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Classification

Motivation

Classification

Question	Answer "y"	
Is this email spam?	no	yes
Is the transaction fraudulent?	no	yes
Is the tumor malignant?	no	yes

y can only be one of two values

"binary classification"

class = category

false true

0 1

useful for classification

"negative class"

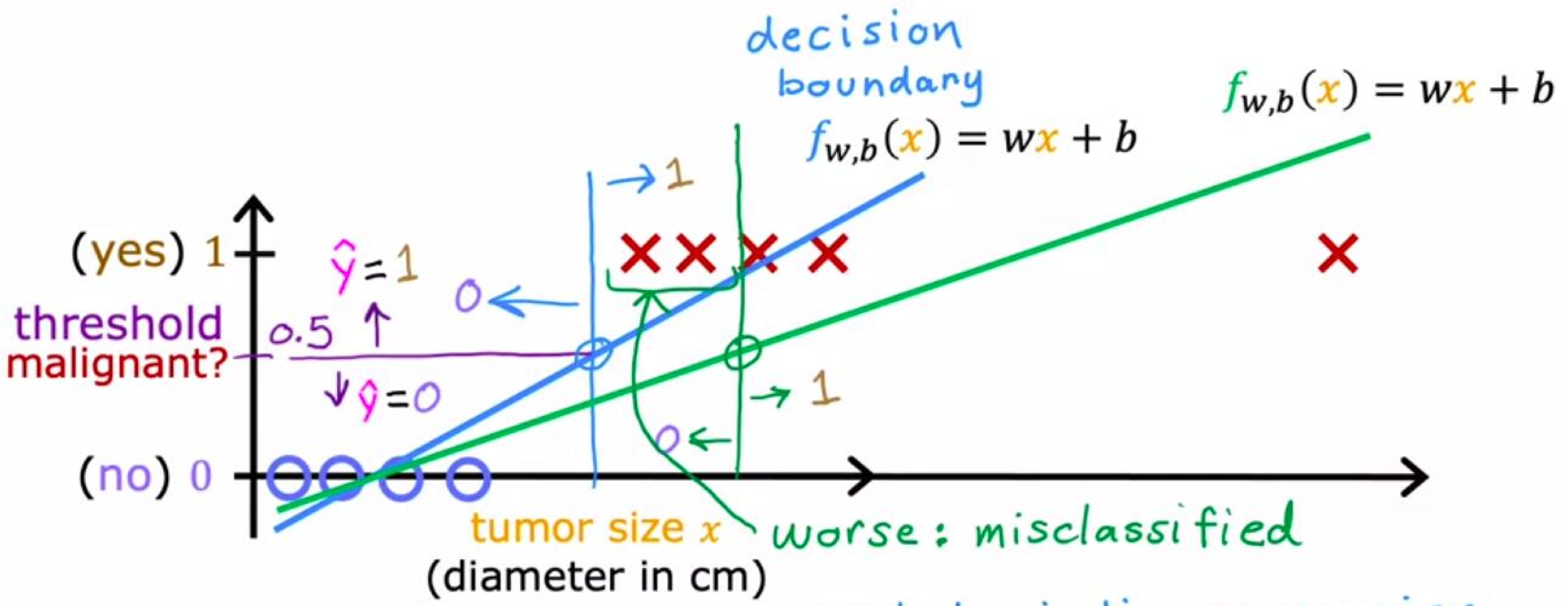
≠ "bad"

absence

"positive class"

≠ "good"

presence



if $f_{w,b}(x) < 0.5 \rightarrow \hat{y} = 0$

if $f_{w,b}(x) \geq 0.5 \rightarrow \hat{y} = 1$

next: logistic $\underbrace{\text{regression}}$ — classification

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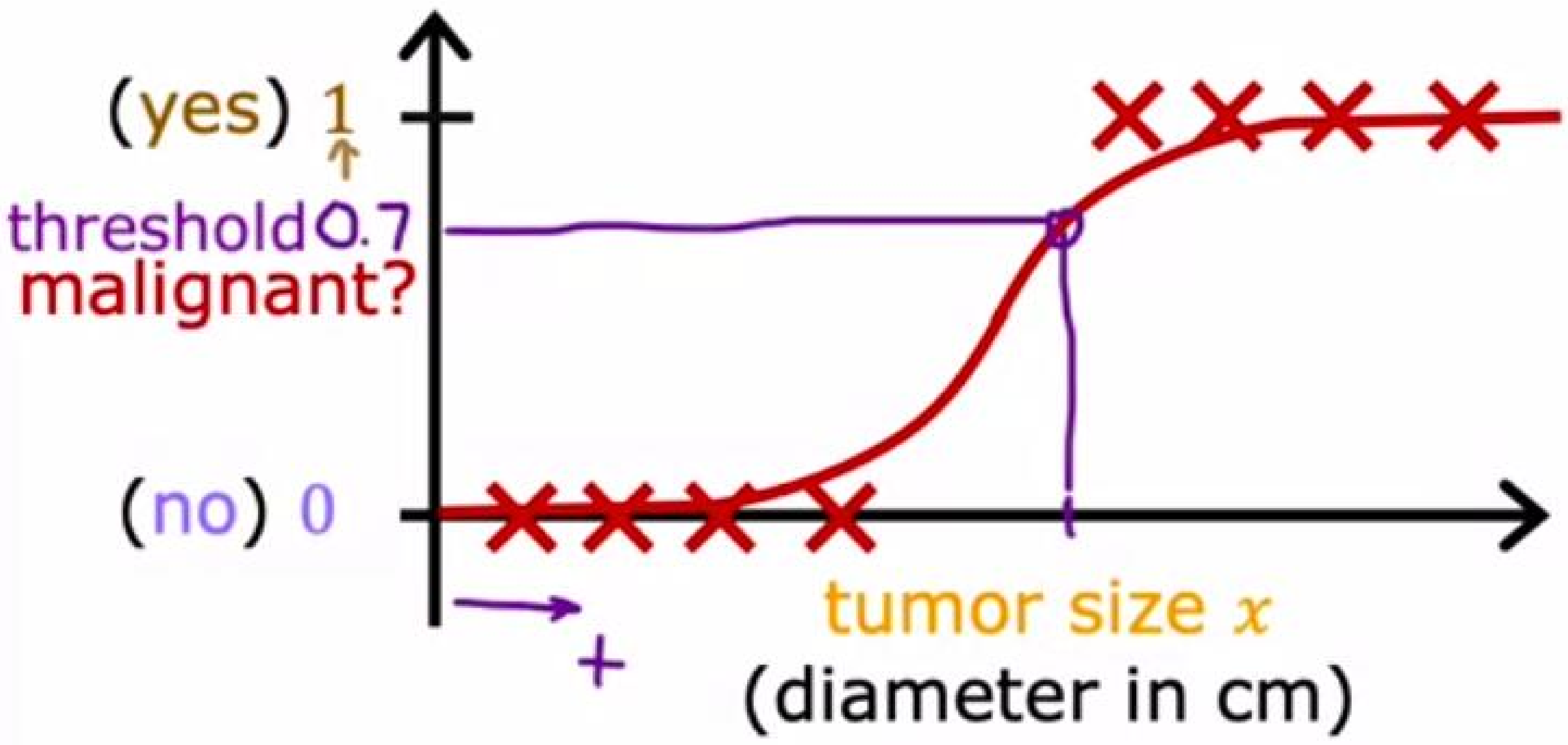
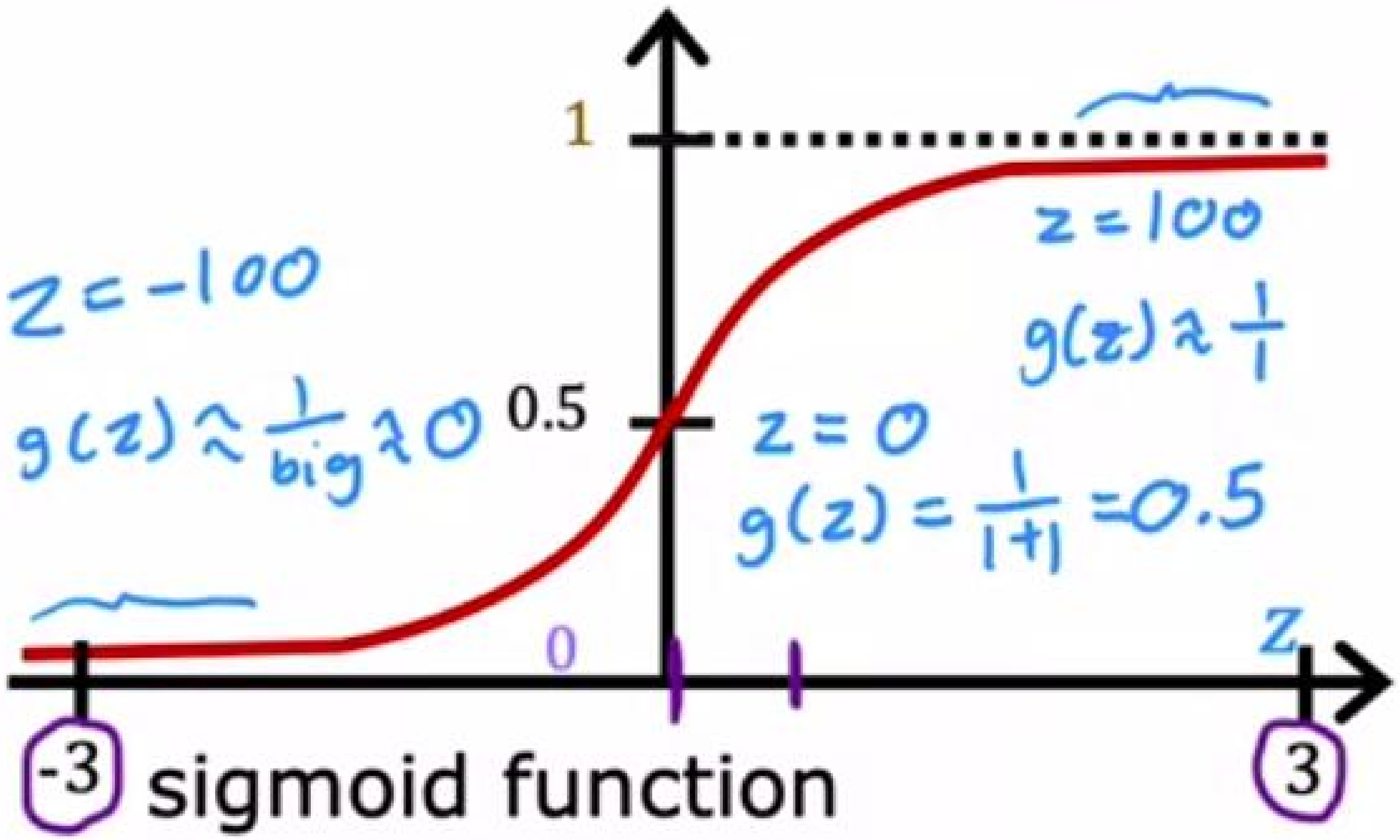
Classification

Logistic Regression

Want outputs between 0 and 1





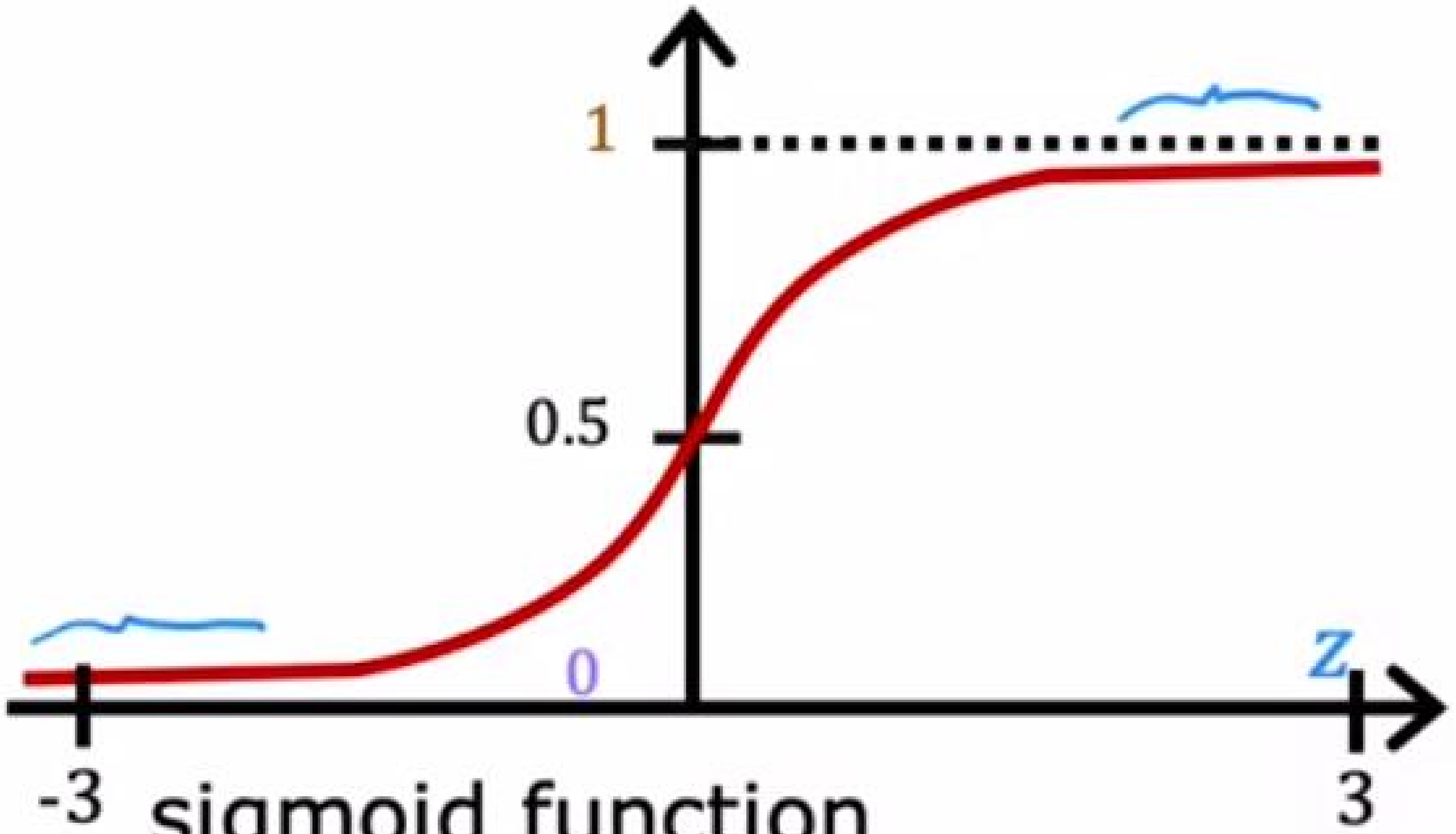
sigmoid function

logistic function

outputs between 0 and 1

$$g(z) = \frac{1}{1+e^{-z}} \qquad 0 < g(z) < 1$$

Want outputs between 0 and 1



sigmoid function

logistic function

$$\boxed{\begin{array}{l}\text{outputs between } 0 \text{ and } 1 \\ g(z) = \frac{1}{1+e^{-z}} \qquad 0 < g(z) < 1\end{array}}$$

$f_{\vec{w},b}(\vec{x})$

$$\boxed{z = \vec{w} \cdot \vec{x} + b}$$

$$\downarrow$$

$$z$$

$$\downarrow$$

$$\boxed{g(z) = \frac{1}{1+e^{-z}}}$$

$$f_{\vec{w},b}(\vec{x}) = g(\underbrace{\vec{w} \cdot \vec{x} + b}_{z}) = \frac{1}{1 + e^{-(\vec{w} \cdot \vec{x} + b)}}$$

"logistic regression"

Interpretation of logistic regression output

$$f_{\vec{w},b}(\vec{x}) = \frac{1}{1+e^{-(\vec{w}\cdot\vec{x}+b)}}$$

"probability" that class is 1

Example:

x is "tumor size"

y is 0 (not malignant)

or 1 (malignant)

$$f_{\vec{w},b}(\vec{x}) = 0.7$$

70% chance that y is 1

P(1)

$$f_{\vec{w},b}(\vec{x}) = P(y = 1|\vec{x};\vec{w},b)$$

Probability that y is 1,
given input $\vec{x}$, parameters $\vec{w},b$

$$P(y=0) + P(y=1) = 1$$

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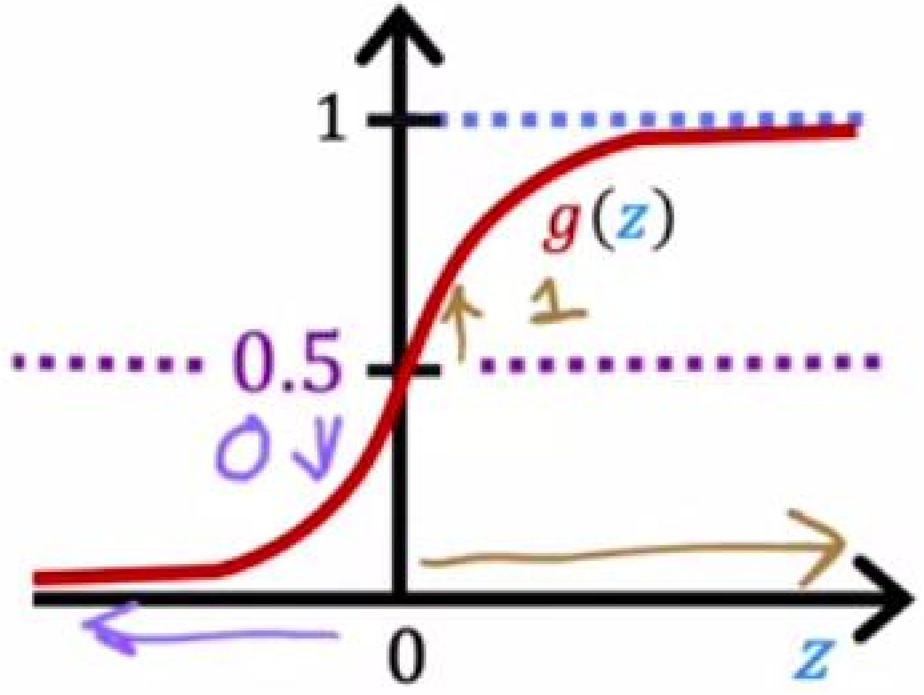
Classification

Decision Boundary



$f_{\vec{w},b}(\vec{x})$

$$\boxed{z = \vec{w} \cdot \vec{x} + b}$$

$$\downarrow$$

$$z$$

$$\downarrow$$

$$\boxed{g(z) = \frac{1}{1+e^{-z}}}$$

$$f_{\vec{w},b}(\vec{x}) = g(\underbrace{\vec{w} \cdot \vec{x} + b}_{z}) = \frac{1}{1 + e^{-(\vec{w} \cdot \vec{x} + b)}}$$

$$= P(y = 1 | x; \vec{w}, b) \quad 0.7 \quad 0.3$$

0 or 1?

Is $f_{\vec{w},b}(\vec{x}) \geq \overbrace{0.5}^{\text{threshold}}$?

Yes: $\hat{y} = 1$ No: $\hat{y} = 0$

When is $f_{\vec{w},b}(\vec{x}) \geq 0.5$?

$$g(z) \geq 0.5$$

$$z \geq 0$$

$$\vec{w} \cdot \vec{x} + b \geq 0 \qquad \vec{w} \cdot \vec{x} + b < 0$$

$$\hat{y} = 1 \qquad \hat{y} = 0$$

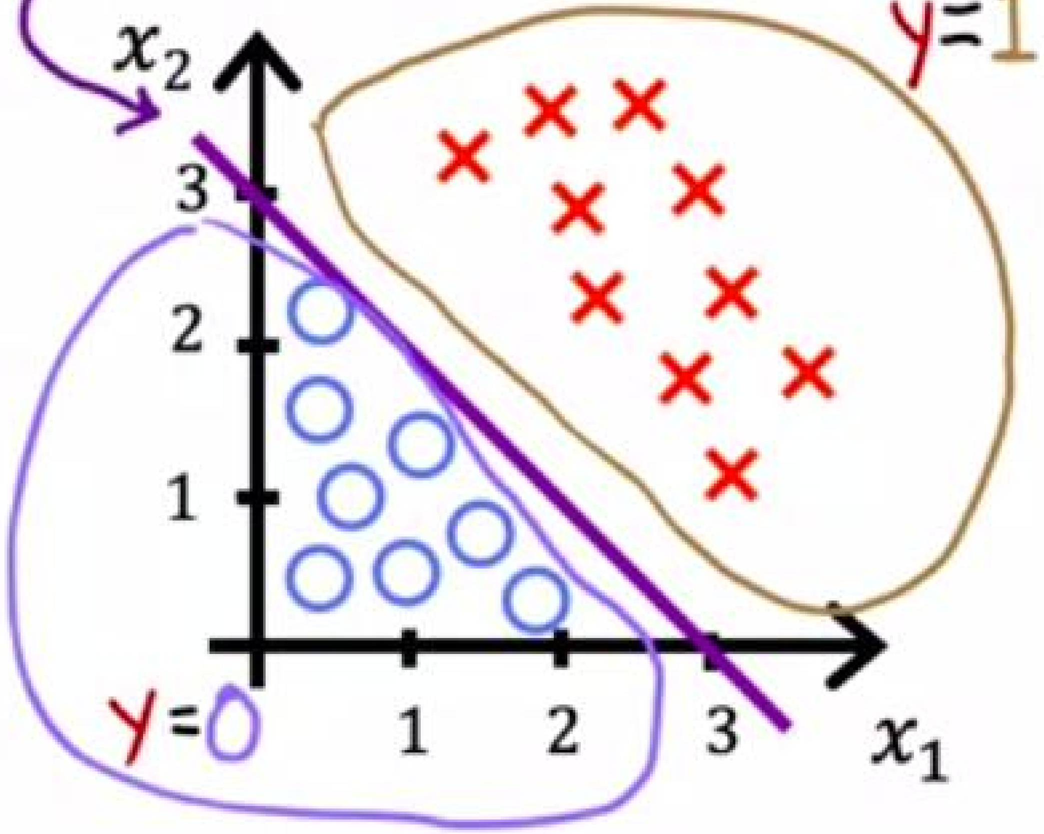
Decision boundary

$$f_{\vec{w},b}(\vec{x}) = g(z) = g(\underbrace{w_1}_{1} x_1 + \underbrace{w_2}_{1} x_2 + \underbrace{b}_{-3})$$

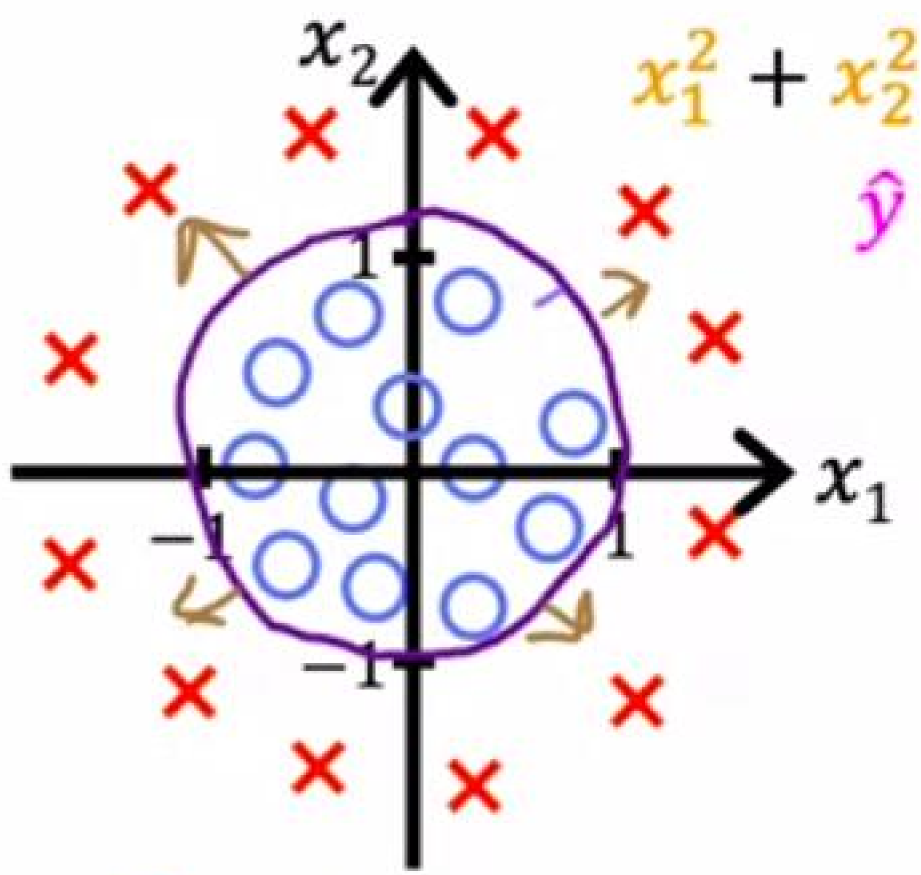
Decision boundary $z = \vec{w} \cdot \vec{x} + b = 0$

$$z = x_1 + x_2 - 3 = 0$$

$$x_1 + x_2 = 3$$



Non-linear decision boundaries



$x_1^2 + x_2^2 \geq 1$

$\hat{y} = 1$

$x_1^2 + x_2^2 < 1$

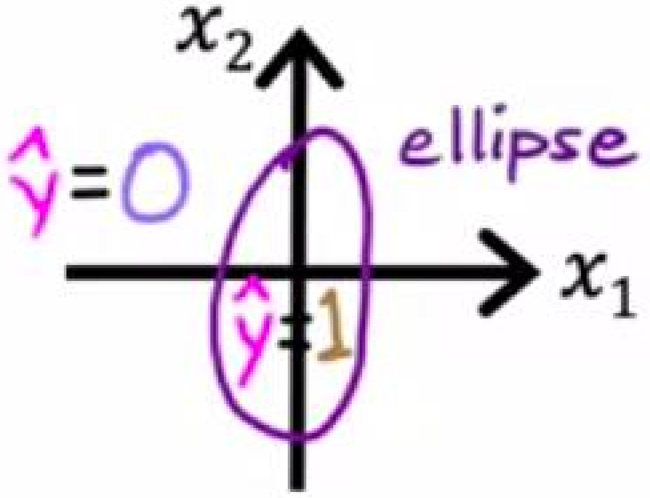
$$f_{\vec{w},b}(\vec{x}) = g(z) = g(\overbrace{\underset{1}{w_1} x_1^2 + \underset{1}{w_2} x_2^2 + \underset{-1}{b}}^{z})$$

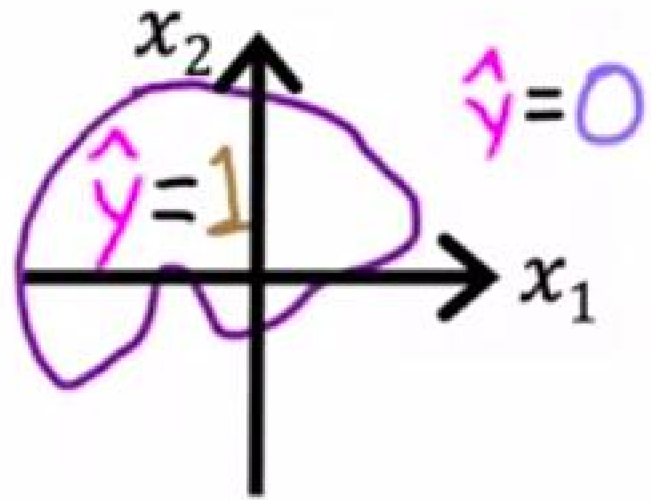
decision boundary

$$z = x_1^2 + x_2^2 - 1 = 0$$

$$x_1^2 + x_2^2 = 1$$

Non-linear decision boundaries





$$
\begin{aligned}
f_{\vec{\mathrm{w}},b}(\vec{\mathrm{x}}) = g(z) = g(&w_1x_1 + w_2x_2 \\
&+w_3x_1^2 + w_4x_1x_2 + w_5x_2^2 \\
&+w_6x_1^3 + \cdots + b)
\end{aligned}
$$

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Cost Function

Cost Function for Logistic Regression

Training set

	tumor size (cm) x_1	...	patient's age x_n	malignant? y
$i=1$	10		52	1
$\vdots$	2		73	0
	5		55	0
	12		49	1
$i=m$	...		...	...

$i = 1, \dots, m$ ← training examples

$j = 1, \dots, n$ ← features

$\boxed{\text{target } y \text{ is } 0 \text{ or } 1}$

$$f_{\vec{w},b}(\vec{x}) = \frac{1}{1 + e^{-(\vec{w}\cdot\vec{x}+b)}}$$

How to choose $\vec{w} = [w_1 \quad w_2 \quad \cdots \quad w_n]$ and b?

Squared error cost

cost

$$J(\vec{w},b) = \frac{1}{m}\sum_{i=1}^{m} \boxed{\frac{1}{2}\left(f_{\vec{w},b}(\vec{x}^{(i)}) - y^{(i)}\right)^2}$$

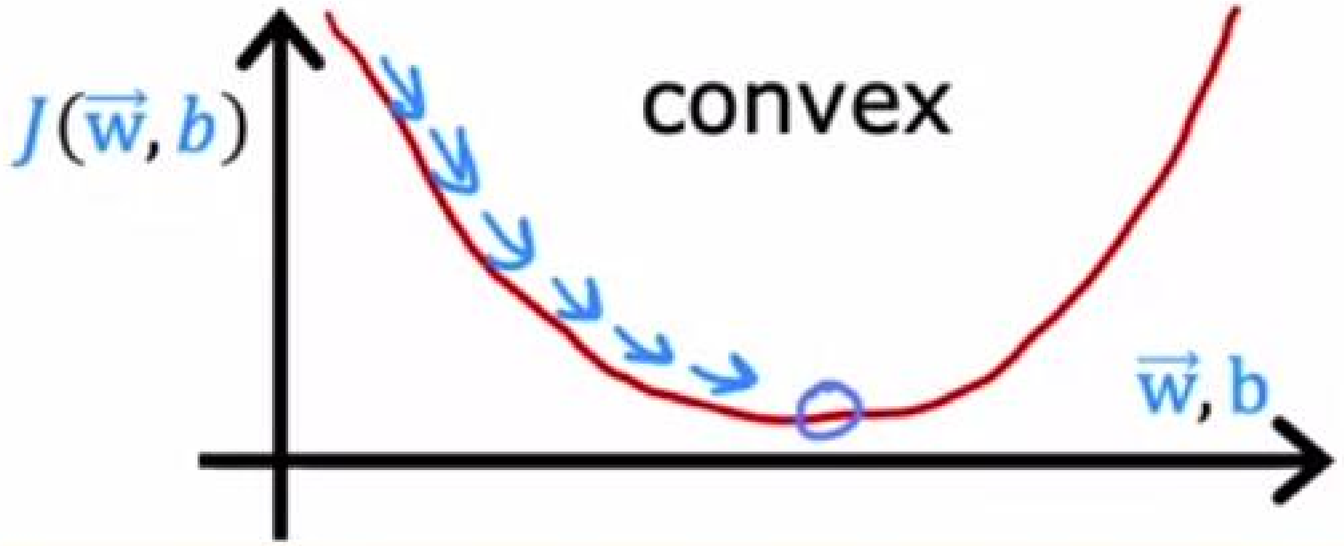
loss $\boxed{L\left(f_{\vec{w},b}(\vec{x}^{(i)}), y^{(i)}\right)}$

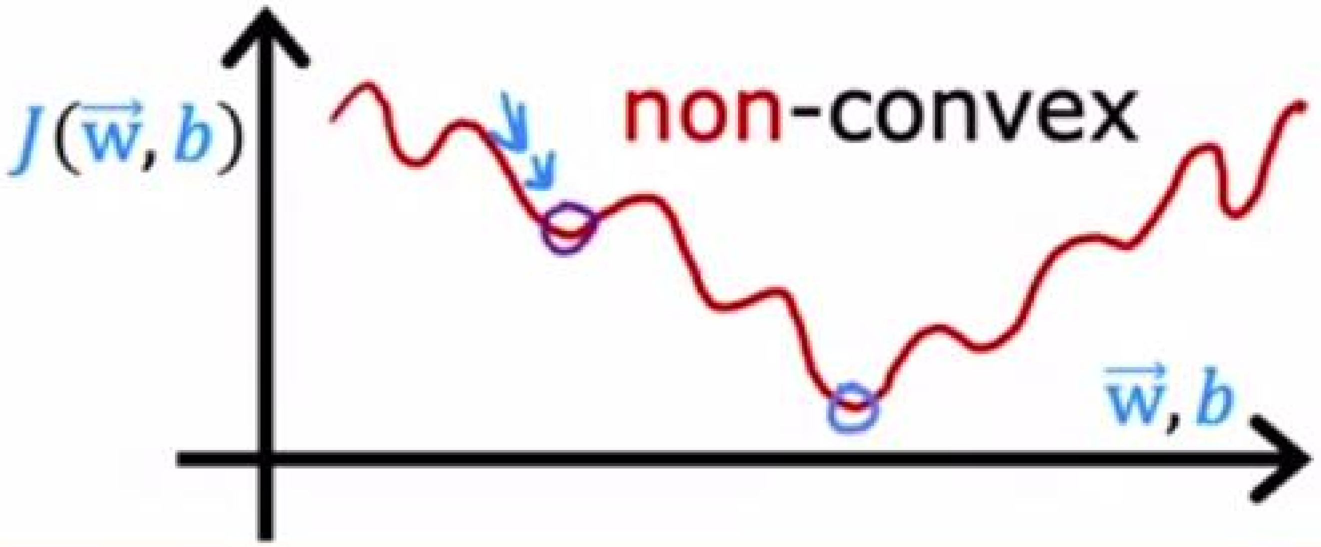
linear regression

$$f_{\vec{w},b}(\vec{x}) = \vec{w} \cdot \vec{x} + b$$

logistic regression

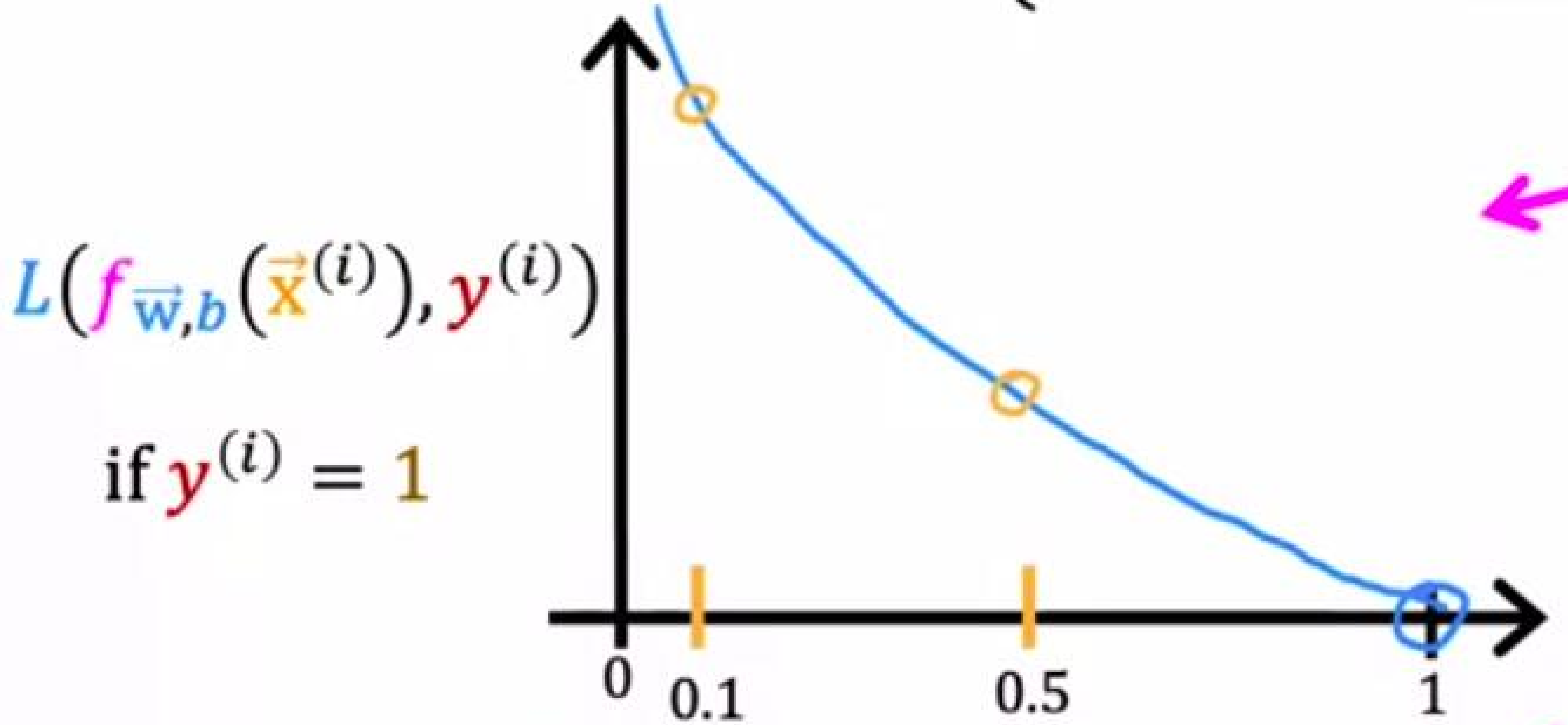
$$f_{\vec{w},b}(\vec{x}) = \frac{1}{1 + e^{-(\vec{w}\cdot\vec{x}+b)}}$$

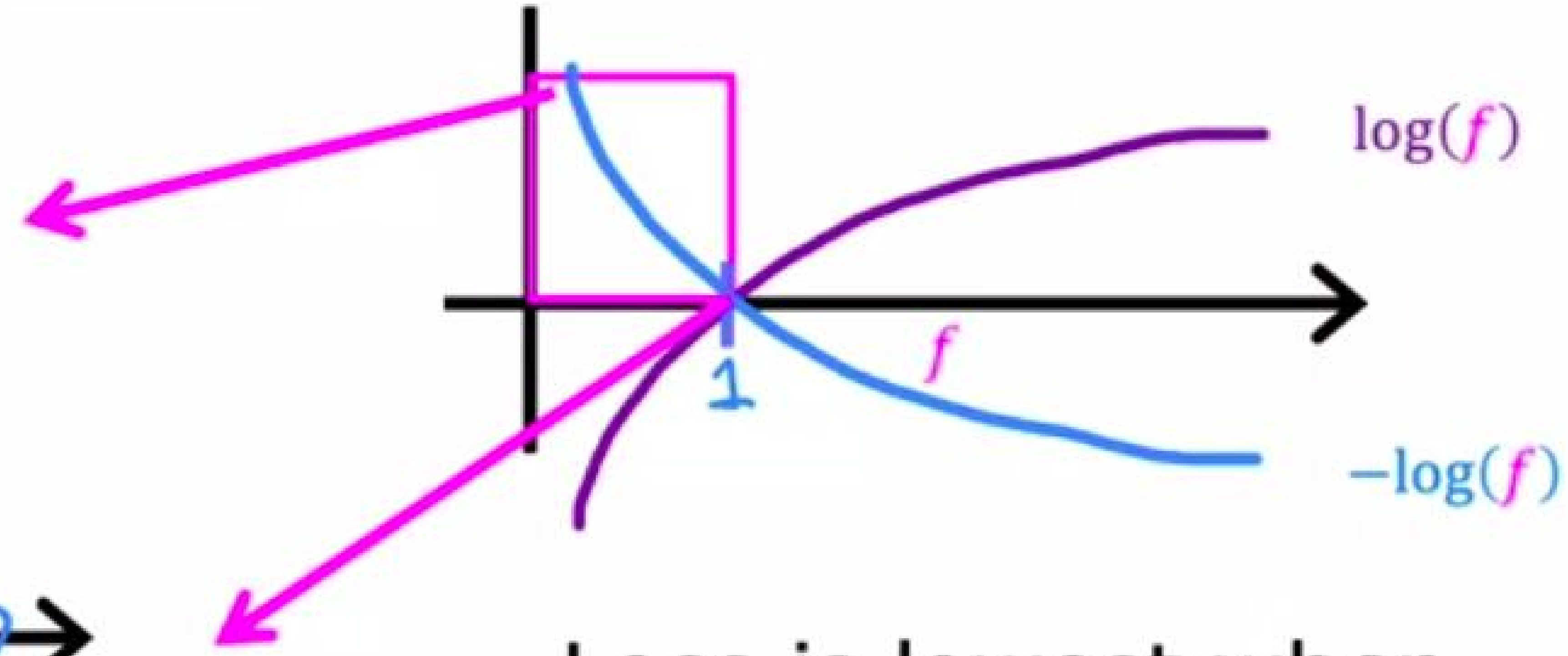




Logistic loss function

$$L\left(f_{\vec{\mathrm{w}},b}\left(\vec{\mathrm{x}}^{(i)}\right), y^{(i)}\right) = \begin{cases} -\log\left(f_{\vec{\mathrm{w}},b}\left(\vec{\mathrm{x}}^{(i)}\right)\right) & \text{if } y^{(i)} = 1 \\ -\log\left(1 - f_{\vec{\mathrm{w}},b}\left(\vec{\mathrm{x}}^{(i)}\right)\right) & \text{if } y^{(i)} = 0 \end{cases}$$





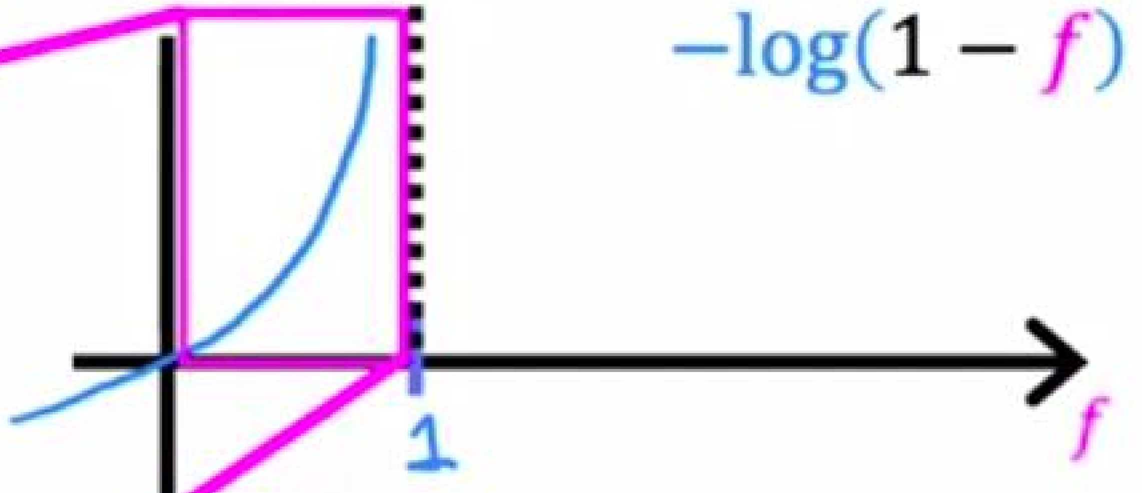
As $f_{\vec{\mathrm{w}},b}\left(\vec{\mathrm{x}}^{(i)}\right) \to 1$ then loss $\to 0$

As $f_{\vec{\mathrm{w}},b}\left(\vec{\mathrm{x}}^{(i)}\right) \to 0$ then loss $\to \infty$

Loss is lowest when $f_{\vec{\mathrm{w}},b}\left(\vec{\mathrm{x}}^{(i)}\right)$ predicts close to true label $y^{(i)}$.

Logistic loss function

$$L\left(f_{\vec{w},b}\left(\vec{x}^{(i)}\right), y^{(i)}\right) = \begin{cases} -\log\left(f_{\vec{w},b}\left(\vec{x}^{(i)}\right)\right) & \text{if } y^{(i)} = 1 \\ -\log\left(1 - f_{\vec{w},b}\left(\vec{x}^{(i)}\right)\right) & \text{if } y^{(i)} = 0 \end{cases}$$

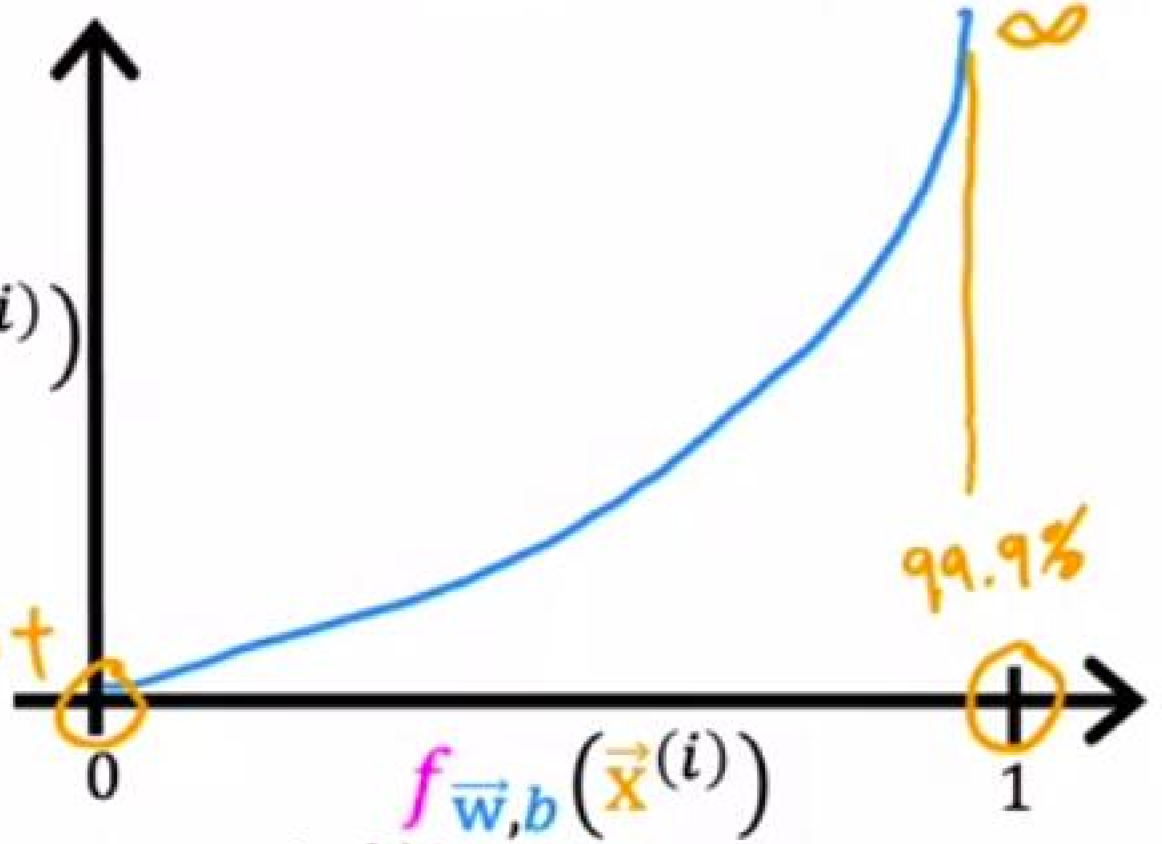


As $f_{\vec{w},b}\left(\vec{x}^{(i)}\right) \rightarrow 0$ then loss $\rightarrow 0$ ☺

$L\left(f_{\vec{w},b}\left(\vec{x}^{(i)}\right), y^{(i)}\right)$

if $y^{(i)} = 0$

not malignant



As $f_{\vec{w},b}\left(\vec{x}^{(i)}\right) \rightarrow 1$ then loss $\rightarrow \infty$ ☹

The further prediction $f_{\vec{w},b}\left(\vec{x}^{(i)}\right)$ is from target $y^{(i)}$, the higher the loss.

Cost

cost

$$J(\vec{\mathrm{w}}, b) = \frac{1}{m} \sum_{i=1}^{m} L\big(\underbrace{f_{\vec{\mathrm{w}},b}(\vec{\mathrm{x}}^{(i)}), y^{(i)}}_{\text{loss}}\big)$$

$$= \begin{cases} -\log\left(f_{\vec{\mathrm{w}},b}(\vec{\mathrm{x}}^{(i)})\right) & \text{if } y^{(i)} = 1 \\ -\log\left(1 - f_{\vec{\mathrm{w}},b}(\vec{\mathrm{x}}^{(i)})\right) & \text{if } y^{(i)} = 0 \end{cases}$$

convex

↳ can reach a global minimum

find w, b that minimize cost J

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Cost Function

Simplified Cost Function

Simplified loss function

$$L\left(f_{\vec{w},b}\left(\vec{x}^{(i)}\right), y^{(i)}\right) = \begin{cases} -\log\left(f_{\vec{w},b}\left(\vec{x}^{(i)}\right)\right) & \text{if } y^{(i)} = 1 \\ \boxed{-\log\left(1 - f_{\vec{w},b}\left(\vec{x}^{(i)}\right)\right)} & \text{if } y^{(i)} = 0 \end{cases}$$

$$\boxed{L\left(f_{\vec{w},b}\left(\vec{x}^{(i)}\right), y^{(i)}\right) = -\underbrace{\cancel{y^{(i)}\log\left(f_{\vec{w},b}\left(\vec{x}^{(i)}\right)\right)}}_{0} - \underbrace{\left(1 - y^{(i)}\right)}_{(1-0)}\log\left(1 - f_{\vec{w},b}\left(\vec{x}^{(i)}\right)\right)}$$

if $y^{(i)} = 1$:

$$L\left(f_{\vec{w},b}\left(\vec{x}^{(i)}\right), y^{(i)}\right) = -1 \log(f(\vec{x}))$$

if $y^{(i)} = 0$:

$$L\left(f_{\vec{w},b}\left(\vec{x}^{(i)}\right), y^{(i)}\right) = \underbrace{-(1-0)\log(1-f(\vec{x}))}$$

Simplified cost function

loss

$$L(f_{\vec{w},b}(\vec{x}^{(i)}), y^{(i)}) = \underbrace{-y^{(i)}\log\left(f_{\vec{w},b}(\vec{x}^{(i)})\right) - (1 - y^{(i)})\log\left(1 - f_{\vec{w},b}(\vec{x}^{(i)})\right)}$$

convex
(single global minimum)

cost

$$J(\vec{w}, b) = \frac{1}{m}\sum_{i=1}^{m}\left[\underbrace{L(f_{\vec{w},b}(\vec{x}^{(i)}), y^{(i)})}\right]$$

$$= -\frac{1}{m}\sum_{i=1}^{m}\left[y^{(i)}\log\left(f_{\vec{w},b}(\vec{x}^{(i)})\right) + (1 - y^{(i)})\log\left(1 - f_{\vec{w},b}(\vec{x}^{(i)})\right)\right]$$

maximum likelihood
(don't worry about it!)

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Gradient Descent

Gradient Descent Implementation

Training logistic regression

Find $\vec{w}, b$

Given new $\vec{x}$, output $f_{\vec{w},b}(\vec{x}) = \frac{1}{1+e^{-(\vec{w}\cdot\vec{x}+b)}}$

$$P(y = 1|\vec{x}; \vec{w}, b)$$

Gradient descent

cost

$$J(\vec{\mathrm{w}},b) = -\frac{1}{m}\sum_{i=1}^{m}\left[y^{(i)}\log\left(f_{\vec{\mathrm{w}},b}(\vec{\mathrm{x}}^{(i)})\right) + (1-y^{(i)})\log\left(1-f_{\vec{\mathrm{w}},b}(\vec{\mathrm{x}}^{(i)})\right)\right]$$

repeat {

$j = 1 \ldots n$

$$w_j = w_j - \alpha \underbrace{\frac{\partial}{\partial w_j}J(\vec{\mathrm{w}},b)}$$

$$\frac{\partial}{\partial w_j}J(\vec{\mathrm{w}},b) = \frac{1}{m}\sum_{i=1}^{m}(f_{\vec{\mathrm{w}},b}(\vec{\mathrm{x}}^{(i)}) - y^{(i)})x_j^{(i)}$$

$$b = b - \alpha \underbrace{\frac{\partial}{\partial b}J(\vec{\mathrm{w}},b)}$$

$$\frac{\partial}{\partial b}J(\vec{\mathrm{w}},b) = \frac{1}{m}\sum_{i=1}^{m}(f_{\vec{\mathrm{w}},b}(\vec{\mathrm{x}}^{(i)}) - y^{(i)})$$

} simultaneous updates

Gradient descent for logistic regression

repeat {

looks like linear regression!

$$w_j = w_j - \alpha \left[\frac{1}{m} \sum_{i=1}^{m} \left(f_{\vec{\mathrm{w}},b}(\vec{\mathrm{x}}^{(i)}) - y^{(i)} \right) x_j^{(i)} \right]$$

$$b = b - \alpha \left[\frac{1}{m} \sum_{i=1}^{m} \left(f_{\vec{\mathrm{w}},b}(\vec{\mathrm{x}}^{(i)}) - y^{(i)} \right) \right]$$

} simultaneous updates

Same concepts:

- Monitor gradient descent (learning curve)
- Vectorized implementation
- Feature scaling

Linear regression $f_{\vec{\mathrm{w}},b}(\vec{\mathrm{x}}) = \vec{\mathrm{w}} \cdot \vec{\mathrm{x}} + b$

Logistic regression $f_{\vec{\mathrm{w}},b}(\vec{\mathrm{x}}) = \dfrac{1}{1 + e^{-(\vec{\mathrm{w}} \cdot \vec{\mathrm{x}} + b)}}$

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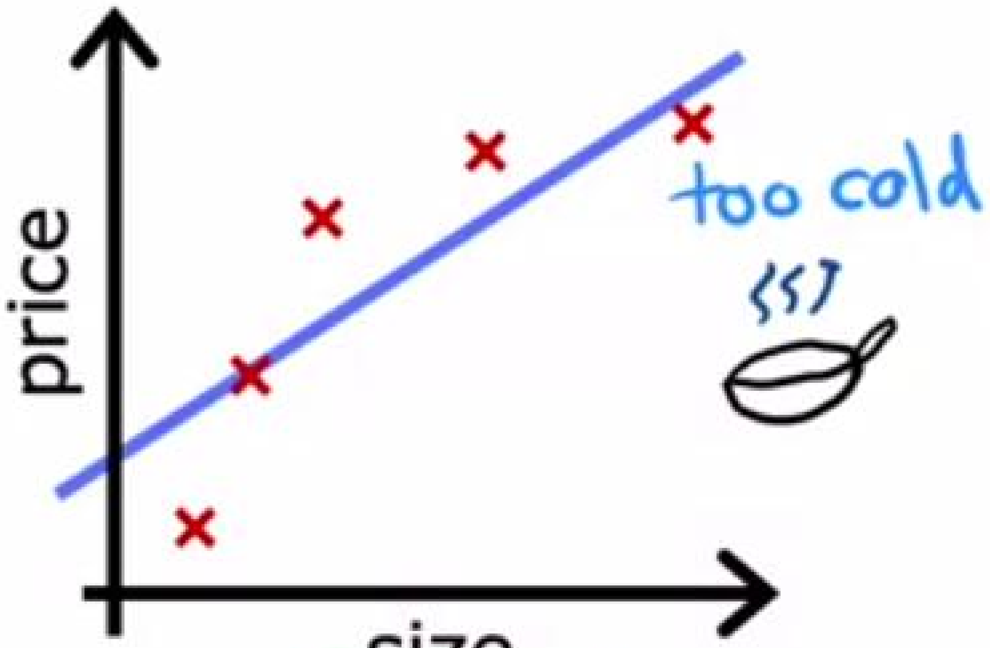
Regularization to Reduce Overfitting

The Problem of Overfitting

Regression example

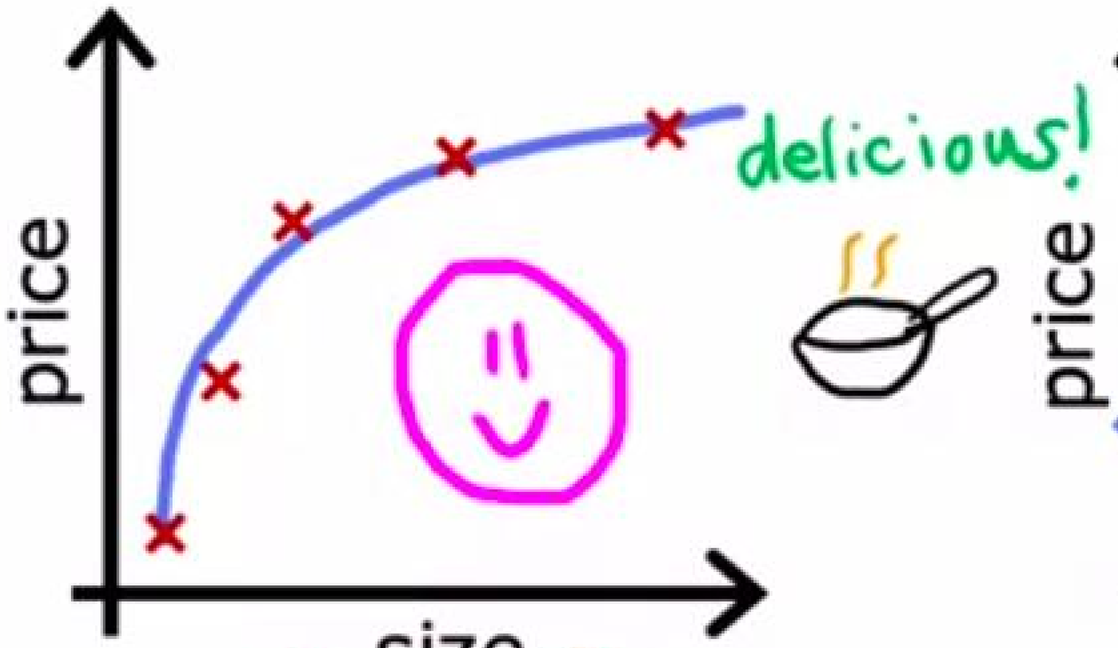


$w_1 x + b$

underfit

- Does not fit the training set well

high bias

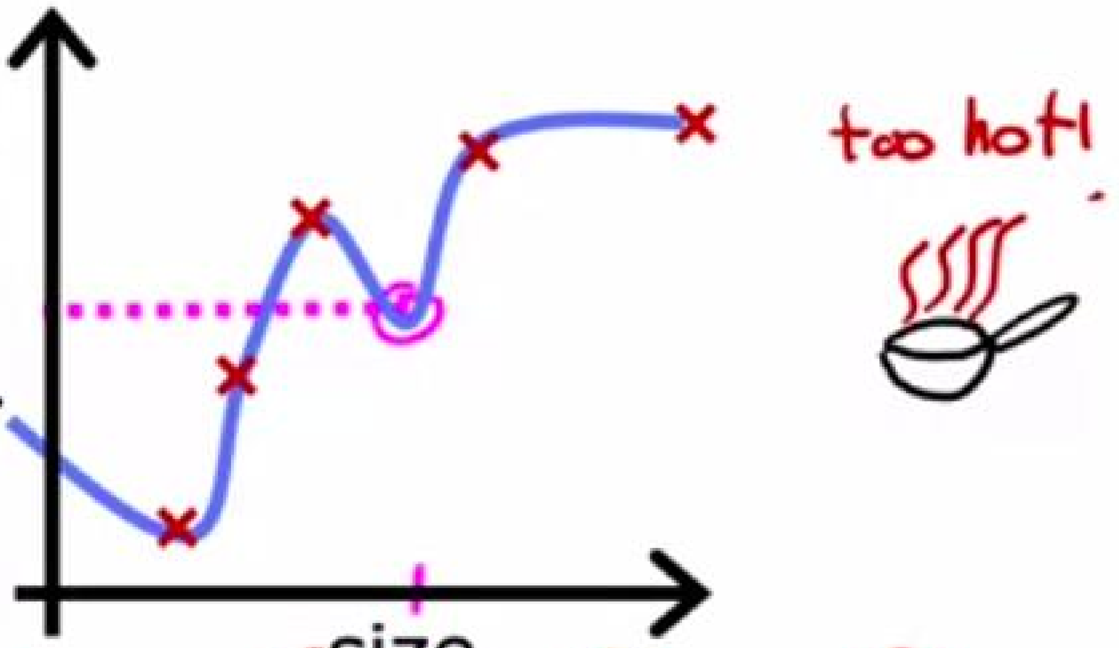


$w_1 x + w_2 x^2 + b$

just right

- Fits training set pretty well

generalization



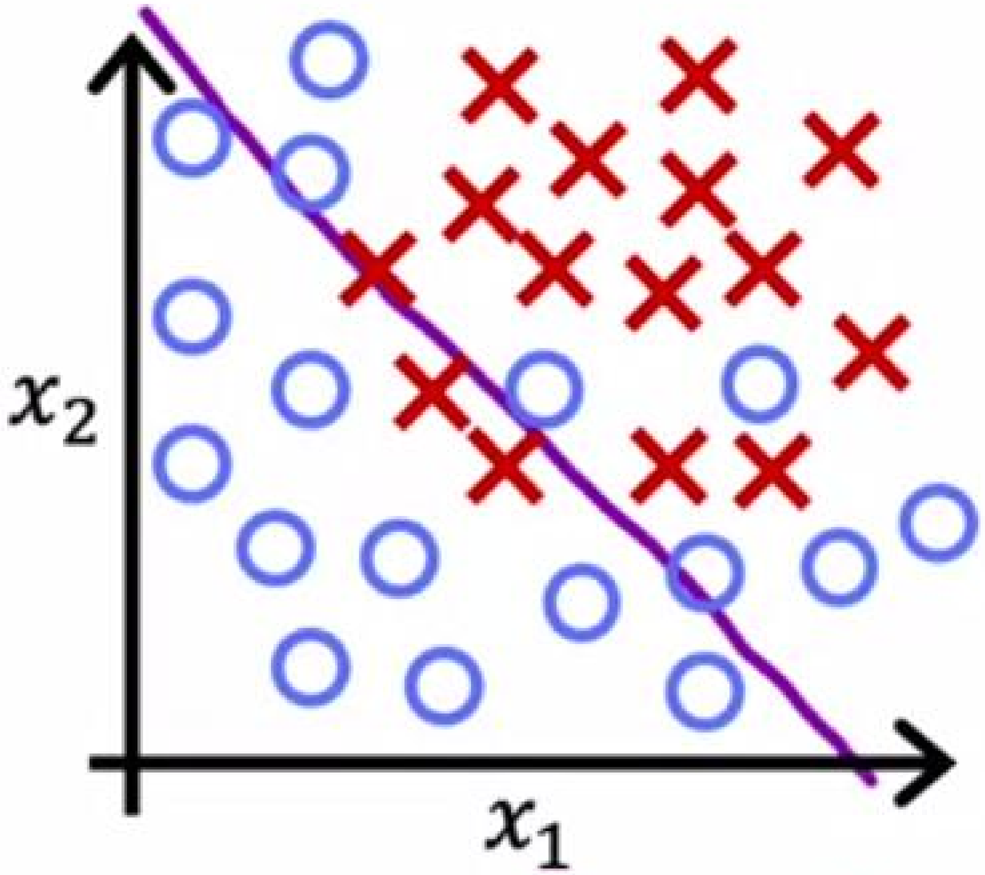
$w_1 x + w_2 x^2 + w_3 x^3 + w_4 x^4 + b$

overfit

- Fits the training set extremely well

high variance

Classification

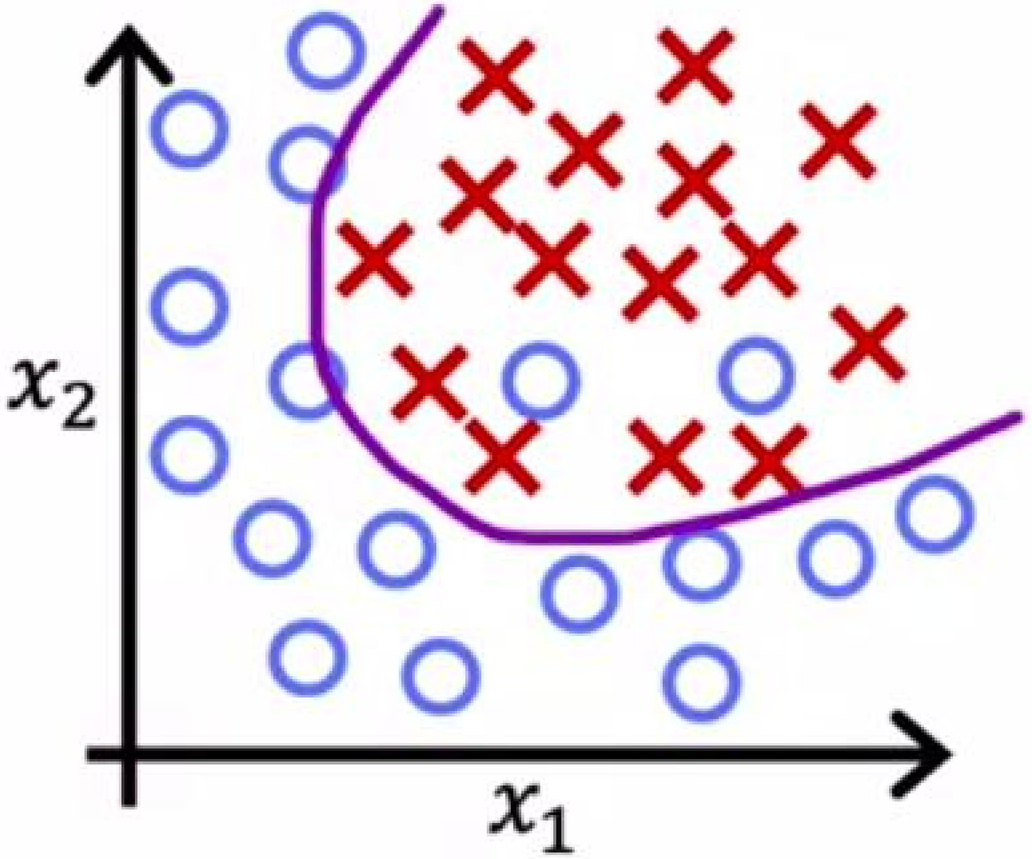


$$z = w_1 x_1 + w_2 x_2 + b$$

$$f_{\vec{w},b}(\vec{x}) = g(z)$$

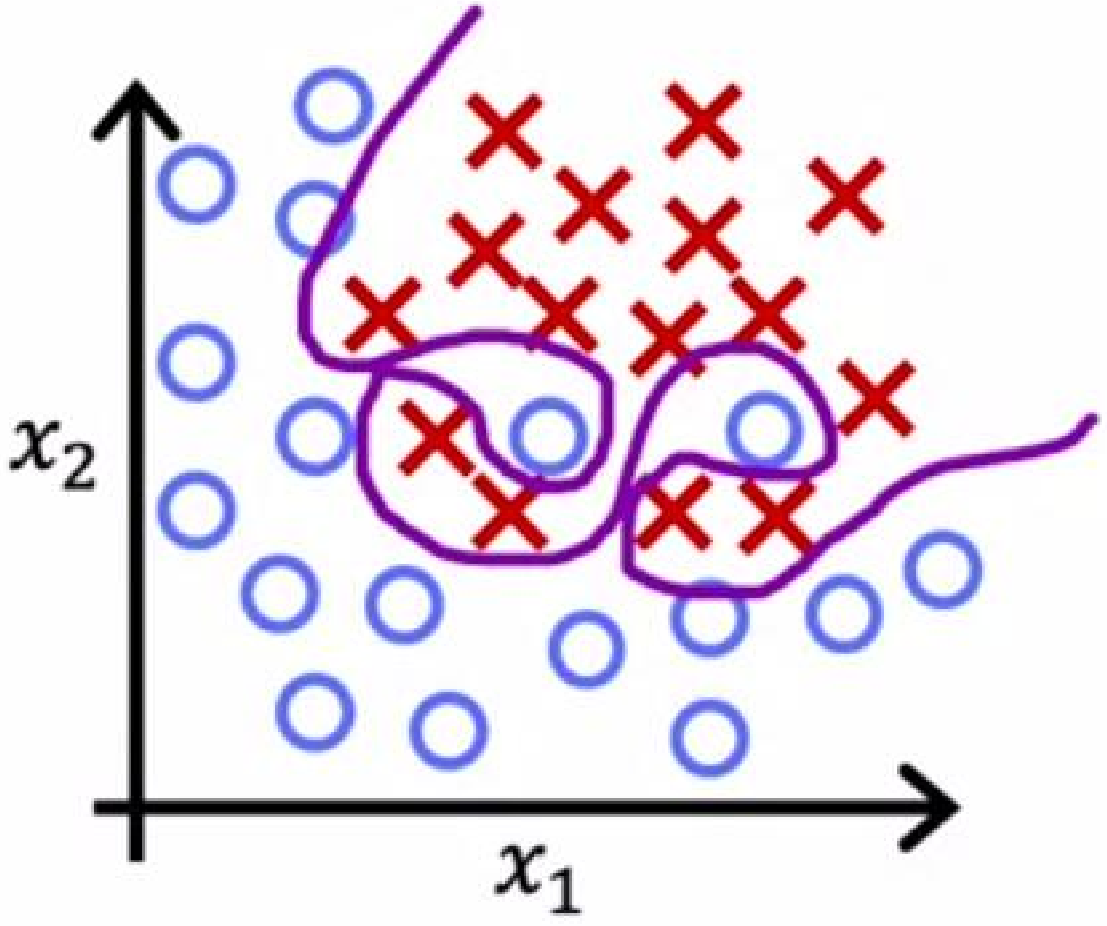
g is the sigmoid function

underfit high bias



$$z = w_1 x_1 + w_2 x_2 + w_3 x_1^2 + w_4 x_2^2 + w_5 x_1 x_2 + b$$

just right



$$z = w_1 x_1 + w_2 x_2 + w_3 x_1^2 x_2 + w_4 x_1^2 x_2^2 + w_5 x_1^2 x_2^3 + w_6 x_1^3 x_2 + \cdots + b$$

overfit

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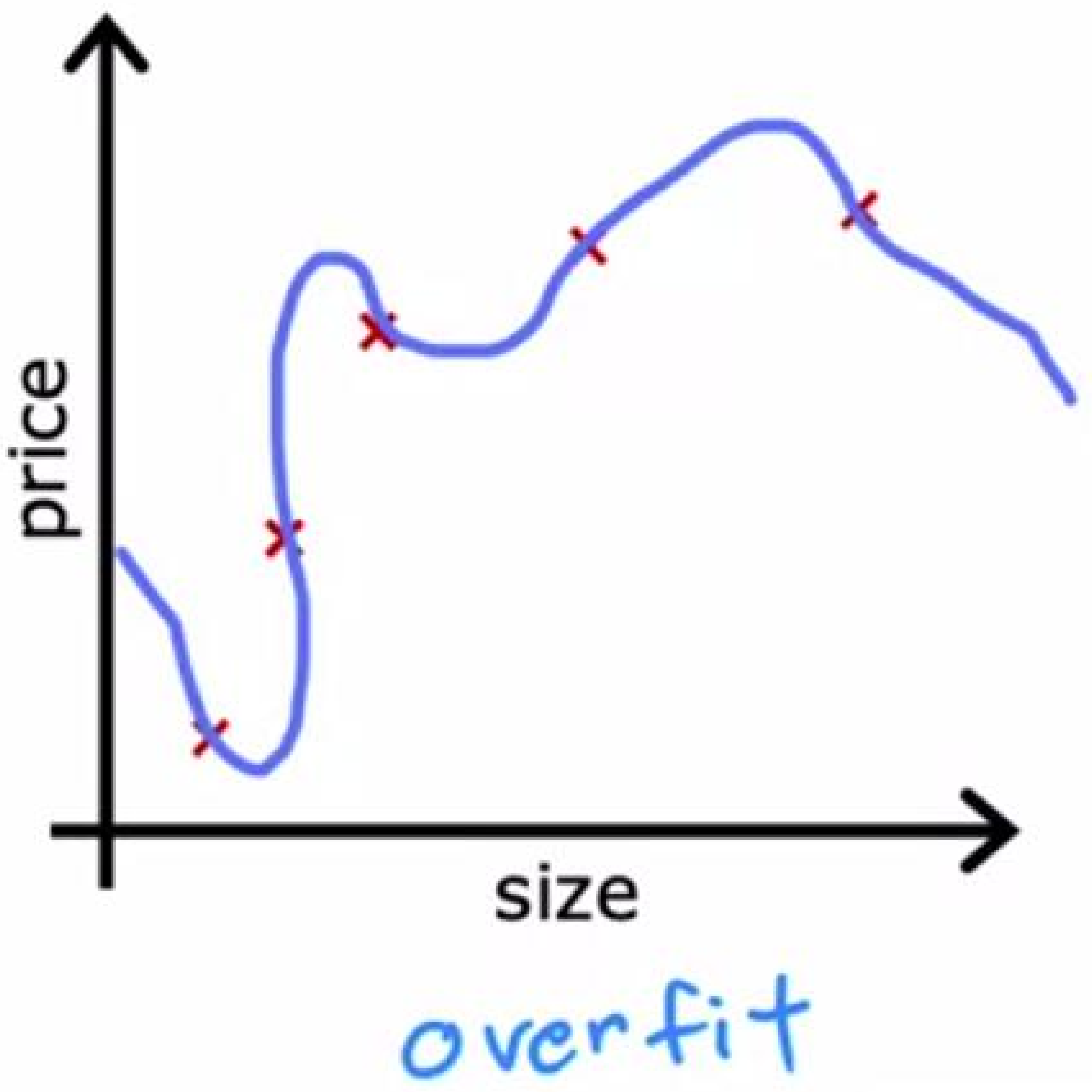
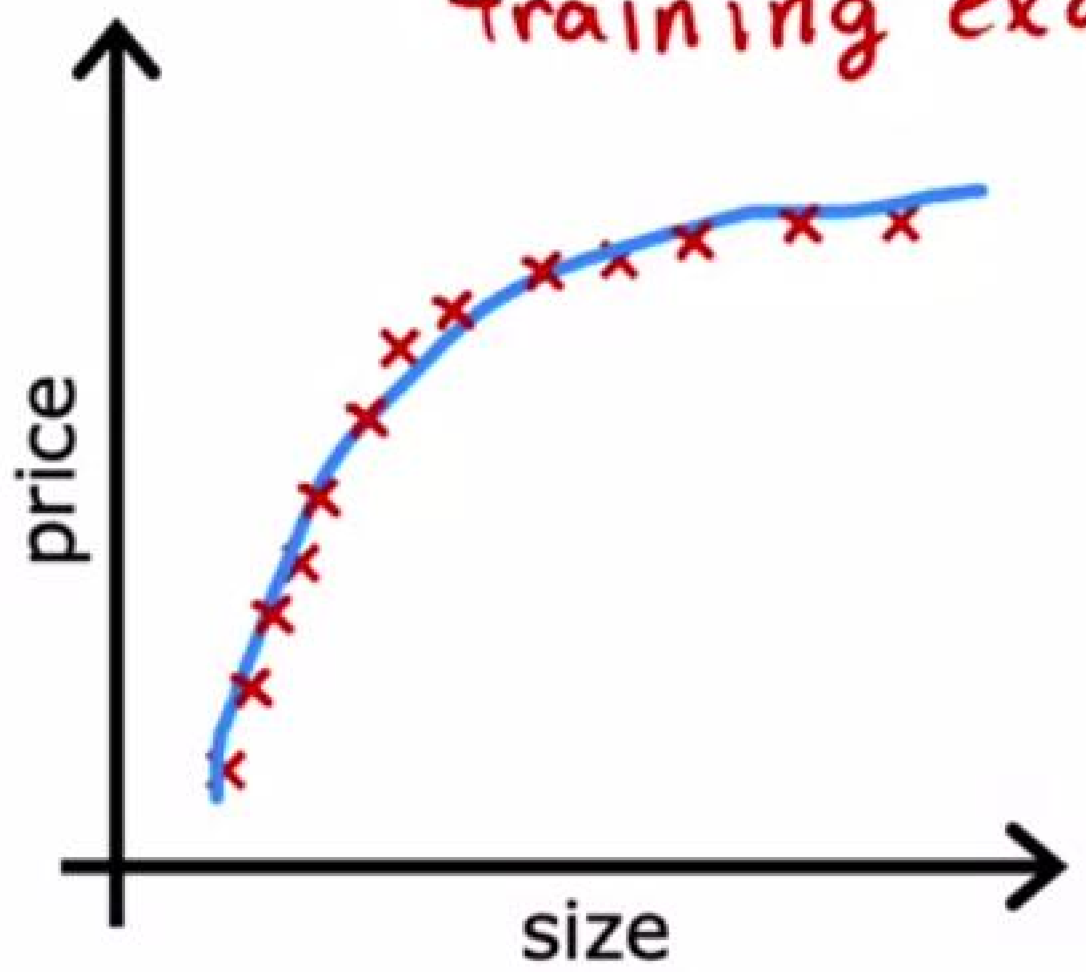
Regularization to Reduce Overfitting

Addressing Overfitting

Collect more training examples





Select features to include/exclude

size	bedrooms	floors	age	avg income	...	distance to coffee shop	price
x_1	x_2	x_3	x_4	x_5		x_{100}	y

all features

+

insufficient data

↓

overfit

selected features

size
bedrooms
age
just right
feature selection

course 2

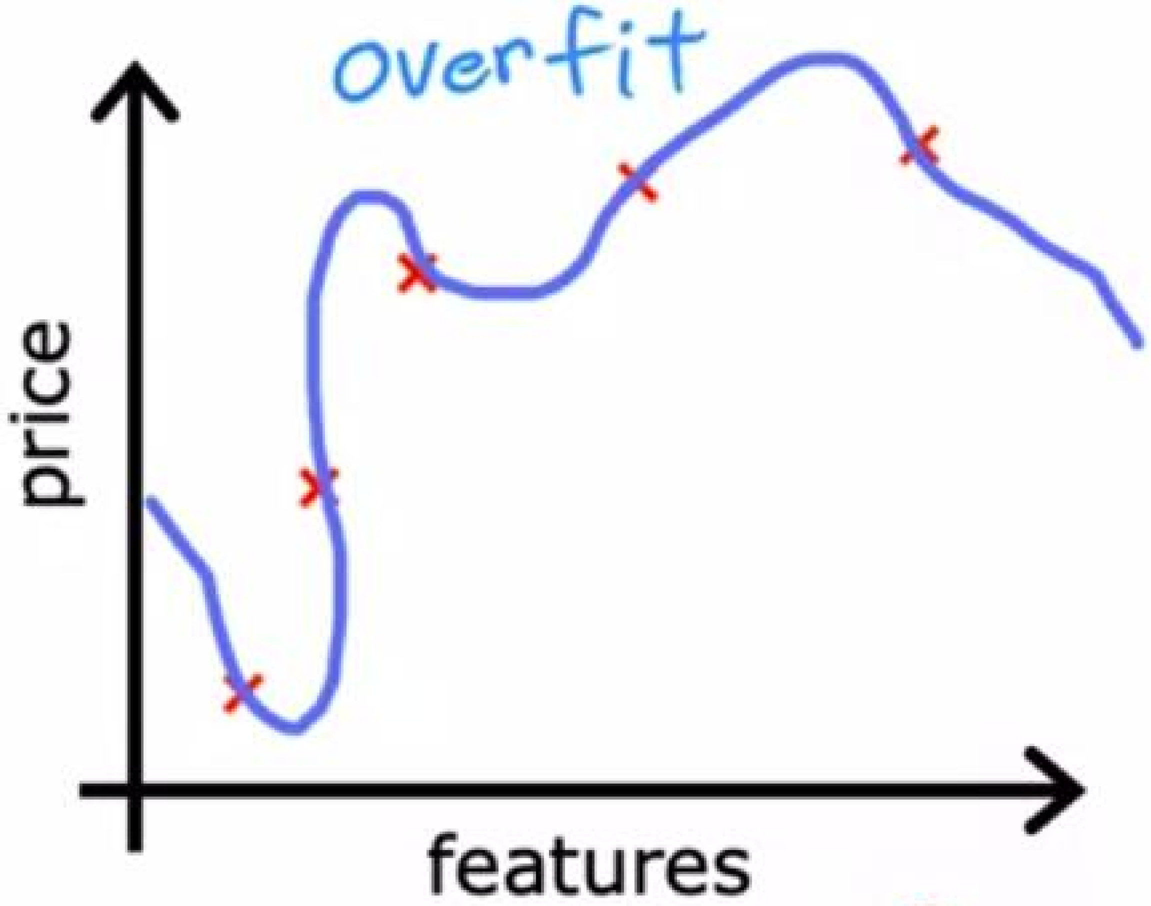
disadvantage

↓

useful features could be lost

Regularization

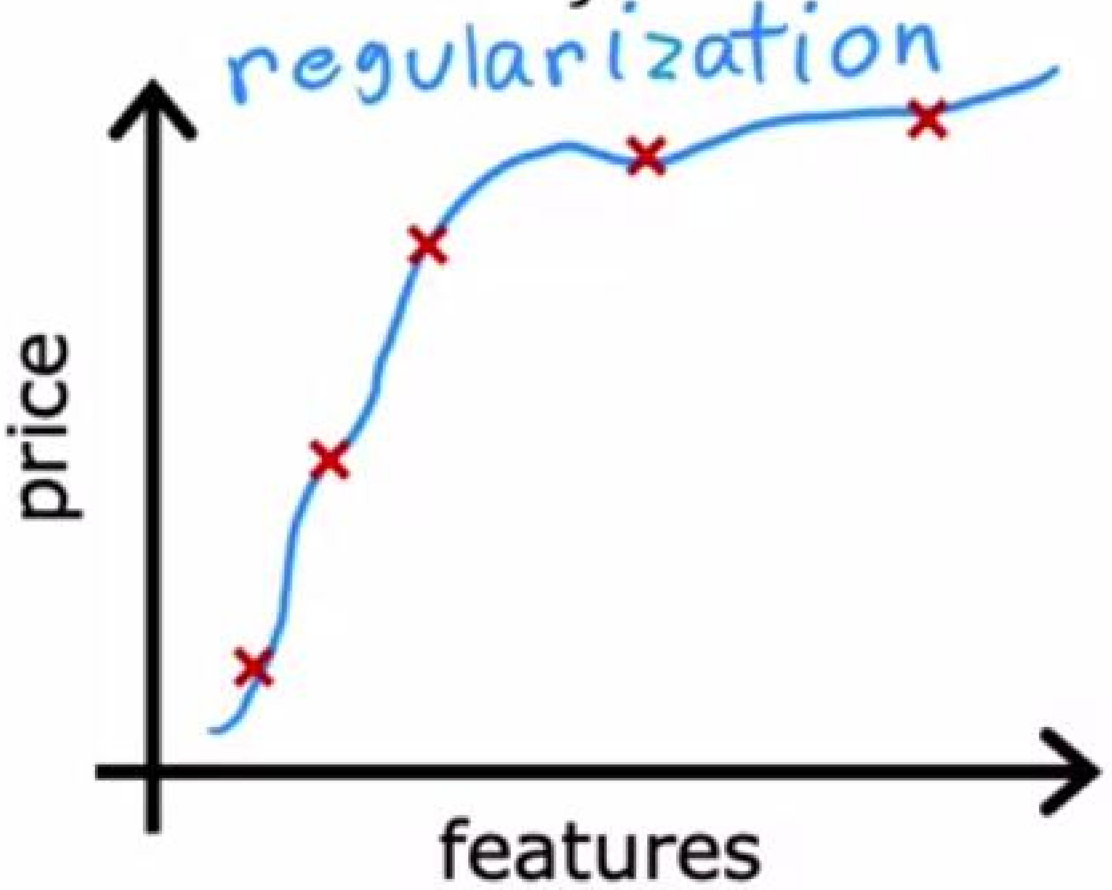
Reduce the size of parameters w_j



$$f(x) = 28x - 385x^2 + 39x^3 - \underbrace{\cancel{174}x^4}_{0} + 100$$

large values for w_j

eliminate feature



$$f(x) = 13x - 0.23x^2 + \underbrace{0.000014}x^3 - \underbrace{0.0001}x^4 + 10$$

small values for w_j

Addressing overfitting

Options

1. Collect more data

2. Select features
   - Feature selection   in course 2

3. Reduce size of parameters
   - "Regularization"   next videos!

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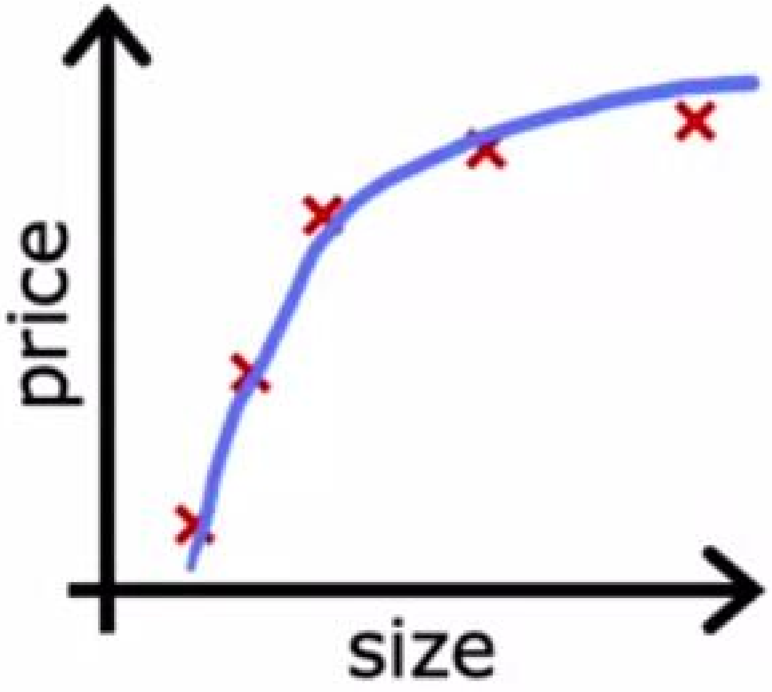
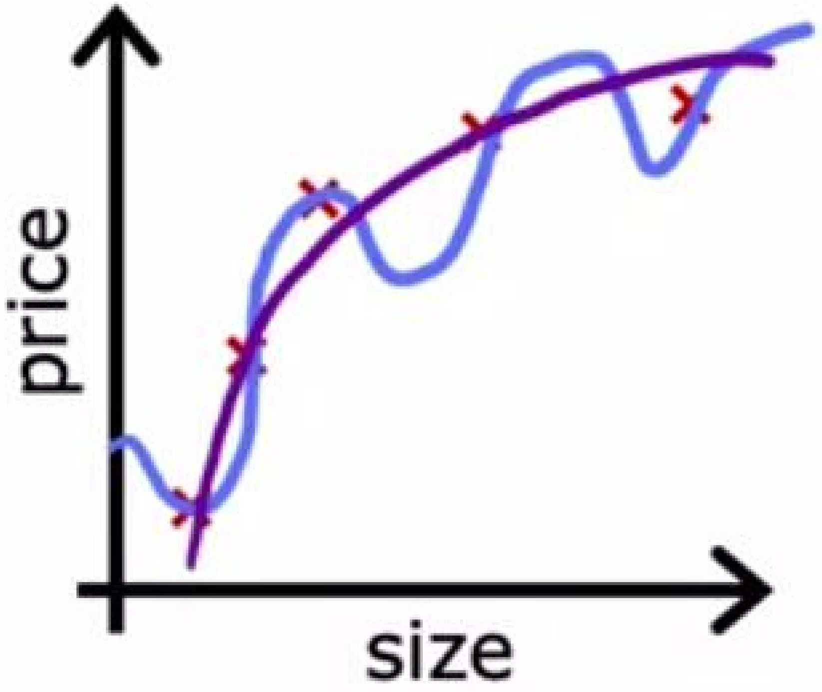
Regularization to Reduce Overfitting

Cost Function with Regularization

Intuition



$$w_1 x + w_2 x^2 + b$$



$$w_1 x + w_2 x^2 + \cancel{\underbrace{w_3}_{\approx 0} x^3} + \cancel{\underbrace{w_4}_{\approx 0} x^4} + b$$

make w_3, w_4 really small (≈ 0)

$$\min_{\vec{w},b} \frac{1}{2m} \sum_{i=1}^{m} \left(f_{\vec{w},b}\left(\vec{x}^{(i)}\right) - y^{(i)}\right)^2 + \cancel{1000\, \underbrace{w_3^2}_{0.001}} + \cancel{1000\, \underbrace{w_4^2}_{0.002}}$$

Regularization

small values $w_1, w_2, \cdots, w_n, b$ — simpler model $w_3 \approx 0$ — less likely to overfit $w_4 \approx 0$

size	bedrooms	floors	age	avg income	...	distance to coffee shop	price
x_1	x_2	x_3	x_4	x_5		x_{100}	y

n features

$n = 100$

$w_1, w_1, w_2, \cdots, w_{100}, b$

$$J(\vec{\mathrm{w}}, b) = \frac{1}{2m}\left[\sum_{i=1}^{m}\left(f_{\vec{\mathrm{w}},b}\left(\vec{\mathrm{x}}^{(i)}\right) - y^{(i)}\right)^2 + \overbrace{\underbrace{\frac{\lambda}{2m}}_{\text{"lambda"}}\sum_{j=1}^{n} w_j^2}^{\text{regularization term}} \quad \overbrace{+ \frac{\lambda}{2m} b^2}\right]$$

can include or exclude b

"lambda" regularization parameter

$\lambda > 0$

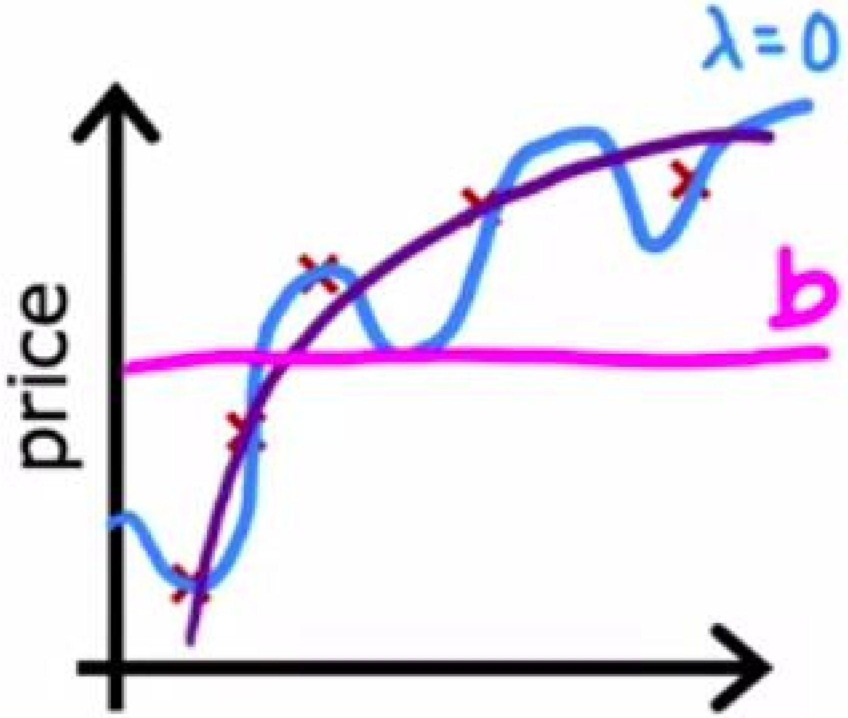
Regularization

$$\min_{\vec{w},b} J(\vec{w},b) = \min_{\vec{w},b}\left[\overbrace{\frac{1}{2m}\sum_{i=1}^{m}\left(f_{\vec{w},b}(\vec{x}^{(i)})-y^{(i)}\right)^2}^{\text{mean squared error}} + \overbrace{\frac{\lambda}{2m}\sum_{j=1}^{n}w_j^2}^{\text{regularization term}}\right]$$

fit data

Keep w_j small

λ balances both goals



choose $\lambda = 10^{10}$

$$f_{\vec{w},b}(\vec{x}) = \underbrace{\cancel{w_1 x}}_{\approx 0} + \underbrace{\cancel{w_2 x^2}}_{\approx 0} + \underbrace{\cancel{w_3 x^3}}_{\approx 0} + \underbrace{\cancel{w_4 x^4}}_{\approx 0} + b$$

$$f(x) = b$$

Choose λ

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Regularization to Reduce Overfitting

Regularized Linear Regression

Regularized linear regression

$$\min_{\vec{w},b} J(\vec{w},b) = \min_{\vec{w},b}\left[\frac{1}{2m}\sum_{i=1}^{m}\left(f_{\vec{w},b}\left(\vec{x}^{(i)}\right) - y^{(i)}\right)^2 + \frac{\lambda}{2m}\sum_{j=1}^{n} w_j^2\right]$$

Gradient descent

repeat {

$$w_j = w_j - \alpha \frac{\partial}{\partial w_j} J(\vec{w},b) = \frac{1}{m}\sum_{i=1}^{m}\left(f_{\vec{w},b}\left(\vec{x}^{(i)}\right) - y^{(i)}\right) x_j^{(i)} + \frac{\lambda}{m} w_j$$

$j = 1, \ldots, n$

$$b = b - \alpha \frac{\partial}{\partial b} J(\vec{w},b) = \frac{1}{m}\sum_{i=1}^{m}\left(f_{\vec{w},b}\left(\vec{x}^{(i)}\right) - y^{(i)}\right)$$

} simultaneous update

don't have to regularize b

Implementing gradient descent

repeat {

$$w_j = w_j - \alpha \left[\frac{1}{m} \sum_{i=1}^{m} \left[\left(f_{\vec{\mathrm{w}},b}\left(\vec{\mathrm{x}}^{(i)}\right) - y^{(i)} \right) x_j^{(i)} \right] + \frac{\lambda}{m} w_j \right]$$

$$b = b - \alpha \frac{1}{m} \sum_{i=1}^{m} \left(f_{\vec{\mathrm{w}},b}\left(\vec{\mathrm{x}}^{(i)}\right) - y^{(i)} \right)$$

} simultaneous update

Implementing gradient descent

repeat {

$$w_j = w_j - \alpha \left[\frac{1}{m} \sum_{i=1}^{m} \left[\left(f_{\vec{\mathrm{w}},b}\left(\vec{\mathrm{x}}^{(i)}\right) - y^{(i)} \right) x_j^{(i)} \right] + \frac{\lambda}{m} w_j \right]$$

$$b = b - \alpha \frac{1}{m} \sum_{i=1}^{m} \left(f_{\vec{\mathrm{w}},b}\left(\vec{\mathrm{x}}^{(i)}\right) - y^{(i)} \right)$$

} simultaneous update $j = 1 \ldots n$

$$w_j = \underbrace{1 w_j - \alpha \frac{\lambda}{m} w_j}_{w_j \left(1 - \alpha \frac{\lambda}{m}\right)} - \underbrace{\alpha \frac{1}{m} \sum_{i=1}^{m} \left(f_{w,b}\left(\vec{x}^{(i)}\right) - y^{(i)} \right) x_j^{(i)}}_{\text{usual update}}$$

$$w_j \underbrace{\left(1 - \alpha \frac{\lambda}{m}\right)}_{\text{shrink } w_j}$$

$$\alpha \frac{\lambda}{m}$$

$$0.01 \frac{1}{50} = 0.0002$$

$$w_j (\underbrace{1 - 0.0002}_{0.9998})$$

How we get the derivative term (optional)

$$\frac{\partial}{\partial w_j} J(\vec{\mathrm{w}}, b) = \frac{\partial}{\partial w_j}\left[\frac{1}{2m}\sum_{i=1}^{m}\left(\underbrace{f(\vec{x}^{(i)})}_{\vec{w}\cdot\vec{x}^{(i)}+b} - y^{(i)}\right)^2 + \frac{\lambda}{2m}\sum_{j=1}^{n} w_j^2\right]$$

$$= \frac{1}{\cancel{2}m}\sum_{i=1}^{m}\left[\left(\vec{w}\cdot\vec{x}^{(i)} + b - y^{(i)}\right)\cancel{2}x_j^{(i)}\right] + \frac{\lambda}{\cancel{2}m}\cancel{2}w_j \quad \text{No } \sum_{j=1}^{n}$$

$$= \frac{1}{m}\sum_{i=1}^{m}\left[\left(\underbrace{\vec{w}\cdot\vec{x}^{(i)} + b}_{f(\vec{x})} - y^{(i)}\right)x_j^{(i)}\right] + \frac{\lambda}{m}w_j$$

$$= \frac{1}{m}\sum_{i=1}^{m}\left[\left(f_{\vec{\mathrm{w}},b}(\vec{\mathrm{x}}^{(i)}) - y^{(i)}\right)x_j^{(i)}\right] + \frac{\lambda}{m}w_j$$

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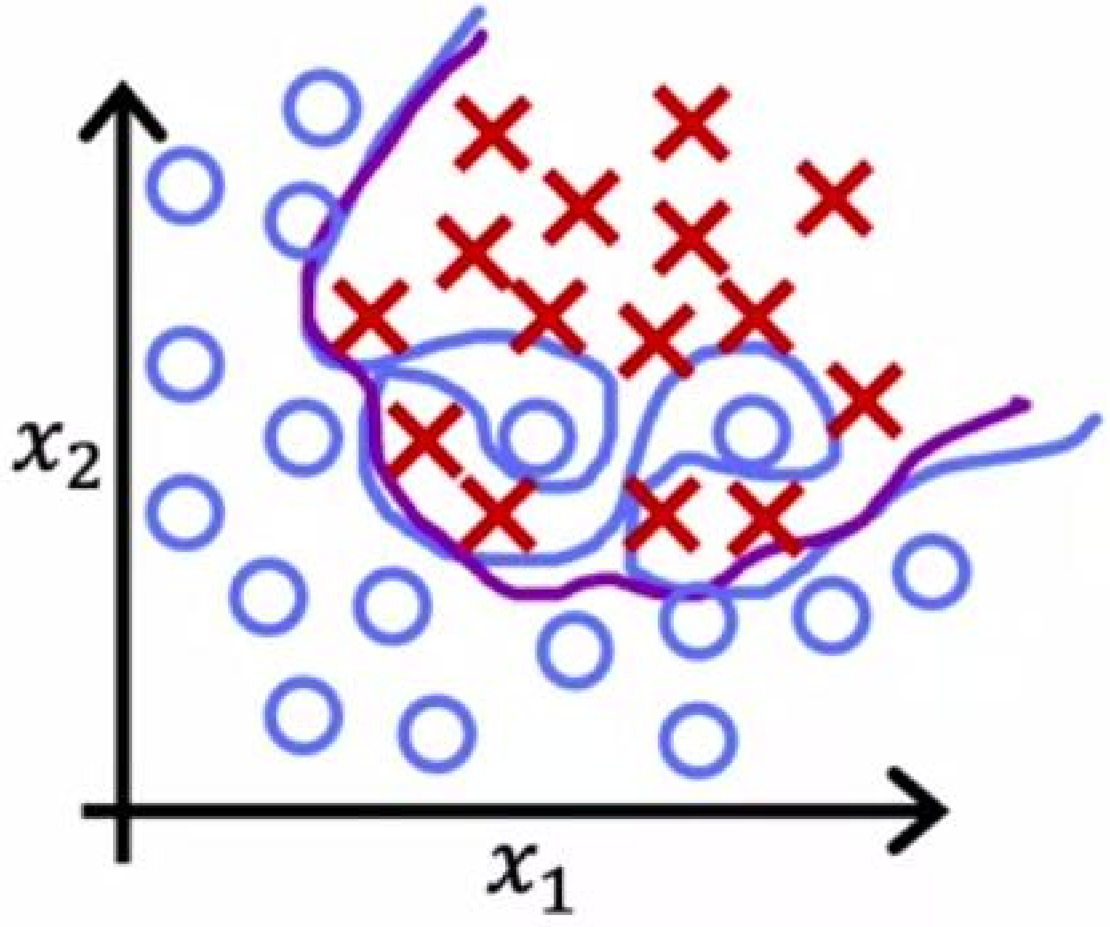
Regularization to Reduce Overfitting

Regularized Logistic Regression

Regularized logistic regression



$$z = w_1x_1 + w_2x_2 + w_3x_1^2x_2 + w_4x_1^2x_2^2 + w_5x_1^2x_2^3 + \cdots + b$$

$$f_{\vec{\mathrm{w}},b}(\vec{\mathrm{x}}) = \frac{1}{1+e^{-z}}$$

Cost function

$$J(\vec{\mathrm{w}},b) = -\frac{1}{m}\sum_{i=1}^{m}\left[y^{(i)}\log\left(f_{\vec{\mathrm{w}},b}(\vec{\mathrm{x}}^{(i)})\right) + (1-y^{(i)})\log\left(1-f_{\vec{\mathrm{w}},b}(\vec{\mathrm{x}}^{(i)})\right)\right] + \frac{\lambda}{2m}\sum_{j=1}^{n} w_j^2$$

$$\min_{\vec{w},b} J(\vec{\mathrm{w}},b) \rightarrow w_j \downarrow$$

Regularized logistic regression

$$J(\vec{\mathrm{w}}, b) = -\frac{1}{m}\sum_{i=1}^{m}\left[y^{(i)}\log\left(f_{\vec{\mathrm{w}},b}\left(\vec{\mathrm{x}}^{(i)}\right)\right) + \left(1 - y^{(i)}\right)\log\left(1 - f_{\vec{\mathrm{w}},b}\left(\vec{\mathrm{x}}^{(i)}\right)\right)\right] + \frac{\lambda}{2m}\sum_{j=1}^{n} w_j^2$$

$\min_{\vec{w},b}$

Gradient descent

Looks same as for linear regression!

repeat {

$$w_j = w_j - \alpha \frac{\partial}{\partial w_j} J(\vec{\mathrm{w}}, b)$$

$j = 1 \ldots n$

$$= \frac{1}{m}\sum_{i=1}^{m}\left[\left(f_{\vec{\mathrm{w}},b}\left(\vec{\mathrm{x}}^{(i)}\right) - y^{(i)}\right) x_j^{(i)}\right] + \frac{\lambda}{m} w_j$$

logistic regression

$$b = b - \alpha \frac{\partial}{\partial b} J(\vec{\mathrm{w}}, b)$$

$$= \frac{1}{m}\sum_{i=1}^{m}\left(f_{\vec{\mathrm{w}},b}\left(\vec{\mathrm{x}}^{(i)}\right) - y^{(i)}\right)$$

}

don't have to regularize