

# Optimization Modeling — Part 2/4

Deterministic models

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Areas of research:

- Multi-agent optimization: Bilevel programs, Game theory
- Optimization modeling: mainly focused on energy and environmental applications

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STOCHASTIC OPTIMIZATION

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# Overview

The main objective of this lecture is to get you to know the **framework and workflow of optimization and modeling**.

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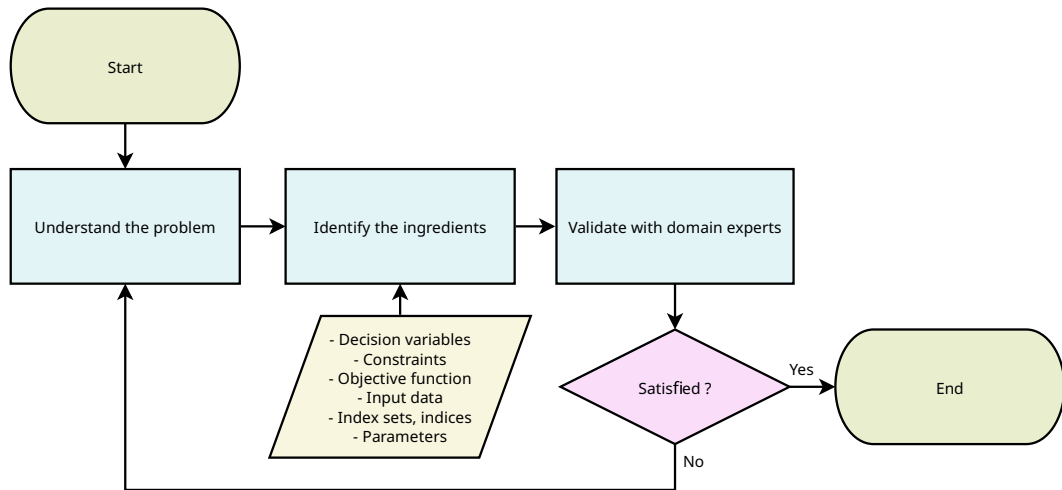
Conclusion

# What is optimization modeling ?

“Modeling is communication.”

Optimization modeling is the process of turning a real-life decision problem into a solvable mathematical program.

# Modeling workflow



# Common modeling mistakes

- Using or putting together wrong units (kW vs MW vs MWh).
- Missing some constraints (e.g., ramping, buffer time)
- Choosing objective that does not reflect the true goal.
- Forgetting integer decisions.
- Forgetting domain bounds.
- Over-simplifying.
- Writing nonlinear constraints when a linear equivalent exists.

# Modeling checklist

- List all ingredients
- Ensure variable units are consistent
- Write a tiny numerical example (mini-case)
- Check the solution for integrity
- Make a table/visualization summarizing everything

## Section 2

### Modeling examples/exercises



## Subsection 1

### Single time-step economic dispatch

# Example 1 — Single time-step economic dispatch

## Story

A system operator must supply a known demand at a single moment (e.g., one hour). There are several generation technologies available:

- gas
- coal
- peaker

Each technology has:

- a cost per MWh,
- a maximum production capacity,

We want to meet the demand at minimum cost.

# Example 1 — Single time-step economic dispatch

To do the modeling, first we fill up the following table.

Notation	Quantifier	Description
<b>Sets</b>		
$I$		The set of technologies, indexed with $i \in I$ .
<b>Parameters (Data)</b>		
$D$		The known demand (MW).
$C_i$	$\forall i \in I$	Cost per MWh of technology $i$ .
$P_i^{\max}$	$\forall i \in I$	Maximum capacity (MW).
<b>Decision variables</b>		
$u_i \geq 0$	$\forall i \in I$	How much the technology $i$ outputs (MW).
<b>Constraints</b>		
$\sum_{i \in I} u_i = D$		The production meets exactly the demand.
$0 \leq u_i \leq P_i^{\max}$	$\forall i \in I$	Capacity limits of each technology.
<b>Objective function</b>		
$\sum_{i \in I} C_i u_i \leftarrow \text{Minimize}$		The total generation cost summing up
$C_i u_i = \underbrace{C_i}_{\text{cost per MWh}} \times \underbrace{u_i}_{\text{power output}} \times \underbrace{1}_{\text{one hour}}$		

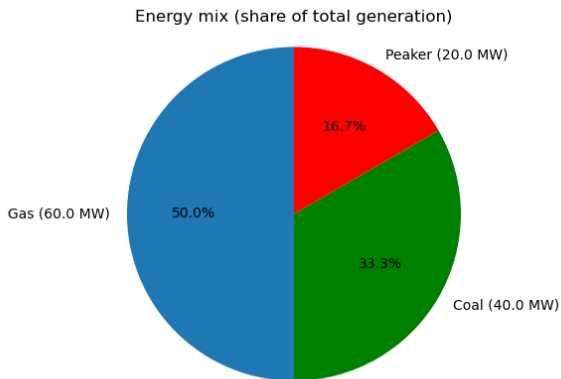
## Example 1 — Single time-step economic dispatch

Once we finished, the parameters section tells us the data we need for the model. In this case, the data is provided in the following table.

Technology	Cost (€/MWh)	Maximum capacity (MW)	Demand (MW)
Gas	20	60	120
Coal	40	40	
Peaker	120	100	

## Example 1 — Single time-step economic dispatch

Once the model is solved, we get this optimal energy mix.



## Subsection 2

Multiple time-step economic dispatch (independent case)

## Example 2 — Multiple time-step economic dispatch (independent case)

In this section, we make the case of Example 1 more complicated.

### Story

A system operator must supply a known demand during each 60 minutes window for a day.

There are several generation technologies available:

- gas
- coal
- peaker
- solar pv (supposed that the solar availability is known)

Each technology has:

- a cost per MWh,
- a maximum production capacity,

We want to meet the demand at minimum cost.

## Example 2 — Multiple time-step economic dispatch (independent case)

Notation	Quantifier	Description
<b>Sets</b>		
$I$		The set of technologies excluding solar pv, indexed with $i \in I$ .
$T$		The set of time-steps, indexed with $t \in T$ .
<b>Parameters (Data)</b>		
$\Delta t$		The duration of between time-steps.
$D_t$	$\forall t \in T$	The known demand at time $t$ (MW).
$C_i$	$\forall i \in I$	Cost per MWh of technology $i$ .
$P_i^{\max}$	$\forall i \in I$	Maximum capacity (MW).
$S_t$	$\forall t \in T$	Available solar pv power (MW) at the time-step $t$ (probably from the forecast).



## Example 2 — Multiple time-step economic dispatch (independent case)

Notation	Quantifier	Description
<b>Decision variables</b>		
$u_{i,t} \geq 0$	$\forall i \in I, \forall t \in T$	How much the technology $i$ outputs during the time-step $t$ (MW).
$u_t^{\text{pv}} \geq 0$	$\forall t \in T$	Solar pv power injected during the time-step $t$ (MW).
<b>Constraints</b>		
$\sum_{i \in I} u_{i,t} + u_t^{\text{pv}} = D_t$	$\forall t \in T$	The production meets exactly the demand at each time-step $t$ .
$0 \leq u_{i,t} \leq P_i^{\text{max}}$	$\forall i \in I, \forall t \in T$	Capacity limits of each technology.
$0 \leq u_t^{\text{pv}} \leq S_t$	$\forall t \in T$	Capacity limit of solar pv.
<b>Objective function</b>		
$\sum_{i \in I} C_i u_i \Delta t \leftarrow \text{Minimize}$		The total production cost.

## Example 2 — Multiple time-step economic dispatch (independent case)

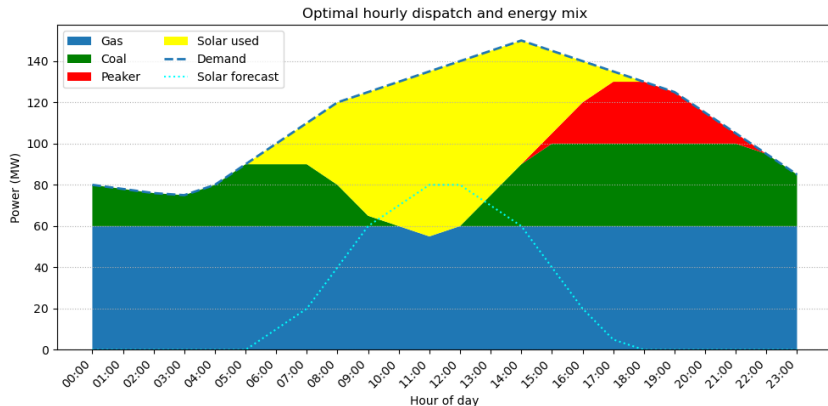
We now need some data, the table earlier and also the hourly demand and solar pv forecast.

Technology	Cost (€/MWh)	Maximum capacity (MW)
Gas	20	60
Coal	40	40
Peaker	120	100

Hour	Demand (MW)	Solar forecast (MW)
00:00	80	0
01:00	78	0
02:00	76	0
03:00	75	0
04:00	80	0
05:00	90	0
06:00	100	10
07:00	110	20
08:00	120	40
09:00	125	60
10:00	130	70
11:00	135	80
12:00	140	80
13:00	145	70
14:00	150	60
15:00	145	40
16:00	140	20
17:00	135	5
18:00	130	0
19:00	125	0
20:00	115	0
21:00	105	0
22:00	95	0
23:00	85	0

## Example 2 — Multiple time-step economic dispatch (independent case)

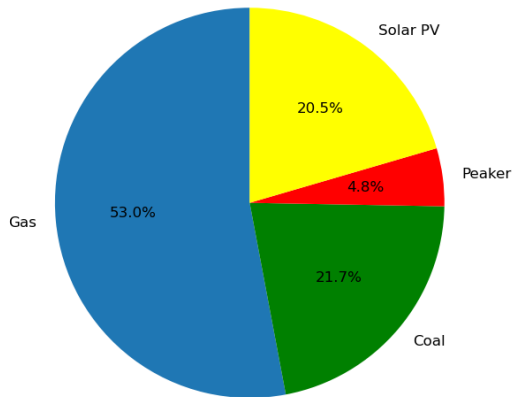
Once the model is solved, we get this optimal energy dispatch.



## Example 2 — Multiple time-step economic dispatch (independent case)

Once the model is solved, we get this optimal energy mix.

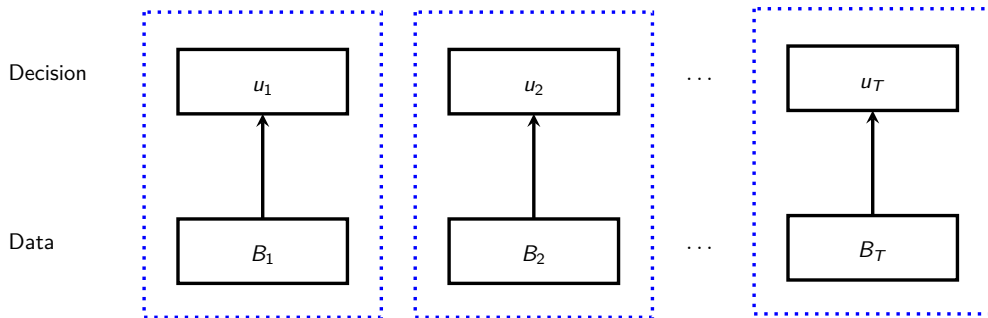
Energy mix over 24 hours



## Example 2 — Multiple time-step economic dispatch (independent case)

The characteristic of this decision model is that the decisions are time-independent, *i.e.*, decision at a time-step  $t$  has no influence on the future time-step  $t + 1$ .

This behavior is illustrated in the following diagram, where each blue dotted box is actually an independent problem.



## Subsection 3

Multiple time-step economic dispatch (dynamic case)

## Example 3 — Multiple time-step economic dispatch (dynamic case)

In this example, we need to introduce an additional ingredient — the **states**.

We write generically a **state space**  $\mathcal{X}$ , with the state element  $x \in \mathcal{X}$ .

Typically, a **current state**  $x \in \mathcal{X}$  and a **decision**  $u \in \mathcal{U}$  together bring us into a new state through a **dynamical equation**

$$x^+ = f(x, u),$$

where  $f : \mathcal{X} \times \mathcal{U} \rightarrow \mathcal{X}$  is the operator that describes this dynamics.

We may take the battery level as an example. In this case, the state  $x \in \mathcal{X} = \mathbb{R}$  is the battery level,  $(u_{\text{charge}}, u_{\text{discharge}}) \in \mathcal{U} = \mathbb{R}^2$  is the decided amounts of charge and discharge and

$$x^+ = f(x, u) = x + u_{\text{charge}} - u_{\text{discharge}}$$

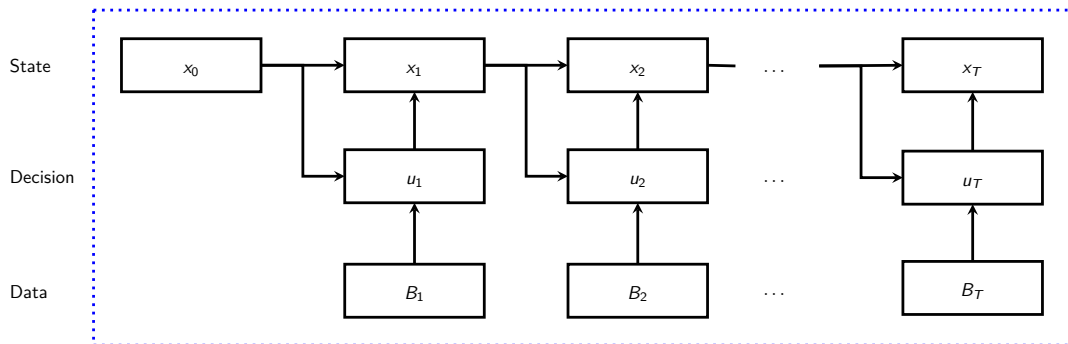
describes the dynamics.

## Example 3 — Multiple time-step economic dispatch (dynamic case)

In this example, we shall explore the case where a decision at time-step  $t$  alters some states that affects a decision at  $t + 1$ .

Here the decision  $u_{t+1}$  is made with the knowledge of the state  $x_t$ , and the state  $x_{t+1}$  is updated with a dynamical equation  $x_{t+1} = f(x_t, u_t)$ .

This is best illustrated by the following diagram.





## Example 3 — Multiple time-step economic dispatch (dynamic case)

### Story

A system operator must supply a known demand during each 60 minutes window for a day.

There are several technologies available:

- gas
- coal
- peaker
- solar pv (supposed that the solar availability is known)
- a battery unit.

Each technology has:

- a cost per MWh,
- a maximum capacity,
- a charge/discharge capacity and efficiency for the battery.

We want to meet the demand at minimum cost.

## Example 3 — Multiple time-step economic dispatch (dynamic case)

Notation	Quantifier	Description
<b>Sets</b>		
$I$		The set of technologies excluding solar pv, indexed with $i \in I$ .
$T$		The set of time-steps, indexed with $t \in T$ .
<b>Parameters (Data)</b>		
$\Delta t$		The duration of between time-steps.
$D_t$	$\forall t \in T$	The known demand at time $t$ (MW).
$C_i$	$\forall i \in I$	Cost per MWh of technology $i$ .
$P_i^{\max}$	$\forall i \in I$	Maximum capacity (MW).
$S_t$	$\forall t \in T$	Available solar pv power (MW) at the time-step $t$ .
$x_0$		The initial energy state of the battery (MWh).
$x^{\max}$		The maximum energy capacity of the battery (MWh).
$P_+^{\max}$		The maximum charging power of the battery (MW).
$P_-^{\max}$		The maximum discharging power of the battery (MW).
$\eta_+ \in [0, 1]$		The charging efficiency of the battery.
$\eta_- \in [0, 1]$		The discharging efficiency of the battery.

## Example 3 — Multiple time-step economic dispatch (dynamic case)

Notation	Quantifier	Description
<b>Decision and state variables</b>		
$u_{i,t} \geq 0$	$\forall i \in I, \forall t \in T$	How much the technology $i$ outputs during the time-step $t$ (MW).
$u_t^{\text{pv}} \geq 0$	$\forall t \in T$	Solar pv power injected during the time-step $t$ (MW).
$u_t^+ \geq 0$	$\forall t \in T$	Charging power of the battery during the time-step $t$ (MW).
$u_t^- \geq 0$	$\forall t \in T$	Discharging power of the battery during the time-step $t$ (MW).
$x_t \geq 0$	$\forall t \in T$	The state of charge of the battery at the time-step $t$ (MWh).

## Example 3 — Multiple time-step economic dispatch (dynamic case)

Notation	Quantifier	Description
<b>Constraints</b>		
$\sum_{i \in I} u_{i,t} + u_t^{\text{pv}} + u_t^- = D_t + u_t^+$	$\forall t \in T$	Power balance at each time-step $t$ .
$0 \leq u_{i,t} \leq P_i^{\text{max}}$	$\forall i \in I, \forall t \in T$	Capacity limits of each technology.
$0 \leq u_t^{\text{pv}} \leq S_t$	$\forall t \in T$	Capacity limit of solar pv.
$0 \leq u_t^+ \leq P_+^{\text{max}}$	$\forall t \in T$	The charging capacity at each time-step $t$ .
$0 \leq u_t^- \leq P_-^{\text{max}}$	$\forall t \in T$	The discharging capacity at each time-step $t$ .
$0 \leq x_t \leq x^{\text{max}}$	$\forall t \in T$	Energy capacity limit at each time-step $t$ .
$x_t = x_{t-1} + \eta_+ u_t^+ \Delta t - \frac{1}{\eta_-} u_t^- \Delta t$	$\forall t \in T$	The battery's state-of-charge dynamical equation.
<b>Objective function</b>		
$\sum_{i \in I} C_i u_i \Delta t \leftarrow \text{Minimize}$		The total production cost.

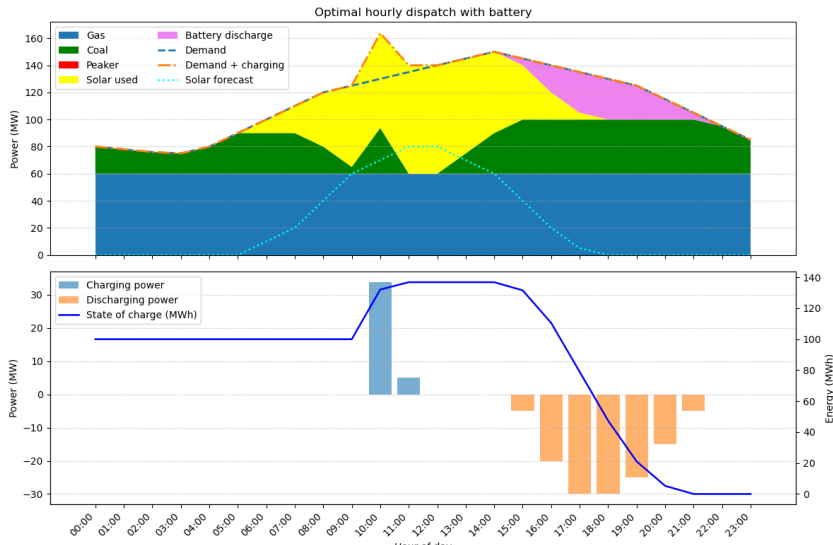
## Example 3 — Multiple time-step economic dispatch (dynamic case)

In addition to the data required in Example 2, we need the following battery data.

Parameter	Value
$x^{\max}$	200 MWh
$p_+^{\max}$	50 MW
$p_-^{\max}$	50 MW
$\eta_+$	0.95
$\eta_-$	0.95
$x_0$	100 MWh

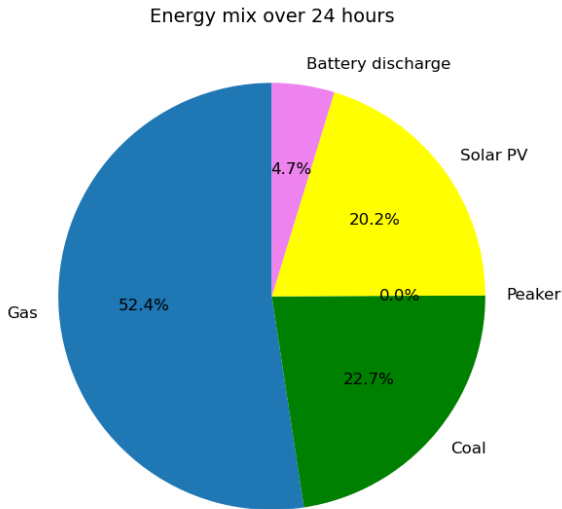
# Example 3 — Multiple time-step economic dispatch (dynamic case)

The following is the visualization of the optimal economic dispatch.



## Example 3 — Multiple time-step economic dispatch (dynamic case)

The following is the visualization of the optimal economic dispatch.



## Subsection 4

### Transmission line expansion



## Example 4 — Transmission line expansion (DC power flow)

### Story

- Some lines are now frequent bottlenecks, forcing power to take longer or less efficient paths and, in extreme cases, causing unavoidable load shedding when flows cannot reach certain regions.
- Building new lines can relieve these bottlenecks, but each project is costly and must be justified by tangible operational improvements.
- The operator needs a systematic way to decide which candidate lines to build, balancing investment costs against gains in reliability, reduced congestion, and avoided load shedding.

## Example 4 — Transmission line expansion (DC power flow)

Notation	Quantifier	Description
<b>Sets</b>		
$N$		Set of buses, indexed by $i$ and $j$ .
$L^{\text{ex}}$		Set of existing transmission lines, indexed by $\ell$
$L^{\text{cand}}$		Set of candidate transmission lines, indexed by $k$
<b>Parameters</b>		
$d_i$	$\forall i \in N$	Demand at bus $i$ (MW).
$g_i$	$\forall i \in N$	Net generation at bus $i$ (MW).
$c_k^{\text{inv}}$	$\forall k \in L^{\text{cand}}$	Investment cost of line $k$ .
$c_i^{\text{shed}}$	$\forall i \in N$	Penalty cost of load shedding.
$\bar{F}_\ell$	$\forall \ell \in L^{\text{ex}}$	Capacity of existing line $\ell$ (MW).
$\bar{F}_k^{\text{new}}$	$\forall k \in L^{\text{cand}}$	Capacity of candidate line $k$ (MW).
$B_\ell$	$\forall \ell \in L^{\text{ex}}$	Susceptance of existing line $\ell$ .
$B_k^{\text{new}}$	$\forall k \in L^{\text{cand}}$	Susceptance of candidate line $k$ .
$r = 1$		Reference bus for angle fixation.

## Example 4 — Transmission line expansion (DC power flow)

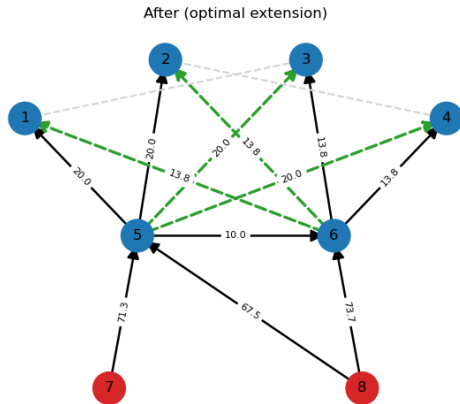
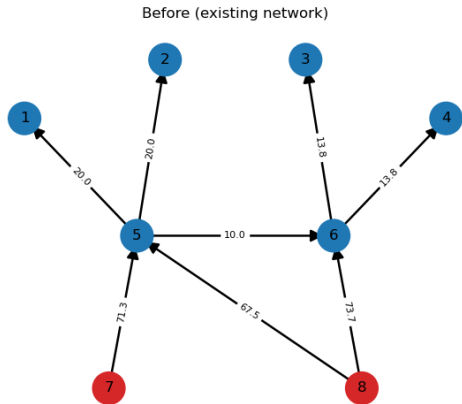
Notation	Quantifier	Description
<b>Decision variables</b>		
$u_i^\theta \in \mathbb{R}$	$\forall i \in N$	Voltage angle at bus $i$ (rad).
$u_\ell^f \in \mathbb{R}$	$\forall \ell \in L^{\text{ex}}$	Power flow on existing line $\ell$ (MW).
$u_k^{f,\text{new}} \in \mathbb{R}$	$\forall k \in L^{\text{cand}}$	Power flow on candidate line $k$ (MW).
$u_i^{\text{shed}} \in \mathbb{R}$	$\forall i \in N$	Load shedding ( $> 0$ ) /spilling ( $< 0$ ) at bus $i$ (MW).
$u_k^{\text{inv}} \in \{0, 1\}$	$\forall k \in L^{\text{cand}}$	1 if line $k$ is built.

## Example 4 — Transmission line expansion (DC power flow)

Notation	Quantifier	Description
<b>Constraints</b>		
$g_i - d_i + u_i^{\text{shed}} = \sum_{\ell=(i,j)} u_\ell^f - \sum_{\ell=(j,i)} u_\ell^f + \sum_{k=(i,j)} u_k^{f,\text{new}} - \sum_{k=(j,i)} u_k^{f,\text{new}}$	$\forall i \in N$	Power balance.
$u_\ell^f = B_\ell (u_i^\theta - u_j^\theta)$	$\forall \ell = (i,j)$	DC flow on existing lines.
$u_k^{f,\text{new}} = B_k^{\text{new}} (u_i^\theta - u_j^\theta)$	$\forall k = (i,j)$	DC flow on candidate lines.
$0 \leq u_\ell^f \leq \bar{F}_\ell$	$\forall \ell \in L^{\text{ex}}$	Capacity of existing lines.
$0 \leq u_k^{f,\text{new}} \leq \bar{F}_k^{\text{new}} u_k^{\text{inv}}$	$\forall k \in L^{\text{cand}}$	Capacity of candidate lines.
$u_r^\theta = 0$		Reference bus angle.
<b>Objective function</b>		
$\sum_{k \in L^{\text{cand}}} c_k^{\text{inv}} u_k^{\text{inv}} + \sum_{i \in N} c_i^{\text{shed}} (u_i^{\text{shed}})^2$	Minimize	Minimize investment + quadratic penalty.

## Example 4 — Transmission line expansion (DC power flow)

The optimal expansion can be illustrated as follows.



## Section 3

## Conclusion

# Key takeaways

- Optimization is a **framework** for turning decision problems into mathematics.
- Modeling is a **workflow**, and it usually requires iterating many times through the processes.
- Start simple and clean, then add complexity.
- A mini-case is usually useful especially in a large-scope problem.
- Always check the results and verify if it is physically realistic.

# What's more ?

- Nonlinear and nonconvex programs (reformulate, simplify, or face it)
- Robustness (sensitivity analysis)
- Uncertainty (stochastic modeling)



-» Continue to **Part 3**.