Supplementary materials to "Joint Estimation of Multiple Network Granger Causal Models"

June 13, 2017

1 Convergence diagnostics of ADMM algorithm.

Here we provide the ADMM algorithm convergence diagnostics via combination of the following four plots ($\boldsymbol{\beta}^{(k)}$ and $\boldsymbol{\gamma}^{(k)}$- estimates of $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$, respectively, resulting from k^{th} update (equations (11) in Section 3.2 of the paper); $f_{obj}^{(k)}$ - minimized value obtained by plugging $\boldsymbol{\beta}^{(k)}$ and $\boldsymbol{\gamma}^{(k)}$ into the optimization criterion (10) from Section 3.2 of the paper):

1. $||\boldsymbol{\beta}^{(k)} - \boldsymbol{\beta}^{(k-1)}||_2^2$, denoted as $Frob(\boldsymbol{\beta}^{(k)} - \boldsymbol{\beta}^{(k-1)})$, against iteration number
2. $||\boldsymbol{\gamma}^{(k)} - L\boldsymbol{\beta}^{(k)}||_2^2$, denoted as $Frob(\boldsymbol{\gamma}^{(k)} - L\boldsymbol{\beta}^{(k)})$, against iteration number
3. $|f_{obj}^{(k)} - f_{obj}^{(k-1)}|$, denoted as $Frob(f_{obj}^{(k)} - f_{obj}^{(k-1)})$, against iteration number
4. $f_{obj}^{(k)}$ against iteration number

In particular, we demonstrate the plots for cases with over 40 iterations until convergence. Parameter ρ was set to be equal to 10 which showed to be a robust choice across the settings during simulation studies. Stopping criterion was chosen to be: $|f_{obj}^{(k)} - f_{obj}^{(k-1)}|/|f_{obj}^{(k-1)}| < 10^{-3}$. Below you can see the Figure 1 for ADMM run for $\lambda_1 = 0.01$, $\lambda_2 = 0.137$, and Figure 2 $-$ for $\lambda_1 = 0.005$, $\lambda_2 = 0.029$. Starting points for the ADMM algorithm: $\boldsymbol{u}^0 = (0,...,0)$, $\boldsymbol{\gamma}^0 = (0,...,0)$. One can see how the sequence of estimates $\{\boldsymbol{\beta}^{(k)}\}$ stabilizes with respect to Frobenius norm (top left panel), the restriction $\boldsymbol{\gamma}^{(k)} - L\boldsymbol{\beta}^{(k)} = 0$, $k = 1,2,...$, of the algorithm always approximately holds (top right panel), the objective function value stabilizes (bottom panels). All of that is indicative of good convergence performance of our ADMM algorithm.

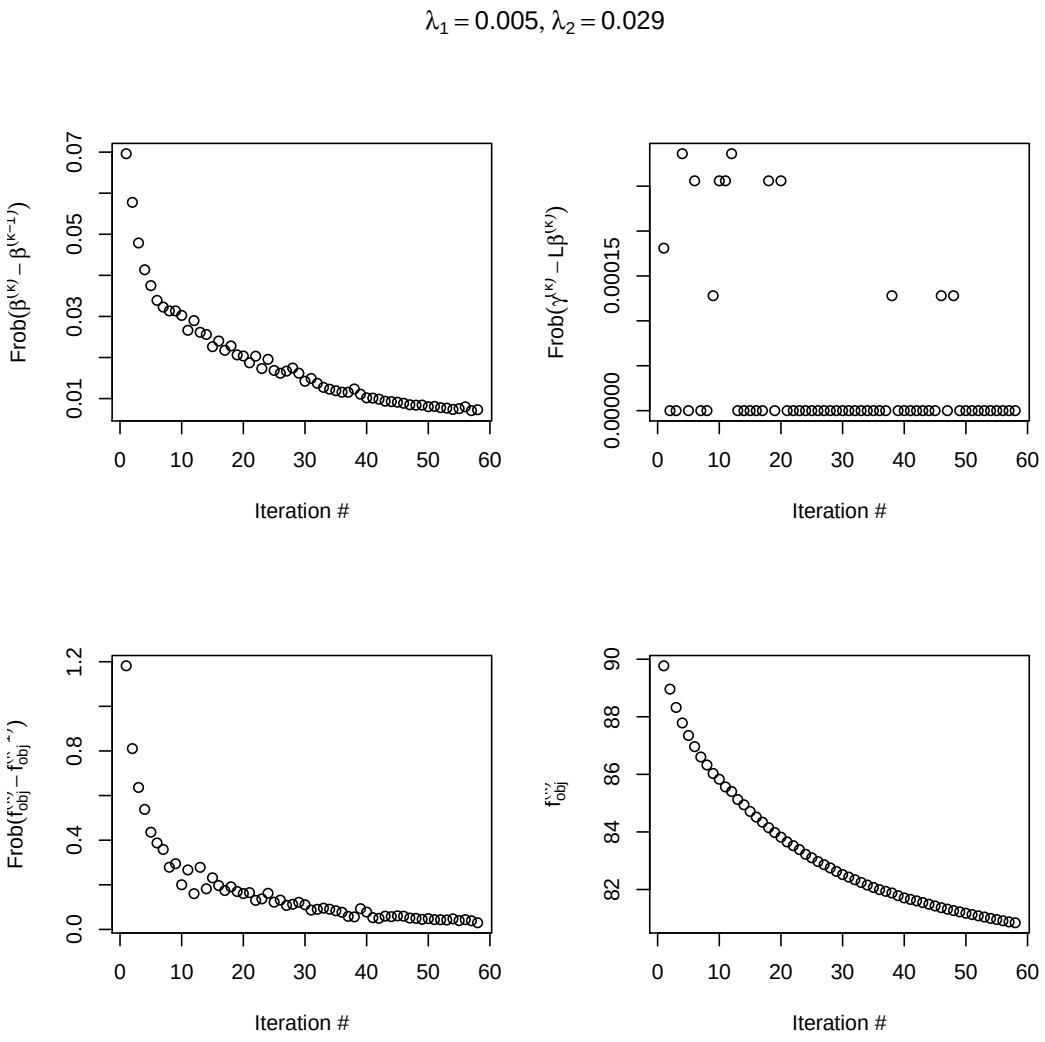

Figure 1: Four measures described above plotted against iteration number for ADMM run for $\lambda_1 = 0.01, \lambda_2 = 0.137$.

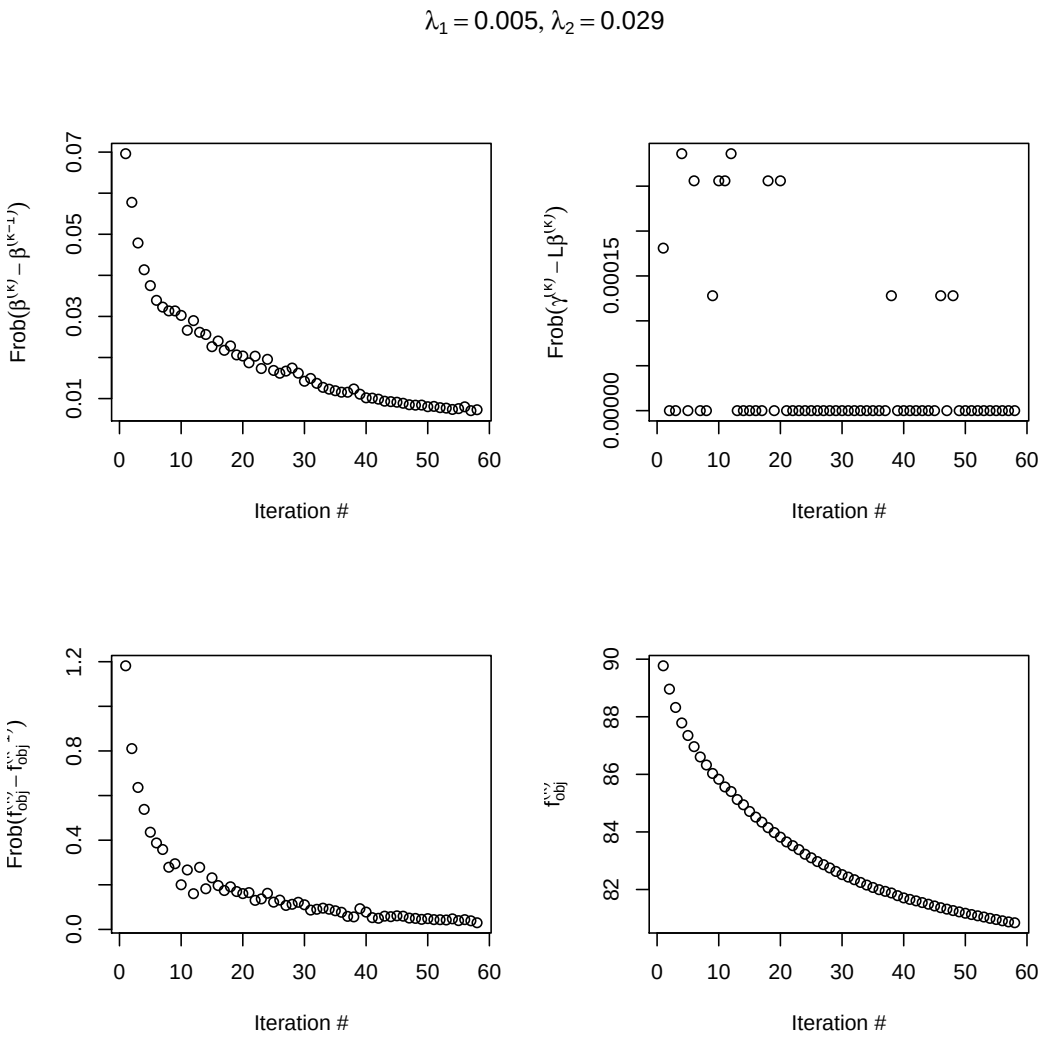

Figure 2: Four measures decribed above plotted against iteration number for ADMM run for $\lambda_1 = 0.005, \lambda_2 = 0.029$.