#### R Notebook

Problem 1 (60 Points) 1. (0 pts) Download the data set Census Income Data for Adults along with its explanation. Note that the data file does not contain header names; you may wish to add those. The description of each column can be found in the data set explanation.

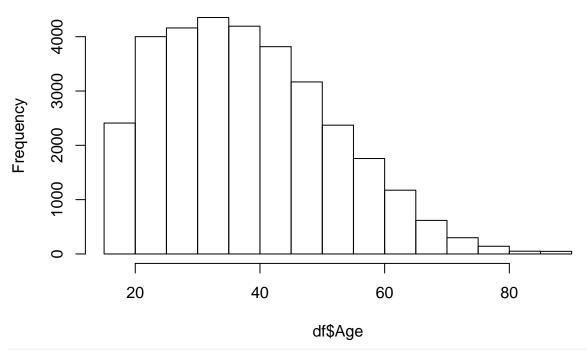
```
library(caret)
## Loading required package: lattice
## Loading required package: ggplot2
library(dplyr)
##
## Attaching package: 'dplyr'
## The following objects are masked from 'package:stats':
##
##
       filter, lag
## The following objects are masked from 'package:base':
##
       intersect, setdiff, setequal, union
#setwd('~/Documents/ML/')
df <- read.csv("adult.data.txt", header = F)</pre>
colnames(df) <- c("Age", "Workclass", "fnlwgt", "Education", "Ed-num", "Marital-status", "Occupation", "Relati</pre>
```

2. (0 pts) Explore the data set as you see fit and that allows you to get a sense of the data and get comfortable with it. Is there distributional skew in any of the features? Is there a need to apply a transform?

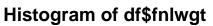
Yes, there is a distribution skew in Age, fnlwgt, Education numbers, Capital loss and capital gain. If we were going to use those features, they would have to be normalized and then converted to categorical features by binning. But since we are not going to use them for modeling, there isn't any need to transform them.

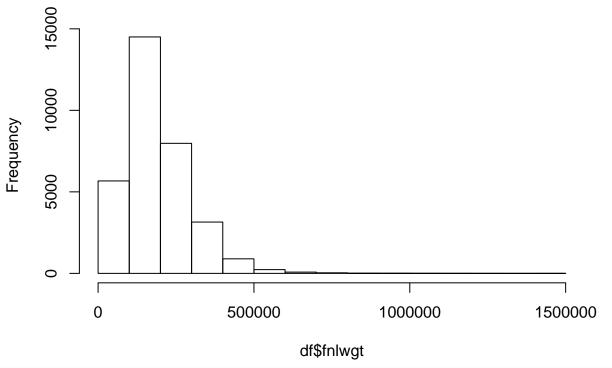
```
hist(df$Age)
```

# Histogram of df\$Age



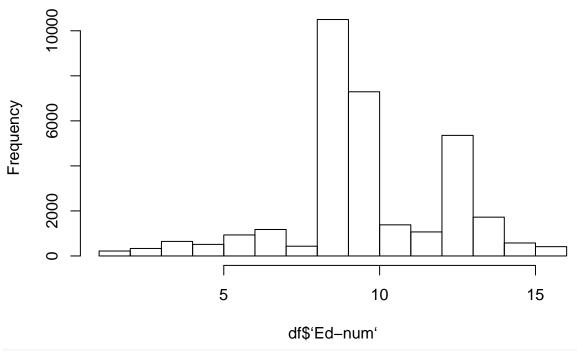
hist(df\$fnlwgt)





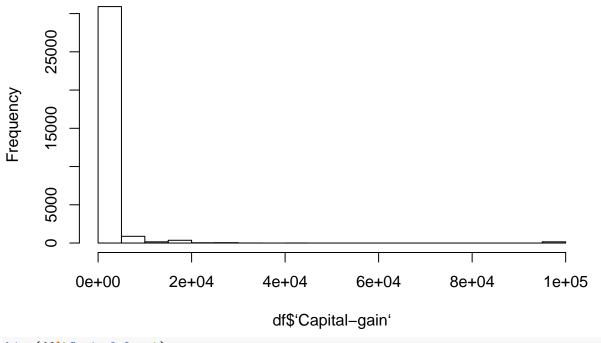
hist(df\$`Ed-num`)

# Histogram of df\$'Ed-num'



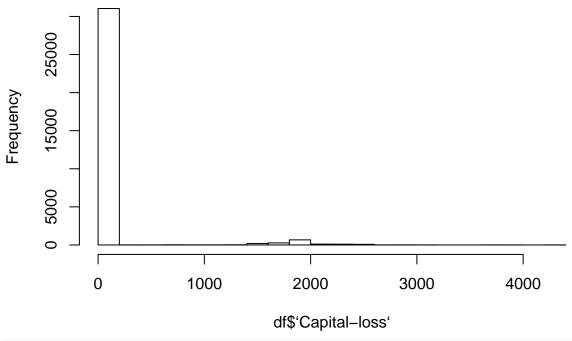
hist(df\$`Capital-gain`)

Histogram of df\$'Capital-gain'



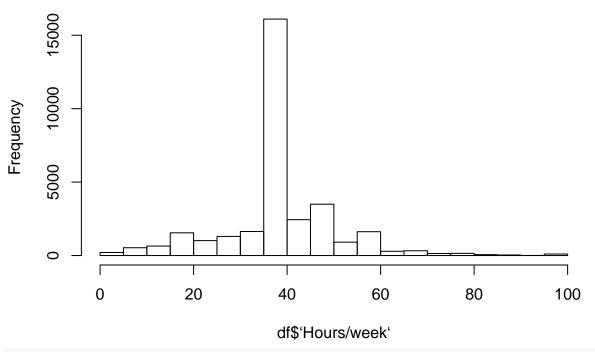
hist(df\$`Capital-loss`)

### Histogram of df\$'Capital-loss'



hist(df\$`Hours/week`)

#### Histogram of df\$'Hours/week'



table(df\$Workclass, df\$Class)

## <=50K >50K

```
1645
##
                                  191
                           589
                                  371
##
      Federal-gov
                           1476
                                  617
##
      Local-gov
##
      Never-worked
                             7
                                    0
##
      Private
                         17733
                                 4963
##
      Self-emp-inc
                           494
                                  622
##
      Self-emp-not-inc
                          1817
                                  724
##
                           945
                                  353
      State-gov
##
      Without-pay
                             14
table(df$Education)
##
##
            10th
                           11th
                                           12th
                                                      1st-4th
                                                                     5th-6th
##
             933
                            1175
                                           433
                                                           168
                                                                          333
##
         7th-8th
                             9th
                                    Assoc-acdm
                                                    Assoc-voc
                                                                   Bachelors
##
             646
                            514
                                           1067
                                                          1382
                                                                         5355
##
       Doctorate
                        HS-grad
                                       Masters
                                                    Preschool
                                                                 Prof-school
##
             413
                          10501
                                           1723
                                                            51
                                                                          576
    Some-college
##
##
            7291
table(df$`Marital-status`)
##
##
                  Divorced
                                 Married-AF-spouse
                                                        Married-civ-spouse
##
                      4443
                                                                      14976
##
    Married-spouse-absent
                                     Never-married
                                                                  Separated
##
                                              10683
                                                                        1025
##
                   Widowed
##
                       993
table(df$0ccupation)
##
                     ?
##
                              Adm-clerical
                                                  Armed-Forces
##
                  1843
                                      3770
##
         Craft-repair
                          Exec-managerial
                                               Farming-fishing
                  4099
                                      4066
##
                                                            994
                                                 Other-service
##
    Handlers-cleaners Machine-op-inspct
##
                  1370
                                      2002
                                                           3295
##
      Priv-house-serv
                           Prof-specialty
                                               Protective-serv
##
                   149
                                      4140
                                                            649
##
                 Sales
                              Tech-support
                                              Transport-moving
##
                  3650
                                       928
                                                           1597
table(df$Relationship)
##
##
           Husband
                      Not-in-family Other-relative
                                                             Own-child
##
              13193
                                                  981
                                                                  5068
                                8305
##
         Unmarried
                                Wife
##
              3446
                                1568
table(df$Race)
##
```

Black

Amer-Indian-Eskimo Asian-Pac-Islander

```
##
                    311
                                        1039
                                                              3124
##
                  Other
                                        White
##
                    271
                                        27816
table(df$Sex)
##
##
    Female
               Male
     10771
             21790
table(df$Native)
##
##
                               ?
                                                      Cambodia
##
                             583
                                                            19
                          Canada
                                                         China
##
##
                             121
                                                            75
                       Columbia
                                                          Cuba
##
##
                                                            95
##
            Dominican-Republic
                                                       Ecuador
##
                                                            28
##
                    El-Salvador
                                                       England
##
                             106
                                                            90
##
                         France
                                                       Germany
##
                              29
                                                           137
##
                         Greece
                                                     Guatemala
##
                              29
##
                           Haiti
                                           Holand-Netherlands
##
                             44
##
                       Honduras
                                                          Hong
##
                              13
                                                            20
##
                        Hungary
                                                         India
                                                           100
##
                             13
##
                           Iran
                                                       Ireland
##
                              43
##
                                                       Jamaica
                           Italy
##
                             73
                                                            81
##
                           Japan
                                                          Laos
##
                              62
                                                            18
##
                         Mexico
                                                    Nicaragua
##
                                                            34
```

## Poland Philippines ## 198 60 ## Portugal Puerto-Rico 37 ## 114 ## Scotland South ## 12 80 Taiwan Thailand ## ## ## Trinadad&Tobago United-States ## 19 29170

Outlying-US(Guam-USVI-etc)

##

##

##

Vietnam

Peru

31

```
table(df$Class)
##
   <=50K
##
            >50K
## 24720
            7841
# Many of the columns have "?" instead of values. They are missing values and will have to be imputed.
# Making function of mode
mode <- function(x){</pre>
  uniq <- unique(x)</pre>
  uniq[which.max(tabulate(match(x,uniq)))]
}
# First convert ? to NA
df[df == " ?"] <- NA
# Now replace with mode
df$Workclass[which(is.na(df$Workclass))] <- mode(df$Workclass)</pre>
df$Occupation[which(is.na(df$Occupation))] <- mode(df$Occupation)</pre>
df$Native[which(is.na(df$Native))] <- mode(df$Native)</pre>
```

3. (10 pts) Create a frequency and then a likelihood table for the categorical features in the data set. Build your own Naive Bayes classifier for those features.

```
# Creating frequency tables for all the features
WC <- table(df$Workclass, df$Class)</pre>
WC <- unclass(WC)</pre>
Ed <- unclass(table(df$Education, df$Class))</pre>
MS <- unclass(table(df$`Marital-status`, df$Class))
OC <- unclass(table(df$Occupation, df$Class))</pre>
Rel <- unclass(table(df$Relationship, df$Class))</pre>
Race <- unclass(table(df$Race, df$Class))</pre>
Sex <- unclass(table(df$Sex, df$Class))</pre>
NT <- unclass(table(df$Native,df$Class))</pre>
calc <- function(data, c1, c2){</pre>
  df <- data
  for(i in 1:nrow(df)){
    c1[i] <- data[i,2]/sum(data[i,1],data[i,2])</pre>
    c2[i] <- data[i,1]/sum(data[i,1],data[i,2])</pre>
  df <- cbind(data,c1,c2)</pre>
  colnames(df)[3] <- ">50K_11"
  colnames(df)[4] <- "<=50K_11"</pre>
}
freq <- function(data){</pre>
  c1 <- nrow(data)
  c2 <- nrow(data)
  calc(data,c1,c2)
# Adding likelihood to the frequency tables
```

```
WC_1 <- freq(WC)</pre>
Ed_1 <-freq(Ed)</pre>
MS_1 <- freq(MS)</pre>
OC_1 <- freq(OC)</pre>
Rel_1 <- freq(Rel)</pre>
Race_1 <- freq(Race)</pre>
Sex_1 <- freq(Sex)</pre>
NT 1 <- freq(NT)
# Transforming the data to display the levels of features as columns and Class as Rows
WC 1 \leftarrow t(WC 1)
Ed_1 <-t(Ed_1)
MS 1 \leftarrow t(MS 1)
OC_1 <- t(OC_1)</pre>
Rel_1 \leftarrow t(Rel_1)
Race_1 <- t(Race_1)</pre>
Sex_1 \leftarrow t(Sex_1)
NT_1 <- t(NT_1)
# Converting to dataframes
WC_1 <- as.data.frame(WC_1)</pre>
Ed_1 <- as.data.frame(Ed_1)</pre>
MS_1 <- as.data.frame(MS_1)</pre>
OC_1 <- as.data.frame(OC_1)</pre>
Rel_l <- as.data.frame(Rel_l)</pre>
Race_1 <- as.data.frame(Race_1)</pre>
Sex 1 <- as.data.frame(Sex 1)</pre>
NT_1 <- as.data.frame(NT_1)</pre>
# Probability of class being >50 or <=50K
t50 <- as.data.frame(unclass(table(df$Class)))
pg50 \leftarrow t50[2,1]/sum(t50[1,1],t50[2,1])
p150 \leftarrow t50[1,1]/sum(t50[1,1],t50[2,1])
# Build Naive Bayes Classifier for all the categorical features
naivebayes<-function(workclass,column,education,column1,occupation,column2,maritalstatus,</pre>
                      column3,relationship,column4,race,column5,sex,column6,nativecountry,column7)
  # the values in function are the likelihood tables of different features and the column which represe
  wg<-workclass[3,column] # probability of workclass of the case having income >50k
  wl<-workclass[4,column] # probability of workclass of the case having income <=50k
  eg<-education[3,column1] # probability of education of the case having income >50k
  el<-education[4,column1] # probability of education of the case having income <=50k
  og<-occupation[3,column2] # probability of the occupation of the case having income >50k
  ol<-occupation[4,column2] \# probability of the occupation of the case having income <=50k
  mg<-maritalstatus[3,column3] # probability of the maritalstatus of the case having income >50k
  ml<-maritalstatus[4,column3] # probability of the maritalstatus of the case having income <=50k
  rg<-relationship[3,column4] # probability of the relationship of the case having income >50k
```

```
rl<-relationship[4,column4] # probability of the relationship of the case having income <=50k
  rcg<-race[3,column5] # probability of the race of the case having income >50k
  rcl<-race[4,column5] # probability of the race of the case having income <=50k
  sg<-sex[3,column6] # probability of the sex of the case having income >50k
  sl<-sex[4,column6] # probability of the sex of the case having income <=50k
  ng<-nativecountry[3,column7] # probability of the native country of the case having income >50k
  nl<-nativecountry[4,column7] # probability of the native country of the case having income <=50k
  lik_g50<-c(wg,eg,og,mg,rg,rcg,sg,ng) # total likelihood of all features having income >50K
  lik_150<-c(wl,el,ol,ml,rcl,sl,nl) # total likelihood of all features having income <=50K
  p_more50<-prod(lik_g50) # Get the product of likelihood for all the features having income >50K
  p_less50<-prod(lik_150) # Get the product of likelihood for all the features having income <=50K
  less_50<-(p_less50*pl50) # Multiply the product of likelihood for all the features having income >50K
  more_50<-(p_more50*pg50) # Multiply the product of likelihood for all the features having income <=50
  final_prob_150<-less_50/(less_50+more_50) #final probability from conditional probability for the giv
  final_prob_150
}
```

4. (30 pts)Predict the binomial class membership for a white female adult who is a federal government worker with a bachelors degree who immigrated from India. Ignore any other features in your model. You must build your own Naive Bayes Classifier – you may not use a package.

The class membership for a white female adult who is a federal government worker with a bachelors degree who immigrated from India is  $\leq 50$ K'

 $\verb|naivebayes1| < -\texttt{function} (\texttt{workclass}, \texttt{column1}, \texttt{education}, \texttt{column2}, \texttt{race}, \texttt{column3}, \texttt{sex}, \texttt{column4}, \texttt{native} \texttt{country}, \texttt{column4}, \texttt{$ 

```
wg<-workclass[3,column1]
wl<-workclass[4,column2]
eg<-education[3,column2]
el<-education[4,column2]

rcg<-race[3,column3]
rcl<-race[4,column3]
sg<-sex[3,column4]
sl<-sex[4,column4]

ng<-nativecountry[3,column5]
nl<-nativecountry[4,column5]

lik_g50<-c(wg,eg,rcg,sg,ng)
lik_l50<-c(wl,el,rcl,sl,nl)

p_more50<-prod(lik_g50)
p_less50<-prod(lik_l50)</pre>
```

```
less_50<-(p_less50*p150)
  more_50<-(p_more50*pg50)
  final prob 150<-less 50/(less 50+more 50)
  final_prob_150
naivebayes1(WC 1,' Federal-gov', Ed 1,' Bachelors', Race 1,' White', Sex 1,' Female',
              NT 1, 'India')
##
       <=50K
## 0.7591904
# Since the probability of the unknown case having income <=50K is 0.759, we can classify it as having in
  5. (20 pts) Perform 10-fold cross validation on your algorithm to tune it and report the final accuracy
     results. The final accuracy is 75%
# This does not give the accuracy. This was my effort to do cross validation. I finally ended up using
# Probability of class being >50 or <=50K
t50 <- as.data.frame(unclass(table(df$Class)))
pg50 \leftarrow t50[2,1]/sum(t50[1,1],t50[2,1])
pl50 <- t50[1,1]/sum(t50[1,1],t50[2,1])
# To make 10-fold cross validations, we need 10 subsets of the data
set.seed(999)
index <- createFolds(df$Class, 10, list = T, returnTrain = F)</pre>
# Cross Validation
for(i in 1:10){
  train <- df[-index[[i]].]
  test <- df[index[[i]],]</pre>
  WC <- table(train$Workclass, train$Class)</pre>
  WC <- unclass(WC)
  Ed <- unclass(table(train$Education, train$Class))</pre>
  MS <- unclass(table(train$`Marital-status`, train$Class))</pre>
  OC <- unclass(table(train$Occupation, train$Class))</pre>
  Rel <- unclass(table(train$Relationship, train$Class))</pre>
  Race <- unclass(table(train$Race, train$Class))</pre>
  Sex <- unclass(table(train$Sex, train$Class))</pre>
  NT <- unclass(table(train$Native,train$Class))</pre>
calc <- function(data, c1, c2){</pre>
  train <- data
  for(i in 1:nrow(train)){
    c1[i] <- data[i,2]/sum(data[i,1],data[i,2])</pre>
    c2[i] <- data[i,1]/sum(data[i,1],data[i,2])</pre>
  train <- cbind(data,c1,c2)</pre>
  colnames(train)[3] <- ">50K 11"
  colnames(train)[4] <- "<=50K_11"</pre>
freq <- function(data){</pre>
  c1 <- nrow(data)</pre>
  c2 <- nrow(data)
```

calc(data,c1,c2)

```
# Adding likelihood to the frequency tables
WC_1 <- as.data.frame(t(freq(WC)))</pre>
Ed_l <-as.data.frame(t(freq(Ed)))</pre>
MS_1 <- as.data.frame(t(freq(MS)))</pre>
OC_1 <- as.data.frame(t(freq(OC)))</pre>
Rel_1 <- as.data.frame(t(freq(Rel)))</pre>
Race_1 <- as.data.frame(t(freq(Race)))</pre>
Sex_1 <- as.data.frame(t(freq(Sex)))</pre>
NT_1 <- as.data.frame(t(freq(NT)))</pre>
prob <- naivebayes1(WC_l,' Federal-gov',Ed_l,' Bachelors',Race_l,' White',Sex_l,' Female',</pre>
             NT_1,' India')
}
prob
##
       <=50K
## 0.7591904
# Since I couldn't figure out how to use test cases in my Naive Bayes function, I used Vaishnavi's code
#Download the data set Census Income Data for Adults along with its explanation. Explore the data set a
# getting the data
data <- file('http://archive.ics.uci.edu/ml/machine-learning-databases/adult/adult.data')</pre>
income_con <- read.table(data, fileEncoding="UTF-16", header = FALSE, sep = ',',</pre>
                         col.names = c('age', 'workclass', 'fnlwgt', 'education', 'education-num',
                                        'marital-status', 'occupation', 'relationship', 'race', 'sex',
                                        'capital-gain', 'capital-loss', 'hours-per-week',
                                        'native-country', 'income-level'), stringsAsFactors = FALSE)
# extracting the columns with categorical features
income_cat <- income_con[, c(2, 4, 6:10, 14:15)]
# transforming the income column as a factor feature
income_cat$income.level <- factor(income_cat$income.level)</pre>
# removing all the rows with missing values, represented as '?' rather than NA in data
fin_income <- income_cat[!(income_cat$workclass == ' ?' | income_cat$occupation == ' ?' |
                             income_cat$native.country == ' ?'), ]
# structure of the transformed dataset
str(fin_income)
## 'data.frame':
                    30162 obs. of 9 variables:
                    : chr " State-gov" " Self-emp-not-inc" " Private" " Private" ...
## $ workclass
                    : chr " Bachelors" " Bachelors" " HS-grad" " 11th" ...
## $ education
## $ marital.status: chr " Never-married" " Married-civ-spouse" " Divorced" " Married-civ-spouse" ...
## $ occupation : chr " Adm-clerical" " Exec-managerial" " Handlers-cleaners" " Handlers-cleaners"
                           " Not-in-family" " Husband" " Not-in-family" " Husband" ...
## $ relationship : chr
## $ race
                    : chr "White" "White" "Black" ...
```

```
: chr " Male" " Male" " Male" " Male" ...
## $ native.country: chr " United-States" " United-States" " United-States" " United-States" ...
## $ income.level : Factor w/ 2 levels " <=50K"," >50K": 1 1 1 1 1 1 1 2 2 2 ...
#Create a frequency and then a likelihood table for the categorical features in the data set. Build you
freq_tbl <- sapply(fin_income[-9], table, fin_income$income.level)</pre>
lap_est <- sapply(freq_tbl, function(x) {</pre>
 apply(x, 1, function(x) {
   x + 1))
# creating a likelihood table by dividing the count with the total sum of that column
lik_tbl <- sapply(lap_est, function(x) {</pre>
 apply(x, 1, function(x) {
   x / sum(x)})})
# taking a transform of the table to get in the naive bayes classifier form
lik_tbl <- lapply(lik_tbl, t)</pre>
head(lik_tbl)
## $workclass
##
            Federal-gov Local-gov Private Self-emp-inc Self-emp-not-inc
##
##
     <=50K
           0.02555051 0.06438374 0.7683244
                                              0.02096112
                                                               0.07881382
##
     >50K
             0.04870259 0.08117099 0.6489687
                                              0.07997339
                                                               0.09514305
##
##
            State-gov Without-pay
     <=50K 0.04130444 0.0006619302
##
     >50K 0.04590818 0.0001330672
##
##
## $education
##
##
                             11th
                  10t.h
                                        12th
                                                  1st-4th
                                                             5th-6th
##
     <=50K 0.033612704 0.043670049 0.015394795 0.0064402294 0.012218791
##
     >50K 0.007974482 0.007974482 0.003987241 0.0009303562 0.001727804
##
##
               7th-8th
                              9th Assoc-acdm Assoc-voc Bachelors
     <=50K 0.023070137 0.019011910 0.03321570 0.04252316 0.1287605
##
     ##
##
##
             Doctorate
                        HS-grad
                                  Masters
                                            Preschool Prof-school
##
     <=50K 0.004234671 0.3627702 0.03131892 0.002029113 0.006043229
     ##
##
##
            Some-college
##
     <=50K
               0.2356859
     >50K
               0.1776980
##
##
## $marital.status
##
##
             Divorced Married-AF-spouse Married-civ-spouse
##
     <=50K 0.16605622
                           0.0005295442
                                                 0.3383346
##
     >50K 0.06027944
                           0.0014637392
                                                 0.8516301
```

##

```
##
             Married-spouse-absent Never-married
                                                     Separated
                                        0.40849918 0.038568466 0.03300825
##
      <=50K
                        0.01500375
      >50K
                        0.00425815
                                        0.06267465 0.008915502 0.01077844
##
##
##
  $occupation
##
##
             Adm-clerical Armed-Forces Craft-repair Exec-managerial
                                             0.1377713
##
               0.14222693 0.0003970355
                                                             0.09070055
      <=50K
##
      >50K
               0.06633874 0.0002658867
                                             0.1208455
                                                              0.25764424
##
##
             Farming-fishing Handlers-cleaners Machine-op-inspct
##
      <=50K
                  0.03860067
                                      0.05593789
                                                         0.07596612
      >50K
                  0.01542143
                                      0.01116724
                                                         0.03270407
##
##
##
             Other-service Priv-house-serv Prof-specialty Protective-serv
##
      <=50K
                0.13591848
                               0.0063084524
                                                  0.09828834
                                                                    0.01919005
##
      >50K
                0.01768147
                               0.0002658867
                                                  0.24089338
                                                                    0.02805105
##
##
                Sales Tech-support Transport-moving
                         0.02801306
                                            0.05532028
##
      <=50K 0.1153609
##
      >50K 0.1290880
                         0.03709120
                                            0.04254188
##
## $relationship
##
##
              Husband Not-in-family Other-relative
                                                        Own-child Unmarried
##
      <=50K 0.2994263
                           0.3046778
                                          0.037731686 0.194307149 0.13239188
##
      >50K 0.7559223
                           0.1096620
                                          0.004791057 0.008650519 0.02848017
##
##
                  Wife
##
      <=50K 0.03146514
      >50K 0.09249401
##
##
## $race
##
             Amer-Indian-Eskimo Asian-Pac-Islander
##
                                                          Black
                                                                       Other
                                          0.02859791 0.10821307 0.009311973
##
      <=50K
                    0.011165541
##
      >50K
                    0.004658592
                                          0.03314255 0.04884866 0.002928258
##
##
                White
##
      <=50K 0.8427115
      >50K 0.9104219
# building a naive bayes classifier
# this classifier calculates the probabilites of a person's income being
# less than >50k
nb <- function(x) {</pre>
  # initializing all the required variables
  t1 <- 0
  t2 <- 0
 li.grt50 <- 0
  li.lss50 <- 0
 pr.1ss50 <- 0
```

```
z1 <- list()</pre>
  z2 <- list()</pre>
 y <- list()
 for (j in 1:nrow(x)) {
    y[[j]] <- colnames(x[j, ] %>% select_if(~!any(is.na(.))))
 for (n in 1:nrow(x)) {
    for (k in 1:length(y[[n]])) {
      t1[k] <- lik_tbl[[y[[n]][k]]][1, x[n, y[[n]][k]]]
    z1[[n]] <- t1
  }
  # similarly, getting the likelihood values for income >50k, again by feeding the
  # column names
  for (n in 1:nrow(x)) {
   for (k in 1:length(y[[n]])) {
      t2[k] <- lik_tbl[[y[[n]][k]]][2, x[n, y[[n]][k]]]
    }
    z2[[n]] \leftarrow t2
  for (m in 1:length(z1)) {
    li.lss50[m] <- prod(z1[[m]])
 for (l in 1:length(z2)) {
   li.grt50[1] <- prod(z2[[1]])
 for (q in 1:nrow(x)) {
    pr.lss50[q] \leftarrow li.lss50[q]/(li.grt50[q] + li.lss50[q])
 return(pr.lss50)
}
#Predict the binomial class membership for a white female adult who is a federal government worker with
# the test case
test <- fin_income[0,-9]</pre>
test[1, ] <- c(' Federal-gov', ' Bachelors', NA, NA, NA, ' White', ' Female', ' India')
# prediciting the binomial class membership for the given case
nb(test)
```

## [1] 0.2203777

```
fin_income2 <- fin_income</pre>
fin_income2$income.level <- if_else(fin_income$income.level == ' >50K', 0, 1)
# predicting the probability of people earning <=50k</pre>
#nb.pred <- nb(fin_income2[-9])</pre>
# person with probability >0.5 is determined to be earning >50k
#nb.pred_class <- ifelse(nb.pred > 0.50, 1, 0)
# checking the accuracy of algorithm
#confusionMatrix(nb.pred_class, fin_income2$income.level)
# cross validation for the predictions
# naive bayes classifier function for cross calidation
nb.cv <- function(x) {</pre>
  # initializing all the required variables
 t1 <- 0
  t2 <- 0
 li.grt50 <- 0
 li.lss50 <- 0
  pr.1ss50 <- 0
  z1 <- list()</pre>
  z2 <- list()</pre>
  y <- list()
  for (j in 1:nrow(x)) {
    y[[j]] <- colnames(x[j, ] %>% select_if(~ !any(is.na(.))))
  # getting the likelihood values for the case of income <=50k for each row for the
  # value it has
  for (n in 1:nrow(x)) {
    for (k in 1:length(y[[n]])) {
      t1[k] <- lik_tbl_cv[[y[[n]][k]]][1, x[n, y[[n]][k]]]</pre>
    z1[[n]] \leftarrow t1
  # similarly, getting the likelihood values for income >50k, again by feeding the
  # column names
  for (n in 1:nrow(x)) {
    for (k in 1:length(y[[n]])) {
      t2[k] <- lik_tbl_cv[[y[[n]][k]]][2, x[n, y[[n]][k]]]
    }
    z2[[n]] \leftarrow t2
```

```
# calculating the overall likelihood value by multiplying the individual likelihoods
  \# when income <=50k
  for (m in 1:length(z1)) {
   li.lss50[m] <- prod(z1[[m]])
  # calculating the overall likelihood value by multiplying the individual likelihoods
  # when income >50k
  for (1 in 1:length(z2)) {
    li.grt50[1] <- prod(z2[[1]])
  # transforming the likelihood into probability by dividing with total likelihood
  for (q in 1:nrow(x)) {
    pr.lss50[q] <- li.lss50[q]/(li.grt50[q] + li.lss50[q])</pre>
  return(pr.1ss50)
# initialize the accuracy vector
accuracy <- rep(0,6)
for (i in 1:6) {
  # indices indicate the interval of the test set
  indices \leftarrow (((i-1) * round((1/10) * nrow(fin_income2))) + 1):((i*round((1/10) * nrow(fin_income2))))
  # training set
  training <- fin_income[-indices,]</pre>
  # test set
  testing <- fin_income2[indices,]</pre>
  # building a frequency and a likelihood table from training set
  freq_tbl_cv <- sapply(training[-9], table, training$income.level)</pre>
  lap_est_cv <- sapply(freq_tbl_cv, function(x) {</pre>
    apply(x, 1, function(x) {
      x + 1))
  lik_tbl_cv <- sapply(lap_est_cv, function(x) {</pre>
    apply(x, 1, function(x) {
      x / sum(x)))
  lik_tbl_cv <- lapply(lik_tbl_cv, t)</pre>
  # make predictions on the test set using the nb.cv function that takes likelihood
  # values from training set
  nb.cv_pred <- nb.cv(testing[-9])</pre>
  nb.cv_pred_class <- ifelse(nb.cv_pred > 0.50, 1, 0)
  # generate the confusion matrix
  conf_mat <- table(testing$income.level, nb.cv_pred_class)</pre>
```

```
# assigning the accuracy of this model to the vector
 accuracy[i] <- sum(diag(conf_mat))/sum(conf_mat)</pre>
accuracy
## [1] 0.7529841 0.7443634 0.7536472 0.7549735 0.7430371 0.7500000
# mean of accuracies
mean(accuracy)
## [1] 0.7498342
Problem 2 (25 Points)
require(rlang)
## Loading required package: rlang
library(readxl)
require(ggplot2)
require(car)
## Loading required package: car
## Loading required package: carData
##
## Attaching package: 'car'
## The following object is masked from 'package:dplyr':
##
##
       recode
#install.packages('corrplot')
require(corrplot)
## Loading required package: corrplot
## corrplot 0.84 loaded
require(psych)
## Loading required package: psych
## Attaching package: 'psych'
## The following object is masked from 'package:car':
##
##
       logit
## The following objects are masked from 'package:ggplot2':
##
       %+%, alpha
##
#install.packages('rms')
require(rms)
## Loading required package: rms
## Loading required package: Hmisc
```

```
## Loading required package: survival
##
## Attaching package: 'survival'
## The following object is masked from 'package:caret':
##
##
       cluster
## Loading required package: Formula
##
## Attaching package: 'Hmisc'
## The following object is masked from 'package:psych':
##
##
       describe
## The following objects are masked from 'package:dplyr':
##
##
       src, summarize
## The following objects are masked from 'package:base':
##
##
       format.pval, units
## Loading required package: SparseM
##
## Attaching package: 'SparseM'
## The following object is masked from 'package:base':
##
       backsolve
##
## Attaching package: 'rms'
## The following objects are masked from 'package:car':
##
       Predict, vif
#install.packages('sqldf')
require(sqldf)
## Loading required package: sqldf
## Loading required package: gsubfn
## Loading required package: proto
## Loading required package: RSQLite
require(reshape2)
## Loading required package: reshape2
#install.packages('mice')
require(mice)
## Loading required package: mice
##
## Attaching package: 'mice'
```

```
## The following objects are masked from 'package:base':
##
      cbind, rbind
##
#install.packages('qmodels')
require(gmodels)
## Loading required package: gmodels
require(e1071)
## Loading required package: e1071
##
## Attaching package: 'e1071'
## The following object is masked from 'package:Hmisc':
##
##
      impute
uff_raw <- as.data.frame(read_excel('uffidata.xlsx'))</pre>
names(uff_raw) <- gsub(x = names(uff_raw), pattern = " ", replacement = "_")</pre>
colnames(uff_raw)[6] <- "Yrs45"</pre>
uff_raw <- uff_raw[-c(1)]
# Analyze the data
str(uff_raw)
## 'data.frame':
                   99 obs. of 11 variables:
                   : num 2009 2009 2011 2011 2010 ...
   $ Year_Sold
## $ Sale_Price
                   : num 76900 78000 79000 80000 82000 84000 84000 84000 85000 85000 ...
## $ UFFI_IN
                          1 1 0 0 1 1 0 0 0 1 ...
                   : num
## $ Brick_Ext
                          0 0 0 0 0 0 0 0 0 1 ...
                   : num
## $ Yrs45
                   : num
                         1 1 1 1 1 1 1 1 1 1 ...
## $ Bsmnt_Fin_SF : num
                          0 154 400 0 157 ...
## $ Lot_Area
                   : num
                          2772 4490 5840 5040 5441 ...
## $ Enc_Pk_Spaces : num 0 0 0 0 0 1 2 0 0 1 ...
                          1018 536 721 513 672 ...
## $ Living_Area_SF: num
## $ Central Air : num
                          0 1 1 0 0 0 0 0 1 0 ...
## $ Pool
                    : num 0000000000...
summary(uff_raw)
##
     Year Sold
                    Sale_Price
                                      UFFI IN
                                                      Brick Ext
##
          :2009
                  Min. : 76900
                                          :0.0000
                                                           :0.0000
  Min.
                                   Min.
                                                   \mathtt{Min}.
  1st Qu.:2011
                  1st Qu.:102000
                                   1st Qu.:0.0000
                                                    1st Qu.:0.0000
## Median :2012
                  Median :115000
                                   Median :0.0000
                                                    Median :0.0000
## Mean
         :2013
                  Mean
                        :124450
                                   Mean
                                          :0.2323
                                                    Mean
                                                           :0.3939
##
   3rd Qu.:2015
                  3rd Qu.:135000
                                   3rd Qu.:0.0000
                                                    3rd Qu.:1.0000
##
  Max.
          :2016
                  Max. :347000
                                   Max.
                                          :1.0000
                                                    Max.
                                                           :1.0000
##
       Yrs45
                     Bsmnt_Fin_SF
                                       Lot_Area
                                                    Enc_Pk_Spaces
## Min.
          :0.0000 Min.
                          : 0.0
                                          : 1800
                                                           :0.0000
                                    Min.
                                                    Min.
## 1st Qu.:1.0000
                    1st Qu.: 0.0
                                    1st Qu.: 4376
                                                    1st Qu.:0.0000
## Median :1.0000
                    Median :248.8
                                    Median: 5205
                                                    Median :1.0000
## Mean :0.8182
                    Mean :248.0
                                    Mean : 5709
                                                    Mean :0.8081
```

```
##
    3rd Qu.:1.0000
                      3rd Qu.:387.2
                                       3rd Qu.: 6509
                                                        3rd Qu.:1.0000
##
           :1.0000
                                       Max.
                                              :11650
                                                               :2.0000
    Max.
                      Max.
                             :915.1
                                                        Max.
##
    Living_Area_SF
                       Central_Air
                                             Pool
           : 431.9
                             :0.0000
                                                :0.0000
##
    Min.
                      Min.
                                        Min.
##
    1st Qu.: 628.5
                      1st Qu.:0.0000
                                        1st Qu.:0.0000
    Median : 750.3
                      Median :1.0000
                                        Median :0.0000
##
           : 858.4
                             :0.5758
                                               :0.0303
##
    Mean
                      Mean
                                        Mean
##
    3rd Qu.:1022.1
                      3rd Qu.:1.0000
                                        3rd Qu.:0.0000
           :2338.7
##
    Max.
                      Max.
                             :1.0000
                                        Max.
                                               :1.0000
OutVals_Liv = boxplot(uff_raw$Living_Area_SF)$out
                                           0
                                           0
                                           0
1500
1000
which(uff_raw$Living_Area_SF %in% OutVals_Liv)
## [1] 95 98 99
OutVals_Lot = boxplot(uff_raw$Lot_Area)$out
                                           8
                                           0
which(uff_raw$Lot_Area %in% OutVals_Lot)
```

## [1] 52 77 84 93 98 99

After reading the case study background information, using the UFFI data set, answer these questions: 1.

(5 pts) Are there outliers in the data set? How do you identify outliers and how do you deal with them? Remove them but create a second data set with outliers removed. Keep the original data set.

```
#As there are multiple feature and imapcting the dependent variable, we shall use the Cook's distance t
#Reference: http://r-statistics.co/Outlier-Treatment-With-R.html

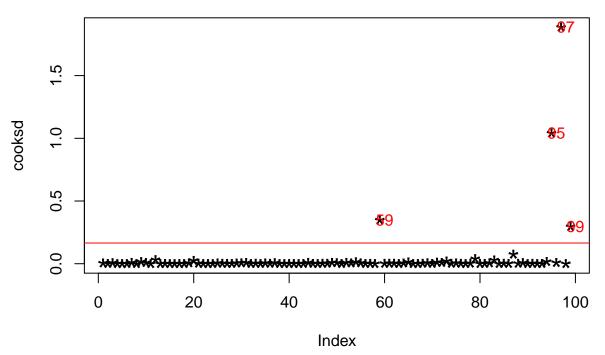
Linear_Model_outlier <- lm(Sale_Price ~ ., data=uff_raw)

cooksd <- cooks.distance(Linear_Model_outlier)

plot(cooksd, pch="*", cex=2, main="Influential Obs by Cooks distance")
abline(h = 4*mean(cooksd, na.rm=T), col="red")

text(x=1:length(cooksd)+1, y=cooksd, labels=ifelse(cooksd>4*mean(cooksd, na.rm=T),names(cooksd),""), co
```

#### Influential Obs by Cooks distance



influential\_outliers <- as.numeric(names(cooksd)[(cooksd > 4\*mean(cooksd, na.rm=T))])
head(uff\_raw[influential\_outliers, ])

```
Year_Sold Sale_Price UFFI_IN Brick_Ext Yrs45 Bsmnt_Fin_SF Lot_Area
##
## 59
            2010
                      121500
                                    0
                                               0
                                                      0
                                                              516.44
                                                                           6500
## 95
            2016
                     200000
                                    0
                                               0
                                                      1
                                                              260.40
                                                                          5544
## 97
            2016
                     347000
                                    0
                                               1
                                                      0
                                                              441.50
                                                                          8190
            2010
                     250000
                                                                0.00
                                                                         11200
## 99
                                    1
                                               1
##
      Enc_Pk_Spaces Living_Area_SF Central_Air Pool
## 59
                   0
                             581.930
## 95
                   2
                            1792.630
                                                 0
                                                       1
                   2
## 97
                            1279.690
                                                 1
                                                       1
## 99
                   2
                            2042.947
car::outlierTest(Linear_Model_outlier)
```

## rstudent unadjusted p-value Bonferonni p ## 97 7.504580 4.9708e-11 4.9211e-09

```
## 95 -4.241616 5.5185e-05 5.4633e-03
```

 $\#From\ the\ observations\ from\ the\ box\ plot,\ linear\ model\ and\ the\ outlier\ test\ we\ will\ eliminate\ obervation uff\_wo\_outliers <-\ uff\_raw[-c(95,97,99),]$ 

2. (2 pts) What are the correlations to the response variable and are there colinearities? Build a full correlation matrix.

```
## 2.Colinearility
correlation <- cor(uff_wo_outliers)</pre>
correlation
##
                  Year_Sold
                             Sale_Price
                                           UFFI_IN
                                                     Brick_Ext
                 1.00000000 0.680124035 -0.202158774
## Year_Sold
                                                    0.212793751
## Sale_Price
                 0.68012403 1.000000000 -0.206732966
                                                    0.156368654
## UFFI IN
                -0.20215877 -0.206732966 1.000000000 -0.024400783
## Brick Ext
                 ## Yrs45
                -0.12564874 -0.148695245 0.058161185 -0.193313916
## Bsmnt_Fin_SF
                 ## Lot_Area
                 0.30503947 \quad 0.417820600 \quad 0.126972876 \quad -0.045619814
## Enc_Pk_Spaces
                 ## Living_Area_SF 0.38191229 0.740844800 0.002296263 0.121199401
                 0.06451258 \quad 0.226899880 \ -0.030271052 \ -0.008563543
## Central Air
## Pool
                -0.11376804 0.004998989 -0.055941445 -0.081248070
##
                      Yrs45 Bsmnt_Fin_SF
                                          Lot_Area Enc_Pk_Spaces
## Year_Sold
                -0.125648742 0.083923242
                                        0.30503947
                                                     0.253447860
## Sale_Price
                -0.148695245 0.131863976 0.41782060
                                                    0.424429425
## UFFI IN
                 0.058161185 -0.039754599 0.12697288 -0.146655065
## Brick Ext
                -0.193313916 -0.083720690 -0.04561981 -0.080654605
## Yrs45
                 1.000000000 -0.461996984 -0.34571978
                                                    0.004249156
## Bsmnt_Fin_SF
                -0.461996984 1.000000000 0.25180102 -0.006314933
## Lot_Area
                -0.345719776  0.251801015  1.00000000
                                                    0.219400345
## Enc_Pk_Spaces
                 0.004249156 -0.006314933 0.21940035
                                                     1.00000000
## Living_Area_SF 0.006167127 -0.058758108 0.33679970
                                                     0.303749439
## Central_Air
                -0.124694832 0.304762289 0.26932099
                                                     0.113460188
## Pool
                -0.221170542   0.114792966   0.04536803   -0.118225345
##
                Living_Area_SF Central_Air
                                                 Pool
## Year_Sold
                   ## Sale_Price
                   0.740844800 0.226899880 0.004998989
## UFFI_IN
                   0.002296263 -0.030271052 -0.055941445
## Brick_Ext
                   0.121199401 -0.008563543 -0.081248070
## Yrs45
                   0.006167127 -0.124694832 -0.221170542
```

0.336799702 0.269320986 0.045368025

1.000000000 0.176247299 -0.085221981

0.176247299 1.000000000 0.088582673

## No id variables; using all as measure variables

## Bsmnt\_Fin\_SF

## Enc\_Pk\_Spaces

## Living\_Area\_SF

## Central Air

## Lot\_Area

```
correlation$absvalue <- abs(correlation$value)</pre>
sqldf("select * from correlation order by absvalue desc")
##
              variable
                                 value
                                           absvalue
## 1
           Sale_Price
                         1.000000000 1.000000000
## 2
      Living_Area_SF
                         0.740844800 0.740844800
## 3
            Year_Sold
                         0.680124035 0.680124035
## 4
        Enc_Pk_Spaces
                         0.424429425 0.424429425
## 5
              Lot_Area
                         0.417820600 0.417820600
## 6
          Central Air
                         0.226899880 0.226899880
## 7
               UFFI_IN -0.206732966 0.206732966
## 8
            Brick Ext 0.156368654 0.156368654
## 9
                 Yrs45 -0.148695245 0.148695245
## 10
         Bsmnt_Fin_SF
                         0.131863976 0.131863976
## 11
                  Pool
                        0.004998989 0.004998989
pairs.panels(uff_wo_outliers)
          100000
                           0.0 0.8
                                           0 600
                                                           0.0 1.5
                                                                           0.0 0.8
   Year_Solo
                             0.21
                     -0.20
                                     -0.13
                                                     0.31
                                                                     0.38
             0.68
                                             0.08
                                                             0.25
                                                                             0.06
                     -0.21
                             0.16
                                     -0.15
                                             0.13
                                                     0.42
                                                             0.42
                                                                     0.74
                                                                             0.23
                                                                                     0.00
                             -0.02
                                             -0.04
                                                     0.13
                                                             -0.15
                                                                     0.00
                                     0.06
                                                                             -0.03
                                                                                     -0.06
                                     -0.19
                                             -0.08
                                                     -0.05
                                                             -0.08
                                                                     0.12
                                                                             -0.01
                                                                                     -0.08
                                             -0.46
                                                                                     -0.22
                                                     -0.35
                                                             0.00
                                                                     0.01
                                                                             -0.12
                                                                                            0.0
                                                     0.25
                                                             -0.01
                                                                     -0.06
                                                                             0.30
                                                                                     0.11
                                                             0.22
                                                                     0.34
                                                                             0.27
                                                                                     0.05
                                                                     0.30
                                                                             0.11
                                                                                     -0.12
                                                                             0.18
                                                                                     -0.09
                                                                                     0.09
  2009
                  0.0
                      0.8
                                  0.0 0.8
                                                  2000
                                                                   500
                                                                                   0.0
                                                                                      0.8
  3. (10 pts) What is the ideal multiple regression model for predicting home prices in this data set using the
     data set with outliers removed? Provide a detailed analysis of the model, including Adjusted R-Squared,
     RMSE, and p-values of principal components. Use backward elimination by p-value to build the model.
## 3.Model
Linear_Model_reg <- lm(Sale_Price ~ ., data=uff_wo_outliers)</pre>
summary(Linear_Model_reg)
##
## Call:
   lm(formula = Sale_Price ~ ., data = uff_wo_outliers)
##
##
```

## Residuals:

```
Min
             1Q Median
                           3Q
                                 Max
## -37226 -8085
                         9009 57418
                   340
##
## Coefficients:
##
                   Estimate Std. Error t value Pr(>|t|)
## (Intercept)
                 -1.066e+07 1.591e+06 -6.703 2.11e-09 ***
## Year Sold
                 5.329e+03 7.918e+02
                                        6.730 1.87e-09 ***
## UFFI_IN
                 -7.497e+03 3.751e+03 -1.999 0.04882 *
## Brick_Ext
                  2.254e+03 3.341e+03
                                        0.675 0.50179
## Yrs45
                  1.959e+01 4.896e+03
                                         0.004 0.99682
## Bsmnt_Fin_SF
                  1.148e+01 7.467e+00
                                         1.537 0.12796
## Lot_Area
                  1.018e+00 9.348e-01
                                         1.089 0.27902
                  6.514e+03 2.428e+03
## Enc_Pk_Spaces
                                        2.683 0.00876 **
## Living_Area_SF 5.257e+01 5.777e+00
                                        9.100 3.39e-14 ***
## Central_Air
                  2.143e+03 3.272e+03
                                         0.655 0.51438
## Pool
                  2.730e+04 1.532e+04
                                         1.782 0.07836 .
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 14440 on 85 degrees of freedom
## Multiple R-squared: 0.7953, Adjusted R-squared: 0.7712
## F-statistic: 33.02 on 10 and 85 DF, p-value: < 2.2e-16
#Removing Yrs45
Linear_Model_reg_1 <- lm(Sale_Price ~ Year_Sold + UFFI_IN + Brick_Ext + Bsmnt_Fin_SF + Lot_Area + Enc_P
summary(Linear_Model_reg_1)
##
## lm(formula = Sale_Price ~ Year_Sold + UFFI_IN + Brick_Ext + Bsmnt_Fin_SF +
      Lot_Area + Enc_Pk_Spaces + Living_Area_SF + Central_Air +
##
      Pool, data = uff_wo_outliers)
##
## Residuals:
             10 Median
     Min
                           3Q
                                 Max
## -37219 -8087
                   338
                         9010 57421
##
## Coefficients:
##
                   Estimate Std. Error t value Pr(>|t|)
## (Intercept)
                 -1.066e+07 1.581e+06 -6.743 1.69e-09 ***
## Year_Sold
                  5.329e+03 7.871e+02
                                        6.771 1.49e-09 ***
## UFFI IN
                 -7.496e+03 3.715e+03 -2.017 0.04677 *
## Brick_Ext
                  2.250e+03 3.161e+03
                                       0.712 0.47857
## Bsmnt_Fin_SF
                  1.147e+01 6.729e+00
                                        1.704 0.09201 .
## Lot_Area
                  1.017e+00 8.814e-01
                                         1.154 0.25165
## Enc_Pk_Spaces
                                         2.699 0.00837 **
                  6.514e+03 2.414e+03
## Living_Area_SF 5.257e+01 5.720e+00
                                         9.191 2.01e-14 ***
## Central_Air
                  2.144e+03 3.239e+03
                                         0.662 0.50974
## Pool
                  2.729e+04 1.489e+04
                                         1.833 0.07026 .
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 14360 on 86 degrees of freedom
## Multiple R-squared: 0.7953, Adjusted R-squared: 0.7738
## F-statistic: 37.12 on 9 and 86 DF, p-value: < 2.2e-16
```

```
#Removing Central_Air
Linear_Model_reg_2 <- lm(Sale_Price ~ Year_Sold + UFFI_IN + Brick_Ext + Bsmnt_Fin_SF + Lot_Area + Enc_P
summary(Linear Model reg 2)
##
## Call:
## lm(formula = Sale_Price ~ Year_Sold + UFFI_IN + Brick_Ext + Bsmnt_Fin_SF +
      Lot_Area + Enc_Pk_Spaces + Living_Area_SF + Pool, data = uff_wo_outliers)
##
## Residuals:
             10 Median
     \mathtt{Min}
                           3Q
                                 Max
## -38574 -8228
                   165
                         8323
                               56184
## Coefficients:
##
                   Estimate Std. Error t value Pr(>|t|)
                 -1.057e+07 1.570e+06 -6.733 1.70e-09 ***
## (Intercept)
## Year_Sold
                  5.285e+03 7.816e+02
                                        6.761 1.50e-09 ***
## UFFI_IN
                 -7.616e+03 3.699e+03 -2.059 0.04250 *
## Brick_Ext
                  2.318e+03 3.149e+03
                                        0.736 0.46367
## Bsmnt_Fin_SF
                  1.268e+01 6.455e+00
                                        1.964 0.05274 .
                                        1.277 0.20507
## Lot_Area
                  1.108e+00 8.678e-01
## Enc_Pk_Spaces
                  6.605e+03 2.402e+03
                                        2.750 0.00725 **
## Living_Area_SF 5.312e+01 5.642e+00 9.415 6.34e-15 ***
## Pool
                  2.790e+04 1.481e+04 1.884 0.06290 .
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 14310 on 87 degrees of freedom
## Multiple R-squared: 0.7942, Adjusted R-squared: 0.7753
## F-statistic: 41.97 on 8 and 87 DF, p-value: < 2.2e-16
#Removing Brick_Ext
Linear_Model_reg_3 <- lm(Sale_Price ~ Year_Sold + UFFI_IN + Bsmnt_Fin_SF + Lot_Area + Enc_Pk_Spaces + L
summary(Linear_Model_reg_3)
##
## Call:
## lm(formula = Sale_Price ~ Year_Sold + UFFI_IN + Bsmnt_Fin_SF +
      Lot_Area + Enc_Pk_Spaces + Living_Area_SF + Pool, data = uff_wo_outliers)
##
## Residuals:
     Min
             1Q Median
                           3Q
                                 Max
## -39949 -8070
                    75
                         7681
                               55390
##
## Coefficients:
                   Estimate Std. Error t value Pr(>|t|)
                 -1.083e+07 1.529e+06 -7.082 3.33e-10 ***
## (Intercept)
## Year_Sold
                  5.410e+03 7.607e+02
                                        7.112 2.89e-10 ***
## UFFI_IN
                 -7.589e+03 3.689e+03 -2.057 0.04264 *
## Bsmnt_Fin_SF
                  1.236e+01 6.423e+00
                                        1.924 0.05760 .
## Lot_Area
                  1.049e+00 8.619e-01
                                        1.218 0.22667
## Enc_Pk_Spaces
                  6.341e+03 2.369e+03
                                         2.677 0.00886 **
## Living_Area_SF 5.350e+01 5.604e+00 9.546 3.08e-15 ***
## Pool
                                         1.854 0.06713 .
                  2.735e+04 1.475e+04
## ---
```

```
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 14280 on 88 degrees of freedom
## Multiple R-squared: 0.7929, Adjusted R-squared: 0.7765
## F-statistic: 48.14 on 7 and 88 DF, p-value: < 2.2e-16
#Removing Lot_Area
Linear_Model_reg_4 <- lm(Sale_Price ~ Year_Sold + UFFI_IN + Bsmnt_Fin_SF + Enc_Pk_Spaces + Living_Area_
summary(Linear_Model_reg_4)
##
## Call:
## lm(formula = Sale_Price ~ Year_Sold + UFFI_IN + Bsmnt_Fin_SF +
       Enc_Pk_Spaces + Living_Area_SF + Pool, data = uff_wo_outliers)
##
## Residuals:
     Min
             1Q Median
                            3Q
                                 Max
## -41956 -9421
                  1138
                         8192 54924
##
## Coefficients:
##
                   Estimate Std. Error t value Pr(>|t|)
                 -1.122e+07 1.498e+06 -7.488 4.83e-11 ***
## (Intercept)
## Year Sold
                  5.607e+03 7.453e+02
                                         7.523 4.09e-11 ***
## UFFI_IN
                 -6.585e+03 3.606e+03 -1.826 0.07115 .
## Bsmnt_Fin_SF
                                        2.339 0.02160 *
                  1.449e+01 6.197e+00
## Enc_Pk_Spaces
                  6.762e+03
                             2.350e+03
                                         2.878 0.00501 **
## Living_Area_SF 5.512e+01 5.457e+00 10.101 < 2e-16 ***
                                        1.981 0.05068 .
                  2.915e+04 1.472e+04
## Pool
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 14310 on 89 degrees of freedom
## Multiple R-squared: 0.7894, Adjusted R-squared: 0.7753
## F-statistic: 55.62 on 6 and 89 DF, p-value: < 2.2e-16
Linear_Model_reg_final <- lm(formula=Sale_Price ~ Year_Sold + UFFI_IN + Bsmnt_Fin_SF + Enc_Pk_Spaces + :
summary(Linear_Model_reg_final)
##
## Call:
## lm(formula = Sale_Price ~ Year_Sold + UFFI_IN + Bsmnt_Fin_SF +
##
      Enc_Pk_Spaces + Living_Area_SF + Pool, data = uff_wo_outliers)
##
## Residuals:
##
     Min
             1Q Median
                           30
                                 Max
## -41956 -9421
                   1138
                         8192 54924
##
## Coefficients:
                   Estimate Std. Error t value Pr(>|t|)
```

7.523 4.09e-11 \*\*\*

-1.826 0.07115 .

2.339 0.02160 \*

2.878 0.00501 \*\*

-1.122e+07 1.498e+06 -7.488 4.83e-11 \*\*\*

3.606e+03

6.197e+00

2.350e+03

5.607e+03 7.453e+02

## Living\_Area\_SF 5.512e+01 5.457e+00 10.101 < 2e-16 \*\*\*

-6.585e+03

1.449e+01

6.762e+03

## (Intercept)

## Bsmnt\_Fin\_SF

## Enc\_Pk\_Spaces

## Year\_Sold

## UFFI\_IN

## [1] 13782.9

4. (3 pts) On average, by how much do we expect UFFI to change the value of a property?

```
# 4.Due to the significance of UFFI index, for every unit of UFFI index, the sales price will be affect
```

5. (5 pts) If the home in question is older than 45 years old, doesn't have a finished basement, has a lot area of 4000 square feet, has a brick exterior, 1 enclosed parking space, 1480 square feet of living space, central air, and no pool, what is its predicted value and what are the 95% confidence intervals of this home with UFFI and without UFFI?

With UFFI, 95% CI is 148623.9 - 204737.3 Without UFFI, 95% CI is 155208.9 - 211322.3

```
# 5.
#Yrs45 = 1
\#Bsmnt\ Fin\ SF = O
\#Lot\_Area = 4000
\#Brick\ Ext = 1
\#Enc_Pk_Spaces = 1
\#Living\_Area\_SF = 1480
\#Central\_Air = 1
\#Pool = 0
#Considering Year_Sold as 2018
#Equation form with UFFI
withuffi < -1.122e+07 + 5.607e+03*(2018) + -6.585e+03*(1) + 1.449e+01*(0) + 6.762e+03*(1) + 5.512e+01*(1) + 
withuffi
## [1] 176680.6
#Upper Bound
withuffi + 1.96*(sqrt(deviance(Linear_Model_reg_final))/df.residual(Linear_Model_reg_final)))
## [1] 204737.3
#Lower Bound
withuffi - 1.96*(sqrt(deviance(Linear_Model_reg_final))/df.residual(Linear_Model_reg_final)))
## [1] 148623.9
#Equation form with UFFI
woithuffi < -1.122e + 07 + 5.607e + 03*(2018) + -6.585e + 03*(0) + 1.449e + 01*(0) + 6.762e + 03*(1) + 5.512e + 01*(0) + 0.762e + 0.762e
woithuffi
## [1] 183265.6
#Upper Bound
woithuffi + 1.96*(sqrt(deviance(Linear_Model_reg_final))/df.residual(Linear_Model_reg_final)))
```

```
## [1] 211322.3
#Lower Bound
woithuffi - 1.96*(sqrt(deviance(Linear Model reg final))/df.residual(Linear Model reg final)))
## [1] 155208.9
Problem 3 (35 Points)
  1. (5 pts) Divide the provided Titanic Survival Data into two subsets: a training data set and a test data
    set. Use whatever strategy you believe it best. Justify your answer.
## Load CSV Files ##
titanic_raw <- read.csv('titanic_data.csv',header = TRUE)</pre>
#titanic_raw \leftarrow titanic_raw[-c(1)]
str(titanic_raw)
## 'data.frame':
                    891 obs. of 12 variables:
    $ PassengerId: int 1 2 3 4 5 6 7 8 9 10 ...
   $ Survived
                 : int
                        0 1 1 1 0 0 0 0 1 1 ...
   $ Pclass
                 : int 3 1 3 1 3 3 1 3 3 2 ...
##
## $ Name
                 : Factor w/ 891 levels "Abbing, Mr. Anthony",..: 109 191 358 277 16 559 520 629 416 58
                 : Factor w/ 2 levels "female", "male": 2 1 1 1 2 2 2 2 1 1 ...
## $ Sex
                 : num 22 38 26 35 35 NA 54 2 27 14 ...
##
   $ Age
## $ SibSp
                 : int 1 1 0 1 0 0 0 3 0 1 ...
## $ Parch
                 : int 000000120 ...
                 : Factor w/ 681 levels "110152","110413",...: 525 596 662 50 473 276 86 396 345 133 ...
## $ Ticket
##
   $ Fare
                 : num 7.25 71.28 7.92 53.1 8.05 ...
                 : Factor w/ 148 levels "", "A10", "A14", ...: 1 83 1 57 1 1 131 1 1 1 ....
## $ Cabin
                 : Factor w/ 4 levels "", "C", "Q", "S": 4 2 4 4 4 3 4 4 4 2 ...
    $ Embarked
summary(titanic_raw)
     PassengerId
                        Survived
                                          Pclass
##
##
                            :0.0000
   Min.
          : 1.0
                    Min.
                                      Min.
                                             :1.000
   1st Qu.:223.5
                    1st Qu.:0.0000
                                      1st Qu.:2.000
                                      Median :3.000
##
  Median :446.0
                    Median :0.0000
##
    Mean
           :446.0
                    Mean
                            :0.3838
                                      Mean
                                             :2.309
##
    3rd Qu.:668.5
                                      3rd Qu.:3.000
                    3rd Qu.:1.0000
    Max.
           :891.0
                    Max.
                            :1.0000
                                      Max.
                                             :3.000
##
##
                                        Name
                                                      Sex
                                                                    Age
##
  Abbing, Mr. Anthony
                                          : 1
                                                  female:314
                                                               Min.
                                                                      : 0.42
## Abbott, Mr. Rossmore Edward
                                                               1st Qu.:20.12
                                          : 1
                                                 male :577
## Abbott, Mrs. Stanton (Rosa Hunt)
                                             1
                                                               Median :28.00
                                          :
  Abelson, Mr. Samuel
                                                                      :29.70
##
                                             1
                                                               Mean
   Abelson, Mrs. Samuel (Hannah Wizosky):
                                                               3rd Qu.:38.00
##
   Adahl, Mr. Mauritz Nils Martin
                                          : 1
                                                               Max.
                                                                       :80.00
##
    (Other)
                                          :885
                                                               NA's
                                                                      :177
##
        SibSp
                                           Ticket
                         Parch
                                                           Fare
##
           :0.000
                            :0.0000
                                                             : 0.00
  Min.
                    Min.
                                      1601
                                              :
                                                      Min.
                                      347082
                                                      1st Qu.: 7.91
##
   1st Qu.:0.000
                    1st Qu.:0.0000
                                                 7
```

Median: 14.45

CA. 2343: 7

## Median :0.000

Median :0.0000

```
##
    Mean
            :0.523
                     Mean
                             :0.3816
                                       3101295 : 6
                                                       Mean
                                                               : 32.20
##
    3rd Qu.:1.000
                     3rd Qu.:0.0000
                                       347088 : 6
                                                       3rd Qu.: 31.00
##
    Max.
           :8.000
                     {\tt Max.}
                             :6.0000
                                       CA 2144 : 6
                                                       Max.
                                                               :512.33
                                       (Other) :852
##
##
            Cabin
                       Embarked
                :687
                        : 2
##
                       C:168
##
   B96 B98
                : 4
    C23 C25 C27:
                       Q: 77
##
                   4
##
    G6
                   4
                       S:644
   C22 C26
##
                   3
##
   D
                   3
    (Other)
##
                :186
# 1.Data Split
#Splitting with createDataPartition to have consistent partition of data. 80%-20% is taken so that we h
titanic_train <- createDataPartition(y=titanic_raw$Survived ,p=0.8, list=F)
titanic_train_data <- titanic_raw[titanic_train,]</pre>
titanic_test_data <- titanic_raw[-titanic_train,]</pre>
  2. (10 pts) Impute any missing values for the age variable using an imputation strategy of your choice.
```

State why you chose that strategy and what others could have been used and why you didn't choose

```
them.
# 2. Imputation using mice
#Package: mice -> Predictive Mean Matching method
# This is used because we need to take into consideration other factors in the data set as well.
titanic imp <- titanic raw
titanic_imp_subset <- subset(titanic_imp, select=c(Age,Sex,Fare,Parch))</pre>
imputed_Data <- mice(titanic_imp_subset, m=1,seed = 500, method = "pmm",maxit = 50)</pre>
##
##
    iter imp variable
##
     1
         1 Age
##
     2
            Age
##
     3
         1
            Age
##
     4
         1
            Age
##
     5
         1
            Age
##
     6
            Age
##
     7
         1
            Age
##
     8
         1
            Age
##
     9
         1
            Age
##
     10
         1
             Age
##
             Age
     11
          1
##
     12
          1
             Age
##
     13
          1
             Age
##
     14
          1
             Age
##
     15
          1
             Age
##
     16
          1
             Age
```

##

##

##

##

##

##

17

18

19

20

21

22

1 Age

1 Age

1 Age

1 Age

1 Age

1 Age

```
##
     24
             Age
         1
##
     25
         1
             Age
##
     26
             Age
         1
##
     27
          1
             Age
##
     28
         1
             Age
##
     29
         1
             Age
##
     30
         1
             Age
##
     31
          1
             Age
##
     32
         1
             Age
##
     33
         1
             Age
##
     34
             Age
         1
##
     35
         1
             Age
##
     36
             Age
##
     37
          1
             Age
##
     38
          1
             Age
##
     39
          1
             Age
##
     40
             Age
         1
##
     41
             Age
        1
##
     42
         1
             Age
##
     43
         1
             Age
##
     44 1
             Age
##
     45 1 Age
##
     46
             Age
          1
##
     47
          1
             Age
##
     48
          1
             Age
##
     49
             Age
          1
     50
          1
             Age
summary(imputed_Data)
## Class: mids
## Number of multiple imputations: 1
## Imputation methods:
     Age
           Sex Fare Parch
                   11 11
## "pmm"
            11 11
## PredictorMatrix:
         Age Sex Fare Parch
##
## Age
           0
              1
                     1
## Sex
               0
           1
                     0
                           1
## Fare
           1
               1
## Parch
                     1
                           0
titanic_imp_sorted <- sqldf("select * from titanic_imp order by Age")</pre>
titanic_imp_sorted_NA <- titanic_imp_sorted[1:177,]</pre>
titanic_imp_sorted_notNA <- titanic_imp_sorted[178:891,]</pre>
titanic_imp_sorted_NA$Age <- imputed_Data$imp$Age[[1]]</pre>
titanic_imp <- rbind(titanic_imp_sorted_NA,titanic_imp_sorted_notNA)</pre>
titanic_imp <- sqldf("select * from titanic_imp order by PassengerId")</pre>
titanic_imp$Embarked[62] <- "S"</pre>
```

23

1 Age

##

```
titanic_imp$Embarked[830] <- "S"
```

3. (10 pts) Construct a logistic regression model to predict the probability of a passenger surviving the Titanic accident. Test the statistical significance of all parameters and eliminate those that have a p-value > 0.05 using stepwise backward elimination.

```
# 3. Model Formulation
titanic_train_data <- titanic_train_data[-c(4,9,11)]</pre>
titanic_test_data <- titanic_test_data[-c(4,9,11)]</pre>
titanic_train_data <- na.omit(titanic_train_data)</pre>
titanic_test_data <- na.omit(titanic_test_data)</pre>
Linear_Model_reg_titanic <- glm(titanic_train_data$Survived~.,data=titanic_train_data,family = binomial
summary(Linear_Model_reg_titanic)
##
## Call:
## glm(formula = titanic_train_data$Survived ~ ., family = binomial,
##
       data = titanic_train_data)
##
## Deviance Residuals:
##
      Min
                1Q
                     Median
                                   30
                                           Max
## -2.6517 -0.6812 -0.3988
                               0.6648
                                        2.4050
##
## Coefficients:
##
                 Estimate Std. Error z value Pr(>|z|)
## (Intercept) 1.680e+01 5.354e+02 0.031 0.97497
## PassengerId 3.806e-04 4.143e-04
                                      0.919 0.35823
## Pclass
              -1.137e+00 1.786e-01 -6.365 1.96e-10 ***
## Sexmale
               -2.553e+00 2.436e-01 -10.478
                                             < 2e-16 ***
## Age
              -4.069e-02 9.169e-03 -4.438 9.08e-06 ***
## SibSp
              -3.729e-01 1.424e-01
                                     -2.619 0.00883 **
## Parch
              -3.657e-02 1.449e-01
                                     -0.252
                                              0.80070
## Fare
               1.432e-03 2.590e-03
                                      0.553
                                              0.58026
              -1.170e+01 5.354e+02 -0.022 0.98256
## EmbarkedC
## EmbarkedQ
              -1.205e+01 5.354e+02 -0.023 0.98204
## EmbarkedS
              -1.198e+01 5.354e+02 -0.022 0.98215
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##
       Null deviance: 771.67 on 571 degrees of freedom
## Residual deviance: 524.52 on 561 degrees of freedom
## AIC: 546.52
##
## Number of Fisher Scoring iterations: 12
#Removing Embarked
Linear_Model_reg_titanic_1 <- glm(titanic_train_data$Survived ~ PassengerId + Pclass + Sex + Age + SibS
summary(Linear_Model_reg_titanic_1)
```

##

```
## Call:
## glm(formula = titanic_train_data$Survived ~ PassengerId + Pclass +
      Sex + Age + SibSp + Parch + Fare, family = binomial, data = titanic_train_data)
##
## Deviance Residuals:
##
      Min
               1Q
                   Median
                                3Q
## -2.7039 -0.6819 -0.3975
                           0.6499
                                     2.4011
##
## Coefficients:
##
               Estimate Std. Error z value Pr(>|z|)
## (Intercept) 4.9155936 0.6871286
                                   7.154 8.44e-13 ***
## PassengerId 0.0003951 0.0004130 0.957 0.33873
## Pclass
             ## Sexmale
             -2.5638828 0.2419191 -10.598 < 2e-16 ***
             -0.0409945 0.0091163 -4.497 6.90e-06 ***
## Age
## SibSp
             -0.3877187 0.1414662
                                   -2.741 0.00613 **
## Parch
             -0.0347173 0.1437894 -0.241 0.80921
## Fare
              0.0019517 0.0025437
                                    0.767 0.44292
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## (Dispersion parameter for binomial family taken to be 1)
##
      Null deviance: 771.67 on 571 degrees of freedom
## Residual deviance: 525.68 on 564 degrees of freedom
## AIC: 541.68
##
## Number of Fisher Scoring iterations: 5
#Removing Fare
Linear_Model_reg_titanic_2 <- glm(titanic_train_data$Survived ~ PassengerId + Pclass + Sex + Age + SibS
summary(Linear_Model_reg_titanic_2)
##
## Call:
## glm(formula = titanic_train_data$Survived ~ PassengerId + Pclass +
      Sex + Age + SibSp + Parch, family = binomial, data = titanic_train_data)
##
## Deviance Residuals:
      Min
           1Q Median
                                30
                                        Max
## -2.6707 -0.6837 -0.3956 0.6404
                                     2.4132
## Coefficients:
               Estimate Std. Error z value Pr(>|z|)
## (Intercept) 5.1296466 0.6320653
                                  8.116 4.83e-16 ***
## PassengerId 0.0003879 0.0004127
                                    0.940 0.34729
## Pclass
             -1.2287669 0.1545023 -7.953 1.82e-15 ***
## Sexmale
             -0.0414687
                        0.0090994 -4.557 5.18e-06 ***
## Age
## SibSp
             -0.3764697   0.1404691   -2.680   0.00736 **
## Parch
             -0.0097668 0.1403997 -0.070 0.94454
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## (Dispersion parameter for binomial family taken to be 1)
```

```
##
##
      Null deviance: 771.67 on 571 degrees of freedom
## Residual deviance: 526.32 on 565 degrees of freedom
## AIC: 540.32
## Number of Fisher Scoring iterations: 5
#Removing Parch
Linear_Model_reg_titanic_3 <- glm(titanic_train_data$Survived ~ PassengerId + Pclass + Sex + Age + SibS
summary(Linear_Model_reg_titanic_3)
##
## Call:
## glm(formula = titanic_train_data$Survived ~ PassengerId + Pclass +
      Sex + Age + SibSp, family = binomial, data = titanic_train_data)
##
##
## Deviance Residuals:
      Min
               1Q
                   Median
                                 3Q
                                        Max
## -2.6744 -0.6832 -0.3949
                             0.6405
                                     2.4134
##
## Coefficients:
               Estimate Std. Error z value Pr(>|z|)
##
## (Intercept) 5.1236506 0.6261720 8.182 2.78e-16 ***
## PassengerId 0.0003864 0.0004121 0.938 0.34843
## Pclass
             -1.2286463 0.1545071 -7.952 1.83e-15 ***
## Sexmale
             -2.5611026 0.2363856 -10.834 < 2e-16 ***
              ## Age
             ## SibSp
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## (Dispersion parameter for binomial family taken to be 1)
      Null deviance: 771.67 on 571 degrees of freedom
## Residual deviance: 526.32 on 566 degrees of freedom
## AIC: 538.32
## Number of Fisher Scoring iterations: 5
#Removing PassengerId
Linear_Model_reg_titanic_4 <- glm(titanic_train_data$Survived ~ Pclass + Sex + Age + SibSp,data=titanic
summary(Linear_Model_reg_titanic_4)
##
## glm(formula = titanic_train_data$Survived ~ Pclass + Sex + Age +
##
      SibSp, family = binomial, data = titanic_train_data)
##
## Deviance Residuals:
      Min
               1Q
                   Median
                                3Q
                                        Max
## -2.6952 -0.6790 -0.4026
                           0.6481
                                     2.3954
##
## Coefficients:
              Estimate Std. Error z value Pr(>|z|)
                       0.596442 8.904 < 2e-16 ***
## (Intercept) 5.310836
```

0.154295 -7.992 1.33e-15 \*\*\*

## Pclass

-1.233083

```
## Sexmale
               -2.549157
                           0.235421 -10.828 < 2e-16 ***
                           0.009071 -4.589 4.46e-06 ***
## Age
               -0.041623
## SibSp
               -0.389311
                           0.133140 -2.924 0.00345 **
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##
       Null deviance: 771.67 on 571 degrees of freedom
## Residual deviance: 527.20 on 567 degrees of freedom
## AIC: 537.2
## Number of Fisher Scoring iterations: 5
Linear_Model_reg_titanic_final <- glm(titanic_train_data$Survived ~ Pclass + Sex + Age + SibSp,data=tit
summary(Linear_Model_reg_titanic_final)
##
## Call:
## glm(formula = titanic_train_data$Survived ~ Pclass + Sex + Age +
       SibSp, family = binomial, data = titanic_train_data)
##
## Deviance Residuals:
##
       Min
                 1Q
                     Median
                                   3Q
                                           Max
                               0.6481
## -2.6952 -0.6790 -0.4026
                                        2.3954
##
## Coefficients:
##
               Estimate Std. Error z value Pr(>|z|)
## (Intercept) 5.310836
                           0.596442
                                     8.904 < 2e-16 ***
## Pclass
               -1.233083
                           0.154295 -7.992 1.33e-15 ***
                           0.235421 -10.828 < 2e-16 ***
## Sexmale
               -2.549157
               -0.041623
                           0.009071 -4.589 4.46e-06 ***
## Age
                           0.133140 -2.924 0.00345 **
## SibSp
               -0.389311
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## (Dispersion parameter for binomial family taken to be 1)
##
##
       Null deviance: 771.67 on 571 degrees of freedom
## Residual deviance: 527.20 on 567 degrees of freedom
## AIC: 537.2
## Number of Fisher Scoring iterations: 5
  4. (5 pts) State the model as a regression equation.
#Equation
#y= 5.941125 - 1.375064(Pclass) - -2.664718(Sex) - 0.050312(Age) - 0.452608(SibSp)
  5. (5 pts) Test the model against the test data set and determine its prediction accuracy (as a percentage
    correct).
# 4.Accuracy
pred <- predict(Linear_Model_reg_titanic_final,titanic_test_data,type = "response")</pre>
pred <- ifelse(pred > 0.5,1,0)
```

```
Accuracydata <- as.data.frame(cbind(as.integer(pred),titanic_test_data$Survived))
Accuracydata <- as.data.frame(Accuracydata)
colnames(Accuracydata) <- c("Predicted","Actual")
Accuracydata$accuracy <- ifelse(Accuracydata$Predicted == Accuracydata$Actual, 1,0)
mean(Accuracydata$accuracy)*100
```

#### ## [1] 83.80282

Problem 4 (10 Points) (10 pts) Elaborate on the use of kNN and Naive Bayes for data imputation. Explain in reasonable detail how you would use these algorithms to impute missing data and why it can work.

kNN: kNN is a machine learning algorithm for classification which can also be used for data imputation. It can be used for continuous, discrete, ordinal and categorical data imputation. There is an underlying assumption that a point can be approximated by values of the points which are nearest to it, based on other features. It matches a given point with its closest neighbours in a multidimensional space based on distances. Distances between different data points are calculated based on distance measures such as the Euclidean, Manhattan, Hamming distance etc. Then the data points are arranged by the distances in a multidimensional space and we consider a given number of closest points(neighbors) for the missing data based on the value of 'k' taken. The selection of k is also quite important. If the value of k is very low, it increases influence of noise and if it is high, it doesn't take local effects in account. Also if the classes are binary, k should be an odd value so that ties can be avoided. Then after considering the k nearest neighbors, any of the aggregation methods such as mean, median or mode are used for imputation of the missing data if the data is numeric and mode if it is categorical.

Naive Bayes: Naive Bayes is a classifier which is based on Bayesian methods which determines the empirical probabilities of each outcome based on frequencies of each of feature values. It is used for categorical data and if the data is numerical, it is first converted to categorical by binning it. When the classifier is then applied to unlabelled cases, it uses the empirical probabilities to predict the most likely case for the unknown class. Naive Bayes uses all the features in the data simultaneously. It makes the assumption that the features are independent of each other. However, even if they are not, Bayes classifier still works really well. For classifying missing data using Naive Bayes, frequency tables of all the features are made for all the categories which are present int the dataset which we will be using for imputation. From the frequency, likelihoods of each of the values in all the features are calculated to build a likelihood table. After doing so, the condidtional probabilities are multiplied and then divided by the total likelihood. This transforms each class likelihood into a probability and then based on the probability, imputation of missing data is done by replacing the missing values with the class having highest probability for the same.

Refernces: Lecture Videos and Textbook https://towardsdatascience.com/the-use-of-knn-for-missing-values-cf33d935c637 http://conteudo.icmc.usp.br/pessoas/gbatista/files/his2002.pdf