Optimization Modeling — Part 3/4

Recalls on probability theory

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Areas of research:
- Multi-agent optimization: Bilevel programs, Game theory
- Optimization modeling: mainly focused on energy and environmental applications

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Overview

The main objective of this lecture is to make a quick summary on the portion of probability theory that is needful for stochastic optimization modeling.

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Section 1

Basic probability

Basic concepts

Probability theory is a branch of mathematics that is used to handle **uncertainty** and describe **likeliness**.

A probability space

To systematically describe likeliness, we need few ingredients.

Notation	**Terminology**	**Description**
Ω	Sample space	The set of all possible outcomes.
$\mathcal{F} \subset 2^{\Omega}$	Information set	Subsets of Ω (called events) that one could actually observe and assign a probability.
$\mathbb{P} : \mathcal{F} \to [0, 1]$	Probability measure	A function that assigns a probability to an event.

A probability space

To rigorously formulate a probability space, we need the following assumptions.

- $\mathcal{F}$ is a σ-field, *i.e.* must satisfy:
  - $\emptyset, \Omega \in \mathcal{F}$.
  - If $\{A_i\}_{i \in I} \subset \mathcal{F}$ is countable, then $\bigcup_{i \in I} A_i \in \mathcal{F}$.
  - If $A \in \mathcal{F}$, then $A^{\complement} \in \mathcal{F}$.
- $\mathbb{P}$ is a measure, *i.e.* must satisfy:
  - $\mathbb{P}(\Omega) = 1$.
  - If $\{A_i\}_{i \in I}$ are disjoint, then $\mathbb{P}\left(\bigcup_{i \in I} A_i\right) = \sum_{i \in I} \mathbb{P}(A_i)$.

The triplet $(\Omega, \mathcal{F}, \mathbb{P})$ is called a **probability space**.

In many cases, Ω is a finite set and $\mathcal{F} = 2^{\Omega}$, *i.e.* every thing is an event and be assigned with a probability.

In some complex cases, we make a sequence of decisions and at each decision we have a different information set (different σ-field).

Some intuitions

Consider a case of **rolling a fair die**.
The sample space is obviously $\Omega = \{1, 2, 3, 4, 5, 6\}$.

What is the information set ?

-» Continue to **Part 3**.