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Research Article

Adaptive Finite-Time Control for Spacecraft Rendezvous under Unknown System Parameters

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In this study, we investigated the sliding mode control (SMC) technology for the spacecraft rendezvous maneuver under unknown system parameters and external disturbance. With no knowledge of the mass and inertial matrix of the pursuer spacecraft, an adaptive SMC approach was devised using the hyperbolic tangent function to realize the control objective of reducing the chattering problem. In addition, the finite-time stability of the relative dynamics and the boundedness of the signals in the closed-loop system were derived under proposed method. The effectiveness and advantages of the proposed method were verified through theoretical analysis and numerical simulations.

1. Introduction

With rapid developments in the aerospace industry, spacecraft rendezvous technology has been extensively applied in space missions, such as deep space exploration, the establishment of space stations, and the detection of various components on Mars. Due to the fact that control systems are one of the most crucial technologies of the rendezvous maneuver, extensive attention of researchers has been attracted in the past decades. To guarantee the success of space missions, various methods are studied for spacecraft rendezvous maneuver. However, designing controllers for the spacecraft rendezvous maneuver is still a challenging task because of the strongly coupled nonlinear dynamics and unknown external environment. Current attitude controls of spacecraft include backstepping control [1, 2], adaptive control [3–5], and sliding mode control (SMC) [6–8].

However, the methods in [1–7] cannot be used in the spacecraft rendezvous maneuver, which severely limits their application. Considerable efforts are required during the controller design process in rendezvous missions to address the effects of complex external disturbance and coupled nonlinear dynamics. To reduce the effect of external disturbances, backstepping-based controllers, in which globally asymptotic stability can be achieved for closed-loop systems, have been investigated in detail in [9–11]. The common drawback in [9–11] is that the convergence rate of the control system is asymptotical, that is, the control objective can be achieved when time is infinite. To ensure finite-time convergence for the entire system, Wang et al. investigated control methods to accomplish the rendezvous mission by using the backstepping design [12]. However, actuator faults in actual spacecraft activities have not been considered in [9–12]. Unexpected and complex failures frequently occur in the actuators in spacecraft rendezvous missions. These failures result in the degradation of control performance. Furthermore, failures may occur during a mission in the event of limited communication bandwidth and transmission delays. To ensure mission success and avoid unexpected failures, adaptive SMC algorithms have been investigated to improve reliability [13, 14]. Another aspect that deserves special attention is collision avoidance during the rendezvous maneuver, which has been ignored in previous articles. Artificial-potential-function-based SMC and backstepping control are proposed in [15, 16], respectively, for avoiding collisions during rendezvous maneuvers.

Results in [9–16] were obtained by regarding the inertial parameters as available valuables. However, inertial parameters are not always available to designers during real missions mainly because of fuel consumption in the pursuer spacecraft. Moreover, solar radiation pressure and disturbances in the space environment influence the inertial parameters. Considering these aspects, an adaptive SMC method was presented in [17] for controlling spacecraft maneuver under unknown inertia. The adaptive SMC of [17] was improved to an adaptive control algorithm based on gain and neural networks in [18] for addressing uncertainties and external disturbances. For spacecraft proximity operations with parametric uncertainties, an integrated adaptive backstepping control method and an adaptive control algorithm using dual quaternions were designed in [19, 20], respectively. However, the results from [17–20] cannot be directly applied to the spacecraft rendezvous maneuver. On the basis of [18–20], the uncertainties of the relative dynamics were compensated using robust adaptive backstepping control in [21] by introducing radial basis function neural networks. In contrast to [21], the problem of time-varying inertial parameters was studied in [22]. Similarly, the continuous adaptive control algorithm was combined with the projection algorithm in [23] for controlling the spacecraft rendezvous maneuver under time-varying inertia parameters and actuator faults.

The effective controllers in [17–23] that address uncertainties are asymptotically stable, that is, the system states converge to equilibrium when time is infinite. Unlike asymptotically stable controllers, finite-time control schemes have been widely studied and applied in spacecraft attitude control because of their high convergence rate and superior control performance [8, 24, 25]. Controllers for the unwinding phenomenon, which was not considered in [24, 25], are presented in [8]. Moreover, input saturation constraints, which influence the performance of spacecraft, were not considered in [8, 24, 25]. Therefore, an adaptive finite-time control algorithm was investigated in [26] to address the problem of unavoidable input saturations for spacecraft. As a continuation of [26], the collision problem between the pursuer spacecraft and the target spacecraft was studied using an adaptive finite-time antisaturation controller in [27].

Excellent finite-time stability can be achieved for the system by using the controllers in [8, 24–27]. However, the chattering phenomenon, which is the main cause of actuator damage, was not considered in these studies. A boundary layer function was incorporated into the controller in [28] to alleviate the chattering phenomenon. Similarly, continuous and chatter-free controllers were introduced in [29] for spacecraft rendezvous and docking. In this study, the finite-time control problem for spacecraft rendezvous was studied in terms of the existing chattering problem in the SMC and the unknown time-varying inertial parameters in real missions. The contributions of this paper are as follows:

(i) In contrast to the existing spacecraft control schemes for known inertia [10–13], unknown parameters are considered in this paper, which considerably extends the application of control methods for the spacecraft rendezvous maneuver

(ii) Unlike the controllers in [17–23], finite-time stability can be achieved for the system with a high convergence rate by using the proposed method even when the inertial parameters are unavailable to designers

(iii) The hyperbolic tangent function can be used in the control law to avoid the chattering problem. Furthermore, the sliding mode method was adopted in this study.

The remainder of this paper is arranged as follows. The dynamics model of the spacecraft is established in Section 2. The finite-time controller is described in Section 3. The effectiveness of the controller is proved through simulations in Section 4. Finally, the conclusion of this paper is presented in Section 5.

2. Spacecraft Model and Preliminaries

2.1. Relative Attitude Dynamic Model. The control equations for the attitude motion of a rigid spacecraft can be established by using the unit quaternion. According to [23], the rotation matrix $\boldsymbol{R} \in SO(3)$ and the unit quaternion $\mathbf{Q} = [q_0, \mathbf{q}^{\mathrm{T}}]^{\mathrm{T}} \in \Xi$ with $\Xi = \{\mathbf{Q} \in \mathbb{R} \times \mathbb{R}^{3\times 3} | q_0^2 + \mathbf{q}^{\mathrm{T}}\mathbf{q} = 1\}$ are applied in attitude formulation. Furthermore, $\mathbf{Q}_p$ and $\mathbf{Q}_t$ represent the attitude of the pursuer and the target, respectively. Consequently, the relative attitude between the pursuer and the target can be expressed as follows:

$$\tilde{\mathbf{Q}} = \left[\tilde{q}_0, \tilde{\mathbf{q}}^{\mathrm{T}}\right]^{\mathrm{T}} = \mathbf{Q}_t^{-1} \odot \mathbf{Q}_p. \tag{1}$$

The relative attitude kinematics model [25] can be expressed as follows:

$$\begin{aligned} \dot{\tilde{q}}_0 &= -\frac{1}{2}\tilde{\boldsymbol{q}}_v{}^T\tilde{\boldsymbol{\omega}}, \\ \dot{\tilde{q}}_v &= \frac{1}{2}(\tilde{\boldsymbol{q}}_v{}^{\times} + \tilde{q}_0 I_3)\tilde{\boldsymbol{\omega}}, \end{aligned} \tag{2}$$

where $\tilde{\boldsymbol{\omega}} = \boldsymbol{\omega}_p - \tilde{\boldsymbol{R}}\boldsymbol{\omega}_t$ is the relative angular velocity, with $\boldsymbol{\omega}_p$ and $\boldsymbol{\omega}_t$ denoting the angular velocity of the pursuer and target, respectively. For a vector $\boldsymbol{a} = [a_1, a_2, a_3]^T$, $\boldsymbol{a}^x$ can be defined using Eq. (3). The rotation matrix is defined in Eq. (4).

$$\boldsymbol{a}^x = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix}, \tag{3}$$

$$\tilde{\boldsymbol{R}} \triangleq \boldsymbol{R}(\tilde{\boldsymbol{q}}) = \left(\tilde{q}^2{}_0 - \tilde{\boldsymbol{q}}_v^T\tilde{\boldsymbol{q}}_v\right)\boldsymbol{I}_3 + 2\tilde{\boldsymbol{q}}_v\tilde{\boldsymbol{q}}_v^T - 2\tilde{\boldsymbol{q}}_0\tilde{\boldsymbol{q}}_v^x. \tag{4}$$

According to the Euler-Newton formulas of the pursuer and target [27], the corresponding attitude dynamics can be expressed as follows:

$$\mathbf{J}_t\dot{\boldsymbol{\omega}}_t + \boldsymbol{\omega}_t{}^{\times}\mathbf{J}_t\boldsymbol{\omega}_t = 0, \tag{5}$$

$$\mathbf{J}\dot{\boldsymbol{\omega}}_p + \boldsymbol{\omega}_p{}^{\times}\mathbf{J}\boldsymbol{\omega}_p = \boldsymbol{\tau} + \boldsymbol{\tau}_d, \tag{6}$$

where $\mathbf{J}_t \in \mathrm{R}^{3\times3}$ and $\mathbf{J} \in \mathrm{R}^{3\times3}$ denote the inertial matrices and $\boldsymbol{\tau} \in \mathrm{R}^3$ and $\boldsymbol{\tau}_d \in \mathrm{R}^3$ represent the control and external disturbance torques, respectively.

The derivative of $\tilde{\boldsymbol{\omega}}$ satisfies the following expression:

$$\dot{\tilde{\boldsymbol{\omega}}} = \dot{\boldsymbol{\omega}}_p - \dot{\tilde{\mathbf{R}}}\boldsymbol{\omega}_t - \tilde{\mathbf{R}}\dot{\boldsymbol{\omega}}_t. \tag{7}$$

Combining Eqs. (5)–(7) and considering that $\dot{\tilde{\mathbf{R}}} = \tilde{\mathbf{R}}\tilde{\boldsymbol{\omega}}^{\times}$, the following equation is obtained:

$$\mathbf{J}\dot{\tilde{\boldsymbol{\omega}}} = -\mathbf{C}_r\tilde{\boldsymbol{\omega}} - \mathbf{n}_r + \boldsymbol{\tau} + \boldsymbol{\tau}_d, \tag{8}$$

where $\mathbf{C}_r = \mathbf{J}(\tilde{\mathbf{R}}\boldsymbol{\omega}_t)^{\times} + (\tilde{\mathbf{R}}\boldsymbol{\omega}_t)^{\times}\mathbf{J} - (\mathbf{J}(\tilde{\boldsymbol{\omega}} + \tilde{\mathbf{R}}\boldsymbol{\omega}_t))^{\times}$ and $\mathbf{n}_r = (\tilde{\mathbf{R}}\boldsymbol{\omega}_t)^{\times}\mathbf{J}\tilde{\mathbf{R}}\boldsymbol{\omega}_t + \mathbf{J}\tilde{\mathbf{R}}\dot{\boldsymbol{\omega}}_t$.

2.2. Relative Orbit Dynamics Model. According to the theory of relative motion, $\mathbf{r_p}$ and $\mathbf{v_p}$ are used to express the position and velocity of the pursuer, as presented in Eqs. (9) and (10), respectively.

$$\mathbf{r_p} = \tilde{\mathbf{r}} + \tilde{\mathbf{R}}(\mathbf{r}_t + \boldsymbol{\sigma}_t), \tag{9}$$

$$\mathbf{v_p} = \tilde{\mathbf{v}} + \tilde{\mathbf{R}}(\mathbf{v}_t + \boldsymbol{\omega}_t^{\times}\boldsymbol{\sigma}_t), \tag{10}$$

where $\mathbf{r}_t$ and $\mathbf{v}_t$ are the position and velocity of the target, respectively; $\tilde{\mathbf{r}}$ and $\tilde{\mathbf{v}}$ are the relative position and relative velocity, respectively; and the constant vector $\boldsymbol{\sigma}_t \in R^3$ denotes the desired rendezvous position. The derivative of Eq. (9) can be expressed as follows:

$$\dot{\mathbf{r}}_\mathbf{p} = \dot{\tilde{\mathbf{r}}} + \dot{\tilde{\mathbf{R}}}(\mathbf{r}_t + \boldsymbol{\sigma}_t) + \tilde{\mathbf{R}}\dot{\mathbf{r}}_t. \tag{11}$$

According to the description of kinematic principles in [30], the following equations are obtained:

$$\dot{\mathbf{r}}_t = \mathbf{v}_t - \boldsymbol{\omega}_t^{\times}\mathbf{r}_t, \tag{12}$$

$$\dot{\mathbf{r}}_\mathbf{p} = \mathbf{v_p} - \boldsymbol{\omega}_p^{\times}\mathbf{r_p}. \tag{13}$$

By combining Eqs. (11) and (13), the following expression is obtained:

$$\dot{\tilde{\mathbf{r}}} + \dot{\tilde{\mathbf{R}}}(\mathbf{r}_t + \boldsymbol{\sigma}_t) + \tilde{\mathbf{R}}\dot{\mathbf{r}}_t = \mathbf{v_p} - \boldsymbol{\omega}_p^{\times}\mathbf{r_p}. \tag{14}$$

From the aforementioned equation, the following expression is obtained:

$$\dot{\tilde{\mathbf{r}}} = \tilde{\mathbf{v}} - \mathbf{C}_t\tilde{\mathbf{r}}, \tag{15}$$

where $\mathbf{C}_t = (\tilde{\boldsymbol{\omega}} + \tilde{\mathbf{R}}\boldsymbol{\omega}_t)^{\times}$. Consequently, the derivative of $\mathbf{v_p}$ is expressed as follows:

$$\dot{\mathbf{v}}_\mathbf{p} = \dot{\tilde{\mathbf{v}}} + \dot{\tilde{\mathbf{R}}}(\mathbf{v}_t + \boldsymbol{\omega}_t^{\times}\boldsymbol{\sigma}_t) + \tilde{\mathbf{R}}(\dot{\mathbf{v}}_t + \dot{\boldsymbol{\omega}}_t^{\times}\boldsymbol{\sigma}_t). \tag{16}$$

According to the theory in [30], Eqs. (17) and (18) define the position dynamics of spacecraft.

$$m_t\dot{\mathbf{v}}_t + m_t\boldsymbol{\omega}_t^{\times}\mathbf{v}_t = 0, \tag{17}$$

$$m_p\dot{\mathbf{v}}_p + m_p\boldsymbol{\omega}_p^{\times}\mathbf{v}_p = \mathbf{f} + \mathbf{f}_d, \tag{18}$$

where m_t and m_p are constants that define the masses of the target and purser, respectively, and $\mathbf{f} \in \mathrm{R}^N$ and $\mathbf{f}_d \in \mathrm{R}^3$ denote the control and external disturbance forces, respectively. From Eqs. (16) and (18), the following equation can be obtained [23]:

$$m_p\left[\dot{\tilde{\mathbf{v}}} + \dot{\tilde{\mathbf{R}}}(\mathbf{v}_t + \boldsymbol{\omega}_t^{\times}\boldsymbol{\sigma}_t) + \tilde{\mathbf{R}}(\dot{\mathbf{v}}_t + \dot{\boldsymbol{\omega}}_t^{\times}\boldsymbol{\sigma}_t)\right] + m_p\boldsymbol{\omega}_p^{\times}\mathbf{v}_p = \mathbf{f} + \mathbf{f}_d. \tag{19}$$

Thus, the following equation is obtained:

$$m_p\dot{\tilde{\mathbf{v}}} = -m_p\mathbf{C}_t\tilde{\mathbf{v}} - m_p\mathbf{n}_t + \mathbf{f} + \mathbf{f}_d, \tag{20}$$

where $\mathbf{n}_t = (\tilde{\mathbf{R}}\boldsymbol{\omega}_t)^{\times}\tilde{\mathbf{R}}\mathbf{v}_t + \tilde{\mathbf{R}}\dot{\mathbf{v}}_t + \tilde{\boldsymbol{\omega}}^{\times}\tilde{\mathbf{R}}\boldsymbol{\sigma}_t^{\times}\boldsymbol{\omega}_t - \tilde{\mathbf{R}}\boldsymbol{\sigma}_t^{\times}\dot{\boldsymbol{\omega}}_t$.

For convenience, the control torques can be expressed as follows:

$$\boldsymbol{\tau} = \boldsymbol{\varepsilon}_r, \mathbf{f} = \boldsymbol{\varepsilon}_t, \tag{21}$$

where $\boldsymbol{\varepsilon}_r \in \mathrm{R}^N$ and $\boldsymbol{\varepsilon}_t \in \mathrm{R}^N$. By combining Eqs. (8), (20), and (21), the relative dynamics can be established as follows:

$$J\dot{\tilde{\boldsymbol{\omega}}} = -\mathbf{C}_r\tilde{\boldsymbol{\omega}} - \mathbf{n}_r + \boldsymbol{\varepsilon}_r + \mathbf{d}_r, \tag{22}$$

$$m\dot{\tilde{\mathbf{v}}} = -m\mathbf{C}_t\tilde{\mathbf{v}} - m\mathbf{n}_t + \boldsymbol{\varepsilon}_t + \mathbf{d}_t, \tag{23}$$

where $\mathbf{d}_r = \boldsymbol{\tau}_d$ and $\mathbf{d}_t = \mathbf{f}_d$.

During the controller design process, the exact motion information of the target spacecraft is assumed to be available to the tracker spacecraft. In this paper, we focus on designing controllers for the dynamics expressed in Eqs. (21) and (22) to ensure the finite-time stability of the closed-loop system even in the presence of unknown inertial parameters and external disturbance.

To facilitate the controller design, the following assumptions were made:

Assumption 1. The dynamics of the target are stable, which implies that $\boldsymbol{\omega}_t$, $\dot{\boldsymbol{\omega}}_t$, $\mathbf{v}_t$, and $\dot{\mathbf{v}}_t$ are bounded and satisfy $\|\boldsymbol{\omega}_t\| \le a_1$, $\|\dot{\boldsymbol{\omega}}_t\| \le a_2$, and $\|\dot{\mathbf{v}}_t\| \le a_4$, where a_1, a_2, a_3, and a_4 are positive constants.

Assumption 2. The inertial matrix $\mathbf{J}$ and the mass of the pursuer m are unknown and satisfy $\lambda_1\mathbf{I}^{3\times3} \le \mathbf{J} \le \lambda_2\mathbf{I}^{3\times3}$ and $m \le \lambda_3$, where λ_i $(i = 1, 2, 3)$ are unknown positive constants.

Assumption 3. The external disturbance $\mathbf{d}_r$ and $\mathbf{d}_t$ are unknown bounded vectors, which satisfy $\|\mathbf{d}_r\| \le D_r < c_1$ and $\|\mathbf{d}_t\| \le D_t < c_2$, where c_1 and c_2 are positive constants.

2.3. Preliminaries

Lemma 4 (see [24]). *For an arbitrary real number $x \in R$, $\mu > 0$, and $\kappa = 0.2785$, the following relation exists:*

$$0 < |x| - x \tanh(\mu x) \le \frac{\kappa}{\mu}. \tag{24}$$

Lemma 5 (see [26]). *For the system expressed in Eqs. (21) and (22), if the Lyapunov function V_1 exists, it satisfies the following expression:*

$$\dot{V}_1 \le -\alpha V_1^p + \sigma, \tag{25}$$

where $\alpha > 0$, $0 < p < 1$, and $0 < \sigma < \infty$, and the system converges to a region in finite time.

3. Attitude Controller Design

3.1. Basic Controller Design. To complete the design of the attitude and orbit control schemes, two sliding mode variables are defined as follows:

$$\begin{aligned} \mathbf{s}_1 &= \tilde{\boldsymbol{\omega}} + k_1\tilde{\mathbf{q}}_v + k_2 \tanh(\tilde{\mathbf{q}}_v), \\ \mathbf{s}_2 &= \tilde{\mathbf{v}} + k_3\tilde{\mathbf{r}} + k_4 \tanh(\tilde{\mathbf{r}}), \end{aligned} \tag{26}$$

where k_1, k_2, k_3, and k_4 are positive constants.

The derivatives of $\mathbf{s}_1$ and $\mathbf{s}_2$ are expressed as follows:

$$\begin{aligned} \mathbf{J}\dot{\mathbf{s}}_1 &= \boldsymbol{J}\dot{\tilde{\boldsymbol{\omega}}} + k_1\boldsymbol{J}\dot{\tilde{\mathbf{q}}}_v + k_2\mathbf{J}\left(1 + \tanh^{\mathrm{T}}(\tilde{\mathbf{q}}_v)\tanh(\tilde{\mathbf{q}}_v)\right)\dot{\tilde{\mathbf{q}}}_v \\ &= -\mathbf{C}_r\tilde{\boldsymbol{\omega}} - \mathbf{n}_r + \boldsymbol{\varepsilon}_r + \mathbf{d}_r \\ &\quad + \frac{1}{2}\boldsymbol{J}\left[k_1 + k_2\left(1 + \tanh^{\mathrm{T}}(\tilde{\mathbf{q}}_v)\tanh(\tilde{\mathbf{q}}_v)\right)\right] \\ &\quad \cdot (\tilde{\mathbf{q}}_v^{\times} + \tilde{q}_0\mathbf{I}_3)\tilde{\boldsymbol{\omega}}, \\ m\dot{\mathbf{s}}_2 &= m\dot{\tilde{\mathbf{v}}} + mk_3\dot{\tilde{\mathbf{r}}} + mk_4\left(1 + \tanh^{\mathrm{T}}(\tilde{\mathbf{r}})\tanh(\tilde{\mathbf{r}})\right)\dot{\tilde{\mathbf{r}}} \\ &= -m\mathbf{C}_t\tilde{\mathbf{v}} - m\mathbf{n}_t + \boldsymbol{\varepsilon}_t + \mathbf{d}_t \\ &\quad + m\left[k_3 + k_4\left(1 + \tanh^{\mathrm{T}}(\tilde{\mathbf{r}})\tanh(\tilde{\mathbf{r}})\right)\right](\tilde{\mathbf{v}} - \mathbf{C}_t\tilde{\mathbf{r}}). \end{aligned} \tag{27}$$

Considering the aforementioned assumptions and the relations $\|\tilde{\mathbf{q}}_v^{\times} + \tilde{q}_0\mathbf{I}_3\| = 1$, $\|\tanh^{\mathrm{T}}(\tilde{\mathbf{q}}_v)\tanh(\tilde{\mathbf{q}}_v)\| \le 1$, $\|1 + \tanh^{\mathrm{T}}(\tilde{\mathbf{r}})\tanh(\tilde{\mathbf{r}})\| \le 1$, and $\tilde{\mathbf{q}}_v^T\tilde{\mathbf{q}}_v + \tilde{q}_0^2 = 1$, the following conditions can be derived:

$$\begin{aligned} \left\|-\mathbf{C}_r\tilde{\boldsymbol{\omega}} - \mathbf{n}_r + \frac{1}{2}\boldsymbol{J}\left[k_1 + k_2\left(1 + \tanh^{\mathrm{T}}(\tilde{\mathbf{q}}_v)\tanh(\tilde{\mathbf{q}}_v)\right)\right](\tilde{\mathbf{q}}_v^{\times} + \tilde{q}_0\mathbf{I}_3)\tilde{\boldsymbol{\omega}}\right\| &\le \alpha_1\varsigma_1, \\ \left\|-m\mathbf{C}_t\tilde{\mathbf{v}} - m\mathbf{n}_t + m\left[k_3 + k_4\left(1 + \tanh^{\mathrm{T}}(\tilde{\mathbf{r}})\tanh(\tilde{\mathbf{r}})\right)\right](\tilde{\mathbf{v}} - \mathbf{C}_t\tilde{\mathbf{r}})\right\| &\le \alpha_2\varsigma_2, \end{aligned} \tag{28}$$

where α_1 and α_2 are unknown positive constants, $\varsigma_1 = \|\boldsymbol{\psi}_r\|$, $\varsigma_2 = \|\boldsymbol{\psi}_t\|$, $\boldsymbol{\psi}_r = [\|\tilde{\boldsymbol{\omega}}\|^2, \|\tilde{\boldsymbol{\omega}}\|, 1]^{\mathrm{T}}$, and $\boldsymbol{\psi}_t = [\|\tilde{\boldsymbol{\omega}}\|\|\tilde{\mathbf{v}}\|, \|\tilde{\boldsymbol{\omega}}\|\|\tilde{\mathbf{r}}\|, \|\tilde{\boldsymbol{\omega}}\|, \|\tilde{\mathbf{v}}\|, \|\tilde{\mathbf{r}}\|, 1]^{\mathrm{T}}$. Control laws for the relative attitude and orbit dynamics can be designed as follows:

$$\boldsymbol{\varepsilon}_r = -k_5\tanh(\mathbf{s}_1) - k_6\mathbf{s}_1 - \widehat{D}_r\tanh\left(\frac{\mathbf{s}_1}{\mu_1}\right) - \widehat{\alpha}_1\varsigma_1\tanh\left(\frac{\varsigma_1\mathbf{s}_1}{\mu_3}\right), \tag{29}$$

$$\boldsymbol{\varepsilon}_t = -k_7\tanh(\mathbf{s}_2) - k_8\mathbf{s}_2 - \widehat{D}_t\tanh\left(\frac{\mathbf{s}_2}{\mu_2}\right) - \widehat{\alpha}_2\varsigma_2\tanh\left(\frac{\varsigma_2\mathbf{s}_2}{\mu_4}\right), \tag{30}$$

where $k_i > 0, i = 5, 6, 7, 8$ and $\mu_i > 0, i = 1, 2, 3, 4$. In the aforementioned equation, $\widehat{D}_r$, $\widehat{D}_t$, $\widehat{\alpha}_1$, and $\widehat{\alpha}_2$ are the estimations of D_r, D_t, α_1, and α_2, respectively. The terms $\dot{\widehat{D}}_r$, $\dot{\widehat{D}}_t$, $\widehat{\alpha}_1$, and $\widehat{\alpha}_2$ are defined as follows:

$$\dot{\widehat{D}}_r = c_1\left(\|\mathbf{s}_1\|\tanh\left(\frac{\|\mathbf{s}_1\|}{3\mu_1}\right) - c_5\widehat{D}_r\right), \tag{31}$$

$$\dot{\widehat{D}}_t = c_2\left(\|\mathbf{s}_2\|\tanh\left(\frac{\|\mathbf{s}_2\|}{3\mu_2}\right) - c_6\widehat{D}_t\right), \tag{32}$$

$$\dot{\widehat{\alpha}}_1 = c_3\left(\varsigma_1\|\mathbf{s}_1\|\tanh\left(\frac{\varsigma_1\|\mathbf{s}_1\|}{3\mu_3}\right) - c_7\widehat{\alpha}_1\right), \tag{33}$$

$$\dot{\widehat{\alpha}}_2 = c_4\left(\varsigma_2\|\mathbf{s}_2\|\tanh\left(\frac{\varsigma_2\|\mathbf{s}_2\|}{3\mu_4}\right) - c_8\widehat{\alpha}_2\right), \tag{34}$$

where $c_i > 0, i = 1, 2, \cdots, 8$. The estimation errors are defined as follows:

$$\begin{aligned} \tilde{D}_r &= D_r - \widehat{D}_r, \\ \dot{\tilde{D}}_t &= D_t - \widehat{D}_t, \\ \tilde{\alpha}_1 &= \alpha_1 - \widehat{\alpha}_1, \\ \tilde{\alpha}_2 &= \alpha_2 - \widehat{\alpha}_2. \end{aligned} \tag{35}$$

Theorem 6. *For the dynamics expressed in Eqs. (22) and (23) with unknown system parameters* $\mathbf{J}$ *and* m*, the finite-time stability of the system can be achieved using the controller proposed in Eqs. (29)–(34).*

Proof. To prove the stability of the system, the Lyapunov function is designed as follows:

$$V_1 = \frac{1}{2}\mathbf{s}_1^{\mathrm{T}}\mathbf{J}\mathbf{s}_1 + \frac{1}{2}\mathbf{s}_2^{\mathrm{T}}m\mathbf{s}_2 + \frac{1}{2c_1}\tilde{D}_r^2 + \frac{1}{2c_2}\tilde{D}_t^2 + \frac{1}{2c_3}\alpha_1^2 + \frac{1}{2c_4}\alpha_2^2. \tag{36}$$

According to the relative system dynamics and control laws, the following expressions are obtained:

$$
\begin{aligned}
\dot{V}_1 &= \mathbf{s}_1^{\mathrm{T}}\mathbf{J}\dot{\mathbf{s}}_1 + \mathbf{s}_2^{\mathrm{T}} m\dot{\mathbf{s}}_2 + \frac{1}{c_1}\tilde{D}_r\dot{\tilde{D}}_r + \frac{1}{c_2}\tilde{D}_t\dot{\tilde{D}}_t + \frac{1}{c_3}\tilde{\alpha}_1\dot{\tilde{\alpha}}_1 + \frac{1}{c_4}\tilde{\alpha}_2\dot{\tilde{\alpha}}_2 \\
&= \mathbf{s}_1^{\mathrm{T}}\mathbf{J}\dot{\mathbf{s}}_1 + \mathbf{s}_2^{\mathrm{T}} m\dot{\mathbf{s}}_2 - \frac{1}{c_1}\tilde{D}_r\dot{\hat{D}}_r - \frac{1}{c_2}\tilde{D}_t\dot{\hat{D}}_t - \frac{1}{c_3}\tilde{\alpha}_1\dot{\hat{\alpha}}_1 - \frac{1}{c_4}\tilde{\alpha}_2\dot{\hat{\alpha}}_2 \\
&= \mathbf{s}_1^{\mathrm{T}}\Big[-\mathbf{C}_r\tilde{\boldsymbol{\omega}} - \mathbf{n}_r + \boldsymbol{\varepsilon}_r + \mathbf{d}_r \\
&\quad + \frac{1}{2}\boldsymbol{J}\left[k_1 + k_2\left(1 + \tanh^{\mathrm{T}}(\tilde{\mathbf{q}}_v)\tanh(\tilde{\mathbf{q}}_v)\right)\right](\tilde{\mathbf{q}}_v^{\times} + \tilde{q}_0\mathbf{I}_3)\tilde{\boldsymbol{\omega}}\Big] \\
&\quad + \mathbf{s}_2^{\mathrm{T}}\Big[-m\mathbf{C}_t\tilde{\mathbf{v}} - m\mathbf{n}_t + \boldsymbol{\varepsilon}_t + \mathbf{d}_t \\
&\quad + m\left[k_3 + k_4\left(1 + \tanh^{\mathrm{T}}(\tilde{\mathbf{r}})\tanh(\tilde{\mathbf{r}})\right)\right](\tilde{\mathbf{v}} - \mathbf{C}_t\tilde{\mathbf{r}})\Big] \\
&\quad - \frac{1}{c_1}\tilde{D}_r\dot{\hat{D}}_r - \frac{1}{c_2}\tilde{D}_t\dot{\hat{D}}_t - \frac{1}{c_3}\tilde{\alpha}_1\dot{\hat{\alpha}}_1 - \frac{1}{c_4}\tilde{\alpha}_2\dot{\hat{\alpha}}_2.
\end{aligned}
\tag{37}
$$

Then, the following equations are derived:

$$
\begin{aligned}
&\mathbf{s}_1^{\mathrm{T}}\left[-\mathbf{C}_r\tilde{\boldsymbol{\omega}} - \mathbf{n}_r + \frac{1}{2}\boldsymbol{J}\left[k_1 + k_2\left(1 + \tanh^{\mathrm{T}}(\tilde{\mathbf{q}}_v)\tanh(\tilde{\mathbf{q}}_v)\right)\right](\tilde{\mathbf{q}}_v^{\times} + \tilde{q}_0\mathbf{I}_3)\tilde{\boldsymbol{\omega}}\right] \\
&\quad \le \alpha_1\varsigma_1\|\mathbf{s}_1\|,
\end{aligned}
\tag{38}
$$

$$
\begin{aligned}
&\mathbf{s}_2^{\mathrm{T}}\left[-m\mathbf{C}_t\tilde{\mathbf{v}} - m\mathbf{n}_t + m\left[k_3 + k_4\left(1 + \tanh^{\mathrm{T}}(\tilde{\mathbf{r}})\tanh(\tilde{\mathbf{r}})\right)\right](\tilde{\mathbf{v}} - \mathbf{C}_t\tilde{\mathbf{r}})\right] \\
&\quad \le \alpha_2\varsigma_2\|\mathbf{s}_2\|.
\end{aligned}
\tag{39}
$$

The following equations are obtained from Eqs. (37)–(39):

$$
\begin{aligned}
\dot{V}_1 &\le \alpha_1\varsigma_1\|\mathbf{s}_1\| + \alpha_2\varsigma_2\|\mathbf{s}_2\| + \mathbf{s}_1^{\mathrm{T}}(\boldsymbol{\varepsilon}_r + \mathbf{d}_r) + \mathbf{s}_2^{\mathrm{T}}(\boldsymbol{\varepsilon}_t + \mathbf{d}_t) \\
&\quad - \frac{1}{c_1}\tilde{D}_r\dot{\hat{D}}_r - \frac{1}{c_2}\tilde{D}_t\dot{\hat{D}}_t - \frac{1}{c_3}\tilde{\alpha}_1\dot{\hat{\alpha}}_1 - \frac{1}{c_4}\tilde{\alpha}_2\dot{\hat{\alpha}}_2.
\end{aligned}
\tag{40}
$$

Substituting the equations of the proposed controllers into Eq. (40), we obtain the following expression:

$$
\begin{aligned}
\dot{V}_1 &\le \alpha_1\varsigma_1\|\mathbf{s}_1\| + \alpha_2\varsigma_2\|\mathbf{s}_2\| + \mathbf{s}_1^{\mathrm{T}}\mathbf{d}_r + \mathbf{s}_2^{\mathrm{T}}\mathbf{d}_t \\
&\quad + \mathbf{s}_1^{\mathrm{T}}\left(-k_5\tanh(\mathbf{s}_1) - k_6\mathbf{s}_1 - \hat{D}_r\tanh\left(\frac{\mathbf{s}_1}{\mu_1}\right)\right. \\
&\quad \left. - \hat{\alpha}_1\varsigma_1\tanh\left(\frac{\varsigma_1\mathbf{s}_1}{\mu_3}\right)\right) + \mathbf{s}_2^{\mathrm{T}}\left(-k_7\tanh(\mathbf{s}_2)\right. \\
&\quad \left. - k_8\mathbf{s}_2 - \hat{D}_t\tanh\left(\frac{\mathbf{s}_2}{\mu_2}\right) - \hat{\alpha}_2\varsigma_2\tanh\left(\frac{\varsigma_2\mathbf{s}_2}{\mu_4}\right)\right) \\
&\quad - \frac{1}{c_1}\tilde{D}_r\dot{\hat{D}}_r - \frac{1}{c_2}\tilde{D}_t\dot{\hat{D}}_t - \frac{1}{c_3}\tilde{\alpha}_1\dot{\hat{\alpha}}_1 - \frac{1}{c_4}\tilde{\alpha}_2\dot{\hat{\alpha}}_2 \\
&\le \alpha_1\varsigma_1\|\mathbf{s}_1\| + \alpha_2\varsigma_2\|\mathbf{s}_2\| + \|\mathbf{s}_1\|D_r + \|\mathbf{s}_2\|D_t - k_5\|\mathbf{s}_1\| \\
&\quad - k_6\|\mathbf{s}_1\|^2 - \mathbf{s}_1^{\mathrm{T}}\hat{D}_r\tanh\left(\frac{\mathbf{s}_1}{\mu_1}\right) - \mathbf{s}_1^{\mathrm{T}}\hat{\alpha}_1\varsigma_1\tanh\left(\frac{\varsigma_1\mathbf{s}_1}{\mu_3}\right) \\
&\quad - k_7\|\mathbf{s}_2\| - k_8\|\mathbf{s}_2\|^2 - \mathbf{s}_2^{\mathrm{T}}\hat{D}_t\tanh\left(\frac{\mathbf{s}_2}{\mu_2}\right) \\
&\quad - \mathbf{s}_2^{\mathrm{T}}\hat{\alpha}_2\varsigma_2\tanh\left(\frac{\varsigma_2\mathbf{s}_2}{\mu_4}\right) - \frac{1}{c_1}\tilde{D}_r\dot{\hat{D}}_r - \frac{1}{c_2}\tilde{D}_t\dot{\hat{D}}_t \\
&\quad - \frac{1}{c_3}\tilde{\alpha}_1\dot{\hat{\alpha}}_1 - \frac{1}{c_4}\tilde{\alpha}_2\dot{\hat{\alpha}}_2 + 3\kappa(k_5 + k_7).
\end{aligned}
\tag{41}
$$

From Lemma 4, we obtain the following condition:

$$
\begin{aligned}
-\hat{D}_r\mathbf{s}_1^{\mathrm{T}}\tanh\left(\frac{\mathbf{s}_1}{\mu_1}\right) &\le -\|\mathbf{s}_1\|\hat{D}_r + 3\hat{D}_r\mu_1\kappa, \\
-\hat{D}_t\mathbf{s}_2^{\mathrm{T}}\tanh\left(\frac{\mathbf{s}_2}{\mu_2}\right) &\le -\|\mathbf{s}_2\|\hat{D}_t + 3\hat{D}_t\mu_2\kappa, \\
-\varsigma_1\hat{\alpha}_1\mathbf{s}_1^{\mathrm{T}}\tanh\left(\frac{\varsigma_1\mathbf{s}_1}{\mu_3}\right) &\le -\varsigma_1\hat{\alpha}_1\|\mathbf{s}_1\| + 3\hat{\alpha}_1\mu_3\kappa, \\
-\varsigma_2\hat{\alpha}_2\mathbf{s}_2^{\mathrm{T}}\tanh\left(\frac{\varsigma_2\mathbf{s}_2}{\mu_4}\right) &\le -\varsigma_2\hat{\alpha}_2\|\mathbf{s}_2\| + 3\hat{\alpha}_2\mu_4\kappa.
\end{aligned}
\tag{42}
$$

Consequently, Eq. (41) can be rewritten as follows:

$$
\begin{aligned}
\dot{V}_1 &\le \alpha_1\varsigma_1\|\mathbf{s}_1\| + \alpha_2\varsigma_2\|\mathbf{s}_2\| + \|\mathbf{s}_1\|D_r + \|\mathbf{s}_2\|D_t - k_5\|\mathbf{s}_1\| \\
&\quad - k_6\|\mathbf{s}_1\|^2 - \|\mathbf{s}_1\|\hat{D}_r + 3\hat{D}_r\mu_1\kappa - \hat{\alpha}_1\varsigma_1\|\mathbf{s}_1\| + 3\hat{\alpha}_1\varsigma_1\mu_3\kappa \\
&\quad - k_7\|\mathbf{s}_2\| - k_8\|\mathbf{s}_2\|^2 - \|\mathbf{s}_2\|\hat{D}_t + 3\hat{D}_t\mu_2\kappa - \hat{\alpha}_2\varsigma_2\|\mathbf{s}_2\| \\
&\quad + 3\hat{\alpha}_2\varsigma_2\mu_4\kappa - \frac{1}{c_1}\tilde{D}_r\dot{\hat{D}}_r - \frac{1}{c_2}\tilde{D}_t\dot{\hat{D}}_t - \frac{1}{c_3}\tilde{\alpha}_1\dot{\hat{\alpha}}_1 - \frac{1}{c_4}\tilde{\alpha}_2\dot{\hat{\alpha}}_2 \\
&\quad + 3\kappa(k_5 + k_7) = \tilde{\alpha}_1\varsigma_1\|\mathbf{s}_1\| + \tilde{\alpha}_2\varsigma_2\|\mathbf{s}_2\| + \|\mathbf{s}_1\|\tilde{D}_r \\
&\quad + \|\mathbf{s}_2\|\tilde{D}_t - k_5\|\mathbf{s}_1\| - k_6\|\mathbf{s}_1\|^2 - k_7\|\mathbf{s}_2\| - k_8\|\mathbf{s}_2\|^2 \\
&\quad - \frac{1}{c_1}\tilde{D}_r\dot{\hat{D}}_r - \frac{1}{c_2}\tilde{D}_t\dot{\hat{D}}_t - \frac{1}{c_3}\tilde{\alpha}_1\dot{\hat{\alpha}}_1 - \frac{1}{c_4}\tilde{\alpha}_2\dot{\hat{\alpha}}_2 \\
&\quad + 3\kappa\left(k_5 + k_7 + \hat{D}_r\mu_1 + \hat{D}_t\mu_2 + \hat{\alpha}_1\varsigma_1\mu_3 + \hat{\alpha}_2\varsigma_2\mu_4\right).
\end{aligned}
\tag{43}
$$

Combining the control laws defined in Eqs. (31)–(34), the following expression can be obtained:

$$
\begin{aligned}
\dot{V}_1 &\le \tilde{\alpha}_1\varsigma_1\|\mathbf{s}_1\| + \tilde{\alpha}_2\varsigma_2\|\mathbf{s}_2\| + \|\mathbf{s}_1\|\tilde{D}_r + \|\mathbf{s}_2\|\tilde{D}_t - k_6\|\mathbf{s}_1\|^2 \\
&\quad - k_8\|\mathbf{s}_2\|^2 - \tilde{D}_r\left(\|\mathbf{s}_1\|\tanh\left(\frac{\|\mathbf{s}_1\|}{3\mu_1}\right) - c_5\hat{D}_r\right) \\
&\quad - \tilde{D}_t\left(\|\mathbf{s}_2\|\tanh\left(\frac{\|\mathbf{s}_2\|}{3\mu_2}\right) - c_6\hat{D}_t\right) \\
&\quad - \tilde{\alpha}_1\left(\varsigma_1\|\mathbf{s}_1\|\tanh\left(\frac{\varsigma_1\|\mathbf{s}_1\|}{3\mu_3}\right) - c_7\hat{\alpha}_1\right) \\
&\quad - \tilde{\alpha}_2\left(\varsigma_2\|\mathbf{s}_2\|\tanh\left(\frac{\varsigma_2\|\mathbf{s}_2\|}{3\mu_4}\right) - c_8\hat{\alpha}_2\right) \\
&\quad + 3\kappa\left(k_5 + k_7 + \hat{D}_r\mu_1 + \hat{D}_t\mu_2 + \hat{\alpha}_1\varsigma_1\mu_3 + \hat{\alpha}_2\varsigma_2\mu_4\right) \\
&= \tilde{\alpha}_1\varsigma_1\|\mathbf{s}_1\| + \tilde{\alpha}_2\varsigma_2\|\mathbf{s}_2\| + \|\mathbf{s}_1\|\tilde{D}_r + \|\mathbf{s}_2\|\tilde{D}_t - k_6\|\mathbf{s}_1\|^2 \\
&\quad - k_8\|\mathbf{s}_2\|^2 - \tilde{D}_r\|\mathbf{s}_1\|\tanh\left(\frac{\|\mathbf{s}_1\|}{3\mu_1}\right) \\
&\quad - \tilde{D}_t\|\mathbf{s}_2\|\tanh\left(\frac{\|\mathbf{s}_2\|}{3\mu_2}\right) + c_5\tilde{D}_r\hat{D}_r + c_6\tilde{D}_t\hat{D}_t \\
&\quad - \tilde{\alpha}_1\varsigma_1\|\mathbf{s}_1\|\tanh\left(\frac{\varsigma_1\|\mathbf{s}_1\|}{3\mu_3}\right) - \tilde{\alpha}_2\varsigma_2\|\mathbf{s}_2\|\tanh\left(\frac{\varsigma_2\|\mathbf{s}_2\|}{3\mu_4}\right) \\
&\quad + c_7\tilde{\alpha}_1\hat{\alpha}_1 + c_8\tilde{\alpha}_2\hat{\alpha}_2 + 3\kappa(k_5 + k_7) + \kappa\hat{D}_r\mu_1 + \kappa\hat{D}_t\mu_2 \\
&\quad + 3\kappa\hat{\alpha}_1\varsigma_1\mu_3 + 3\kappa\hat{\alpha}_2\varsigma_2\mu_4 \le \tilde{\alpha}_1\varsigma_1\|\mathbf{s}_1\| + \tilde{\alpha}_2\varsigma_2\|\mathbf{s}_2\| \\
&\quad + \|\mathbf{s}_1\|\tilde{D}_r + \|\mathbf{s}_2\|\tilde{D}_t - k_6\|\mathbf{s}_1\|^2 - k_8\|\mathbf{s}_2\|^2 - \tilde{D}_r\|\mathbf{s}_1\| \\
&\quad + 3\tilde{D}_r\mu_1\kappa - \tilde{D}_t\|\mathbf{s}_2\| + 3\tilde{D}_t\mu_2\kappa - \tilde{\alpha}_1\varsigma_1\|\mathbf{s}_1\| + 3\tilde{\alpha}_1\varsigma_1\mu_3\kappa \\
&\quad - \tilde{\alpha}_2\varsigma_2\|\mathbf{s}_2\| + 3\tilde{\alpha}_2\varsigma_2\mu_4\kappa + c_5\tilde{D}_r\hat{D}_r + c_6\tilde{D}_t\hat{D}_t + c_7\tilde{\alpha}_1\hat{\alpha}_1 \\
&\quad + c_8\tilde{\alpha}_2\hat{\alpha}_2 + 3\kappa\left(k_5 + k_7 + \hat{D}_r\mu_1 + \hat{D}_t\mu_2 + \hat{\alpha}_1\varsigma_1\mu_3 + \hat{\alpha}_2\varsigma_2\mu_4\right) \\
&\le -k_6\|\mathbf{s}_1\|^2 - k_8\|\mathbf{s}_2\|^2 + c_5\tilde{D}_r\hat{D}_r + c_6\tilde{D}_t\hat{D}_t + c_7\tilde{\alpha}_1\hat{\alpha}_1 \\
&\quad + c_8\tilde{\alpha}_2\hat{\alpha}_2 + 3\kappa(k_5 + k_7 + D_r\mu_1 + D_t\mu_2 + \alpha_1\varsigma_1\mu_3 + \alpha_2\varsigma_2\mu_4) \\
&\le -k_6\|\mathbf{s}_1\|^2 - k_8\|\mathbf{s}_2\|^2 - \left(D_r - \hat{D}_r\right)^2 - \left(D_t - \hat{D}_t\right)^2 \\
&\quad - (\alpha_1 - \hat{\alpha}_1)^2 - (\alpha_2 - \hat{\alpha}_2)^2 + \Delta_1,
\end{aligned}
\tag{44}
$$

where $\Delta_1 = (c_5 D_r^2/4(c_5-1)) + (c_6 D_t^2/4(c_6-1)) + (c_7\alpha_1^2/4(c_7-1)) + (c_8\alpha_2^2/4(c_8-1)) + 3\kappa(k_5+k_7+D_r\mu_1+D_t\mu_2+\alpha_1\varsigma_1\mu_3+\alpha_2\varsigma_2\mu_4)$..

Equation (44) can be further rewritten as follows:

$$\begin{aligned}\dot{V}_1 &\le -\frac{2k_6}{\lambda_{\max}(\mathbf{J})}\left(\frac{1}{2}\mathbf{s}_1^{\mathrm{T}}\mathbf{J}\mathbf{s}_1\right) - \frac{2k_8}{m}\left(\frac{1}{2}\mathbf{s}_2^{\mathrm{T}}m\mathbf{s}_2\right) - 2c_1\left(\frac{1}{2c_1}\tilde{D}_r^2\right) \\ &\quad - 2c_2\left(\frac{1}{2c_2}\tilde{D}_t^2\right) - 2c_3\left(\frac{1}{2c_3}\alpha_1^2\right) - 2c_4\left(\frac{1}{2c_4}\alpha_2^2\right) + \Delta_1 \\ &\le -\rho_1 V_1 + \Delta_1,\end{aligned} \tag{45}$$

where $\rho_1 = \min\{(2k_6/\lambda_{\max}(\mathbf{J})), (2k_8/m), 2c_1, 2c_2, 2c_3, 2c_4\}$.

Consequently, according to the aforementioned equation, we conclude that $\mathbf{s}_1$, $\mathbf{s}_2$, $\tilde{D}_r$, $\tilde{D}_t$, $\tilde{\alpha}_1$, and $\tilde{\alpha}_2$ exponentially converge to a bounded region with respect to Δ_1. Then, the positive constants $\bar{D}_r$, $\bar{D}_t$, $\bar{\alpha}_1$, and $\bar{\alpha}_2$ that satisfy the $\bar{D}_r \ge D_r$, $\bar{D}_r \ge \widehat{D}_r$, $\bar{D}_t \ge D_t$, $\bar{D}_t \ge \widehat{D}_t$ and $\bar{\alpha}_1 \ge \alpha_1$, $\bar{\alpha}_1 \ge \widehat{\alpha}_1$, $\bar{\alpha}_2 \ge \alpha_2$, $\bar{\alpha}_2 \ge \widehat{\alpha}_2$ conditions must exist.

To illustrate the finite-time stability, the Lyapunov function is defined as follows:

$$\begin{aligned}V_2 &= \frac{1}{2}\mathbf{s}_1^{\mathrm{T}}\mathbf{J}\mathbf{s}_1 + \frac{1}{2}\mathbf{s}_2^{\mathrm{T}}m\mathbf{s}_2 + \frac{1}{c_1}\left(\bar{D}_r - \widehat{D}_r\right)^2 + \frac{1}{c_2}\left(\bar{D}_t - \widehat{D}_t\right)^2 \\ &\quad + \frac{1}{c_3}(\bar{\alpha}_1 - \widehat{\alpha}_1)^2 + \frac{1}{c_4}(\bar{\alpha}_2 - \widehat{\alpha}_2)^2.\end{aligned} \tag{46}$$

Combining the aforementioned schemes, the derivative of V_2 satisfies the following expression:

$$\begin{aligned}\dot{V}_2 &= \mathbf{s}_1^{\mathrm{T}}\mathbf{J}\dot{\mathbf{s}}_1 + \mathbf{s}_2^{\mathrm{T}}m\dot{\mathbf{s}}_2 - \frac{2}{c_1}\left(\bar{D}_r - \widehat{D}_r\right)\dot{\widehat{D}}_r \\ &\quad - \frac{2}{c_2}\left(\bar{D}_t - \widehat{D}_t\right)\dot{\widehat{D}}_t - \frac{2}{c_3}(\bar{\alpha}_1 - \widehat{\alpha}_1)\dot{\widehat{\alpha}}_1 \\ &\quad - \frac{2}{c_4}(\bar{\alpha}_2 - \widehat{\alpha}_2)\dot{\widehat{\alpha}}_2 \\ &= \mathbf{s}_1^{\mathrm{T}}[-\mathbf{C}_r\tilde{\boldsymbol{\omega}} - \mathbf{n}_r + \boldsymbol{\varepsilon}_r + \mathbf{d}_r \\ &\quad + \frac{1}{2}\boldsymbol{J}\left[k_1 + k_2\left(1 + \tanh^{\mathrm{T}}(\tilde{\mathbf{q}}_v)\tanh(\tilde{\mathbf{q}}_v)\right)\right] \\ &\quad \cdot(\tilde{\mathbf{q}}_v^{\times} + \tilde{q}_0\mathbf{I}_3)\tilde{\boldsymbol{\omega}}\Big] + \mathbf{s}_2^{\mathrm{T}}\left[-m\mathbf{C}_t\tilde{\mathbf{v}} - m\mathbf{n}_t + \boldsymbol{\varepsilon}_t + \mathbf{d}_t\right. \\ &\quad \left. + m\left[k_3 + k_4\left(1 + \tanh^{\mathrm{T}}(\tilde{\mathbf{r}})\tanh(\tilde{\mathbf{r}})\right)\right](\tilde{\mathbf{v}} - \mathbf{C}_t\tilde{\mathbf{r}})\right] \\ &\quad - \frac{2}{c_1}\left(\bar{D}_r - \widehat{D}_r\right)\dot{\widehat{D}}_r - \frac{2}{c_2}\left(\bar{D}_t - \widehat{D}_t\right)\dot{\widehat{D}}_t \\ &\quad - \frac{2}{c_3}(\bar{\alpha}_1 - \widehat{\alpha}_1)\dot{\widehat{\alpha}}_1 - \frac{2}{c_4}(\bar{\alpha}_2 - \widehat{\alpha}_2)\dot{\widehat{\alpha}}_2 \\ &\le \alpha_1\varsigma_1\|\mathbf{s}_1\| + \alpha_2\varsigma_2\|\mathbf{s}_2\| + \mathbf{s}_1^{\mathrm{T}}(\boldsymbol{\varepsilon}_r + \mathbf{d}_r) \\ &\quad + \mathbf{s}_2^{\mathrm{T}}(\boldsymbol{\varepsilon}_t + \mathbf{d}_t) - \frac{2}{c_1}\left(\bar{D}_r - \widehat{D}_r\right)\dot{\widehat{D}}_r \\ &\quad - \frac{2}{c_2}\left(\bar{D}_t - \widehat{D}_t\right)\dot{\widehat{D}}_t - \frac{2}{c_3}(\bar{\alpha}_1 - \widehat{\alpha}_1)\dot{\widehat{\alpha}}_1 \\ &\quad - \frac{2}{c_4}(\bar{\alpha}_2 - \widehat{\alpha}_2)\dot{\widehat{\alpha}}_2.\end{aligned} \tag{47}$$

By using the proposed control scheme, the following expression can be obtained:

$$\begin{aligned}\dot{V}_2 &\le \alpha_1\varsigma_1\|\mathbf{s}_1\| + \alpha_2\varsigma_2\|\mathbf{s}_2\| + \|\mathbf{s}_1\|D_r + \|\mathbf{s}_2\|D_t \\ &\quad + \mathbf{s}_1^{\mathrm{T}}\left(-k_5\tanh(\mathbf{s}_1) - k_6\mathbf{s}_1 - \widehat{D}_r\tanh\left(\frac{\mathbf{s}_1}{\mu_1}\right)\right. \\ &\quad \left. - \widehat{\alpha}_1\varsigma_1\tanh\left(\frac{\varsigma_1\mathbf{s}_1}{\mu_3}\right)\right) + \mathbf{s}_2^{\mathrm{T}}\left(-k_7\tanh(\mathbf{s}_2) - k_8\mathbf{s}_2\right. \\ &\quad \left. - \widehat{D}_t\tanh\left(\frac{\mathbf{s}_2}{\mu_2}\right) - \widehat{\alpha}_2\varsigma_2\tanh\left(\frac{\varsigma_2\mathbf{s}_2}{\mu_4}\right)\right) \\ &\quad - \frac{2}{c_1}\left(\bar{D}_r - \widehat{D}_r\right)\dot{\widehat{D}}_r - \frac{2}{c_2}\left(\bar{D}_t - \widehat{D}_t\right)\dot{\widehat{D}}_t \\ &\quad - \frac{2}{c_3}(\bar{\alpha}_1 - \widehat{\alpha}_1)\dot{\widehat{\alpha}}_1 - \frac{2}{c_4}(\bar{\alpha}_2 - \widehat{\alpha}_2)\dot{\widehat{\alpha}}_2 \\ &\le \alpha_1\varsigma_1\|\mathbf{s}_1\| + \alpha_2\varsigma_2\|\mathbf{s}_2\| + \|\mathbf{s}_1\|D_r + \|\mathbf{s}_2\|D_t \\ &\quad - k_5\|\mathbf{s}_1\| - k_6\|\mathbf{s}_1\|^2 - \widehat{D}_r\mathbf{s}_1^{\mathrm{T}}\tanh\left(\frac{\mathbf{s}_1}{\mu_1}\right) \\ &\quad - \widehat{\alpha}_1\varsigma_1\mathbf{s}_1^{\mathrm{T}}\tanh\left(\frac{\varsigma_1\mathbf{s}_1}{\mu_3}\right) - k_7\|\mathbf{s}_2\| - k_8\|\mathbf{s}_2\|^2 \\ &\quad - \widehat{D}_t\mathbf{s}_2^{\mathrm{T}}\tanh\left(\frac{\mathbf{s}_2}{\mu_2}\right) - \widehat{\alpha}_2\varsigma_2\mathbf{s}_2^{\mathrm{T}}\tanh\left(\frac{\varsigma_2\mathbf{s}_2}{\mu_4}\right) \\ &\quad - \frac{2}{c_1}\left(\bar{D}_r - \widehat{D}_r\right)\dot{\widehat{D}}_r - \frac{2}{c_2}\left(\bar{D}_t - \widehat{D}_t\right)\dot{\widehat{D}}_t \\ &\quad - \frac{2}{c_3}(\bar{\alpha}_1 - \widehat{\alpha}_1)\dot{\widehat{\alpha}}_1 - \frac{2}{c_4}(\bar{\alpha}_2 - \widehat{\alpha}_2)\dot{\widehat{\alpha}}_2 + 3\kappa(k_5 + k_7) \\ &\le \alpha_1\varsigma_1\|\mathbf{s}_1\| + \alpha_2\varsigma_2\|\mathbf{s}_2\| + \|\mathbf{s}_1\|D_r + \|\mathbf{s}_2\|D_t - k_5\|\mathbf{s}_1\| \\ &\quad - k_6\|\mathbf{s}_1\|^2 - \widehat{D}_r\|\mathbf{s}_1\| + 3\widehat{D}_r\mu_1\kappa - \widehat{\alpha}_1\varsigma_1\|\mathbf{s}_1\| \\ &\quad + 3\widehat{\alpha}_1\varsigma_1\mu_3\kappa - k_7\|\mathbf{s}_2\| - k_8\|\mathbf{s}_2\|^2 - \widehat{D}_t\|\mathbf{s}_2\| \\ &\quad + 3\widehat{D}_t\mu_2\kappa - \widehat{\alpha}_2\varsigma_2\|\mathbf{s}_2\| + 3\widehat{\alpha}_2\varsigma_2\mu_4\kappa \\ &\quad - \frac{2}{c_1}\left(\bar{D}_r - \widehat{D}_r\right)\dot{\widehat{D}}_r - \frac{2}{c_2}\left(\bar{D}_t - \widehat{D}_t\right)\dot{\widehat{D}}_t \\ &\quad - \frac{2}{c_3}(\bar{\alpha}_1 - \widehat{\alpha}_1)\dot{\widehat{\alpha}}_1 - \frac{2}{c_4}(\bar{\alpha}_2 - \widehat{\alpha}_2)\dot{\widehat{\alpha}}_2 + 3\kappa(k_5 + k_7) \\ &\le -k_5\|\mathbf{s}_1\| - k_7\|\mathbf{s}_2\| + \varsigma_1\|\mathbf{s}_1\|(\bar{\alpha}_1 - \widehat{\alpha}_1) \\ &\quad + \varsigma_2\|\mathbf{s}_2\|(\bar{\alpha}_2 - \widehat{\alpha}_2) + \|\mathbf{s}_1\|\left(\bar{D}_r - \widehat{D}_r\right) \\ &\quad + \|\mathbf{s}_2\|\left(\bar{D}_t - \widehat{D}_t\right) - \frac{2}{c_1}\left(\bar{D}_r - \widehat{D}_r\right)\dot{\widehat{D}}_r \\ &\quad - \frac{2}{c_2}\left(\bar{D}_t - \widehat{D}_t\right)\dot{\widehat{D}}_t - \frac{2}{c_3}(\bar{\alpha}_1 - \widehat{\alpha}_1)\dot{\widehat{\alpha}}_1 \\ &\quad - \frac{2}{c_4}(\bar{\alpha}_2 - \widehat{\alpha}_2)\dot{\widehat{\alpha}}_2 \\ &\quad + 3\kappa\left(k_5 + k_7 + \widehat{D}_r\mu_1 + \widehat{D}_t\mu_2 + \widehat{\alpha}_1\varsigma_1\mu_3 + \widehat{\alpha}_2\varsigma_2\mu_4\right).\end{aligned} \tag{48}$$

According to the designed adaptive laws, the following expression can be obtained:

$$
\begin{aligned}
\dot{V}_2 \le{}& -k_5\|\mathbf{s}_1\| - k_7\|\mathbf{s}_2\| + \varsigma_1\|\mathbf{s}_1\|(\bar{\alpha}_1 - \hat{\alpha}_1) + \varsigma_2\|\mathbf{s}_2\|(\bar{\alpha}_2 - \hat{\alpha}_2) \\
&+ \|\mathbf{s}_1\|(\bar{D}_r - \hat{D}_r) + \|\mathbf{s}_2\|(\bar{D}_t - \hat{D}_t) \\
&- 2(\bar{D}_r - \hat{D}_r)\left(\|\mathbf{s}_1\| \tanh\left(\frac{\|\mathbf{s}_1\|}{3\mu_1}\right) - c_5\hat{D}_r\right) \\
&- 2(\bar{D}_t - \hat{D}_t)\left(\|\mathbf{s}_2\| \tanh\left(\frac{\|\mathbf{s}_2\|}{3\mu_2}\right) - c_6\hat{D}_t\right) \\
&- 2(\bar{\alpha}_1 - \hat{\alpha}_1)\left(\varsigma_1\|\mathbf{s}_1\| \tanh\left(\frac{\varsigma_1\|\mathbf{s}_1\|}{3\mu_3}\right) - c_7\hat{\alpha}_1\right) \\
&- 2(\bar{\alpha}_2 - \hat{\alpha}_2)\left(\varsigma_2\|\mathbf{s}_2\| \tanh\left(\frac{\varsigma_2\|\mathbf{s}_2\|}{3\mu_4}\right) - c_8\hat{\alpha}_2\right) \\
&+ 3\kappa(k_5 + k_7 + \hat{D}_r\mu_1 + \hat{D}_t\mu_2 + \hat{\alpha}_1\varsigma_1\mu_3 + \hat{\alpha}_2\varsigma_2\mu_4) \\
={}& -k_5\|\mathbf{s}_1\| - k_7\|\mathbf{s}_2\| + \varsigma_1\|\mathbf{s}_1\|(\bar{\alpha}_1 - \hat{\alpha}_1) + \varsigma_2\|\mathbf{s}_2\|(\bar{\alpha}_2 - \hat{\alpha}_2) \\
&+ \|\mathbf{s}_1\|(\bar{D}_r - \hat{D}_r) + \|\mathbf{s}_2\|(\bar{D}_t - \hat{D}_t) \\
&- 2(\bar{D}_r - \hat{D}_r)\|\mathbf{s}_1\| \tanh\left(\frac{\|\mathbf{s}_1\|}{3\mu_1}\right) \\
&- 2(\bar{D}_t - \hat{D}_t)\|\mathbf{s}_2\| \tanh\left(\frac{\|\mathbf{s}_2\|}{3\mu_2}\right) \\
&- 2(\bar{\alpha}_1 - \hat{\alpha}_1)\varsigma_1\|\mathbf{s}_1\| \tanh\left(\frac{\varsigma_1\|\mathbf{s}_1\|}{3\mu_3}\right) \\
&- 2(\bar{\alpha}_2 - \hat{\alpha}_2)\varsigma_2\|\mathbf{s}_2\| \tanh\left(\frac{\varsigma_2\|\mathbf{s}_2\|}{3\mu_4}\right) \\
&+ 2c_5\hat{D}_r(\bar{D}_r - \hat{D}_r) + 2c_6\hat{D}_t(\bar{D}_t - \hat{D}_t) \\
&+ 2c_7\hat{\alpha}_1(\bar{\alpha}_1 - \hat{\alpha}_1) + 2c_8\hat{\alpha}_2(\bar{\alpha}_2 - \hat{\alpha}_2) \\
&+ 3\kappa(k_5 + k_7 + \hat{D}_r\mu_1 + \hat{D}_t\mu_2 + \hat{\alpha}_1\varsigma_1\mu_3 + \hat{\alpha}_2\varsigma_2\mu_4).
\end{aligned} \tag{49}
$$

According to Lemma 4, the following expression can be obtained:

$$
\begin{aligned}
\dot{V}_2 \le{}& -k_5\|\mathbf{s}_1\| - k_7\|\mathbf{s}_2\| + \varsigma_1\|\mathbf{s}_1\|(\bar{\alpha}_1 - \hat{\alpha}_1) \\
&+ \varsigma_2\|\mathbf{s}_2\|(\bar{\alpha}_2 - \hat{\alpha}_2) + \|\mathbf{s}_1\|(\bar{D}_r - \hat{D}_r) + \|\mathbf{s}_2\|(\bar{D}_t - \hat{D}_t) \\
&- 2(\bar{D}_r - \hat{D}_r)\|\mathbf{s}_1\| + 6\mu_1\kappa(\bar{D}_r - \hat{D}_r) - 2(\bar{D}_t - \hat{D}_t)\|\mathbf{s}_2\| \\
&+ 6\mu_2\kappa(\bar{D}_t - \hat{D}_t) - 2(\bar{\alpha}_1 - \hat{\alpha}_1)\varsigma_1\|\mathbf{s}_1\| + 6\mu_3\varsigma_1\kappa(\bar{\alpha}_1 - \hat{\alpha}_1) \\
&- 2(\bar{\alpha}_2 - \hat{\alpha}_2)\varsigma_2\|\mathbf{s}_2\| + 6\mu_4\varsigma_2\kappa(\bar{\alpha}_2 - \hat{\alpha}_2) + 2c_5\hat{D}_r(\bar{D}_r - \hat{D}_r) \\
&+ 2c_6\hat{D}_t(\bar{D}_t - \hat{D}_t) + 2c_7\hat{\alpha}_1(\bar{\alpha}_1 - \hat{\alpha}_1) + 2c_8\hat{\alpha}_2(\bar{\alpha}_2 - \hat{\alpha}_2) \\
&+ 3\kappa(k_5 + k_7 + \hat{D}_r\mu_1 + \hat{D}_t\mu_2 + \hat{\alpha}_1\varsigma_1\mu_3 + \hat{\alpha}_2\varsigma_2\mu_4) \\
={}& -k_5\|\mathbf{s}_1\| - k_7\|\mathbf{s}_2\| - (\|\mathbf{s}_1\| - 3\mu_1\kappa)(\bar{D}_r - \hat{D}_r) \\
&- (\|\mathbf{s}_2\| - 3\mu_2\kappa)(\bar{D}_t - \hat{D}_t) - (\varsigma_1\|\mathbf{s}_1\| - 3\mu_3\varsigma_1\kappa)(\bar{\alpha}_1 - \hat{\alpha}_1) \\
&- (\varsigma_2\|\mathbf{s}_2\| - 3\mu_4\varsigma_2\kappa)(\bar{\alpha}_2 - \hat{\alpha}_2) + 2c_5\hat{D}_r(\bar{D}_r - \hat{D}_r) \\
&+ 2c_6\hat{D}_t(\bar{D}_t - \hat{D}_t) + 2c_7\hat{\alpha}_1(\bar{\alpha}_1 - \hat{\alpha}_1) + 2c_8\hat{\alpha}_2(\bar{\alpha}_2 - \hat{\alpha}_2) \\
&+ 3\kappa(k_5 + k_7 + \bar{D}_r\mu_1 + \bar{D}_t\mu_2 + \bar{\alpha}_1\varsigma_1\mu_3 + \bar{\alpha}_2\varsigma_2\mu_4) \\
={}& -k_5\|\mathbf{s}_1\| - k_7\|\mathbf{s}_2\| - (\|\mathbf{s}_1\| - 3\mu_1\kappa)(\bar{D}_r - \hat{D}_r) \\
&- (\|\mathbf{s}_2\| - 3\mu_2\kappa)(\bar{D}_t - \hat{D}_t) - (\varsigma_1\|\mathbf{s}_1\| - 3\mu_3\varsigma_1\kappa)(\bar{\alpha}_1 - \hat{\alpha}_1) \\
&- (\varsigma_2\|\mathbf{s}_2\| - 3\mu_4\varsigma_2\kappa)(\bar{\alpha}_2 - \hat{\alpha}_2) + \Delta_2,
\end{aligned} \tag{50}
$$

where $\Delta_2 = (c_5/2)\bar{D}_r^2 + (c_6/2)\bar{D}_t^2 + (c_7/2)\bar{\alpha}_1^2 + (c_8/2)\bar{\alpha}_2^2 + 3\kappa(k_5 + k_7 + \bar{D}_r\mu_1 + \bar{D}_t\mu_2 + \bar{\alpha}_1\varsigma_1\mu_3 + \bar{\alpha}_2\varsigma_2\mu_4)$.

Equation (50) can be rewritten as follows:

$$
\begin{aligned}
\dot{V}_2 \le{}& -k_5\sqrt{\frac{2}{\lambda_{\max}(\mathbf{J})}}\left(\frac{1}{2}\mathbf{s}_1^{\mathrm{T}}\mathbf{J}\mathbf{s}_1\right)^{1/2} - k_7\sqrt{\frac{2}{m}}\left(\frac{1}{2}\mathbf{s}_2^{\mathrm{T}}m\mathbf{s}_2\right)^{1/2} \\
&- \sqrt{c_1}(\|\mathbf{s}_1\| - 3\mu_1\kappa)\left(\frac{1}{c_1}(\bar{D}_r - \hat{D}_r)^2\right)^{1/2} \\
&- \sqrt{c_2}(\|\mathbf{s}_2\| - 3\mu_2\kappa)\left(\frac{1}{c_2}(\bar{D}_t - \hat{D}_t)^2\right)^{1/2} \\
&- \sqrt{c_3}(\varsigma_1\|\mathbf{s}_1\| - 3\mu_3\varsigma_1\kappa)\left(\frac{1}{c_3}(\bar{\alpha}_1 - \hat{\alpha}_1)^2\right)^{1/2} \\
&- \sqrt{c_4}(\varsigma_2\|\mathbf{s}_2\| - 3\mu_4\varsigma_2\kappa)\left(\frac{1}{c_4}(\bar{\alpha}_2 - \hat{\alpha}_2)^2\right)^{1/2} + \Delta_2 \\
\le{}& -\rho_2\mathbf{V}_2^{1/2} + \Delta_2,
\end{aligned} \tag{51}
$$

where $\rho_2 = \min\{k_5\sqrt{2/\lambda_{\max}(\mathbf{J})}, k_7\sqrt{2/m}, \sqrt{c_1}(\|\mathbf{s}_1\| - 3\mu_1\kappa), \sqrt{c_2}(\|\mathbf{s}_2\| - 3\mu_2\kappa), \sqrt{c_3}(\varsigma_1\|\mathbf{s}_1\| - 3\mu_3\varsigma_1\kappa), \sqrt{c_4}(\varsigma_2\|\mathbf{s}_2\| - 3\mu_4\varsigma_2\kappa)\}$.

Consequently, $\mathbf{s}_1$ and $\mathbf{s}_2$ converge to two small regions Θ_1 and Θ_2 in finite time, where Θ_1 and Θ_2 are positive constants. The following equations can be obtained from the definition of $\mathbf{s}_1$ and $\mathbf{s}_2$:

$$
\begin{aligned}
\tilde{\omega}_i + k_1\tilde{q}_{vi} + k_2\tanh(\tilde{q}_{vi}) = \zeta_i,\ |\zeta_i| \le \Theta_1 \quad i = 1, 2, 3, \\
\tilde{v}_i + k_3\tilde{r}_i + k_4\tanh(\tilde{r}_i) = \vartheta_i,\ |\vartheta_i| \le \Theta_2 \quad i = 1, 2, 3.
\end{aligned} \tag{52}
$$

Thus, the following expressions can be obtained:

$$
\begin{aligned}
\tilde{\omega}_i + \left(k_1 - \frac{\zeta_i}{2\tilde{q}_{vi}}\right)\tilde{q}_{vi} + \left(k_2 - \frac{\zeta_i}{2\tanh(\tilde{q}_{vi})}\right)\tanh(\tilde{q}_{vi}) = 0 \quad i = 1, 2, 3, \\
\tilde{v}_i + \left(k_3 - \frac{\vartheta_i}{2\tilde{r}_i}\right)\tilde{r}_i + \left(k_4 - \frac{\vartheta_i}{2\tanh(\tilde{r}_i)}\right)\tanh(\tilde{r}_i) = 0 \quad i = 1, 2, 3.
\end{aligned} \tag{53}
$$

Consequently, the relative dynamic system is stabilized in finite time under the conditions $k_1 - \zeta_i/2\tilde{q}_{vi} > 0, k_2 - \zeta_i/2\tanh(\tilde{q}_{vi}) > 0$, and $k_3 - \vartheta_i/2\tilde{r}_i > 0, k_4 - \vartheta_i/2\tanh(\tilde{r}_i) > 0$, which implies that the errors $\tilde{q}_{vi}$ and $\tilde{r}$ converge to the following region in finite time:

$$
\begin{aligned}
\Delta_{\tilde{q}_v} &= \max\left(\frac{\zeta_i}{2k_1}, \frac{1}{2}\ln\left(\frac{k_2 - \zeta_i}{k_2}\right) - \frac{1}{2}\right), \\
\Delta_{\tilde{r}} &= \max\left(\frac{\vartheta_i}{2k_3}, \frac{1}{2}\ln\left(\frac{k_4 - \vartheta_i}{k_4}\right) - \frac{1}{2}\right).
\end{aligned} \tag{54}
$$

Furthermore, the relative angular velocity $\tilde{\omega}_i$ and relative velocity $\tilde{v}_i$ converge to the following regions in finite time:

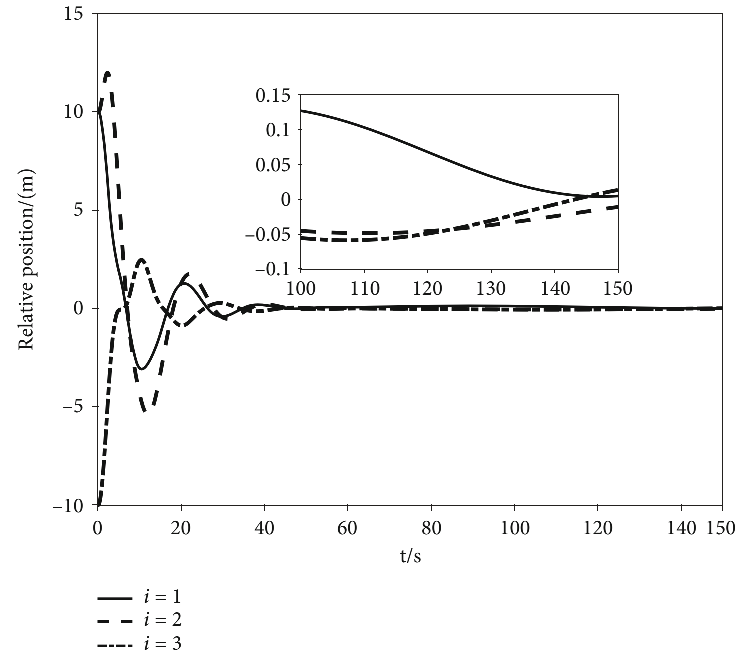

Figure 1: Relative position.

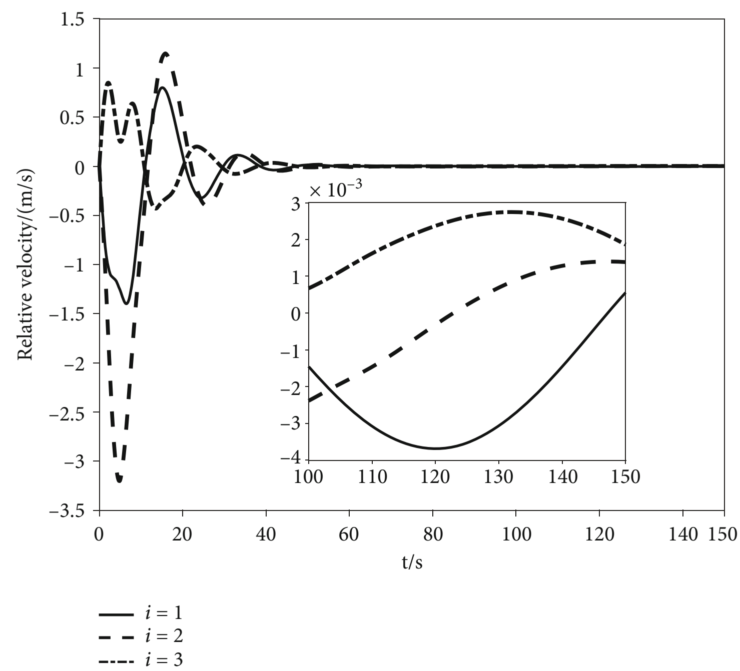

Figure 2: Relative velocity.

$$
\begin{aligned}
|\tilde{\omega}_i| &\le |\zeta_i| + k_{1i}|\tilde{q}_{vi}| + k_2|\tanh\left(\tilde{q}_{vi}\right)| \\
&\le \Theta_1 + k_1\Delta_{\tilde{q}_v} + k_2 \tanh\left(\Delta_{\tilde{q}_v}\right) = \Theta_{q_v}, \\
|\tilde{v}_i| &\le |\vartheta_i| + k_{3i}|\tilde{r}_i| + k_{4i}|\tanh\left(\tilde{r}_i\right)| \\
&\le \Theta_2 + \bar{k}_3\Delta_{\tilde{r}} + \bar{k}_4 \tanh\left(\Delta_{\tilde{r}}\right) = \Theta_r.
\end{aligned} \tag{55}
$$

Therefore, Theorem 6 is proved.

4. Simulation Results

In this section, the effectiveness and advantage of the developed control strategy are presented. Detailed information regarding the orbit and these two spacecrafts is presented as follows [23]. In the simulation scenarios, a pursuer spacecraft is forced to rendezvous with the target spacecraft in an elliptical orbit with a perigee altitude $r_{pa} = 400\text{km}$ and

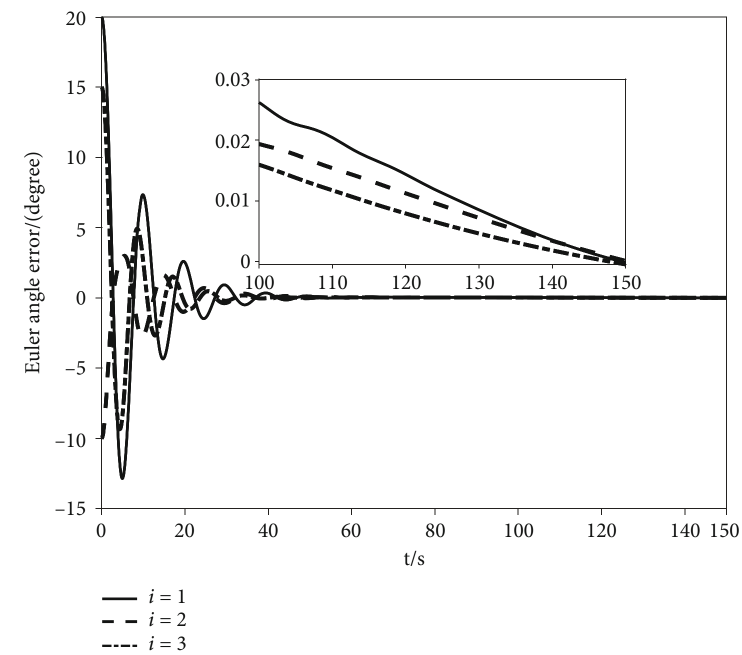

Figure 3: Relative Euler angle.

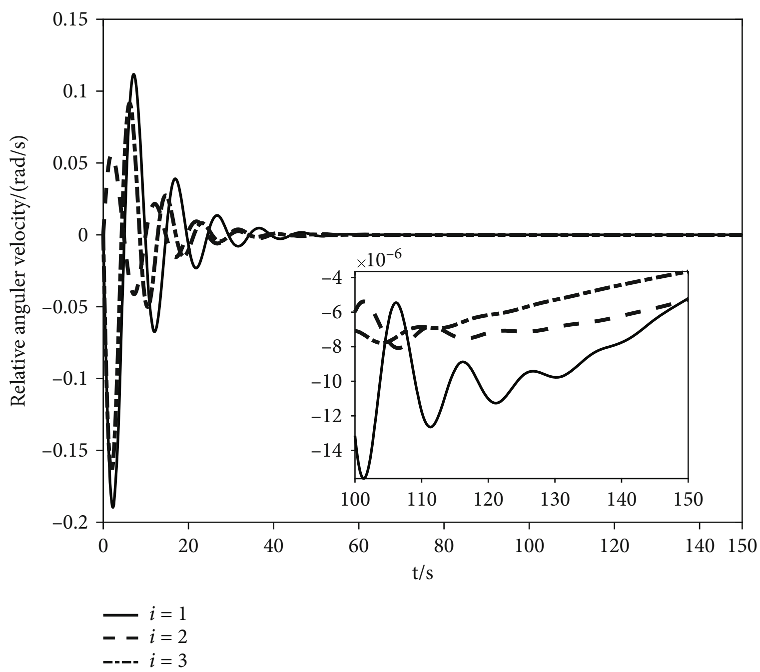

Figure 4: Relative angular velocity.

eccentricity $e = 0.3$. The gravitational constant is $\mu = 3.986 \times 10^{14} \mathrm{Nm}^2/\mathrm{kg}$ and the radius of Earth is $R_E = 6371 \mathrm{km}$. The mass and initial matrix of the purser spacecraft and the target spacecraft are chosen as $m = 90 \mathrm{kg}$, $\mathbf{J} = \mathrm{diag}\,(20, 20, 15) \mathrm{kg} \cdot \mathrm{m}^2$ and $m_t = 150 \mathrm{kg}$, $\mathbf{J}_t = \mathrm{diag}\,(26, 16, 21) \mathrm{kg} \cdot \mathrm{m}^2$, respectively. The initial true anomaly is $\nu(0) = 10°$.

The target spacecraft is supposed to service with the following position:

$$\mathbf{r}_t = [r_t, 0, 0]^{\mathrm{T}}, r_t = \frac{a(1 - e^2)}{1 + e \cos \nu}, \tag{56}$$

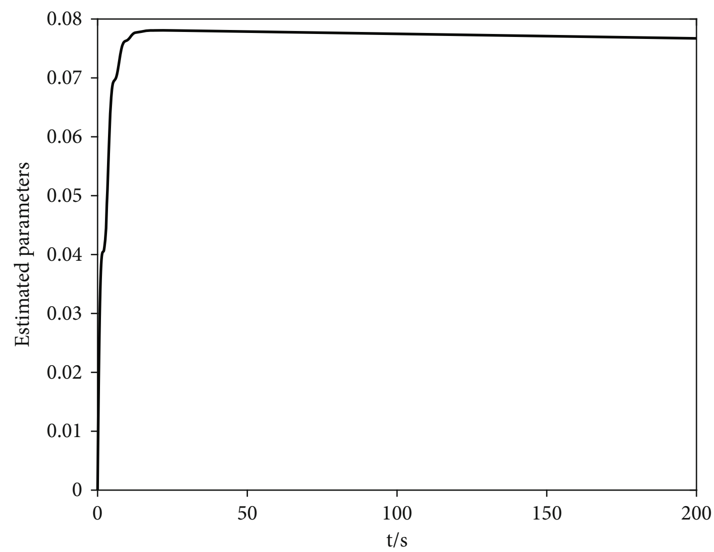

FIGURE 5: Estimation of β_r.

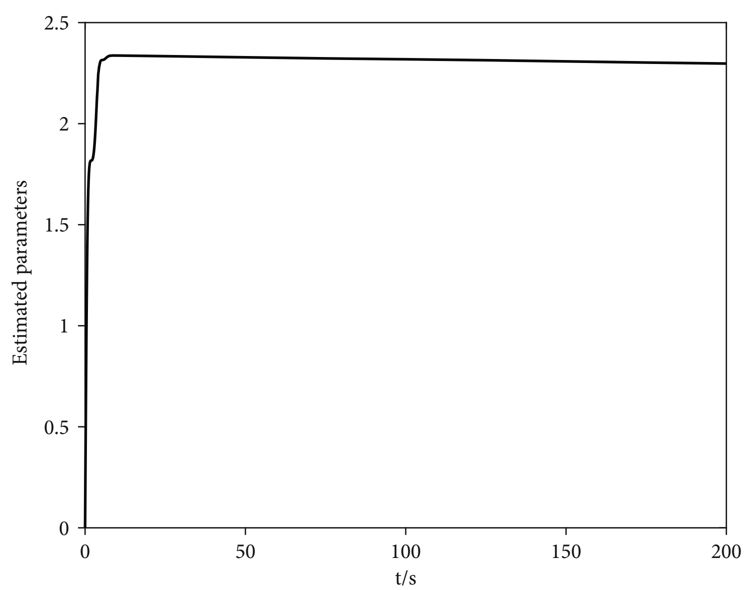

FIGURE 6: Estimation of β_t.

where $a = R_E + (r_{pa}/1 - e)$ denotes the semimajor axis. The parameter v is expressed as follows:

$$\dot{v} = \frac{n(1 + e\cos v)^2}{(1 - e^2)^{3/2}}, \ddot{v} = \frac{2n^2 e(1 + e\cos v)^3 \sin v}{(1 - e^2)^3}, \tag{57}$$

where $n = \sqrt{u/a^3}$. For the pursuer spacecraft, its rendezvous position is expressed as $\boldsymbol{\delta}_t = [0, 5, 0]^{\mathrm{T}}$ in the body coordinate frame of the target. The angular velocity of the target and the external disturbances are expressed as follows:

$$\begin{aligned}\boldsymbol{\tau}_d &= 0.02 \times \left(1 + \cos\left(\frac{\pi}{150}t\right) + \sin\left(\frac{\pi}{150}t\right)\right)[1\,;1\,;1]^{\mathrm{T}}\mathrm{N}\cdot\mathrm{m},\\ \mathbf{f}_d &= 0.05 \times \left(1 + \cos\left(\frac{\pi}{150}t\right) + \sin\left(\frac{\pi}{150}t\right)\right)[1\,;1\,;1]^{\mathrm{T}}\mathrm{N}\cdot\mathrm{m}.\end{aligned} \tag{58}$$

The initial states are set as follows: the initial Euler angle error is $\Theta(0) = [19.9984 - 9.9987\ 15.0050]^{\mathrm{T}}$ deg, $\tilde{\boldsymbol{\omega}} = [0\,0\,0]^{\mathrm{T}}$, $\tilde{\mathbf{r}}(0) = [10, 10, -10]^{\mathrm{T}}$, and $\tilde{\mathbf{v}}(0) = [0\,0\,0]^{\mathrm{T}}$.

The design parameters are set as follows: $k_1 = 2$, $k_2 = 2$, $k_3 = 0.5$, $k_4 = 0.1$, $k_5 = 0.05$, $k_6 = 4$, $k_7 = 10$, $k_8 = 10$, $c_1 = 0.02$, $c_2 = 0.001$, $c_3 = 0.05$, $c_4 = 0.001$, $c_5 = 1.1$, $c_6 = 5$, $c_7 = 1.1$, and

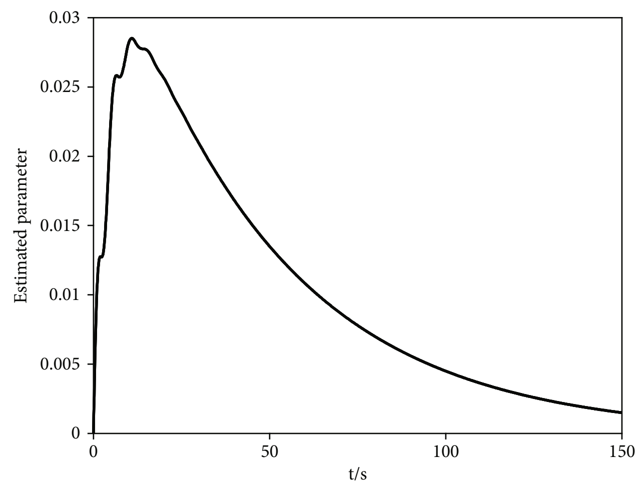

Figure 7: Estimation of D_r.

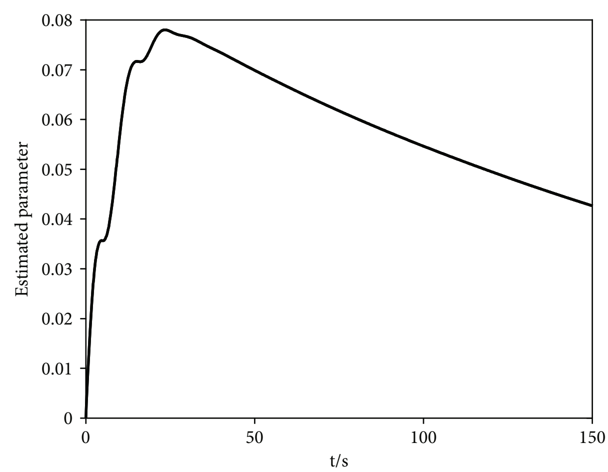

Figure 8: Estimation of D_t.

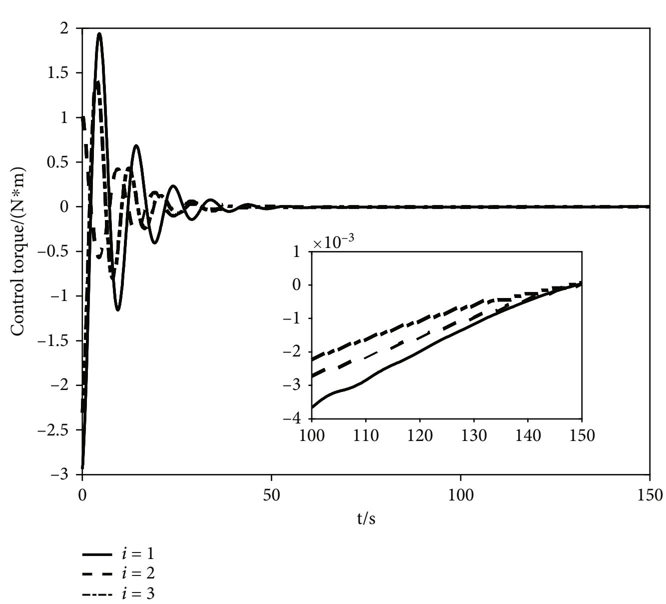

Figure 9: Control torque $\boldsymbol{\tau}_r$.

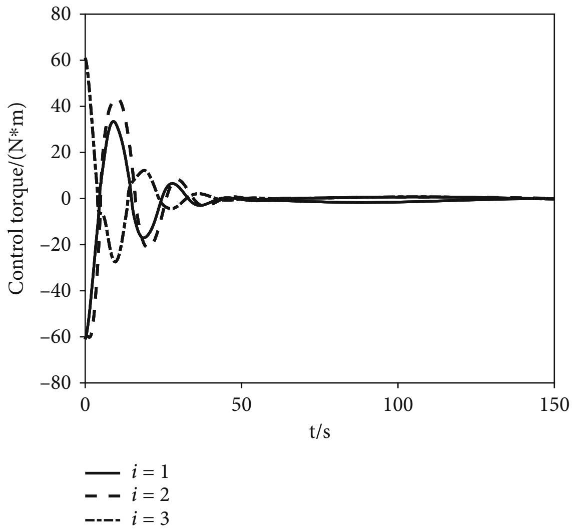

FIGURE 10: Control torque $\boldsymbol{\tau}_t$.

$c_8 = 5$. The simulation results are illustrated in Figures 1–10. Figures 1–4 indicate that the relative attitude dynamics can be stabilized within 40 s and that the finite-time stability for the relative orbit dynamics can be ensured in 50 s. Figures 5–8 present the estimated valuables, which have an upper bound under the proposed adaptive laws. Figures 9 and 10 display the control torques of the developed controllers. These simulation results indicate that the control objective can be achieved with satisfactory performance and that the chattering phenomenon does not occur in the controllers.

5. Conclusion

In this study, we focus on the finite-time model-free tracking control problem for the spacecraft rendezvous maneuver under external disturbances. By resorting to a well-defined sliding mode surface, finite-time convergence of the tracking errors is ensured, and all closed-loop signals are upper bounded. Adaptive laws are embedded into the control schemes such that external disturbances and system uncertainties could be compensated. Moreover, the chattering issue inherent in sliding mode technologies could be tackled based on the hyperbolic tangent function. Upon utilizing the proposed method, rendezvous maneuver could be accomplished with satisfactory performance even when system parameters remain inaccessible to designers. Theoretical analysis and simulation results have been presented to illustrate the effectiveness of the proposed method.

Data Availability

No data were used to support this study.

Conflicts of Interest

Authors declare that there are no conflicts of interest regarding publication of this article.

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