How does the temperature and volume of water in a wine glass affect the pitch of the sound created by a glass harp?

Introduction

I have been interested in music for a long time and I have also been studying its history and theory along with practicing an instrument. My interest in physics, specifically, the physics behind music instruments grew when I came across a video that showed the vibrations on a drum snare when it is struck. I was intrigued to know more about the glass harp, it being an instrument that isn't very well known and is interesting because of how different it is. I was more intrigued to know about sound pitches in a glass harp when I watched a show where a musician played happy birthday on a glass harp.

Background Information

Richard Pockrich was an Irish musician who invented the glass harp in 1741. He called it the 'angelic organ' and it built his success as he had founded something never seen before (Zeitler). Initially, He struck the glasses with a stick, while he later discovered that using a moist finger formed a better sound. The glass harp is played by adding water into a wine glass and slowly sliding your moistened finger over its rim, creating a sound wave with a pitch dependent on the volume of water in the glass, the temperature, the glass shape, etc.

To understand the way that a glass harp works, we must first understand what stick-slip friction is. Stick-slip friction is a type of friction between two surfaces, where both surfaces alternate between sticking to each other, not moving at all, and sliding over each other smoothly (Science Buddies). The surfaces alternate between these at a high frequency. This is what occurs when you slide a moist finger over the rim of the glass. Your finger and the glass rim undergo stick-slip friction, causing the molecules of the glass to vibrate at their natural frequencies (Science Buddies). The natural frequency of an object is the frequency at which it vibrates when it is struck. In context, when the glass rim is rubbed by a finger, it rapidly vibrates at its natural frequency. When the frequency of the force, which is your finger moving, matches the natural frequency of the glass, resonance occurs. The wine glass vibrates initially at its natural frequency, however, because of the continuous stick-slip friction it undergoes, it continues to vibrate at this frequency, causing resonance. Resonance occurs when an object vibrates at the same natural frequency as another object close to it, causing the second object to vibrate, thus causing the objects to vibrate at a higher frequency due to the superposition of those two waves (The Physics Classroom). The vibrations within the wine glass cause standing waves on the surface of the water close to the edge of the glass, also contributing to the resonance. This explains why the experimenter cannot hold the glass at the bowl (wider section) of the wine glass, as it is an antinode, and holds the glass at the stem (thin section), the node. Holding the glass at its bowl would resist the standing waves, interfering with the amplitude of the sound wave and not cause any resonance.

The formula for frequency is $\frac{speed}{wavelength}$ which shows us the factors that will affect this frequency. The speed relates to the speed at which my finger moves along the rim of the wine glass. The wavelength depends on the medium (refractive index) thus the volume of water in the glass. To have an accurate investigation, I will attempt to control the speed of my finger along the rim of the glass. The wavelength can be affected by the medium and its properties, therefore, I also aim to control some properties of the room the experiment is performed in, such as temperature or external sound.

Because I was interested in how I could control the frequency of the sound wave produced by the resonance, I commenced this investigation to see how the volume of water and the temperature will change the frequency of the sound produced by the glass harp. As explained above, the reason why both of these factors would have an effect on the frequency of the sound wave produced is that they both affect the medium in

which the standing waves and resonance is created. Hence, it affects the wavelength of the waves created by the wine glass by definition of the formula of the frequency of a wave ($f = \frac{v}{\lambda}$, where f is frequency, v is speed of wave so speed of finger rubbing and λ is wavelength, which depends on the medium).

Aim

The aim for my investigation is to investigate the relationship between the volume and temperature of water in a wine glass as a glass harp and the frequency of the sound wave produced. I will be doing this by playing the glass harp by adding different volumes of water at different temperatures and measuring the frequency of the sound wave using the Logger Pro software and a Vernier microphone connected to my computer. The reason why I have chosen to change these two variables and investigate their effects on the frequency is because frequency depends on speed and wavelength, as per definition. Factors that affect the wavelength of a wave are the properties of the medium the wave travels in, which is a major and can not be ignored. The temperature of the medium and the volume of water (refractive index of the medium) both have an effect on the wavelength, hence frequency. I will be controlling the volume of the water by using a syringe to add water into the glass. I will be controlling the temperature by using a hot plate to heat up the water.

Research Question

How does the temperature and volume of water in a glass harp affect the frequency of the pitch of the sound wave created?

Variables

<u>Variable</u>	Type	Justification
% volume filled with water (horizontal axis/x-axis)	Independent	I will be changing the % volume of the glass that is filled with water by 10% for every trial. I will make use of a syringe to add exactly 10% more water per trial (total volume of glass is 200 cm ³ → 200 ml, adding 20 ml per trial). This range was chosen as 200 ml was the maximum capacity of the glass and adding 20 ml will give us 10 trials for volume. This will help identify any possible relationship between the variables. I will make use of markings on the glass and use a syringe to improve the accuracy of the results and findings
Temperature of water (horizontal axis / x-axis)	Independent	I will also be changing the temperature of the water in the glass harp by 10 degrees Celsius (starting from 20 to 80). The reason for this range of temperature is that I believe that going over 80 degrees Celsius could cause a hazard for myself or even damage the glass. I will change the temperature of the water using a hot plate, which I can set at a specific temperature. This can lead to a well controlled temperature for every trial.
Frequency of the sound wave created when glass harp is struck	Dependent	The frequency of the sound wave will change as the volume of water and temperature of water will change in the glass harp. I will measure this variable using a Vernier microphone held close to the glass harp using a set-up consisting of clamps and a clamp stand. This will be measured in Hertz.

Speed of rubbing	Controlled	The speed of rubbing my finger along the rim of the wine glass affects the speed the waves travel at which also affects the frequency of the sound wave as per the definition of frequency of a wave $(f = \frac{v}{\lambda})$. I will be controlling this by using a timer to visually control the speed to be 2 seconds per full circle. This number is not chosen at random, but tested to fit within the range of actually creating sound. This means that rubbing a finger too slow or too fast along the rim of the glass will not produce a sound at all due to too much or too less friction. 2 seconds per full revolution does produce a sound wave.
Material of glass	Controlled	The material of the glass that the wine glass consists of will be constant. The reason for this is that different types of glass can have a direct effect on the stick-slip friction coefficient which will result in a different force on the rim of the glass. This will result in a different frequency produced or different amplitude, which may cause inaccuracies in my data. I will control this by making use of the same glass for every trial of my experiment and in the case that the glass is damaged, I will have another set of identical glasses ready for use.
Distance between Vernier microphone and glass	Controlled	I will be controlling the distance between the vernier microphone and the wine glass. The reason for this is that if we look at the wine glass as the source of the sound waves and the vernier microphone as the observer, any movement could cause the doppler effect, causing an inaccuracy in the frequency of the sound wave recorded. This will be controlled by holding the Vernier microphone using a clamp stand and clamps and keeping the wine glass at rest as well.
Thickness of glass	Controlled	The thickness of the glass should remain constant as it influences the amplitude of the standing waves produced near the edge of the glass, which cause the resonance. Different thicknesses for the glass could resist the amplitude and give different values for frequency. I will control this by again making use of one wine glass that has the same thickness across the whole glass.
Sound generated by the atmosphere / surroundings	Controlled	I will control the sound waves in the environment of my experiment. The reason for this is that high amplitude sounds could cause interference which will cause inaccuracies and imprecisions in my recorded values for frequencies. I will control this by performing my experiment in a quiet room with low sound produced to avoid interference and collect accurate and precise data.

Hypothesis

I predict that as the temperature of the water increases, the frequency of the sound wave will increase linearly as we have seen that molecules at a higher temperature will vibrate quicker, at a higher frequency, causing the frequency of the sound wave to be higher, which creates a sound wave with a higher frequency / pitch. The temperature of the water will be the independent variable (x-axis), and the frequency of the sound wave will be the dependent variable (y-axis). I assume that sound waves in the atmosphere may interfere with the sound wave produced by the harp, interfering with the data collected by the microphone, however, I will create a method to control this.

Apparatus

- Wine Glass
- Water
- Glass Marker
- Clamps
- Clamp Stand
- Vernier Microphone
- Syringe
- Hot Plate
- Measuring Cylinder
- stopwatch
- Thermometer
- Device with Logger Pro Software

Procedure

Diagram

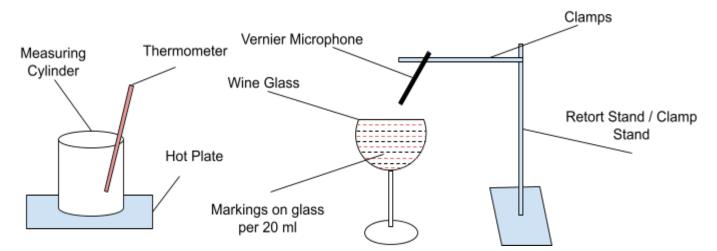


Fig. 1, Diagram showing the setup of the experiment

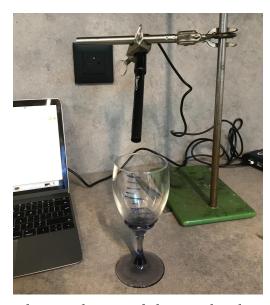


Fig. 2, Image showing the type of glass used and setup in real life

Method

Prior to conducting the experiment, I constructed the diagram above using the necessary apparatus. I verified the markings on the wine glass by adding 20 ml of water each time using a syringe for accuracy. I started up the hot plate and set the temperature to 90 degrees to start with. As the hot plate heated up, I filled the measuring cylinder with no more than 300 ml of water and placed it on the hot plate and placed the thermometer inside the cylinder. Once the temperature of the water was 80 °C, I used a syringe to transfer 20 ml of water from the measuring cylinder into the wine glass. I began recording the sound using the Vernier software and microphone and rubbed my finger on the edge of the glass, playing the glass harp. I recorded the frequency measured by the software and redid this 2 more times. I continued to add 20 ml of water into the wine glass and conducted three trials for each volume of water. After the wine glass was full (200 ml), I transferred all the water back into the measuring cylinder and decreased the temperature of the hot plate by 10 °C. I continued transferring 20 ml of water, recording data and decreasing the temperature of the hot plate until I conducted the experiment 3 times for all temperature levels (80 °C, 70 °C, 60 °C, 50 °C, 40 °C, 30 °C and 20 °C \rightarrow ± 1 °C) and all levels of volume of the water (20 ml, 40 ml, 60 ml, 80 ml, 100 ml, 120 ml, 140 ml, 160 ml, 180 ml, and 200ml)

Safety Measures

When playing the glass harp, I will make sure to be careful to not damage the glass with the high frequencies produced or any other way as a glass shard can be dangerous and can cause severe pain. The hot plate must be placed away from any electronics and the glass as well as it may heat it up, causing it to shatter. The hot water will be used with caution and not kept near any electronics and will also make use of it carefully, not causing any burns. I will be wearing a lab coat and safety goggles as a precaution to any damage with the wine glass or hot water.

Data Tables & Graphs

Table 1: Raw data collected from experiment with Vernier microphone - 3 trials of Frequency per temperature per volume of water in wine glass

	Frequency (f) in Hertz (Hz) ± 1 Hz														
Temperature (T) in C		Volume (V) of water in milliliters (ml)													
		20			40			60		80					
20	1098	1098	1097	1105	1105	1104	1126	1129	1130	1169	1169	1168			
30	1099	1098	1099	1105	1106	1106	1128	1131	1129	1170	1170	1170			
40	1098	1100	1101	1106	1106	1108	1131	1132	1131	1172	1171	1170			
50	1100	1103	1100	1107	1109	1107	1133	1134	1132	1173	1174	1174			
60	1102	1102	1103	1110	1110	1108	1134	1134	1135	1175	1175	1175			
70	1104	1104	1104	1110	1111	1111	1136	1136	1136	1176	1176	1177			
80	1105	1106	1105	1112	1112	1113	1138	1135	1137	1177	1178	1178			

	Frequency (f) in Hertz (Hz) \pm 1 Hz														
Temperature (T) in C		Volume (V) of water in milliliters (ml)													
		100			120			140		160					
20	1232	1232	1234	1186	1188	1189	1089	1090	1089	958	955	955			
30	1234	1233	1233	1189	1188	1189	1090	1091	1091	957	958	958			
40	1234	1235	1236	1190	1191	1192	1092	1092	1093	959	959	959			
50	1236	1237	1237	1192	1192	1193	1093	1094	1094	960	959	961			
60	1238	1238	1238	1194	1193	1193	1094	1095	1096	961	962	962			
70	1238	1238	1239	1195	1196	1196	1094	1097	1098	963	963	964			
80	1239	1240	1241	1197	1197	1197	1098	1099	1098	964	965	964			

Frequency (f) in Hertz (Hz) ± 1 Hz												
Temperature (T) in C	Volume of water in milliliters (ml)											
		180		200								
20	845	840	843	752	753	756						
30	845	844	845	754	754	755						
40	846 846	847	755	756	756							
50	848	847	848	757	757	758						
60	848	851	849	758	758	758						
70	850	851	852	759	760	761						
80	852	852	853	761	763	761						

Raw data - average frequency per temperature per volume of water

$$f_{av} = \frac{f_1 + f_2 + f_3}{3}$$

$$20 \text{ °C, } 20 \text{ ml} \Rightarrow f_{av} = \frac{1098 + 1098 + 1097}{3}$$

$$20 \text{ °C, } 20 \text{ ml} \Rightarrow \Delta f_{av} = \frac{1098 - 1097}{2}$$

$$f_{av} = 1097. \overline{6} \approx 1098$$

$$20 \text{ °C, } 20 \text{ ml} \Rightarrow \Delta f_{av} = \frac{1098 - 1097}{2}$$

$$\Delta f_{av} = 0.5$$

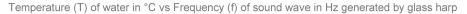
Because the uncertainty for raw data collected is \pm 1 Hz, any uncertainty calculated less than 1 using the formula above will not be considered and rounded to \pm 1. Thus, for 20 °C, 20 ml \Rightarrow f_{av} = 1098 and Δf_{av} = 1.0. For a calculated frequency uncertainty of 1.5, uncertainty is rounded to 2 as the value for frequency is given as a whole number (no decimal places).

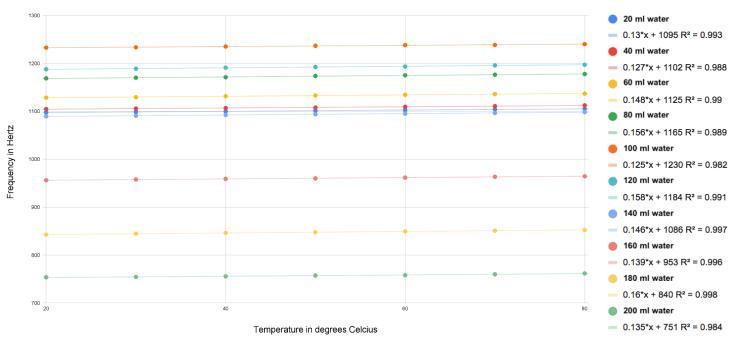
Table 2: Processed data from experiment with Vernier microphone - Average of trials of Frequency per temperature per volume of water in wine glass with calculated uncertainty

Volume of water in wine glass (max. 200ml)	20		40		60		80		100		120		140		160		180		200	
Temperature (°C) ± 0.5		Frequency (Hz) $\pm \Delta f$																		
20.0	1098	1.0	1105	1.0	1128	2.0	1169	1.0	1233	1.0	1188	2.0	1089	1.0	956	2.0	843	1.0	754	2.0
30.0	1099	1.0	1106	1.0	1129	2.0	1170	1.0	1233	1.0	1189	1.0	1091	1.0	958	1.0	845	1.0	754	1.0
40.0	1100	2.0	1107	1.0	1131	1.0	1171	1.0	1235	1.0	1191	1.0	1092	1.0	959	1.0	846	1.0	756	1.0
50.0	1101	2.0	1108	1.0	1133	1.0	1174	1.0	1237	1.0	1192	1.0	1094	1.0	960	1.0	848	1.0	757	1.0
60.0	1102	1.0	1109	1.0	1134	1.0	1175	1.0	1238	1.0	1193	1.0	1095	1.0	962	1.0	849	2.0	758	1.0
70.0	1104	1.0	1111	1.0	1136	1.0	1176	1.0	1238	1.0	1196	1.0	1096	2.0	963	1.0	851	1.0	760	1.0
80.0	1105	1.0	1112	1.0	1137	2.0	1178	1.0	1240	1.0	1197	1.0	1098	1.0	964	1.0	852	1.0	762	1.0

Graph 1

This graph shows the frequency in Hz for increasing temperatures of water for different values of volume of the water. The horizontal uncertainties, \pm 1 °C, and vertical uncertainties, different for every point (given in table 2), are too small to be seen on the graph. Trendlines are given for every volume series of data points to show the relationship between the temperature and frequency.



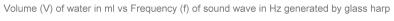


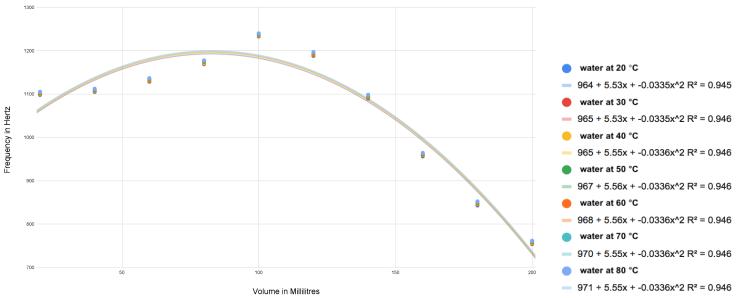
<u>Interpretation of Graph 1</u>

The x-axis shows the temperature of the water in the wine glass in degrees Celcius and the y-axis shows the frequency of the sound wave produced by the glass harp in Hertz. We can see using the equations of the trendlines for chosen volumes of water that as the temperature increases, the frequency of the sound wave increases as well, at a slow, constant rate. Thus there is a linear dependency between the temperature of the water and the frequency of the wave, showing proportionality between the two variables. We can deduce that the coefficient of the frequency is a constant. This means that it does not change for different values of temperature. The R^2 values are different for various volumes of water, however, they are all close to 1, assuring the reliability of the trendline for my raw data. The vertical and horizontal uncertainties are too small to be seen on the graph (vertical uncertainties are given in the table and horizontal uncertainties are \pm 0.5 for every point on the graph). Also, the percentage uncertainty for all data points is less than 5%, due to the high frequency values. This shows the precision of the data. Because the two variables have a linear relationship, they follow the equation y = mx + c where m, the gradient is 0.1424 ± 0.0175 by averaging the gradient values of the given equations of trend lines shows next to the graph. This value shows the proportion between the two variables as it is the coefficient of the frequency. However, we do see that because of the very small gradient, the relationship between the two variables is weak, hence temperature is not very relevant. Because the aim of my investigation was to find a relationship between the two variables, this constant justifies it and shows the relationship / proportionality.

Graph 2

This graph shows the frequency in Hz for increasing volumes of water for different values of temperature of the water. The horizontal uncertainties, ± 1 ml, and vertical uncertainties, different for every point (given in table 2), are too small to be seen on the graph. Data points for different temperatures are overlapping, thus difficult to see on the graph as well. Trendlines are given for every temperature series of data points to show the relationship between the volume and frequency.

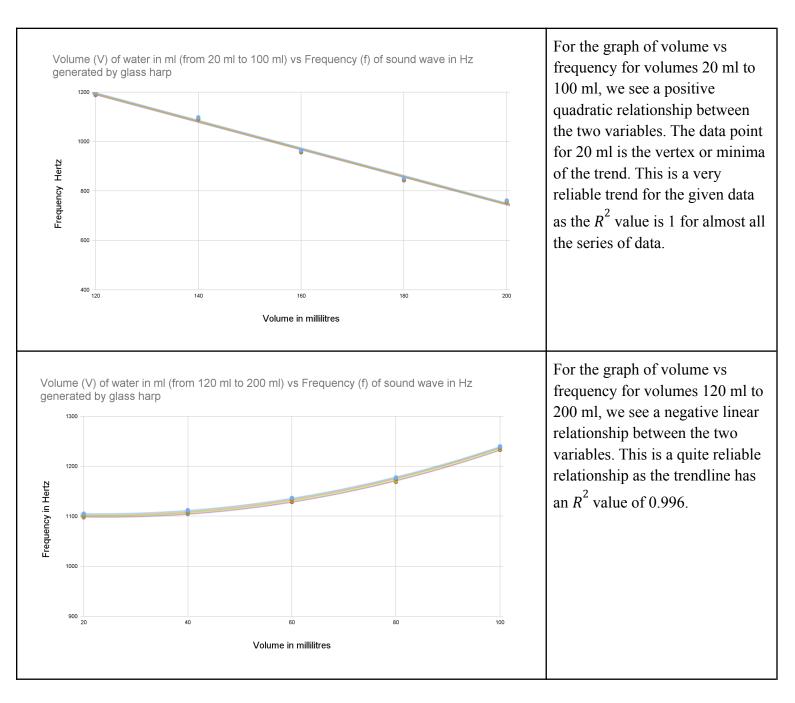




Interpretation of Graph 2

The x-axis shows the volume of the water in the wine glass in milliliters and the y-axis shows the frequency of the sound wave produced by the glass harp in Hertz. We can observe on the graph that as the volume increases, the frequency of the sound wave increases as well, but at a changing, exponential rate until 50% or 100 ml. After this point, we see a linear decrease in frequency for increasing volumes of water. I will be looking at this separately later in my investigation. The trendlines suggest a quadratic relationship, however solely analyzing the points on the graph, we see a different trend. On average, the R^2 value is 0.946, which is reliable, but could indeed be improved. The vertical and horizontal uncertainties are too small to be seen on the graph (vertical uncertainties are given in the table and horizontal uncertainties are \pm 1 for every point on the graph). The percentage uncertainty for this data plotted on this graph is also less than 5% for all data points. This again verifies the precision for the collected data.

If we split graph 2 in half, we can see two different trends between the variables. The x-axis again shows the volume of the water in the wine glass in milliliters and the y-axis shows the frequency of the sound wave produced by the glass harp in Hertz. On the left we see the graph for the range of volume of water from 20 ml to 100 ml. On the right we see the graph for the range of volume of water from 120 ml to 200 ml.



This new perspective shows us how there may be a level of doubt for the accuracy rather than precision for the data collected and the calculated relationship between the volume of water in the glass and frequency of the sound wave. The reasoning behind the linear relationship is that as volume increases, medium is more water, thus higher. Higher refractive index, results in a higher wavelength and lower frequency as per the formula for frequency. However, the first graph, that shows a positive quadratic relationship may be due to the increasing space within the glass (radius), which increases medium and decreases wavelength, increasing frequency.

Because, according to the overall trendline, the two variables have a quadratic relationship, there is another variable involved, which looking at my data, I believe is the width or radius of the wine glass at every point as the radius of the glass also increases until 50% and then decreases again. Thus, this may be evidence that the shape of the glass does have an influence on the frequency. This is because the radius of the glass also controls the medium within the glass and the smaller the medium, the more superposition and resonance occurs. This can give use various peaks, for frequency. The aim of my investigation was again to find a relationship between these two variables as well, I have shown a relationship and the dependency between the two variables, while explaining the relationship and its possible cause. This justifies the aim of my investigation.

Conclusion

My goal for this experiment was to investigate whether there is a relationship, and if there is, what kind of relationship there is between the temperature of the water in a glass harp, the volume of water inside the glass harp and the frequency of the sound wave produced by the glass harp. My results show a relationship between the variables. The relationship between the temperature of the water in the glass and the frequency of the wave created is linear, increasing at a constant rate. As the temperature increases, the sound wave's frequency increases. The relationship between the volume of the water and the frequency of the wave created is following a quadratic relationship, where its vertex lies between 40% and 60% of the total volume of the glass. The quadratic relationship indicates that there are two variables that affect the frequency values, which are the volume of the water in milliliters and the radius of the wine glass.

Something I did not consider in my experiment was how the radius of the wine glass had affected my results, which could potentially produce a function with the two variables, volume in milliliters and radius in millimeters, to calculate a value for the frequency produced by the glass harp.

To conclude, the data collected allows me to answer my research question, specifically, what the relationship is between the frequency of sound waves produced by the glass harp and the temperature and volume of the water in the glass. The relationship between temperature and frequency is linear with a constant proportion of 0.15 ± 0.02 . The relationship between volume of water and frequency was rather quadratic. Because a true value is not known, I cannot assess the accuracy of my data and results, however, I can conclude that my data held a high precision, which verifies the success of my experiment.

Evaluation

Reflecting on my data and conclusion, the temperature of the water in the glass harp and the frequency produced by the glass harp are directly proportional to each other, with a constant proportionality of 0.15 ± 0.02 . The volume of water in the glass harp and the frequency produced by the glass harp have a quadratic proportional relationship, where the second variable related to the proportionality is the radius of the wine glass. These are both shown on the graph. This shows that my hypothesis about temperature and frequency was correct, however, my hypothesis about volume and frequency was incorrect.

Systematic errors were caused by the apparatus used during this experiment. The thermometer to measure the temperature of the water had an uncertainty of \pm 0.5 °C. The percentage uncertainty decreases as the temperature of the water increases. For 20 °C, 30 °C, 40 °C, 50 °C, 60 °C, 70 °C and 80 °C, the percentage uncertainties are 2.5%, 1.7%, 1.3%, 1.0%, 0.8%, 0.7% and 0.6% respectively. I controlled the temperature of the water to a smaller percentage uncertainty, improving the reliability of my raw data and making the uncertainty of the hot plate negligible over those of the thermometer. The syringe used to add exactly 20 ml of water to the glass harp had an uncertainty of \pm 0.5 ml. The percentage uncertainty for the volume of 20 ml, 40 ml, 60 ml, 80 ml, 100 ml, 120 ml, 140 ml, 160 ml, 180 ml and 200 ml are 2.5%, 1.3%, 0.8%, 0.6%, 0.5%, 0.4%, 0.4%, 0.3%, 0.3% and 0.3% respectively (rounded to 2 significant figures). The Vernier microphone and the software used to receive data about the frequency of the sound produced by the glass harp has an uncertainty of \pm 0.5 Hz, for which the percentage uncertainty for each value recorded was < 0.1%. All the systematic uncertainties for this experiment are extremely small, the largest one is 2.5%, which verifies the reliability of my experiment and results.

The systematic error for this experiment was extremely small due to the scientific apparatus that was used throughout this experiment, however, it still may have an effect on the accuracy of the relationship between the variables rather than the type of proportionality. The systematic errors can be altered by isolating the experiment so no heat is lost when performing the experiment due to the cold environment or the temperature of the glass. Heat and volume of water could also be lost as the temperature increased due to evaporation. Also, more accurate apparatus can be used during the experiment to decrease the uncertainty. However, during this experiment, I have made use of very accurate instruments to measure the volume and temperature of water, with less room for improvement.

Random errors are caused by the experimenter and their inability to repeat movements the same way repetitively, causing uncertainties. In the context of my experiment, random errors were caused when playing the glass harp. The pressure I applied by my finger on the rim of the glass or the speed at which my finger traveled across the rim of the glass could not be kept constant as temperature changed. This can produce sounds and different frequencies or sounds at lower amplitudes which increases the uncertainty of the microphone due to interference of other sounds. I can improve this by removing the human element and replacing my finger with a mechanism that can mimic the motion of my finger with silicone. This can increase the consistency of the pressure and speed immensely.

The precision of my data was high as the three trials showed very similar results with barely any anomalies. This can ensure correct results, whether there may be other uncontrolled variables that affect the accuracy of my data. The accuracy of my data was good, but can certainly be improved given that the exact dependency between the variables could not be found.

Through this investigation, we found that as the temperature increases, the frequency of the sound wave produced by the glass harp increases. And as the volume increases, the frequency also increases until 50% volume, where after this point it decreases again, as it has a negative quadratic relationship $(y = -x^2)$. This data can be further used in research related music for great musicians to recreate sounds using a glass harp. On a larger scale, it can be used to study the behavior of waves at different temperatures or different indices as a higher frequency means a higher number of waves traveling past a point per second. However, to completely benefit from this research, we must find the exact relationship or the influence the radius of the wine glass has on the frequency.

Personally, I believe that this experiment was a success as my method fulfilled the aim for my investigation by finding relationships between three variables, however, it would be more satisfactory if my data and interpretation could give me an exact relationship between the volume and frequency or better show how radius influences the frequency. This lab is useful as it could be used in studies that could investigate the relationship between medium and frequency. Although we know that it has an effect on the wavelength, it may be useful to know how big of a difference refractive index really has on frequency of sound to improve practices regarding, for instance, noise pollution.

Extension

I have noticed during my experiment that the shape of the glass possibly also had an effect on the pitch of the sound wave. I am curious to know more about how the shape of the wine glass, more specifically, how the location of at what height the wine glass is the widest, affects the pitch of the frequency. And also whether there are other variables, such as the changing radius or height of the glass, that affect the sound wave of the pitch to generate a formula we can use to calculate the frequency of the sound wave created by substituting the fixed values (information) of the other variables.

Bibliography

- Science Buddies. "Singing Glasses." Scientificamerican.com, Scientific American, 26 Nov. 2015, www.scientificamerican.com/article/singing-glasses1/.
- The Physics Classroom. "Physics Tutorial: Resonance." Www.physicsclassroom.com, www.physicsclassroom.com/class/sound/Lesson-5/Resonance.
- Zeitler, William. "The Glass Armonica Benjamin Franklin's Magical Musical Invention." Glassarmonica.com, 2009, glassarmonica.com/armonica/pockrich.htm.