Stats - Assignment - 1

1. mean $= \frac{\sum_{i=1}^{n} x_i}{n} = \frac{6+7+5+7+7+8+7+6+9+7+4+10+6+8+8+9+5+6+4+8}{20}$

$= 6.85$

median $= 5.5$

mode $= 7$

standar deviation $\sigma = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \mu)^2}{n}}$

$= \sqrt{\frac{(6-6.85)^2 + (7-6.85)^2 + \cdots (8-6.85)^2}{20}}$

$= 2.52$

2. mean $= 1881.5$

median $= 194$

mode $= 75$

3. $x = 0, 1, 2, 3, 4, 5$

$f(x) = 0.09, 0.15, 0.40, 0.25, 0.10, 0.01$

$\mu = \sum x f(x) = 2.15$

$\sigma^2 = \sum (x-\mu)^2 p(x)$

$= 1.20$

4. $P(d > 12.60) = \int_{12.6}^{\infty} f(x)\,dx = \int_{12.6}^{\infty} 20e^{-20(d-12.5)}\,dx = 0.135$

$$\therefore f(x) = \begin{cases} 0 \;; & \text{for } x < 12.5 \\ 1 - e^{-20(d-12.5)} \;; & \text{for } x \geq 12.5 \end{cases}$$

5. $N = 6,\; n = 2,\; p(\text{faulty}) = 0.30,\; q = 0.70$

$$p(n|N) = \frac{6!}{2!\,(4!)} \times (0.30)^2 \times (0.70)^4$$

$$= 0.324$$

6. $\mu_g = 8 \times \frac{75}{100} = 6, \quad \mu_B = 12 \times \frac{45}{100} = 5.4$

$$P(x=5) = P_g(x=5) + P_B(x=5)$$

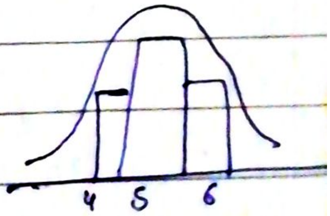
$$= \frac{e^{-6}\,6^5}{5!} + \frac{e^{-5.4}\,(5.4)^5}{5!}$$

$$= 0.33$$

$$P(x=4) = P_g(x=4) + P_B(x=4)$$

$$= \frac{e^{-6}(6)^4}{4!} + \frac{e^{-5.4}(5.4)^4}{4!}$$

$$= 0.29$$



$$P(x=6) = P_g(x=6) + P_B(x=6)$$

$$= \frac{e^{-6}(6)^6}{6!} + \frac{e^{-5.4}(5.4)^6}{6!}$$

$$= 0.31$$

Hence we can see it follows a normal distributⁿ.

7. A Customers arrive per min = $72/60 = 1.2$
Customers arrive in 4 min = $1.2 \times 4 = 4.8$

(i) $P(x = 5 \mid \mu = 4.8) = \dfrac{e^{-4.8} (4.8)^5}{5!}$ ~~0.008~~

$= \dfrac{15.18}{120}$

$= \underline{\underline{0.1265}}$

(ii) $P(x \leq 3 \mid \mu = 4.8) = \dfrac{e^{-4.8} (4.8)^1}{1!} + \dfrac{e^{-4.8} (4.8)^2}{2!} + \dfrac{e^{-4.8} (4.8)^3}{3!} + \dfrac{e^{-4.8} (4.8)^0}{0!}$

$= 0.039 + 0.09 + 0.151 + 0.008$

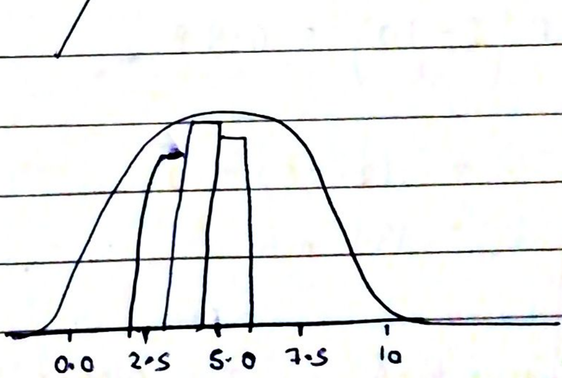
$= \underline{\underline{0.29}}$

(iii) $P(x \geq 3)$ $= \dfrac{e^{-4.8} (4.8)^4}{4!} + \dfrac{e^{-4.8} (4.8)^5}{5!}$

$= 1 - 0.29$ $= 0.81 + 0.17$

$= \underline{\underline{0.70}}$ $= 0.98$

pictorial representation -



It's given that he writes 77 words/ min having 6 error/hr

$\Rightarrow$ 77 X 60 words / hour

$\Rightarrow$ 4620 words / hour with 6 errors

$\Rightarrow$ 6 errors / 4620 words

$\Rightarrow$ 0.001 error / 1 word

$\mu_{455\,words} =$ 0.001 X 455 error / 455 words

0.59 error / 455 words

$$\therefore \; p(x=2) = \frac{e^{-0.59}(0.59)^2}{2!}$$

$$= \underline{\underline{0.09}}$$

$\mu_{1000\,words} =$ 0.001 X 1000 error / 1000 words

= 1 error / 1000 words

$$\therefore \; p(x=2) = \frac{e^{-1}(1)^2}{2!} = \underline{\underline{0.18}}$$

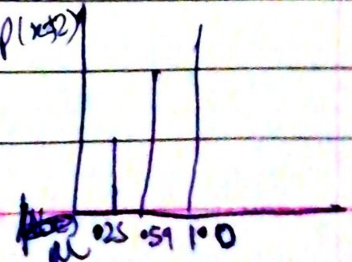
$\mu_{255\,words} =$ 0.001 X 255 error / 255 words

= 0.25 error / 255 words

$$p(x=2) = \frac{e^{-0.25}(0.25)^2}{2!} = \underline{\underline{0.02}}$$

$$\therefore \; \lambda_{255} < \lambda_{455} < \lambda_{1000}$$

hence with the increase in no. of words, probablity of the error increases



Problem 10

a) $P(Z > 1.26)$

$P(z > 1.26) = 1 - P(z \leq 1.26)$

$= 1 - .896 = 0.104$

$P(z < -0.86) = .1949$

$P(-1.25 < Z < 0.37) = P(Z < 0.37) - P(Z < -1.25)$

$= .6443 - .1056$

$= 0.5387$

b) $P(Z > z) = 0.05$

$z = -1.59$

c) $P(-z < Z < z) = 0.99$

Total probablity $= 1$

then $1 - 0.99 = 0.01$

devide by 2 , $\frac{0.01}{2} = 0.005$

$P(Z < 0.005) = .5000$

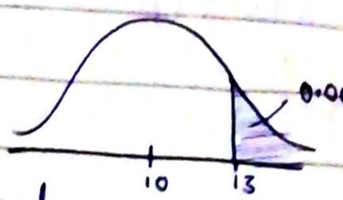
$\therefore P(-0.50 < Z < 0.50) = 0.99$

11. $\mu = 10mA$, $\sigma^2 = 4mA^2 \Rightarrow \sigma = 2mA^2$

$x = 13mA$

$P(x>13) = 1 - P\left(\frac{13-10}{2}\right) = 1 - P\left(\frac{3}{2}\right) = 1 - P(1.5) = 1 - 0.93$

$P(x>13) = 0.06$



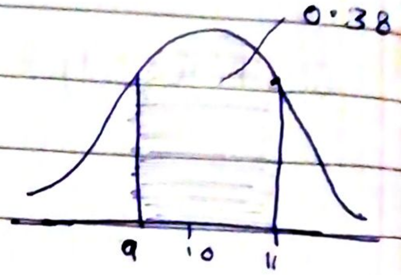
There is 6% probablity that current measure exceed 13

$P(9 < x < 11) = P(x<11) - P(x<9)$

$= P\left(\frac{11-10}{2}\right) - P\left(\frac{9-10}{2}\right)$

$= P(0.5) - P(-0.5)$

$= 0.38$



There is 38% probablity that current measurement in b/w 9 & 11 mA

$P(Z = z) = 0.98$

$P\left(\frac{z-10}{2}\right) = 0.98$

$x = 2(2.05) + 10$

$= 14.1 \text{ mA}$

12. $\mu = 0.25$, $\sigma = 0.0005$ inch

specification of the shaft are - 0.2500 ± 0.0015

$\therefore$ $(0.2500 - 0.0015) = 0.248$

$(0.2500 + 0.0015) = 0.251$

$\therefore P(0.248 < Z < 0.251) = P(z < 0.251) - P(Z \leq 0.248)$

$P(Z < 0.248) = P\left(Z < \dfrac{0.248 - 0.250}{0.0005}\right)$

$= P(Z < -1.5) = 0.06$

$\therefore P(0.0248 < Z < 0.251) = 0.97 - 0.06 = 0.91$

$\therefore$ 91% of shafts are in sync with specification

If $\mu = 0.2500$, then also 91% of shaft are in sync