$C=\{2,4,5,8\}$, $dP_0=1$, $\forall 1\le i\le 10 \rightarrow dP_i=0$: initial states

rhs = 1 ;

$\forall i: c_i>1 \rightarrow dP_1=0$] no card was picked

rhs = 2

$2=2 \rightarrow dP_{2_{new}} = \underbrace{dP_2}_{\text{not pick 2}} + \underbrace{dP_0}_{\text{Pick 2}} = 0+1=1$]

$4,5,8>2$

rhs = 3

$2<3 \rightarrow dP_{3_{new}} = \underbrace{dP_3}_{\text{not pick 2}} + \underbrace{dP_1}_{\text{Pick 2}} = 0+0=0$]

$4,5,8>3$

rhs = 4 $\rightarrow dP_{4_{new}} = \underbrace{dP_4}_{\text{not pick 2,4}} + \underbrace{dP_2}_{\text{pick 2}} + \underbrace{dP_0}_{\text{Pick 4}} = 0+1+1=2$] picked

$2,4\le 4$, $5,8>4$

rhs = 5 $\rightarrow dP_{5_{new}} = \underbrace{dP_5}_{\text{not pick 2,4,5}} + \underbrace{dP_3}_{\text{Pick 2}} + \underbrace{dP_1}_{\text{Pick 4}} + \underbrace{dP_0}_{\text{Pick 5}} = 0+0+0+1=1$]

$2,4,5\le 5, 8>5$

rhs = 6 $\rightarrow dP_{6_{new}} = \underbrace{dP_6}_{\text{not pick 2,4,5}} + \underbrace{dP_4}_{\text{Pick 2}} + \underbrace{dP_2}_{\text{Pick 4}} + \underbrace{dP_1}_{\text{Pick 5}} = 0+2+1+0=3$]

$2,4,5\le 6, 8>6$

rhs = 7 $\rightarrow dP_{7_{new}} = \underbrace{dP_7}_{\text{not pick 2,4,5}} + \underbrace{dP_5}_{\text{Pick 2}} + \underbrace{dP_3}_{\text{Pick 4}} + \underbrace{dP_2}_{\text{Pick 5}} = 0+1+0+1=2$]

$2,4,5\le 6, 8>7$

rhs = 8 $\rightarrow dP_{8_{new}} = \underbrace{dP_8}_{\text{not pick any}} + \underbrace{dP_6}_{\text{Pick 2}} + \underbrace{dP_4}_{\text{Pick 4}} + \underbrace{dP_3}_{\text{Pick 5}} + \underbrace{dP_0}_{\text{Pick 8}} = 0+3+2+0+1=6$]

$2,4,5,8\le 8$

rhs = 9 $\rightarrow dP_{9_{new}} = \underbrace{dP_9}_{\text{not pick any}} + \underbrace{dP_7}_{\text{Pick 2}} + \underbrace{dP_5}_{\text{Pick 4}} + \underbrace{dP_4}_{\text{Pick 5}} + \underbrace{dP_1}_{\text{Pick 8}} = 0+2+1+2+0=5$]

rhs = 10 $dP_{10_{new}} = \underbrace{dP_{10}}_{\text{not pick}} + \underbrace{dP_8}_{\text{pick 2}} + \underbrace{dP_6}_{\text{pick 4}} + \underbrace{dP_5}_{\text{pick 5}} + \underbrace{dP_2}_{\text{Pick 8}} = 0+6+3+1+1=11$]

$\Rightarrow$ there are 11 ways to pick cards with value $\in\{2,4,5,8\}$ that sum of cards is 10 and arrenge them on a line so

$$\sum \frac{(x_1+x_2+x_3+x_4)!}{x_1!\,x_2!\,x_3!\,x_4!} = 11 \qquad \begin{cases} 2x_1+4x_2+5x_3+8x_4=10 \\ x_i\ge 0 \text{ , int} \end{cases}$$