Multiphysics Reduced Order Modelling

Reduced Basis Method for Lead-Cooled Fast Reactor

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Motivation

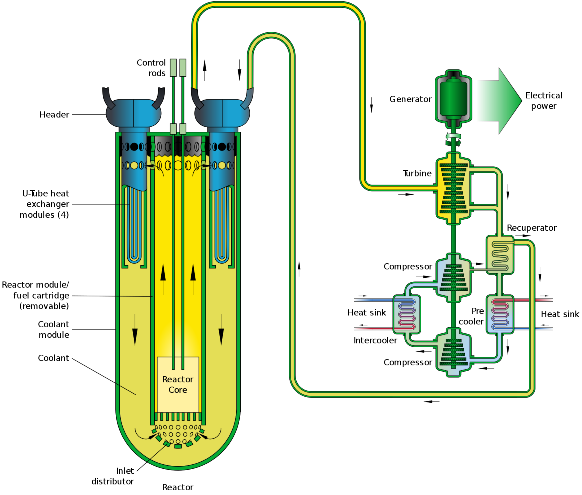
The full order simulation of nuclear reactors is extremely costly!

Can we reduce the computational time and cost associated with these simulations without compromising with the accuracy?

Objective

Develop a Reduced Basis method, with basis functions sampled by a Proper Orthogonal Decomposition technique, to develop a reduced order model of a parametrised Lead-cooled Fast Reactor single-channel.

Lead-cooled Fast Reactor (LFR)



Neutron Flux

Neutron balance

$$\frac{\partial \phi}{\partial t} = -\nabla \cdot D\nabla\phi + \nu\Sigma_f\phi - (\Sigma_a + \Sigma_s)\phi$$

ϕ - Neutron Flux

D - Diffusion Coefficient

Σ_a - Macroscopic Absorption Cross Section

Σ_s - Macroscopic Scattering Cross Section

ν - Fissile Yield (No. of neutrons produced per fission)

Σ_f - Macroscopic Fission Cross Section

Neutron Flux

Rate of change of neutron flux

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Rate of change of neutron flux

Neutron production through fission

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Rate of change of neutron flux

Neutron production through fission

$$\frac{\partial \phi}{\partial t} = -\nabla \cdot D\nabla\phi + \nu\Sigma_f\phi - (\Sigma_a + \Sigma_s)\phi$$

Diffusion of neutrons

Loss of neutrons due to absorption and scattering

Heat Transfer

Energy Balance

$$\frac{\partial T}{\partial t} = \nabla \cdot [(K + K_T)\nabla T] - \rho C_p \boldsymbol{v} \cdot \nabla T + Q$$

T - Temperature
K - Thermal Conductivity
K_T - Turbulent Thermal Conductivity
C_p - Heat Capacity
$\boldsymbol{v}$ - Velocity of fluid
Q - Source Term

Heat Transfer

Rate of change of temperature

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Diffusion term

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Rate of change of temperature

Advection term

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Heat Transfer

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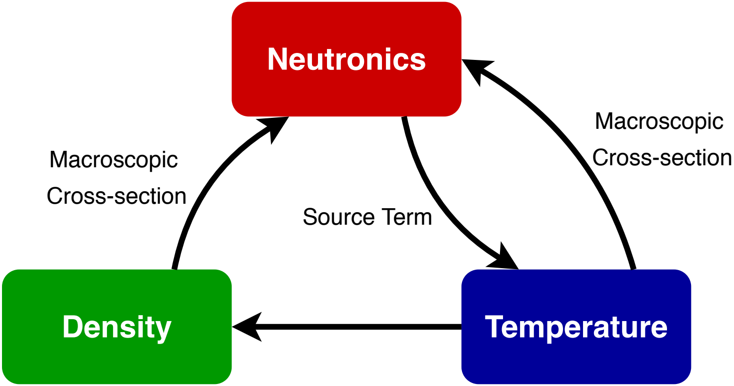
Diffusion term

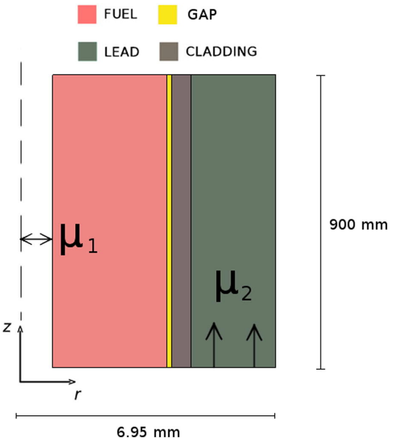
Source term

Multiphysics Model



Parametrised Multiphysics Model

- Parametrised LFR single channel model.
- Rotational symmetry along fuel channel (r-z model).
- Time-independent settings.
- Parameters:
  - Inner radius of fuel pellet $\mu_1 \in [0.1, 0.43]$ (cm)
  - Nominal coolant flow rate $\mu_2 \in [0.8, 1.6]$ (m/s)



Neutronics

The multi-group neutron diffusion equation in stationary formulation is a generalised eigenvalue problem.

$$\left(-\nabla \cdot \overline{\overline{D}}\nabla + \overline{\overline{\Sigma}}_a + \overline{\overline{\Sigma}}_s\right)\overline{\Phi} = \lambda_{eff}\overline{\chi}\overline{F}'\overline{\Phi}$$

$$\overline{\Phi} = \begin{bmatrix} \Phi_1(\mathbf{r}) \\ \vdots \\ \Phi_6(\mathbf{r}) \end{bmatrix} \quad \overline{\overline{D}} = \begin{bmatrix} D_1(\mathbf{r}) & \dots & 0 \\ & \ddots & \\ & \dots & D_6(\mathbf{r}) \end{bmatrix} \quad \overline{\overline{\Sigma}}_a = \begin{bmatrix} \Sigma_a^1(\mathbf{r}) & \dots & 0 \\ & \ddots & \\ & \dots & \Sigma_a^6(\mathbf{r}) \end{bmatrix}$$

$$\overline{\overline{\Sigma}}_s = \begin{bmatrix} \sum_{g'\neq 1} \Sigma_s^{1\to g'}(\mathbf{r}) & \dots & -\Sigma_s^{6\to 1} \\ & \ddots & \\ -\Sigma_s^{1\to 6} & \dots & \sum_{g'\neq 6} \Sigma_s^{6\to g'} \end{bmatrix} \quad \overline{\chi} = \begin{bmatrix} \chi_1 \\ \vdots \\ \chi_6 \end{bmatrix} \quad \overline{F} = \begin{bmatrix} \nu\Sigma_f^1(\mathbf{r}) \\ \vdots \\ \nu\Sigma_f^6(\mathbf{r}) \end{bmatrix}$$

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Fuel domain

$$\Sigma(T,\rho) = \frac{\rho}{\rho_0}\left[\Sigma_0 + \alpha \log\left(\frac{T}{T_0}\right)\right]$$

Lead domain

$$\Sigma(T,\rho) = \frac{\rho}{\rho_0}\Sigma_0$$

Neutronics

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Boundary conditions

- Albedo boundary conditions provide a good compromise between accuracy and computational requirements.

$$\overline{n} \cdot (D_g \nabla \Phi_g) = -\gamma_z \phi_g$$

$$\overline{n} \cdot (D_g \nabla \Phi_g) = -\gamma_r \phi_g$$

Heat Transfer

Within the cladding and gap domains the heat transfer is purely conductive.

$$-\nabla \cdot (K \nabla T) = 0$$

Within the fuel domain the heat transfer is purely conductive but has a source term.

$$-\nabla \cdot (\nabla T) = \sum_{g=1}^{6} \Sigma_g^f E_f \Phi_g$$

Heat Transfer

Within the lead domain, the heat transfer is has an advective and a conductive term.

$$-\nabla \cdot [(K + K_T)\nabla T] = \rho C_p \boldsymbol{v} \cdot \nabla T$$

Boundary conditions

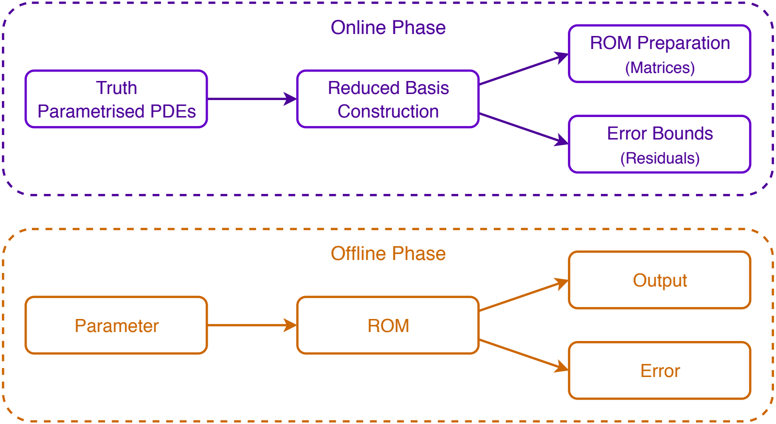
- Symmetry conditions at inner radius of the fluid domain and and outerradius of fluid domain.
- Homogeneous Neumann conditions on top and bottom of the domain.

Back to Objective

The full-fidelity model can have $\mathcal{O}(10^6 - 10^9)$ degrees of freedom.

We want to capture the behaviour of the system without using a high-fidelity, computationally expensive discretisation scheme.

Offline/Online Strategy



Finite Element Discretisation

For a given $\mu \in \mathcal{D}$, find $\overline{\Phi}(\mu) \in V^{\mathcal{N}}$ and $T(\mu) \in W^{\mathcal{N}}$ such that

$$\mathrm{a}\left(\overline{\Phi}, \overline{\nu}; \mu, \Sigma(T)\right) = \lambda_{eff}\,\mathrm{m}\left(\overline{\Phi}, \overline{\nu}; \mu, \Sigma(T)\right) \qquad \forall \overline{\nu} \in V^{\mathcal{N}}$$

$$\mathrm{d}\left(T, w; \mu\right) = \mathrm{f}\left(w; \mu, S(\overline{\phi})\right) \qquad \forall w \in W^{\mathcal{N}}$$

$$\mathrm{a}\left(\overline{\Phi}, \overline{\nu}; \mu, \Sigma(T)\right) = \int_{\Omega} \overline{\overline{D}}(\nabla\overline{\Phi} \cdot \nabla\overline{\nu}) + (\overline{\overline{\Sigma_a}} + \overline{\overline{\Sigma_s}})\overline{\Phi}\overline{\nu} + \int_{\Gamma_{side}} \gamma_r \overline{\Phi}\overline{\nu} + \int_{\Gamma_{top}+\Gamma_{bot}} \gamma_z \overline{\Phi}\overline{\nu}$$

$$\mathrm{m}\left(\overline{\Phi}, \overline{\nu}; \mu, \Sigma(T)\right) = \int_{\Omega} \overline{\chi}\overline{F}'\overline{\Phi}\overline{\nu}$$

Finite Element Discretisation

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$$\mathrm{d}\left(T, w; \mu\right) = \mathrm{f}\left(w; \mu, S(\overline{\phi})\right) \qquad \forall w \in W^{\mathcal{N}}$$

$$\mathrm{d}\left(T, w; \mu\right) = \int_{\Omega} K \nabla T \nabla w + \int_{\Omega_{lead}} K_T \nabla T \nabla w$$

$$\mathrm{f}\left(w; \mu, S(\overline{\phi})\right) = \int_{\Omega_{fuel}} Q w + \int_{\Omega_{lead}} \rho C_p (\boldsymbol{v} \cdot \nabla T) w$$

Affine Parametric Dependence

$$\mathrm{a}\left(\overline{\Phi}, \overline{\nu}; \mu, \Sigma(T)\right) = \sum_{q=1}^{Q_a} \Theta_a^q(\mu)\mathrm{a}^q\left(\overline{\Phi}, \overline{\nu}; \Sigma(T)\right)$$

$$\mathrm{m}\left(\overline{\Phi}, \overline{\nu}; \mu, \Sigma(T)\right) = \sum_{q=1}^{Q_m} \Theta_m^q(\mu)\mathrm{m}^q\left(\overline{\Phi}, \overline{\nu}; \Sigma(T)\right)$$

$$\mathrm{d}\left(T, w; \mu\right) = \sum_{q=1}^{Q_d} \Theta_d^q(\mu)\mathrm{d}^q\left(T, w\right)$$

$$\mathrm{f}\left(w; \mu, S(\overline{\phi})\right) = \sum_{q=1}^{Q_f} \Theta_f^q(\mu)\mathrm{f}^q\left(w; S(\overline{\phi})\right)$$

Reduced Basis Model

The RB method is aimed at constructing reduced order solutions $\overline{\Phi}^{N_f}$ and T^{N_T} such that

$$\mathrm{a}\left(\overline{\Phi}^{N_f}, \overline{\nu}; \mu, \Sigma(T^{N_T})\right) = \lambda_{eff} \mathrm{m}\left(\overline{\Phi}^{N_f}, \overline{\nu}; \mu, \Sigma(T^{N_T})\right)$$

$$\forall \overline{\nu} \in V^{N_f} \subset V^{\mathcal{N}}$$

$$\mathrm{d}\left(T^{N_T}, w; \mu\right) = \mathrm{f}\left(w; \mu, S(\overline{\phi}^{N_f})\right)$$

$$\forall w \in W^{N_T} \subset W^{\mathcal{N}}$$

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The reduced spaces V^{N_f} and W^{N_T} have dimensions N_f and N_T respectively and are defined as:

$$V^{N_f} = \mathrm{span}\left\{\xi_1^f, \ldots, \xi_{N_f}^f\right\} \qquad W^{N_T} = \mathrm{span}\left\{\xi_1^T, \ldots, \xi_{N_T}^T\right\}$$

Reduced Basis Model

$$\Phi^{N_f} = \sum_{i=1}^{N_f} \phi_{N,i} \xi_i^f \qquad T^{N_T} = \sum_{j=1}^{N_T} T_{N,j} \xi_j^T$$

$$\mathrm{a}(\mathcal{Z}_f \overline{\phi_N}, \overline{\nu}; \mu, \Sigma(\mathcal{Z}_T \overline{T}_N)) = \lambda_{eff}^N \mathrm{m}(\mathcal{Z}_f \overline{\phi_N}, \overline{\nu}; \mu, \Sigma(\mathcal{Z}_T \overline{T}_N))$$

$$\mathrm{d}(\mathcal{Z}_f \overline{T_N}, w; \mu) = \mathrm{f}(w; \mu, S(\mathcal{Z}_f \overline{\phi_N}))$$

$$\mathcal{Z}_f = [\xi_1^f | \ldots | \xi_{N_f}^f] \qquad \mathcal{Z}_T = [\xi_1^T | \ldots | \xi_{N_T}^T]$$

Offline Phase

Offline

- Run high fidelity computations to get snapshots.
- Construct matrices $\mathcal{Z}_f$ and $\mathcal{Z}_T$ required to build the matrices of ROM
- Matrices corresponding to the bilinear form of affine decompositions.

Online

- Multiply reduced matrices by proper coefficients and sum together.
- Solve the ROM.

Expected Outcomes and Results

- Evaluate the errors between the reactivity provided by ROM and FE solution and error vs. number of basis functions.

$$\epsilon_\lambda(\mu) = |\lambda_N(\mu) - \lambda_{\mathcal{N}}(\mu)|$$

- Evaluate the error between the neutron flux provided by the two models.

$$\epsilon_f = \int_\Omega (\Phi^{N_f} - \overline{\Phi})^2$$

- Evaluate the error between the temperature field provided by the two models.

$$\epsilon_T = \int_\Omega (T^{N_f} - T)^2$$

Expected Outcomes and Results

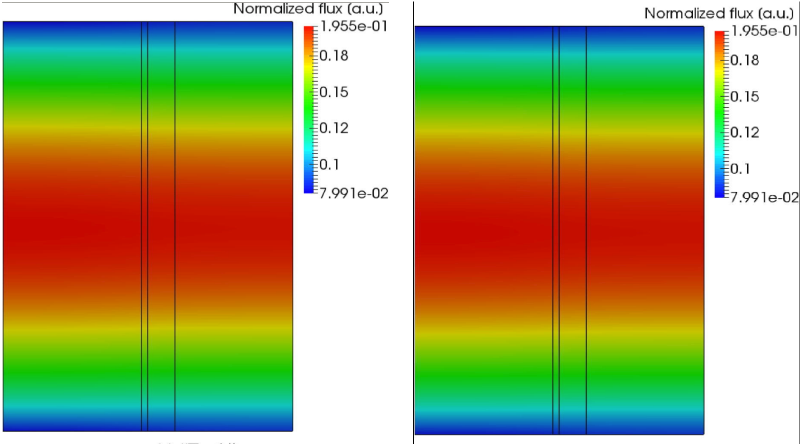
- Comparison of computational cost of ROM and FE model.

$$\text{Speed-up} = \frac{\text{Wall time of ROM calculation}}{\text{Wall time of FE simulation}}$$

- Computational break-even.

$$\text{Break-even} = \frac{\text{Total Offline time}}{\text{Time for 1 FE simulation}}$$

Sample Results



(a) "Truth" (b) ROM

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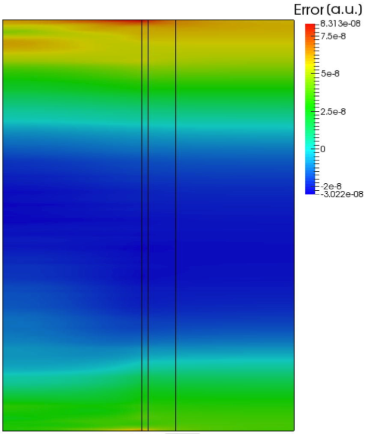

(c) Error