INDUSTRIAL APPLICATION

Multiobjective optimization for crash safety design of vehicles using stepwise regression model

Xingtao Liao · Qing Li · Xujing Yang · Weigang Zhang · Wei Li

Abstract In automotive industry, structural optimization for crashworthiness criteria is of special importance. Due to the high nonlinearities, however, there exists substantial difficulty to obtain accurate continuum or discrete sensitivities. For this reason, metamodel or surrogate model methods have been extensively employed in vehicle design with industry interest. This paper presents a multiobjective optimization procedure for the vehicle design, where the weight, acceleration characteristics and toe-board intrusion are considered as the design objectives. The response surface method with linear and quadratic basis functions is employed to formulate these objectives, in which optimal Latin hypercube sampling and stepwise regression techniques are implemented. In this study, a nondominated sorting genetic algorithm is employed to search for Pareto solution to a full-scale vehicle design problem that undergoes both the full frontal and 40% offset-frontal crashes. The results demonstrate the capability and potential of this procedure in solving the crashworthiness design of vehicles.

Keywords Crashworthiness · Multiobjective optimization · Stepwise regression · Finite element method · Genetic algorithm

X. Liao · X. Yang · W. Zhang
State Key Laboratory of Advanced Design and Manufacture for Vehicle Body, Hunan University,
Changsha 410082, China

Q. Li (✉) · W. Li
School of Aerospace, Mechanical and Mechatronic Engineering,
The University of Sydney,
Room S509, Building J07,
Sydney, NSW 2006, Australia
e-mail: Qing.Li@aeromech.usyd.edu.au

1 Introduction

Crashworthiness is a very important yet highly demanding design requirement while developing high-quality and low-cost industrial products involving potential impact. In the automotive industry, advents of high performance computer as well as advanced numerical algorithms provide a possibility to simulate a full range of laboratory crash events (Kodiyalam et al. 2004a, b), e.g., roof crush (Mao et al. 2006), interior squash (Hamza et al. 2004), as well as frontal (Rudenko et al. 2002; Yang et al. 2005), side (Gu et al. 2001; Youn et al. 2004) and rear impacts (Latchford and Chirwa 2000), with acceptable accuracy and efficiency. To date, nonlinear explicit finite element analysis (FEA) has been extensively employed for the design of vehicle to meet various safety guidelines (Fang et al. 2005a, b). However, the implicit relation and the great complexity of sensitivity analysis in a context of material and geometric nonlinearities as well as frictional contact dynamics could largely compromises the feasibility of practical application of prevalent mathematical programming techniques (Li et al. 2005; Fang et al. 2005a, b). As an effective (sometimes the unique) alternative, such surrogate or metamodel techniques as response surface method (RSM) have been exhaustively adopted in industrial designs (Simpson et al. 2001; Yang et al. 2005).

The metamodel techniques have been a hot topic in relation to the crashworthiness design of the vehicle models. In 2001, Gu et al. (2001) presented a nonlinear RSM to solve for crashworthiness optimization with a full-scale vehicle model under a side impact, where the weight was set as the objective function while peak force, velocity and deflection were the constraints with 11 design variables. In their procedure, the Latin Hyper Cube Sampling and Stepwise regression techniques were implemented for constructing the response surfaces in an economical fashion. Marklund and Nilsson (2001) designed a full car model by minimizing the weight subject to velocity constraints, in which linear and quadratic basis functions were employed to construct the corresponding response surfacesr, respectively. Yu et al. (2001) also considered the weight of reinforcement members as the objective while the toe-board intrusion and the deformation of A–C pillar space as constraints in a full-scale car crashworthiness design, where a multilevel RSM was presented for the so-called Statistical Design Support System. Later, Redhe and Nilsson (2004) took a Saab automobile model as an example to minimize the intrusion subject to the constraints of stop-time and the door displacements. After comparing RSM with the stochastic optimization (SO) methods, they found that RSM outperformed if the design variables are fewer and suggested that a combined procedure of RSM and SO (Redhe and Nilsson (2004) could be useful for optimizing the vehicle frontal structure. To reduce the number of full-scale FE runs, Craig et al. (2005) adopted variable screening technique in a full front impact. Fang et al. (2005a, b) recently found that conventional quadratic polynomials do provide a good approximation to model the energy absorption, but the radial basis function (RBF; Lanzi et al. 2004) could perform better to model such highly nonlinear objectives as peak acceleration. In addition to these deterministic solutions, Youn et al. (2004) proposed a reliability-based design optimization methodology with help of the response surface technique for vehicle impacts, where a number of reliability constraints were presented to take into account the issues of manufacturing imperfection and tolerances. Fu and Sahin (2004) also developed a stochastic method for vehicle impact, in which the Monte Carlo sampling and sequential quadratic programming algorithms were adopted to form the inner and outer loops of the optimization procedure.

The above studies to a certain extent showed that the construction of an accurate surrogate or metamodel is by no means easy, which generally requires considerable computing resources (Kodiyalam et al. 2004a, b; Fang et al. 2005a, b). This problem can become particularly severe when the number of design variables is great. For this reason, various modification techniques have been presented to achieve best possible tradeoff between the design accuracy and computing efficiency (Simpson et al. 2001). In this regard, Yang et al. (2005) thoroughly compared the performance of the stepwise regression, moving least square, Kriging interpolation, multiquadratic and adaptive/interactive modelling system methods through a real-world full vehicle model. Their studies revealed that there is no best single metamodel available to accommodate all scenarios of crashworthiness requirements and consequently a "hybrid metamodelling strategy" was suggested.

Furthermore, crashworthiness design of real-world vehicles involves optimization of a number of objectives. It is more often than not that some of these objectives conflict each other, which to a certain extent, any further improvement in one objective must lead to worsening one or more other objectives. Thus, it is only possible to search for a solution in a Pareto sense. As an alternative, indeed, the multidisciplinary and/or multiobjective optimizations in vehicle crashworthiness design were previously addressed in a form of single "primary" objective function $f(\boldsymbol{x})$ with multiple functional $g_i(\boldsymbol{x})$ and dimensional $h_j(\boldsymbol{x})$ constraints as follows (e.g., Sobieszczanski-Sobieski et al. 2001; Redhe et al. 2004).

$$\begin{aligned} &\min F(\mathbf{x}) = f(\mathbf{x}) \\ &s.t.\, g_i(\mathbf{x}) \leq 0,\, i = 1,\, 2, \cdots, I \\ &\quad h_j(\mathbf{x}) \leq 0,\, j = 1,\, 2, \cdots, J \end{aligned} \tag{1}$$

However, it is not easy even for most experienced design engineers to identify a primary or predominant objective from a long list of design requirements $f_i(\boldsymbol{x})$. Instead, it is largely desirable to address these requirements in a multiobjectve framework, which could lead to a more thorough understanding of the optimal space consisting of different design objectives. To take the advantage of the single-objective optimization, one of the simplest approaches is to aggregate these different objectives into a single cost function in terms of weight average by w_k, as presented by Fang et al. (2005a, b) in a full-scale vehicle model for weight, peak acceleration and energy-absorption objectives, as well as by Hamza et al. (2004) in a multibody dynamics model with multiple flexion objectives,

$$\begin{aligned} &\min F(\mathbf{x}) = w_1 f_1(\mathbf{x}) + w_2 f_2(\mathbf{x}) + \cdots + w_m f_m(\mathbf{x}) \\ &s.t. g_i(\mathbf{x}) \leq 0,\, i = 1, 2, \cdots, I \\ &\quad h_j(\mathbf{x}) \leq 0,\, j = 1, 2, \cdots, J \end{aligned} \tag{2}$$

This technique provides flexibility in making use of various algorithms that were developed for single-objective optimization problems. When the Pareto space is convex as the weighted criteria method is able to search for the Pareto frontier (Athan and Papalambros 1999).

However, the complex nature between the design objectives and variables may not always guarantee the convexity of Pareto space, in which the strategy of formulating a single cost function may become infeasible or difficult. For this reason, some evolutionary search methods have been developed (e.g., Deb 2001). Using the RBFs to construct response surfaces, Lanzi et al. (2004) presented a multiobjective Genetic Algorithm (GA) to optimize composite absorber shape under different crashworthiness requirements. In the context of vehicle design, based on multibody dynamics models, Hong et al. (2001) presented five biomechanical objectives including the head injury criterion, chest acceleration, chest deflection, and peak loads in two femurs, in which the quadratic response surfaces were used. Later, Dias and Pereira (2004) also adopted the multibody dynamics technique to model the multicar train collision, where the optimization of acceleration and deformation were searched by using the genetic algorithm. Rudenko et al. (2002) sought for the minimization of weight and maximization of energy absorption with a number of mechanical and acoustic constraints in the design of the front end part, where a nondominated sorting genetic algorithm II (NSGA-II; Deb 2001; Deb et al. 2002) was employed to search for the Pareto space. Recently, Yoshimura et al. (2005) further demonstrated the efficiency and effectiveness of NSGA-II in solving multiobjective optimization problems.

This paper adopts the evolutionary multiobjective optimization method, more specifically NSGA-II, by cooperating the surrogate modelling techniques with the Latin Hypercube Sampling and stepwise regression (Krishnaiah 1982). A full-scale vehicle model is developed to simulate the full frontal crash and a 40% offset-frontal crash. To address several important safety requirements of crashworthiness design, energy-absorption, acceleration integration and toe-board intrusion are taken as the objective functions. The optimization generates a Pareto solution space, which provides engineers with a flexibility to make their final design decision.

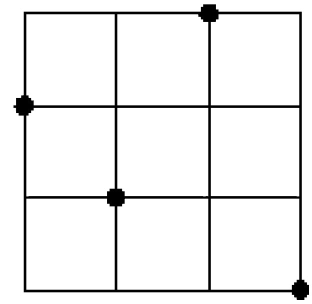
2 Surrogate model method

The surrogate model method uses some basis functions to approximate the highly complicated objective and constraint functions involved in the design problems. The major steps of a surrogate model involve (a) choosing a design of experiment (DOE) for generating sampling data; (b) choosing a mathematical model to represent the data; and (c) best fitting the model to the sampling data.

2.1 Optimal Latin Hypercube Sampling (OLHS)

Among the DOE techniques, the optimal Latin Hypercube Sampling (OLHS) technique (Currin et al. 1991) is employed to construct the surrogate models for the crashworthiness criteria of vehicle. Unlike the conventional factorial DOE, the OLHS method is capable of capturing the higher order of nonlinearity with relatively fewer design points. To ensure the uniformity of the sampling points within the region of interest, a combinatorial optimization algorithm is taken into account based on an entropy criterion to minimize the bias of mean square error. Figures 1 and 2 illustrate the sampling strategies of the LHS and the OLHS methods, respectively. The number of

Fig. 1 LHS sampling points



runs in OLHS is determined by the total number of factors including control variables and noisy variables in the model (Kodiyalam et al. 2004a, b). The minimum number of runs selected in this paper is $3n$, where n denotes the total number of design variables. It is noted that the number of sampling points can vary according to the availability of computing resources.

2.2 Response surface model

The response surface model is one of the typical surrogate models that use some simple basis functions to formulate the complex global objective and constraint functions in the design space. The selection of such basis functions is essential to describe the real responses (Fang et al. 2005a, b). These basis functions can be polynomials of any order or other different simpler functions, e.g., sine and cosine functions. The approximate response can be consequently defined in terms of basis function as:

$$y(\mathbf{x}) = \sum_{j=1}^{N} a_j \varphi_j(\mathbf{x}) \tag{3}$$

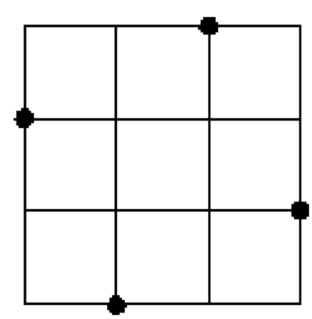
where N represents the numbers of basis functions $\varphi_j(\mathbf{x})$, a_j represents the tuning parameters. When using a quadratic model, the full set of the second-order polynomials of $\varphi_j(\mathbf{x})$ are given as,

$$1, x_1, x_2, \cdots, x_n, x_1^2, x_1x_2, \cdots, x_1x_n, \cdots, x_n^2 \tag{4}$$

and the surrogate model could be thus defined as,

$$y(\mathbf{x}) = a_0 + \sum_{i=1}^{n} a_i x_i + \sum_{i=1}^{n} a_{ii} x_i^2 + \sum_{j<i}^{n} a_{ji} x_j x_i \tag{5}$$

Fig. 2 OLHS sampling points



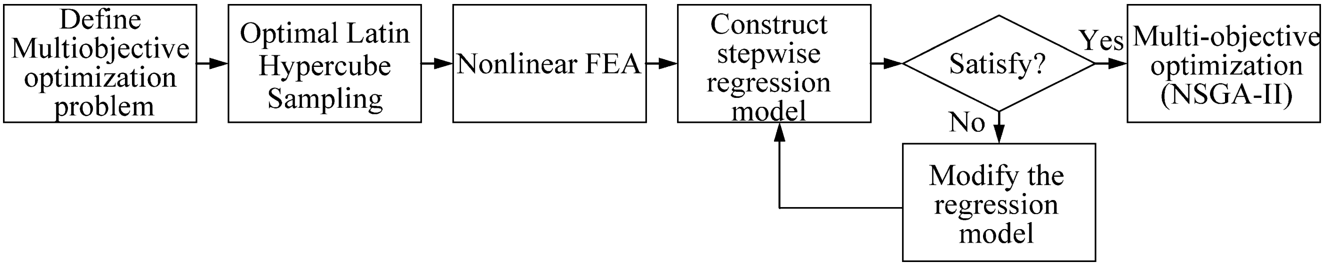


Fig. 3 Flowchart of multiobjective optimization procedure

If the finite element analysis results $\mathbf{f} = \left(f^{(1)}\ f^{(2)}\ \cdots\ f^{(M)}\right)^T$ at the selected M design points (M>N) are obtained, the unknown tuning parameters $\boldsymbol{a}$= ($a_1\ a_2 ...\ a_N$) can be determined by means of the least-squares method. At the ith design point x_i, the error between the finite element simulation $f^{(i)}$ and surrogate approximation $y^{(i)}$is expressed as

$$\varepsilon_i = f^{(i)} - y^{(i)} = f^{(i)} - \sum_{j=1}^{N} a_j \varphi_j \left(x^{(i)}\right) \tag{6}$$

And the sum of the square of the errors is given as

$$E(a) = \sum_{i=1}^{M} \varepsilon_i^2 = \sum_{i=1}^{M} \left[y^{(i)} - \sum\nolimits_{j=1}^{N} a_j \varphi_j \left(x^{(i)}\right)\right]^2 \tag{7}$$

To minimize the overall error $E(\boldsymbol{a})$, the least square method is employed to estimate coefficients a_j, which leads to,

$$\mathbf{a} = \left(\mathbf{X}^T\mathbf{X}\right)^{-1}\left(\mathbf{X}^T\mathbf{y}\right) \tag{8}$$

where $\boldsymbol{X}$ is the matrix consisting of basis functions evaluated at the sample points and is denoted as,

$$\mathbf{X} = [Xui] = [\phi i(\mathbf{x}u)] \tag{9}$$

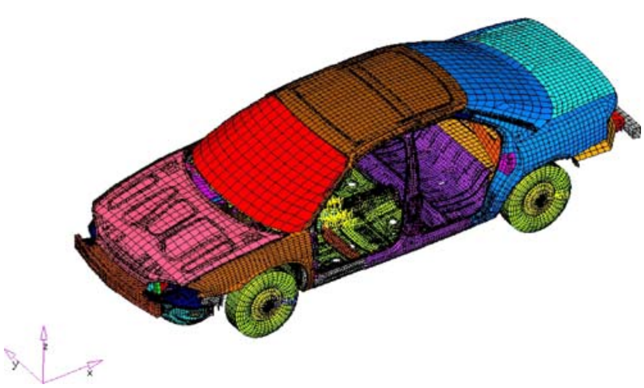


Fig. 4 Finite element model of vehicle

2.3 Stepwise regression

In view of the quadratic response surface model in (5), it is necessary to perform finite element (FE) analysis more than $\frac{(N+1)(N+2)}{2}$ times to obtain the tuning parameters $\boldsymbol{a}$. However, each FE crashing analysis may take a number of hours and thus the computing cost can be very high to get the full set of tuning parameters $\boldsymbol{a}$. A robust method to tackle this problem is to use "stepwise regression" (Krishnaiah 1982) to screen the terms in the quadratic functions which have relatively little influence in the response model.

The stepwise regression model in accordance with the quadratic response surface model in (5) is given as

$$g(\mathbf{x}) = \beta^T\mathbf{x} \tag{10}$$

where vector β includes the unknown coefficients of the basis functions a_o, a_i, a_{ii}, a_{ji}, and the vector $\boldsymbol{x}$ includes all the terms of these basis functions in (5).

In general, the stepwise regression model is constructed recursively by adding or deleting one independent prediction at each time. In a forward procedure, the first step is to choose one predictor, which provides the best fit. The second independent predictor to be added to the regression model is the one that provides the best fit in conjunction with the first predictor. Given that the other predictors are already in the model, next optimum predictor is then added at each step in a recursive fashion. Alternatively, a backward elimination can be used which starts from a model with a full set of basis functions and then gradually drops out the predictor with the least effect in the model in a stepwise fashion. Indeed, after adding certain number of predictors into the model in the forward procedure, their effects may largely interact. It becomes necessary to gradually drop out some predictors from the model whose contribution reduces most significantly. As a result, the stepwise regression model is built by combining the techniques of the forward selection with backward eliminations.

In this paper, the quadratic polynomial for the regression is employed to represent the nonlinear responses of acceleration characteristics and toe-board intrusion for the full frontal crash and 40% offset-frontal crash models, respectively. But a linear basis function is used for the

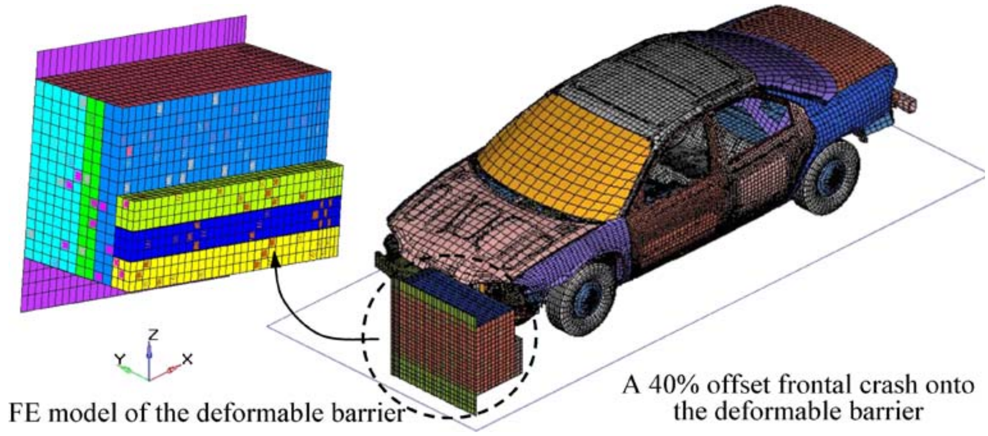


Fig. 5 FE modelling for 40% offset-frontal crashing onto a deformable barrier

vehicle mass as reported in literature (e.g., Marklund and Nilsson 2001).

3 Multiobjective optimization and nondominated sorting genetic algorithm-II

A general multiobjective optimization problem can be expressed as

$$\begin{aligned} \min F(\mathbf{x}) &= [f_1(x), f_2(x), \cdots, f_m(x)] \\ s.t. g_i(\mathbf{x}) &\le 0 \quad i = 1, 2, \cdots, I \\ h_j(\mathbf{x}) &\le 0, \quad j = 1, 2, \cdots, J \\ \mathbf{x} &\in \mathbf{S} \end{aligned} \tag{11}$$

where $f_1(\boldsymbol{x}), f_2(\boldsymbol{x}), ..., f_m(\boldsymbol{x})$ are the m objective functions, $\boldsymbol{x}=(x_1, x_2, ..., x_n)^{\mathrm{T}}$ are the n design variables, $\boldsymbol{x} \in \mathrm{S}$ defines the design space.

In the multiobjective optimization, it is often hardly possible to achieve all such objectives simultaneously (Fang et al. 2005a, b). To some stage, any further improvement in one objective requires a clear tradeoff with at least one other objective. This defines a Pareto optimum, in which there exists no feasible solution $\boldsymbol{x}$ that can decrease some objective functions without causing at least one other objective function to increase. In this sense, the Pareto optimum represents a set of solutions and a multiobjective approach should lead to the identification of a Pareto set (Yoshimura et al. 2005; Deb 2001; Deb et al. 2002; Rudenko et al. 2002; Lanzi et al. 2004; Li et al. 2005).

To conduct multiobjective optimization as defined in (11), the multiobjective evolutionary algorithm, more specifically named NSGA-II (Deb 2001 and Deb et al. 2002) is employed, which enables an efficient implementation of multiobjective GA. In NSGA-II, a Pareto-based ranking of the existing designs in the population is applied for the stochastic selection process involved. As such, there exists a set of nondominated designs for each population. Recall that a design is Pareto-dominant when there is no other design in the population that is strictly better in terms of all objectives. As the search progresses from generation to generation, the nondominated designs are driven gradually close to the true Pareto set of the problem. As a result, near-Pareto designs can be generated without the need for the use of scaling weights to combine the different criteria into a single objective.

4 Integrated multiobjective optimization procedure

As mentioned above, the techniques of OLHS, the stepwise regression, the NSGA-II as well as the FEA based RSM are

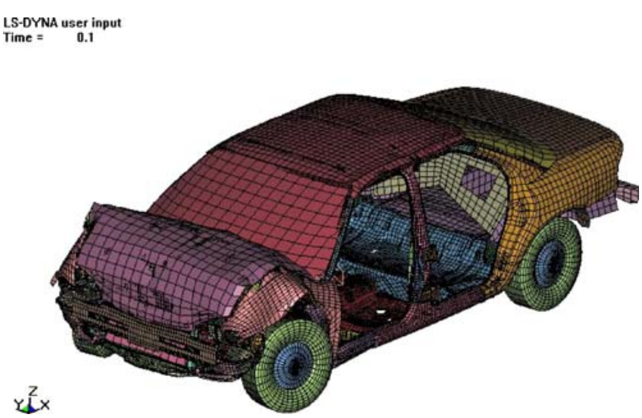


Fig. 6 The deformed results of the full frontal impact

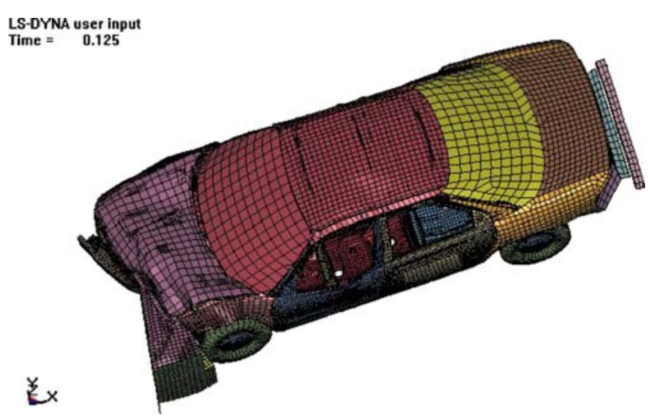


Fig. 7 The deformed results subject to the offset-frontal impact

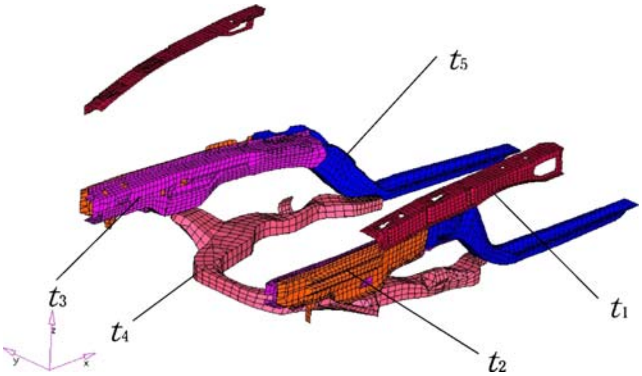


Fig. 8 Design variables of the vehicle model

integrated into a multiobjective optimization procedure. Firstly, the optimization problem including the objectives, constraints and design variables is defined. A set of sample points is computed to construct the stepwise regression models to approximate to the accurate FE solutions. After the fitting accuracy of the surrogate models is validated, the NSGA-II procedure is then performed to search for the Pareto solution to the multiobjective optimization. Figure 3 gives a flowchart to summarize the procedure for multi-objective optimization.

5 Multiobjective optimization for crashworthiness design of vehicles

A full-width frontal crash test and an offset-frontal crash test are taken into account in this study. The full-width impact usually results in a higher deceleration comparing with the offset-frontal impact. It is known that the high deceleration can cause serious biomechanical injuries to the occupants while the offset test is demanding for the structural integrity of a vehicle. In practice, to improve the overall crashworthiness of a frontal structure of a vehicle, these two crashing scenarios should be considered simultaneously.

5.1 Finite element modeling

The finite element model considered contains approximately 90000 elements of a National Highway Transportation and Safety Association vehicle (National Crash Analysis Center 2001) as shown in Fig. 4. To perform a full frontal crash, the vehicle is subjected to a rigid wall impact with the initial velocity of 50 km/h. To simulate a 40% offset-frontal crash, the deformable barrier is constructed abiding by the rules ECE R94.1 (United Nations Economic Commission for Europe 1998). The major part of the deformable barrier is a low-density honeycomb material ("Main honeycomb block") with the crush strength of 0.342 MPa and a high-density honeycomb ("bumper element") with the crush strength of 1.711 MPa. LS-DYNA constitutive law MAT-26 is used to mathematically model the honeycomb materials as an energy absorber. Figure 5 shows the finite element model of the deformable barrier and vehicle crashing settings. The finite element meshes of the vehicle and the deformable barrier are generated in Hypermesh (Altair Engineering 2004) and are solved in LS-DYNA. Figures 6 and 7 show the simulation results in the scenarios of the full frontal crash and the 40% offset-frontal crash, respectively.

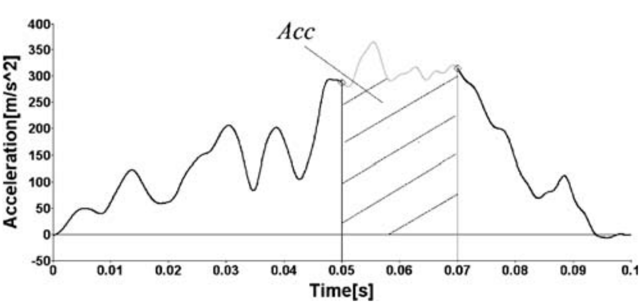


Fig. 9 Acceleration characteristic

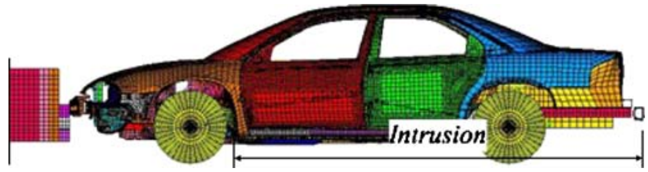


Fig. 10 The toe board intrusion of offset-frontal crash

5.2 Multiobjective optimization problem

As shown in Fig. 8, the thickness of five reinforced members around the frontal structure is chosen as the design variables which could significantly affect the crash safety. For the consideration of lightweight, the mass of the vehicle is set as the first design objective. To take into account the worst scenario of acceleration-induced biomechanical damage of occupants (Yang et al. 2005), an integration of collision acceleration between t_1=0.05 s and t_2=0.07 s in the "full frontal crash" (shown in Fig. 9) was chosen as the second objective function as

$$A_{in} = \int_{t_1}^{t_2} \mathrm{adt} \tag{12}$$

Finally, to consider the most severe mechanical injury, the toe board intrusion in the "offset-frontal crash" (shown in Fig. 10) is regarded as the third objective. It should be pointed out herein that the second and third objectives are constructed, respectively, from the data in two different crash conditions to reflect the extreme crashworthiness

Table 1 Pareto set of the optimization result

	Mass (kg)	A_{in} (m/s)	Intrusion (m)
1	1,698.74	5.59567	0.055533
2	1,696.3	5.55174	0.066563
3	1,675.58	4.47847	0.262609
4	1,675.47	4.48006	0.263222
5	1,677.31	5.76637	0.074325
6	1,690.62	6.357	0.050618
7	1,669.14	6.67743	0.062413
8	1,697.93	5.64378	0.055403
9	1,695	5.72354	0.039277
10	1,679.62	4.88909	0.212811
11	1,665.69	6.42572	0.068167
12	1,672.03	6.51818	0.060617
13	1,688.4	5.07193	0.165519
14	1,687.36	6.4424	0.055686
15	1,678.59	5.67282	0.061767
16	1,681.79	4.63313	0.227072
17	1,666.89	5.30448	0.100443
18	1,661.71	6.63164	0.070684
19	1,670.62	6.80367	0.05471
20	1,693.31	5.68448	0.042852
21	1,667.9	6.75112	0.059584
22	1,685.35	6.1099	0.059836
23	1,682.55	4.6518	0.221514
24	1,673.2	6.46793	0.061608
25	1,691.55	5.65053	0.052289
26	1,689.11	6.4764	0.052537
27	1,664.01	6.58728	0.070601
28	1,676.82	5.72882	0.074664
29	1,675.28	4.55464	0.256598
30	1,694.99	5.72333	0.039335

scenarios. As such, the multiobjective optimization problem is formulated as,

$$\begin{array}{c} \min F(\mathbf{x}) = [Mass, A_{in}, Intrusion] \\ s.t.\ 1mm \le \mathbf{x} \le 3mm \\ \text{where } \mathbf{x} = (t_1, t_2, t_3, t_4, t_5)^T \end{array} \tag{13}$$

5.3 Optimization process and result

As mentioned before, the car has to undergo the two crash conditions to improve overall crashworthiness of a frontal structure. Therefore, the surrogate models used for the optimization have to be constructed based on the data acquired from both the full frontal crash model and the offset-frontal crash model, respectively.

In the full frontal crash and the 40% offset-frontal crash, 15 sampling points are, respectively, generated using the Optimal Latin Hypercube Sampling technique. The surrogate models of the objective functions of the acceleration integration and intrusion are constructed in the quadratic formulae by using the stepwise regression, while the vehicle mass is formulated in a linear function as follows:

$$\begin{array}{l} Mass = 1640.2823 + 2.3573285t_1 + 2.3220035t_2 \\ +4.5688768t_3 + 7.7213633t_4 + 4.4559504t_5 \end{array} \tag{14}$$

$$\begin{array}{l} Ain = 6.5856 + 1.15t_1 - 1.0427t_2 + 0.9738t_3 \\ +0.8364t_4 - 0.3695t_1t_4 + 0.0861t_1t_5 + 0.3628t_2t_4 \\ -0.1106t_1^2 - 0.3437t_3^2 + 0.1764t_4^2 \end{array} \tag{15}$$

$$\begin{array}{l} Intrusion = -0.0551 + 0.0181t_1 + 0.1024t_2 \\ +0.0421t_3 - 0.0073t_1t_2 + 0.024t_2t_3 - 0.0118t_2t_4 \\ -0.0204t_3t_4 - 0.008t_3t_5 - 0.0241t_2^2 + 0.0109t_4^2 \end{array} \tag{16}$$

It is essential to evaluate the accuracies of such stepwise regression surrogate models obtained. In this paper we use the coefficient of multiple determination R^2 and the adjusted coefficient of multiple determination R^2_{adj} as a measure of the accuracy of the models, as defined as follows:

$$R^2 = \frac{\sum_{i=1}^{P} (\widehat{y}i - \overline{y}i)^2}{\sum_{i=1}^{P} (yi - \overline{y}i)^2} \tag{17}$$

$$R^2_{adj} = 1 - \frac{\sum_{i=1}^{P} (y_i - \widehat{y}_i)^2 (P-1)}{\sum_{i=1}^{P} (y_i - \overline{y}_i)^2 (P-m-1)} \tag{18}$$

where P is the number of design points and $\overline{y}_i$, $\widehat{y}_i$ and y_i represent the mean of the responses, the predicted response, and the actual response, respectively. These indicators, which vary between 0 and 1, represent the ability of the stepwise regression surrogate models to identify the variability of the design response. It is noted that the total errors between the surrogate model and the FE analysis, i. e., R^2 for the *Mass*, A_{in} and *Intrusion* are 99.88, 97.87, and 99.74%, respectively, R^2_{adj} are 97.87, 92.54 and 99.08%, respectively. Hence, the accuracies of the surrogate models are considered adequate for this study.

Based on these three surrogate models a multiobjective optimization is performed using the NSGA-II algorithm. The population size chosen is 30, and the NSGA-II is run 50 generations. Table 1 shows the optimal results of the 30 Pareto's points.

It should be noted that the establishment of Pareto-set is crucial to fully understand the solution space. Indeed, the Pareto-set provides designer with a large number of optimal solutions for their decision-make. Take this design problem as an example, the designers may choose the 11*th*, the 18th or the $27th$ as the final solution if they would like to pay more attention to the mass of the vehicle. While if the designer would like to emphasize the toe board intrusion, they may consider the $9th$, 20^{th}, or $30th$ solutions.

6 Conclusions

This paper presents an integrated, efficient and effective multiobjective optimization procedure for the multiobjective design of vehicles crashworthiness. Based on an explicit dynamic FE model, simple stepwise regression models have been created to mathematically capture the performance of the vehicle during impact. Furthermore, with the help of multiobjective GA NSGA-II, the multiobjective design problem is directly sought based on the simplified surrogate models. The method is feasible to search for the Pareto solution.

A vehicle crash safety design example is presented to demonstrate the effectiveness and the capability of the integrated optimization method in this paper. The vehicle undergoes two different crash conditions and three objectives are taken into account simultaneously, where the objective of the acceleration characteristics is constructed from the most decelerated condition of full frontal crash and the objective of toe-board intrusion is formulated from most displaced condition of an offset-frontal crush. With the simplified stepwise regression models of the full frontal and offset-frontal crash, the Pareto optimal space can be obtained via NSGA-II. This result provides the design engineers with a set of solution points on the Pareto front to help their decision making.

Acknowledgements The support of National Natural Science Foundation of China (10372029) is gratefully acknowledged.

References

Altair Engineering (2004) Hypermesh 7.0 Tutorial

Athan TW, Papalambros PY (1999) A note on weighted criteria methods for compromise solutions in multi-objective optimization. Eng Opt 27(2):155–176

Craig KJ, Stander N, Dooge DA, Varadappa S (2005) Automotive crashworthiness design using response surface-based variable screening and optimization. Eng Comput 22(1–2):38–61

Currin C, Mitchell T, Morris M, Ylvisaker D (1991) Bayesian prediction of deterministic functions, with applications to the design and analysis of computer experiments. J Am Stat Assoc 86(416):953–963

Deb K (2001) Multi-objective optimization using evolutionary algorithms. Wiley, New York

Deb K, Pratap A, Agarwal S, Meyarivan T (2002) A fast and elitist multiobjective genetic algorithm: NSGA-II. IEEE Trans Evol Comput 6(2):182–197

Dias JP, Pereira MS (2004) Optimization methods for crashworthiness design using multibody models. Comput Struct 82(17–19):1371–1380

Fang H, Rais-Rohani M, Liu Z, Horstemeyer MF (2005a) A comparative study of metamodeling methods for multiobjective crashworthiness optimization. Comput Struct 83(25–26):2121–2136

Fang H, Solanki K, Horstemeyer MF (2005b) Numerical simulations of multiple vehicle crashes and multidisciplinary crashworthiness optimization. Int J Crashworthiness 10(2):161–171

Fu Y, Sahin KH (2004) Better optimization of nonlinear uncertain systems (BONUS) for vehicle structural design. Ann Oper Res 132(1–4):69–84

Gu L, Yang RJ, Tho CH, Makowski M, Faruque O, Li Y (2001) Optimization and robustness for crashworthiness of side impact. Int J Veh Des 26(4):348–360

Hamza K, Hossoy I, Reyes-Luna JF, Papalambros PY (2004) Combined maximization of interior comfort and frontal crashworthiness in preliminary vehicle design. Int J Veh Des 35 (3):167–185

Hong JH, Mun MS, Song SH (2001) An optimum design methodology development using a statistical technique for vehicle occupant safety. Proc Inst Mech Eng D J Automot Eng 215:795–801

Hou SJ, Li Q, Long SY, Yang XJ, Li W (2007) Design optimization of regular hexagonal thin-walled columns with crashworthiness criteria. Finite Elem Anal Des 43(6–7):555–565

Kodiyalam S, Yang RJ, Gu L (2004a) High-performance computing and surrogate modeling for rapid visualization with multidisciplinary optimization. AIAA J 42(11):2347–2354

Kodiyalam S, Yang RJ, Gu L, Tho CH (2004b) Multidisciplinary design optimization of a vehicle system in a scalable, high performance computing environment. Struct Multidiscipl Optim 26(3–4):256–263

Krishnaiah PR (1982) Selection of variables under univariate regression models. Handbook of statistics, vol 2

Lanzi L, Castelletti LML, Anghileri M (2004) Multi-objective optimisation of composite absorber shape under crashworthiness requirements. Compstruct 65(3–4):433–441

Latchford J, Chirwa EC (2000) Airbag head restraint system. Proc Inst Mech Eng D J Automot Eng 214(D3):229–241

Li W, Li Q, Steven GP, Xie YM (2005) An evolutionary shape optimization for elastic contact problems subject to multiple load cases. Comput Methods Appl Mech Eng 194:3394–3415

Mao M, Chirwa EC, Wang W (2006) Assessment of vehicle roof crush test protocols using FE models: inverted drop tests versus updated FMVSS no. 216. Int J Crashworthiness 11(1):49–63

Marklund PO, Nilsson L (2001) Optimization of a car body component subjected to side impact. Struct Multidiscipl Optim 21(5):383–392

Myers RH, Montgomery DC (2002) Response surface methodology: process and product optimization using designed experiments. Wiley, New York

Redhe M, Nilsson L (2004) Optimization of the new Saab 9–3 exposed to impact load using a space mapping technique. Struct Multidiscipl Optim 27(5):411–420

Redhe M, Giger M, Nilsson L (2004) An investigation of structural optimization in crashworthiness design using a stochastic approach — A comparison of stochastic optimization and the response surface methodology. Struct Multidisc Optim 27(6): 446–459

Rudenko O, Schoenauer M, Bosio T, Fontana R (2002) A multi-objective evolutionary algorithm for car front end design. In: P Collet, C Fonlupt, J-K Hao, E Lutton, and M Schoenauer (eds) Artificial Evolution. 5th International Conference, Evolution

Artificielle, EA'2001. Lecture notes in computer science, vol 2310. Springer, Berlin pp 205–216 (Selected Papers)

Simpson TW, Peplinski JD, Koch PN, Allen JK (2001) Metamodels for computer-based engineering design: survey and recommendations. Eng Comput 17(2):129–150

Sobieszczanski-Sobieski J, Kodiyalam S, Yang RY (2001) Optimization of car body under constraints of noise, vibration, and harshness (NVH), and crash. Struct Multidiscipl Optim 22 (4):295–306

United Nations Economic Commission for Europe (1998) ECE R94: uniform provisions concerning the approval of vehicles with regard to the protection of the occupants in the event of a frontal collision

Yang RJ, Wang N, Tho CH, Bobineau JP (2005) Metamodeling development for vehicle frontal impact simulation. J Mec Des 127(5):1014–1020

Yoshimura M, Nishiwaki S, Izui K (2005) A multiple cross-sectional shape optimization method for automotive body frames. J Mec Des 127:49–57

Youn BD, Choi KK, Yang RJ, Gu L (2004) Reliability-based design optimization for crashworthiness of vehicle side impact. Struct Multidisc Optim 26(3–4):272–283

Yu Q, Koizumi N, Yajima H, Shiratori M (2001) Optimum design of vehicle frontal structure and occupant restraint system for crashworthiness (A multilevel approach using SDSS). JSME Int J Ser A Solid Mech Mater Eng 44(4):594–601