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Dynamic Forces Induced by a Single Pedestrian: A Literature Review

With the use of lighter construction materials, more slender architectural designs, and open floor plans resulting in low damping, vibration serviceability has become a dominant design criterion for structural engineers worldwide. In principle, assessment of floor vibration serviceability requires a proper consideration of three key issues: excitation source, system, and receiver. Walking is usually the dominant human excitation for building floors. This paper provides a comprehensive review of a considerable number of references dealing with experimental measurement and mathematical modeling of dynamic forces induced by a single pedestrian. The historical development of walking force modeling—from single harmonic loads to extremely complex stochastic processes—is discussed. As a conclusion to this effort, it is suggested that less reliance should be made by the industry on the deterministic force models, since they have been shown to be overly conservative. Alternatively, due to the random nature of human walking, probabilistic force models seem to be more realistic, while more research is needed to achieve enough confidence to implement in design practice. [DOI: 10.1115/1.4036327]

Keywords: floor vibrations, pedestrian loading, dynamic forces, building floors, serviceability

1 Introduction

Excessive vibrations sometimes create significant problems for the occupants of various types of structures, including stadia [1], theaters [2], building floors [3,4], footbridges [5,6], and staircases [7]. These problems are naturally generated due to the implementation of lighter construction materials, more slender and aesthetic architectural designs, and open floor plans with few full-height nonstructural partitions, which results in low damping [8–10]. Therefore, vibration serviceability has become a dominant criterion in structural design over the last few years [11–14] and has been increasingly the focus of researchers worldwide. However, it should also be mentioned that floor vibration problems have been known about since olden times, which is evident from the following comment by Tredgold [15]:

> Girders for long bearings should always be made as deep as the timber can be obtained; an inch or two taken from the height of a room is of little consequence compared with a ceiling disfigured with cracks, besides the inconvenience of not being able to move without shaking everything in the room.

Any structural vibration serviceability analysis may be broken down into three key constituents, as indicated in Fig. 1. These are the input (excitation source), the system (floor structure), and the output (vibration receiver) [16–20], which is either a human occupant or vibration-sensitive equipment. This paper focuses on the first aspect of the analysis, which is the dynamic loading. Human activities such as walking [21], jogging [22], jumping [23], and running [24] are common sources of dynamic excitations leading to vibration problems.

The two main parameters typically used to quantitatively describe vibration response are amplitude and frequency [25]. Receivers are often more sensitive to vibration at some frequencies than at others. If the amplitude, described either by peak or root-mean-square (RMS) value, exceeds a tolerance limit at the frequency of vibration, then vibration is considered to be excessive [11]. As a consequence, building occupants become annoyed and adverse comments may result. In a similar manner, floors may

Fig. 1 Components of vibration serviceability analysis

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Manuscript received August 24, 2016; final manuscript received March 22, 2017; published online April 10, 2017. Assoc. Editor: Francois Barthelat.

fail to accommodate vibration-sensitive equipment in medical and industrial facilities.

Despite the importance of vibration serviceability in structural design, human-induced excitation of floors remains relatively poorly understood, and there is a proliferation of contradictory dynamic load models [26]. In all likelihood, this is due to many uncertainties associated with human walking. These uncertainties make it challenging to reliably predict the load using computer modeling and simplified manual calculation approaches [27]. Despite the preceding vast research efforts in this area, these uncertainties still require further examination to enhance the reliability of the assessment of floor vibrations during the design stage.

For building floors, especially ones in quiet environments such as offices, walking is the most frequently occurring and often dominant type of human-induced excitation [17,28,29]. Therefore, this paper aims to provide the reader with a solid background on modeling the dynamic load induced on building floors by a single pedestrian, bearing in mind that the ground or the supporting structure is assumed to be rigid. In effect, excellent state-of-the-art reviews pertaining to vibration serviceability were published by Pavic and Reynolds [16,17] for building floors, Živanović et al. [20] for footbridges, and Jones et al. [1] for sports stadia. In addition, a significant research effort has been dedicated in Ref. [19] with the purpose of comprehending experimental and analytical studies related to the walking forces, with more concentration on the biomechanics of human walking. However, the original contribution of this paper can be outlined in the following points: (a) Considering the elapsed time and the large body of work carried out since the previous review papers, this effort comprises more recent research studies in the field of vibration serviceability in general and human walking force modeling in particular. Furthermore, the latest review article known to authors and specifically concerning floor vibration was published over a decade ago [16,17,26], with the exception of Ref. [30] that was only focused on the response of high-frequency floors; (b) this paper is designed to be more focused toward the direction of the vibration serviceability for building floors. Hence, the assumptions, considerations, and the discussions made in this paper are actually more relevant to the dynamics of building floors. For instance, even when including footbridge-related studies as in Ref. [20], only the "vertical forces" part was extracted for discussion in this review article owing to its applicability in the case of building floors.

At the outset, the human walking process will be explained along with the definition of its relevant key concepts. Then, discussions of studies related to experimental measurement of vertical walking forces will be presented. After that, the historical evolution of the force models proposed for human walking will be discussed, starting from the simplest which assumes the load to be harmonic and ending with the most complex stochastic models. Finally, a pertinent study is presented to compare different types of walking force models.

2 Understanding the Walking Process

2.1 Walking Cycle, Step Phases, and Body Support Stages. During walking, each foot passes through two phases during a single step: the *swing phase* and the *stance (or contact) phase*. The swing phase refers to the period when the foot is off the ground, while the stance phase refers to the period during which the foot contacts the floor. The stance phase starts when the heel strikes the ground and ends by the act of the foot toe-off [31].

Concurrently, the human body passes through two stages during the walking process: the *double-* and the *single-support stage*. The double-support stage occurs when both feet are in contact with the floor (i.e., one foot contacts the ground while the other foot is still in a contact) and comprises not more than 20% of the walking cycle [32]. It has been verified that as the walking speed increases, this stage comprises a smaller percentage of the walking time. Conversely, the human body is at the single-support stage when one foot is in contact, while the other one is off the ground [33]. In the case of running, which can be considered as an extreme form of walking in terms of speed, the human body passes regularly through a no-support stage during which both feet are off the ground [34].

A *complete cycle* consists of the human motion that occurs from the start of a step with one foot to the start of the next step with that same foot, as illustrated in Fig. 2, and explained through a series of consecutive events as follows [35–38]:

(1) The left foot moves off the floor ("left toe-off" in Fig. 2) and through the air in a forward direction. Simultaneously, the right foot is providing the support for the human body. Hence, the left foot is in a swing phase, while the human body is in a right single-support stage.

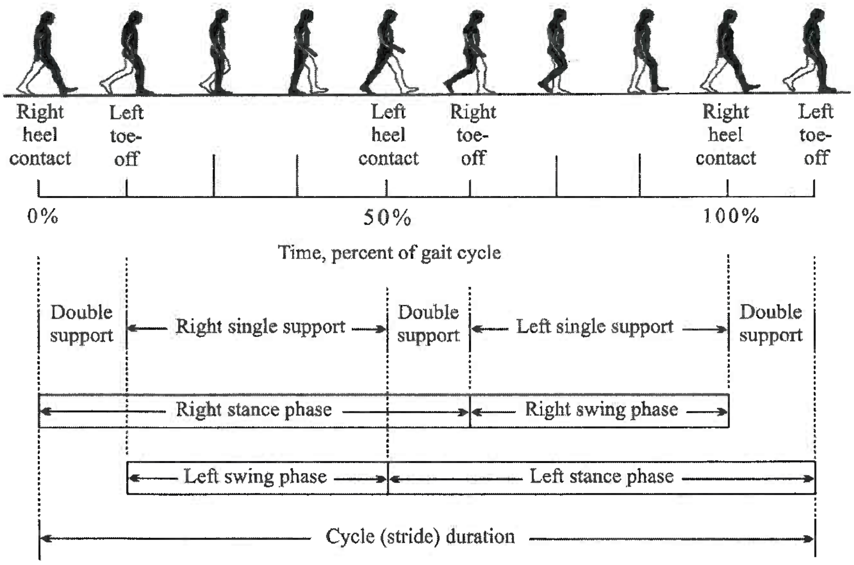

Fig. 2 A single walk cycle. (Reprinted with permission from Racic et al. [19]. The original version was published by Inman et al. [35]. Copyright 1981 by Williams & Wilkins.)

(2) Subsequently, the left foot contacts the floor ("left heel contact" in Fig. 2) and is starting its contact phase, while the human body passes again through a stage of double support.
(3) Next, the right foot moves off the floor and starts a swing phase, while the left foot is still in a contact phase and providing left single support for the human body.
(4) Next, the right foot contacts the floor and the body passes through a double support phase.
(5) Finally, the left foot moves off the floor (left toe-off in Fig. 2), and the cycle is complete.

2.2 Walking Parameters. Two types of parameters are used to describe the walking routine: temporal (time) and spatial (distance or location) parameters [39].

Walking speed, cycle time, and pacing rate (walking frequency) are typically measured time parameters. *Walking speed* is the magnitude of horizontal velocity in the gait direction, usually expressed in meters per second. *Pacing rate*, also known as step frequency, is the number of steps in a specific duration, usually expressed in steps per minute or Hz. *Cycle time* represents the period of time during which a complete cycle occurs [37].

Commonly used spatial parameters are step length, step width, and stride length (or cycle length). *Step length* is defined as the distance measured during a single step between both heels in the direction of walking. *Step width* is the distance measured transversely between the two lines which describe the paths taken by the right and the left foot, where these lines pass through heel midpoints. *Stride length* is the distance measured along the gait direction between two consecutive contacts of the same foot. It also represents the total distance traveled during one cycle period [38].

Two types of randomness exist in human walking: intersubject and intrasubject variability [40]. Intersubject variability exists because different persons will have different key parameters which are directly related to the induced forces, such as subject weight, step frequency, walking speed, and so on [41]. Consequently, the resultant walking forces vary from person to person. However, intrasubject variability exists because an individual never repeats two identical steps in sequence; therefore, a person produces forces which are different at each footfall [42].

3 Measurement of Walking-Induced Vertical Load

In general, a walking person causes dynamic forces which have components in three directions: (a) vertical, (b) horizontal–parallel, and (c) horizontal–transverse to the direction of movement [43]. In this paper, research studies considering mainly the vertical component will be the focus because:

(1) The vertical force has the highest magnitude, so it is the most important and has been most often investigated in the literature [16].
(2) The primary concern is vibration serviceability of building floors, on which the effect of the horizontal components of vibration response on occupants are almost always insignificant [17].

3.1 Vertical Forces Due to a Single Step. Several researchers, starting with Harper [44] and followed by Galbraith and Barton [34], Blanchard et al. [45], Ohlsson [46], Andriacchi et al. [47], and Kerr [48], used force transducers to measure the dynamic forces resulting from individual footfalls. Their measurements indicate that the vertical force typically has two peaks and a trough as shown in Fig. 3.

In the figure, the vertical force is expressed as a percentage of the body weight, and the time is normalized with respect to the stance phase (see Fig. 2) duration. Once the heel strikes the floor, the dynamic force increases until it approaches a peak value of F_1

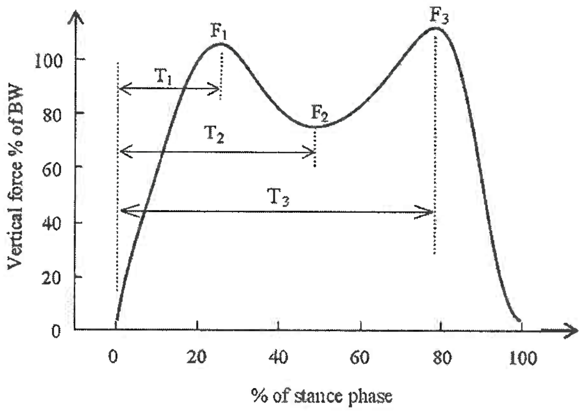

Fig. 3 Vertical force resulted from a single step. (Reprinted with permission from Racic et al. [19]. Copyright 2009 by Elsevier.)

at time, T_1, which is at approximately 25–30% of the stance phase duration. After that, the dynamic force decreases until the mid-stance point, T_2, at which both the heel and toe are in contact with the ground, while the opposite foot is in a swing phase. Subsequently, the heel rises and the vertical force increases until it reaches another peak, F_3, at time, T_3, near the end of the stance phase and heel strike of the other foot. Finally, the foot rises from the floor and the force decreases quickly to zero at the stance phase completion [19].

Single-step force measurements were followed by more advanced recordings of multiple step walking force waveforms. Blanchard et al. [45] developed the "gait machine," while Rainer et al. [49] employed a floor strip on which ground reaction forces were continuously measured during walking and other activities. More recently, Ebrahimpour et al. [50] utilized a short walkway that consisted of several individual force plates. In studies by these researchers, it was observed that the measured time histories were approximately periodic, with a typical cycle time equal to the reciprocal of the average step frequency.

3.2 Effect of Walking Speed. Galbraith and Barton [34] investigated the effect of walking speed on the vertical force due to individual footfalls on a force plate. They reported a number of differences between the running and walking dynamic forces resulting from a single step. Essentially, the dynamic force graph induced by running had typically a single peak, dissimilar to the double-peaked shape recorded from walking. Moreover, two main factors were thought to influence the amplitude of the peak force: subject weight and pacing rate. It was observed, from the results of Galbraith and Barton [34], that increasing those factors leads to higher peak forces.

Assume that single-step forces are identical, combining single-step graphs in a successive manner resulted in artificial time histories for the forces excited by continuous walking and running, as shown in Fig. 4. A typical short period can be observed in the running time history, during which both feet are off the floor, and thus, no force is excited. On the other hand, through the continuous walking time, there is a frequent overlapping between the left and the right feet forces, which is noticed the moment both feet are on the floor.

Wheeler [51,52] conducted a wide-ranging study, which is recognized as a considerable step forward into the research in human-induced forces. The author organized the work of previous investigators and classified different categories of human moving, arranged from slow walking to running. Each of these categories had a single-step dynamic force that was unique in peak amplitude, shape, and stance period, as shown in Fig. 5. Furthermore, the same author stated that several parameters such as stride

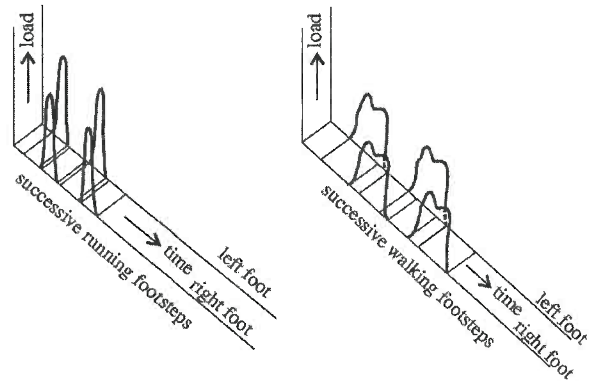

Fig. 4 Typical walking and running time histories. (Reprinted with permission from Racic et al. [19]. The original version was published by Galbraith and Barton [34]. Copyright 1970 by Acoustical Society of America.)

length, walking speed, contact time, peak force, and step frequency are related. For instance, for a constant stride length, the walking speed tends to increase as the step frequency rises. Moreover, increasing the step frequency results in shortening the contact time as well as boosting the peak force.

It should be highlighted that even though human-induced walking force is mainly focused on, running is sometimes mentioned in this paper since it is regarded as an extreme case of walking in terms of movement speed.

4 Modeling of Human Walking Forces

Section 3 has discussed research studies describing the measurement of the vertical dynamic forces induced by a walking pedestrian and some important relationships between measured force and walking parameters. The next step is to develop reliable mathematical representations of these forces, a process termed *force modeling* [53]. The resulting models can be used to predict vibration during the design phase.

Mathematical modeling of human-induced dynamic forces is challenging for a number of reasons. First, there is high variability of force waveform shapes [52] (e.g., the differences observed in Fig. 5) due to many parameters which have high inter- and intra-subject variability [54]. Moreover, the dynamic forces induced by a single person are narrowband random processes (as will be discussed later) that are not well understood and thus are difficult to accurately represent [55].

Nevertheless, force models have been presented by researchers within the past few decades based on reasonable assumptions, and some are employed in the widely used design guides. Two types of models have been proposed for human-induced walking excitation: time- and frequency-domain force models; the former is currently much more widely used.

Time-domain force models are subdivided into two main groups in the literature: deterministic and probabilistic. Deterministic models aim to generate a uniform force model for any individual without directly considering the natural variability between people [56]. However, the probabilistic model takes into account the fact that each individual has a unique set of parameters directly influencing the produced forces such as static weight, pacing rate, and so on, which is referred to as intersubject variability. This kind of unpredictability is described for each parameter via its probability density function, and thus considered in the force model by means of probability of occurrence [57].

In Secs. 4.1–4.3, the main types of force models are presented according to their progression through the history of research in this area, as illustrated in Fig. 6. Time-domain deterministic force models are discussed first because they are the oldest and the most common. This type of force model is presented from the simplest to the most advanced ones that are utilized in the current design guides. After that, frequency-domain representation of walking

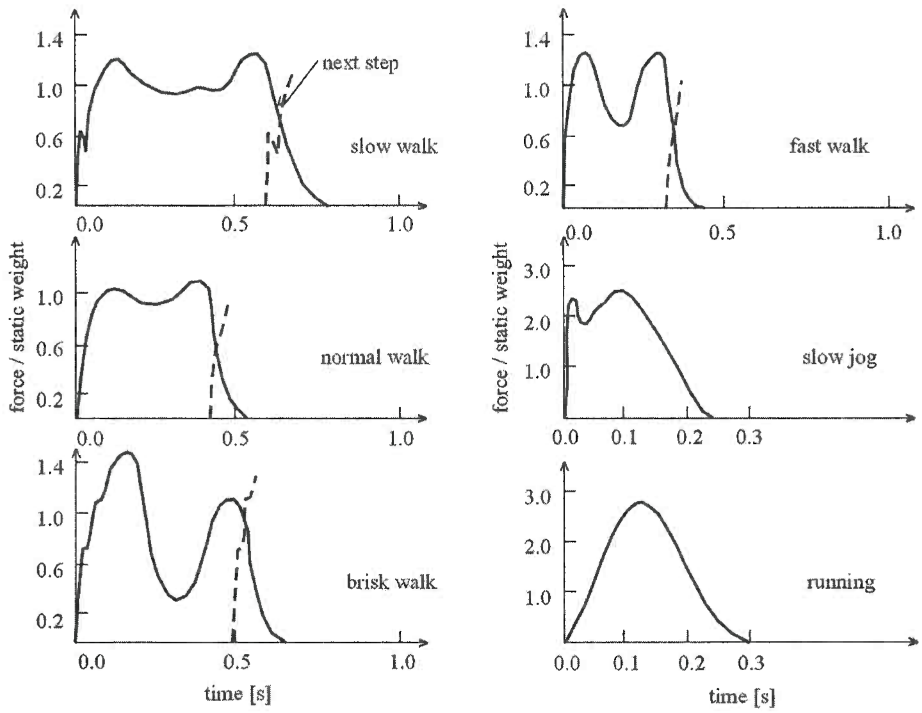

Fig. 5 Vertical force patterns for different modes of movement activity. (Reprinted with permission from Živanović et al. [20]. The original version was published by Wheeler [51]. Copyright 1980 by National Academy of Sciences.)

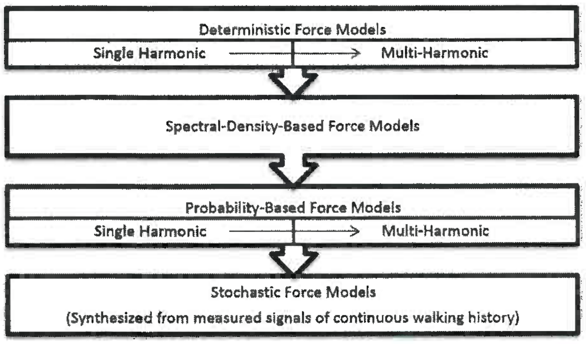

Fig. 6 Historical development of force modeling approaches

force is introduced, and some relevant research studies are discussed. Finally, some recent complex stochastic force models are explained.

4.1 Time-Domain Deterministic Force Models. With the consideration of deterministic force models, computing vibration responses is typically based on key assumptions such as perfectly periodic induced forces and linear elastic system [12]. The magnitude of human-induced vibration has been considered to depend significantly on the ratio of walking frequency of the pedestrian and lowest vertical natural frequency of the floor. Two types of floors are presented in the literature according to their natural frequency: low- and high-frequency floors [19]. Low-frequency floors are typically regarded as floors having at least one responsive natural frequency lower than 9–10 Hz, while the lowest natural frequency of high frequency floors is higher than 9–10 Hz. A large body of research related to modeling of walking forces has concluded that the dynamic response in the two cases can be considerably different.

Previous studies have demonstrated that in low-frequency floors, responses to successive heel strikes tend to build up leading to the state of (near-)resonance [58]. This phenomenon takes place when the natural frequency of the floor is close to the typical human walking pace (1.5–2.5 Hz) or an integer multiple of the walking frequency. On the other hand, high-frequency floors typically do not exhibit resonance under successive footfalls. As will be detailed later, the response of such floors resembles a series of impulse responses to each footstep.

So far, there is no deterministic walking force model that can be used for prediction of both low- and high-frequency floor responses. In all likelihood, this is due to the different response behavior assumed for each type. Therefore, researchers have developed different force models for each, which are discussed in Secs. 4.1.1 and 4.1.2.

4.1.1 Models Proposed for Low-Frequency Floors. Being assumed as periodic, the vertical dynamic force induced by a walking pedestrian $F_p(t)$ can be expressed in the time domain by a summation of harmonic components (i.e., Fourier series), as per the following equation [56]:

$$F_p(t) = G + \sum_{i=1}^{n} G\alpha_i \sin(2\pi i f_p t - \phi_i) \tag{1}$$

where G is the individual's static weight (N), i is the order number of the harmonic, n is the number of all contributing harmonics, f_p is the step frequency (Hz), ϕ_i is the phase shift of the ith harmonic (rad), and α_i is the Fourier coefficient of the ith harmonic usually known as the dynamic load factor (DLF), and sometimes referred to as the dynamic coefficient.

Therefore, based on this equation and with the assumption of linearly elastic and proportionally damped structure, the vibration response of the floor is computed by the summation of responses to all contributing harmonics. Obtaining appropriate values of DLF (α_i) for the contributing harmonics is the key to generate an accurate deterministic force model. Several researchers have used experimental studies to estimate DLFs over the years, resulting in the force models in the major design guides in use today.

One of the first deterministic force models was published by Blanchard et al. [45] who proposed a simple model for the load exerted on footbridges by a pedestrian. This model considered that resonance would occur in the first vibration mode due to the first harmonic of the dynamic load. Two main scenarios, depending on the footbridge's natural frequency, were considered as follows:

(1) If the fundamental frequency does not exceed 4 Hz, resonance is assumed to occur due to the first harmonic of the walking force with $\alpha_1 = 0.257$ and $G = 700$ N.
(2) If the fundamental frequency is between 4 and 5 Hz, resonance is assumed to occur due to the second harmonic of the walking force. Therefore, considering the lower amplitude of the second harmonic, the authors introduced reduction factors to apply to the DLF.

A few years later, Kajikawa (according to Yoneda [59]) developed an enhanced walking force model by considering the first harmonic DLF and including the "correction coefficient" illustrated in Fig. 7. The author introduced a direct relationship between the dynamic force and walking pace (step/s). Moreover, walking speed (m/s) was also included in his model as an output dependent on step frequency.

As a decent forward step, Bachmann and Ammann [43] reported the Fourier coefficient values for the first five harmonics of the walking force. Also, they indicated that the walking-induced load is strongly related to step frequency. The force components in all directions (vertical, midlateral, and longitudinal) were included in their effort. The authors suggested vertical force DLFs of 0.4 and 0.5 for the first harmonic DLF at frequencies of 2.0 and 2.4 Hz, respectively, with linear interpolation for the frequencies in between. Regarding the second and third harmonics, a DLF value of 0.1 was proposed if the step frequency is close to 2.0 Hz.

In 1988, Rainer et al. [49] added a significant contribution by including further types of human activities. The authors measured the continuous force waveform induced by walking, running, and jumping on an instrumented platform, and then Fourier transformed each waveform to a force spectrum with peaks that provide estimates of the DLFs. Like others, they confirmed the strong relation between the DLFs and the activity frequency. The authors included the first four harmonics of the activity in their proposed force models, while the DLFs were observed to decrease with

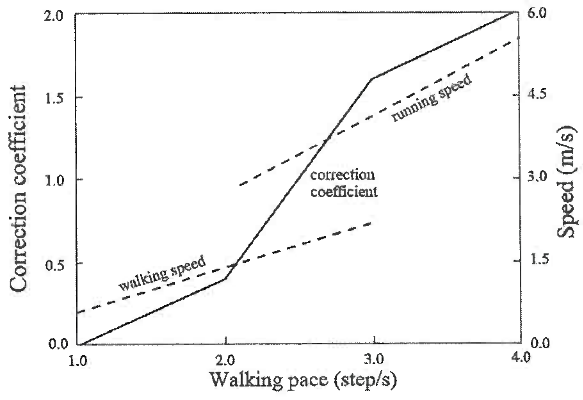

Fig. 7 Walking speed and DLF as a function of pacing rate. (Reprinted with permission from Živanović et al. [20]. The original version was published by Yoneda [59]. Copyright 2002 by AFGC.)

higher order of harmonics. Interestingly, the dynamic forces reported for walking were found to be considerably higher than the ones suggested by previous researchers. According to Refs. [19,20], this study was not statistically rigorous because it included measurements from only three test individuals.

As a part of his Ph.D. thesis, Kerr [48] conducted one of the most comprehensive studies relevant to human-induced walking forces. He engaged a total number of 40 test subjects and requested that each perform a single footfall on a force plate. Different step frequencies were used by the test subjects, ranging from very low (1 Hz) to very high (3 Hz), a significantly larger range than that of typical step frequencies. Consequently, nearly 1000 single-step force measurements were collected, generating a very large database of walking forces.

Each footstep was repeatedly appended and summed to synthesize a waveform of a walking event, which was Fourier transformed to a spectrum to estimate the DLFs. For only the first harmonic, it was noticeable that the dynamic forces increase with step frequency. However, for the other harmonics, the DLF values were largely scattered and thus statistically described simply by mean values and standard deviations. Racic et al. [19] stated that the main shortcoming of this study was the incapability of its data to represent the intrasubject variability. The force model proposed by Kerr did not consider the fact that a single individual is improbable to produce two identical steps in sequence.

Subsequent to Kerr's effort, Young [60] as well as Willford et al. [61] took a valued step and conducted a wide-ranging study to develop a reliable guide for modeling walking forces. The authors assembled the data published from several researchers, as presented in Fig. 8, and performed statistical regression. As a result, they proposed mean and design values of DLF for the first four harmonics of a walking force.

The following set of equations was suggested by Young [60] for the design values (i.e., suitable for serviceability assessment) as a function of step frequency:

$$
\begin{aligned}
&\alpha_1 = 0.41\ (f - 0.95) \leq 0.56; && 1\ \text{Hz} \leq f \leq 2.8\ \text{Hz}\\
&\alpha_2 = 0.069 + 0.0056\, f; && 2\ \text{Hz} \leq f \leq 5.6\ \text{Hz}\\
&\alpha_3 = 0.033 + 0.0064\, f; && 3\ \text{Hz} \leq f \leq 8.4\ \text{Hz}\\
&\alpha_4 = 0.013 + 0.0065\, f; && 4\ \text{Hz} \leq f \leq 11.2\ \text{Hz}
\end{aligned}
\tag{2}
$$

where f is the frequency of the relevant harmonic, which apparently equals an integer multiple of the pacing rate. As shown in Eq. (2), Young considered the pacing rate to range between 1 and 2.8 Hz, reasonably assuming that step frequencies out of this range are not likely to occur in reality. He stated that the recommended design values have 25% probability of exceedance [60], which makes him one of the earliest investigators to consider the probabilistic nature of human walking.

Table 1 presents the DLFs for vertical human-induced forces reported from several authors. The table is extracted from the Živanović et al. literature review about vibration serviceability of footbridges [20]. This table is a brief outline of the efforts mentioned earlier in this section.

The force modeling approaches previously discussed in this section were all based on experimental measurement of walking

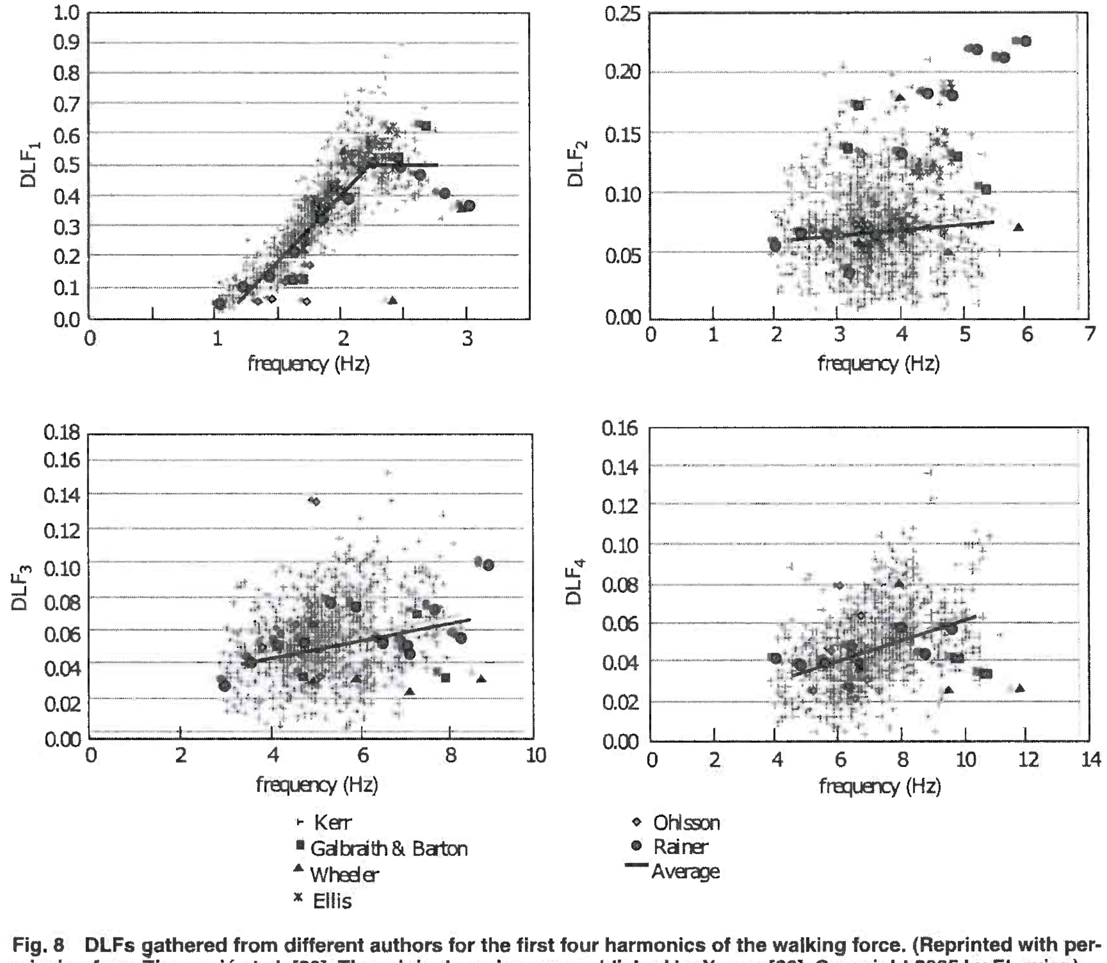

Fig. 8 DLFs gathered from different authors for the first four harmonics of the walking force. (Reprinted with permission from Zivanović et al. [20]. The original version was published by Young [60]. Copyright 2005 by Elsevier.)

Table 1 DLFs proposed by different authors for single-person walking vertical force models. (Extracted with permission from Živanović et al. [20] with modifications. Copyright 2005 by Elsevier.)

Investigator(s)	DLFs for relevant harmonics	Remark	Activity type
Blanchard et al. [45]	$\alpha_1 = 0.257$	DLF is reduced for frequencies between 4 and 5 Hz	Walking
Kajikawa (according to Yoneda [59])	α_1	DLF is frequency dependent (Fig. 7)	Walking and running
Bachmann and Ammann [43]	$\alpha_1 = 0.4$–0.5	Between 2.0 Hz and 2.4 Hz	Walking
	$\alpha_2 = \alpha_3 = 0.1$	At approximately 2.0 Hz	
Schulze (after Bachmann and Ammann [43])	$\alpha_1 = 0.37$, $\alpha_2 = 0.10$, $\alpha_3 = 0.12$, $\alpha_4 = 0.04$, and $\alpha_5 = 0.08$	At 2.0 Hz	Walking
Rainer et al. [49]	α_1, α_2, α_3, and α_4	DLFs are frequency dependent	Walking, running, and jumping
Bachmann et al. [62]	$\alpha_1 = 0.4/0.5$, $\alpha_2 = \alpha_3 = 0.1$	At 2.0/2.4 Hz	Walking
	$\alpha_1 = 1.6$, $\alpha_2 = 0.7$, and $\alpha_3 = 0.2$	At 2.0–3.0 Hz	Running
Kerr [48]	α_1, $\alpha_2 = 0.07$, $\alpha_3 \approx 0.06$	α_1 is frequency dependent	Walking
Young [60]; Willford et al. [61]	$\alpha_1 = 0.37(f - 0.95) \leq 0.5$ $\alpha_2 = 0.054 + 0.0044f$ $\alpha_3 = 0.026 + 0.0050f$ $\alpha_4 = 0.010 + 0.0051f$	These are mean values for DLFs (Fig. 8)	Walking

loads on rigid surfaces. It means that these models might not be accurate for simulating the response of flexible floors subjected to human-induced dynamic excitation. For instance, Baumann and Bachmann [63] reported that the magnitude of the walking load on stiff floors was approximately 10% more compared to the one measured on the flexible counterparts. Likewise, Pimentel [64] reported that walking forces measured on flexible footbridges were much lower than previously recorded ones on stiff surfaces.

Bocian et al. [65] stated that there is an inevitable shortcoming resulted from disregarding the human–structure dynamic interaction in the proposed force models. This shortcoming results from the scarcity of measurement techniques that are used to track the dynamic forces in the presence of structural motion. To address this problem, the same authors [65] proposed a biomechanically inspired inverted-pendulum force model that can represent the dynamic interaction between the pedestrian and the footbridge, which was proved to have a significant effect in the case of moving crowds.

4.1.2 Models Proposed for High-Frequency Floors. Floors with no responsive natural frequency below 9–10 Hz (i.e., higher than four times the average walking frequency) are referred to as high-frequency floors. Figure 9 illustrates the difference between the response behavior in high- and low-frequency floors. Unlike the resonant behavior exhibited in the response of low-frequency counterparts, the response of high-frequency floors under footfall loads has a transient (impulsive) profile [30]. At first, a heel impact produces an initial peak response. Afterward, the floor oscillates at its natural frequency with a decaying rate associated with the damping ratio of the fundamental mode [66]. Likewise, as soon as the subsequent footfall strikes the floor, another impulse response is generated. Hence, the vibration responses of successive steps do not build up in the case of high-frequency floors due to structural damping decaying effect.

The literature on walking force modeling for high-frequency floors is limited when compared with low-frequency floors because most vibration serviceability problems are due to human discomfort on low-frequency floors. A main relevant study on high-frequency floor force models was conducted by Willford et al. [67]. The analytical force model introduced in this study was formulated based on the similarity between the transient response of high-frequency floors under walking forces and the response of single degree-of-freedom systems under a series of impulse forces. In order to generate this model, Willford et al. [67] simulated the responses of a large number of single degree-of-freedom (SDOF) systems, which had a unit modal mass as well as different natural frequencies, under continuous walking forces synthesized from Kerr's single-step measurements. For each simulation, the peak velocity of the SDOF system was computed and used to obtain a quantity called the *effective impulse*. Since each SDOF system has a unit modal mass, the effective impulse is numerically equal to the initial (i.e., peak) velocity.

The results of this study suggested that effective impulse increases as the walking frequency decreases, and it also decreases with higher natural frequencies of the floor. Based on statistical analysis of the results presented in Fig. 10, the following equation was developed for calculating the effective impulse I_{eff}, with N s units, on a high-frequency floor [67]:

$$I_{\text{eff}} = 54\frac{f^{1.43}}{f_n^{1.30}} \tag{3}$$

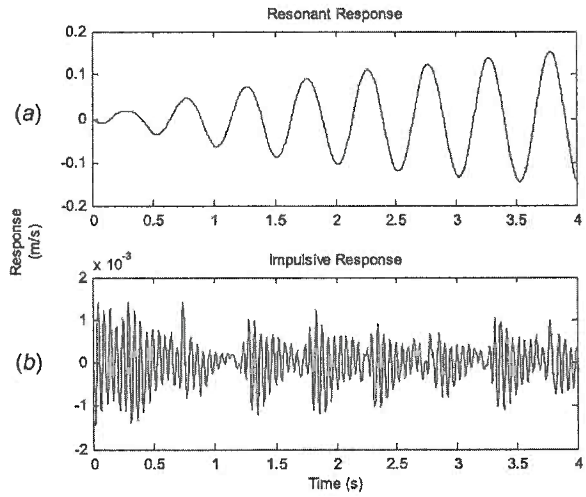

Fig. 9 Comparison of the response behavior in (*a*) low- and (*b*) high-frequency floor due to successive steps. (Reprinted with permission from Middleton and Brownjohn [30]. Copyright 2010 by Elsevier.)

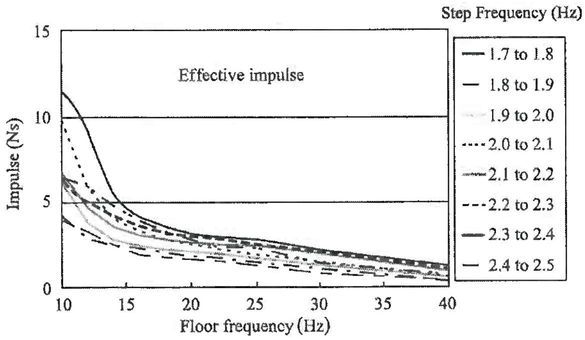

Fig. 10 Effective impulse proposed as a function of pacing rate and floor's natural frequency. (Reprinted with permission from Racic et al. [19]. The original version was published by Willford et al. [67]. Copyright 2005 by SPIE.)

where f is the walking frequency (in Hz) and f_n is the floor's natural frequency (in Hz). I_{eff} computed according to this equation has a 25% probability of exceedance.

4.1.3 Drawbacks of Commonly Applied Deterministic Force Modeling. Being the most common approach to the mathematical representation of walking force, the reliability of deterministic force models has been extensively tested by investigators and structural engineers. It has been shown that deterministic modeling approaches are not very accurate or precise, often leading to overestimation of the floor vibration response [55], and less commonly leading to underestimation of the response [68]. In view of that, the following points were identified as possible weaknesses of deterministic force modeling:

(1) Deterministic models do not explicitly consider the inter- and intrasubject variability of human walking.
(2) For low-frequency structures, there is usually an assumption of precise resonance of at least one vibration mode, while in reality, it is unlikely that this condition will precisely be achieved.
(3) Classification of floors by their natural frequency into two categories (i.e., high- and low-frequency) and then selecting the modeling approach accordingly have been shown to not be accurate in certain circumstances. A floor with a fundamental frequency close to the 9–10 Hz demarcation between low- and high-frequency floors probably exhibits a response in between a resonant and impulse response [68].
(4) Since the force model is deterministic, the computed vibration response will be a numeral usually compared with a tolerance limit. This will result in a binary pass/fail assessment criterion which does not provide enough information for the design engineer to make an informed decision. This pass/fail design criterion is more suitable when dealing with structural safety, but the vibration response is commonly agreed to be rather a serviceability issue [69].

4.2 Frequency-Domain Force Models. Continuous walking forces can be alternatively represented in the frequency domain. The walking time history can be expressed in terms of the amplitudes obtained for the corresponding sine and cosine waves via Fourier transform decomposition [70,71]. Ohlsson [46] was one of the earliest to research frequency-domain representations of human-induced forces. By measuring a single footfall force and then repeating it, assuming perfect periodicity, Ohlsson created an artificial time history of a continuous walking force. He found that a single step holds most of its excitation energy in the frequency range between 0 and 6 Hz. Moreover, he considered the walking force as a transient signal resulting from a series of perfectly repeated footfalls, and accordingly determined the corresponding autospectral density (ASD). Since Ohlsson's study was concentrated on the behavior of high-frequency floors, he focused on the frequency content of the ASD in a relatively high-frequency range (6–50 Hz).

After that, Eriksson [72] conducted a more concentrated study aimed to investigate the continuous walking force in the frequency domain considering the case of low-frequency floors. Eriksson produced the ASD for the indirect measurements of walking forces, focusing on the frequency range (0–6 Hz). As can be observed from Fig. 11, the content of excitation energy at integer multiples of step frequency is smeared into the adjacent frequencies, producing what is known as *leakage* at each peak. Deeply rooted in digital signal processing, this leakage indicates that the human walking force is not perfectly periodic [70]. Therefore, the same author stated that human walking should be rather described as a random process [72].

It should be mentioned that the existence of such imperfections in walking periodicity was previously indicated by Rainer et al. [49] during their investigation of the dynamic load induced on footbridge by a walking pedestrian. The authors compared the measured values of response produced by pedestrians attempting to excite resonance with the analytical sinusoidal steady-state ones which were based on the first walking harmonic. Interestingly, the authors reported that the measured responses were approximately one-half of its analytically predicted peers, indicating the significance of the temporal imperfections in the walking force [49].

As a significant step forward, Brownjohn et al. [55] employed the spectral density approach for modeling continuous vertical walking forces on pedestrian structures. As shown in Fig. 12, the authors utilized a treadmill instrument to record the real walking forces in a continuous manner rather than artificially repeating a measured single footfall. Three test subjects were requested to walk at a self-selected pacing rate with different speeds ranging between 2.5 and 7.5 m/s. Consequently, each test subject produced a sequence of 1-min walking time histories along with the corresponding pacing rates and gait parameters. In spite of the small number of test subjects, the authors argued that their study aimed to focus on the effect of the intrasubject rather than the intersubject variability nature, which had been already studied in detail by other researchers.

After that, these records were compared with those produced by deterministic force models based on the assumption of perfect periodicity. Figure 13 shows the Fourier amplitudes, normalized with respect to body weight, for deterministic force modeling at a pacing rate of 1.91 Hz. The figure illustrates that the force function is estimated with its harmonic components using Eq. (1) [55].

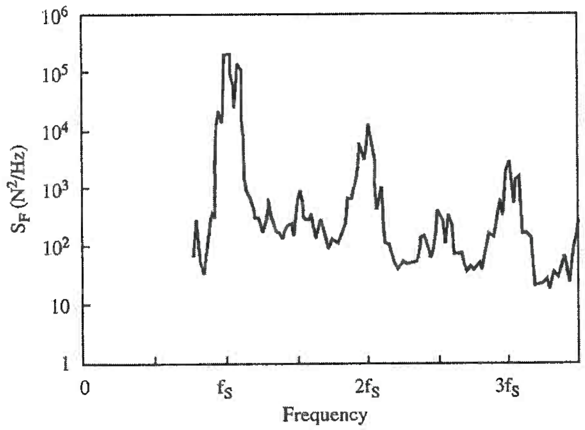

Fig. 11 Autospectral density of the walking force. (Reprinted with permission from Živanović et al. [20]. The original version was published by Eriksson [72]. Copyright 1994 by Chalmers Publication Library.)

Fig. 12 Measurement of continuous walking force using an instrumented treadmill. (Reprinted with permission from Brownjohn et al. [55]. Copyright 2004 by National Research Council Canada.)

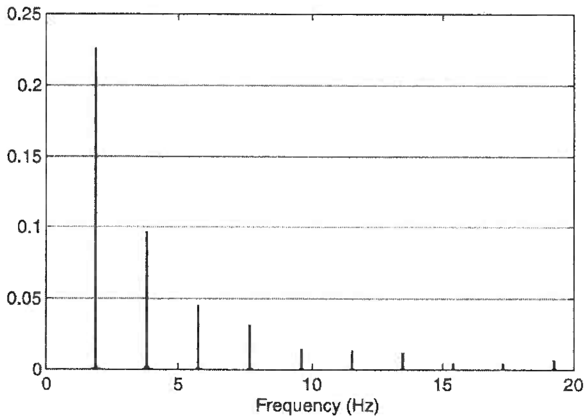

Fig. 13 Representation of simulated deterministic walking force signal in the frequency domain. (Reprinted with permission from Brownjohn et al. [55]. Copyright 2004 by National Research Council Canada.)

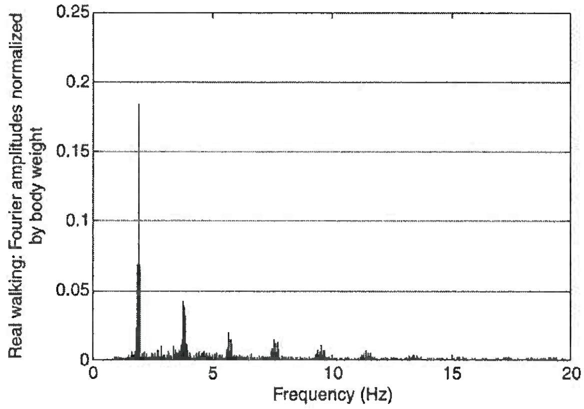

Fig. 14 Representation of real continuous walking force in the frequency domain. (Reprinted with permission from Brownjohn et al. [55]. Copyright 2004 by National Research Council Canada.)

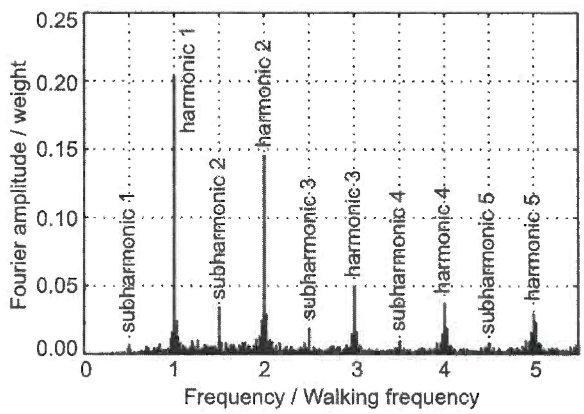

Fig. 15 Appearing of the subharmonic amplitudes of the walking force in the frequency domain. (Reprinted with permission from Živanović et al. [69]. Copyright 2007 by Elsevier.)

On the other hand, Fig. 14 presents the Fourier amplitude spectrum for the real walking force signal recorded for the same subject weight and step frequency.

Comparing Figs. 13 and 14, it is observed that there is a leakage of excitation energy from the deterministic force case to the actually measured one, which appears to be more significant for higher harmonics. This indicates that modeling of walking force using the classical deterministic approach is overconservative and inaccurate. Alternatively, the authors proposed a frequency-domain modeling approach for both single individual and crowds, which was based on the ASD function to represent the walking imperfections, similar to the use of stochastic analysis in wind load modeling [55].

In another study, Sahnaci and Kasperski [73] stated that the imperfections in pedestrian walking pattern result in what is known as *subharmonics*, which are actually intermediate load amplitudes holding a relatively significant portion of excitation energy between the main harmonic frequency bands ($0.5\,f_w$, $1.5\,f_w$, and so on), similar to what is shown in Fig. 15. The authors suggested that the main reason of this phenomenon is the inevitable difference between left and right feet in terms of walking parameters, such as step length and pacing rate. Furthermore, based on studying a relevant example, they stated that the dynamic response predicted without taking these subharmonics into consideration deviates significantly from the actual one [73].

4.3 Probabilistic Force Models. As mentioned before, the currently applied vibration serviceability assessment of floors is usually based on the assumption that a single person walks at a step frequency corresponding with a natural frequency of the structure [54]. This practice has been shown to significantly overestimate the floor dynamic responses due to neglecting the leakage associated with imperfect periodicity of human walking. Therefore, probabilistic modeling has been introduced in the literature as a more reliable approach to mathematical representation of human-induced forces. In this manner, human activity is considered to be rather a random or stochastic process with nondeterministic behavior (i.e., the next state is not predictable knowing the current state) [74].

Based on the probabilistic approach to force modeling, each person has a unique set of key parameters such as subject's weight, stride length, and step frequency, which is referred to as intersubject variability [75]. In order to achieve a reasonable statistical description of these parameters, a considerable number of measurements are first required [76]. These variables, which describe human-induced force, are then defined in the model via their probability density functions. Furthermore, it is also taken into account that a single pedestrian will never produce

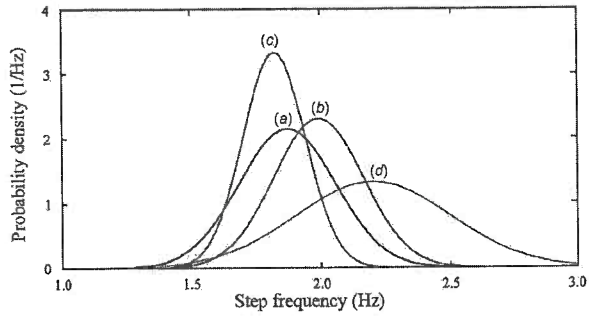

Fig. 16 Normal distribution of the step frequency for normal walking, reported after (*a*) Živanović et al. [69], (*b*) Matsumoto et al. [82], (*c*) Kasperski and Sahnaci [83], and (*d*) Kramer and Kebe [85]. (Reprinted with permission from Pedersen and Frier [54] with modifications. Copyright 2010 by Elsevier.)

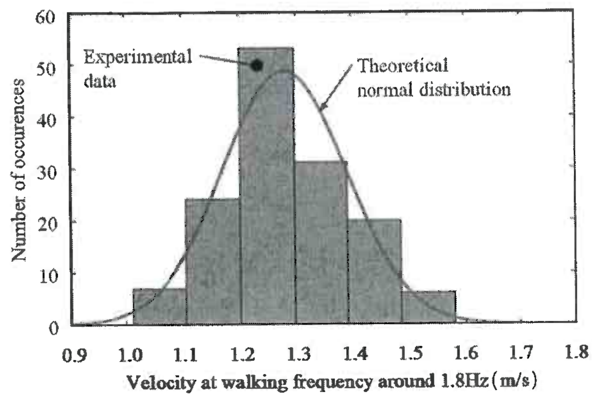

Fig. 17 Normal distribution of walking speed at 1.8 Hz of step frequency. (Reprinted with permission from Racic et al. [19]. The original version was published by Zivanovic [86]. Copyright 2006 by the University of Sheffield.)

consecutively two identical steps (i.e., intrasubject variability). Hence, the walking force time histories recorded during repeated experiments for the same person will be different [77].

As a final result, instead of producing a single vibration response value, which allows only a pass/fail assessment, the vibration response can be expressed as a probability that it will not exceed a certain value. This would be more convenient since it provides more space for engineering judgment since vibration, in general, is considered to be a serviceability rather than a safety design issue [78].

4.3.1 Statistical Description of Walking Parameters. Several researchers have worked to achieve a reliable statistical description of human pacing rate [54]. At first, Leonard [79] suggested a range of 1.7–2.3 Hz for normal speed walking frequency. After that, Matsumoto et al. [80] conducted a comprehensive study on a sample of 505 individuals walking at a self-selected speed. They concluded that step frequency typically followed a normal distribution with a mean value of 1.99 Hz and standard deviation of 0.173 Hz. Afterward, Kerr and Bishop [81] examined 40 individuals, resulting in a mean value of 1.9 Hz for walking frequency. More recently, Živanović et al. [69] investigated around 2000 pedestrians crossing a footbridge located in Montenegro. The aim of this study was to achieve probability density functions for various walking parameters such as pacing rate. Like the preceding researchers [79,81,82], the authors reported the normal distribution for pacing rate, but with a different mean value (1.87 Hz) and standard deviation (0.186 Hz). Also, Kasperski and Sahnaci [83], Pachi and Ji [84], as well as Kramer and Kebe [85] reported average pacing rates of 1.82, 1.8, and 2.2 Hz, respectively. Figure 16 shows the normal distribution function proposed by different authors for the step frequency of regular walking.

There is a notable difference between mean step frequencies from several studies in the literature. Zivanovic [86] explained such inconsistency by the variety of cultures. For instance, Wiseman stated that people in Singapore tend to walk faster than the citizens of the UK [87]. Therefore, Racic et al. [19] suggested that in order to perform a reliable statistical analysis of walking parameters, a random sample of people from different genders, nations, and cultures should be collected.

In addition, Pachi and Ji [84] have introduced the environment as a factor affecting not only the step frequency but also the walking speed. They studied about 200 walkers, of both genders, on two shopping floors and two footbridges. From statistical analysis of the nearly 800 measurements, they concluded that human pacing rate is usually higher in the case of shopping malls rather than footbridges. Even though the normal probability distribution was described for step frequency in both cases, the mean values of the shopping malls and footbridges were 2.0 and 1.8 Hz, respectively. Regarding the walking speed, a slightly higher value was reported in the case of shopping malls (1.4 m/s) compared with the footbridges (1.3 m/s) [84]. Also, the authors proposed a linear relation between walking velocity v (m/s) and step frequency f (Hz), which is expressed as follows:

$$v = L_s f \tag{4}$$

where L_s is the step length, which has mean values of 0.75 m for males, 0.67 m for females, and 0.71 m in general.

Interestingly, via statistical analysis of the data collected by Pachi and Ji [84], Zivanovic [86] noted that walking speed follows a normal distribution pattern at a certain pacing rate, as shown in Fig. 17. Moreover, Živanović et al. [69] independently investigated the stride length of pedestrians from the data collected on Podgorica footbridge. It was observed that the stride length measurements fit a normal distribution with a mean value of 0.71 m and standard deviation of 0.071 m, similar to what had been proposed by Pachi and Ji [84].

Table 2, extracted from Pedersen and Frier [54], presents the mean value and standard deviation of step frequency reported by different authors.

A relevant comprehensive biomechanical study was conducted by Yamasaki et al. [89] to investigate the relationship between

Table 2 Mean value and standard deviation of step frequency reported after different authors. (Extracted with permission from Pedersen and Frier [54] with modifications. Copyright 2010 by Elsevier.)

Authors	Mean value (Hz)	Standard deviation (Hz)	Subjects tested (no.)
Matsumoto et al. [82]	1.99	0.173	505
Schulze [88]	2.0	0.13	Not known to authors
Kramer and Kebe [85]	2.2	0.3	Not known to authors
Zivanovic et al. [86]	1.87	0.186	1976
Kerr [48]	1.9	Not known to authors	40
Kasperski and Sahnaci [83]	1.82	0.12	250

walking parameters such as speed, step length, and stance period taking gender into consideration. Interestingly, the authors reported a nonlinear relationship between walking speed v and step length L_s, which was in contradiction with several previous studies. This nonlinear behavior becomes more clearly observed at faster walking speeds as illustrated in Fig. 18. Moreover, the authors indicated that the step length for females is typically shorter compared with males especially at high walking speeds. Therefore, females tend to increase speed by increasing pacing rate, while males extend their stride length for the same purpose [89].

4.3.2 Probabilistic Force Models Available in the Literature. One of the first attempts to include the randomness effect into human-induced walking forces was performed in 1996 by Ebrahimpour et al. [90]. In their study on dynamic loads induced by crowds, they observed the significant effect of the erratic nature walking on the results. Furthermore, by including the probability distribution function of the time delay between several pedestrians, the authors achieved a design-oriented simple model that can be used to determine the first harmonic DLF provided the step frequency and the number of people, as shown in Fig. 19. On the other hand, they did not introduce such a comprehensive force model that can be practically applied to predict the structural response. More recently, the stochastic nature of walking was further emphasized in Refs. [11], [60], and [67] by the proposed DLF's of the first four harmonics of the human walking force (with 25% probability of exceedance).

Zivanovic [86] proposed an original framework based on probability theory which can be used to predict the vertical dynamic response of a footbridge excited by a single walking pedestrian. First, statistical analysis has been performed for the main walking parameters such as stride length, step frequency, force amplitudes, and walking imperfections. After that, the probability density functions of these parameters have been incorporated, making it applicable to calculate the cumulative probability that the vibration response will not exceed any certain limit, typically as shown in Fig. 20.

This way, the outcome can be used as a key factor which enables engineers to make a decision regarding the vibration serviceability of footbridge. It should be noted that the author [86] recommended to utilize this probabilistic model for the design of

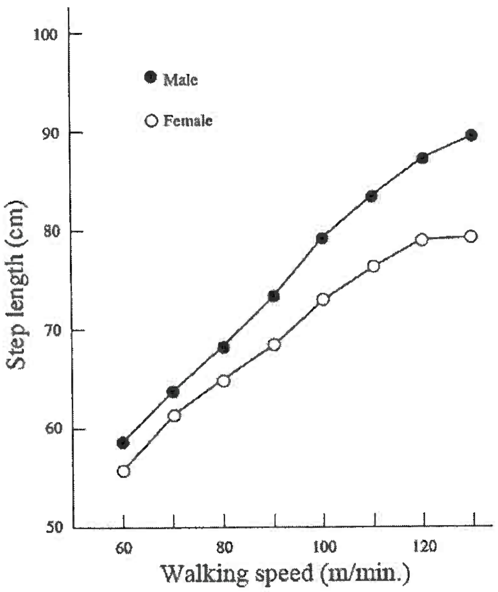

Fig. 18 Nonlinear observed relationship between step length and walking speed. (Reprinted with permission from Racic et al. [19]. The original version was published by Yamasaki et al. [89]. Copyright 1991 by Springer.)

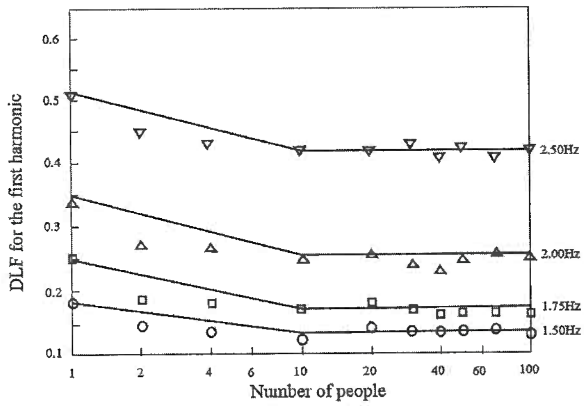

Fig. 19 DLFs of the first harmonic crowd-walking force as a function of the number of persons and step frequency. (Reprinted with permission from Živanović et al. [20]. The original version was published by Ebrahimpour et al. [90]. Copyright 1996 by ASCE.)

footbridges which are subjected to light traffic, where a single walking pedestrian loading case is predominantly expected. Nevertheless, a reliable representation for a single-person excitation was believed to be the first necessary step toward developing a prospective probability-based model for multipedestrian loading scenario.

According to Zivanovic [86], the main weakness of the proposed model in the beginning was being limited to calculate the dynamic response considering only the first harmonic of walking force that is applied to a single vibration mode of the structure. This caused the model to be applicable only to certain types of structural systems, bearing in mind that some structures may respond to human-induced excitation significantly in several vibration modes at the same time. As a consequence, Živanović et al. [69] have extended this probability-based model to cover not only the main harmonics of the walking force commonly considered but also the subharmonics appeared in the frequency domain, as illustrated in Fig. 21.

In this model, the intersubject variability was considered via the probability density functions for walking frequency, step length, pedestrian weight, and main harmonic amplitudes. Because of the lack of measurement database for subharmonics, the corresponding force amplitudes were simply included as a function of the first harmonic's DLF, which might be argued as a weakness point of that model. Moreover, the intrasubject variability was essentially included in the force model through the phase angle, which was represented in the frequency domain with a random pattern uniformly distributed between $(-\pi, \pi)$.

With knowing the dynamic properties of each vibration mode and applying the concept of modal supervision, the authors [69] established a comprehensive model which was able to calculate the vertical multimode response corresponding to multiharmonic walking excitation of a single pedestrian. The accuracy of this model was proved to be adequate based on the results collected from two footbridge case studies: One was simulated and the other was at full scale. The methodology proposed in Ref. [69] had the potential to form the basis of more reliable and modern design guides for vibration serviceability assessment.

A more advanced stochastic model was proposed by Racic and Brownjohn [92] for vertical human-induced walking excitations. At first, the authors generated a comprehensive database of measured continuous vertical walking load signals. A total of 824 vertical walking time histories were collected, utilizing an instrumented treadmill, from about 80 test subjects from both genders and with different weights, heights, and ages. Each individual was requested to walk at a self-selected pacing rate with a different walking speed for each measurement, resulting in at least ten actual recorded signals for human walking forces. Figure 22

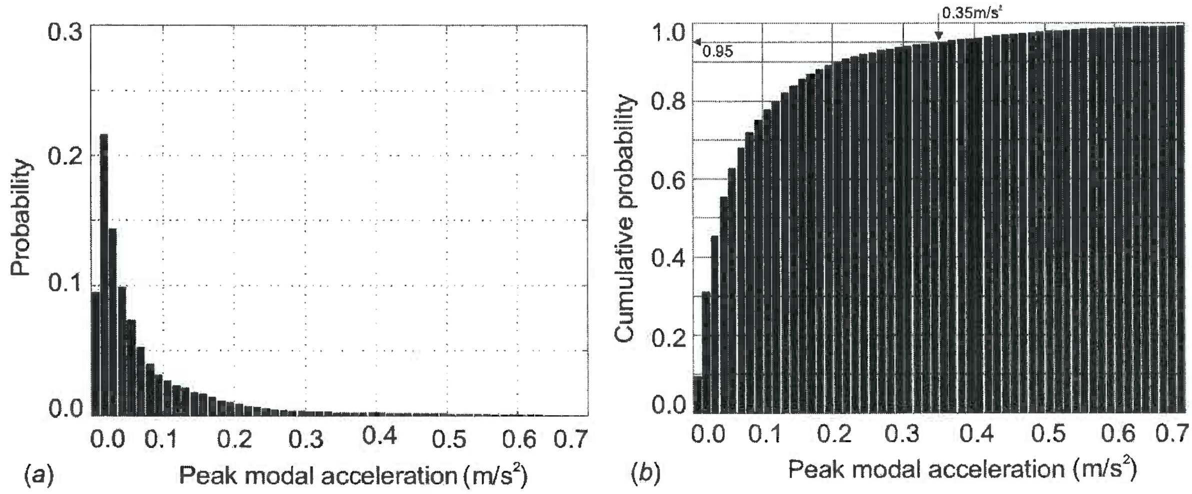

Fig. 20 **(*a*) Probability of the peak modal acceleration excited by a single pedestrian; (*b*) cumulative probability that the peak modal acceleration is less than or equal the value specified in the *x*-axis. (Reprinted with permission from Živanović et al. [91]. Copyright 2007 by Society for Experimental Mechanics.)**

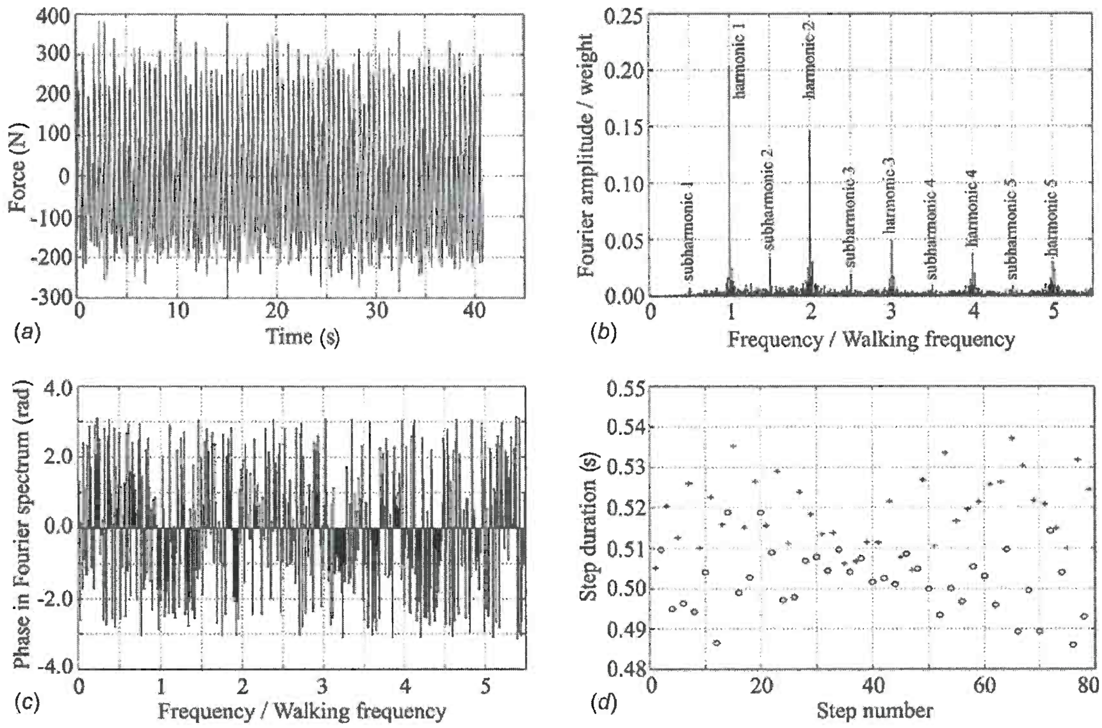

Fig. 21 **(*a*) The 80-step walking force time history measured by an instrumented treadmill for a single pedestrian, (*b*) DLFs appearing for the main and subharmonics in the frequency domain, (*c*) phase angle of forces in Fourier spectrum, and (*d*) period of walking steps. (Reprinted with permission from Živanović et al. [69]. Copyright 2007 by Elsevier.)**

shows a portion of actually measured force signal of 40 s period, normalized to subject weight. Moreover, Racic and Brownjohn [92] categorized the walking signals according to the pacing rate into 20 clusters, each of 0.1 Hz interval, as shown in Fig. 23. It was observed by the same authors that the class of 1.88–1.98 Hz range of pacing rates was the most frequent, indicating that it is generally the most comfortable for walking pedestrians. At each cluster, the signals data were separately analyzed, and the key modeling parameters such as autoregression coefficients were computed.

It should be mentioned that, in order to account for the walking imperfections (i.e., intrasubject variability), the authors in Ref. [92] represented the variations in the cycle period (T) of the signal by a series of dimensionless numbers τ_i, which is expressed as [92]

$$\tau_i = \frac{T_i - \mu_T}{\mu_T}; \quad \mu_T = \text{mean } (T_i) \tag{5}$$

In addition, the authors' algorithm included calculation of the cycle impulse, which is defined as the integral of walking force over cycle period, distinctively for each single footfall of the signal. After that, the algorithm computed the normalized impulse,

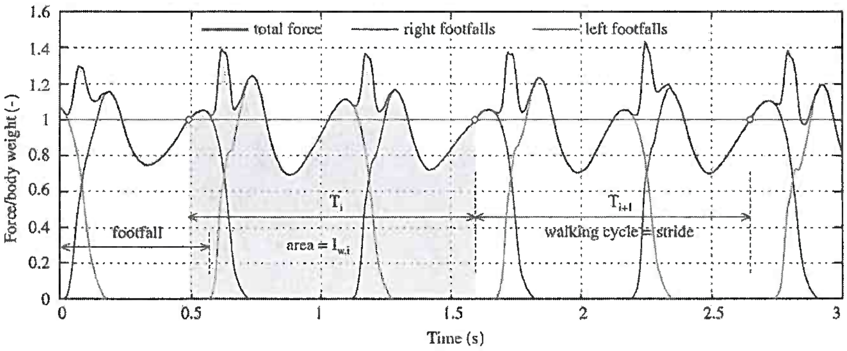

Fig. 22 A portion of actually measured continuous walking force in a 40-s period. (Reprinted with permission from Racic and Brownjohn [92]. Copyright 2011 by Elsevier.)

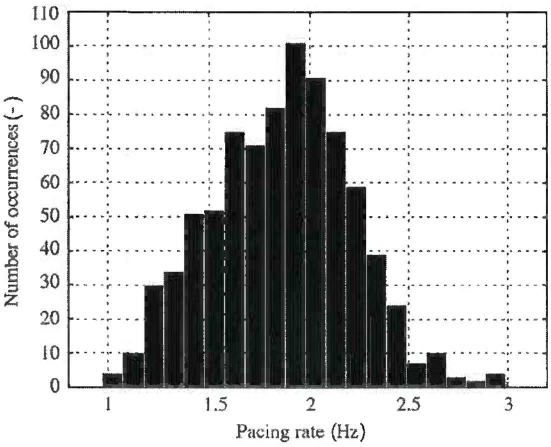

Fig. 23 Frequency-based categorization of actually measured walking force signals. (Reprinted with permission from Racic and Brownjohn [92]. Copyright 2011 by Elsevier.)

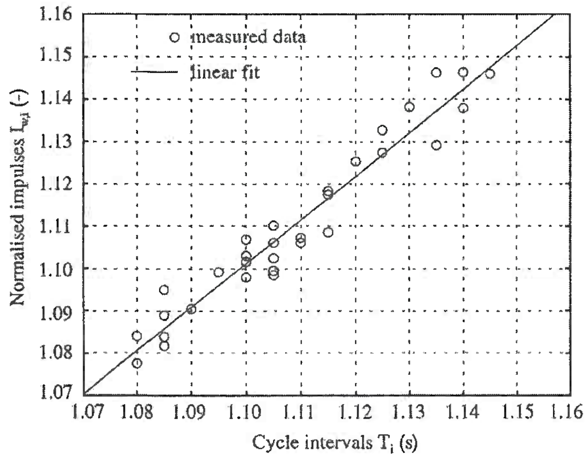

Fig. 24 Linear relationship trend between normalized impulse and cycle time. (Reprinted with permission from Racic and Brownjohn [92]. Copyright 2011 by Elsevier.)

which refers to the ratio of the cycle impulse to the subject weight. Figure 24 shows a possible linear relation between the normalized impulse and cycle time which can be represented as

$$I_{w,i} = a_0 + a_1 T_i + \varepsilon_i \tag{6}$$

where $I_{w,i}$ is the normalized impulse of the ith cycle, ε_i is the subsequent error at the ith cycle, and a_0 and a_1 are the regression parameters of values 0.05 and 1.05, respectively.

In summary, the complete procedures of generating the walking force signal for a given set of pacing rate and walking period are explained in Fig. 25. In the proposed algorithm, the number of cycles is initially calculated, followed by a selection of the key parameters according to the frequency cluster and ending with generating the force signal for this described walking activity.

The main shortcoming of this modeling approach is being numerically complicated, which makes it impractical for the design engineers compared to the simplified hand calculations utilized in the current design guides. The authors argued that the application of this model could be easy to use by the design engineers through a computer program which has a friendly user graphical interface. For instance, by simply entering the walking path, pacing rate, and walking duration, the designer will get the corresponding generated force signal as an outcome, and then, vibration response of the floor is consequently calculated.

4.4 Force Models Under Comparison: A Case of Vibration Serviceability Assessment of a Building Floor. Živanović and Pavić [68] conducted a comparison study on a primary school structural floor of which the dynamic properties had been experimentally identified in an earlier effort [93]. As shown in Fig. 26, the analyzed floor comprised four nominally identical parts, while each was made of prestressed beams, lightweight blocks, and nonreinforced screed layer on top. The vertical floor vibration responses were estimated from five different force models of a single walking pedestrian, and the results out of these models were analyzed and compared. The force models adopted in this study were as follows:

Model 1: Actually measured forces, obtained using an instrumented treadmill, for a single pedestrian walking through the longer direction on a path matching with the center line of the floor.

Model 2: A set of continuous walking force measurements obtained previously by Brownjohn et al. [55] using an instrumented treadmill.

Model 3: The deterministic force model proposed for low-frequency floors, which is defined in vibration serviceability design guide: CSTR43-AppG. The force model utilized the first four harmonics of the walking excitation [11].

Model 4: The deterministic force model proposed for high-frequency floors, which is also defined in the design guide: CSTR43-AppG. This model assumes the transient behavior of walking excitation and deals with each heel drop as a separate force signal [11].

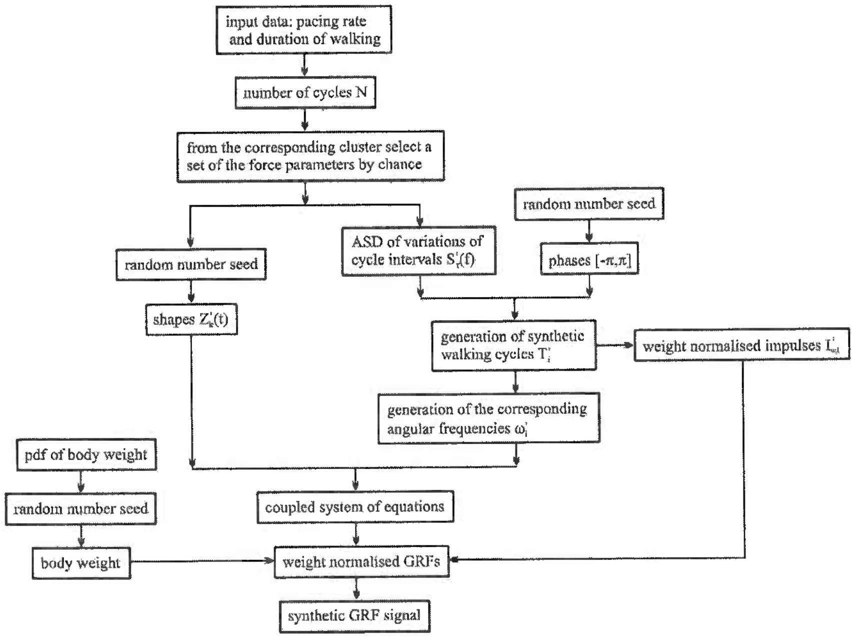

Fig. 25 Algorithm for generating synthesized walking force signals. (Reprinted with permission from Racic and Brownjohn [92]. Copyright 2011 by Elsevier.)

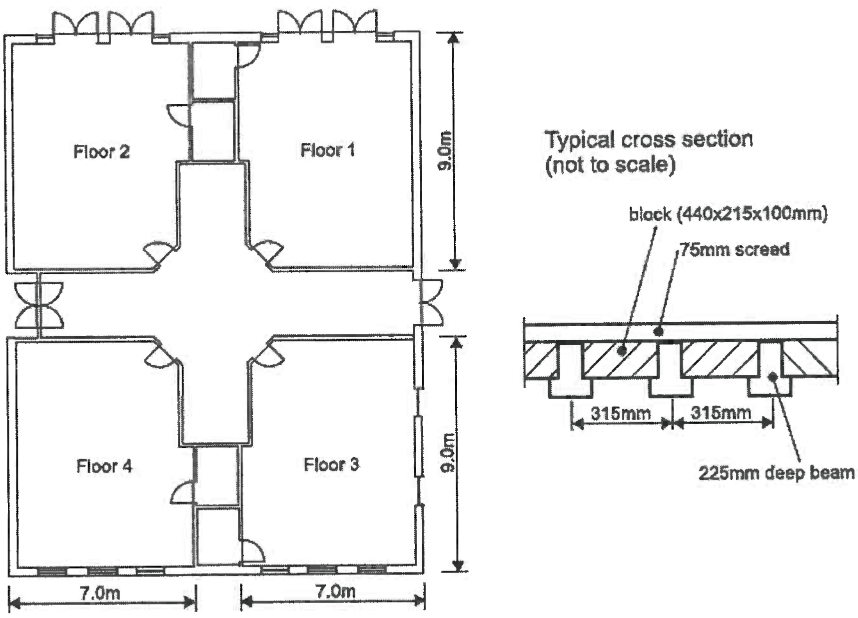

Fig. 26 Description of the four-parted building floor selected for the comparative study. (Reprinted with permission from Živanović and Pavić [68]. Copyright 2009 by ASCE.)

The reason behind including force models for both high- and low-frequency floors was that the fundamental frequency of the structure was slightly lower than the 10 Hz limit, which was suggested in the design guide as a separation line between the two floor types. Therefore, it would not be clear which of the two behaviors the floor would respond to, so the authors decided to utilize both for investigation.

Model 5: The force model suggested by the authors, which was of probability-based nature as previously described in this section. Moreover, this model suggested that the floor response shall be collectively taken from the contribution of two models. The first one is similar to model 3 for the modes of natural frequency less than 10 Hz, while the second is close to model 4 and used for higher vibration modes [68].

As a conclusion to their effort, Živanović and Pavić [68] stated that, at certain circumstances, it could be misleading to select the methodology of vibration serviceability assessment based on simply the floor classification according to the fundamental frequency as suggested by most of widely used contemporary design guides. As a rationalization of this problem, the probabilistic force model suggested by the authors was strongly nominated as an alternative due to several reasons. First, it combined the effects of force modeling approaches for both low- and high-frequency floors, which makes it applicable for any floor with a natural frequency exceeding 3 Hz, and thus leads to the elimination of floor-frequency classification. Moreover, the outcome of this model is presented as a probability of nonexceedance, which allows more space for engineering judgment.

4.5 Comments on Widely Used Floor Vibration Design Guides. Two main requirements are needed to achieve a force model which is practical to utilize at the design stage:

(a) To be simple and easy to use by engineers.
(b) To be as accurate as possible.

However, because of the random nature of human activities in general, it has been challenging for the researchers in the past decades to come up with design steps satisfying both conditions at once. As a consequence, there is no such design code known to be "enforced" for vibration serviceability assessment of floors [19]. What have been published so far are some "design guides," which generally include simple instructions recommended by experts to deal with human-induced floor vibrations in the design stage. These guides explain design procedures and different methods of handling the problem along with their limitations. Even though they are officially published by recognized institutions, the engineer has some freedom not to strictly follow these guides in case of providing reasonable arguments, such as introducing new materials or structural systems [19].

The most common guides, including the AISC Design Guide 11 (U.S., steel-framed floors) [13], Concrete Society Technical Report 43 (CSTR43) Appendix G (UK, posttensioned concrete) [11], SCI P354 (UK, composite floors) [12], HIVOSS (EU, steel-framed/composite) [94], and the UK Concrete Center Design Guide [11], utilize the deterministic approach to walking force modeling, with some differences such as the number of included harmonics and magnitudes of DLFs. For instance, the design value of a DLF is generally considered in the AISC Design Guide 11 as a constant dependent on the range within which the corresponding harmonic's frequency lies [13]. On the other hand, the Concrete Society Technical Report 43 (CSTR43) Appendix G [11], the UK Concrete Center Design Guide [11], and the SCI P354 [12] consider the DLF as a linear function of the corresponding harmonic's frequency as per the model presented in Eq. (2). A slight difference is observed in the design values of DLFs proposed in SCI P354 [12] due to the consideration of different reliability factors. On the contrary, the design philosophy of HIVOSS [94] requires first to determine the value of a quantity called "90% one-step RMS" using the floor dynamic properties and provided set of graphs. This value is then compared with the allowable limits set by the design guide for different floor destinations [94]. What is more, probabilistic models have not yet been adopted in a design guide [5,19,68,95].

In addition, the authors perceive that having a large number of alternative floor vibration design guides with different methods of assessment contained therein has not served the international structural engineering community particularly well. This issue would become more noticeable when structural engineers are working across international boundaries. What is required as a rationalization of these design guides is an internationally recognized "best practice" to provide reliable prediction and assessment of floor vibrations under human actions.

5 Summary and Conclusions

This paper has considered a large body of prior scientific literature dealing with experimental measurement and mathematical modeling of vertical dynamic forces induced by a walking person. The earliest approaches were based on the assumption that measured loads were periodic but not harmonic. This could be observed in spectra of both load measurements and structural responses due to such loading. It was recognized that harmonics of pedestrian loading higher than the fundamental are also a potential cause of vibration problems. Hence, much effort was expended to defining pedestrian loading as a Fourier series and determining appropriate frequencies and amplitudes.

Two main types of building floors are presented in the literature in terms of vibration attributes. Low-frequency floors have at least one responsive natural frequency below 9–10 Hz, which is within reach of one of the first four harmonics of the walking force, thus allowing resonant build-ups. Fourier series representations are useful for predicting resonant responses of these floors. In contrast, high-frequency floors do not undergo resonant build-ups. Effective impulse representations are useful for predicting impulse responses of these.

In contrast to the traditional belief, human walking was recently demonstrated to be a narrowband random process. Therefore, it becomes necessary to employ stochastic analysis to achieve more accurate force modeling. Some excellent studies, aimed to stochastically represent these forces, have been conducted in the past few years. The established probabilistic force models have the potential to revolutionize the classical methods adopted in the current design guides, which are shown to be not very accurate. However, the authors believe that such an all-inclusive probabilistic force model does not exist yet, and further research is needed.

6 Recommendations for Future Work

The following points are further suggested by the authors as key research needs:

- More realistic data are required on statistical distributions of loadings for typical building floors.
- More research effort should be spent to rationalize the sheer number of floor vibration design guides currently available, leading to an internationally recognized best practice that can be used by the structural design community worldwide.
- It is required to develop a comprehensive stochastic model for floor dynamic loading taking into account pedestrian inter- and intrasubject variability together with typical patterns of floor usage.
- Start points, end points, and walking paths on the floor should be determined utilizing a random model originally based on an analysis of video monitoring data. This model should include the effect of floor nonstructural details such as furniture, partition walls, electrical kits, and so on.
- The duration needed by a single person to walk on the building floor at different timing of the day should be estimated using a random model developed from video monitoring data.

Acknowledgment

The financial support for this research was provided by Qatar National Research Fund, QNRF (a member of Qatar Foundation) via the National Priorities Research Program (NPRP), Project No. NPRP8-836-2-353. The statements made herein are solely the responsibility of the authors.

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