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**Algorithm 1** Expectation-Maximization Data-centric (EM-DC) over  $\mathcal{D}$ 


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1: INPUT  $\mathcal{D}$ : data,  $k$ : cluster number.
2: OUTPUT  $\mathcal{C}_1, \dots, \mathcal{C}_k$ :  $k$ -normal distributions/clusters.
3: // Each  $\mathcal{C}_j^i \sim \mathcal{N}(\mu_j^i, \Sigma_j^i)$ ,  $P(\mathcal{C}_j^i), w_{\mathbf{x}_j}^i, X_j^i \in \mathcal{C}_j^i$ .
4: //  $i$ : iteration number,  $j$ : cluster number.
5: //  $\mu_j^i$ : mean,  $\Sigma_j^i$ : covariance,  $P(\mathcal{C}_j^i)$ : prior.
6: //  $w_{\mathbf{x}_j}^i \in \mathbb{R}$ : likelihood and  $X_j^i \subseteq \mathcal{D}$ .
7: // Binary Search Trees (BSTs):  $\mathbf{T}^i = \{\mathcal{T}_1^i, \mathcal{T}_2^i, \dots, \mathcal{T}_k^i\}, \cup_{t=1}^k \mathcal{T}_t = \mathcal{D}$ 
8: randomly construct  $\mathbf{C}^0 = \{\mathcal{C}_1^0, \mathcal{C}_2^0, \dots, \mathcal{C}_k^0\}$ 
9:  $i \leftarrow 0$ 
10: //  $\mathcal{D}_{HE}$ : high expressive data. All data points are high expressive at the first iteration.
11:  $\mathcal{D}_{HE} \leftarrow \mathcal{D}$ 
12: repeat
13:   for  $\mathbf{x} \in \mathcal{D}$  do
14:     // E-step:
15:     for  $\mathcal{C}_j^i \in \mathbf{C}^i$  do
16:        $\mathcal{C}_j^i.w_{\mathbf{x}_j}^i \leftarrow P(\mathcal{C}_j^i | \mathbf{x})$ 
17:     end for
18:     // If  $i = 0$ , build BSTs, otherwise, update them—depends on  $w_{\mathbf{x}_j}^i$ .
19:      $\mathcal{C}_j^i.\mathcal{T}_j^i.insert(\mathbf{x}, w_{\mathbf{x}_j}^i) \leftarrow (\mathbf{x}, w_{\mathbf{x}_j}^i)$ 
20:   end for
21:   //  $\mathcal{D}_{HE}$ : A temporary variable used to store the HE data at each iteration.
22:    $\mathcal{D}'_{HE} \leftarrow \emptyset$ 
23:   // M-step:
24:   for  $\mathcal{C}_j^i \in \mathbf{C}^i$  do
25:      $\mathcal{C}_j^{i+1}.\mu_j^i \leftarrow \Sigma_{\mathbf{x} \in \mathcal{C}_j^i.X_j^i}(\mathbf{x} \cdot \mathcal{C}_j^i.w_{\mathbf{x}_j}^i / (\Sigma \mathcal{C}_j^i.w_{\mathbf{x}_j}^i))$ 
26:      $\mathcal{C}_j^{i+1}.\Sigma_j^i \leftarrow \Sigma_{\mathbf{x} \in \mathcal{C}_j^i.X_j^i}(\mathcal{C}_j^i.w_{\mathbf{x}_j}^i (\mathbf{x} - \mathcal{C}_j^i.\mu_j^i)(\mathbf{x} - \mathcal{C}_j^i.\mu_j^i)^T / (\Sigma_j^i \mathcal{C}_j^i.w_{\mathbf{x}_j}^i))$ 
27:      $\mathcal{C}_j^{i+1}.P(\mathcal{C}_j^i) \leftarrow \Sigma(\mathcal{C}_j^i.w_{\mathbf{x}_j}^i) / |\mathcal{C}_j^i.X_j^i|$ 
28:     // Using BSTs to determine new HE data and storing them in  $\mathcal{D}'_{HE}$ 
29:     //  $\mathcal{C}_j^i.\mathcal{T}_j^i.flush(\Sigma)$  represents separation of HE data from BST
30:      $\mathcal{D}'_{HE} \leftarrow \mathcal{C}_j^i.\mathcal{T}_j^i.flush(\Sigma)$ 
31:      $\mathbf{C}^{i+1} \leftarrow \cup \{\mathcal{C}_j^{i+1}\}$ 
32:   end for
33:    $i \leftarrow i + 1$ 
34:   // Updating final HE data
35:    $\mathcal{D}_{HE} \leftarrow \mathcal{D}'_{HE}$ 
36:   //  $d: \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$ , Stopping criterion (Convergence over structure, BSTs):
37:   // Node-wise hamming distance among BSTs between two consecutive iterations.
38: until threshold on  $d(\mathbf{C}^{i-1}, \mathbf{C}^i) \leq \epsilon$ 

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