A Thesis on Collision-free navigation and control of satellite using APF guidance law and Sliding Mode Controller

Project-I (AE) report submitted to

Indian Institute of Technology Kharagpur

in partial fulfilment for the award of the degree of

Bachelor of Technology

in

Aerospace Engineering

by

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Autumn Semester, 2024-25

November 12, 2024

DECLARATION

I certify that

(a) The work contained in this report has been done by me under the guidance of my supervisor.

(b) The work has not been submitted to any other Institute for any degree or diploma.

(c) I have conformed to the norms and guidelines given in the Ethical Code of Conduct of the Institute.

(d) Whenever I have used materials (data, theoretical analysis, figures, and text) from other sources, I have given due credit to them by citing them in the text of the thesis and giving their details in the references. Further, I have taken permission from the copyright owners of the sources, whenever necessary.

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DEPARTMENT OF AEROSPACE ENGINEERING

INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR

KHARAGPUR - 721302, INDIA

CERTIFICATE

This is to certify that the project report entitled "**A Thesis on Collision-free navigation and control of satellite using APF guidance law and Sliding Mode Controller**" submitted by **Paridhi D Choudhary** (Roll No. 21AE3AI02) to Indian Institute of Technology Kharagpur towards partial fulfilment of requirements for the award of degree of Bachelor of Technology in Aerospace Engineering is a record of bona fide work carried out by him under my supervision and guidance during Autumn Semester, 2024-25.

Date: November 12, 2024
Place: Kharagpur

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Abstract

Name of the student: **Paridhi D Choudhary** Roll No: **21AE3AI02**
Degree for which submitted: **Bachelor of Technology**
Department: **Department of Aerospace Engineering**
Thesis title: **A Thesis on Collision-free navigation and control of satellite using APF guidance law and Sliding Mode Controller**
Thesis supervisor: **Professor Manoranjan Sinha**
Month and year of thesis submission: **November 12, 2024**

With the surge in space exploration missions, there is a growing accumulation of space debris orbiting the earth. This poses a growing risk of collision of the operational satellites and future space missions. To counter these risks, the development of an effective and precise guidance law for satellites is essential. Various guidance algorithms have been explored in the literature including, the Artificial Potential Field-based guidance algorithm (APF) which is widely used for real-time path adjustments to avoid the collision. However, this algorithm is prone to local minima near obstacles. To tackle with this local minima issue, many methods are available in literature, one of which has been used in this thesis (Rafal Szczepanski), modeling the spacecraft as a rigid body in Lie Space (Lee) and using Sliding Mode Controller (smc) to control the spacecraft to reach the desired goal. Furthermore, it is desired to implement a robust Model Predictive Controller, along with a new devised approach of APF to perform the aforementioned task.

Acknowledgements

This project owes its inception and development to the profound inspiration derived from Professor Manoranjan Sinha. His visionary insights, unwavering support, and invaluable guidance have been instrumental in shaping every facet of this endeavor. I am profoundly grateful for his mentorship, which has not only enriched the project but also significantly contributed to my personal and professional growth.

As a student studying Aerospace Engineering at the Indian Institute of Technology, Kharagpur, I extend our gratitude and appreciation towards the teaching faculty of the Aerospace Engineering Department for helping me reach where I am today and for putting up with my queries. I also appreciate our fellow batch mates for their assistance over the past few years.

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To everyone involved, for all of their kind cooperation during this project, in each aspect of its work, we extend to you our thanks.

Finally, I must express my very profound gratitude to my parents for providing me with unfailing support and continuous encouragement throughout my years of study and through the process of researching and writing this thesis. This accomplishment would not have been possible without them. Thank you.

Contents

Declaration **i**

Certificate **ii**

Abstract **iii**

Acknowledgements **iv**

Contents **v**

List of Figures **vii**

1 Introduction **1**

2 Literature Review **3**

2.1 Spacecraft Modeling using Lie Algebra and Control using SMC 3

. . . . 3

Sliding Mode Controller (SMC) 4

2.2 Artificial Potential Field Algorithm 5

2.2.1 Local Minima Problem 5

2.2.2 Adaptive Artificial Safe Potential Field Algorithm 6

3 Mathematics And Implementation **8**

3.1 Dynamics Modeling and Sliding Mode Controller 8

3.1.1 Leader Dynamics 8

3.1.2 Follower Dynamics 9

3.1.3 Relative Dynamics 10

3.1.4 Sliding Mode Controller Formulation and overall Dynamics . . . 12

3.2 Artificial Potential Field Algorithm(APF) 13

3.2.1 Attractive Potential Gradient 14

3.2.2 Repulsive Potential Gradient 14

3.2.3 Adaptive Safe Artificial Potential Field Algorithm 15

3.2.3.1 Adaptive Scaling of Attractive Potential 15

3.2.3.2 Adaptive Scaling of Repulsive Potential 15
3.3 Combining Guidance Law with Dynamics 17

4 Simulation Results **18**
4.1 Dynamics and Sliding Mode Controller 18
4.2 Obstacle avoidance using ASAPF in 3d including Dynamics and SMC . . 18

5 Further works **20**
5.1 Work yet to be done 20

Bibliography **21**

List of Figures

2.1 Sliding Mode Controller 4
2.2 Artificial Potential Field Algorithm 6
2.3 Artificial Potential Field Algorithm 7

4.1 Dynamics and SMC simulation (Matlab) 18
4.2 SMC with dynamics and Guidance Law 19
4.3 SMC with dynamics and Guidance Law 19

Chapter 1

Introduction

As spacecraft rendezvous and docking missions become increasingly complex, the need for advanced guidance laws has surged. Vital to ensuring secure rendezvous and docking of spacecraft is the task of obstacle and collision avoidance. A huge amount of additional debris is created as the cause of a major collision, thereby making collision avoidance absolutely necessary.

A lot of work has been done in the field of developing path planning algorithms with a proper balance of complexity and computational expense with robustness and performance of the solution. Analytical methods including glideslope, artificial potential function and Lyapunov functions are often used. These methods, though computationally inexpensive , don't produce optimal results, hence, the trend is to either use pure optimal methods, which though produce optimal results but are computationally expensive or to use these methods with a modification to produce relatively better results. In recent years, techniques involving use of Reinforcement Learning to tackle spacecraft rendezvous guidance challenges have been in prominent use. (rl) describes one such method. (rl) also utilises a similar approach for safe rendezvous and docking with dynamic obstacle avoidance.

Artificial Potential Field Algorithm is a popular choice among analytical path planning methods, which primarily works by assigning a repulsive potential to obstacles and an attractive potential to goal. This method is straightforward and computationally easy to implement, however, fails to give optimal results due to local minima issue. Quite a lot of variations of this algorithm have been employed to avoid local minima. (R. Szczepanski and Tarczewski) uses an an augmented reality approach, while (9) uses Reinforcement Learning to solve the same issue. Smoothness improvement was proposed in (J. Song and Su) and (Lin and Tsukada).

[We have proposed a guidance algorithm to address the limitations of traditional collision avoidance techniques, particularly the local minima issues in standard APF methods.

In this thesis, first the dynamics of spacecraft is modeled using Lie Group (SE3), which is special Euclidean group, which is the set of positions and orientations of the spacecraft moving in three dimensional Euclidean space. Then a Sliding Mode Controller(SMC) is used for tracking of a spacecraft to a desired attitude and position by using a virtual leader. Thereafter, an already existent Adaptive Artificial Safe Potential Field Algorithm (Rafal Szczepanski) has been used in 3d using aforementioned dynamics and SMC to control it.

Chapter 2

Literature Review

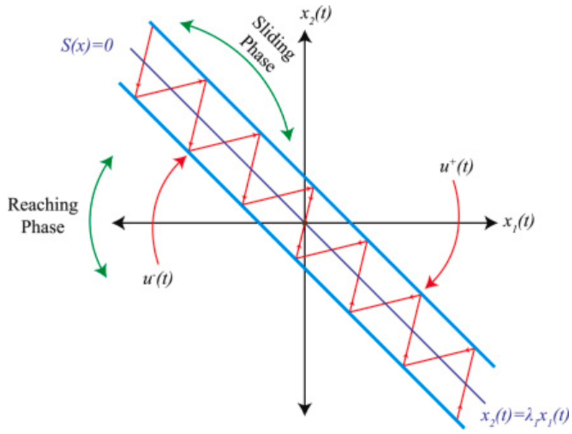
2.1 Spacecraft Modeling using Lie Algebra and Control using SMC

In the context of spacecraft control, Lie algebra is often used to analyze and control the relative movements between spacecraft in a leader-follower setup. By applying Lie algebra, one can formulate the dynamics of each spacecraft in a way that respects the geometric constraints of rotational and translational motion, which is particularly useful when dealing with coupled systems. The use of Lie algebra enables the decomposition of complex dynamics into simpler components, making it easier to design control laws that ensure stability and synchronization among spacecraft.

The paper (Lee) , applies Lie algebra within a coupled tracking control strategy for multiple spacecraft systems. A sliding mode control (SMC) framework is employed to achieve robust tracking of a leader spacecraft by several followers, even under input saturation constraints. The control strategy is designed to ensure that the followers maintain a specified formation around the leader by using sliding mode principles to handle uncertainties and disturbances. Lie algebra in this context is instrumental in formulating the relative dynamics between the leader and followers, allowing the authors to construct a control law that respects the geometric constraints and actuator limits of each spacecraft.

The aim of this work, however, is to model the spacecraft using SE3 dynamics and to use a new approach of Artificial Potential Field Algorithm in order to ensure collision free navigation of spacecraft for rendezvous and docking purposes. Hence, only one spacecraft is modeled using this dynamics and then sliding mode controller is used to reach a desired orientation. The detailed dynamics and implementation procedure along with the simulation results are shown in the Mathematics and Implementation section.

Sliding Mode Controller (SMC) A sliding mode controller (SMC) is a prominent nonlinear controller that brings the state of the system from any arbitrary position to the equilibrium position. Rather than using a continuous control law, SMC traditionally uses discontinuous law for sliding around the designed sliding manifold depicted below:



(a) (smc)

FIGURE 2.1: Sliding Mode Controller

In sliding mode control (SMC), the state-feedback control law is characterized by its switching between different continuous structures, rather than being a continuous function over time. This makes SMC a type of variable structure control. The control law is crafted to ensure that the system's trajectory continuously moves toward adjacent regions with distinct control structures, leading to a transition across multiple control structures rather than staying within a single one. Consequently, the system "slides" along the boundaries of these control regions, a behavior termed as the sliding mode. These boundaries, defining the sliding motion, are referred to as the sliding (hyper)surface. In contemporary control theory, an SMC-governed system can be viewed as a hybrid dynamical system, as it operates in a continuous state space while alternating between discrete control modes. Since it is a non linear controller, a Lyapunov stability has to be applied to keep the non linear system under control.

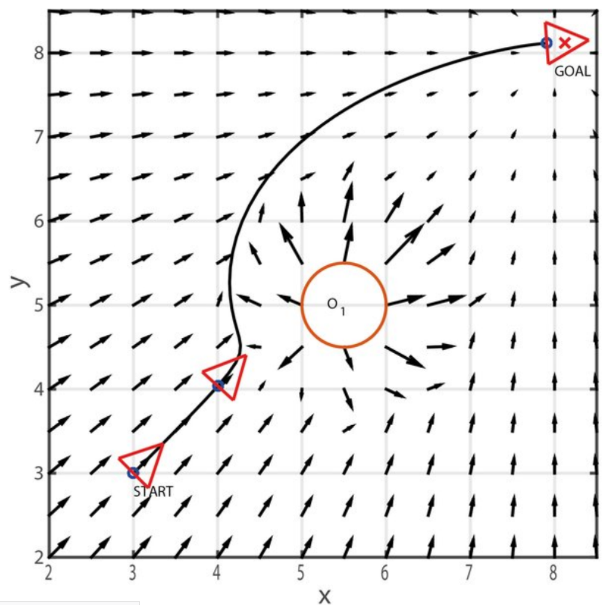
2.2 Artificial Potential Field Algorithm

A lot of work has been done in the field of developing path planning algorithms with a proper balance of complexity and computational expense with robustness and performance of the solution. Analytical methods including glideslope, artificial potential function and Lyapunov functions are often used. These methods, though computationally inexpensive, don't produce optimal results, hence, the trend is to either use pure optimal methods, which though produce optimal results but are computationally expensive or to use these methods with a modification to produce relatively better results. In recent years, techniques involving use of Reinforcement Learning to tackle spacecraft rendezvous guidance challenges have been in prominent use. (rl) describes one such method.(rl) also utilises a similar approach for safe rendezvous and docking with dynamic obstacle avoidance.

Artificial Potential Field Algorithm has been in popular use for various path planning algorithms since past couple of years. This algorithm primarily employs an artificial potential field to regulate a robot in a certain space. It assigns an artificial potential field to every point in the world using defined potential field functions. The potential field generates attractive or repulsive forces, and the robot or the UAV is pulled towards certain points in the environment due to attractive forces or pushed away from certain points due to the repulsive forces. Obstacles produce a repulsive force while the goal produces an attractive force on the robot. Although straightforward to understand and implement, this method poses a major problem of local minima.

2.2.1 Local Minima Problem

Situations in which problem of local minima is faced can majorly be categorised into three cases. First, when the robot, obstacle and goal are along the same line, obstacle being between the goal and the robot. In this case, the attractive force may be equal to the repulsive force, thereby making the total force negligible, due to which the robot might stop or hit the obstacle. Second, when the goal is in front of the robot and there are two obstacles on the left and right sides of the robot, forming a narrow passage, due to which there is a balance of net force, trapping the robot. Third, in situations when the goal is placed in between the range of obstacles, due to which the repulsive field rapidly increases, while the attractive field decreases as the robot moves towards the goal as elucidated in 2.3(a).

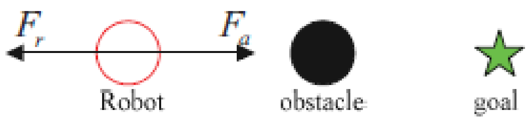


(a) (apf)

FIGURE 2.2: Artificial Potential Field Algorithm

2.2.2 Adaptive Artificial Safe Potential Field Algorithm

To remove the local minima error, (Rafal Szczepanski) uses the Safe Artificial Potential Field algorithm. This approach provides smooth movement, operation in narrow corridors, and simple local minima avoidance. Moreover, the computational effort is not significant due to a lack of prediction or optimization algorithms. The algorithm maintains the required distance from the obstacle during the movement. There are scaling factors in original APF, which are changed adaptively. The adaptation mechanism modifies one of the scaling factors or provides a switching potential function.



(a) (Li Zhou)

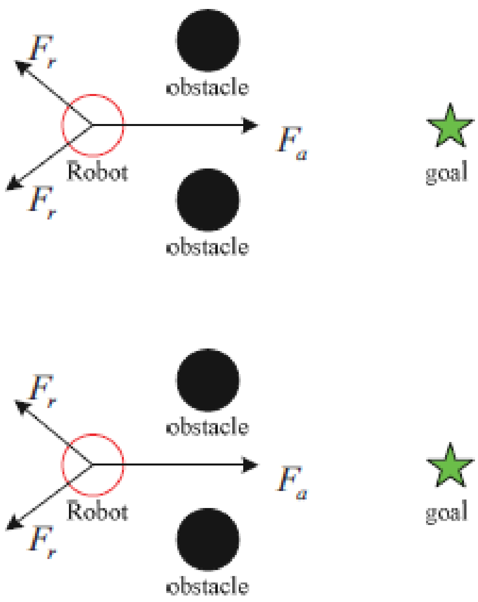


FIGURE 2.3: Artificial Potential Field Algorithm

Chapter 3

Mathematics And Implementation

3.1 Dynamics Modeling and Sliding Mode Controller

As discussed above, the spacecraft is modeled as rigid body using Lie Algebra and following a leader follower approach.

3.1.1 Leader Dynamics

The Leader spacecraft is modeled as a rigid spacecraft orbiting the Earth with its central gravity field. The configuration space of leader is SE(3). The leader attitude is represented by the rotation matrix $R^0 \in \mathrm{SO}(3)$ that transforms the leader body-fixed frame to the Earth-centered inertial (ECI) frame. The overall dynamics of the leader in compact form are given as follows:

$$I_0\dot{\xi}_0 = \mathrm{ad}^*_{\xi_0} I_0\xi_0 + \varphi^0_g$$

Here ^{0} notation refers to the states of leader. g^0 refers to the configuration of the leader on SE(3) which is represented by the subsequent $4X4$ matrix:

$$g_0 = \begin{pmatrix} R_0 & b_0 \\ 0 & 1 \end{pmatrix}$$

while the vector of velocities of the body are:

$$\xi_0 = \begin{pmatrix} \omega^0 \\ v^0 \end{pmatrix}$$

where the leader's position is given by the inertial position vector $\mathbf{b}_0 \in \mathbb{R}^3$, representing the position from the origin of the Earth-Centered Inertial (ECI) frame to the center of mass of the leader. The translational and angular velocities of the leader are represented by the vectors $\nu_0 \in \mathbb{R}^3$ and $\mathbf{\Omega}_0 \in \mathbb{R}^3$, respectively, as observed in the leader's body-fixed frame.

Subsequently, the kinematics of the leader will be expressed as

$$\dot{g}_0 = g_0(\xi_0)^\vee$$

where

$$(\xi_0)^\vee = \begin{pmatrix} (\Omega_0)_\times & \nu_0 \\ 0 & 0 \end{pmatrix} \in \mathfrak{se}(3)$$

The mass and inertia properties of the leader, a 6×6 matrix, and the vector of gravitational forces and moments, a 6×1 vector, are expressed as

$$\phi_0^g = \begin{pmatrix} M_0^g \\ F_0^g \end{pmatrix} \in \mathbb{R}^6$$

and

$$I_0 = \begin{pmatrix} J_0 & 0 \\ 0 & m_0 I \end{pmatrix} \in \mathbb{R}^{6\times6}$$

$$\mathrm{ad}_{\xi_0} = \begin{pmatrix} (\Omega_0)_\times & 0 \\ (\nu_0)_\times & (\Omega_0)_\times \end{pmatrix} \in \mathbb{R}^{6\times6}$$

$$\mathrm{ad}_{\xi_0} = (\mathrm{ad}_{\xi_0})^T$$

3.1.2 Follower Dynamics

The configuration of the follower spacecraft is represented by the position vector $\mathbf{b} \in \mathbb{R}^3$ from the origin of the geocentric inertial frame to the center of mass, and the attitude by the rotation matrix $R \in \mathrm{SO}(3)$. The kinematics are given by

$$\dot{\mathbf{b}} = R\nu, \quad \dot{R} = R(\Omega)_\times$$

or equivalently,

$$\dot{g} = g(\xi)^\vee$$

where

$$(\xi)^\vee = \begin{pmatrix} (\Omega)_\times & \nu \\ 0 & 0 \end{pmatrix} \in \mathfrak{se}(3)$$

with $g \in \mathrm{SE}(3)$ representing the configuration, $\nu \in \mathbb{R}^3$ the translational velocity, and $\Omega \in \mathbb{R}^3$ the angular velocity of the follower in its body frame.

The dynamics of the spacecraft are:

$$m\dot{\nu} = m(\nu)_\times \Omega + F_g(\mathbf{b}, R) + mR^T a_{J2}(\mathbf{b}) + \phi_c(\mathbf{b}, R, \nu, \Omega) + \phi_d$$

$$J\dot{\Omega} = J(\Omega)_\times \Omega + M_g(\mathbf{b}, R) + \tau_c(\mathbf{b}, R, \nu, \Omega) + \tau_d$$

where F_g and $M_g \in \mathbb{R}^3$ are the gravity force and gradient moment, ϕ_c and $\tau_c \in \mathbb{R}^3$ are control force and moment, and ϕ_d and $\tau_d \in \mathbb{R}^3$ are external disturbances.

In compact form, the dynamics are:

$$I\dot{\xi} = \mathrm{ad}^*_\xi I\xi + \phi_g + \phi_c + \phi_d$$

where

$$\phi_g = \begin{pmatrix} M_g \\ F_g + mR^T a_{J2} \end{pmatrix} \in \mathbb{R}^6, \quad \phi_c = \begin{pmatrix} \tau_c \\ \phi_c \end{pmatrix} \in \mathbb{R}^6, \quad \phi_d = \begin{pmatrix} \tau_d \\ \phi_d \end{pmatrix} \in \mathbb{R}^6$$

and

$$I = \begin{pmatrix} J & 0 \\ 0 & mI \end{pmatrix} \in \mathbb{R}^{6\times 6}$$

where $\phi_g \in \mathbb{R}^6$ is the vector of gravity inputs, $\phi_c \in \mathbb{R}^6$ is the vector of control inputs, and $\phi_d \in \mathbb{R}^6$ represents external disturbances on the follower.

3.1.3 Relative Dynamics

Let the relative configuration between the leader and follower be $h \in \mathrm{SE}(3)$ and the desired relative configuration be $h_f \in \mathrm{SE}(3)$. The states of the follower spacecraft are denoted by $(g, \xi) \in \mathrm{SE}(3) \times \mathbb{R}^6$. The desired follower states are given by

$$g_d = g_0 h_f, \quad \xi_d = \mathrm{Ad}_{h_f^{-1}} \xi_0 \quad \text{for} \quad t \geq t_0$$

The configuration error is

$$h = g_0^{-1} g \quad \text{for} \quad t \geq t_0$$

The tracking error in exponential coordinates is

$$(\tilde{\eta})^\vee = \log_m [h_f^{-1} h] = \log_m [g_d^{-1} g]$$

where $\log_m : \mathrm{SE}(3) \to \mathfrak{se}(3)$ is the logarithm map. This leads to the relative configuration vector

$$\tilde{\eta} = \begin{pmatrix} \Theta \\ \beta \end{pmatrix} \in \mathbb{R}^6$$

where $\Theta \in \mathbb{R}^3$ and $\beta \in \mathbb{R}^3$ represent the attitude and position tracking errors, respectively.

The relative velocity of the follower with respect to the leader is

$$\tilde{\xi} = \xi - \mathrm{Ad}_{h^{-1}} \xi_0 = \xi - \mathrm{Ad}_{h_f^{-1}} \xi_d$$

where $h_f = g_d^{-1} g = \exp[(\tilde{\eta})^\vee]$.

The kinematics in exponential coordinates is given by

$$\dot{\tilde{\eta}} = G(\tilde{\eta}) \tilde{\xi}$$

with

$$G(\tilde{\eta}) = \begin{pmatrix} A(\Theta) & 0 \\ T(\Theta, \beta) & A(\Theta) \end{pmatrix}$$

where $A(\Theta)$ and $T(\Theta, \beta)$ are specific functions defined as

$$A(\Theta) = I + \frac{1}{2}(\Theta)_\times + \frac{1 - \cos\theta}{\theta^2}(\Theta_\times)^2, \quad T(\Theta, \beta) = \ldots$$

The relative velocity can also be expressed as

$$\tilde{\xi} = \xi - \hat{\xi}, \quad \hat{\xi} = \mathrm{Ad}_h^{-1} \xi_0$$

and the relative acceleration is

$$\dot{\tilde{\xi}} = \dot{\xi} + \mathrm{ad}_\xi \, \mathrm{Ad}_h^{-1} \xi_0 - \mathrm{Ad}_h^{-1} \dot{\xi}_0$$

Substituting the dynamics equations yields

$$I\dot{\xi} = \mathrm{ad}_{\xi}^{*} I\xi + \phi_g + \phi_c + \phi_d + I[\mathrm{ad}_{\xi}\,\mathrm{Ad}_{h}^{-1}\xi_0 - \mathrm{Ad}_{h}^{-1}\dot{\xi}_0]$$

3.1.4 Sliding Mode Controller Formulation and overall Dynamics

The Sliding Mode Control (SMC) for the spacecraft formation is designed to ensure that each spacecraft in the formation tracks its desired trajectory, maintaining a constant relative pose or configuration with respect to the virtual leader. The external disturbances are bounded, and the control law for the spacecraft achieves almost global asymptotic tracking of the desired state trajectory. The control law is given by:

$$\phi_{kc} = -\phi_k^g - \mathrm{ad}_{\xi_k}^{*} I_k \xi_k - I_k \left[\mathrm{ad}_{\xi_k} A_d(h_k^{-1})\xi_0 - A_d(h_k^{-1})\dot{\xi}_0\right] + CG(\eta_k)\xi_k - Ps_k - K\mathrm{sgn}(s_k)$$

Where $s_k = \eta_k + C\xi_k$ is the sliding surface, and C, P, and K are positive gain matrices. The gains are designed such that $k_i > F_i$, ensuring boundedness of the control law.

The feedback system described by this law is guaranteed to asymptotically track the desired trajectory (g_k, ξ_k), and the domain of attraction is almost global in the state space. The Lyapunov function for the system is:

$$V_k(\xi_k, h_k, \xi_0, \eta_k) = \frac{1}{2} s_k^T I_k s_k$$

The time derivative of the Lyapunov function is negative semidefinite, confirming stability. The dynamics of the feedback system are described by:

$$\dot{\eta}_k = G(\eta_k)\xi_k$$

$$\dot{\xi}_k = -CG(\eta_k)\xi_k - I_k^{-1}[Ps_k + K\mathrm{sgn}(s_k)]$$

Once the sliding surface s_k converges to zero, both η_k and ξ_k approach zero, indicating that the relative configuration and velocities converge to the desired values. The control input is limited to prevent actuator saturation:

$$\phi_{cs} = \begin{cases} \phi_c & \text{if } \|\phi_c\| < \phi_m \\ \frac{\phi_m}{\|\phi_c\|}\phi_c & \text{if } \|\phi_c\| \geq \phi_m \end{cases}$$

Additionally, to avoid chattering, a continuous saturation function is used to replace the signum function in the control law:

$$\text{sat}(s_i, \epsilon) = \begin{cases} \frac{s_i}{\epsilon} & \text{if } |s_i| < \epsilon \\ \text{sgn}(s_i) & \text{if } |s_i| \geq \epsilon \end{cases}$$

Finally, the relative configuration and velocities are updated as:

$$h_k = h_k^f \exp(\eta_k^\vee)$$

$$g_k = g_0 h_k = g_0 h_k^f \exp(\eta_k^\vee)$$

$$\xi_k = \tilde{\xi}_k + A_d(h_k^{-1})\xi_0$$

This control scheme ensures that the spacecraft follow the desired trajectory while maintaining relative configuration.

3.2 Artificial Potential Field Algorithm(APF)

The Artificial Potential Field (APF) algorithm is inspired by electromagnetic particle behavior: particles with identical potentials repel each other, while those with opposite potentials attract each other. Using this principle, a vehicle is attracted to the goal position while being repelled by obstacles. The potential function for APF is given by:

$$U(\mathbf{q}) = U_{att}(\mathbf{q}) + U_{rep}(\mathbf{q})$$

where: - $\mathbf{q}$ is the position of the vehicle, - $U_{att}(\mathbf{q})$ is the attractive potential function, - $U_{rep}(\mathbf{q})$ is the repulsive potential function.

The gradient of the total potential function is calculated to derive the force acting on the robot:

$$\nabla U(\mathbf{q}) = \nabla U_{att}(\mathbf{q}) + \nabla U_{rep}(\mathbf{q})$$

3.2.1 Attractive Potential Gradient

The gradient of the attractive potential $\nabla U_{att}(\mathbf{q})$ is defined as:

$$\nabla U_{att}(\mathbf{q}) = \begin{cases} \zeta(\mathbf{q} - \mathbf{q}^*) & \text{if } d_g \leq d_g^* \\ \frac{d_g^* \zeta(\mathbf{q}-\mathbf{q}^*)}{d_g} & \text{otherwise} \end{cases}$$

where: - $\mathbf{q}^*$ is the goal position, - $d(\mathbf{x}, \mathbf{y})$ is a function calculating the Euclidean distance between points, - ζ is the scaling factor of the attractive potential, - d_g^* is a threshold distance for selecting between a quadratic or conic potential, reducing the attractive force when the goal is far away, - $d_g = d(\mathbf{q}, \mathbf{q}^*)$ is the current distance to the goal.

3.2.2 Repulsive Potential Gradient

The gradient of the repulsive potential $\nabla U_{rep}(\mathbf{q})$ is the sum of gradients for each obstacle O_i in the environment:

$$\nabla U_{rep}(\mathbf{q}) = \sum_{i=1}^{N} \nabla U_{O_i}^{rep}(\mathbf{q})$$

where N is the number of obstacles. For an individual obstacle O_i, the repulsive gradient is defined as:

$$\nabla U_{O_i}^{rep}(\mathbf{q}) = \begin{cases} \eta \left(\frac{1}{Q^*} - \frac{1}{d_{O_i}} \right) \frac{1}{d_{O_i}^2} \nabla d_{O_i} & \text{if } d_{O_i} \leq Q^* \\ 0 & \text{otherwise} \end{cases}$$

where: - η is the scaling factor for the repulsive potential, - Q^* is the obstacle reaction margin within which repulsive forces act, - $d_{O_i} = d(\mathbf{q}, O_i)$ is the distance from the robot to obstacle O_i.

3.2.3 Adaptive Safe Artificial Potential Field Algorithm

The major difference between APF and Safe APF is that Safe APF switches between vortex potential general repulsive potential of APF in case of an obstacle, thereby, dealing with the issue of local minima. This adaptive Safe Artificial Potential Field (ASAPF) approach improves traditional APF by dynamically adjusting the repulsive potential scaling factor η to ensure safe navigation with a constant desired distance from obstacles. By applying an adaptive scaling factor, the spacecraft can effectively avoid obstacles without oscillations and maintain smooth, goal-directed motion, even in narrow corridors or environments with varying obstacle proximity.

3.2.3.1 Adaptive Scaling of Attractive Potential

The scaling factor of the attractive potential ζ is calculated using an assumed deceleration for goal-reaching movement, with the assumption that there are no obstacles close to the vehicle:

$$\zeta = \sqrt{\frac{2a_{max}d_g^*}{d_g^*}}$$

where a_{max} is the value of the assumed maximum deceleration.

3.2.3.2 Adaptive Scaling of Repulsive Potential

The adaptation of the repulsive potential scaling factor η is based on the Widrow-Hoff rule, a gradient-descent-based adaptation mechanism. This adaptation enables the spacecraft to maintain a safe distance d_{safe} from obstacles while preventing issues such as goal inaccessibility and excessive repulsive forces. The adaptive scaling factor is computed as follows:

$$\eta_{k+1} = \eta_k + \mu \cdot e_d \cdot d_{min} \cdot \nabla d$$

where: - η_k is the current repulsive scaling factor, - μ is the adaptation gain, - $e_d = d_{vort} - d_{min}$ is the distance error, where d_{vort} is the desired distance from the obstacle, and d_{min} is the minimum distance to any obstacle, - ∇d is the gradient of the distance to obstacles.

This adaptive rule continuously updates η to maintain the desired distance from obstacles. Additionally, to prevent large values that could overly influence vehicle motion, η is constrained within bounds:

$$\eta_{\min} \leq \eta \leq \eta_{\max}$$

where: - $\eta_{\min} = f_\eta(d_{safe}, -v_{max})$ ensures that the robot moves away with maximum velocity when at d_{safe}, - $\eta_{\max} = f_\eta(Q^* - d_{safe}, v_{max})$, ensuring the robot reaches the goal efficiently when far from obstacles.

Control Signal Computation

The ASAPF algorithm utilizes the gradient of the potential function as a velocity signal. Linear velocity v^* and orientation θ^* are adjusted using adaptive scaling for collision-free navigation:

$$v^* = \begin{cases} \min(\alpha \cdot \|\nabla U(\mathbf{q})\|, v_{max}), & \text{if } |\theta_{error}| \leq \theta_{max}^{error} \\ 0, & \text{otherwise} \end{cases}$$

$$\theta^* = \text{atan2}(-\nabla U_y(\mathbf{q}), -\nabla U_x(\mathbf{q}))$$

where: - $\alpha = \frac{\theta_{max}^{error} - |\theta_{error}|}{\theta_{max}^{error}}$ scales the velocity based on the robot's orientation error θ_{error}, - v_{max} is the maximum allowable linear velocity.

3.3 Combining Guidance Law with Dynamics

In order to utilise the ASAPF guidance law with the SE(3) Dynamics of the spacecraft along with Sliding Mode Controller as discussed above, the total Force was taken from ASAPF and it was added in the control input of sliding mode controller ϕ_{kc}.

$$\boldsymbol{\phi}_r = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \nabla U \end{bmatrix}$$

where ∇U represents the total force derived from ASAPF. The control input thus becomes:

$$\phi_{kc} = -\phi_k^g - \mathrm{ad}^*_{\xi_k} I_k \xi_k - I_k \left[\mathrm{ad}_{\xi_k} A_d(h_k^{-1})\xi_0 - A_d(h_k^{-1})\dot{\xi_0} \right] + CG(\eta_k)\xi_k - Ps_k - K\mathrm{sgn}(s_k) + \phi_r$$

The dynamics, hence, is changed accordingly.

Chapter 4

Simulation Results

4.1 Dynamics and Sliding Mode Controller

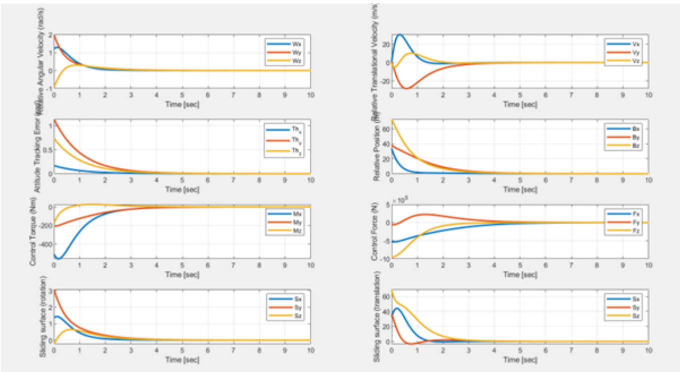


FIGURE 4.1: Dynamics and SMC simulation (Matlab)

4.2 Obstacle avoidance using ASAPF in 3d including Dynamics and SMC

Combining the dynamics and SMC along with ASAPF guidance law gave following results in simulation:

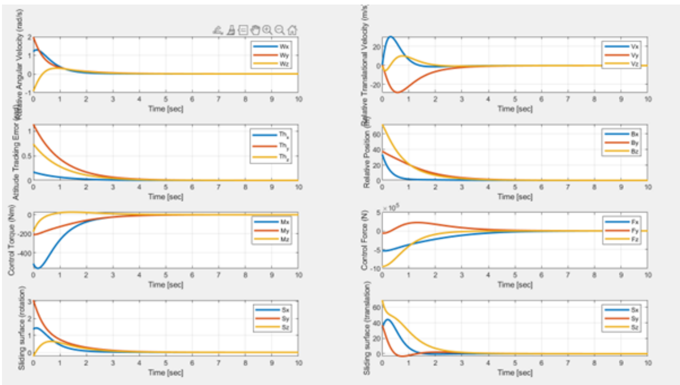


FIGURE 4.2: SMC with dynamics and Guidance Law

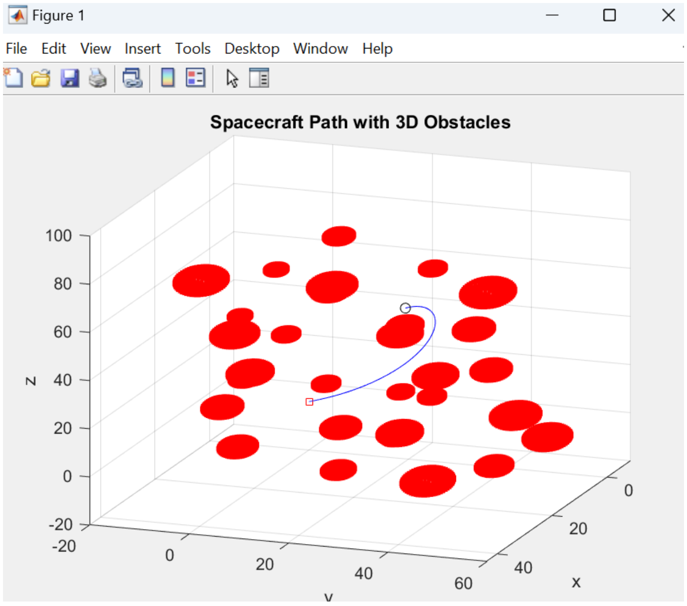


FIGURE 4.3: SMC with dynamics and Guidance Law

Chapter 5

Further works

5.1 Work yet to be done

In future work, the primary focus will be on implementing Model Predictive Control (MPC) within the ASAPF framework to optimize the control of the system dynamics. This integration will enable more efficient trajectory planning by incorporating real-time system constraints and optimizing performance over a finite prediction horizon. Alongside this, a new approach will be developed to address the challenge of local minima, which often lead to suboptimal solutions in optimization problems. The proposed method aims to enhance the robustness of the MPC by preventing it from getting trapped in local minima, thereby improving the overall solution quality. This novel approach will be incorporated into the MPC framework, and the integrated system will undergo thorough testing to evaluate its performance in dynamic environments. The goal is to refine the method, ensuring both global optimality and reliable real-time execution in complex scenarios.

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