Announcements

Pset 3 solutions up

Pset 4 out

Undecidability

A_{TM}, E_{TM}

$EQ_{TM} = \{\langle M_1, M_2\rangle : M_1, M_2 \text{ are TMs and } L(M_1) = L(M_2)\}$.

Claim: this is undecidable.

Proof: Suppose that D decides EQ_{TM}.

To decide E_{TM}:

"On input $\langle M\rangle$ where M is a TM:

1. Let T be a TM that rejects everything.
2. Run D on $\langle M, T\rangle$.
3. If D accepts, accept.
   If D rejects, reject."

$\Rightarrow$ D cannot exist (b/c E_{TM} is undecidable)

$\Rightarrow$ EQ_{TM} is undecidable.

Properties of TM languages

A property of TM langs, P, is a set of TM descriptions so that for any 2 TMs M_1 & M_2 with $L(M_1) = L(M_2)$ then either

(1) $\langle M_1 \rangle, \langle M_2 \rangle \in P$ ←

or (2) $\langle M_1 \rangle, \langle M_2 \rangle \notin P$. ←

A nontrivial property has at least 1 TM that is not in P, and another that is in P.

$\{\langle M \rangle : \text{---}\}$ ✓

$\{\langle M_1, M_2 \rangle : \text{---}\}$ ✗

Rice's Theorem: every nontrivial property of TM langs is undecidable.

$REGULAR_{TM} = \{\langle M \rangle : M \text{ is a TM and } L(M) \text{ is regular}\}$.

(1) This is a property of TM langs.

If $L(M_1) = L(M_2)$, so either both are reg or neither are.

(nontrivial) (2) Let M_1 be a TM that rejects everything.

$\Rightarrow \langle M_1 \rangle \in REGULAR_{TM}$.

Let M_2 be a TM that accepts the language $\{0^n 1^n : n \geq 0\}$.

$\Rightarrow \langle M_2 \rangle \notin REGULAR_{TM}$.

By Rice's Thm, $REGULAR_{TM}$ is undecidable.

Office Hours/Problem Solving

Daily Exercise:

T/F: If L is a CFL, then $\overline{L}$ is decidable.

ALL CFLs are decidable.
Decidable langs closed under complement.

Pset 4: