Analysis and Modeling of Maximum Likelihood Detection for a MIMO Antenna System

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Abstract

In this project, we will provide an analysis of the performance of maximum likelihood detection (MLD) over flat-fading channels in a wireless multiple input–multiple output (MIMO) antenna system. We will introduce a tight union bound with an asymptotic form on the probability of symbol error rate (SER) for MIMO MLD systems with two-dimensional signal constellations (such as QAM and PSK). Using this analytic bound, we will demonstrate the performance of the MIMO antenna system quantitatively with respect to channel estimation, constellation size, and antenna configuration.

I. Introduction

WIRELESS multiple input–multiple output (MIMO) systems promise improved performance compared to conventional systems. Techniques for achieving these advantages include zero-forcing (ZF), minimum mean square error (MMSE), and maximum likelihood detection (MLD). Among these techniques, MLD is optimum in terms of minimizing the overall error probability, with small numbers of transmit antennas and low-order constellations, the complexity of MLD is not overwhelming.

In this project,we provide a performance analysis of MLD over flat fading channels. A tight union bound and an asymptotic bound on the SER are developed to the MIMO configuration, with 2-D constellations. These bounds are then utilized to demonstrate the performance of MLD quantitatively. Our approach of deriving the pairwise symbol error probability might be extended to evaluate the pairwise block error probability of the MLD for a coded system.

II. Methodology

Our investigation begins with a comprehensive definition of key parameters characterizing our **MIMO** (Multiple Input Multiple Output) system, including the number of transmit antennas **(K)**, receive antennas **(L)**, constellation size **(M)**, and other essential variables such as average symbol energy, noise power spectral density **(No)**, channel estimation accuracy (σ_v), and the correlation coefficient (ρ_{hv}). This foundational step lays the groundwork for our subsequent analytical endeavors, providing a holistic understanding of the system's configuration and characteristics.

Subsequently, we proceed to simulate the transmission process by generating random channel gain matrices (H) based on the specified transmit and receive antenna configurations (K and L, respectively). These complex Gaussian matrices serve to encapsulate the spatial diversity inherent in MIMO systems, reflecting the intricate propagation characteristics of wireless communication channels.

With the transmission simulated, we embark on computing received signal vectors (y) using the classic model **y = Hd + n**, where 'n' represents the additive white Gaussian noise. This critical step enables us to emulate the reception process, accounting for noise and channel fading effects. Following this, we calculate the Maximum Likelihood (ML) metric (Lambda) using the provided formula, serving as a pivotal performance indicator. Our subsequent implementation of a union bound on the Symbol Error Rate (SER) offers insights into system resilience across varying channel estimation accuracies and antenna configurations, guiding optimization efforts and informing design decisions. Through systematic analysis, we unravel nuanced performance trends, facilitating a comprehensive understanding of the system's behavior and offering valuable insights for further system refinement and optimization.

III. System Model

We consider a MIMO system with $\boldsymbol{K}$ transmit and $\boldsymbol{L}$ receive antennas, where the transmitted signals are assumed to be independent in time as well as space. The transmitted signal vector at a particular time instant is written as $\boldsymbol{d}$ and consists of $\boldsymbol{K}$ QAM or PSK symbols each with a constellation size of $\boldsymbol{M}$ and average symbol energy $\boldsymbol{Es}$. The received signal vector $\boldsymbol{y}$ is given by:

$$\boldsymbol{y} = \boldsymbol{Hd} + \boldsymbol{n} \tag{1}$$

where $\boldsymbol{H}$ is an $L\ X\ K$ channel gain matrix for the flat fading channel, whose elements are independent zero-mean complex Gaussian random variables with unit variance, and the elements of vector $\boldsymbol{n}$ are samples of independent complex additive white Gaussian noise (AWGN) processes with single-sided power spectral density ***No.***

Channel estimation is determined by channel state information (CSI) and we assume that the estimate of true channel gain matrix H is denoted by V which also consists of independent zero-mean complex Gaussian random variables, with variance

σ_v^2. Let ρ_{hv} denote the correlation coefficient between corresponding elements of H and V and, since they are jointly Gaussian distributed with independent components, we can write

$$H = \beta_{hv} \cdot V + E \tag{2}$$

where $\beta_{hv} = \frac{\rho_{hv}}{\sigma_v}$ is the coefficient for MMSE estimation of V and H, and E is a zero mean Gaussian distributed error matrix with the variance $(1 - |\rho_{hv}|^2)$. It is assumed that $|\beta_{hv}| = 1$ and note that, with perfect CSI, $\rho_{hv} = 1$ and $\beta_{hv} = 1$.

IV. Dataset Description

The conditional probability density function (pdf) of the received ***y***, given the channel estimate ***V*** and the candidate data vector ***d***, is given by:

$$P_y(y/d, V) = \frac{1}{(2\pi)^L \cdot (\sigma_y^2)^L} \cdot \exp\left(-\frac{\mu}{2\sigma_y^2}\right) \tag{3}$$

where σ_y^2 is given by $\sigma_y^2 = (1 - |\rho_{hv}|^2) \cdot ||d||^2 + N_o$ and the Euclidean distance metric μ can be expressed as

$$\mu = \sum_{l=1}^{L} (y_l - \beta_{hv} v_l d)^2 \tag{4}$$

where y_l is the lth received signal and v_l denotes the lth row of V.

Neglecting hypothesis-independent terms, the ML metric to be minimized is given by:

$$\Lambda = L \ln(\sigma_y^2) + \frac{\mu}{2\sigma_y^2} \tag{5}$$

V. UNION BOUND ON SER FOR MIMO MLD

A tight union bound on the SER of the kth (k=1 to K) transmitted signal stream can be found by applying the results in [6] and [7] to the MIMO configuration for 2-D constellations under the channel estimate. It is assumed that all the possible symbols are equally probable. We define $\{s_m\}$ as the set of all M possible symbols transmitted at a particular antenna, and $\{d\}$ as the set of all possible symbol vectors from the K transmit antennas. We also let $\{d_j\}$ denote a subset of $\{d\}$ in which vectors have s_m as their kth element so that in total there M^{K-1} are vectors in $\{d_j\}$. We also define $\{d_i\}$ as the set of transmission vectors that differ in their th position from $\{d_j\}$ so that there are a total of $M^K - M^{K-1}$ such vectors. The distance metrics of d_i and d_j are denoted by μ_i and μ_j , respectively, and a pairwise error occurs when the detector chooses the erroneous d_i over d_j if D_{ij} = μ_i - μ_j < 0. Hence, the union bound on the SER of the signal stream transmitted by the kth antenna is

$$P_s \leq M^{-K} \cdot \sum_m \sum_j \sum_i P_{s_{m,i,j}} \tag{6}$$

where $P_{s_{m,i,j}}$ denotes the pairwise error probability between d_i and d_j, given that s_m is transmitted by the kth antenna. There can be up to $M^K \cdot (M^K - M^{K-1})$ pairwise error probabilities but symmetry in the constellation allows simplification.

The pairwise error probability is determined by

$$P_s^{m,i,j} = \frac{1}{(1 + r_s^{m,i,j})^{2L-1}} \sum_{l=0}^{L-1} ((2L-1)C(l))(r_s^{m,i,j})^l \tag{7}$$

$$r_s^{m,i,j} = a_s^{m,i,j} \cdot T_s^{m,j} + \sqrt{(a_s^{m,i,j} \cdot T_s^{m,j})^2 + 2 \cdot (a_s^{m,i,j} \cdot T_s^{m,j})} + 1 \tag{8}$$

$$a_s^{m,i,j} = \frac{||d_i - d_j||^2}{2 \cdot E_s} \tag{9}$$

$$T_s^{m,j} = \frac{\gamma_c \cdot |\rho_{hv}|^2}{\left(\frac{\gamma_c \cdot (1 - |\rho_{hv}|^2)}{E_s} \cdot ||d_j||^2\right) + 1} \tag{10}$$

with $\gamma_c = \frac{E_s}{N_o}$ denoting the average symbol SNR per diversity branch since variance of the channel gain has been normalized to be unity.

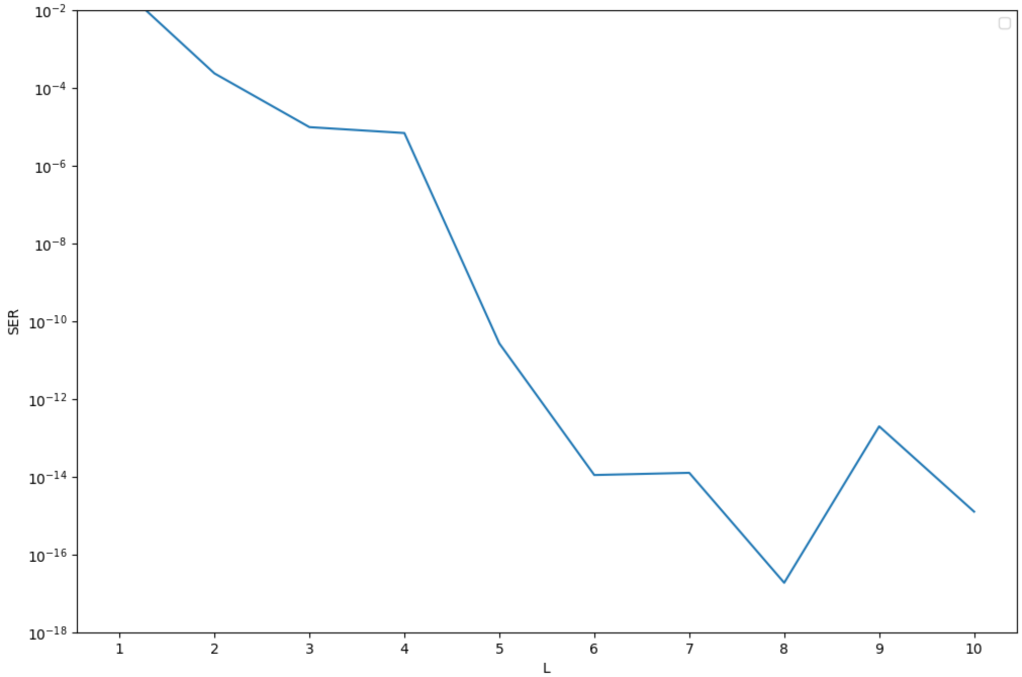

Fig. 1: Union Bound on SER

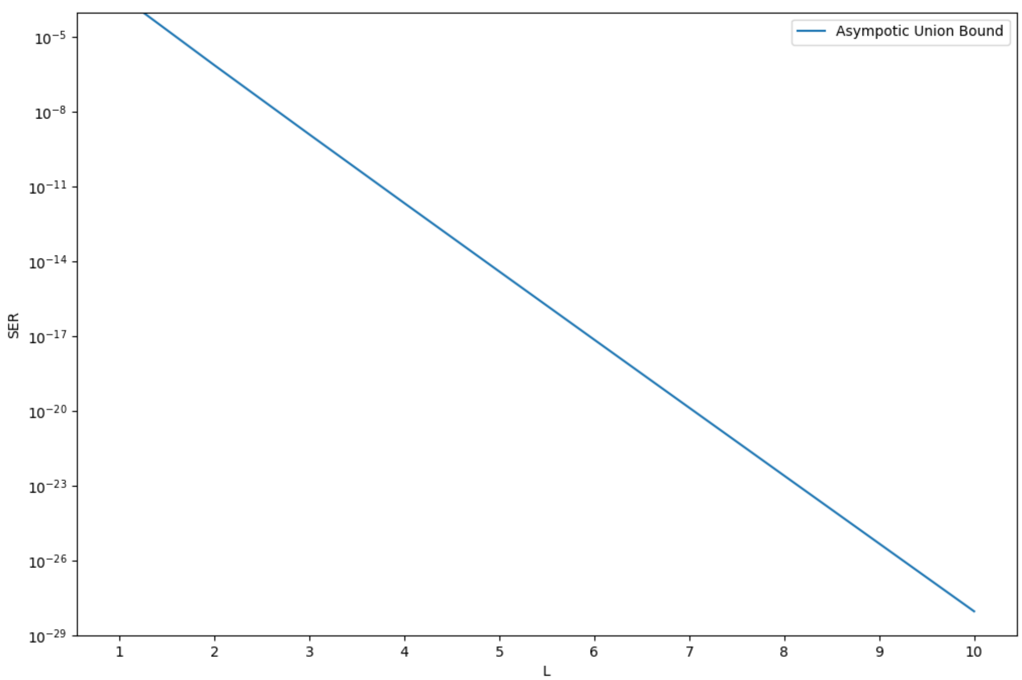

Fig. 2: Asymptotic Union Bound on SER

VI. Asymptotic Union Bound

When SNR becomes high, the asymptotic form of pairwise probability can be written as

$$P_s^{m,i,j,asymp} = (r_s^{m,i,j})^{-L} \cdot (2L-1)C(L-1) \tag{11}$$

where $r_s^{m,i,j} = 2 \cdot a_s^{m,i,j} \cdot T_s^{m,j}$ and is an extension of the results in [7].

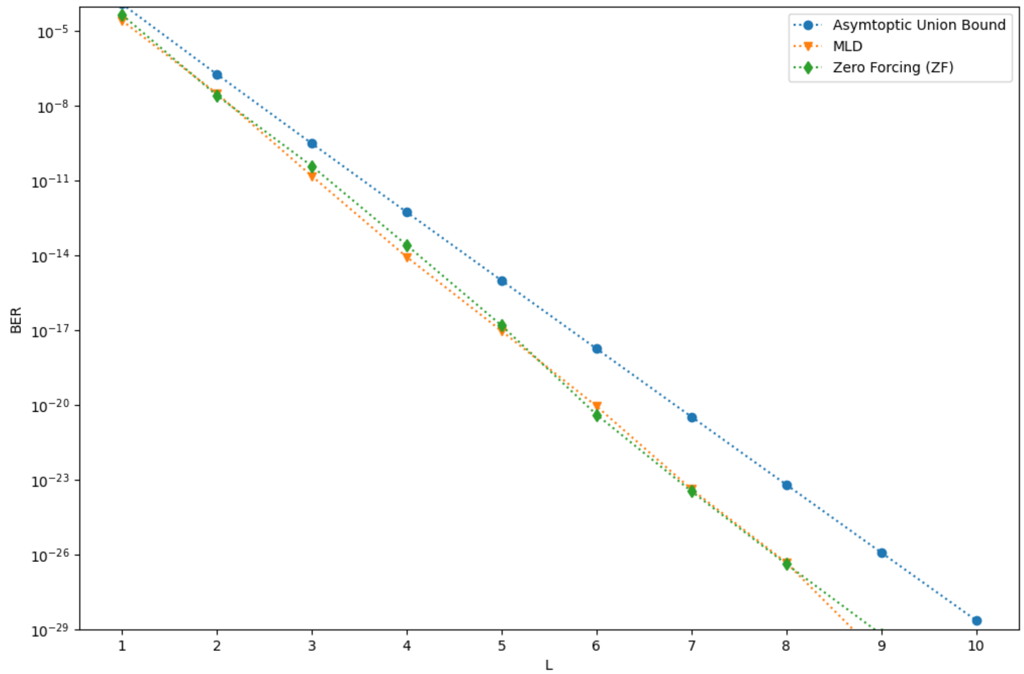

Fig. 3: Comparison of Zero Forcing(ZF) Maximum Likelihood Detection (MLD)

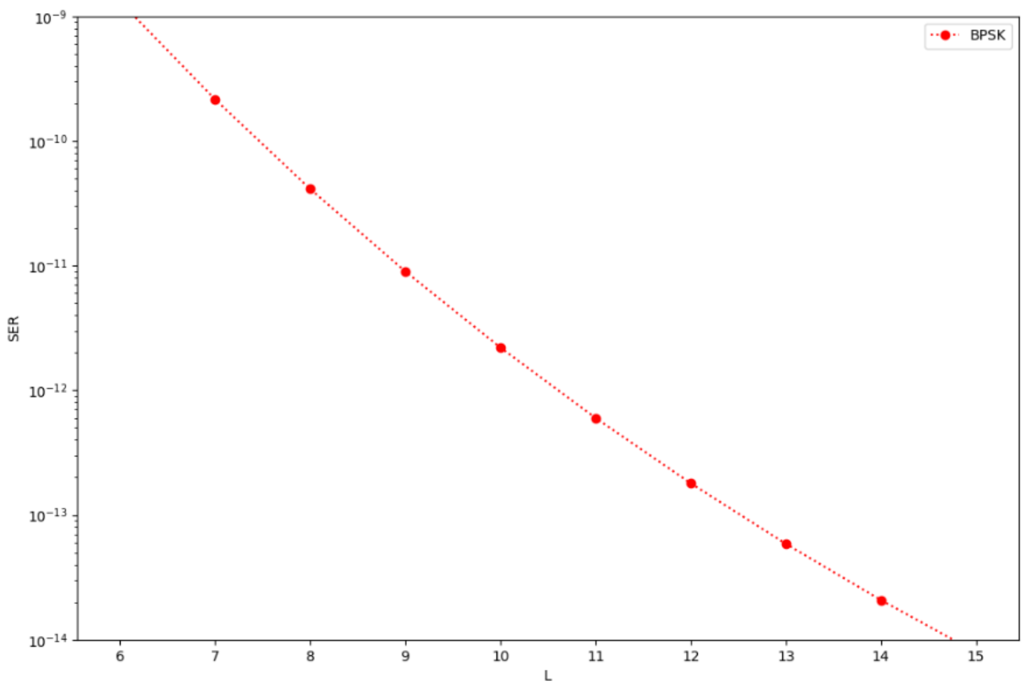

Fig. 4: Variation of SER with ’L’ for BPSK

VII. Performance Analysis

A. *Effect of Antenna Configuration*

Our project delved into the influence of antenna configuration on Bit Error Rate (BER) through manipulation of the number of receiving antennas, denoted as ’L.’ As ’L’ escalates, we observed a corresponding decrease in BER [Figure 2], indicating a significant relationship between antenna quantity and error rates. For BPSK, a similar pattern is observed for SER [Figure 4]. These findings underscore the critical role antenna configuration plays in communication system performance, offering valuable insights for optimizing BER in practical applications.

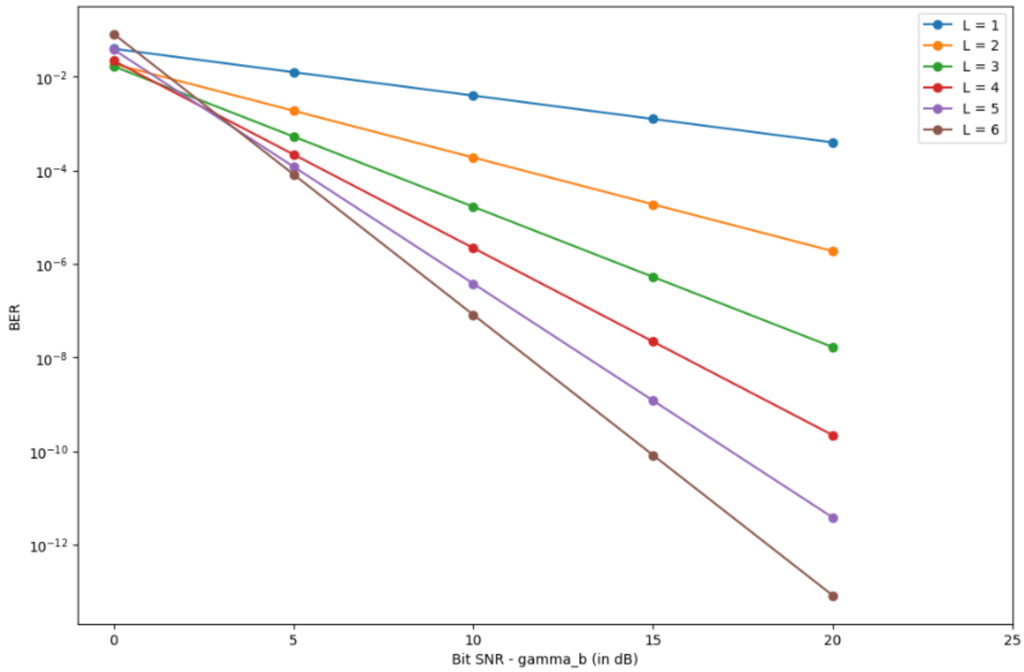

Fig. 5: Variation of BER with γ_b

B. *Effect of Imperfect CSI on Performance*

It has been shown approximately that imperfect CSI degrades the SNR by an asymptotic factor of **Kb+1** , independent of the number of receive antennas. Using our union bound given by (6) and (7), the effect of imperfect CSI on the performance has been investigated with b=0.199 (implying $\rho_{hv} = 1$ when $\gamma_c = 20dB$). We found that with two transmit antennas this leads to an SNR penalty of about 1.4 dB for both BPSK and 16QAM and matches our conclusion from the asymptotic form of the union bound.

C. *Effect of bit SNR (γ_b) on BER*

It can be ascertained that for systems with equal bit SNR (γ_b) performance, higher receive antenna counts necessitate lower total transmit power. Furthermore, a direct relationship emerges between increasing γ_b and decreasing BER [Figure 5], highlighting the pivotal role of signal-to-noise ratio in error rate mitigation.

D. *Diversity Order*

It can be deduced that with a relatively high SNR (i.e., BER is below a specific level such as 0.01), the error probability is proportional to the inverse of the SNR to the power of L. This implies that the diversity order of MLD is equal to the number of receive antennas, independent of the number of transmit antennas. Furthermore, in this case the SNR penalty due to the increased number of transmit antennas plays a major role in the performance change.

This implies that (and without regard to complexity) we can achieve an arbitrary high data rate with a low SNR penalty when the number of receive antennas is sufficiently large.

E. *Performance Comparison Between MLD and ZF*

The performance of both Zero Forcing and Maximum Likelihood Detection is compared concerning the Asymptotic Union Bound and is plotted in Fig. 3.

It can be seen that there is not much difference between conventional detection techniques.

VIII. CONCLUSION

This project presents a tight union bound and an asymptotic expression for the Symbol Error Rate (SER) in a Multiple Input Multiple Output (MIMO) Maximum Likelihood Detection (MLD) system utilizing two-dimensional signal constellations. The analysis reveals that as the number of receive antennas (L) increases, a remarkably high data rate can be attained with minimal Signal-to-Noise Ratio (SNR) degradation, and the diversity order of MLD aligns with the number of receive antennas (L). Additionally, we conduct a performance evaluation comparing different two-dimensional constellations and contrasting MLD with Zero Forcing (ZF) detection.

IX. References

1) Xu Zhu and R.D. Murch,"Performance analysis of maximum likelihood detection in a MIMO antenna system"
2) B. A. Bjerke and J. G. Proakis, "Multiple-antenna diversity techniques for transmission over fading channels"

Appendix

Code for MBSA Project ('Group5_MBSA_Project.ipynb' in linked Github Repository)