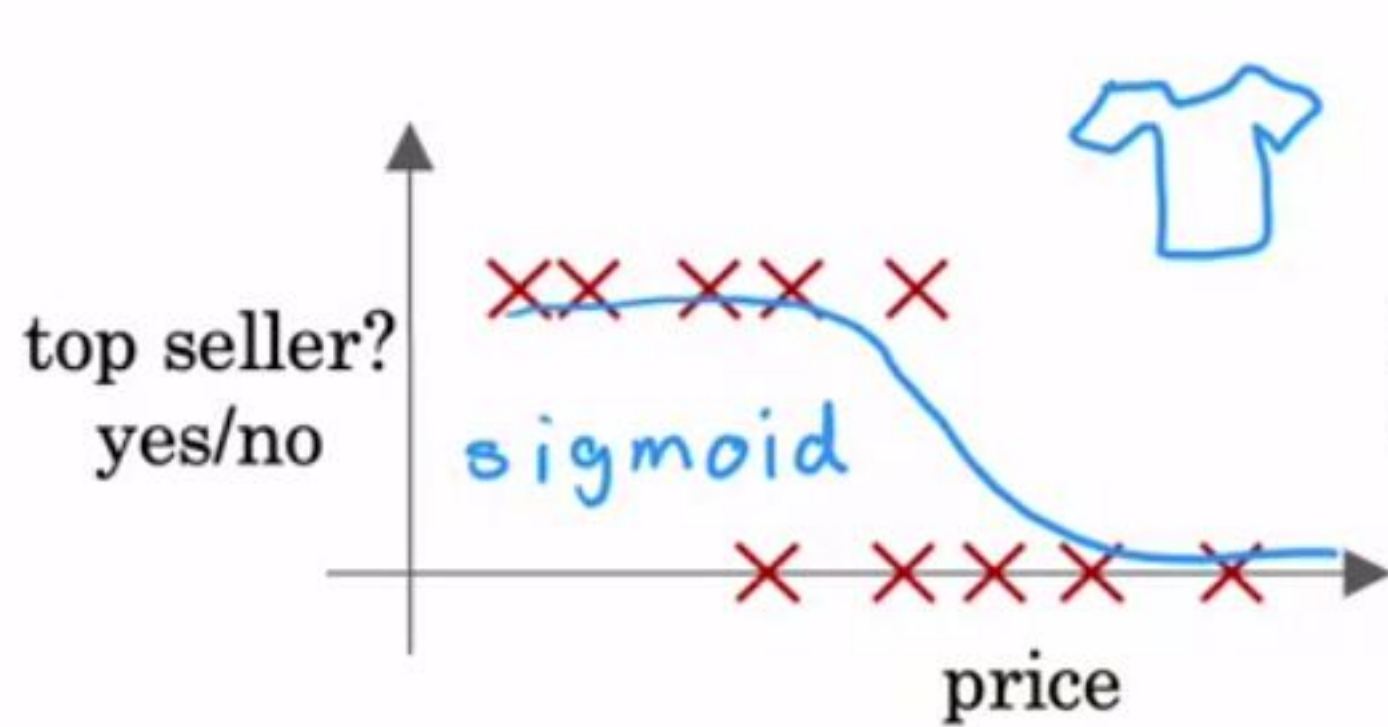
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Neural Network Intuition

Demand Prediction

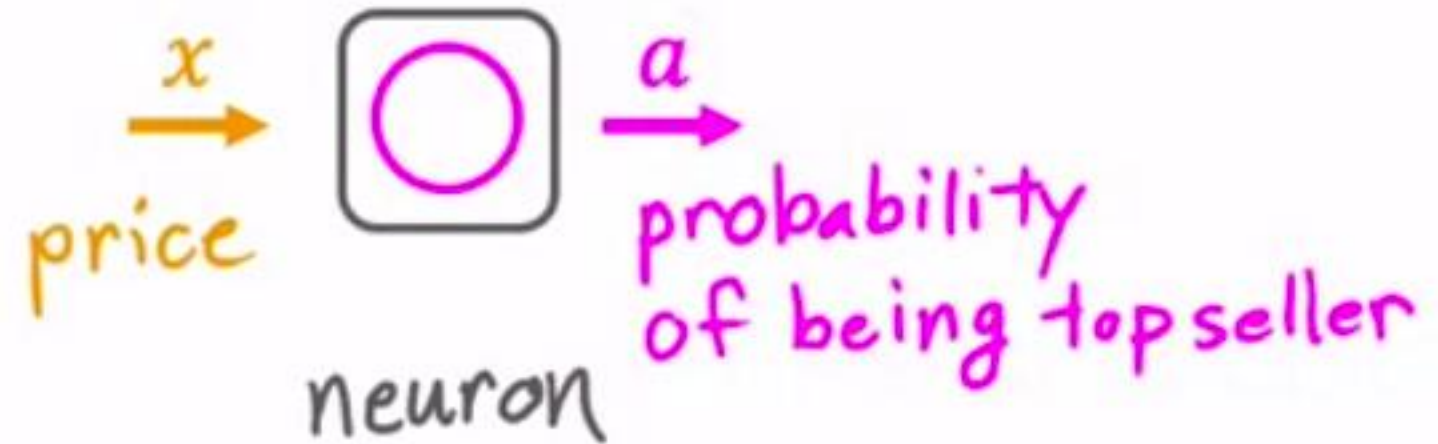
Demand Prediction



$x = \text{price}$ input

$a = f(x) = \frac{1}{1 + e^{-(wx+b)}}$ output

activation



Demand Prediction



feature engineering
 $x_1 x_2$

$\vec{x} (x, y)$

input layer

price

shipping cost

marketing

material

"hidden layer"
layer \leftarrow can have multiple neurons

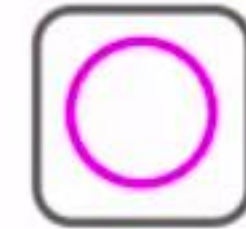
"activations"

affordability \leftarrow

awareness \leftarrow

perceived
quality \leftarrow

layer \leftarrow output layer



a \rightarrow probability of
being a top seller

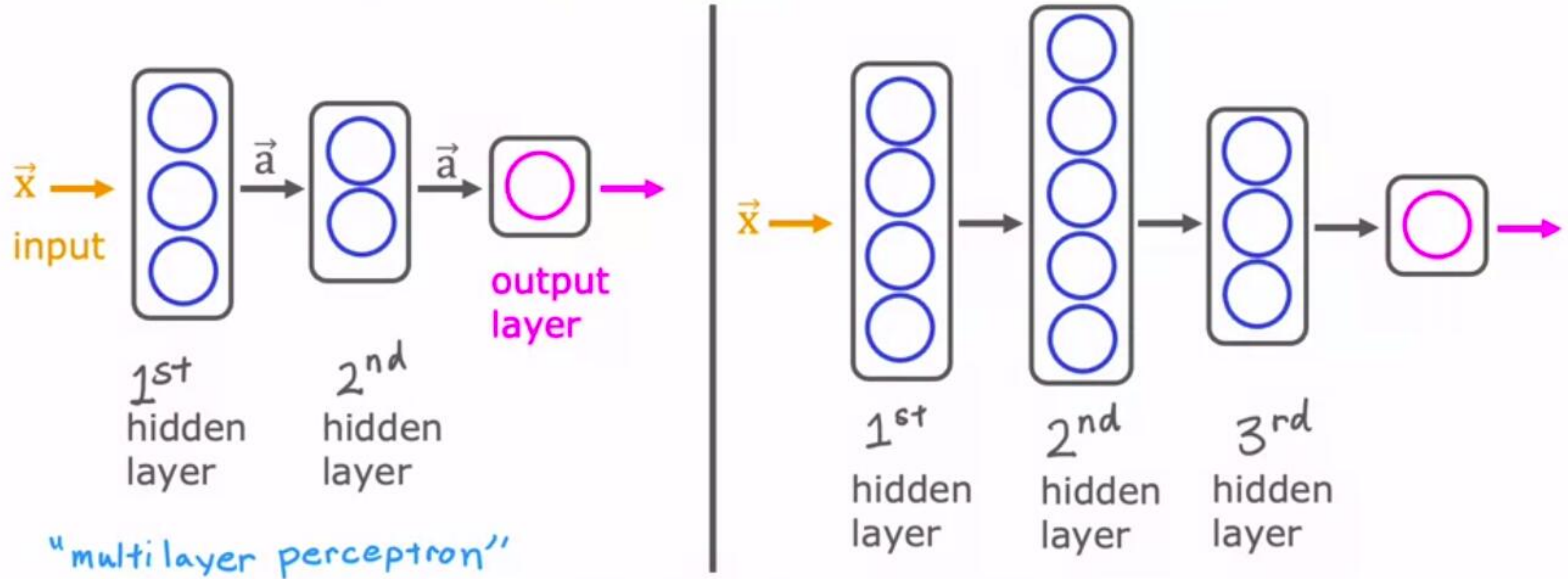
activation values

4 numbers

3 numbers

1 number

Multiple hidden layers



"multilayer perceptron"

neural network architecture



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Neural Networks Intuition

**Example:
Recognizing Images**

Face recognition

1000 pixels



1000 pixels

1000 columns

1000 rows

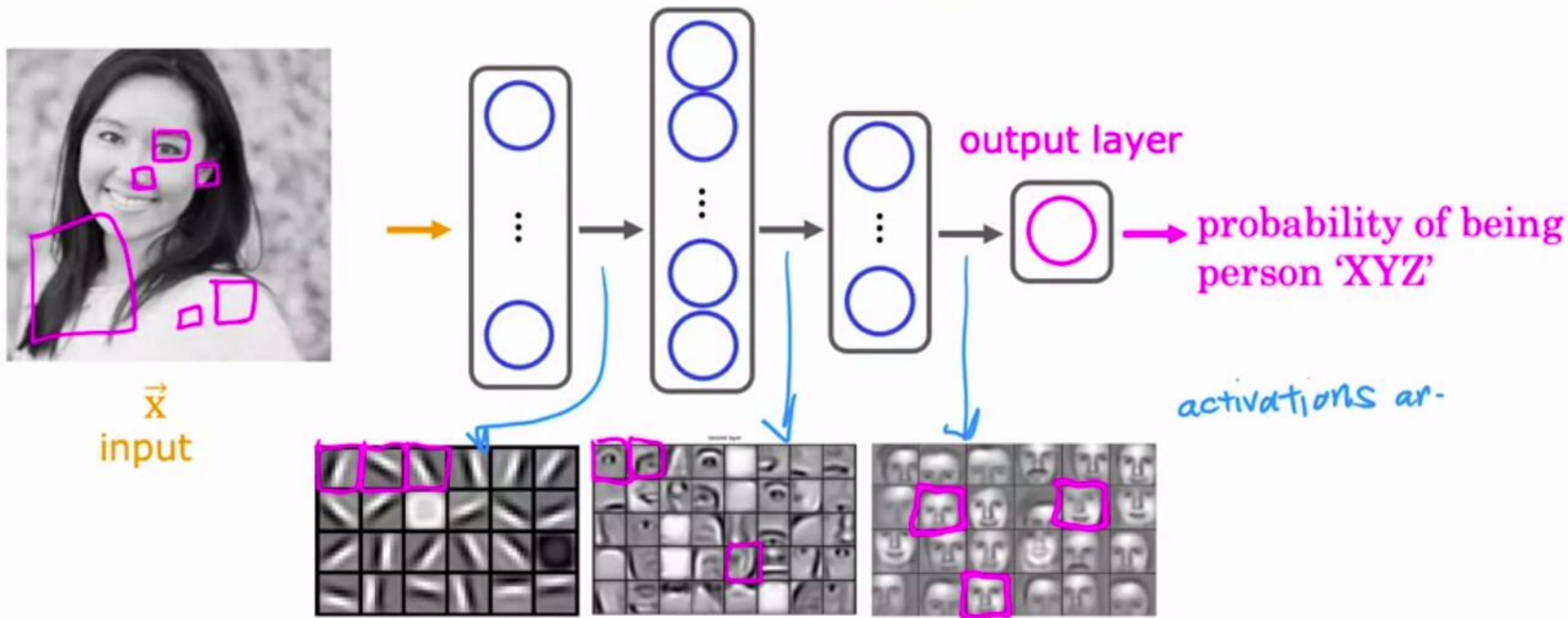
197	185	203
...	57	64	92	...
⋮				
		...	187	214

$\vec{x} =$

197
185
203
⋮
57
64
92
⋮
187
214

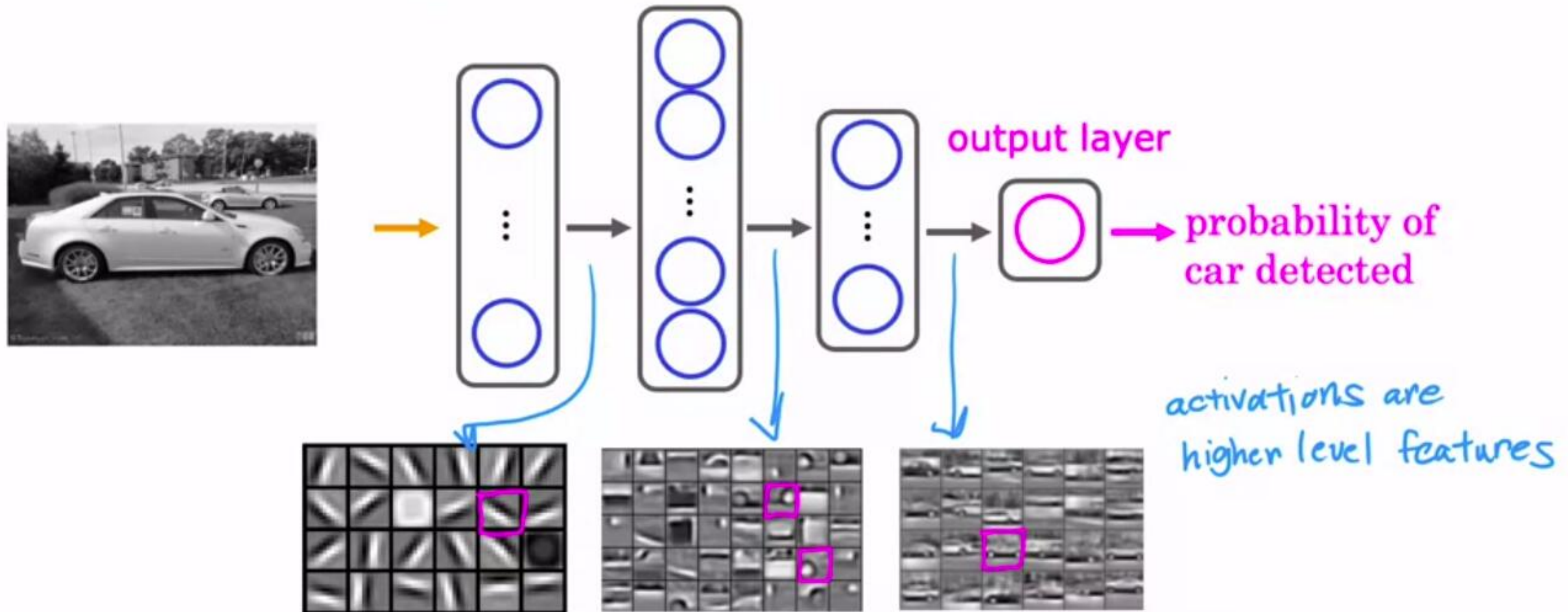
1 million

Face recognition



source: Convolutional Deep Belief Networks for Scalable Unsupervised Learning of Hierarchical Representations
by Honglak Lee, Roger Grosse, Ranganath Andrew Y. Ng

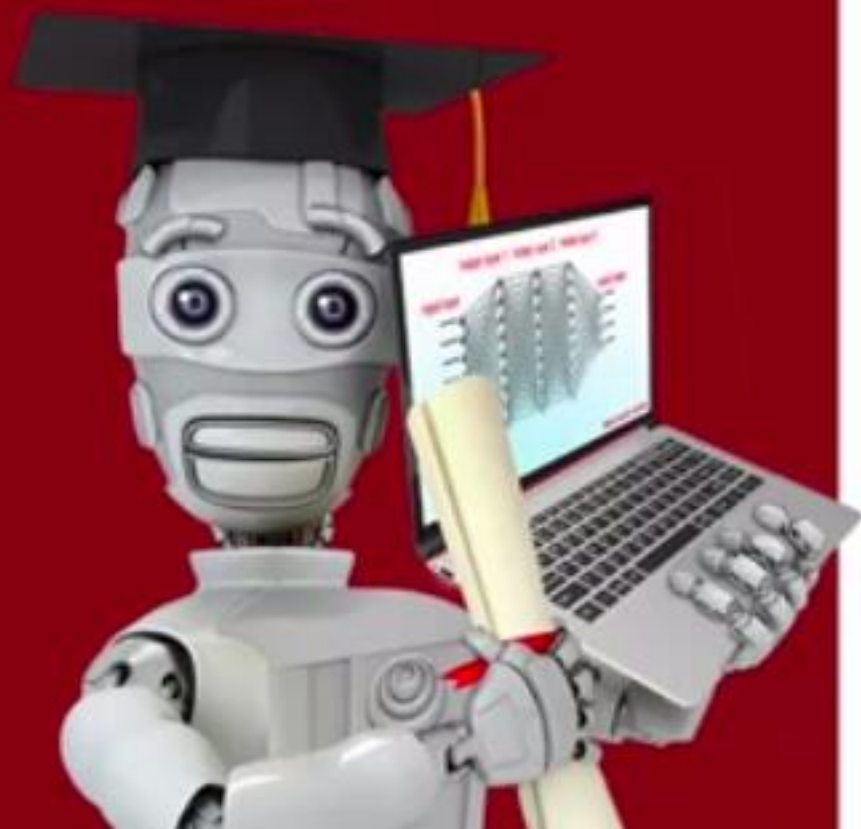
Car classification



source: Convolutional Deep Belief Networks for Scalable Unsupervised Learning of Hierarchical Representations
by Honglak Lee, Roger Grosse, Ranganath Andrew Y. Ng

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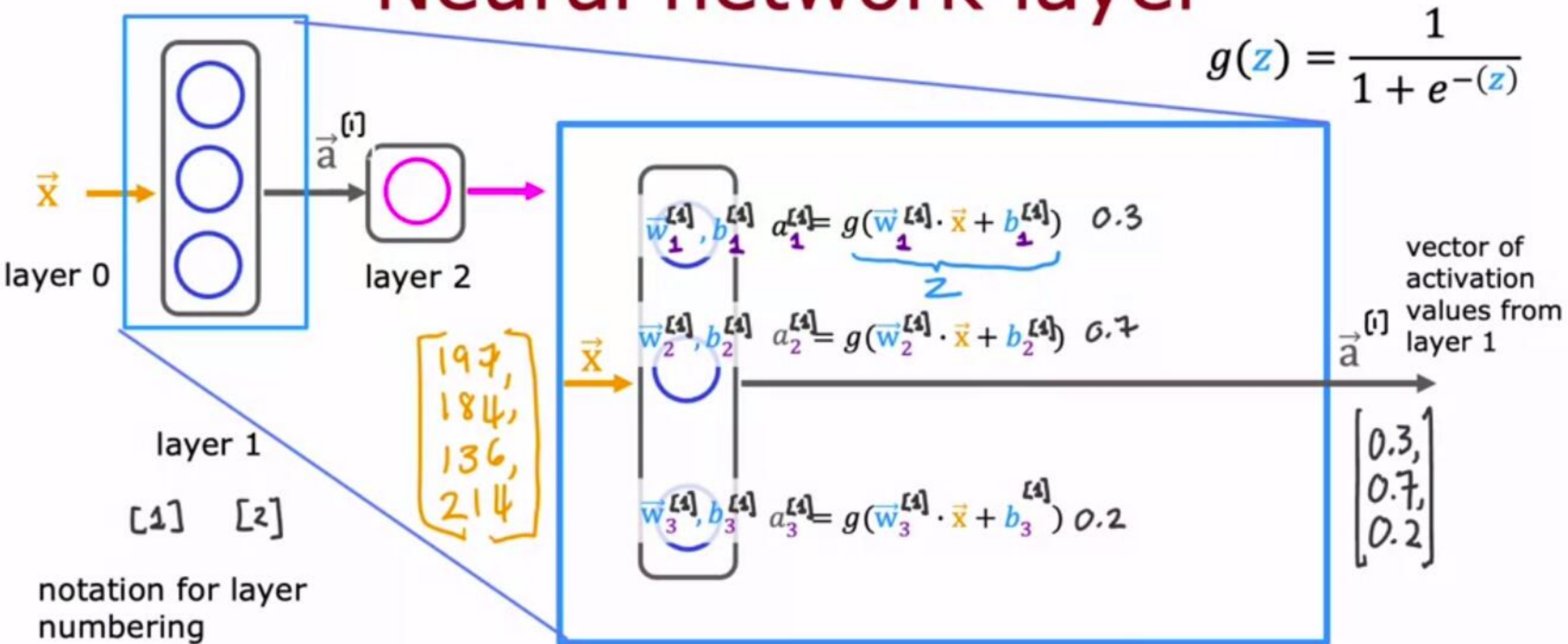
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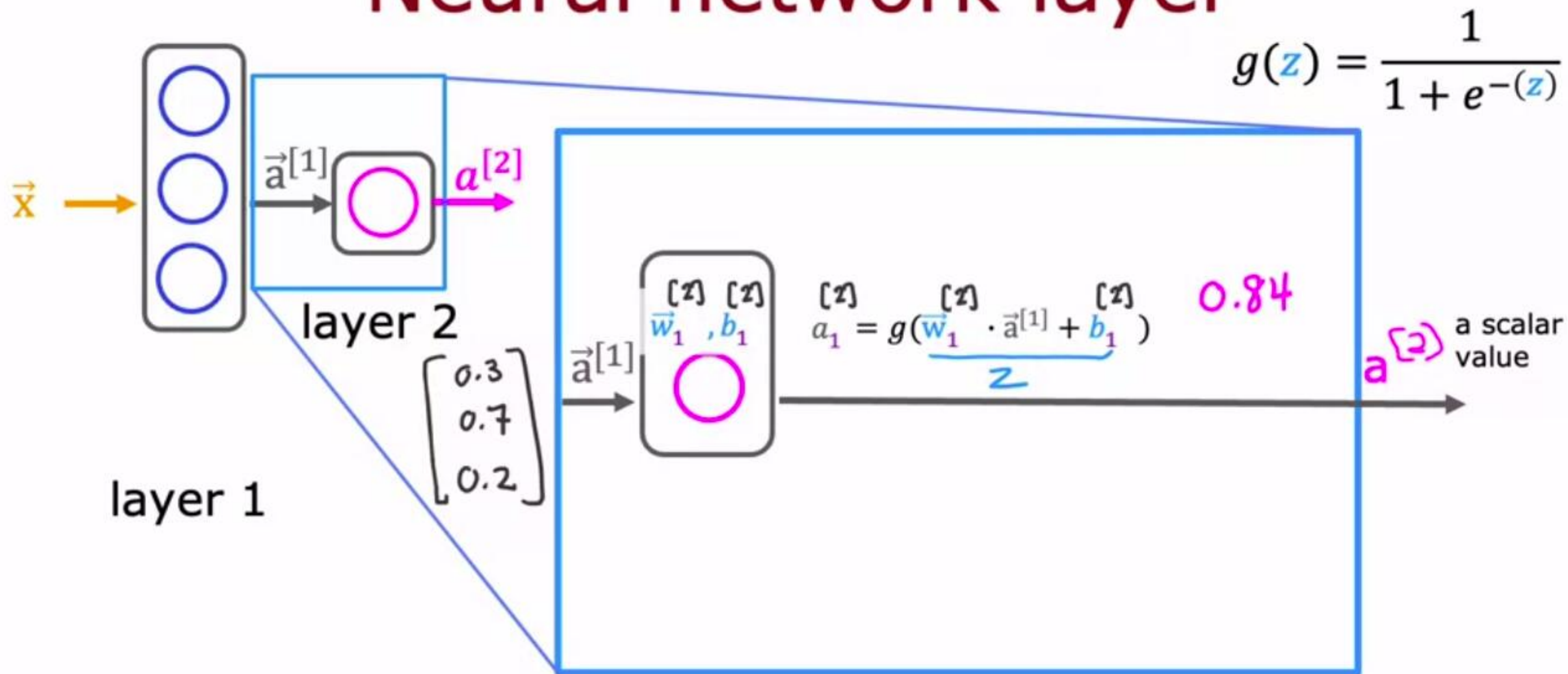
Neural network model

Neural network layer

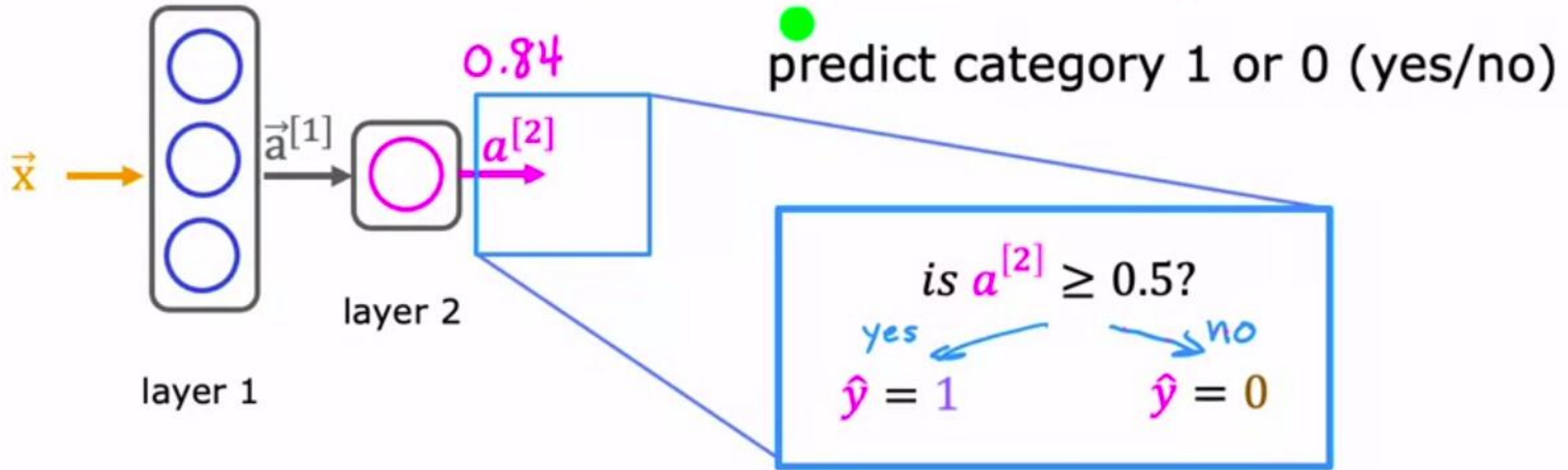
Neural network layer



Neural network layer

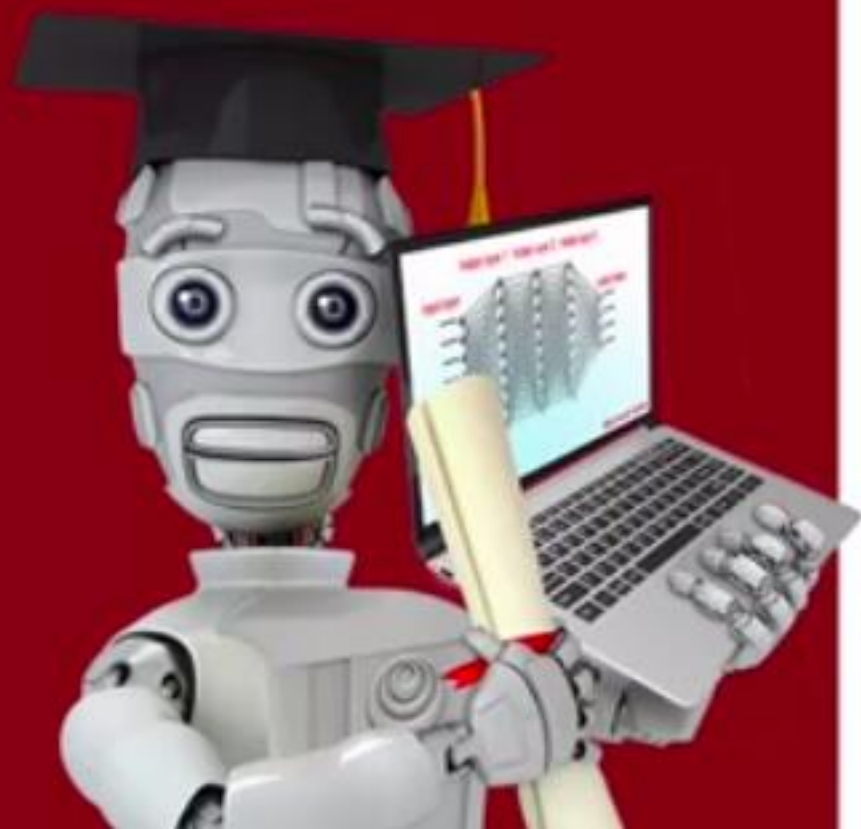


Neural network layer



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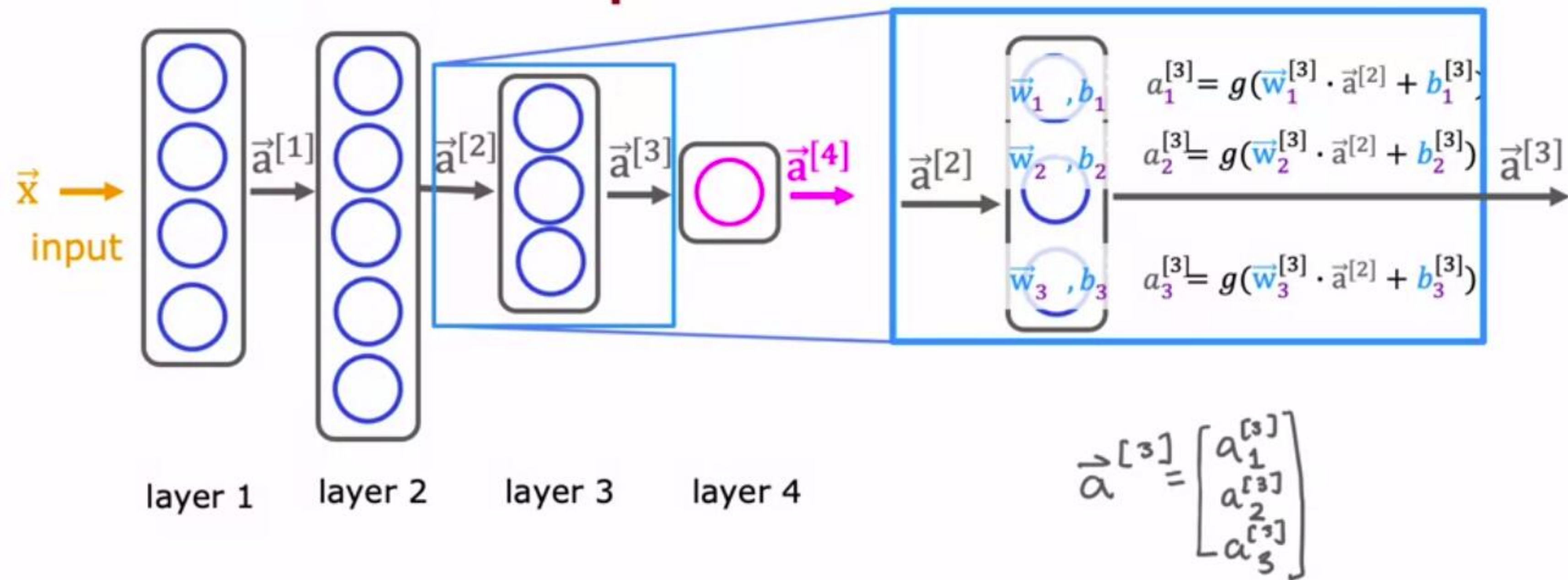
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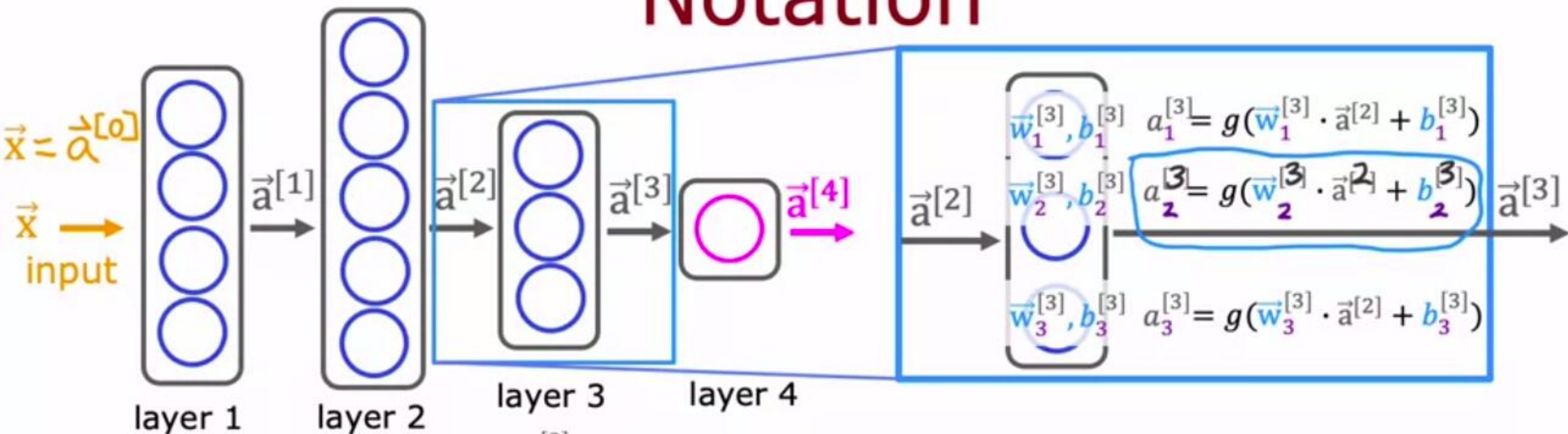
Neural Network Model

More complex neural networks

More complex neural network



Notation



Activation value of
layer l , unit(neuron) j

$$a_2^{[3]} = g(\bar{w}_2^{[3]} \cdot \bar{a}^{[2]} + b_2^{[3]})$$

output of layer $l - 1$
(previous layer)

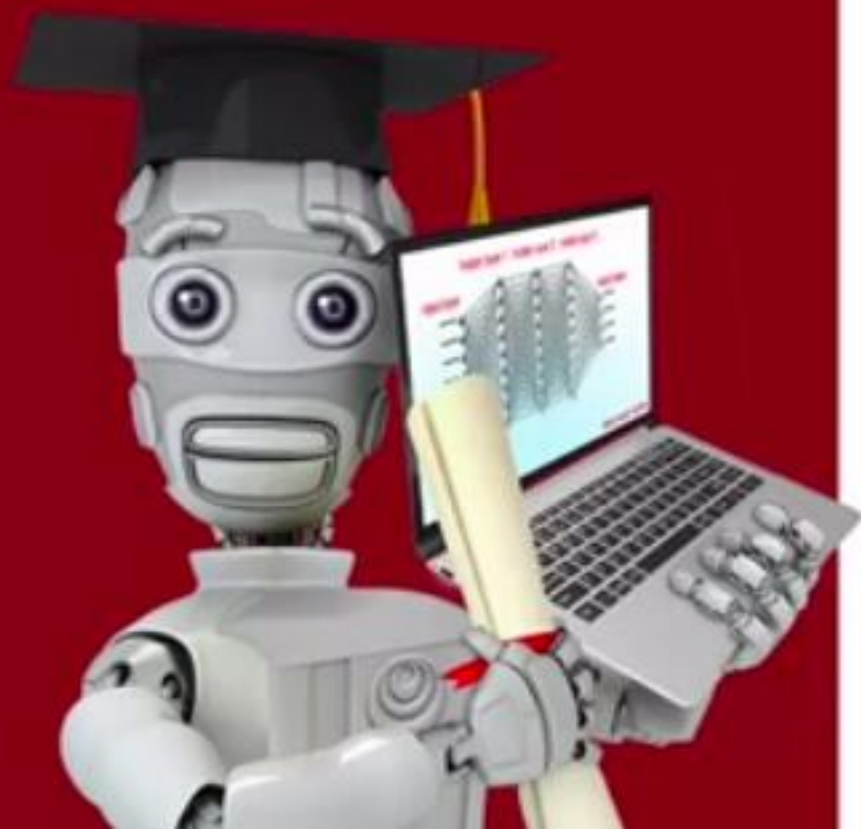
$$a_j^{[l]} = g(\bar{w}_j^{[l]} \cdot \bar{a}^{[l-1]} + b_j^{[l]})$$

sigmoid
"activation function"

Parameters w & b of layer l , unit j

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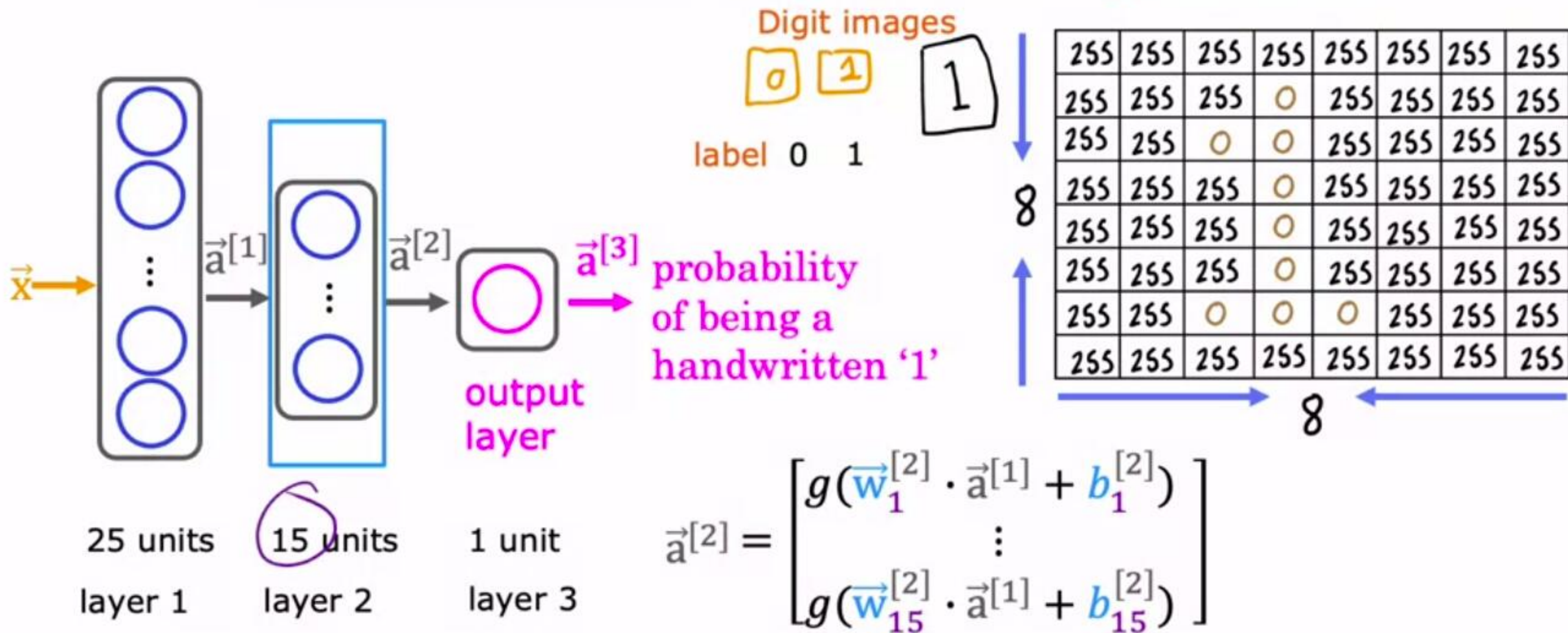
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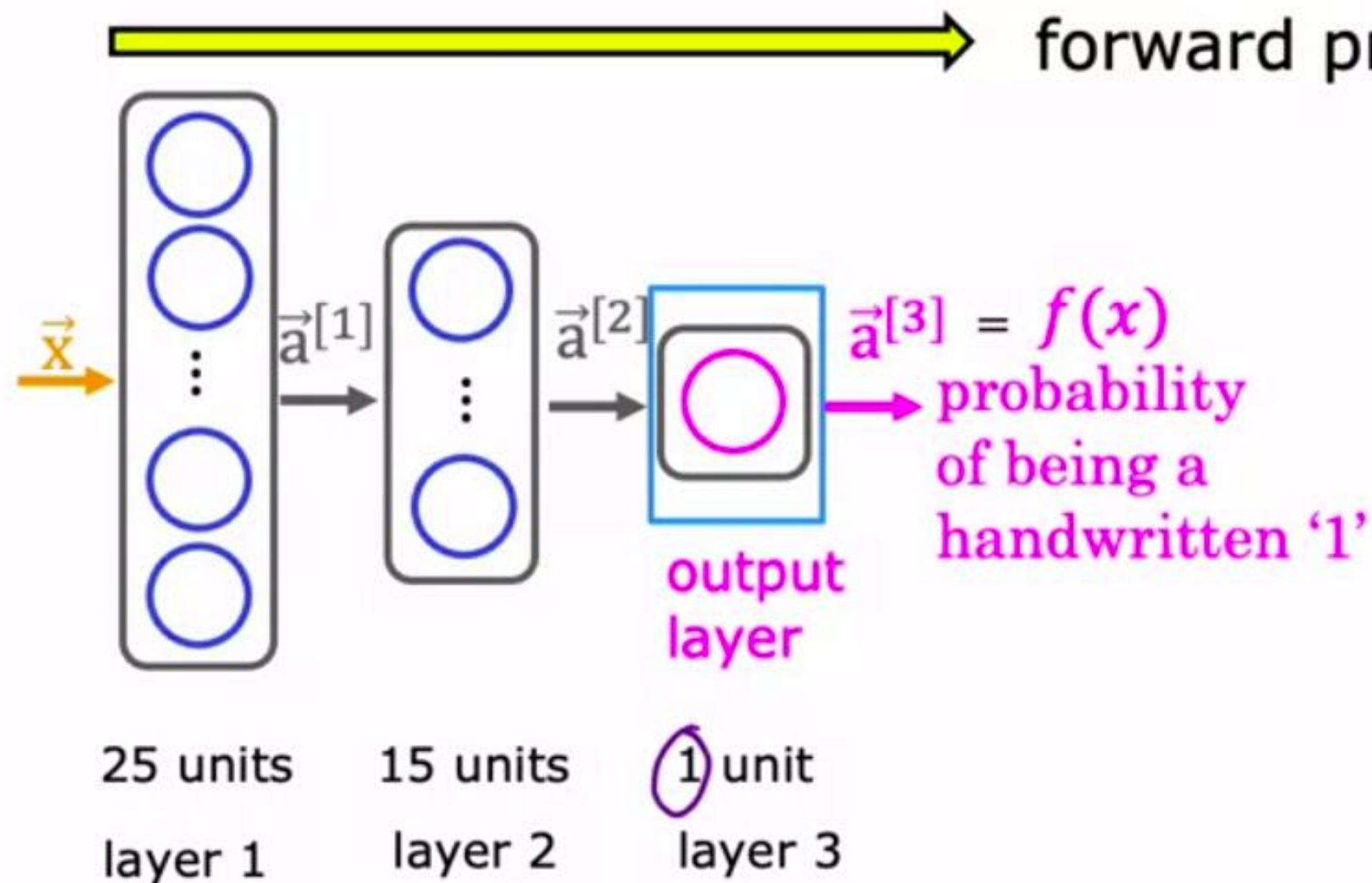
Neural Network Model

**Inference: making predictions
(forward propagation)**

Handwritten digit recognition



Handwritten digit recognition



$$\vec{a}^{[3]} = \left[g \left(\vec{w}_1^{[3]} \cdot \vec{a}^{[2]} + b_1^{[3]} \right) \right]$$

$$\text{is } a_1^{[3]} \geq 0.5?$$

yes

$$\hat{y} = 1$$

image is digit 1

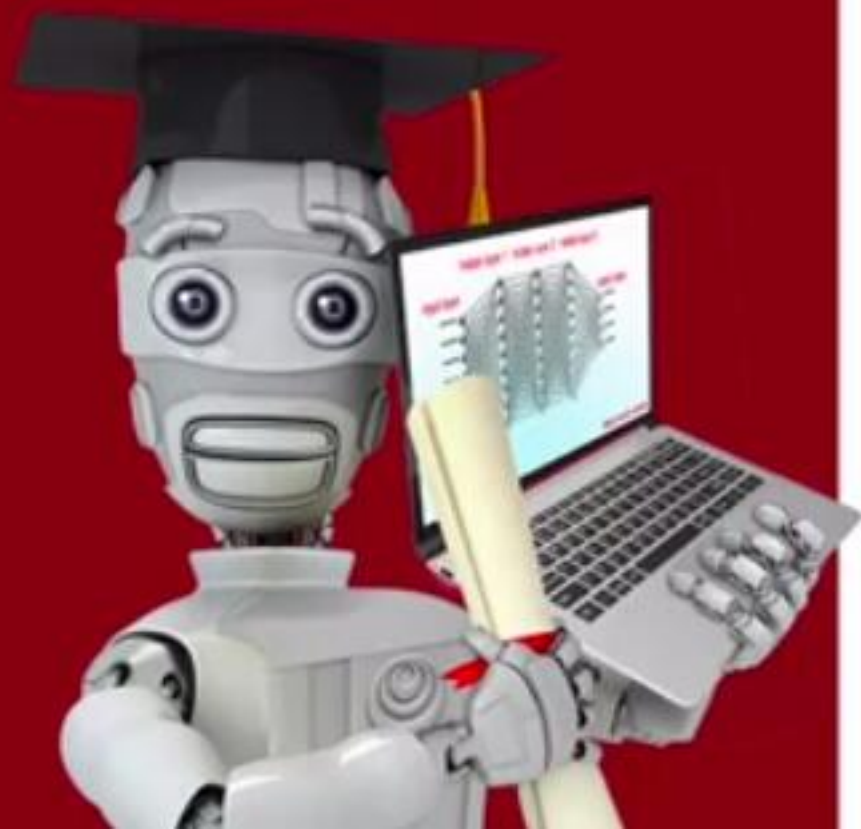
no

$$\hat{y} = 0$$

image isn't digit 1

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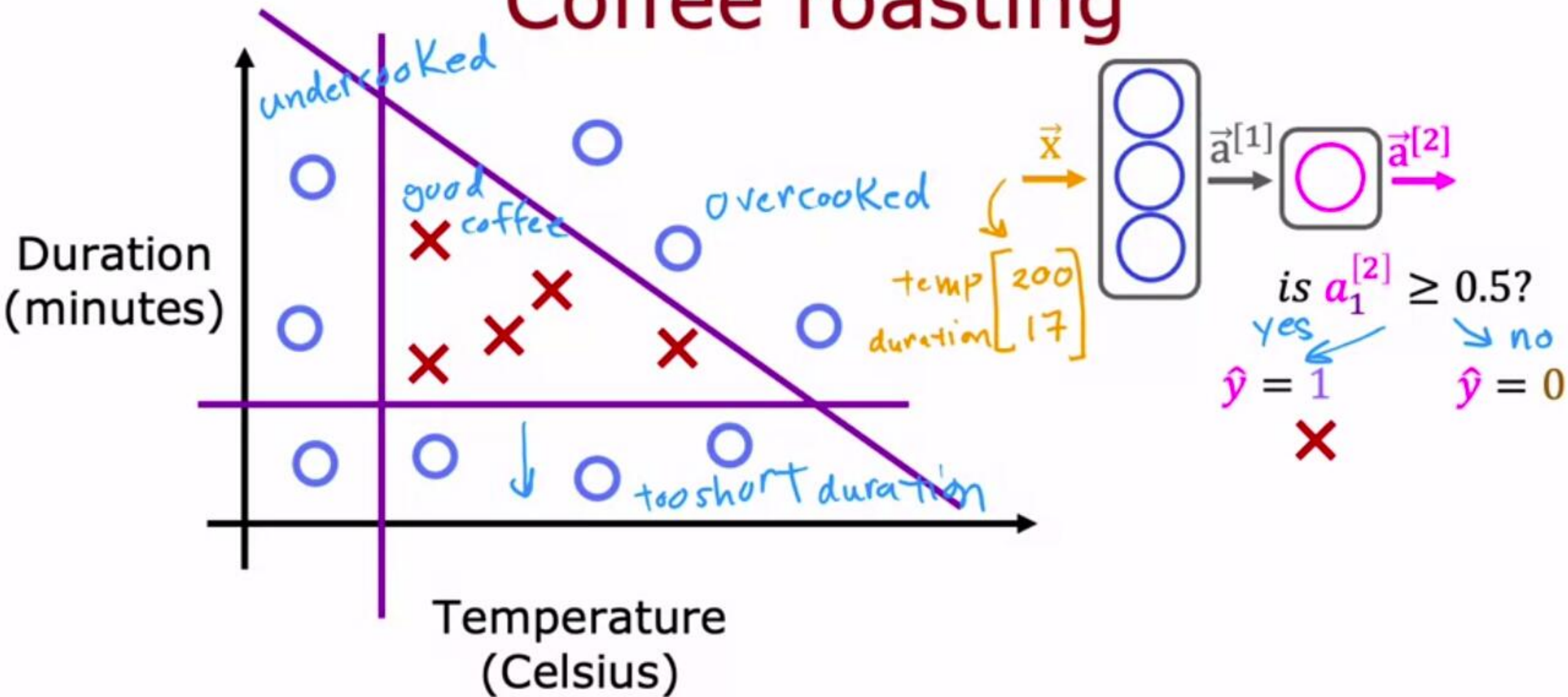
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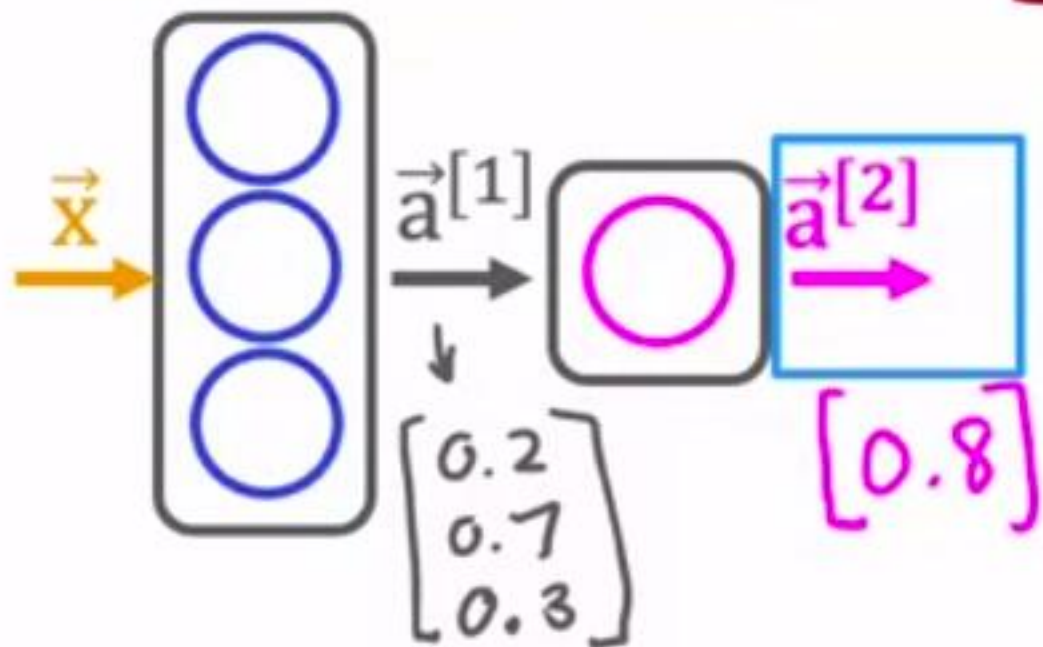
TensorFlow implementation

Inference in Code

Coffee roasting



Build the model using TensorFlow



```
x = np.array([[200.0, 17.0]])  
layer_1 = Dense(units=3, activation='sigmoid')  
a1 = layer_1(x)
```

```
layer_2 = Dense(units=1, activation='sigmoid')  
a2 = layer_2(a1)
```

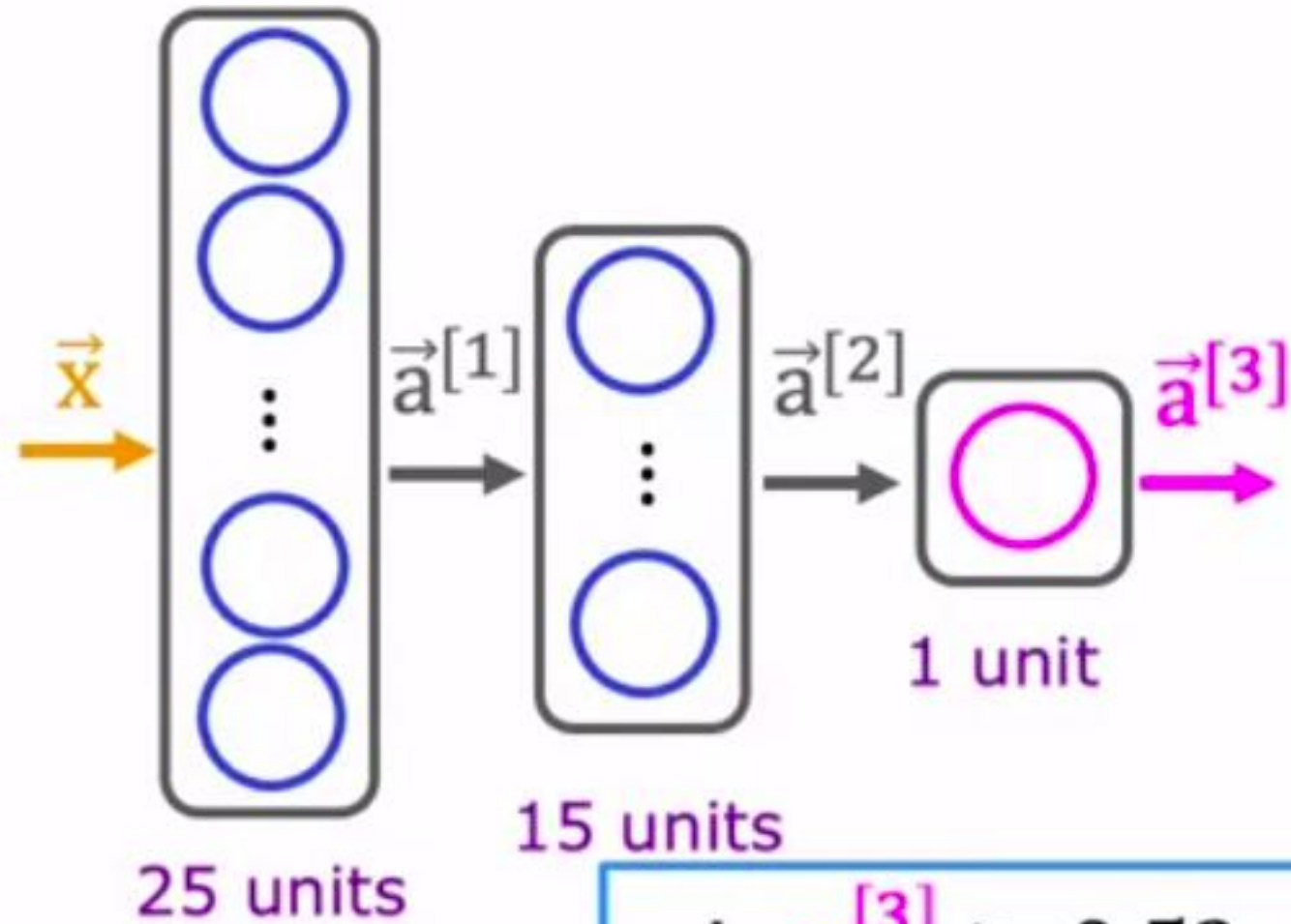
is $a_1^{[2]} \geq 0.5$?

yes $\hat{y} = 1$ \times

no $\hat{y} = 0$ \circ

```
if a2 >= 0.5:  
    yhat = 1  
else:  
    yhat = 0
```

Model for digit classification



```
x = np.array([[0.0, ..., 245, ..., 240, ..., 0]])  
layer_1 = Dense(units=25, activation='sigmoid')  
a1 = layer_1(x)
```

```
layer_2 = Dense(units=15, activation='sigmoid')  
a2 = layer_2(a1)
```

```
layer_3 = Dense(units=1, activation='sigmoid')  
a3 = layer_3(a2)
```

is $a_1^{[3]} \geq 0.5$?

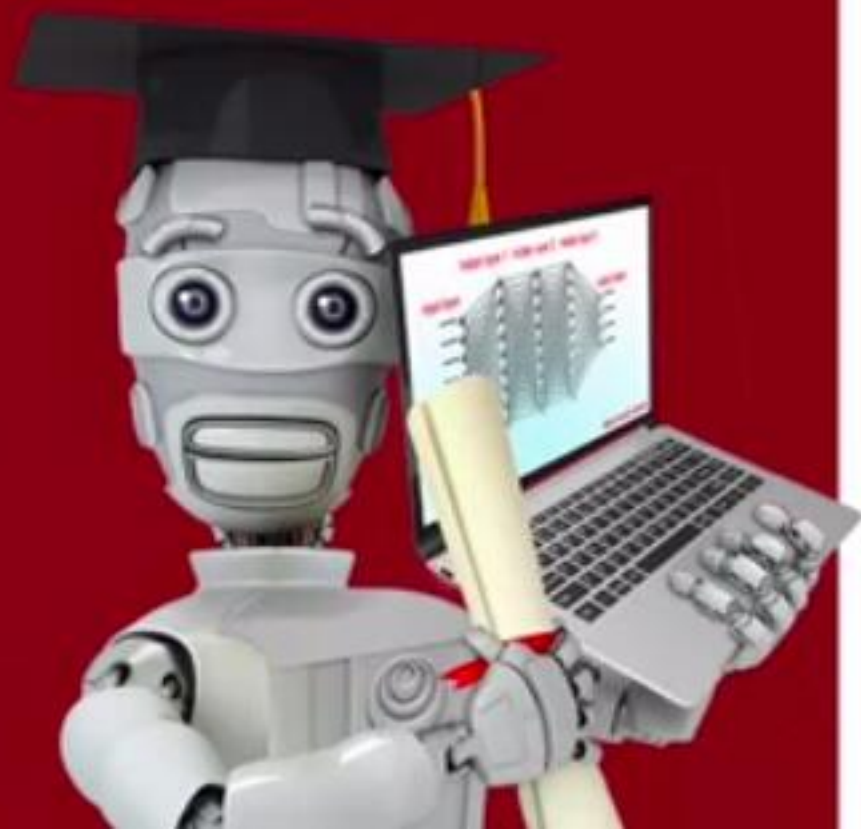
$\hat{y} = 1$
X

$\hat{y} = 0$
O

```
if a3 >= 0.5:  
    yhat
```


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TensorFlow implementation

Data in TensorFlow

Feature vectors

temperature (Celsius)	duration (minutes)	Good coffee? (1/0)
200.0	17.0	1
425.0	18.5	0
...

```
x = np.array([[200.0, 17.0]])
```

←

[[200.0, 17.0]]

why?

Note about numpy arrays

2 rows
3 columns
 $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$
2 x 3 matrix

4 rows
2 columns
 $\begin{bmatrix} 0.1 & 0.2 \\ -3 & -4 \\ -5 & -6 \\ 7 & 8 \end{bmatrix}$
4 x 2 matrix

```
x = np.array([[1, 2, 3],  
              [4, 5, 6]])
```

2D array

```
x = np.array([[0.1, 0.2],  
              [-3.0, -4.0,],  
              [-0.5, -0.6,],  
              [7.0, 8.0,]])
```

```
[[0.1, 0.2],  
 [-3.0, -4.0,],  
 [-0.5, -0.6,],  
 [7.0, 8.0,]]
```

2 x 3

4 x 2

1 x 2

2 x 1

Note about numpy arrays

`x = np.array([[200, 17]])` → $\begin{bmatrix} 200 & 17 \end{bmatrix}$ 1×2

`x = np.array([[200],
[17]])` → $\begin{bmatrix} 200 \\ 17 \end{bmatrix}$ 2×1

→ `x = np.array([200, 17])`

1D
"vector"

Feature vectors

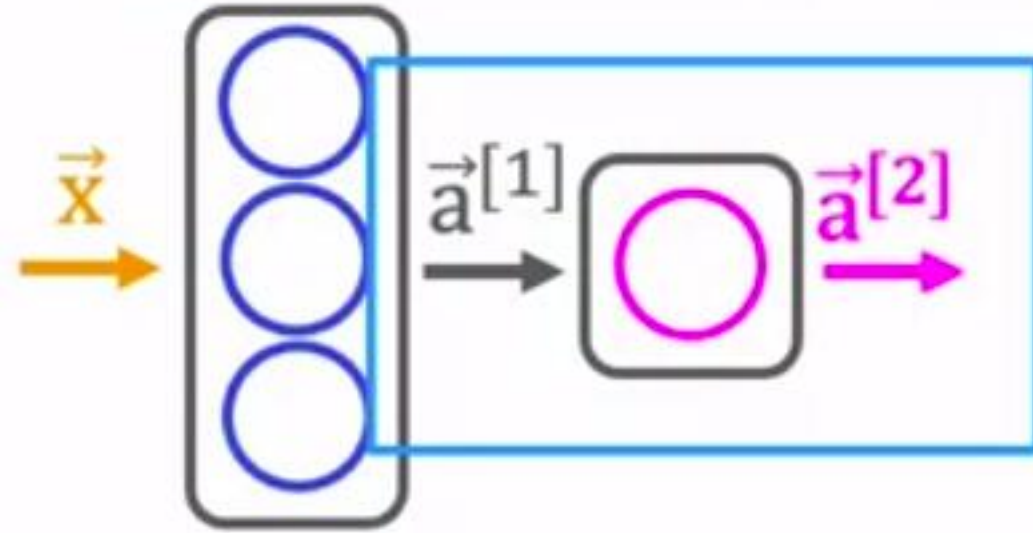
temperature (Celsius)	duration (minutes)	Good coffee? (1/0)
200.0	17.0	1
425.0	18.5	0
...

`x = np.array([[200.0, 17.0]])` ←

`[[200.0, 17.0]]`

↓ ↓
→ `[200.0 17.0]` 1 x 2

Activation vector



```
→ layer_2 = Dense(units=1, activation='sigmoid')  
→ a2 = layer_2(a1)
```

↪ $[[0.8]]$ ←

1 x 1

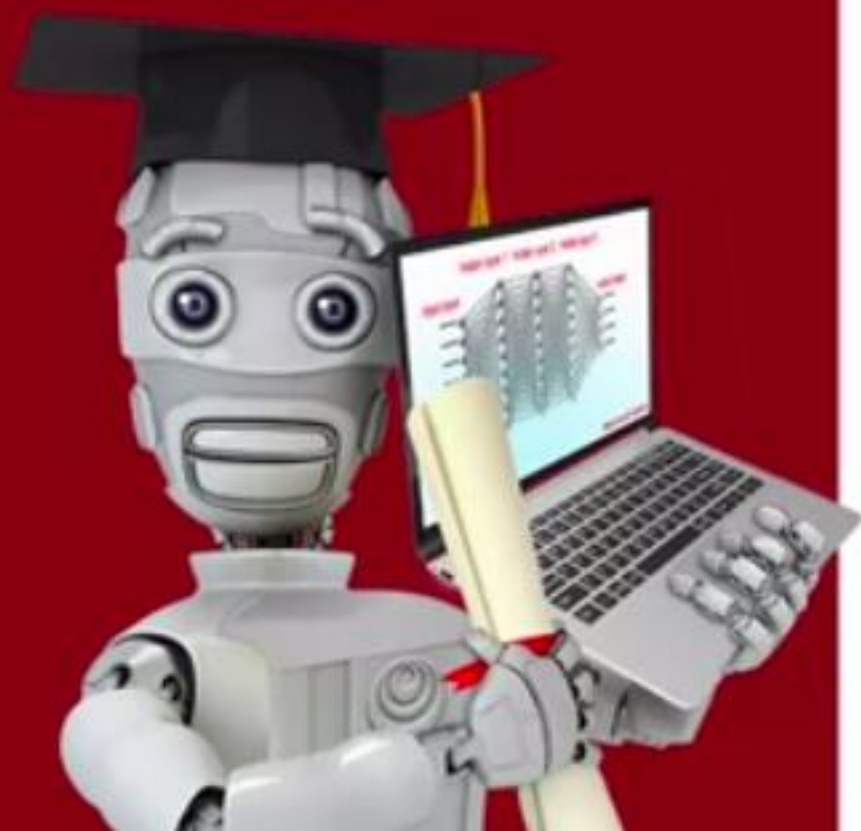
```
→ tf.Tensor([[0.8]], shape=(1, 1), dtype=float32)
```

```
→ a2.numpy()
```

```
array([[0.8]], dtype=float32)
```


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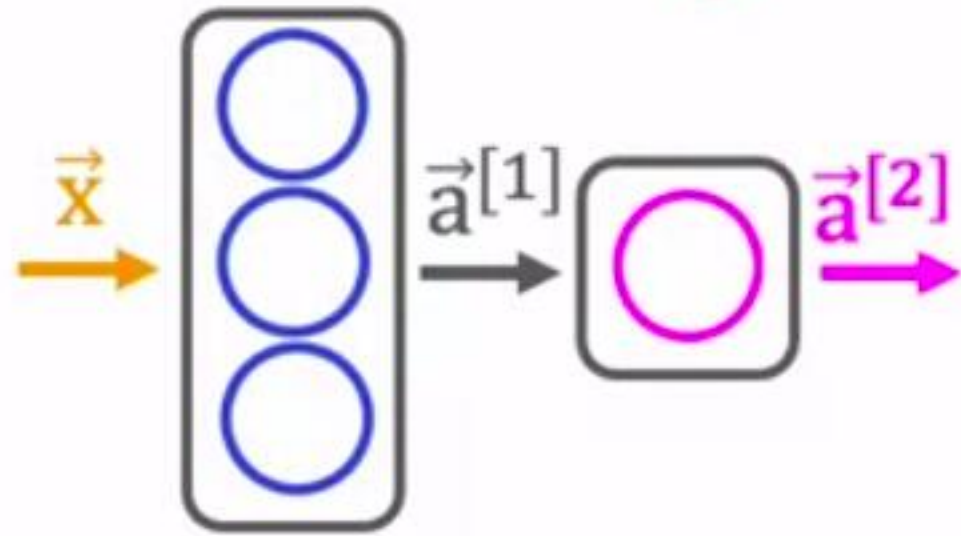
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TensorFlow implementation

Building a neural network

Building a neural network architecture



```
→ model = Sequential([  
→   Dense(units=3, activation="sigmoid"),  
→   Dense(units=1, activation="sigmoid")])
```

		y
200	17	1
120	5	0
425	20	0
212	18	1

```
x = np.array([[200.0, 17.0],  
              [120.0, 5.0],  
              [425.0, 20.0],  
              [212.0, 18.0]])
```

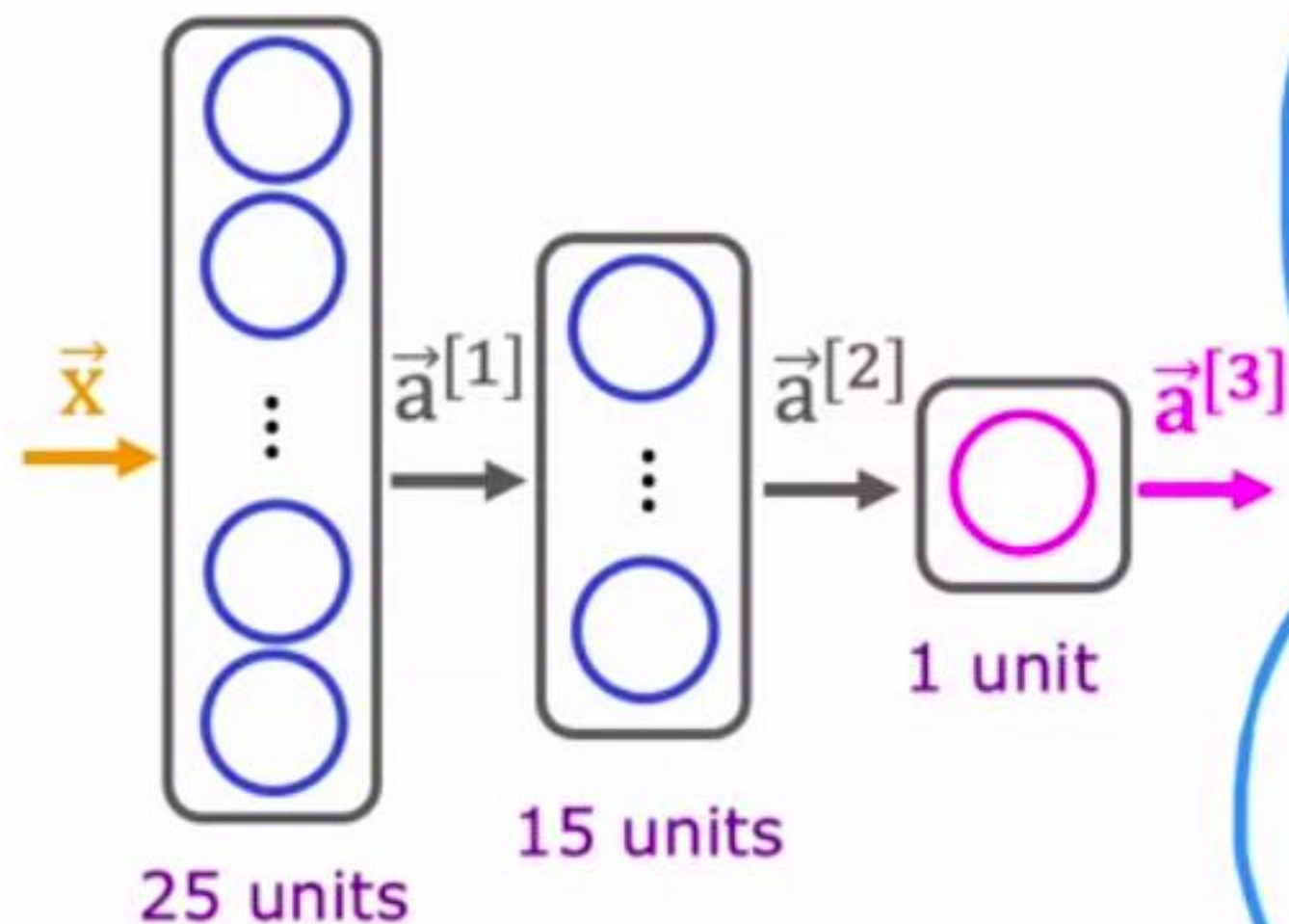
4 x 2

targets `y = np.array([1,0,0,1])`

```
model.compile(...) ← more about this next week!  
model.fit(x,y)
```

```
→ model.predict(x_new) ←
```


Digit classification model



```
→ layer_1 = Dense(units=25, activation="sigmoid")  
→ layer_2 = Dense(units=15, activation="sigmoid")  
→ layer_3 = Dense(units=1, activation="sigmoid")  
→ model = Sequential([layer_1, layer_2, layer_3])  
model.compile(...)
```

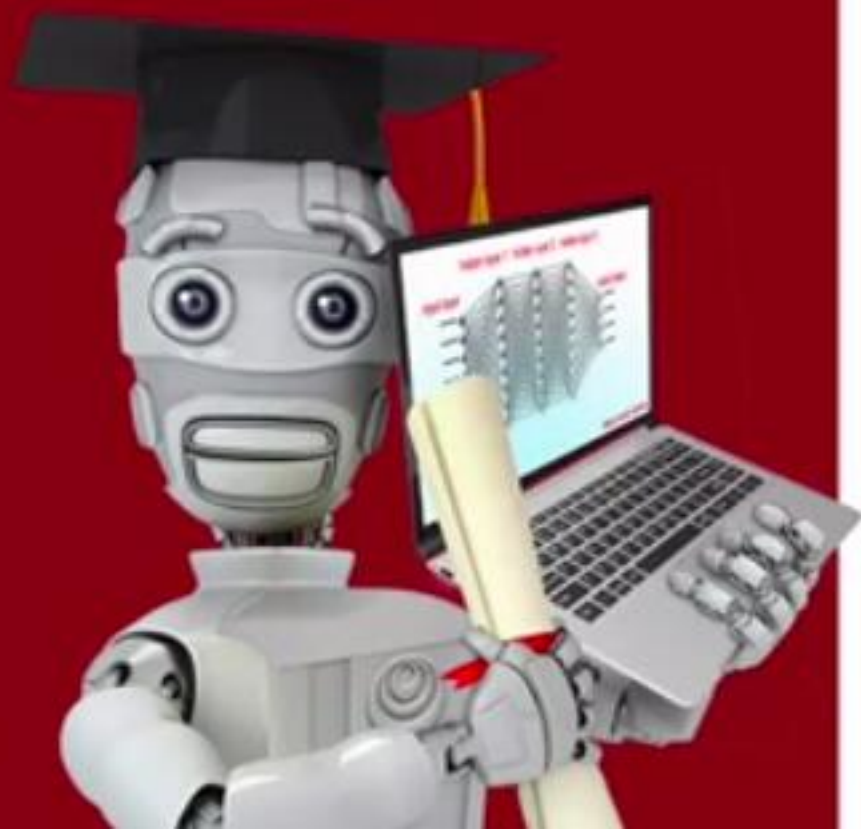
```
x = np.array([[0..., 245, ..., 17],  
              [0..., 200, ..., 184]])  
y = np.array([1,0])
```

```
model.fit(x,y) ← more about this next week!
```

```
→ model.predict(x_new)
```

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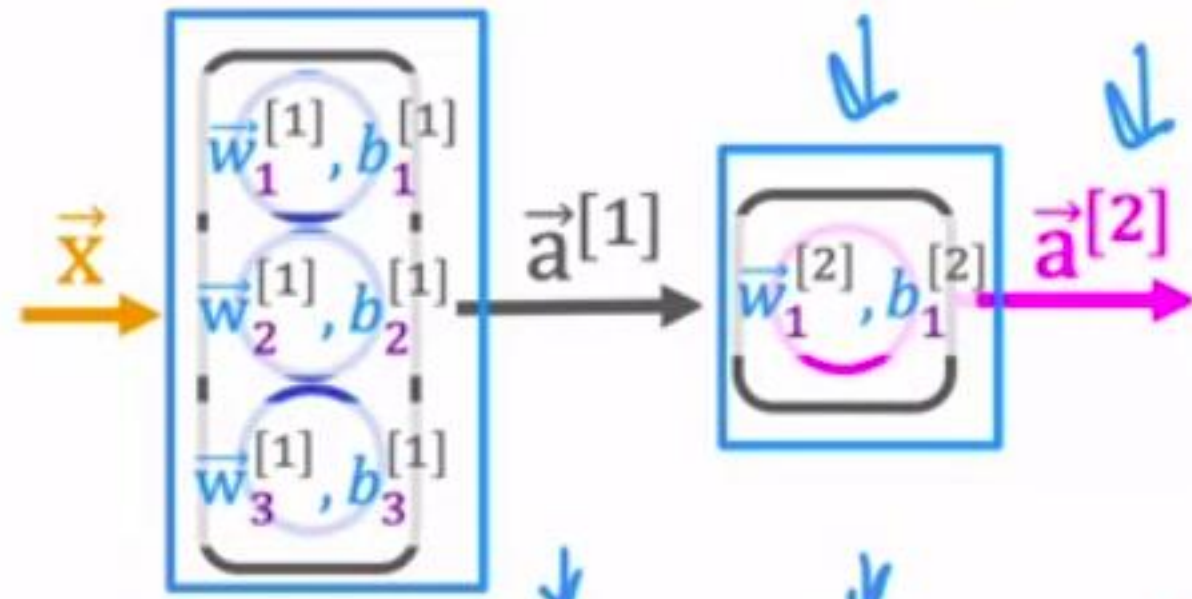
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Neural network implementation in Python

Forward prop in a single layer

forward prop (coffee roasting model)



$x = \text{np.array}([200, 17])$

$$a_1^{[1]} = g(\bar{w}_1^{[1]} \cdot \vec{x} + b_1^{[1]})$$

$w1_1 = \text{np.array}([1, 2])$

$b1_1 = \text{np.array}([-1])$

$z1_1 = \text{np.dot}(w1_1, x) + b1_1$

$a1_1 = \text{sigmoid}(z1_1)$

1D arrays

$$a_2^{[1]} = g(\bar{w}_2^{[1]} \cdot \vec{x} + b_2^{[1]})$$

$w1_2 = \text{np.array}([-3, 4])$

$b1_2 = \text{np.array}([1])$

$z1_2 = \text{np.dot}(w1_2, x) + b1_2$

$a1_2 = \text{sigmoid}(z1_2)$

$a1 = \text{np.array}([a1_1, a1_2, a1_3])$

$$a_1^{[2]} = g(\bar{w}_1^{[2]} \cdot \vec{a}^{[1]} + b_1^{[2]})$$

$\Rightarrow w2_1 = \text{np.array}([-7, 8, 9])$

$\Rightarrow b2_1 = \text{np.array}([3])$

$\Rightarrow z2_1 = \text{np.dot}(w2_1, a1) + b2_1$

$\Rightarrow a2_1 = \text{sigmoid}(z2_1)$

$w_1^{[2]}$ $w2_1$

$$a_3^{[1]} = g(\bar{w}_3^{[1]} \cdot \vec{x} + b_3^{[1]})$$

$w1_3 = \text{np.array}([5, -6])$

$b1_3 = \text{np.array}([2])$

$z1_3 = \text{np.dot}(w1_3, x) + b1_3$

$a1_3 = \text{sigmoid}(z1_3)$

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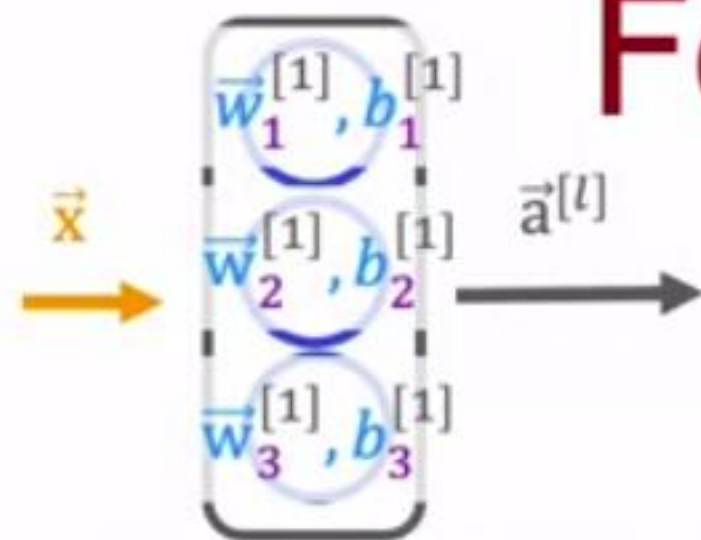
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Neural network implementation in Python

General implementation of
forward propagation

Forward prop in NumPy



$$\bar{w}_1^{[1]} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad \bar{w}_2^{[1]} = \begin{bmatrix} -3 \\ 4 \end{bmatrix} \quad \bar{w}_3^{[1]} = \begin{bmatrix} 5 \\ -6 \end{bmatrix}$$

W = np.array([
 $\begin{bmatrix} 1 & -3 & 5 \\ 2 & 4 & -6 \end{bmatrix}$]) 2 by 3

$$b_1^{[l]} = -1 \quad b_2^{[l]} = 1 \quad b_3^{[l]} = 2$$

b = np.array([-1, 1, 2])

$$\vec{a}^{[0]} = \vec{x}$$

a_in = np.array([-2, 4])

```
def dense(a_in, W, b):
    3 units = W.shape[1]
    a_out = np.zeros(units)
    for j in range(units):
        w = W[:, j]
        z = np.dot(w, a_in) + b[j]
        a_out[j] = g(z)
    return a_out
```

Note: g() is defined outside of dense().
 (see optional lab for details)

```
def sequential(x):
    a1 = dense(x, W1, b1)
    a2 = dense(a1, W2, b2)
    a3 = dense(a2, W3, b3)
    a4 = dense(a3, W4, b4)
    f_x = a4
    return f_x
```

capital W refers to a matrix

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Vectorization (optional)

How neural networks are
implemented efficiently

For loops vs. vectorization

```
x = np.array([200, 17])  
W = np.array([[1, -3, 5],  
              [-2, 4, -6]])  
b = np.array([-1, 1, 2])
```

```
def dense(a_in, W, b):  
    units = W.shape[1]  
    a_out = np.zeros(units)  
    for j in range(units):  
        w = W[:,j]  
        z = np.dot(w, a_in) + b[j]  
        a_out[j] = g(z)  
    return a_out
```

[1,0,1]

```
X = np.array([[200, 17]])  
W = np.array([[1, -3, 5],  
              [-2, 4, -6]])  
B = np.array([[-1, 1, 2]])
```

```
def dense(A_in, W, B):  
    Z = np.matmul(A_in, W) + B  
    A_out = g(Z)  
    return A_out  
[[1,0,1]]
```

vectorized

2D array

same

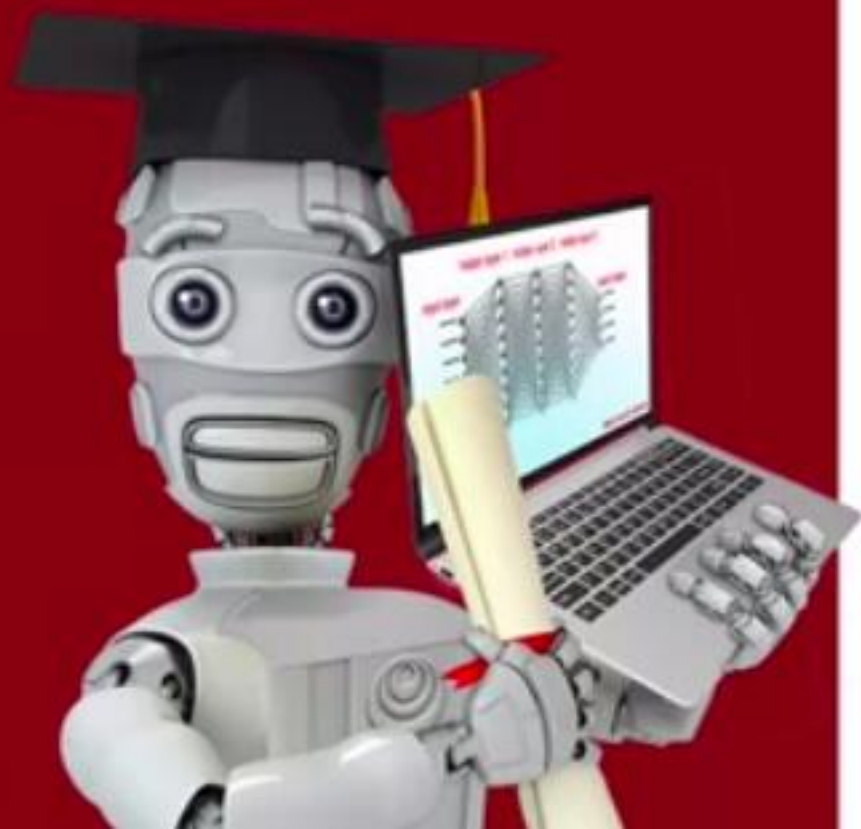
1x3 2D array

all 2D arrays

matrix multiplication

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**Vectorization
(optional)**

Matrix multiplication

Dot products

example

$$\begin{bmatrix} 1 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

$$z = (1 \times 3) + (2 \times 4)$$

3 + 8
11

in general

$$\begin{bmatrix} \uparrow \\ \vec{a} \\ \downarrow \end{bmatrix} \cdot \begin{bmatrix} \uparrow \\ \vec{w} \\ \downarrow \end{bmatrix}$$

$$z = \vec{a} \cdot \vec{w}$$

transpose

$$\vec{a} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\vec{a}^T = [1 \quad 2]$$

vector vector multiplication

$$\begin{bmatrix} \leftarrow \vec{a}^T \rightarrow \end{bmatrix} \begin{bmatrix} \uparrow \\ \vec{w} \\ \downarrow \end{bmatrix} \quad \begin{matrix} 1 \times 2 \\ 2 \times 1 \end{matrix}$$

$$z = \vec{a}^T \vec{w}$$

useful for understanding
matrix multiplication

equivalent

Vector matrix multiplication

$$\vec{a} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\vec{a}^T = \begin{bmatrix} 1 & 2 \end{bmatrix}$$

$$w = \begin{bmatrix} 3 \\ 4 \end{bmatrix} \begin{bmatrix} 5 \\ 6 \end{bmatrix}$$

$$Z = \vec{a}^T w$$

1 by 2

$$[\leftarrow \vec{a}^T \rightarrow] \begin{bmatrix} \uparrow & \uparrow \\ \vec{w}_1 & \vec{w}_2 \\ \downarrow & \downarrow \end{bmatrix}$$

$$Z = [\vec{a}^T \vec{w}_1 \quad \vec{a}^T \vec{w}_2]$$

$$\begin{aligned} (1 * 3) + (2 * 4) \\ 3 + 8 \\ 11 \end{aligned}$$

$$\begin{aligned} (1 * 5) + (2 * 6) \\ 5 + 12 \\ 17 \end{aligned}$$

$$Z = [11 \quad 17]$$

matrix matrix multiplication

$$A = \begin{bmatrix} 1 & -1 \\ 2 & -2 \end{bmatrix} \quad A^T = \begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix} \quad W = \begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix} \quad Z = A^T W = \begin{bmatrix} \leftarrow \vec{a}_1^T \rightarrow & \leftarrow \vec{a}_2^T \rightarrow \end{bmatrix} \begin{bmatrix} \uparrow \vec{w}_1 \downarrow & \uparrow \vec{w}_2 \downarrow \end{bmatrix}$$

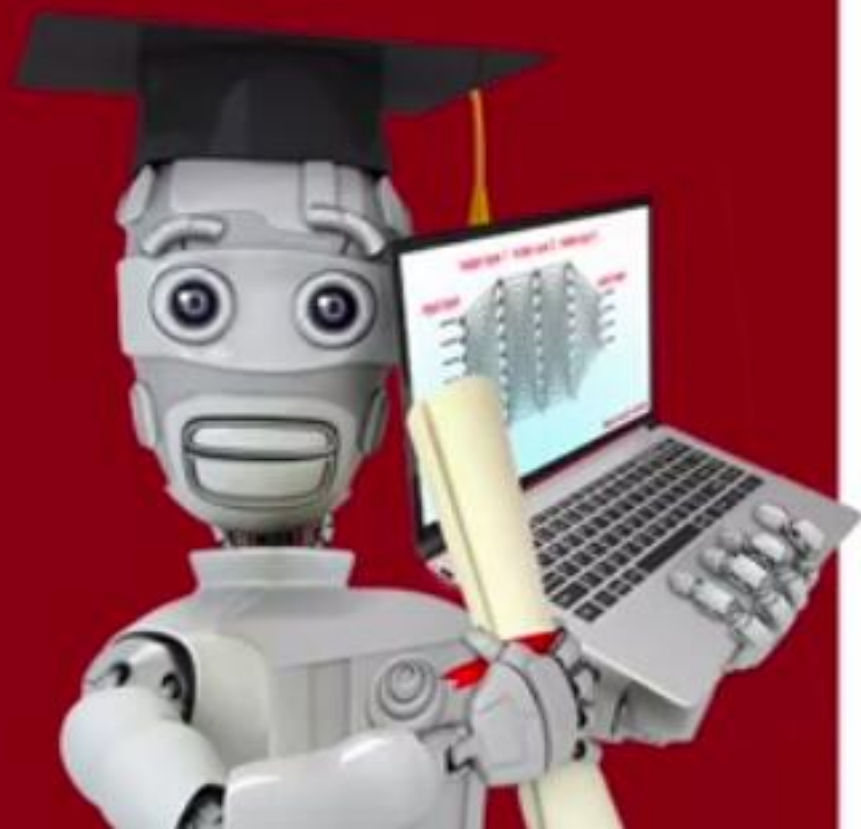
rows columns

$$\begin{array}{l} \text{row 1 col 1} = \begin{bmatrix} \vec{a}_1^T \vec{w}_1 & \vec{a}_1^T \vec{w}_2 \\ \vec{a}_2^T \vec{w}_1 & \vec{a}_2^T \vec{w}_2 \end{bmatrix} \begin{array}{l} \text{row 1 col 2} \\ \text{row 2 col 2} \end{array} \\ \text{row 2 col 1} \quad \downarrow \quad \downarrow \\ (-1 \times 3) + (-2 \times 4) \quad (-1 \times 5) + (-2 \times 6) \\ -3 \quad + \quad -8 \quad \quad -5 \quad + \quad -12 \\ -11 \quad \quad \quad -17 \\ = \begin{bmatrix} 11 & 17 \\ -11 & -17 \end{bmatrix} \end{array}$$

general rules for
matrix multiplication
↪ next video!

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Vectorization (optional)

Matrix multiplication rules

Matrix multiplication rules

$$A = \begin{bmatrix} 1 & -1 & 0.1 \\ 2 & -2 & 0.2 \end{bmatrix} \quad A^T = \begin{bmatrix} 1 & 2 \\ -1 & -2 \\ 0.1 & 0.2 \end{bmatrix} \quad W = \begin{bmatrix} 3 & 5 & 7 & 9 \\ 4 & 6 & 8 & 0 \end{bmatrix} \quad Z = A^T W = \begin{bmatrix} 11 & 17 & 23 & 9 \\ -11 & -17 & -23 & -9 \\ 1.1 & 1.7 & 2.3 & 0.9 \end{bmatrix}$$

3×2 2×4

can only take dot products
of vectors that are same length


$$[0.1 \ 0.2]$$

length 2

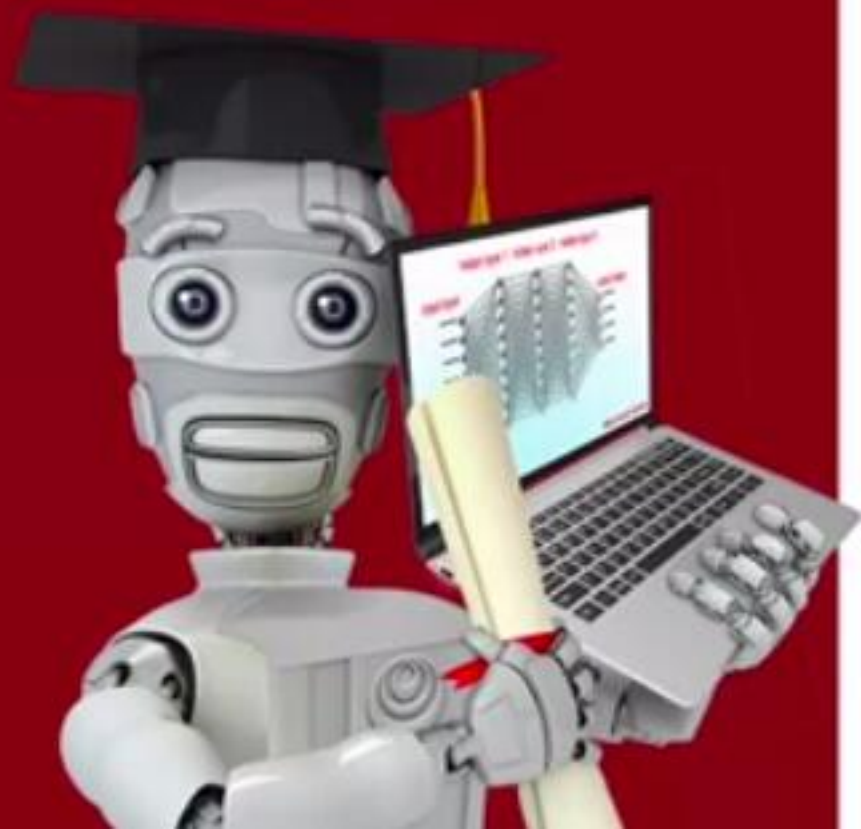
$$\begin{bmatrix} 5 \\ 6 \end{bmatrix}$$

length 2

3 by 4 matrix
↳ same # rows as A^T
↳ same # columns as W

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Vectorization (optional)

Matrix multiplication code

Matrix multiplication in NumPy

$$A = \begin{bmatrix} 1 & -1 & 0.1 \\ 2 & -2 & 0.2 \end{bmatrix} \quad A^T = \begin{bmatrix} 1 & 2 \\ -1 & -2 \\ 0.1 & 0.2 \end{bmatrix} \quad W = \begin{bmatrix} 3 & 5 & 7 & 9 \\ 4 & 6 & 8 & 0 \end{bmatrix} \quad Z = A^T W = \begin{bmatrix} 11 & 17 & 23 & 9 \\ -11 & -17 & -23 & -9 \\ 1.1 & 1.7 & 2.3 & 0.9 \end{bmatrix}$$

```
A=np.array([[1,-1,0.1],  
            [2,-2,0.2]])
```

```
W=np.array([[3,5,7,9],  
            [4,6,8,0]])
```

```
Z = np.matmul(AT,W)
```

or
→ $Z = AT @ W$

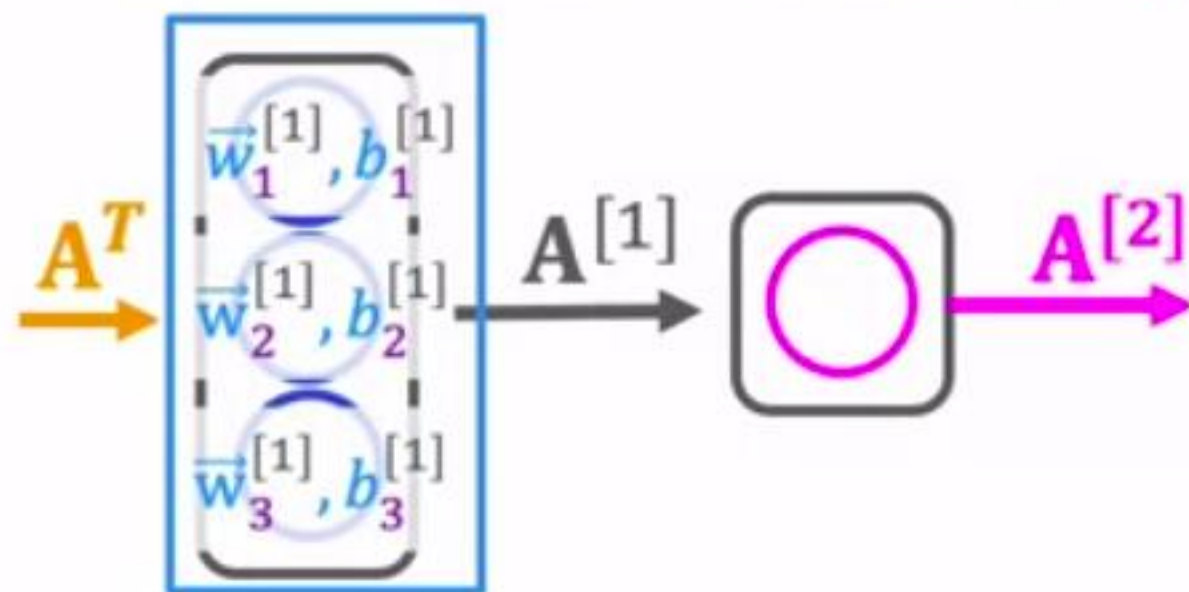
```
AT=np.array([[1,2],  
             [-1,-2],  
             [0.1,0.2]])
```

```
AT=A.T  
↳ transpose
```

result

```
[[11,17,23,9],  
 [-11,-17,-23,-9],  
 [1.1,1.7,2.3,0.9]]
```

Dense layer vectorized



$$A^T = \begin{bmatrix} 200 & 17 \end{bmatrix}$$

1 × 2

$$W = \begin{bmatrix} 1 & -3 & 5 \\ -2 & 4 & -6 \end{bmatrix}$$

2 × 3

$$B = \begin{bmatrix} -1 & 1 & 2 \end{bmatrix}$$

1 × 3

$$Z = A^T W + B$$

$$\begin{bmatrix} 165 & -531 & 900 \end{bmatrix}$$

$z_1^{[1]} \quad z_2^{[1]} \quad z_3^{[1]}$

$$A = g(Z)$$

$$\begin{bmatrix} 1 & 0 & 1 \end{bmatrix}$$

A

```
AT = np.array([[200, 17]])
W = np.array([[1, -3, 5],
              [-2, 4, -6]])
b = np.array([[ -1, 1, 2]])
```

a_{in}

```
def dense(AT, W, b):
    z = np.matmul(AT, W) + b
    a_out = g(z)  $a_{in}$ 
    return a_out
```

$[[1, 0, 1]]$ \downarrow