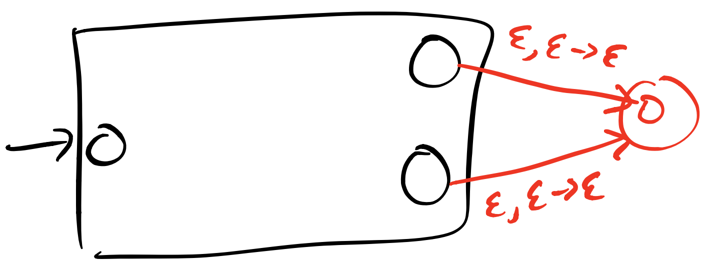
Announcements

Pset 3 out

Pset 2 – extend to Thursday 11:59 P.M.

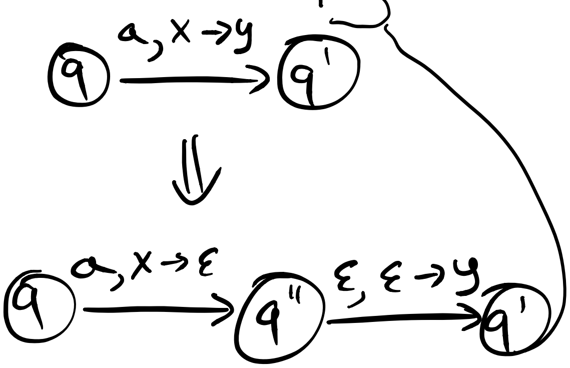
PDA → CFG

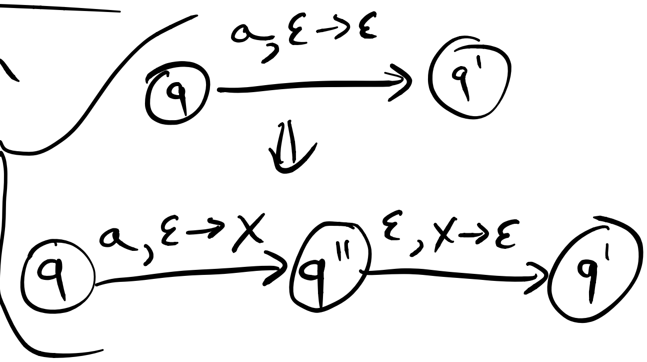
Q: Can any PDA be converted into one that has exactly 1 final state?



Q: Can any PDA be converted into one that on <u>every</u> transition either pushes or pops but not both or neither?

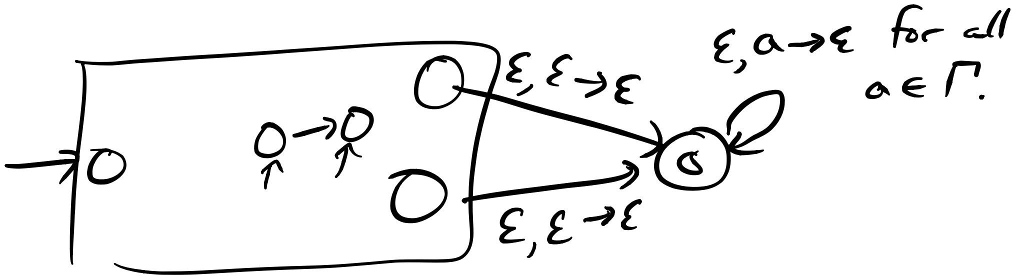
		pop: yes	no
push	yes	↑	✓
	no	✓	✓

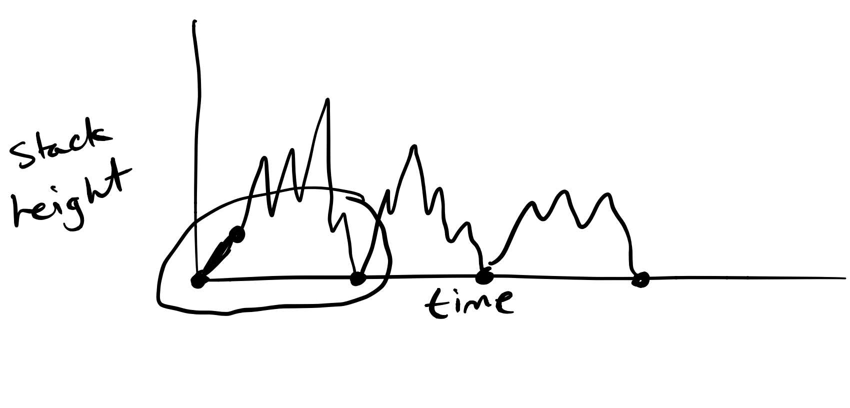


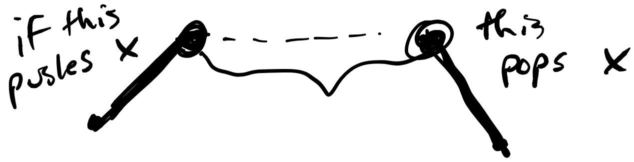


Q: Can every PDA be converted into one that if it has an acceptating computation has another w/ the stack ending empty?

Yes:







Create variables A_{pq} where $p, q \in Q$.

A_{pq} will generate all strings that go from state p to state q, empty stack to empty stack.

Start variable is $A_{q_0 q_f}$

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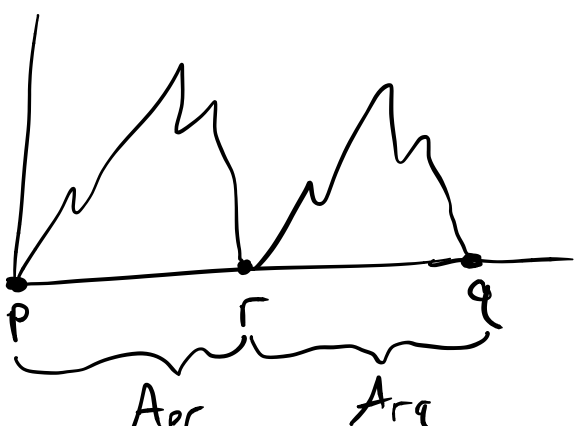
① $A_{pp} \to \varepsilon$ for all $p \in Q$.

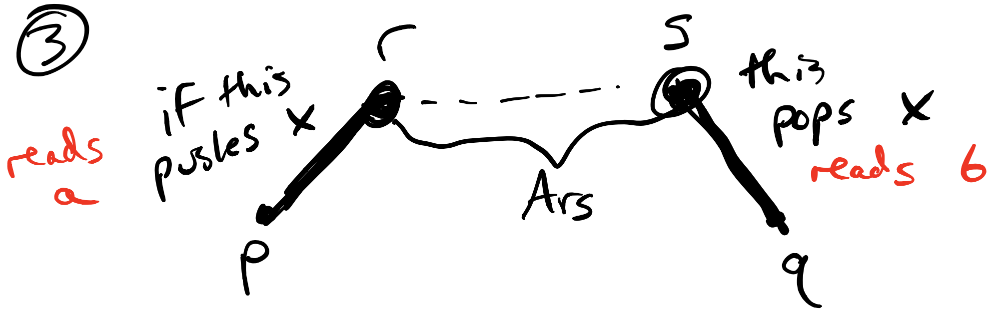
②



for all $p, q, r \in Q$

add rule $A_{pq} \to A_{pr} A_{rq}$

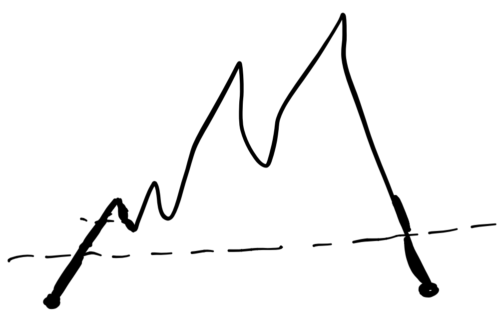
③

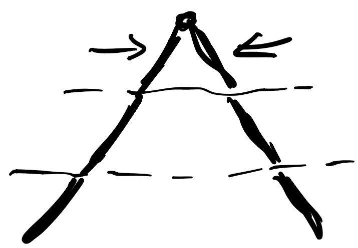


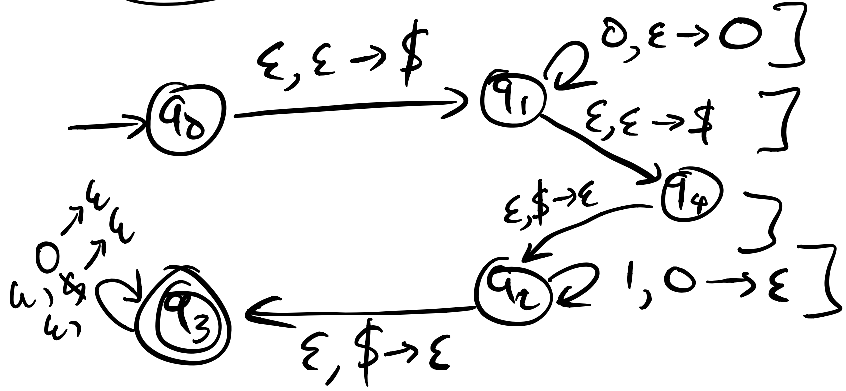
add rule

$$A_{pq} \to \underline{a}\, A_{rs}\, \underline{b}$$

for all $p, q, r, s \in Q$ and having valid push/pop transitions as above.







① $A_{q_0 q_0} \to \varepsilon$

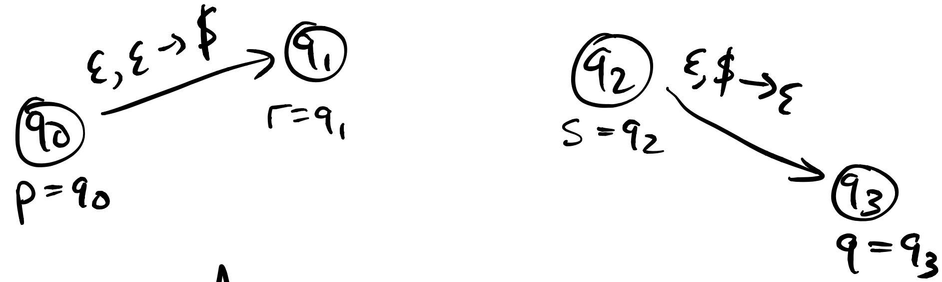
$A_{q_2 q_2} \to \varepsilon$

$A_{q_4 q_4} \to \varepsilon$

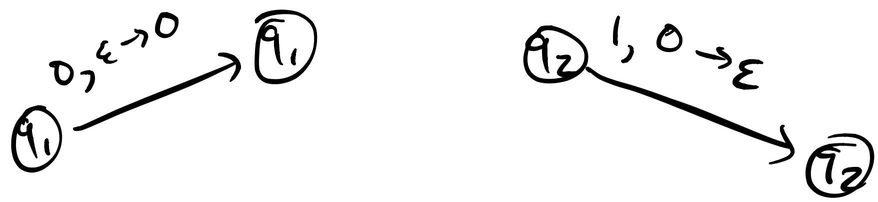
(2) $A_{q_0 q_3} \to A_{q_0 q_4} A_{q_4 q_3}$

$A_{q_2 q_0} \to A_{q_2 q_1} A_{q_1 q_0}$

(3)



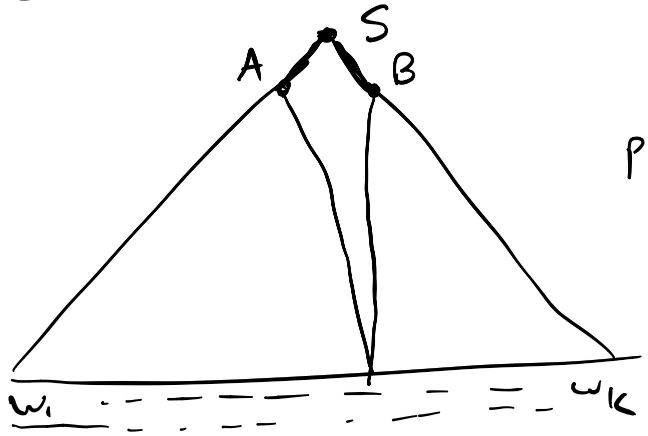
$A_{q_0 q_3} \to A_{q_1 q_2}$



$A_{q_1 q_2} \to 0 A_{q_1 q_2} 1$

$\{0^n 1^n 2^n : n \geq 0\}$.

Let G be an arbitrary CFG in CNF.

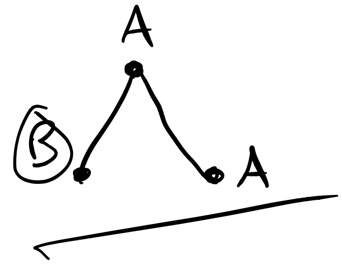


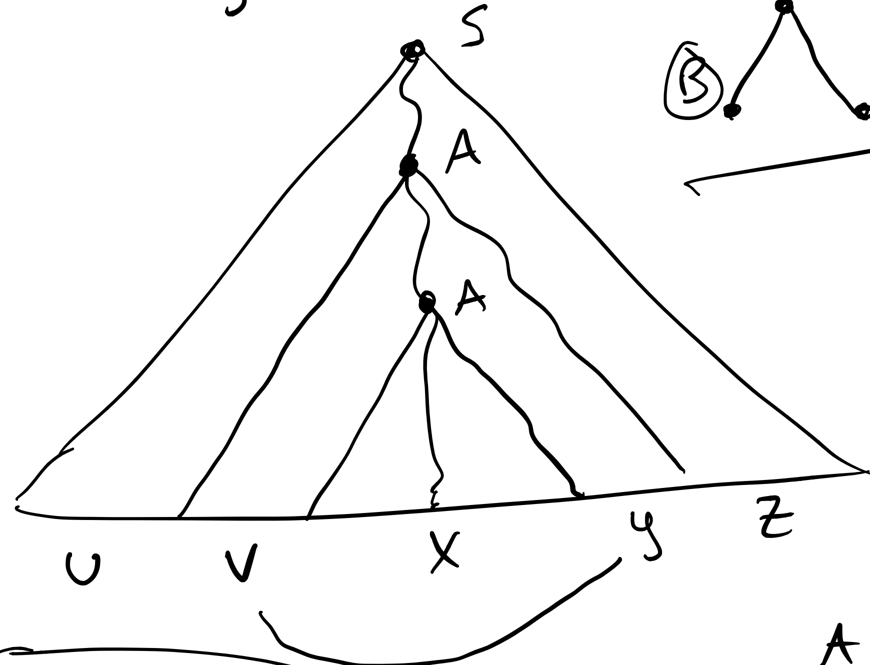
parse tree for some $w \in L(G)$

n variables in G.

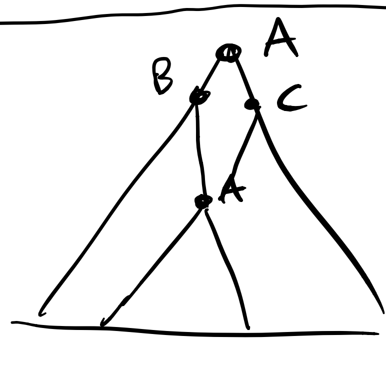
If we can guarantee height of parse tree is $\geq n+1$, then the (wlog) longest root-to-leaf path in the tree repeats a variable.

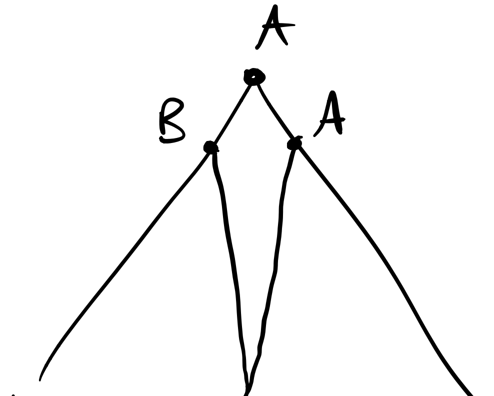
If the parse tree has $\geq 2^n + 1$ leaves, then height $\geq n+1$.

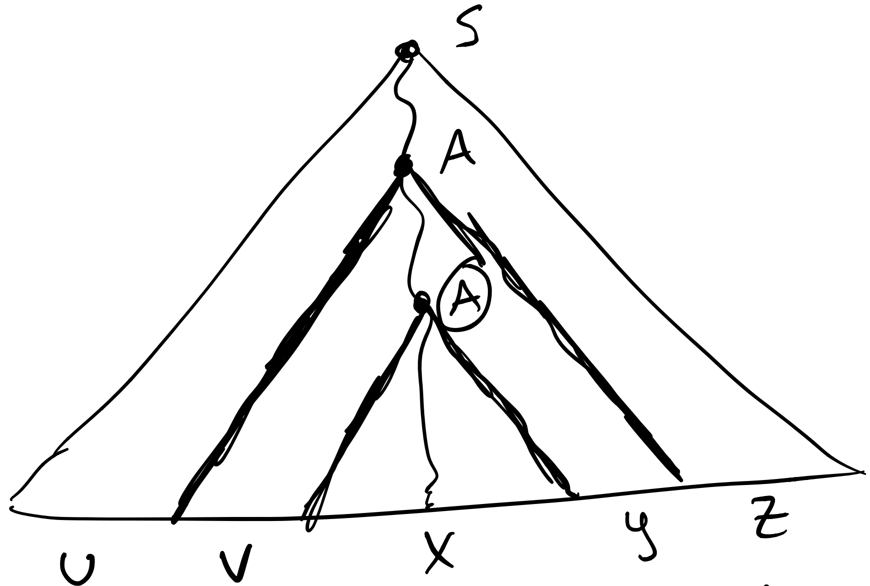




$|vy| \geq 1$ $(vy \neq \varepsilon)$







$$A \Rightarrow^* x$$
$$A \Rightarrow^* vAy$$
$$S \Rightarrow^* uAz$$

$$S \Rightarrow^* uvAyz \Rightarrow^* uv^2Ay^2z$$
$$S \Rightarrow^* uv^ixy^iz$$

$uv^ixy^iz \in L$ for all $i \geq 0$.

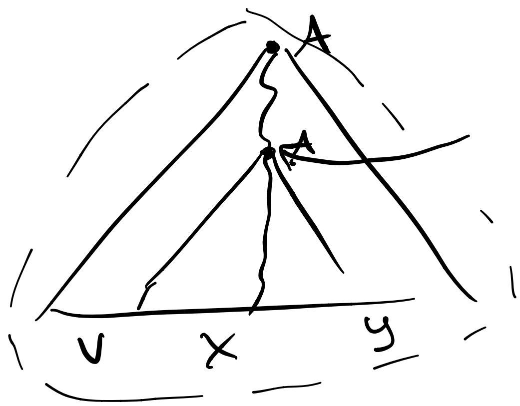
Pumping Lemma for CFLs

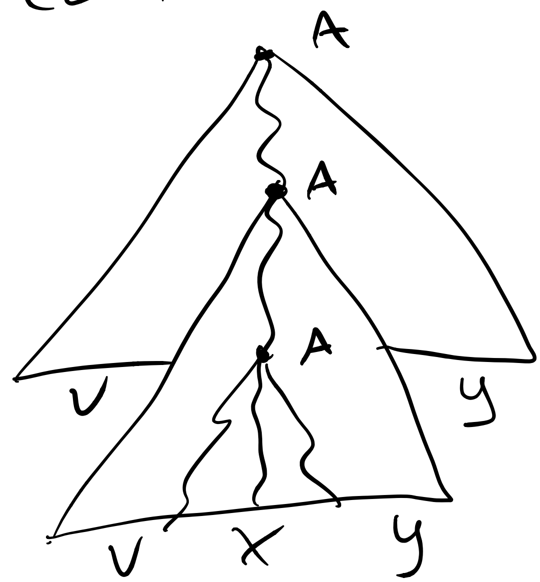
Let L be a CFL. Then there exists a pumping constant p for L so ~~that~~ that for all $w \in L$ with $|w| \geq p$ there exists a way to write w as $uvxyz$ so that

1. $|vxy| \leq p$

2. $|vy| \geq 1$

3. $uv^ixy^iz \in L$ for all $i \geq 0$.





Show that $L = \{0^n 1^n 2^n : n \geq 0\}$ is not a CFL.

Proof: Assume L is a CFL.

$\Rightarrow \exists$ a p for L.

Choose $w = 0^p 1^p 2^p$.

For any decomposition of w into $uvxyz$,
we can see that vxy cannot have a 0, 1, and a 2.

Consider $s = uv^2xy^2z$: if it is in L, then
$\#_0(s) = \#_1(s) = \#_2(s)$

Because $|vy| \geq 1$, one of these three values increased by at least 1.

But because vxy cannot contain a 0 and a 2, then vxy only contains 1's.

This implies $\#_1(s)$ increased, but $\#_0(s)$ did not.

So $s \notin L$.

Therefore, L is not a CFL.

$S \to S_1 \mid S_2$ for union

$S \to S_1 S_2$ for concat

$S \to \varepsilon \mid S_1 S$ for star

Intersection?

$\{0^n 1^n 2^n : n \geq 0\}$

$L_1 = \{0^n 1^n 2^m : n, m \geq 0\}$

$L_2 = \{0^n 1^m 2^m : m, n \geq 0\}$

$S \to AB$

$A \to 0A1 \mid \varepsilon$

$B \to 2B \mid \varepsilon$

$S \to AB$

$A \to 0A \mid \varepsilon$

$B \to 1B2 \mid \varepsilon$

$L_1 \cap L_2 = \{0^n 1^n 2^n : n \geq 0\}$

but this is not a CFL.

Complement?

$$L_1 \cap L_2 = \overline{\overline{L_1} \cup \overline{L_2}}$$ (De Morgan's Law)

$\Longrightarrow$ no closure for complement

$\overline{\{0^n 1^n 2^n : n \geq 0\}}$

Office Hours/Problem Solving

Closure properties

	union	intersect	complement	concat	star
reg lang	✓	✓	✓	✓	✓
CFL	✓	X	X	✓	✓

intersect w/ reg langs?

Yes!

	union	intersect	complement	concat	star
non-reg langs	X	X	✓	X	X

Let L be any non-reg lang.

$L \cup \overline{L} = \Sigma^* \leftarrow$ reg.

$0 \circ 1 \circ 0$ R' handles all these

[R'] R' _ _ _

$\{ wxw : w, x \in \{0,1\}^* \}$

[. - - -] ∘ [. - - -]

[0 - - -] ∘ 0

Product Construction closure under union/intersect
for reg. langs.

Powerset Construction — NFA → DFA

NFA → reg grammar
reg grammar → NFA

GNFA method: NFA → regex

Regex → NFA

CFG → PDA
PDA → CFG

---

g represents star

Rg*

O***

$L = \{0^i 1^j 2^k : 0 \le i \le j \underline{\le k}\}$

is not a CFL.

Assume L is a CFL.

$\Rightarrow \exists$ a p for L.

Choose $w = 0^p 1^p 2^p$. uv^ixy^iz $i=0$

If vy contain only 1s or 2s, then pump down.
Then $\#_2(s) < \#_0(s)$ or $\#_2(s) < \#_1(s)$.

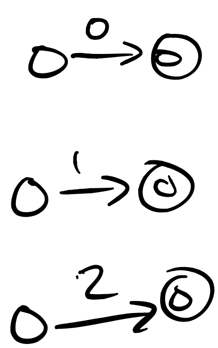
If vy contains only 0's or 1's, then pump up.
Then $\#_0(s) > \#_2(s)$ or $\#_1(s) > \#_2(s)$.

In all cases, we leave L.

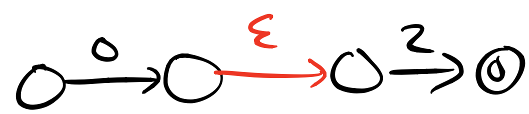
$\Rightarrow$ L is not a CFL.

Regex → NFA

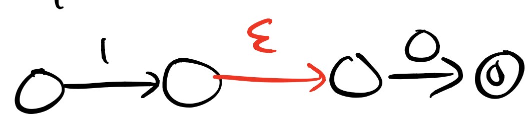
$$(02 \cup 10)^*$$



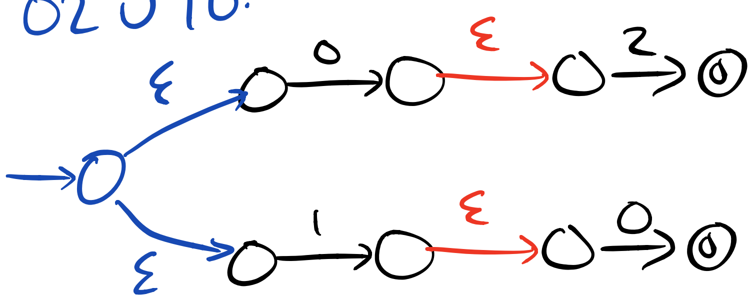
02:

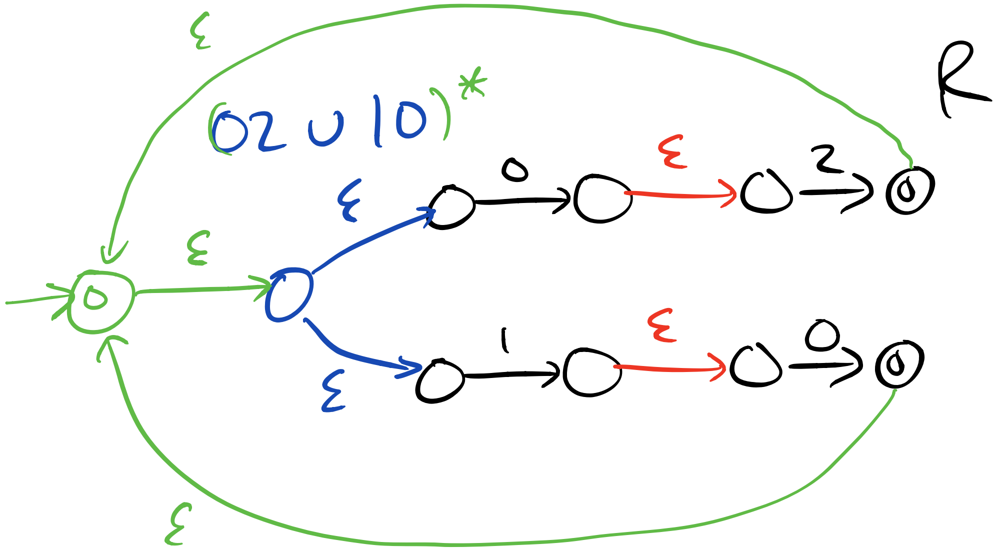


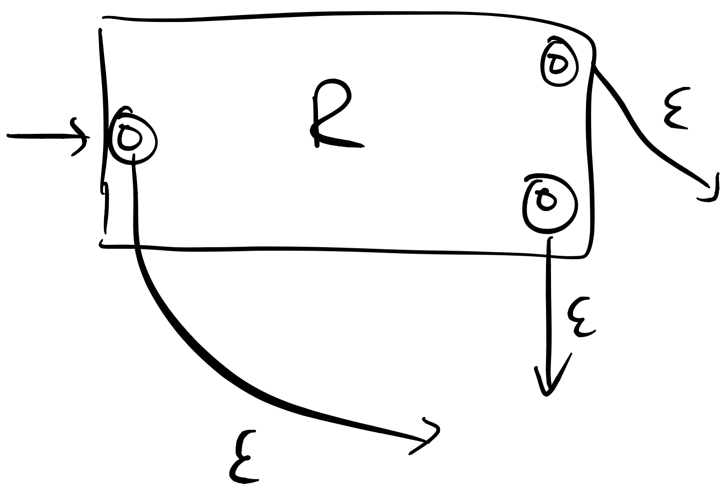
10:

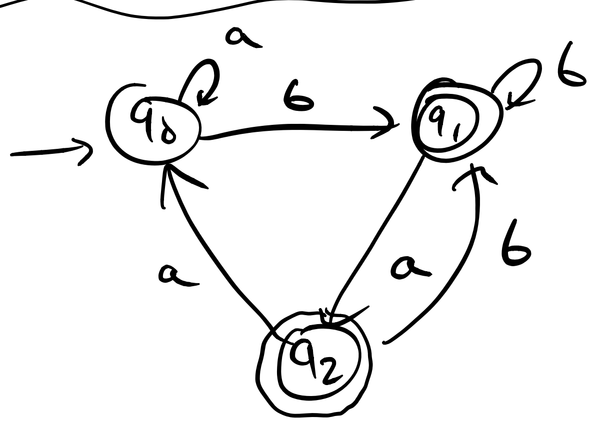


$02 \cup 10$:

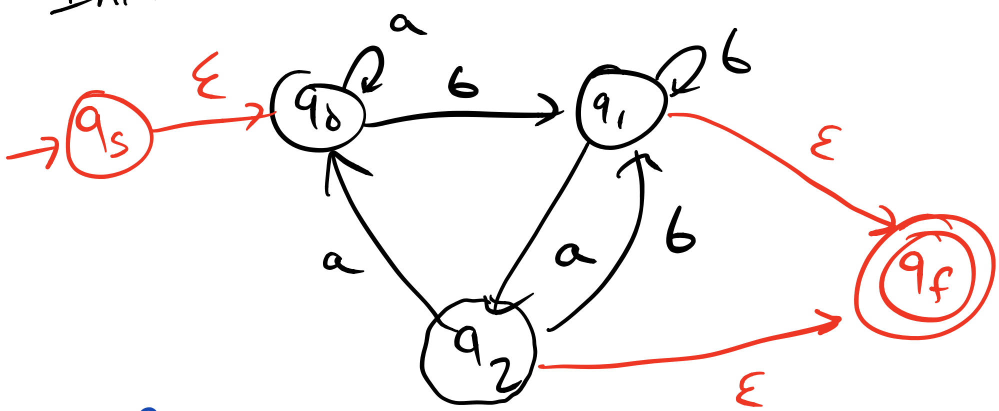



R
(02 ∪ 10)*
ε
0
2
1
0

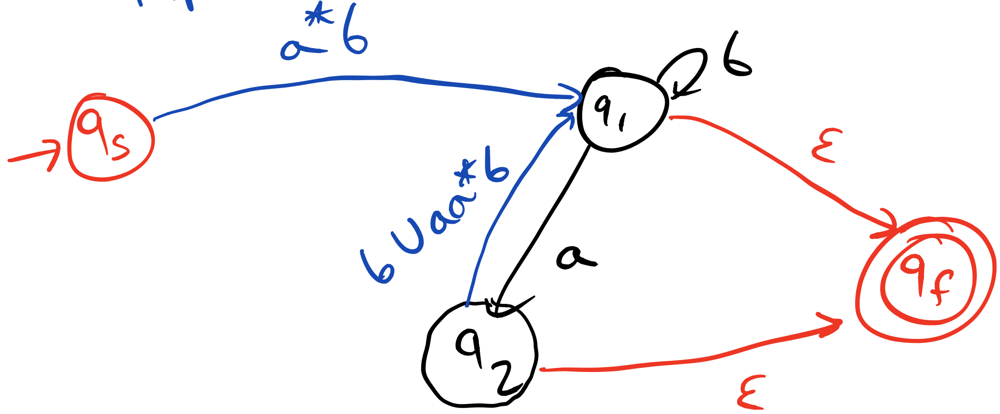

R
ε
ε
ε


q0
q1
q2
a
b
b
a
a
b

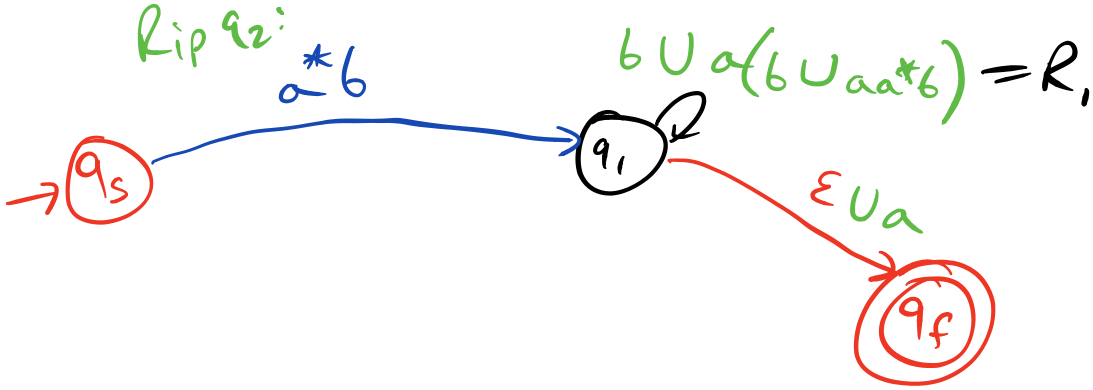
Initial GNFA:



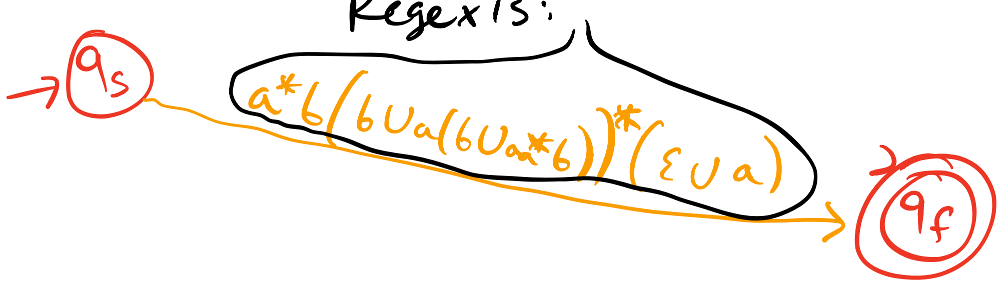
Rip q_0:

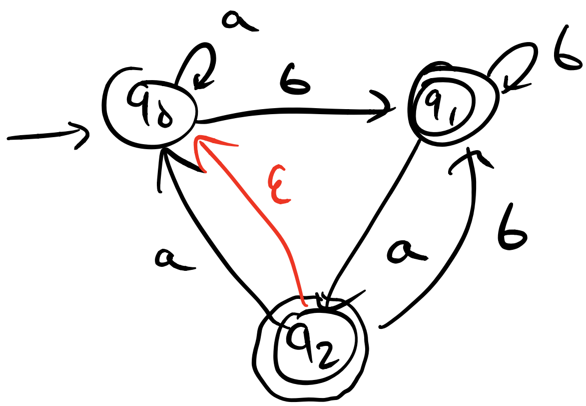


Rip q_2:



Regex is:





$q_0 \to a q_0 \mid b q_1$

$q_1 \to b q_1 \mid a q_2 \mid \varepsilon \mid a$

$q_2 \to b q_1 \mid a q_0 \mid \varepsilon \mid q_0$