Project 2: Learning to Rank using Linear Regression

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Problem Statement:

In this project we are using machine learning to solve a famous problem of information retrieval i.e. Learning to Rank (LeToR) problem. We implement the linear regression model on two given data set. The learn to Rank (LeToR) data set has 69623 query-document pairs (rows) along with 46 features for each row. We also implement the model on the given synthetic data set which has 20000 entries. We first fit the linear regression model to each data set using the closed-form solution and then by using Stochastic Gradient Descent (SGD) Solution. After fitting the model we find the minimum difference of the ERMS value between the training and the validation data set and then report the test error.

We perform the below 4 steps to complete the task on both LeToR Data Set and Synthetic Data Set:

STEP #1 Data Partition

Here we partition both the data set. First, the LeToR data set is imported using 'genfromtext' function from the library 'NUMPY'. Then the loaded data is partitioned as per the below ratio:

Training Data Set – 80%

Validation Data Set - 10%

Test Data Set – 10%

Below table shows the exact distribution of the data:

| Set | Start | End |
|-------------------|--|--|
| Training Set | 0 | floor(0.8 * #records) |
| Validation Set | floor(0.8 * #records)+1 | floor(0.8 * #records) + floor(0.1* #records |
| Testing Set | floor(0.8 * #records) + floor(0.1* #records) +1 | End of File |

STEP #2

Find the Closed form Solution

To find the closed form solution for both the data set we perform the below steps:

- 1) Find the clusters for the given data.
- 2) Find the spreads for the given data.
- 3) Compute the design matrix.
- 4) Start with a random value for the hyper parameters and then keep updating as we do the hyper parameter tuning (in step 4).
- 5) Find the Closed form solution using the model and the hyper parameters.
- 6) Compute the ERMS values for the training, validation and test data set.

STEP #3

Find the Stochastic Gradient Solution (Using Early Stopping Algorithm)

- 1) Design matrix that has been computed earlier.
- 2) A functions for determining the model parameters has been defined. This function uses early stopping algorithm where the program stops when it hits the minimum validation error after the patience parameter is overridden.
- 3) Using the model parameters, we predict the target values of the test data.
- 4) Find the ERMS values using the predicted target values from the above model.

Parameters used for Early Stopping: patience = 10, validation Steps = 10, learning_rate=0.1, epochs=100

STEP #4

Perform Hyper Parameter Tuning

- 1) Parameter Tuning for both Closed Form and SGS is done and the values are updated.
- 2) Parameter Tuning is done by finding the min value of the difference between ERMS values of Training and Validation data sets. When the values for hyper parameters are obtained, ERMS for the test data is determined and is reported.

Hyper Parameters

We use the validation data set to perform hyper tuning of parameters. Finally, we take the below values for the various hyper parameters after performing the tuning.

For LeToR Data Set (Closed):

| Hyper Parameters | Symbol | Value |
|---------------------|--------|-------|
| Number of clusters | M/K | 14 |
| Regularization Term | λ | 0.01 |
| Learning Rate | η (τ) | 0.1 |

For LeToR Data Set (SGD):

| Hyper Parameters | Symbol | Value | |
|------------------------------------|---------|-------|--|
| Number of clusters | M/K | 12 | |
| Regularization Term | λ | 0.01 | |
| Learning Rate | η (τ) | 0.1 | |
| Parameters Used for Early Stopping | | | |
| Patience | Р | 10 | |
| Validation Steps | V steps | 10 | |
| Epoch | е | 100 | |

For Synthetic Data Set (Closed):

| Hyper Parameters | Symbol | Value |
|---------------------|--------|-------|
| | | |
| Number of clusters | M/K | 18 |
| | | |
| Regularization Term | λ | 0.01 |
| | | |
| Learning Rate | η(τ) | 0.1 |
| | " | |
| | | |

For Synthetic Data Set (SGD):

| Hyper Parameters | Symbol | Value | |
|------------------------------------|-----------|-------|--|
| Number of clusters | M/K | 10 | |
| Regularization Term | λ | 0.01 | |
| Learning Rate | η (τ) | 0.1 | |
| Parameters used for Early Stopping | | | |
| Patience | р | 10 | |
| Epoch | epoch | 100 | |
| Validation Steps | Val steps | 10 | |

ERMS Values:

As described in the steps above we computed the ERMS values as below:

For LeToR Data Set

| DATA SET | Closed Form Solution | Stochastic Gradient Solution |
|------------|----------------------|------------------------------|
| Training | 0.5708 | 0.569 |
| Validation | 0.5679 | 0.577 |
| Test | 0.5633 | 0.5673 |

For Synthetic Data Set

| DATA SET | Closed Form Solution | Stochastic Gradient Solution |
|------------|----------------------|------------------------------|
| | | |
| Training | 0.72 | 0.77 |
| | | |
| Validation | 0.73 | 0.79073 |
| | | |
| Test | 0.74 | 0.7940 |
| | | |

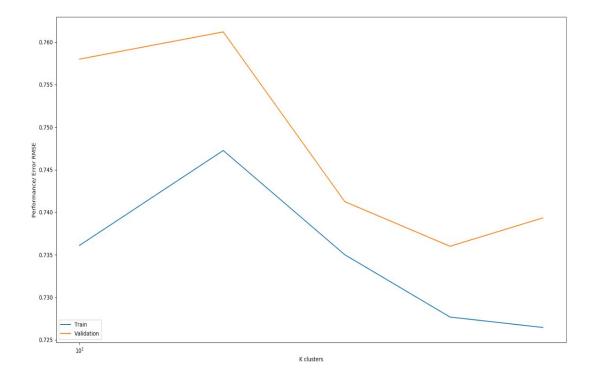
Printed out results from code:

```
personnumber: 5024/649
erms val close syn 0.7351207694405604
erms tr close syn 0.7258052639249162
erms test close syn 0.7463548141553978
KClose syn 18
LambdaCloseSyn 0.01
erms val sgd Syn 0.7907331989948544
erms train sgd Syn 0.7797079616439568
erms test sgd syn 0.7940543388466381
KSGD syn 10
Lambda SGD syn 0.31
erms_val_close_letor 0.5679089679521918
erms tr close letor 0.5708503755929183
erms test close letor 0.563320041561073
KClose syn 14
LambdaClose letor 0.01
^[^Terms_val_sgd_letor_0.5681804473998012
erms train sqd letor 0.5714685929325389
erms test sgd letor 0.5639527017881113
KSGD letor 14
Lambda SGD letor 0.01
```

```
erms_val_sgd_letor 0.5773629969727536
erms_train_sgd_letor 0.5696317424445312
erms_test_sgd_letor 0.5673284267930686
KSGD_letor 12
Lambda SGD letor 0.01
```

Train and Validation Error Comparison:

For fixed lambda =0.01



For Lambda = 0.01 to 1

