Announcements

Pset 0 being graded

Pset 1 out

Pset 2 will post Tuesday

Midterm Exam #1 Wednesday

Generalized NFAs (DFA/NFA → regex)

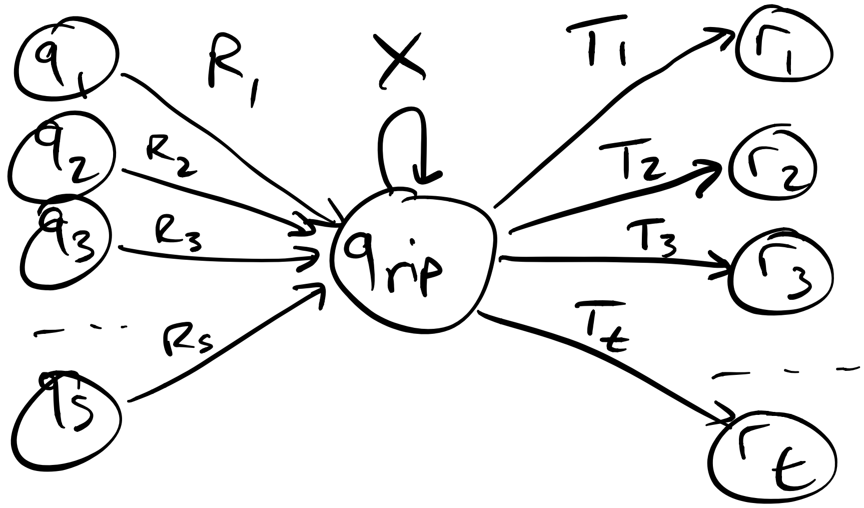
Let G be a GNFA.

let Q be its set of states,

& let q_0 be its start state

and q_f be its only final state.

So let $q_{rip} \in Q \setminus \{q_0, q_f\}$.



for all $1 \le i \le s$
and $1 \le j \le t$:

add transition $q_i \xrightarrow{R_i X^* T_j} r_j$

Q: what if there exists a transition from q_i to r_j already?

A: union $R_i X^* T_j$ with it

Q: what if there is no loop on q_{rip}?

A: $q_i \xrightarrow{R_i T_j} r_j$

Formal def'n: all undefined transitions are $\emptyset$.

$L = \{ w \in \{0,1\}^* :$ w has the same number of 0's and 1's$\}$.

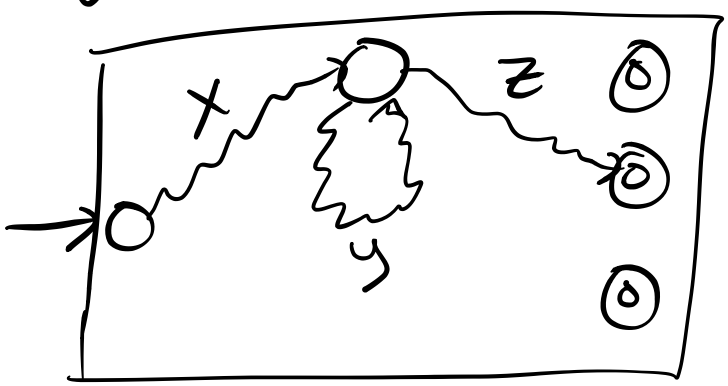
Try ?

Some strings: ε, 01, 0011, 000111, ...

If we had a DFA with m states

$0^m 1^m$ ✓

$\text{exponent} = \underbrace{00 \cdots 0}_{m \text{ times}}$



$\rightsquigarrow$ = a bunch of transitions

input $= xyz$

$\underbrace{(q_0, q_2, q_3, \ldots)}_{m+1}$

Q: is $xz \in L$?

Q: is $xyyz \in L$?

Q: is $xy^iz \in L$ for all $i \geq 0$?

Q: What do we know about y?

A: $|y| \geq 1$ (or $y \neq \varepsilon$)

$$(\ldots\ldots, q_1, q_2, \ldots) \in$$

repeated state

Q: How early can we guarantee the first repetition of the input be?

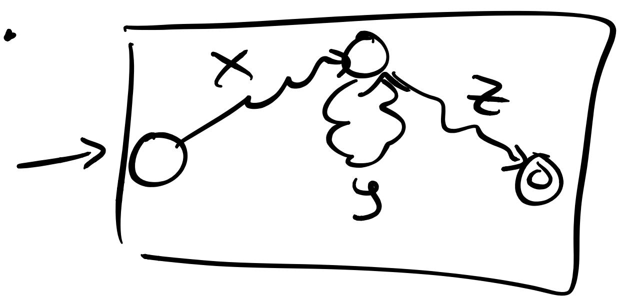
A: after m characters

So, $|xy| \leq m$.

Pumping lemma for Regular Languages

Let L be a regular language.
Then, there exists a "pumping constant" p (or "pumping length") for L, so that for <u>all</u> $w \in L$ with $|w| \geq p$, there exists a way to write w as xyz so that:

1. $xy^iz \in L$ for all $i \geq 0$.
2. $|y| \geq 1$ (required)
3. $|xy| \leq p$.



L regular $\Rightarrow$ (_ _ _ . _)

$\neg$(_ _ _) $\Rightarrow$ L is not regular

Steps to show L is not reg:

1. Assume L is reg.
2. $\exists$ a pumping const p for L.
3. Pick a string $w \in L$ with $|w| \geq p$ "intelligently."
4. Want to show there is <u>no way</u> to write w as xyz with $|y| \geq 1$ and $|xy| \leq p$ satisfies $xy^iz \in L$ for all $i \geq 0$.

   (Pick a value of i and show that all decompositions of xy^iz have it being $\notin L$).

$L_1 = \{ w \in \{0,1\}^*$ s.t. w has the same number of 0's and 1's$\}$.

Claim: L_1 is not regular.

Proof: Assume L_1 is regular.

$\Rightarrow$ exists a pumping constant p for L_1.

Bad string: $w = 0011$

Bad string: $w = 0^p 1$

Good string: $w = 0^p 1^p$

$$\underbrace{0 \cdots}_{y} 0\,1 \cdots 1$$

We can see that

$$x = 0^a \qquad a \geq 0 \qquad (a+b \leq p)$$

$$\rightarrow y = 0^b \qquad b \geq 1$$

$$z = 0^{p-a-b} 1^p.$$

$i=2$:

$$xy^2z = 0^{a} 0^{2b} 0^{p\cancel{-a-b}} 1^p$$

$$= 0^{p+b} 1^p$$

$\Rightarrow b = 0$, a contradiction.

$\Rightarrow L_1$ is not regular.

$i=0$:

$$xy^0z$$

$$= xz$$

$$= 0^a 0^{p-a-b} 1^p.$$

$$= 0^{p-b} 1^p.$$

$\Rightarrow b = 0$

$L_2 = \{0^n 1^n : n \geq 0\}$.

Claim: L_2 is not regular.

$L_1 \subseteq \Sigma^*$

$L_1 \cap 0^* 1^* = L_2$

if regular — regular

If L_1 were regular, then L_2 is regular.

L_2 is not regular

$\Longrightarrow L_1$ is not regular.

$L_2 \cap 0^* 1^* \neq L_1$

$L_3 = \{0^{n^2} : n \geq 0\}$. "perfect square" number of 0's

ε, 0, 0000, 000000000, ...

Claim: L_3 is not regular.

Proof: Assume L_3 is regular.

$\Longrightarrow \exists$ a p for L_3.

Choose $w = 0^{p^2}$.

$$\left.\begin{aligned} x &= 0^a \\ y &= 0^b \quad b \geq 1 \\ z &= 0^{p^2-a-b} \end{aligned}\right]$$

$|xy| \leq p$

$|y| \leq p$

Choose $i=2$:

$$xy^2z = 0^{p^2+6}$$

$$p^2 < p^2+1 \le p^2+6 \le p^2+p < (p+1)^2 = p^2+2p+1$$

length of resulting string

$\Rightarrow$ p^2+6 is not a perfect square

$\Rightarrow$ L_3 is not regular.

$L_4 = \{ 0^p : p \text{ is prime} \}$.

Claim: L_4 is not regular.

Proof: Assume L_4 is regular.

$\Rightarrow$ $\exists$ a p for L_4.

Choose $w = 0^r$ where r is the smallest prime at least $p+2$.

$$|xy^iz| = |xz| + |y^i|$$
$$= |xz| + i \cdot |y|$$

$|xyz| \ge p+2$

$|xz| \geq 2$

$$= \underbrace{|xz|}_{\geq 2} \underbrace{(1 + |y|)}_{\geq 2}$$

Choose

$i = \underbrace{|xz|}$?

Cannot possibly be prime

L_4 is not regular.