$n^k = (2m)^k$
$= 2^k m^k$

Announcements

Midterm 2 returned

Pset 4 out

Extra Office hours next Friday (after class)

Properties of TM languages

A property of TM langs, P, is a set of TM descriptions so that for any 2 TMs M_1 & M_2 with $L(M_1) = L(M_2)$ then either

(1) $\langle M_1 \rangle, \langle M_2 \rangle \in P$ ←

or (2) $\langle M_1 \rangle, \langle M_2 \rangle \notin P$. ←

A nontrivial property has at least 1 TM that is not in P, and another that is in P.

$\{\langle M \rangle : \ldots\}$ ✓

$\{\langle M_1, M_2 \rangle : \ldots\}$ ✗

Rice's Theorem: every nontrivial property of TM langs is undecidable.

Proof: let P be an arbitrary nontrivial property of TM langs.

Suppose that P is decidable.

We will try to decide A_{TM} as follows:

"On input $\langle M, w \rangle$:

1. Construct TM
   $M' =$ "On input x:
   (a) Simulate M on w.
   If M rejects w, reject x.
   (b) Run T on x, and
   accept if T accepts."
2. Let D be the decider for P.
   Run D on $\langle M' \rangle$.
3. If D accepts, accept.
   If D rejects, reject."

Suppose $M_\emptyset$ is a TM that has $L(M_\emptyset) = \emptyset$.
And suppose $\langle M_\emptyset \rangle \notin P$.
Suppose T is a TM and $\langle T \rangle \in P$.

$THREE_{TM} = \{ \langle M \rangle : M \text{ is a TM } \& \ |L(M)| \leq 3 \}$.

Claim: this is undecidable.

property of TM langs:
if $L(M_1) = L(M_2)$ then either both have at most 3 strings or neither do.

nontrivial:
Let M_1 be a TM that rejects everything.
$\Rightarrow \langle M_1 \rangle \in THREE_{TM}$.
Let M_2 ______ accepts everything.
$\Rightarrow \langle M_2 \rangle \notin THREE_{TM}$.

By Rice's Thm, $THREE_{TM}$ is undecidable.

A computation history of TM M on w is a sequence of configurations $C_0, C_1, \ldots, C_m$ where

(1) C_0 is the start config,
(2) for all $0 \le i < m$, C_i yields C_{i+1}.

A comp. hist. is accepting if C_m is accepting.
——— rejecting ——— rejecting.

A linear bounded automaton (LBA) is exactly like a TM but can never move right past its input.

$$A_{LBA} = \{ \langle M, w \rangle : M \text{ is an LBA and } w \in L(M) \}$$

Claim: this is decidable.

Proof: suppose input has n characters.
Suppose there are q states, g tape symbols.

There are g^n possible tape contents.

$g^n q n + 1$

"On input $\langle M, w \rangle$ (M is an LBA):
1. Run M on w for $g^n q n + 1$ transitions, or until M stops.
2. If M has not halted on w by now, reject.

3. Output what M said."

$E_{LBA} = \{ \langle M \rangle : M$ is an LBA and $L(M) = \emptyset \}$.

Claim: this is undecidable.

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write a comp. hist. $C_0, C_1, \ldots, C_m$

by $\#C_0\#C_1\# \ldots \#C_m\#$.

Q: Given a TM M & w, can an LBA figure out whether this comp. hist. is accepting?

1. Check if C_0 is a valid start config.
2. Check if C_m is accepting.
3. Check if C_i yields C_{i+1} for all $0 \leq i < m$.

To decide A_{TM}:

"On input $\langle M, w \rangle$:

1. Build an LBA B using M & w as described above.
2. Run the supposed decider for E_{LBA}, E, on $\langle B \rangle$.
3. If E accepts, reject.
   If E rejects, accept."

Since A_{TM} is undecidable, E_{LBA} is also.

$ALL_{CFG} = \{ \langle G \rangle : G$ is a CFG and $L(G) = \Sigma^* \}$

Claim: this is undecidable.

Q: Given TM M & w, how can a CFG recognize <u>all</u> strings that are <u>not</u> ACH of M on w?

1. C_0 is not a valid config

$$\underbrace{\Gamma^*}_{\text{reg}} \cap \overline{\underbrace{\{q_0 w_1 w_2 \cdots w_n\}}_{\text{reg}}} \rightarrow \text{reg}$$

2. C_m is <u>not</u> an accepting config $\rightarrow$ reg

3. $\_\_\_\_\_\_ \# C_i \# C_{i+1}^{rev} \# \_\_\_\_$

$w \# w^{rev}$

A reversed ACH is of the form

$$\# C_0 \# C_1^{rev} \# C_2 \# C_3^{rev} \# \cdots \# C_m \#$$

$$\cdots \# \underbrace{\square \quad a1}_{C_2} \# \underbrace{1a \quad \square}_{C_3^{rev}} \# \cdots$$

this is a CFL!

Reg $\cup$ Reg $\cup$ CFL $\rightarrow$ a CFL

$\Rightarrow$ a CFG can recognize all strings that are <u>not</u> ACHs.

"On input $\langle M, w \rangle$:
1. Build a CFG, G, as above. (based on M, w).
2. Run the supposed decider D for ALL_{CFG} on $\langle G \rangle$.
3. If D accepts, reject.
   If D rejects, accept."

Chapter 7

Q: what types of resources do we want to restrict?

- Time
- Space

Def: $TIME(f(n))$ = set of all languages decided by TMs that run in $O(f(n))$ time for all inputs of size n.

Ex: $\{0^n 1^n : n \geq 0\} \in TIME(n^2)$

all CFLs $\in TIME(n^3)$.

Def: $NTIME(f(n))$ = ____________________

________ NTMs ______________

________________________.

Say an NTM runs in time $O(f(n))$ if <u>all</u> computations take at most $O(f(n))$ transitions

Def: $P = \bigcup_{k \geq 0} TIME(n^k)$

$NP = \bigcup_{k \geq 0} NTIME(n^k)$.

Conj: $P \neq NP$.

$5n + o(1)$