Combinatorial Optimization

Paris Gavriilidis, Dionysios Kotanidis

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1 Problem Formulation

1.1 Mathematical Model

Sets

- I: Set of production facilities.
- J: Set of substations.
- K: Set of customers.
- S: Set of seasons.

Parameters

- $PC_{i,s}$: Production cost for facility i in season s.
- $Cap_{i,s}$: Production capacity for facility i in season s.
- FC_j: Fixed cost of opening substation j.
- $SC_{j,s}$: Capacity of substation j in season s.
- $D_{k,s}$: Demand of customer k in season s.
- α_{ij}: Heat loss coefficient between facility i and substation j.
- β_{jk}: Heat loss coefficient between substation j and customer k.
- M_{jk}: Large constant for assignment constraints.

Variables

- y_j: Binary variable indicating if substation j is open.
- x_{jk}: Binary variable for assigning customer k to substation j.
- $f_{ij,s}$: Flow from facility i to substation j in season s.
- $f_{jk,s}$: Flow from substation j to customer k in season s.

Objective Function

The objective is to minimize the total cost, which includes the fixed costs of opening substations and the production costs associated with flows from facilities to substations:

$$Minimize \sum_{j \in J} FC_j \cdot y_j + \sum_{i \in I} \sum_{j \in J} \sum_{s \in S} PC_{i,s} \cdot f_{ij,s}$$

Constraints

1. Flow Conservation at Substations

The total adjusted flow arriving at each substation j in season s from all facilities must equal the flow leaving to customers:

$$\sum_{i \in I} f_{ij,s} \cdot \alpha_{ij} = \sum_{k \in K} f_{jk,s}, \quad \forall j \in J, \forall s \in S$$

2. Demand Satisfaction

The flow arriving at each customer k in season s, adjusted by heat loss, must meet or exceed their demand:

$$\sum_{j \in J} f_{jk,s} \cdot \beta_{jk} \geq D_{k,s}, \quad \forall k \in K, \forall s \in S$$

3. Production Capacity Limits

The total flow sent from each facility i to substations in season s must not exceed the facility's production capacity:

$$\sum_{j \in J} f_{ij,s} \leq Cap_{i,s}, \quad \forall i \in I, \forall s \in S$$

4. Substation Capacity Limits

The flow through each substation j in season s must not exceed its capacity if it is open:

$$\sum_{k \in K} f_{jk,s} \leq SC_{j,s} \cdot y_j, \quad \forall j \in J, \forall s \in S$$

5. Customer Assignment

Each customer k must be assigned to exactly one substation:

$$\sum_{j \in J} x_{jk} = 1, \quad \forall k \in K$$

6. Assignment Only to Open Substations

Customers can only be assigned to substations that are open:

$$x_{jk} \leq y_j, \quad \forall j \in J, \forall k \in K$$

7. Flow Only Through Assigned Substations

The flow from a substation j to a customer k in season s is limited by a large constant M_{jk} only if the substation serves the customer:

$$f_{jk,s} \leq M_{jk} \cdot x_{jk}, \quad \forall j \in J, \forall k \in K, \forall s \in S$$