On Witsenhausen's paradigm to multi-agent optimization

with a touch of energy applications

Emerges from a PGMO project with
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Areas of research:

- Bilevel programs, Multi-agent network optimization, Games
- Optimization modeling for energy and environmental applications
- Nonsmooth geometry in optimization

What is and isn't here.

You will see ...

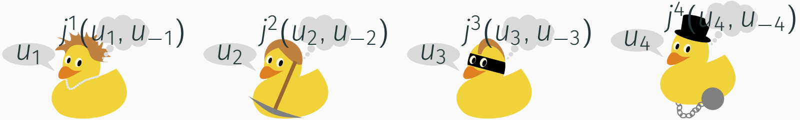
- some background on multi-agent optimization, [10 minutes]
- bilevel structure, [5 minutes]
- Witsenhausen's modeling framework, [30 minutes]
- some electricity demand response models. [15 minutes]

You will not see ...

- proofs,
- compiled models,
- numerics,
- too many references.

Multi-agent problems configuration

Multi-agent problem



Important features of a multi-agent problems:

- Agents are decision-makers.
  - Agents $\mathsf{a} \in \mathsf{A}$.
  - Decision variable $u_\mathsf{a} \in \mathbb{U}_\mathsf{a} = \mathbb{R}^{n_\mathsf{a}}$, constrained to $\mathbb{U}_\mathsf{a}^{\mathrm{ad}} \subset \mathbb{U}_\mathsf{a}$ or more genearlly, to $\mathbb{U}_\mathsf{a}^{\mathrm{ad}} : \mathbb{U}_{-\mathsf{a}} \to 2^{\mathbb{U}_\mathsf{a}}$.
- An agent's decision affects other agents' outcomes.
  - Cost/payoff function $j^\mathsf{a} : \mathbb{U}_\mathsf{a} \times \mathbb{U}_{-\mathsf{a}} \to \mathbb{R}$.
- Many possible configurations.
  - Nash or Stackelberg or etc.

The first configuration — Simultaneous decisions

Nash equilibrium — The choices that each agent has no incentive to deviate.

$$\min_{u_1 \in \mathbb{U}_1^{\mathrm{ad}}} / \max \ j^1(u_1, \ldots, u_N)$$

$\leftrightarrow$

$$\min_{u_2 \in \mathbb{U}_2^{\mathrm{ad}}} / \max \ j^2(u_1, \ldots, u_N)$$

$\leftrightarrow \cdots \leftrightarrow$

$$\min_{u_N \in \mathbb{U}_N^{\mathrm{ad}}} / \max \ j^N(u_1, \ldots, u_N)$$

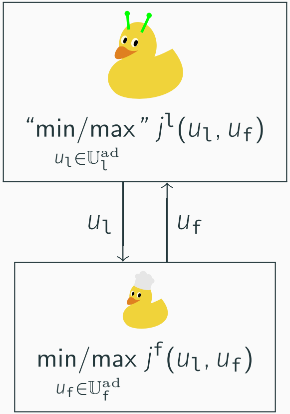
- Agents: $\mathbf{a} \in \mathsf{A} = \{1, \ldots, N\}$.

A toy example

A couple (Husband and Wife) is deciding whether to go to Paris or staying at home by maximizing their payoff functions.

Husband \ Wife	Paris	Home
Paris	$(100, 100)$	$(20, -100)$
Home	$(120, 30)$	$(\mathbf{50}, \mathbf{50})$

Stackelberg equilibrium — The follower react optimally after observing the leader's move.



- Agents: $\mathtt{A} = \{\mathtt{l}, \mathtt{f}\}$, classified into a leader $\mathtt{l}$ and a follower $\mathtt{f}$.

The Paris-Home example revisited (Wife leads)

Husband \ Wife	Paris	Home
Paris	$(100, 100)$	$(20, -100)$
Home	$(120, 30)$	$(\mathbf{50}, \mathbf{50})$

The Paris-Home example revisited (Husband leads)

Husband \ Wife	Paris	Home
Paris	(**100**, **100**)	(20, −100)
Home	(120, 30)	(50, 50)

The third configuration — Sequential simultaneous decisions — Multi-leader, multi-follower

$$\text{“}\min/\max\text{”}_{u_{l_1}\in\mathbb{U}^{\mathrm{ad}}_{l_1}} \; j^{l_1}\big((u_l)_{l\in L},(u_f)_{f\in F}\big)$$

$$\leftrightarrow \cdots \leftrightarrow$$

$$\text{“}\min/\max\text{”}_{u_{l_N}\in\mathbb{U}^{\mathrm{ad}}_{l_N}} \; j^{l_N}\big((u_l)_{l\in L},(u_f)_{f\in F}\big)$$

$$u_L = (u_l)_{l\in L} \downarrow \qquad \uparrow u_F = (u_f)_{f\in F}$$

$$\min/\max_{u_{f_1}\in\mathbb{U}^{\mathrm{ad}}_{f_1}} \; j^{f_1}\big((u_l)_{l\in L},(u_f)_{f\in F}\big)$$

$$\leftrightarrow \cdots \leftrightarrow$$

$$\min/\max_{u_{f_N}\in\mathbb{U}^{\mathrm{ad}}_{f_N}} \; j^{f_N}\big((u_l)_{l\in L},(u_f)_{f\in F}\big)$$

- Agents: $A = L \cup F$, $\quad L = \{l_1, \dots, l_N\}$, $\quad F = \{f_1, \dots, f_M\}$.

The special case of third configuration — Sequential simultaneous decisions — Single-leader, multi-follower

$$\underset{u_{\mathtt{l}} \in \mathbb{U}^{\mathrm{ad}}_{\mathtt{l}}}{\text{“}\min/\max\text{”}} \; j^{\mathtt{l}}\big(u_{\mathtt{l}}, (u_{\mathtt{f}})_{\mathtt{f} \in \mathtt{F}}\big)$$

$$u_{\mathtt{l}} \downarrow \qquad \uparrow u_{\mathtt{F}} = (u_{\mathtt{f}})_{\mathtt{f} \in \mathtt{F}}$$

$$\min_{u_{\mathtt{f}_1} \in \mathbb{U}^{\mathrm{ad}}_{\mathtt{f}_1}}/\max \; j^{\mathtt{f}_1}\big((u_{\mathtt{l}})_{\mathtt{l} \in \mathtt{L}}, (u_{\mathtt{f}})_{\mathtt{f} \in \mathtt{F}}\big) \quad \leftrightarrow \cdots \leftrightarrow$$

$$\min_{u_{\mathtt{f}_N} \in \mathbb{U}^{\mathrm{ad}}_{\mathtt{f}_N}}/\max \; j^{\mathtt{f}_N}\big((u_{\mathtt{l}})_{\mathtt{l} \in \mathtt{L}}, (u_{\mathtt{f}})_{\mathtt{f} \in \mathtt{F}}\big)$$

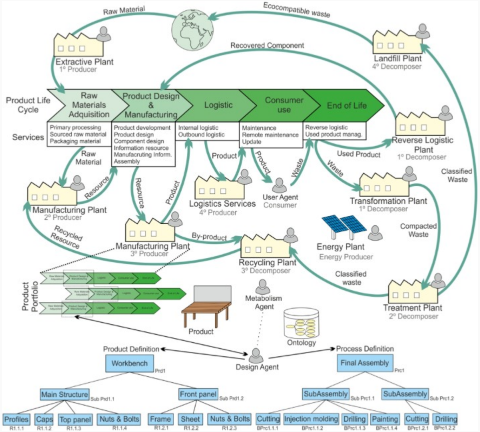
- Agents: $\mathtt{A} = \mathtt{L} \cup \mathtt{F}, \quad \mathtt{L} = \{\mathtt{l}\}, \quad \mathtt{F} = \{\mathtt{f}_1, \ldots, \mathtt{f}_M\}$.

Research programs

- Existence and stability of solutions.
- Optimality conditions.
- Reformulation techniques and solutioin methods.
- Numerical techniques.
- Applications. [Some of my projects in the next slides.]

Eco-Industrial Park (EIP) water network: NRCT^{granted}, ENSIACET

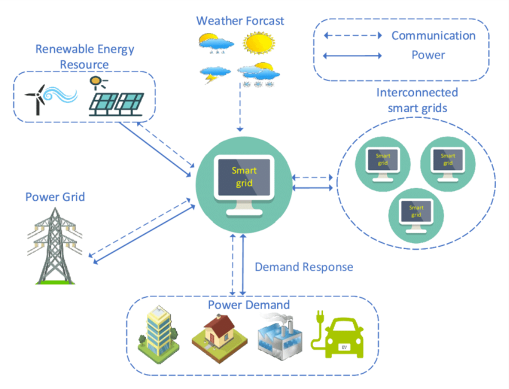
Design circulatory scheme in an industrial park and manage flows of resources.



- Leader: EIP authority
  - Objective: Minimize clean resources and contamination (Ecological).
  - Constraints: Resouce limitation, Site limitation, etc.
- Followers: Factories
  - Objective: Minimize payment (Economical).
  - Constraints: Circulatory limitation, Contamination level, Inventory, etc.

Transition to clean electricity: proposed to EGAT, MEA, PEA.

A model that interconnects seller (EGAT), load aggregators (MEA and PEA) and end users.

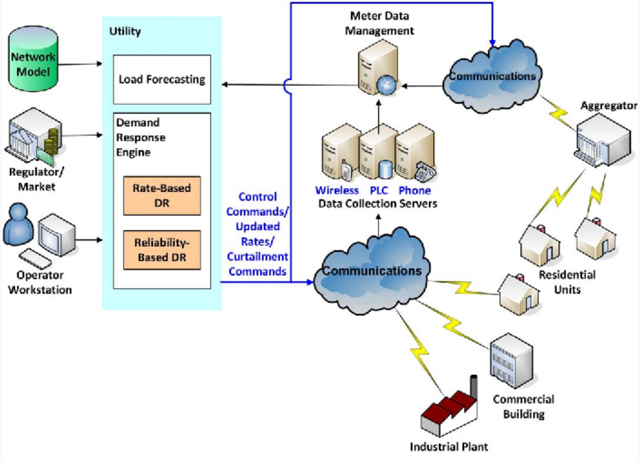


- Leader: Seller
  - Objective: Maximize marginal profit (Economical).
  - Constraints: Technology, Capacity, Supply level, etc.
- Followers: Consumers.
  - Objective: Minimize payment.
  - Constraints: Needs, Ability to shift, Technology, etc.

Efficient train control: WCE, NRCT^{granted}

Bilevel timetable design w.r.t. minimized mechanical efforts in the traction.

Information and knowledge structure modeling in the process of demand response.



Bilevel structure again

Mathematical formulation with single leader and single follower

The formal definition of an optimistic bilevel program.

$$
\begin{aligned}
\min_{u_{\mathtt{l}}, \hat{u}_{\mathtt{f}}} \quad & j^{\mathtt{l}}(u_{\mathtt{l}}, \hat{u}_{\mathtt{f}}) \\
\text{s.t.} \quad & u_{\mathtt{l}} \in \mathbb{U}_{\mathtt{l}}^{\mathrm{ad}} \\
& \hat{u}_{\mathtt{f}} \in \mathrm{Opt}(u_{\mathtt{l}}) \\
& \mathrm{Opt}(u_{\mathtt{l}}) := \operatorname*{argmin}_{u_{\mathtt{f}}} \{ j^{\mathtt{f}}(u_{\mathtt{l}}, u_{\mathtt{f}}) \mid u_{\mathtt{f}} \in \mathbb{U}_{\mathtt{f}}^{\mathrm{ad}}(u_{\mathtt{l}}) \}.
\end{aligned}
$$

Multiple timesteps

Suppose that a decision is made at each time step:

$$u_{\mathtt{a}} = (u_{\mathtt{a},1}, u_{\mathtt{a},2}, \dots, u_{\mathtt{a},T}) \quad \mathtt{a} = \mathtt{l}, \mathtt{f},$$
$$t \in \mathbb{T} = \{1, 2, \dots, T\}.$$

Then the bilevel program reads

$$\begin{aligned}
\min_{(u_{\mathtt{l},t}, \hat{u}_{\mathtt{f},t})_{t\in\mathbb{T}}} \quad & j^{\mathtt{l}}((u_{\mathtt{l},t}, \hat{u}_{\mathtt{f},t})_{t\in\mathbb{T}}) \\
\text{s.t.} \quad & (u_{\mathtt{l},t})_{t\in\mathbb{T}} \in \mathbb{U}_{\mathtt{l}}^{\mathrm{ad}} \\
& (\hat{u}_{\mathtt{f},t})_{t\in\mathbb{T}} \in \mathrm{Opt}((u_{\mathtt{l},t})_{t\in\mathbb{T}}) \\
& \mathrm{Opt}((u_{\mathtt{l},t})_{t\in\mathbb{T}}) := \operatorname*{argmin}_{(u_{\mathtt{f},t})_{t\in\mathbb{T}}} \{ j^{\mathtt{f}}((u_{\mathtt{l},t}, u_{\mathtt{f},t})_{t\in\mathbb{T}}) \mid u_{\mathtt{f}} \in \mathbb{U}_{\mathtt{f}}^{\mathrm{ad}}((u_{\mathtt{l},t})_{t\in\mathbb{T}}) \}.
\end{aligned}$$

The model acts as if **the leader makes a decision for all $t \in \mathbb{T}$ ahead of time and signals all of them to the follower.**

Dissolution for resolution

A classical way to solve a bilevel program numerically is to replace the lower-level problem with its KKT conditions.

$$\begin{aligned} \min_{u_\mathfrak{l}, \hat{u}_\mathfrak{f}} \quad & j^\mathfrak{l}(u_\mathfrak{l}, \hat{u}_\mathfrak{f}) \\ \text{s.t.} \quad & u_\mathfrak{l} \in \mathbb{U}_\mathfrak{l}^{\mathrm{ad}} \\ & \hat{u}_\mathfrak{f} \in \mathrm{Opt}(u_\mathfrak{l}) \end{aligned} \quad \underset{\text{Lower-level convexity}}{\overset{\text{Lower-level C.Q.}}{\Longrightarrow \atop \Longleftarrow}} \quad \begin{aligned} \min_{u_\mathfrak{l}, \hat{u}_\mathfrak{f}} \quad & j^\mathfrak{l}(u_\mathfrak{l}, \hat{u}_\mathfrak{f}) \\ \text{s.t.} \quad & u_\mathfrak{l} \in \mathbb{U}_\mathfrak{l}^{\mathrm{ad}} \\ & \hat{u}_\mathfrak{f} \in \mathrm{KKT}[\mathrm{Opt}(u_\mathfrak{l})] \end{aligned}$$

Non-tractable **MPCC: Nonconvex**

Questions

Can we do better . . . by

- considering **informational constraints**, *e.g.*

$$\sigma(u_{\mathtt{f},t}) \subset \sigma(u_{\mathtt{l},t}, u_{\mathtt{l},t-1} u_{\mathtt{f},t-1}, \dots, u_{\mathtt{l},1}, u_{\mathtt{f},1}),$$

- then, dissolving the **bilevel problem** (vertical) into a **Nash** (horizontal) ?

What we have seen . . .

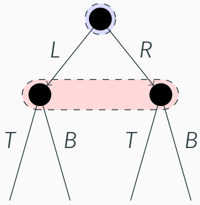
- Configurations of multi-agent optimization problems (**vertical** and **horizontal**).
- Some solution concepts.
- Some difficulties from both **Modeling** and **Technical** perspectives.

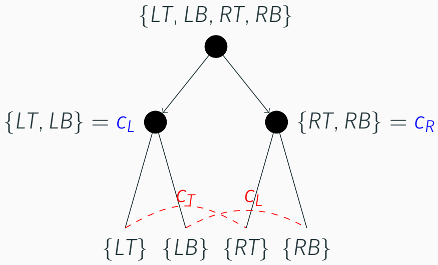
What's next . . . ?

Witsenhausen model with information structures.

Witsenhausen models (W-models)

Some facts





- A Witsenhausen model (W-model) is an extension of Kuhn's extensive form games.
- Belongs the game theory with information (not classically to optimization).
- Pretty much unknown.
- Provides a flexible framework for modeling with information.
- Does not help with numerics.

The program

- Game form (Interactions)
  - Agents, Players, Information, Actions, Strategies, Playability
- Preferences (Optimization)
  - Criterion function, Beliefs, Normal-form games

Witsenhausen models (W-models)

Game form

Agents and Players

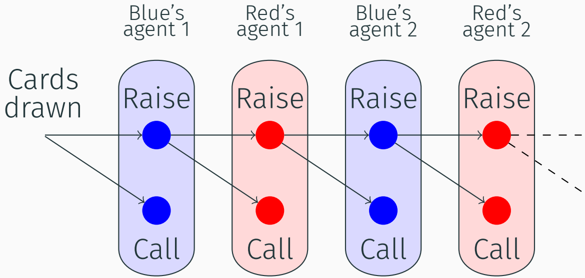
Agent $\neq$ Player

- **Agents** are **decision makers**.
- Each agent makes a decision at most **once**.
- A **player** is composed her **agents** (possibly one).

Agents and Players

Two players: Blue and Red.

Each player has multiple **agents**.



W-models (Agents, Players, Uncertainty, Actions)

- Agents and players
  - The set A of agents a.
  - The set P of players p, which is a partition of A.
- Uncertainty (Nature)
  - **Exogenous nature:** a measurable space $(\Omega^{\mathsf{e}}, \mathcal{F}^{\mathsf{e}})$.
  - **Player's type:** a measurable space $(\Omega^{\mathsf{p}}, \mathcal{F}^{\mathsf{p}})$
- Actions
  - The measurable space $(\mathbb{U}_{\mathsf{a}}, \mathcal{U}_{\mathsf{a}})$ of actions of each $\mathsf{a} \in \mathsf{A}$,
    where the σ-field $\mathcal{U}_{\mathsf{a}}$ represents what could be observed externally.

All the ingredients boils down to creating a configuration (the state-of-the-art) of a system.

- Configuration
  - The **configuration space** is defined as the product

$$\mathbb{H} = \Omega^{e} \times \prod_{p \in P} \Omega^{p} \times \prod_{a \in A} \mathbb{U}_{a}$$

together with the **information field**

$$\mathcal{H} = \mathcal{F}^{e} \otimes \bigotimes_{p \in P} \mathcal{F}^{p} \otimes \bigotimes_{a \in A} \mathcal{U}_{a}.$$

An agent makes a decision at the disposal of her knowledge (information).

- Information fields
  - The information of an agent **a** is represented by a subfield $\mathcal{I}_{\mathsf{a}}$ of $\mathcal{H}$.
  - We insist that $\mathcal{I}_{\mathsf{a}}$ is a σ-field over the configuration space $\mathbb{H}$.

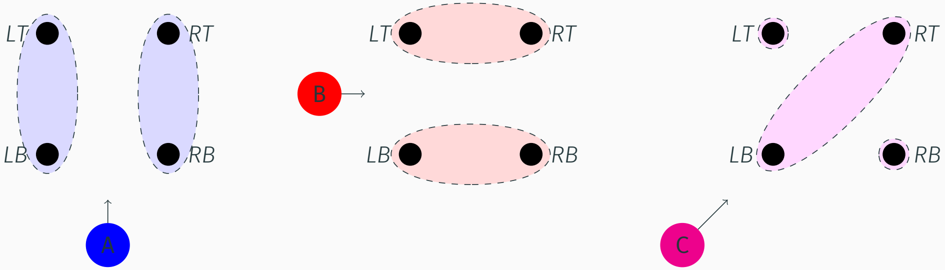


Figure 1: Left: $\mathcal{I}_{\mathsf{A}}$, Middle: $\mathcal{I}_{\mathsf{B}}$, Right: $\mathcal{I}_{\mathsf{C}}$.

W-models (Information fields)

For example … An agent $\mathsf{a} = (\mathsf{p}, t)$ that belongs to a player p, who decides along the timesteps $t \in \mathbb{T}$, may have an information field given by

$$\mathcal{I}_{\mathsf{p},t} = \underbrace{\{\emptyset, \Omega^{\mathsf{e}}\}}_{\text{Does not know nature.}} \otimes \underbrace{\mathcal{F}^{\mathsf{p}}}_{\text{Knows her owner's type.}} \otimes \underbrace{\bigotimes_{\mathsf{q}\in P\setminus\{\mathsf{p}\}} \{\emptyset, \Omega^{\mathsf{q}}\}}_{\text{Does not know other players' types.}}$$

$$\otimes \underbrace{\bigotimes_{s=1,\ldots,t-1} \mathcal{U}_{\mathsf{p},s}}_{\text{Knows the past.}} \otimes \underbrace{\bigotimes_{r=t,\ldots,T} \{\emptyset, \mathbb{U}_{\mathsf{p},r}\}}_{\text{Undecided at the present time.}} \otimes \underbrace{\bigotimes_{\substack{\mathsf{q}\in P\setminus\{\mathsf{p}\} \\ t\in\mathbb{T}}} \{\emptyset, \mathbb{U}_{\mathsf{q},t}\}}_{\text{Does not observe other players' agents.}}$$

$$\subset \mathcal{H} = \mathcal{F}^{\mathsf{e}} \otimes \bigotimes_{\mathsf{p}\in P} \mathcal{F}^{\mathsf{p}} \otimes \bigotimes_{\mathsf{a}\in A} \mathcal{U}_{\mathsf{a}} .$$

Principle-Agent (Leader-Follower) Information Structure.

Two players: $\mathtt{l}$ and $\mathtt{f}$ Each is a single agent.

Then the leader's information field could be

$$\mathcal{I}_{\mathtt{l}} \subset \underbrace{\mathcal{F}^{\mathtt{e}}}_{\text{Observe the nature.}} \otimes \underbrace{\mathcal{F}^{\mathtt{l}}}_{\text{Knows her own type.}} \otimes \underbrace{\mathcal{F}^{\mathtt{f}}}_{\text{Knows her follower.}} \otimes \underbrace{\{\emptyset, \mathbb{U}_{\mathtt{l}}\} \otimes \{\emptyset, \mathbb{U}_{\mathtt{f}}\}}_{\text{Not yet decided.}},$$

and the follower's information field could be

$$\mathcal{I}_{\mathtt{f}} \subset \underbrace{\mathcal{F}^{\mathtt{e}}}_{\text{Observe the nature.}} \otimes \underbrace{\{\emptyset, \Omega^{\mathtt{l}}\}}_{\text{Doesn't know the leader's type.}} \otimes \underbrace{\mathcal{F}^{\mathtt{f}}}_{\text{Knows her own type.}}$$

$$\otimes \underbrace{\mathcal{U}_{\mathtt{l}}}_{\text{Signalled by the leader.}} \otimes \underbrace{\{\emptyset, \mathbb{U}_{\mathtt{f}}\}}_{\text{Not yet decided.}} .$$

The selfessness axiom

- **The selflessness axiom**
  - The information field $\mathcal{I}_a$ of an agent **a** is said to satisfy the **selflessness axiom** if

$$\mathcal{I}_a \subset \mathcal{F}^e \otimes \bigotimes_{p \in P} \mathcal{F}^p \otimes \{\emptyset, \mathbb{U}_a\} \otimes \bigotimes_{b \in A \setminus \{a\}} \mathcal{U}_b .$$

W-models (Strategies)

Action $\neq$ Strategy

$$\underbrace{\text{“If } \overbrace{\text{this happens,}}^{\text{A configuration}} \text{ then I will } \overbrace{\text{do that.”}}^{\text{An action}}}_{\text{Strategy}}$$

- **Strategies** (or **Pure W-Strategies**)
  - A strategy of an agent **a** is a measurable map $\lambda_{\mathsf{a}} : (\mathbb{H}, \mathcal{I}_{\mathsf{a}}) \to (\mathbb{U}_{\mathsf{a}}, \mathcal{U}_{\mathsf{a}})$.
  - Under the **selflessness axiom**, agent **a**'s strategies **never depend on herself**.
  - The set of all strategies of **a** is denoted with Λ_{a}.
  - The set of all strategy profiles of all agents is $\Lambda := \prod_{\mathsf{a} \in A} \Lambda_{\mathsf{a}}$.
  - When an agent **a** is emphasized, we present a profile $\lambda \in \Lambda$ with $(\lambda_{\mathsf{a}}, \lambda_{-\mathsf{a}})$.

W-models (Playability)

From now on, we always assume that all information fields satisfy the selflessness axiom.

- **Playability**
  - A strategy profile $\lambda \in \Lambda$ is **playable** if for any $h = (\underbrace{\omega}_{=(\omega^e, (\omega^p)_{p \in P})}, (u_a)_{a \in A}) \in \mathbb{H}$, the following (closed-loop) equations hold
  $$u_a = \lambda_a(\omega, \cancel{u_a}, u_{-a}) = \tilde{\lambda}_a(\omega, u_{-a}) \quad \forall a \in A.$$
  - We denoted by $\Lambda^* \subset \Lambda$ the set of all **playable** strategies.
  - For any $\lambda \in \Lambda^*$, we associate a **solution map** $S_\lambda : \underbrace{\Omega}_{=\Omega^e \times \prod_{p \in P} \Omega^p} \to \mathbb{H}$ given by
  $$S_\lambda(\omega) = (\omega, (u_a)_{a \in A})$$
  where $u_a = \tilde{\lambda}_a(\omega, u_{-a})$ for all $a \in A$.

W-models (Playability)

Theorem 1
*A **Principle-Agent** (**Sequential**, **Causal**, resp.) strategy is always playable.*

Remark
Any game writable in Kuhn's extensive form is causal.

W-models (Causal and Sequential systems)

(Informally . . .) a causal system is the one that agents make decisions one after another, with non-anticipative information, but the order may depend on the course of earlier actions.

A sequential system is the one that agents make decisions one after another, with non-anticipative information, and the order is fixed.

A non-sequential causal system

A company P wants to make a contract with one of the two applicants, a and b but couldn't make a straight decision.

- The company randomly calls one of the applicants.
- If she accepts, then the contract is made and the game ends here.
- Otherwise, the company calls the remaining applicant.
- If she accepts, then the contract is made and the game ends here.
- The applicants do not know if they were the first who got called.
- If none accepts, then the game ends without a contract.

Agents: $A = \{\overbrace{P_1, P_2}^{\text{Agents of P who makes the first and second calls.}}, a, b\}$

The order could be either (P_1, a, P_2, b) or (P_1, b, P_2, a).

Witsenhausen models (W-models)

Preferences

The agents play, and their players shall evaluate.

- Criterion functions
  - Each player p could be equipped with a **criterion function** $j^{\mathsf{p}} : \mathbb{H} \to \bar{\mathbb{R}}$, which is measurable.
  - The criterion j^{p} may depend on nature, all players' types, and all agents' actions.
  - Each player p tries either to minimize (when j^p is the cost) or to maximize (when j^p is the payoff), **not with her agents' actions** . . .

but rather with strategies.

I could make a good decision because I have a good statistics.

- Beliefs (probably made with statistics)
  - A player p's **belief** is any probability measure β^{p} over the measurable space $(\Omega, \mathcal{F})$.
  - Usually, p makes her beliefs over each uncertainty:

$$\beta^{\mathsf{p},\mathsf{e}}(d\omega^{\mathsf{e}}) \text{ over } (\Omega^{\mathsf{e}}, \mathcal{F}^{\mathsf{e}}),$$

$$\beta^{\mathsf{p},\mathsf{q}}(d\omega^{\mathsf{q}}) \text{ over } (\Omega^{\mathsf{q}}, \mathcal{F}^{\mathsf{q}}), \qquad \forall \mathsf{q} \in \mathsf{P},$$

$$\text{hence} \ldots \ \beta^{\mathsf{p}}(d\omega) = \beta^{\mathsf{p},\mathsf{e}}(d\omega^{\mathsf{e}}) \otimes \bigotimes_{\mathsf{q}\in\mathsf{P}} \beta^{\mathsf{p},\mathsf{q}}(d\omega^{\mathsf{q}}) \text{ over } \Omega = \Omega^{\mathsf{e}} \times \prod_{\mathsf{q}\in\mathsf{P}} \Omega^{\mathsf{q}}.$$

Finally, a player p could make an evaluation of her decision, taken into account the information structure, using the following **normal-form** objective function $J^{\mathsf{p}} : \Lambda^* \to \mathbb{R}$ defined as

$$J^{\mathsf{p}}(\lambda^{\mathsf{p}}, \lambda^{-\mathsf{p}}) = \mathbb{E}_{\beta^{\mathsf{p}}}[j^{\mathsf{p}} \circ S_{\lambda^{\mathsf{p}}, \lambda^{-\mathsf{p}}}] = \int_{\Omega} j^{\mathsf{p}} \circ S_{\lambda^{\mathsf{p}}, \lambda^{-\mathsf{p}}}(\omega) d\beta^{\mathsf{p}}(\omega).$$

W-models (Normal-form Nash equilibrium)

- A **Nash equilibrium** associated to the W-model we have created is any profile $\bar{\lambda} \in \Lambda^*$ such that for each p, the following optimization problem is solved at $\bar{\lambda}^{\mathsf{p}}$

$$\begin{aligned} \min_{\lambda^{\mathsf{p}} \in \Lambda}/\max \quad & J^{\mathsf{p}}(\lambda^{\mathsf{p}}, \bar{\lambda}^{-\mathsf{p}}) \\ \text{s.t.} \quad & (\lambda^{\mathsf{p}}, \bar{\lambda}^{-\mathsf{p}}) \in \Lambda^*. \end{aligned}$$

- Due to the structure of the problem, its resolution amounts to **stochastic control/Dynamic programming**.

What we have seen . . . , and what could be considered ? . . .

What we have seen . . .

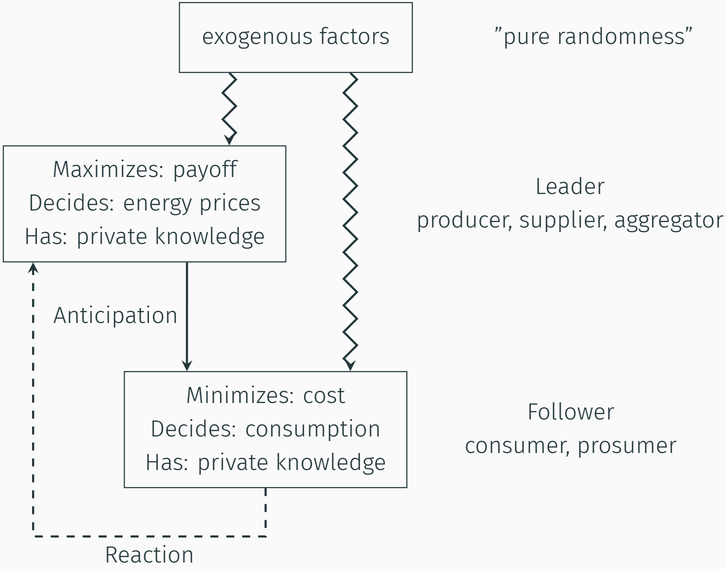
- Witsenhausen's modeling framework.
- . . . especially the information structure.
- It covers many classes of problems — Bilevel programs (single- or multi-follower), Sequential models, Causal models, etc.

What's more ? . . . (but we shall not cover them here.)

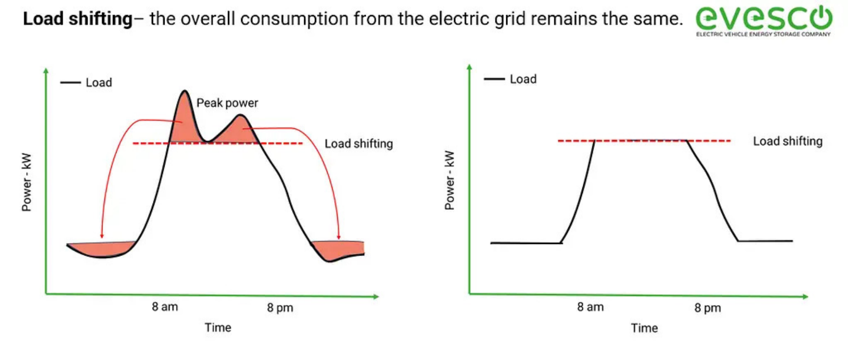
- Other risk measures (not just expectation) in the normal-form objectives.
- Mixed strategies and behaviorial strategies (Aumann's formalisms).
- W- version of perfect recall, backwards induction, subgames, etc. (pretty much not done).

Applications to energy

Multi-agent energy management



Demand response



Cause: Demand is too high to be covered (technically or economically).

Insight: The high demand occurs only during some hours.

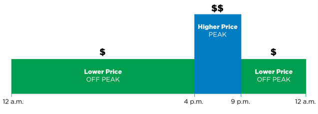
Remedy: Force high demand (during the **peak hours**) to shift to other period (during the **off-peak hours**) using **price mechanism**. ← Demand Response

W-Modeling outline for Demand Response (Simple case)

- Agents
  - $\mathtt{A} = \{\mathtt{l}, \mathtt{f}\}$
  - $\mathtt{l}$ represents the producer.
  - $\mathtt{f}$ represents the consumer.
- Actions
  - **Leader:** $u_{\mathtt{l}} = (\underline{u}_{\mathtt{l}}, \overline{u}_{\mathtt{l}}) \in \mathbb{U}_{\mathtt{l}} = \mathbb{R}^2_+$,
    the peak price and the off-peak price, resp.
  - **Follower:** $u_{\mathtt{f}} = (\underline{u}_{\mathtt{f}}, \overline{u}_{\mathtt{f}}) \in \mathbb{U}_{\mathtt{f}} = \{(\alpha, \beta) \in \mathbb{R}^2_+ \mid \alpha + \beta = 1\}$,
    the consumption ratio during peak and off-peak hours.
  - Equip both action sets with the Borel field.

W-Modeling outline for Demand Response (Simple case)

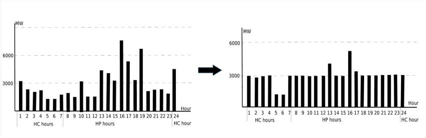
Decides: (peak, off-peak) prices



Leader (agent)

electricity producer

Decides: consumption shift



Follower (agent)

consumer

W-Modeling outline for Demand Response (Simple case)

- Natures
  - **Nature** is given as

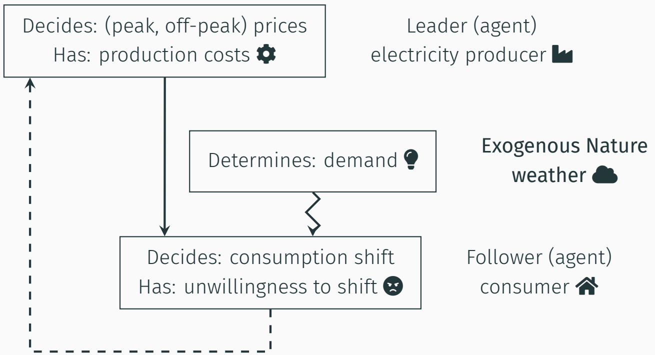
$$\underbrace{\Omega^{\mathsf{e}}}_{\text{Exogenous}} \times \underbrace{\Omega^{\mathsf{l}}}_{\text{Leader's type}} \times \underbrace{\Omega^{\mathsf{f}}}_{\text{Follower's type}} = \mathbb{R}_+ \times \mathbb{R}_+ \times \mathbb{R}_+$$

    with the σ-field

$$\mathcal{F} = \mathcal{F}^{\mathsf{e}} \otimes \mathcal{F}^{\mathsf{l}} \otimes \mathcal{F}^{\mathsf{f}} = \mathcal{B}(\mathbb{R}_+) \otimes \mathcal{B}(\mathbb{R}_+) \otimes \mathcal{B}(\mathbb{R}_+).$$

  - Exogenous nature $\omega^{\mathsf{e}} \in \Omega^{\mathsf{e}}$ represents the **demand (kWh)** caused by the weather.
  - Leader's type $\omega^{\mathsf{l}} \in \Omega^{\mathsf{l}}$ represents its **unitary production cost (€)**.
  - Follower's type $\omega^{\mathsf{f}} \in \Omega^{\mathsf{f}}$ represents its **unwillingness to shift consumption (€/kWh)**.

W-Modeling outline for Demand Response (Simple case)



- Configuration space
  - The **configuration space** is then constructed as

$$\mathbb{H} = \underbrace{\Omega^{\mathsf{e}} \times \Omega^{\mathsf{l}} \times \Omega^{\mathsf{f}}}_{\Omega} \times \mathbb{U}_{\mathsf{l}} \times \mathbb{U}_{\mathsf{f}}$$

with σ-field

$$\mathcal{H} = \underbrace{\mathcal{F}^{\mathsf{e}} \otimes \mathcal{F}^{\mathsf{l}} \otimes \mathcal{F}^{\mathsf{f}}}_{\mathcal{F}} \otimes \mathcal{U}_{\mathsf{l}} \otimes \mathcal{U}_{\mathsf{f}} = \mathcal{B}(\mathbb{R}^3_+) \otimes \mathcal{B}(\mathbb{R}^2_+) \otimes \mathcal{B}(\{(\alpha, \beta) \mid \alpha + \beta = 1\}).$$

W-Modeling outline for Demand Response (Simple case)

- Information fields
  - Leader:

$$\mathcal{I}_{\mathrm{l}} \subset \overbrace{\{\emptyset, \Omega^{\mathrm{e}}\}, \mathcal{F}^{\mathrm{l}} \otimes \{\emptyset, \Omega^{\mathrm{f}}\}}^{\text{Observes partially the nature.}} \otimes \overbrace{\{\emptyset, \mathbb{U}_{\mathrm{l}}\}}^{\text{Selflessness.}} \otimes \overbrace{\{\emptyset, \mathbb{U}_{\mathrm{f}}\}}^{\text{Does not see future.}} .$$

  - Follower:

$$\mathcal{I}_{\mathrm{f}} \subset \underbrace{\mathcal{F}^{\mathrm{e}} \otimes \{\emptyset, \Omega^{\mathrm{l}}\}, \mathcal{F}^{\mathrm{f}}}_{\text{Observes partially the nature.}} \otimes \underbrace{\mathcal{U}_{\mathrm{l}}}_{\text{Signalled.}} \otimes \underbrace{\{\emptyset, \mathbb{U}_{\mathrm{f}}\}}_{\text{Selflessness.}} .$$

W-Modeling outline for Demand Response (Simple case)

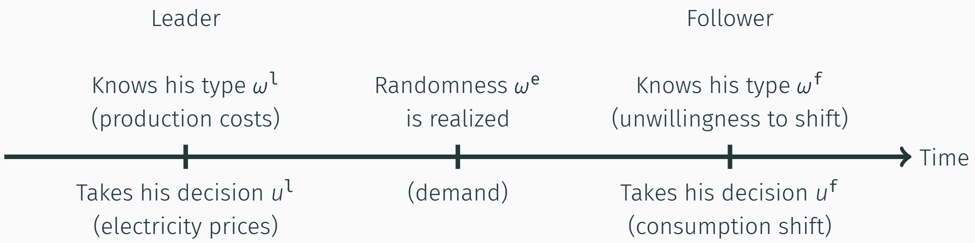


Figure 2: Visualization of the information structure

W-Modeling outline for Demand Response (Simple case)

Figure 3: Building the solution map

W-Modeling outline for Demand Response (Simple case)

- Criterion functions
  - Leader: $j^{\mathtt{l}} \equiv \text{sales} - \text{productioncosts}$ (Payoff)
  - Follower: $j^{\mathtt{f}} \equiv \text{bills} + \text{unwillingnesscosts}$ (Cost)
- Beliefs
  - Leader:

$$\beta^{\mathtt{l}} = \beta^{\mathtt{l},\mathtt{e}} \otimes \delta_{\bar{\omega}^{\mathtt{l}}} \otimes \beta^{\mathtt{l},\mathtt{f}}$$

  - Follower:

$$\beta^{\mathtt{f}} = \delta_{\bar{\omega}^{\mathtt{e}}} \otimes \beta^{\mathtt{f},\mathtt{l}} \otimes \delta_{\bar{\omega}^{\mathtt{f}}} .$$

W-Modeling outline for Demand Response (Simple case)

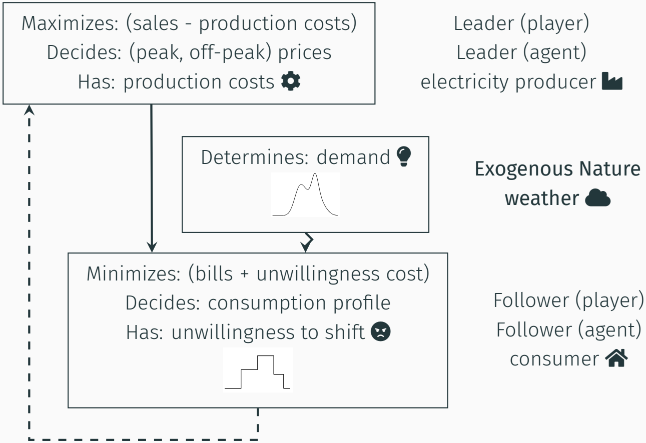


Figure 4: Illustration of the leader's belief

W-Modeling outline for Demand Response (Simple case)

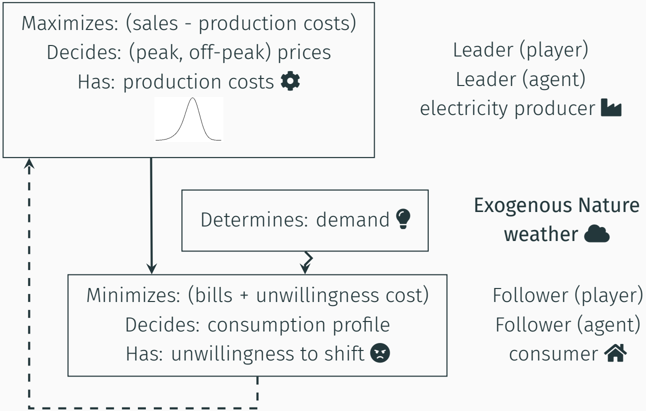
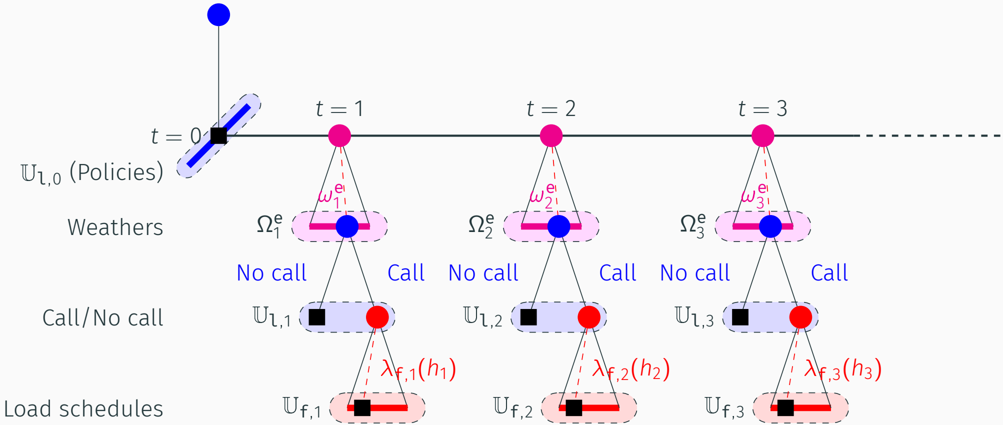


Figure 5: Illustration of the follower's belief

W-Modeling outline for Demand Response (Another type, not going into details)



Takeaways

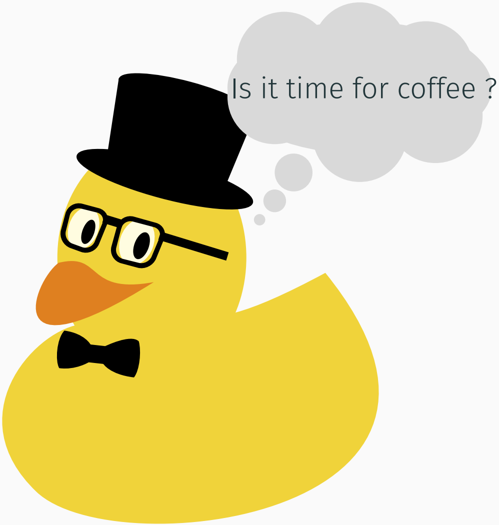
Takeaways

W-model . . .

- treats game theory with information beyond the ones of Kuhn's,
- is a flexible and modular modeling approach,
- dissolves vertical structure into horizontal structure,
- amounts to stochastic control in resolution,
- allows the language of game theory,
- pretty much open in many aspects.

Thanks to . . .

- the **organizers** and **sponsors** of **CSSS2025** for the wonderful event,
- **KMUTT** for the amazing environment,
- **Electricité de France** for the grant and supervision,
- **my family** for their love and moral support.


Is it time for coffee ?

Lopburi, 2023

☻ Thank you for your attendance.

contact: parin.cha@kmutt.ac.th