Announcements

Pset 2 out
Recitation 1 grades up?

Chomsky Normal Form

$S \to \varepsilon$
$A \to a$
$A \to BC \quad (BC \neq S)$

$S, A, B, C \in V$

$S \to (S) \mid SS \mid \varepsilon \leftarrow$

Step 1 (of 5): Make a new start variable S_0 with rule $S_0 \to S$

$S_0 \to S$
$S \to (S) \mid SS \mid \varepsilon \leftarrow$

( )

$S_0 \Rightarrow S \Rightarrow \varepsilon$

Step 2: "Eliminate" ε-rules.

Call a variable A nullable if

- $A \to \varepsilon$ is a rule
- $A \to X_1 X_2 \cdots X_n$ where each X_i is a nullable variable

$S \to TA$
$T \to \varepsilon$
$A \to \varepsilon$

Ex: $B \to aCDADD$ (C, D, A are nullable)

add rules: $B \to aDADD$
$B \to aDD$
$B \to aCA$
$B \to aA$

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nullable vars: S, S_0

$S_0 \to S$
$S \to (S) \mid SS \mid \varepsilon$

$S_0 \to S \mid \varepsilon$

$S \to (S) \mid SS \mid ()$

Step 3: "Eliminate" unit rules $(A \to B)$

$S \to T$

$T \to A$

$A \to B$

$B \to a$

Form a directed graph where the vertices are the variables, and add a directed edge from A to B if $A \to B$ is a unit rule.

if $A \to A$ is a unit rule, delete it.

for all rules $A \to uBy$, then if B has a path in the graph to C then add rule $A \to uCy$.

$S_0 \to S \mid \varepsilon$

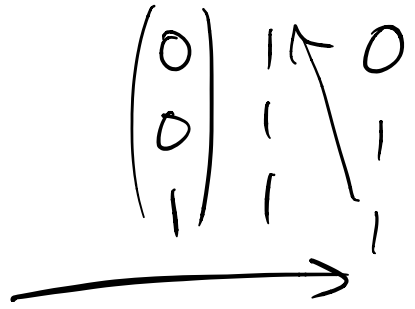
$S \to (S) \mid SS \mid ()$

$S_0 \to \varepsilon \mid (S) \mid SS \mid ()$

$S \to (S) \mid SS \mid ()$

Office Hours / Problem Solving

Pset 1:



Proof of reversal!

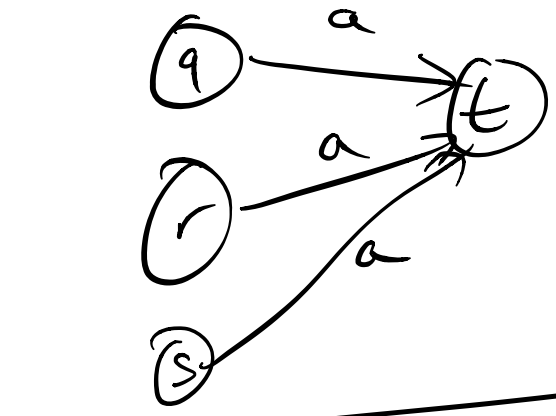
DFA D for L.

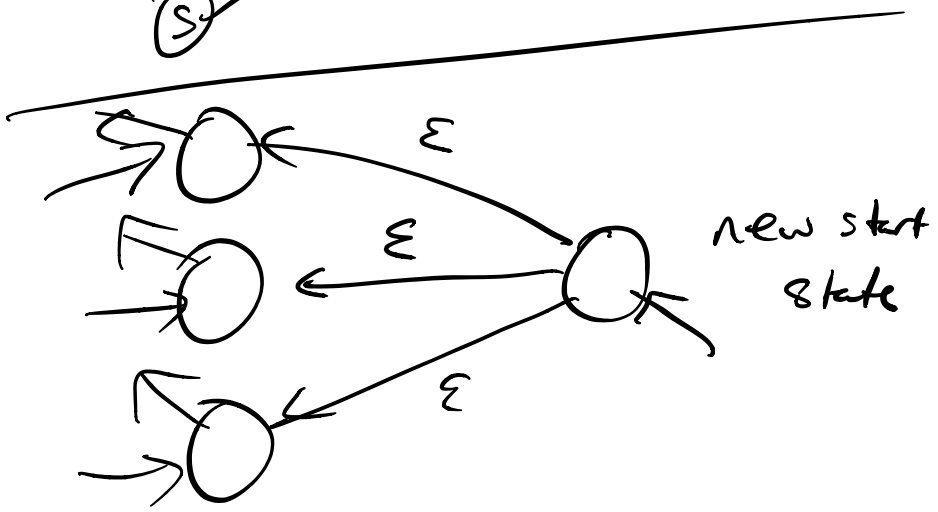
Want DFA/NFA/.. for L^R.

if $w = w_1 w_2 \cdots w_n$

then $w^R = w_n w_{n-1} \cdots w_2 w_1$

$L^R = \{w^R : w \in L\}$

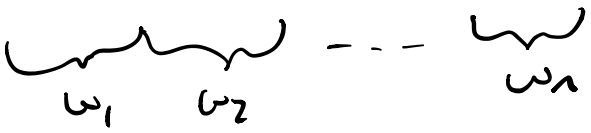




now make original start state final

Regexes if R is a regex for L:

1. if $R = a$ then $rev(R) = a$
2. if $R = \varepsilon$ ——— ε
3. if $R = \emptyset$ ——— $\emptyset$
4. if $R = R_1 \cup R_2$ ——— $rev(R_1) \cup Rev(R_2)$
5. if $R = R_1 R_2$ ——— $rev(R_2)\, rev(R_1)$
6. if $R = R_1^*$ ——— $rev(R_1^*)$



w_n^{Rev} w_{n-1}^{Rev} —— w_1^{Rev}

(Note: extra scratch paper)

$$L = \{a^m b^n : \underline{m < \sqrt{n}} \text{ or } \underline{n < \sqrt{m}}\}.$$

Claim: L is not reg.

Proof: Assume it is

$\Rightarrow \exists$ a p for L.

Choose $w = a^p b^{p^2+1}$

$$x = a^{\alpha}$$

$$y = a^{\beta}$$

$$z = a^{p-\alpha-\beta} b^{p^2+1}$$

Pump up: $xy^2z = a^{p+\beta} b^{p^2+1}$

($m < \sqrt{n}$)

$p+\beta < \sqrt{p^2+1}$

$(p+\beta)^2 < p^2+1$

$2p\beta + \beta^2 < 1$

$\Rightarrow$ contradiction!

($n < \sqrt{m}$)

$p^2+1 < \sqrt{p+\beta}$

$\leq \sqrt{2p}$

$\Rightarrow$ contradiction!

$$\delta : Q \times \Sigma \to Q \times \{L, R\}$$