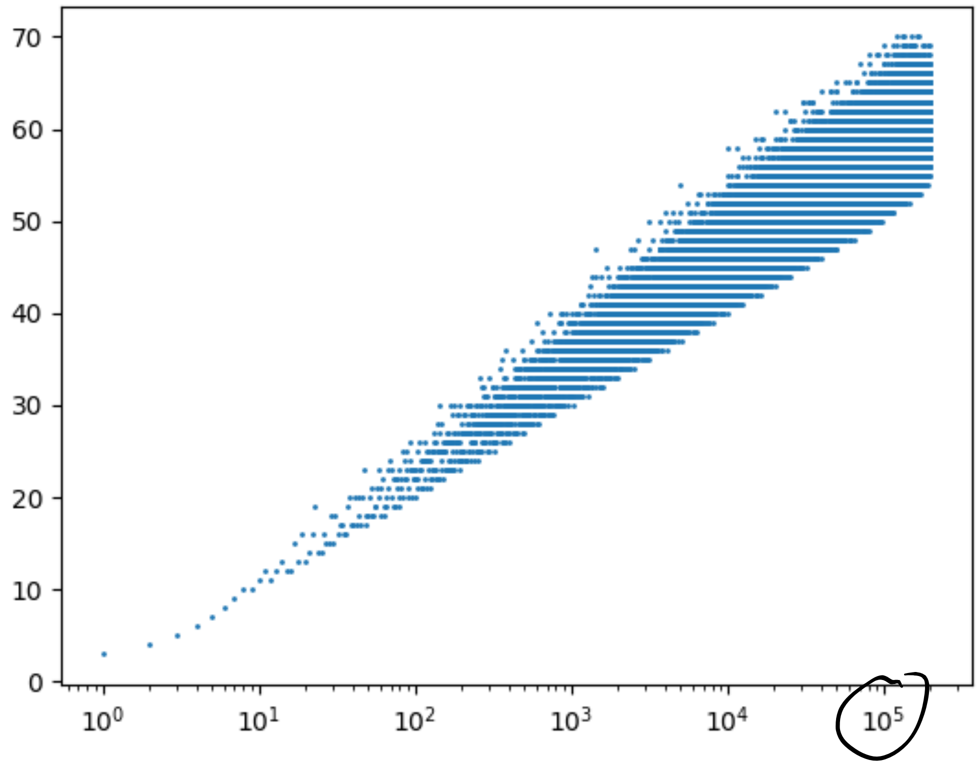


Announcements

Pset 2 due tomorrow

Pset 3 out

$$L = \{ 0^{2^n} : n \geq 0 \}$$

Assume L is a CFL.

$\Rightarrow \exists$ a p for L. $\leftarrow$

Choose $w = 0^{2^p}$. $\quad 2^p \geq p$ for all $p \geq 0$? ✓

Let's suppose $\underline{vy} = 0^a$. $\quad a \geq 1$

Consider $uv^2xy^2z = \underline{0^{2^p + a}}$

$\Rightarrow 2^p + a$ is a power of 2.

not 2^p because $a \geq 1$

2^{p+1}?

$$2^p + a \leq 2^p + p = 2^{p+1}?$$

$$p = 2^p.$$

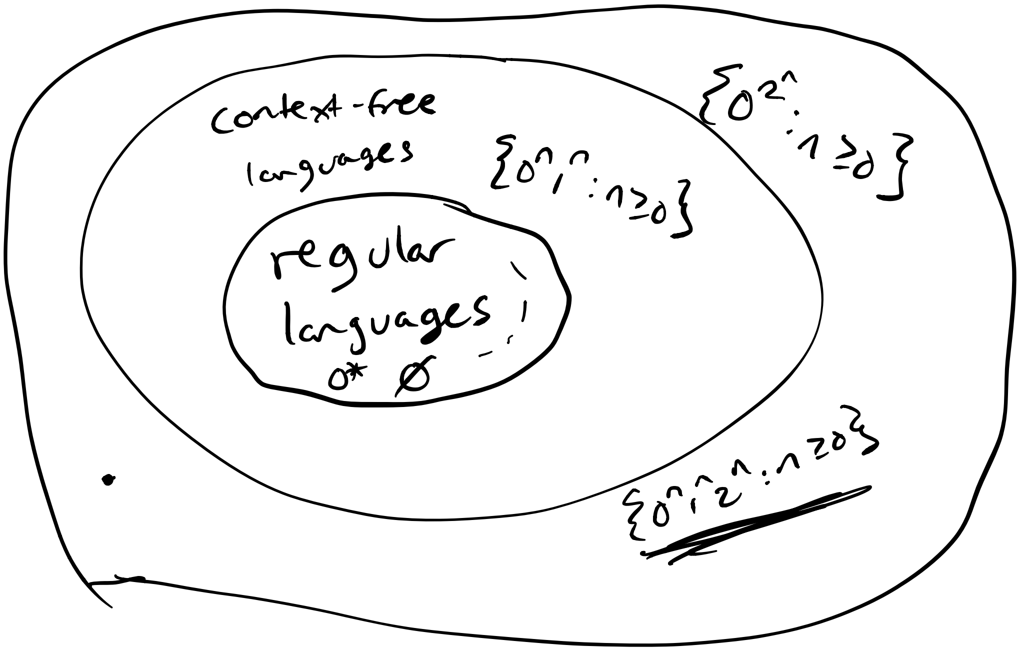
never true

We can see that $p < 2^p$ for all p.

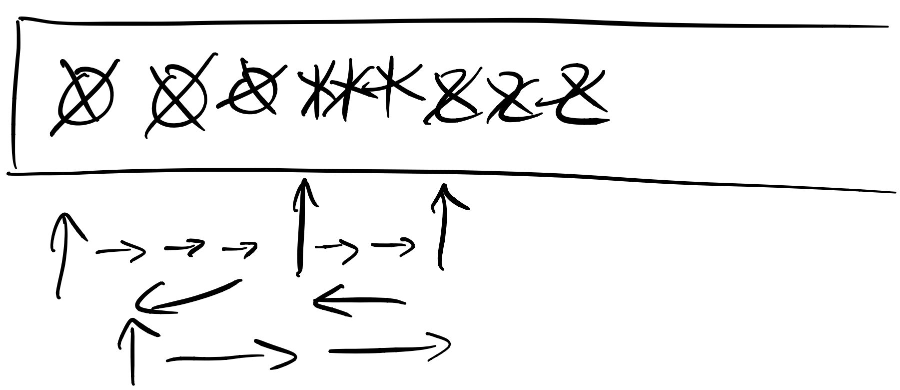
Therefore $2^p < 2^p + a < 2^{p+1}$.

So $2^p + a$ is not a power of 2.

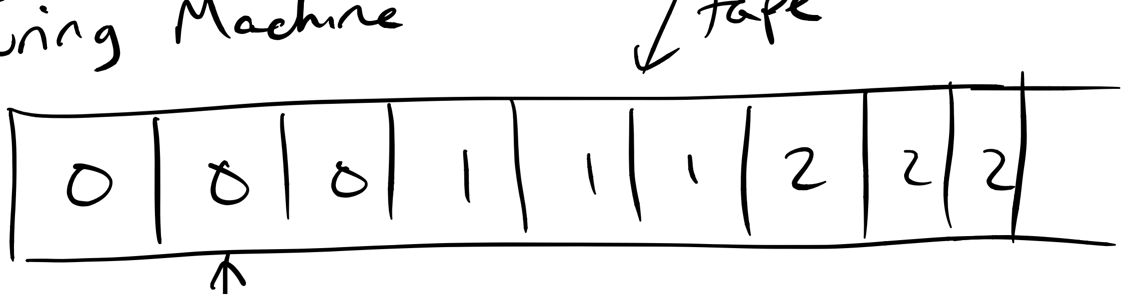
$\Rightarrow 0^{2^p + a} \notin L \Rightarrow L$ is not a CFL.



$$\{0^n 1^n 2^n : n \geq 0\}$$



Turing Machine



| tape head

Transition: examine current state,
current tape symbol,
and then determine what is the
next state,
symbol to write in that cell,
whether to move left or right.

Want: deterministic

A Turing Machine is a -tuple

$$(Q, \Sigma, \Gamma, \delta, q_0, q_{accept}, q_{reject})$$

Q = finite set of states $\quad \sqcup \in \Gamma \quad \Sigma \subset \Gamma$

Σ = finite input alphabet $\quad \sqcup \notin \Sigma$

Γ = finite tape alphabet

q_0 = start state

$\sqcup^{10^{10^{100}}} \; w_1 w_2 \cdots w_n$

q_{accept} = accept state

q_{reject} = reject state $(q_{accept} \neq q_{reject})$

$\sqcup$ = "blank character"

$w_1 w_2 \cdots\cdots w_n \sqcup \sqcup \sqcup \cdots\cdots$

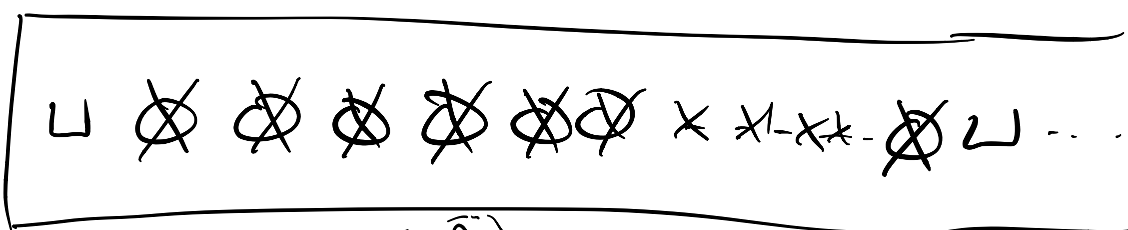
$$\delta : Q' \times \Gamma \to Q \times \Gamma \times \{L, R\}$$

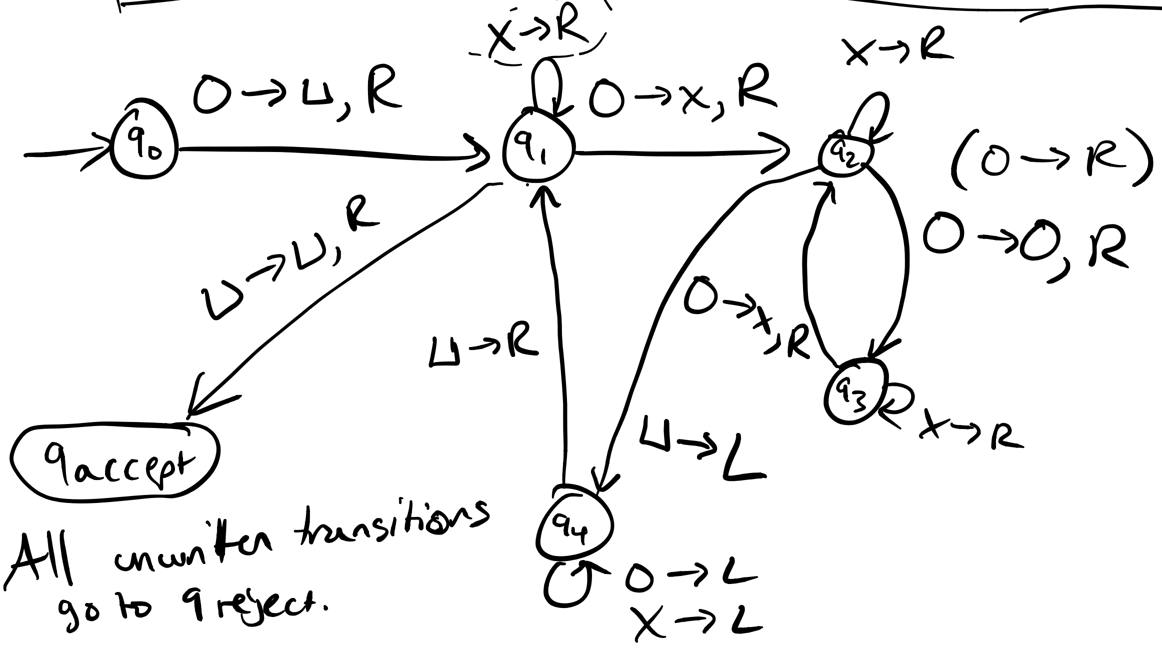
total function $\quad \left[\, Q' = Q \setminus \{q_{accept}, q_{reject}\} \,\right]$

2 scenarios:

- "bouncing" on the Left end of tape]
- the computation stops and rejects

$$L = \{ 0^{2^n} : n \geq 0 \}$$





All unwritten transitions go to q_{reject}.

0000
q_0

$\sqcup\, 0\, 0\, 0$
q_1

$\sqcup\, X\, 0\, 0$
q_2

$\sqcup\, X\, 0\, 0$
q_3

$\sqcup\, X\, 0\, X\, \sqcup$
q_2

$\sqcup\, X\, 0\, X\, \sqcup$
q_4

$\sqcup\, X\, 0\, X\, \sqcup$
q_4

$\sqcup\, X\, 0\, X\, \sqcup$
q_4

$\sqcup\, X\, 0\, X\, \sqcup$
q_4

$\sqcup\, X\, 0\, X\, \sqcup$
q_1

$\sqcup\, X\, 0\, X\, \sqcup$
q_1

$\sqcup\, X\, X\, X\, \sqcup$
q_2

A configuration contains the tape contents, tape head location, and current cell.
(a string in $\Gamma^* Q \Gamma^*$).

Starting configuration: $q_0 0000$

next configuration $\sqcup q_1 000$

$$\sqcup X 0 0 q_3 0$$
$$\Downarrow \quad \leftarrow \text{"yields"}$$
$$\sqcup X 0 0 X q_2 \sqcup$$

Say that config C_1 <u>yields</u> C_2 if after 1 transition from C_1, the resulting config is C_2.

Starting config: $q_0 w$.

Say that a config C is <u>accepting</u> if the single state in it is q_{accept}.

______ <u>rejecting</u> ______

______ q_{reject}.

Computation is a sequence of configs $C_0, \ldots, C_m$
where (1) C_0 is the start config,
(2) C_i yields C_{i+1} for all i.

Say computation is accepting if C_m is accepting.
______ rejecting ______ rejecting.
______ halting if C_m is accepting or rejecting.

$L(M)$ where M is a TM is
$\{w : M$ has an accepting computation on $w\}$.

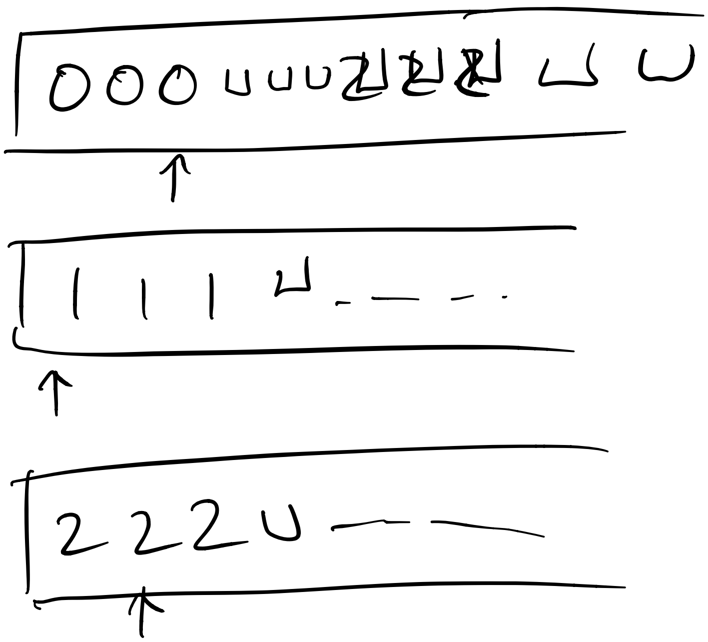
Say a language L is <u>recognizable</u> if it is the language of some TM.

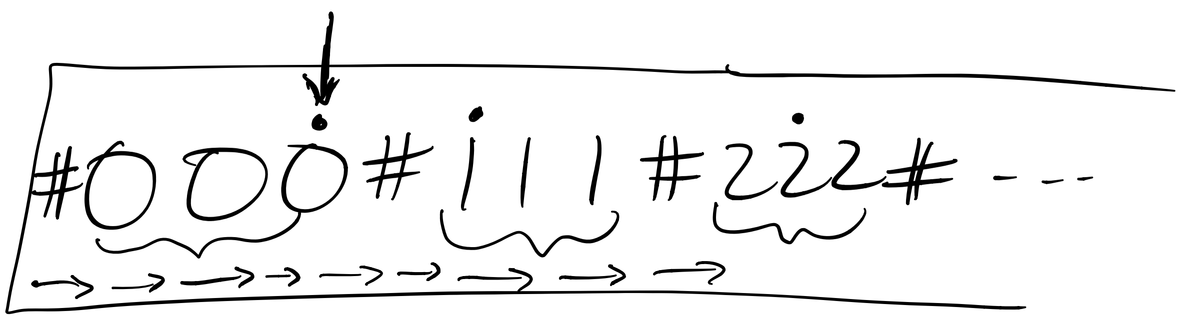
If a TM halts on <u>all</u> inputs, then its language is <u>decidable</u>.

$\underline{\{0^n 1^n 2^n : n \geq 0\}}$ $\qquad$ $\{0^{2^n} : n \geq 0\}$

Multitape TMs:

k=3 tapes





"dotting" for every symbol $a \in \Gamma$

add a new symbol $\dot{a}$

To carry out a transition, start at LHS and start scanning right, remembering which k symbols were dotted (in states).

Then start scanning left, carrying out the appropriate transition and dotting symbols as necessary.

If any transition moves R into a #, push rest of tape contents R one position, and then mark original cell as ⊔.

"Simulation"

Ex 7.49

If a single-tape TM runs in $o(n \log n)$ time for all inputs of size n, its language is regular.

"Stay-Put" TM

in addition to L, R can also do an S instruction where the tape head does not move.

if a S instruction is found, simulate it with a R then L transition w/o changing the tape.

"left-reset" TM

- can do L, R transitions but also do a RESET transition, where the tape head instantly moves to the leftmost tape cell

to simulate a LRTM, push input right 1 position and (before that) mark the first cell with special character.

To execute a RESET transition, scan left until special symbol is found, then move R.

Office Hours / Problem Solving

$\{ww : w \in \{0\}^* \}$

$\{0,1\}^*$

$\{wvw : w \in \{0,1\}^*, |w| \geq 1, |v| \geq 1\}$.

$w = 0^p$ $\quad$ wvw into xyz

$v = 1$

xy^0z

$\underline{wvw = 0^p 1 0^p}$ $\quad$ instead of

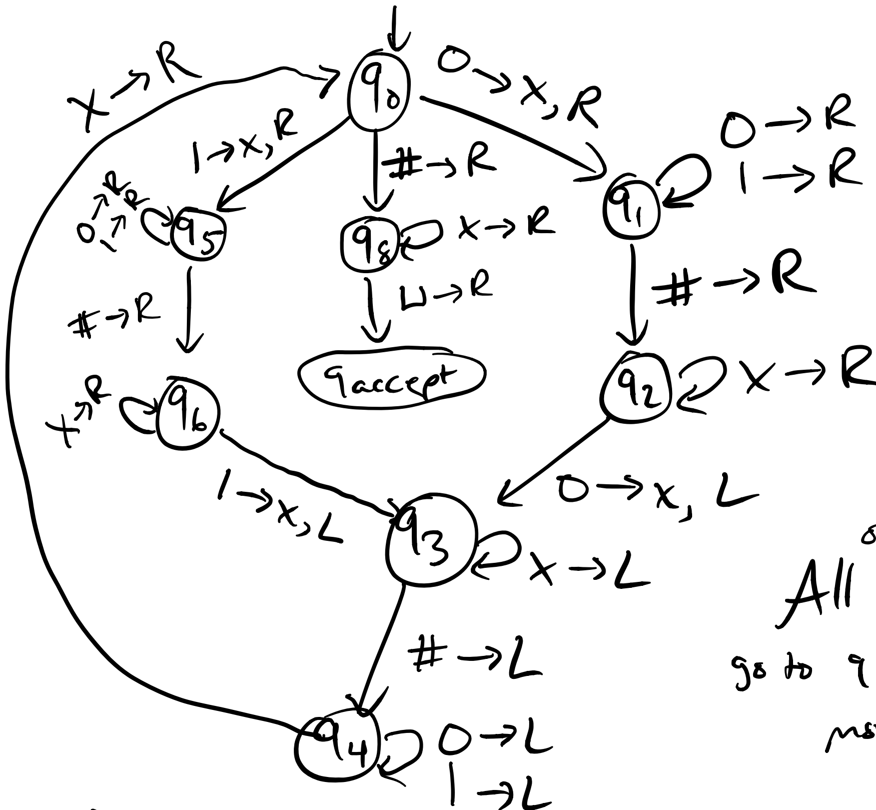
$s =$

$xy^2z]$

$\{wvw : v, w \in \{0,1\}^* \}$

$L = \{w \# w : w \in \{0,1\}^*\}$

$011\#011$



All other transitions go to q_{reject} and move R.

X