Analysis and Generation of Baseband Pulse Shapes for Digital Communications

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1 Introduction

In digital communications, pulse shaping is the process of changing the waveform of transmitted pulses. Its purpose is to make the signal better suited to its communication channel, typically by limiting the effective bandwidth of the transmission and controlling Intersymbol Interference (ISI).

This project's goal was to implement a single, flexible MATLAB function, `pulseShape`, capable of generating several of the most common pulse shapes. The function signature is: `[p, t] = pulseShape(pulseName, fs, nSymbolSamples, varargin)` where `pulseName` is a string identifying the pulse, `fs` is the sampling frequency, and `nSymbolSamples` is the number of samples per symbol (T_s). The `varargin` input is used to pass additional parameters, such as the roll-off factor (β) and pulse span, for more complex pulse shapes.

A critical requirement for all generated pulses is that they must be normalized to have a total energy of 1.

2 Causal Time-Limited Pulses

The first category of pulses are "causal" in the context of our simulation, meaning they are defined only for $t \geq 0$. They are strictly time-limited to a single symbol period, $t \in [0, T_s]$.

2.1 Rectangular Pulse

The rectangular pulse is the simplest pulse shape. It represents a symbol with a constant amplitude for the entire symbol duration. Its unnormalized definition is:

$$p_0(t) = \begin{cases} 1 & 0 \leq t \leq T_s \\ 0 & \text{o.w.} \end{cases}$$

While simple to generate, its frequency response is a sinc function, which has very large sidelobes. This causes significant interference in adjacent frequency channels.

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2.2 Triangular Pulse

The triangular pulse ramps up linearly to a peak at $T_s/2$ and ramps back down to zero at T_s. This shape has faster-decaying spectral sidelobes than the rectangular pulse. Its unnormalized definition is:

$$p_0(t) = \begin{cases} 1 - \frac{2|t - T_s/2|}{T_s} & 0 \leq t \leq T_s \\ 0 & \text{o.w.} \end{cases}$$

2.3 Half-Sine Pulse

The half-sine pulse uses the first half-cycle of a sinusoid as its shape. Its spectrum is more concentrated at the main lobe than the rectangular or triangular pulses. The unnormalized definition is:

$$p_0(t) = \begin{cases} \sin\left(\frac{\pi t}{T_s}\right) & 0 \leq t \leq T_s \\ 0 & \text{o.w.} \end{cases}$$

3 Non-Causal ISI-Controlling Pulses

This second category of pulses is non-causal (symmetric around $t = 0$) and is designed specifically to meet the Nyquist criterion for zero ISI. They are theoretically infinite in duration but are truncated in practice to a finite length specified by a `spanInSymbol` parameter.

3.1 Raised-Cosine (RC) Pulse

The Raised-Cosine pulse is the most common pulse that satisfies the Nyquist criterion. Its main advantage is that its frequency spectrum is strictly band-limited. Its roll-off factor, β, controls the excess bandwidth beyond the Nyquist minimum.

The time-domain definition of the RC pulse is:

$$p(t) = \text{sinc}\left(\frac{t}{T_s}\right) \frac{\cos\left(\frac{\pi \beta t}{T_s}\right)}{1 - \left(\frac{2\beta t}{T_s}\right)^2}$$

Its famous frequency-domain definition is:

$$P(f) = \begin{cases} T_s & |f| \leq \frac{1-\beta}{2T_s} \\ \frac{T_s}{2}\left(1 + \cos\left[\frac{\pi T_s}{\beta}\left(|f| - \frac{1-\beta}{2T_s}\right)\right]\right) & \frac{1-\beta}{2T_s} < |f| \leq \frac{1+\beta}{2T_s} \\ 0 & |f| > \frac{1+\beta}{2T_s} \end{cases}$$

3.2 Root-Raised-Cosine (RRC) Pulse

In a practical system, the optimal way to control ISI and maximize signal-to-noise ratio is to use a matched filter. This is achieved by "splitting" the RC filter between the transmitter and receiver. Each one uses a Root-Raised-Cosine (RRC) filter, so that their combined response in cascade is a full RC filter.

The RRC pulse time-domain definition is:

$$h_{rrc}(t) = \frac{1}{\sqrt{T_s}} \left(\frac{\sin\left(\pi \frac{t}{T_s}(1-\beta)\right) + 4\beta \frac{t}{T_s} \cos\left(\pi \frac{t}{T_s}(1+\beta)\right)}{\pi \frac{t}{T_s}\left(1 - \left(4\beta \frac{t}{T_s}\right)^2\right)} \right)$$

This formula has singularities at $t = 0$ and $t = \pm T_s/(4\beta)$ that must be handled separately using L'Hôpital's rule.

3.3 Gaussian Pulse

Gaussian filters are used extensively in systems like GSM (GMSK modulation) because they have a very compact frequency spectrum (no sidelobes) and a smooth time-domain response, which helps reduce interference. Unlike RC pulses, they do not satisfy the Nyquist criterion perfectly, so they introduce a small, controlled amount of ISI.

The pulse shape is defined by the standard Gaussian function:

$$p_0(t) = \exp(-a^2 t^2)$$

The parameter a is related to the bandwidth-time product BT_s (which is passed as the β parameter in the project) by the formula:

$$a = \frac{\pi(BT_s)}{T_s\sqrt{\ln(2)}}$$

4 Energy Normalization Requirement

A key requirement of the project is that all generated pulses must have unit energy ($E = 1$). This is essential for fair comparison of their spectral properties and for correct power calculations in a simulation.

For a continuous-time signal $p_0(t)$, the energy is $E = \int_{-\infty}^{\infty} |p_0(t)|^2 dt$. For our discrete-time sampled pulse $p_0[n]$, the integral is approximated by a sum, scaled by the sampling period $T_{samp} = 1/f_s$:

$$E = \sum_n |p_0[n]|^2 \cdot T_{samp}$$

To create the final, normalized pulse $p[n]$, the unnormalized pulse $p_0[n]$ is divided by the square root of its energy:

$$p[n] = \frac{p_0[n]}{\sqrt{E}} = \frac{p_0[n]}{\sqrt{T_{samp}\sum_n |p_0[n]|^2}}$$

This normalization is the final step performed by the `pulseShape` function before returning the output pulse p. For example, the continuous-time energy of the unnormalized rectangular pulse is $\int_0^{T_s} 1^2 dt = T_s$. The normalization factor is thus $1/\sqrt{T_s}$, which matches the formula given in the project description for the rectangular pulse.

5 Frequency Domain Analysis

The project also required analyzing the frequency response of these pulses. This was accomplished by taking the Fast Fourier Transform (FFT) of the generated time-domain pulse p. As specified, a 256-point FFT was used:

$$P[k] = \text{FFT}(p, 256)$$

To center the resulting spectrum around 0 Hz (DC), the `fftshift` command is used. The resulting plots of $|P[k]|$ versus frequency confirm the properties discussed:

- **Rectangular**: Slow-decaying sinc sidelobes.
- **RC/RRC/Gaussian**: The plots for different β values clearly show that as β (the roll-off or BT_s product) increases, the bandwidth occupied by the pulse also increases.

6 Conclusion

This project successfully covered the design, implementation, and analysis of a comprehensive pulse-shaping function. The mathematical definitions for several key pulse shapes were reviewed, and the critical step of energy normalization was successfully derived and implemented. The resulting function is a flexible and accurate tool for simulating and analyzing baseband digital communication systems.