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ENGINEERING STRUCTURES
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Probability-based prediction of multi-mode vibration response to walking excitation

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Received 1 February 2006; received in revised form 28 June 2006; accepted 10 July 2006
Available online 15 September 2006

Abstract

In vibration serviceability checks of footbridges, a force induced by a single person walking is usually modelled as a harmonic force having a frequency that matches one of the footbridge natural frequencies. This approach assumes that, among the infinite number of harmonics a walking force is composed of, only a single harmonic is important for a vibration serviceability check. Another usual assumption is that the footbridge can be modelled as an SDOF system, implying that only vibration in a single mode is of interest. In addition, due to the deterministic nature of this approach, it cannot take into account inter- and intra-subject variabilities in the walking force that are now well documented in the literature. To account for these variabilities, a novel probabilistic approach to carry out a vibration serviceability check is developed in this paper. Factors such as the probability distribution of walking frequencies, step lengths and amplitude of walking force for its five lowest harmonics and subharmonics are taken into account. Using walking force time histories measured on a treadmill, the frequency content of the force was investigated, resulting in the formulation of a multi-harmonic force model. This model can be used to estimate the multi-mode response in footbridges. This was verified successfully on an as-built catenary footbridge structure. Although only the vibration response of footbridges was analysed in this paper, the force model proposed has the potential to be implemented in the estimation of floor vibration as well, where multi-mode response occurs more frequently. The model is easily programmable and as such could present a powerful tool for estimating efficiently the probability of various levels of vibration response due to single person walking. Therefore, the proposed probability-based methodology has the potential to revolutionise the philosophy of the current codes of practice dealing with vibration serviceability of structures under human-induced vibration.

Keywords: Vibration serviceability; Walking; Probabilistic design model

1. Introduction

Vibration serviceability checks for slender footbridges that are potentially lively in the vertical direction are usually based on the assumption that an average single pedestrian is crossing the bridge with a step frequency matching one of the structural natural frequencies. In these analyses, it is assumed that the human-induced walking force is a sinusoidal force and that a footbridge structure responds in a single vibration mode [1–6]. This practice can significantly overestimate the footbridge vibration response [7]. This occurs due to neglecting inter- and intra-subject variability in the walking force induced [8–11]. The former term implies that different pedestrians generate different dynamic forces, and the latter means that even a single pedestrian induces a walking force that differs with each step. Therefore, the walking force is not a sinusoidal force as assumed in the mathematical model used widely for the vibration serviceability check of footbridges. Rather, it is a considerably more complex narrow band random process, as demonstrated by Brownjohn et al. [10].

Variability in the walking force can be taken into account via probability-based modelling [12]. In this approach, variables that describe the human-induced force can be defined via their probability density functions. These can be defined for the parameters characterising both inter-subject variability (such as the walking frequency, step length and force amplitude) and intra-subject variability, for example the inability of people to repeat the same force in each step. As a logical consequence of this probabilistic approach, the estimate of the vibration response can be expressed as a probability that a certain level

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0141-0296/$ - see front matter
doi:10.1016/j.engstruct.2006.07.004

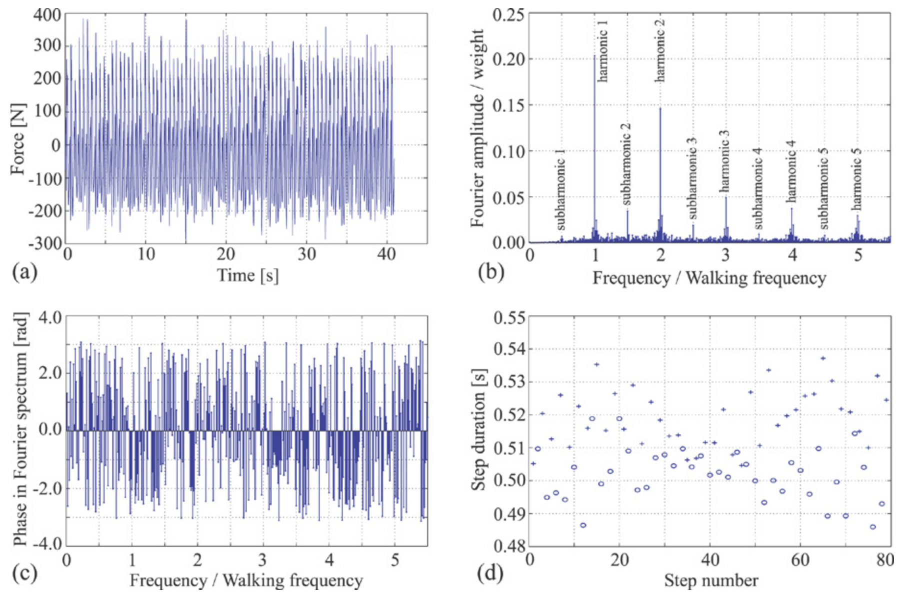

Fig. 1. (a) Dynamic part of a force induced by walking. (b) Amplitude of force Fourier spectrum. (c) Phase of force Fourier spectrum. (d) Period of each step.

of vibration, considered to be a limiting value for a vibration serviceability check, will not be exceeded. Rather than ending up with a single number and a binary pass or fail outcome, the novelty of the proposed approach is that a range of possible vibration responses and the probability of their occurrence is produced. An assessment of these is a much more logical way of judging vibration serviceability.

The previously defined probability-based model [12] was developed for the case when a single force harmonic and the corresponding single mode response were sufficient for the vibration serviceability check. However, there are footbridge and other structures, even with very simple configurations, such as catenary footbridges (as will be shown in this study), that respond to pedestrian-induced excitation in several vibration modes simultaneously, with more than one of them being important. These modes often can be excited by energy around different force harmonics. Therefore, it is necessary to take into account all relevant harmonics of the walking force. By doing this, and knowing the modal properties of the relevant vibration modes, the multi-mode response of the structure can be found via the mode superposition principle [13].

This paper aims to formulate a multi-harmonic force model for calculation of the multi-mode structural response to a single person walking across a footbridge. This will be done using a probability-based framework developed in the previous stage of the research [12]. Therefore, a single harmonic force model will be extended to a multi-harmonic force model. This new model will contain not only the main harmonics usually dealt with in the literature, but also subharmonics that appear between main harmonics in the force spectrum [11].

In the first part of the paper, the appearance of the subharmonics in the force spectrum is explained. After this, probability density functions describing inter-subject variability in the walking force are defined. Then, the intra-subject variability, that is the imperfections in human-induced force, is analysed to formulate a time domain force model. This force model is then experimentally verified, leading to the main conclusions presented at the end of the paper.

2. Subharmonics in walking force

Human-induced walking force is not a periodic, but rather it is a narrow band random process [10]. This means that there is a leaking of energy around the main harmonics in the force spectrum. To illustrate this, 80 steps of a dynamic part of the walking force, measured on an instrumented treadmill [10] and shown in Fig. 1(a), are analysed. The average number of steps per second made by a test subject during this measurement was 1.96 steps/s, that is the walking frequency f_s was 1.96 Hz. The force presented was transformed into the frequency domain. Fourier amplitudes and phases are shown in Fig. 1(b) and (c), respectively. The aforementioned leaking of energy around the main harmonics can clearly be seen in Fig. 1(b). Here, the main harmonics are those present at frequencies equal to the average walking frequency (1.96 Hz) and its integer multiples. However, it can be seen that there are also some subharmonics appearing at frequencies between the main harmonics. This phenomenon has recently been reported in detail by Sahnaci and Kasperski [11]. The explanation for this lies in the fact that the fundamental period of the force time history is equal to the time required to make two successive steps, rather than one, as has been widely accepted in the literature. In this way, the fundamental period is actually approximately two times higher than when analysing one step only and consequently the fundamental frequency of the walking force is approximately two times lower than that for a single step. The reason for this is that the walking process for two legs can be described by slightly different parameters (walking frequency/period and step length) meaning that one leg is ‘stronger’ than another [11]. An illustration of this is the differences in the periods for the two feet that are presented in Fig. 1(d). Crosses represent the

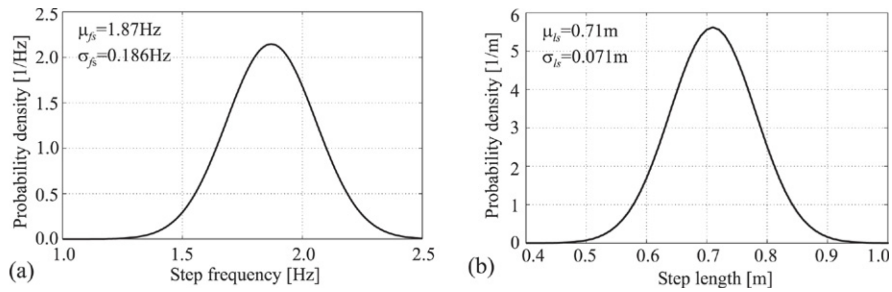

Fig. 2. Probability density functions for: (a) walking frequency and (b) step length.

period for the left foot while circles represent the period for the right foot. It can be seen that time required for the left foot to make one step is consistently longer than that for the right foot.

Based on this analysis, it would be more appropriate to call the harmonic appearing at a frequency of $0.5f_s$ (Fig. 1(b)) the fundamental harmonic. However, for the sake of consistency with the literature published in the last 40 years and bearing in mind the narrow-band nature of the walking force, the terminology used is as follows. The energy around peaks that appear at frequencies that are integer multiples of the average step frequency will be called 'harmonics' (or 'main harmonics') of the walking force, while the energy corresponding to peaks between these will be called 'subharmonics'.

3. Inter-subject variability during walking

Parameters that describe the variability in walking forces induced by different pedestrians are walking frequency, step length and magnitude of the walking force. These parameters all have a certain influence on the level of footbridge vibration response [12] and they are described in this section.

3.1. Walking frequency and step length

Walking frequency f_s and step length l_s can be considered as two independent modelling parameters [12]. It is necessary to have information about them when doing force modelling. f_s in fact defines the forcing fundamental frequency and together with l_s is required for the calculation of time T_c required for footbridge crossing. This time can be obtained from:

$$T_c = \frac{L}{f_s l_s}, \tag{1}$$

where L is the length of the footbridge, and basically it defines the duration of the walking force. Therefore, it is useful to know the probability distributions of f_s and l_s. These were described by Živanović [12] as normal distributions and are shown in Fig. 2(a) and (b), respectively. The mean values of walking frequency and step length in Fig. 2(a) are denoted as μ_{fs} and μ_{ls}, respectively, while their standard deviations are denoted as σ_{fs} and σ_{ls}, respectively. Letters μ and σ will be used throughout the paper to describe the mean value and standard deviation of a variable whose name will be given as their subscript.

3.2. Force magnitude

The third parameter required for force description is the magnitude of the walking force. Since this force is composed of harmonics and subharmonics (Fig. 1(b)), it is necessary to define the amplitude of each of them. This is not a straightforward task because of energy spreading around the main harmonics and subharmonics. However, for each of them a sinusoidal force can be defined in such a way that its power is equal to the power of the (sub)harmonic analysed, taking into account its neighbouring frequency lines, say those in the range of $\pm 0.25f_s$ around the (sub)harmonic. The amplitude of this sinusoid, divided by the test subject's weight, is the value widely accepted for characterisation of the strength of each (sub)harmonic. This value is called the dynamic loading factor (DLF).

3.2.1. DLFs for main harmonics

Different people generate different values of DLFs, even when walking at the same frequency [14,9]. For the first harmonic, Kerr [9] found that the mean value of its DLF is dependent on the walking frequency f_s, as follows:

$$\mu_{\mathrm{DLF1}} = -0.2649f_s^3 + 1.3206f_s^2 - 1.7597f_s + 0.7613. \tag{2}$$

The distribution of DLF_1 around its mean value, for a particular walking frequency, can be obtained via multiplication of the mean value by a normally distributed factor MF [12] shown in Fig. 3(a) by the solid line.

For a complete description of the walking force, the probability distributions of DLFs for higher harmonics are also required. This paper deals with the first five harmonics of the walking force, since it is believed that harmonics higher than the fifth are not capable of inducing perceptible vibrations in footbridges.

Kerr [9] reported a mean value for DLF_2 equal to 0.07 (Fig. 3(b)) regardless of the walking frequency. Under an assumption of normal distribution of DLF_2 for each walking frequency and assuming that DLF_2 is independent from DLF_1, a normal distribution of a multiplication factor for getting DLF_2 from its mean value can be presented based on Kerr's data (dashed line in Fig. 3(a)). It should be noted that, due to large standard deviation, some negative values of DLF_2 tend to appear in Fig. 3(a). These do not have physical meaning and as such should be removed from the calculation procedure by replacing them with zero values. It can be seen that the

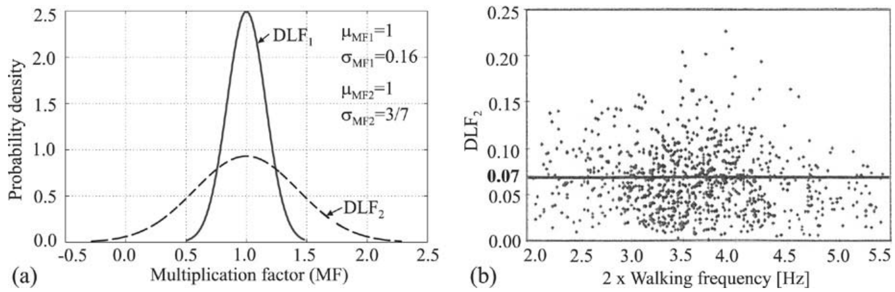

Fig. 3. (a) Probability density function for multiplication factor for DLF_1 (solid line) and DLF_2 (dashed line). (b) DLF_2 measured by Kerr [9].

Table 1
Parameters describing normal distribution of DLFs for higher harmonics

Harmonic #	Mean	Standard deviation
2	0.07	0.030
3	0.05	0.020
4	0.05	0.020
5	0.03	0.015

scattering of DLF_2 is much higher in comparison with that for the first harmonic.

Based on Kerr's research [9], the normal distribution for the third and forth harmonics can be defined in a similar way as was done for the second harmonic. By analysing 95 force time histories measured by Brownjohn et al. [10], the data related to the fifth harmonic have also been collected. The mean values and standard deviations describing the normal distributions of the second, third, fourth and fifth harmonics are listed in Table 1. Similarly to DLF_2, all negative values of DLFs that appear in the probability distributions due to large scatter should be replaced by zero values.

3.2.2. DLFs for subharmonics

This section aims to define DLFs for the first five subharmonics in the walking force. These data are missing from the literature available. Because of this, the 95 time histories measured by Brownjohn et al. [10] for nine test subjects walking on a treadmill were transformed into the frequency domain. Then the power for each subharmonic was calculated in the frequency range $(i - 0.5)f_s \pm 0.25f_s$, where i is the subharmonic considered ($i = 1, 2, 3, 4, 5$). After this, the amplitude of a sinusoidal force having the same power was calculated and divided by the weight of the test subject to get the DLF. However, since force time histories for only nine test subjects taking part in 95 measurements with different walking speeds were available, it is not prudent to construct probability density functions for these subharmonics as there were insufficient data points around each walking frequency. Instead, it is possible to establish a relationship between DLFs for the subharmonics and, say, the first walking harmonic DLF_1. These relationships are presented in Fig. 4, based on the DLFs obtained for 95 force time histories. Also, a linear fit in the least square sense is presented for each graph in the figure. Therefore, for modelling purposes only the relative relationship between subharmonics and the first harmonic is adopted. In this way the magnitude of the DLF for the subharmonics can be obtained only after the magnitude of DLF_1 is known.

It is worth commenting here that Fig. 4 implies that the magnitudes of the DLFs for subharmonics are generally higher when the magnitude of DLF_1 is higher. However, it might occur that for a sample of test subjects that is larger than the nine used here when obtaining Fig. 4, this relationship is actually nondeterministic (random), i.e. the DLFs for subharmonics can be considered as independent from DLF_1. This should be verified when more data become available. This is especially important if the force model is to be used for structures that are sensitive even to small force magnitudes present at frequencies typical for subharmonics.

3.2.3. Pedestrian's weight

An additional random parameter that influences the force modelling but is not considered as a random variable in this paper is the pedestrian's weight W. There are indications that increasing the pedestrian's weight leads to increased DLFs [15]. However, there is no quantification of this dependence known to the authors. Therefore, it is currently not possible to construct a joint probability density function for these two, apparently dependent, variables defining the force amplitude $DLF \cdot W$ for each (sub)harmonic. This was the reason to decide to include the pedestrian's average weight of 750 N [16] in the formulation of the force model.

4. Intra-subject variability during walking

Due to the inability of human beings to walk in the same way when making every single step, the walking force is not a perfectly periodic process. Imperfections in the human walking force can be described via slight changes of the walking frequency (that is the reciprocal value of the period presented in Fig. 1(d)), amplitude (Fig. 1(a)) and phase lag in each step. These changes can be taken into account via investigation of the force in the frequency domain that is an inherent part of the force modelling described in the next section. These imperfections change the level of footbridge vibration response in comparison with that obtained due to the corresponding sinusoidal force. This is especially so when considering higher harmonics of the walking force [10].

5. Force modelling

In this section, the frequency content of the measured walking forces is analysed first, followed by a description of

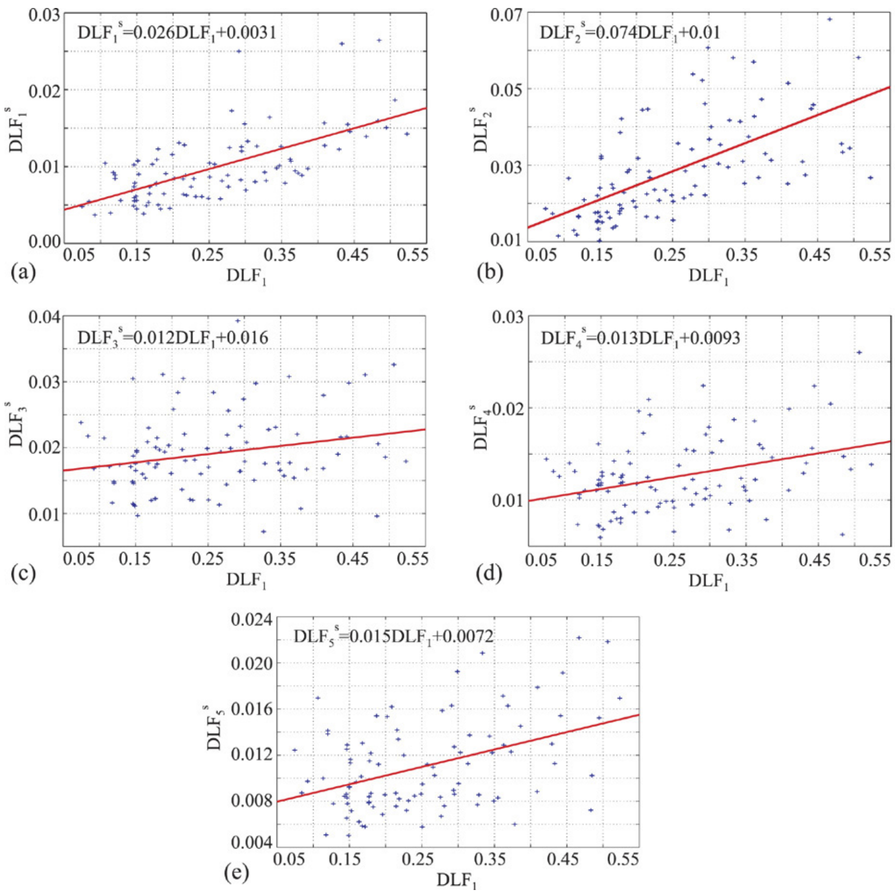

Fig. 4. DLFs for subharmonics as functions of DLF_1.

the force model adopted. Then a procedure for model usage is briefly summarised.

5.1. Force description in the frequency domain

The force induced by walking can be presented in the frequency domain via its amplitudes and phases characterising each frequency line in the force spectrum (Fig. 1(b) and (c)). Since a decision to cover the frequency range of the walking force related to the first five harmonics and subharmonics was made, a force model covering the frequency range $0.25 f_s$–$5.25 f_s$ will be formulated.

To formulate this force model, 95 time histories measured on a treadmill set to a constant speed [10] were analysed. The Fourier spectrum of exactly 80 steps for each time history was found. Amplitudes of this spectrum in the range $\pm 0.25 f_s$ around each (sub)harmonic were divided with the corresponding DLF for this (sub)harmonic. In this way the normalised amplitude spectra were obtained. They were overlayed for each (sub)harmonic, and are presented as grey lines in Fig. 5. After this, the mean functions for all spectra were found and fitted in the least square sense. The normalised amplitudes $\overline{\text{DLF}_i}(\bar{f}_j)$ for the ith harmonic are fitted by a function that is available as a built-in function in MATLAB [17]:

$$\overline{\text{DLF}_i}(\bar{f}_j) = a_{i,1}\mathrm{e}^{-\left(\frac{\bar{f}_j - b_{i,1}}{c_{i,1}}\right)^2} + a_{i,2}\mathrm{e}^{-\left(\frac{\bar{f}_j - b_{i,2}}{c_{i,2}}\right)^2} + a_{i,3}\mathrm{e}^{-\left(\frac{\bar{f}_j - b_{i,3}}{c_{i,3}}\right)^2}, \tag{3}$$

where $a_{i,k}$, $b_{i,k}$ and $c_{i,k}$ $(k = 1, 2, 3)$ are nine fitting parameters for the ith harmonic. $\bar{f}_j$ is the frequency ratio between the current frequency line and step frequency f_s, and it belongs to the interval $[i - 0.25, i + 0.25)$, with step $\frac{1}{80}$, including its left limit. Therefore, for the first harmonic variable $\bar{f}_j$ is in the range 0.75–1.25, for the second harmonic 1.75–2.25, for the third 2.75–3.25, for the fourth 3.75–4.25 and for the fifth 4.75–5.25. Therefore, the spectrum width of $0.5 f_s$ around each harmonic is taken into account when defining the normalised DLF for that harmonic. Since every time history analysed contained 80 steps, this was the reason to have spacing between frequency lines equal to $\frac{f_s}{80}$. When the spectrum width of $0.5 f_s$ used for each harmonic is divided by the frequency spacing, it follows that each harmonic is described by 40 lines. The nine parameters for each of the first five harmonics are listed in Table 2.

The normalised amplitudes for subharmonics $\overline{\text{DLF}_i^s}$ were fitted by a function:

$$\overline{\text{DLF}_i}^s(\bar{f}_j^s) = a_{i,1}^s\mathrm{e}^{-\left(\frac{\bar{f}_j^s - b_{i,1}^s}{c_{i,1}^s}\right)^2} + a_{i,2}^s\mathrm{e}^{-\left(\frac{\bar{f}_j^s - b_{i,2}^s}{c_{i,2}^s}\right)^2}, \tag{4}$$

where $a_{i,k}^s$, $b_{i,k}^s$ and $c_{i,k}^s$ $(k = 1, 2)$ are six fitting parameters for ith subharmonic. $\bar{f}_j^s$ is the frequency ratio between the

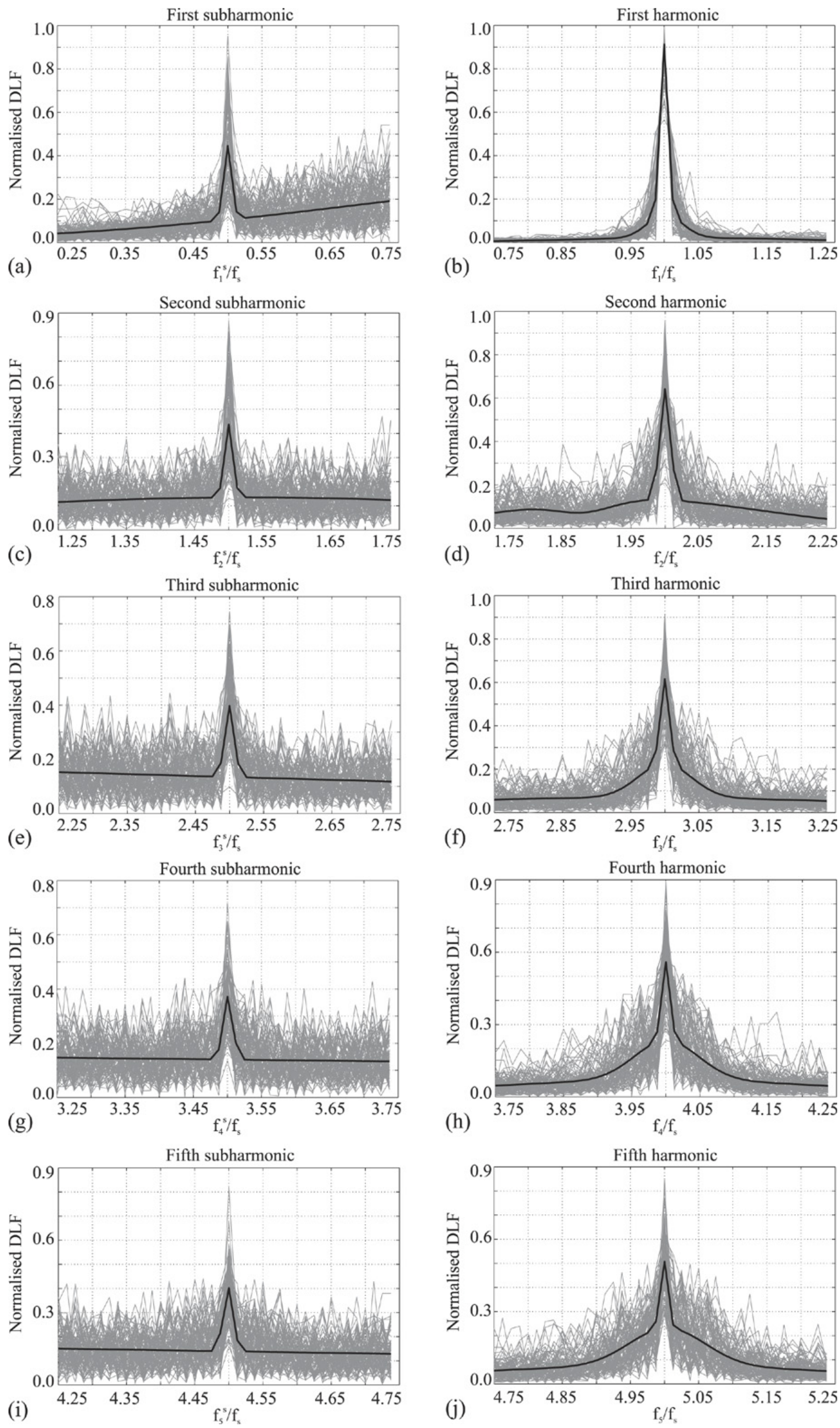

Fig. 5. Normalised DLFs for the first five subharmonics (left column) and harmonics (right column).

current frequency line and step frequency f_s, and it belongs to the interval $[i - 0.75, i - 0.25)$ including its left limit. So, for the first subharmonic this variable is in the range 0.25–0.75, for the second subharmonic it is 1.25–1.75, and so on. As in the case of the main harmonics, 40 lines are used for the description of a subharmonic. The six parameters for each of the five subharmonics are listed in Table 3.

The functions in Eqs. (3) and (4) used to fit the mean functions for the harmonics and subharmonics are shown as black lines in Fig. 5. It can be seen that the fit for subharmonics is quite similar for all of them in terms of normalised amplitude and shape of the fitting function. In the case of main harmonics, it is evident that the higher ones are weaker in amplitude and broader in frequency content than the lower

Table 2
Fitting parameters for five harmonics

i	1	2	3	4	5
$a_{i,1}$	0.785200	0.513000	0.390800	0.325500	0.280600
$b_{i,1}$	0.999900	2.000000	3.000000	4.000000	4.999000
$c_{i,1}$	0.008314	0.011050	0.009560	0.008797	0.007939
$a_{i,2}$	0.020600	0.133000	0.156700	0.164700	0.158400
$b_{i,2}$	1.034000	1.957000	3.000000	4.001000	5.004000
$c_{i,2}$	0.252400	0.263200	0.055250	0.066410	0.078250
$a_{i,3}$	0.107400	−0.049840	0.068660	0.068880	0.072890
$b_{i,3}$	1.001000	1.882000	2.957000	3.991000	4.987000
$c_{i,3}$	0.036530	0.058070	0.560700	0.375000	0.450100

Table 3
Fitting parameters for five subharmonics

i	1	2	3	4	5
$a^s_{i,1}$	0.340600	0.302400	0.262700	0.234400	0.264500
$b^s_{i,1}$	0.498800	1.500000	2.500000	3.501000	4.499000
$c^s_{i,1}$	0.008337	0.008735	0.009748	0.009898	0.010190
$a^s_{i,2}$	0.280300	0.134500	0.245600	0.235500	0.238900
$b^s_{i,2}$	1.133000	1.532000	0.231200	−1.576000	1.153000
$c^s_{i,2}$	0.638800	0.723300	2.932000	7.050000	4.561000

ones. This indicates a higher degree of randomness for higher harmonics.

Having a model representing amplitudes in the spectrum of walking force, additional information about the phase for each frequency line is required for accurate force representation in the time domain. For this purpose, the phase spectra of measured forces were examined. It was noticed that the phases for all frequency lines in the range $0.25 f_s$–$5.25 f_s$ are uniformly distributed in the interval $[-\pi, +\pi]$ for any force time history analysed. An example of this distribution for the phase diagram presented in Fig. 1(c) is shown by a histogram in Fig. 6. Any interdependence between phase changes around the main harmonics (where the amplitudes are the highest and most important) as well as between different harmonics could not be noticed. This was the reason to adopt a uniformly distributed random phase in the force model.

It should be said here that the modelling presented in this section is an extension of a model formulated by Brownjohn et al. [10], as:

- subharmonics are now included into the force model,
- phase information is taken into account, and
- the complete frequency content of the force spectrum, up to the frequency of 5.25 f_s, is now included into the model without any discontinuities.

With all frequency lines considered and phase information added, the reconstruction of the force in the time domain is possible. This, in turn, makes it possible to weight the human-induced force by mode shape in order to take into account that the force moves across the footbridge and has limited duration. This weighted (modal) force can then be used to calculate the structural modal response in a particular mode of vibration. This kind of analysis is not possible in the frequency domain, which is a feature of the model formulated by Brownjohn et al. [10].

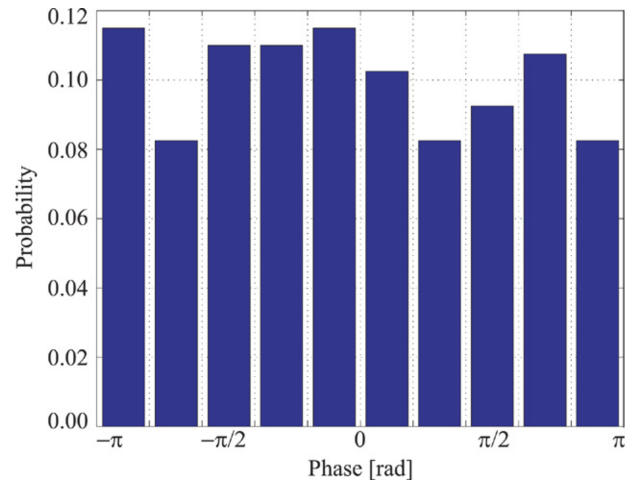

Fig. 6. Distribution of phases for 400 lines in the force spectrum.

5.2. Time domain force model

For the ith harmonic, occurring at frequency if_s, the force can be reconstructed in the time domain via the following formula:

$$F_i(t) = W \cdot \mathrm{DLF}_i \times \sum_{\bar{f}_j=i-0.25}^{i+0.25} \overline{\mathrm{DLF}}_i(\bar{f}_j)\cos(2\pi \bar{f}_j f_s t + \theta(\bar{f}_j)), \tag{5}$$

while for the ith subharmonic it would be

$$F^s_i(t) = W \cdot \mathrm{DLF}^s_i \times \sum_{\bar{f}^s_j=i-0.75}^{i-0.25} \overline{\mathrm{DLF}}^s_i(\bar{f}^s_j)\cos(2\pi \bar{f}^s_j f_s t + \theta(\bar{f}^s_j)). \tag{6}$$

Here, i is the (sub)harmonic considered, and $\bar{f}_j f_s$ is a frequency line within the energy range of the harmonic analysed, while $\theta(\bar{f}_j)$ is the phase assigned to the current line in the spectrum. This assignment is based on a uniform distribution of phases in the interval $[-\pi, +\pi]$. DLF_i is the DLF for the harmonic analysed, and $\overline{\mathrm{DLF}}_i(\bar{f}_j)$ is the normalised amplitude for the same harmonic for each line, while $W = 750$ N is the average weight of a pedestrian. Variables containing superscript s are related to subharmonics. Finally, the total force can be obtained as:

$$F(t) = \sum_{i=1}^{5} F_i(t) + \sum_{i=1}^{5} F^s_i(t). \tag{7}$$

To demonstrate the quality of the force model presented, an attempt to model the force shown in Fig. 1(a) has been made. The fundamental frequency of this force as well as its DLFs are already known. The frequency is 1.96 Hz while DLFs for all harmonics and subharmonics are obtained based on the procedure explained in Section 3.2. These values were

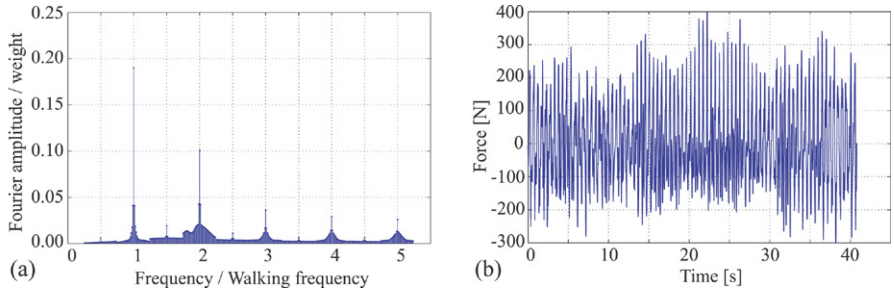

Fig. 7. (a) Force spectrum for force shown in Fig. 1(a) according to the force model adopted. (b) Force reconstructed in the time domain.

used as input values for defining normalised amplitudes in the frequency domain for the force analysed. The force spectrum obtained in this way is presented in Fig. 7(a). The peaks in this spectrum are a bit attenuated in comparison with the real spectrum (Fig. 1(b)) due to using an average spectrum of walking forces defined by Eqs. (3) and (4) and Fig. 5. After obtaining the spectrum of amplitudes, uniformly distributed random phases were generated for all 400 lines in the spectrum and the force was reconstructed in the time domain (Fig. 7(b)). It can be noticed that the force model (Fig. 7(b)) differs from that in Fig. 1(a). The two forces could visually be, in general, both more similar and more different from each other than obtained here, depending on the randomly generated phase values for each case. Since the probability-based response calculation will be based on a large sample of generated forces, it can be assumed that the phase influence on the results in such a sample is not significant. It should be noticed that the energy of the force is not influenced by the phase values.

5.3. *Procedure for response simulations*

This section reviews briefly the key steps required for estimation of a modal response of a footbridge when using the force model formulated. To simplify the explanation, it is assumed that simulations for 2000 individual pedestrians walking on their own across a bridge will be conducted. It is further assumed that only a single mode is relevant for the calculation, and that its modal properties are known. In the case that more than one mode is relevant, the modal responses obtained for individual vibration modes should be summed according to the mode superposition principle. The procedure for a single mode response can be described as follows:

1. Generate the walking frequency and step length for each of 2000 pedestrians (Fig. 2(a) and (b)).
2. Calculate μ_{DLF1} for each walking frequency (Eq. (2)).
3. Calculate the crossing time for each person via Eq. (1).
4. Generate multiplication factors for each DLF_1 (Fig. 3(a)) and multiply them with μ_{DLF1} one by one to get 2000 DLFs for the first harmonic.
5. Generate 2000 DLFs for higher harmonics (Table 1).
6. Generate 2000 DLFs for each subharmonic depending on the DLF_1 values (Fig. 4).
7. Generate functions for normalised DLFs for harmonics and subharmonics by using the functions defined in Eqs. (3) and (4).
8. For each pedestrian do the following:
   - Generate 400 random phase values for each frequency line in the 400-line force spectrum.
   - Reconstruct the force in the time domain by using Eqs. (5)–(7). The force should be reconstructed for time equal to the crossing time for the person analysed.
   - Calculate the modal force by multiplying the individual forces $F(t)$ by the mode shape.
   - Calculate the footbridge modal response to each of 2000 modal forces.
   - Calculate the peak and/or RMS value of the response and save it.
9. Find the cumulative probability that a certain footbridge vibration level will be less than or equal to a prescribed limiting value.

6. Verification of the force model

Verification of the force model was conducted by analysing the response of two structures to walking excitation. The first ‘structure’ is an imaginary 3DOF bridge while the second one is an as-built real-life footbridge near Sheffield in the UK.

6.1. *Imaginary footbridge*

The imaginary 3DOF footbridge is a lightly damped ‘structure’ that responds to dynamic excitation in three vibration modes. It is assumed that all vibration modes have maximum displacement at the same point. This allows a simple summation of the responses in individual modes in order to get the total response of the structure. The modal properties of the mode shapes were chosen to be:

- Modal mass equal to 10 000 kg in each vibration mode.
- Natural frequencies equal to 1.9 Hz, 3.8 Hz and 5.7 Hz, for the first, second and third mode respectively.
- Modal damping equal to 0.3% for all modes.

The natural frequencies of vibration modes were chosen to correspond with the dominant walking frequency and its integer multiples. Therefore, it is expected that all three modes of vibration will be excited since their frequencies are matched by the dominant frequencies for the first three harmonics. Since the three harmonics will all generate structural responses that cannot be neglected, it is expected that phase difference between different harmonics will play an important role in the estimation of the total vibration response.

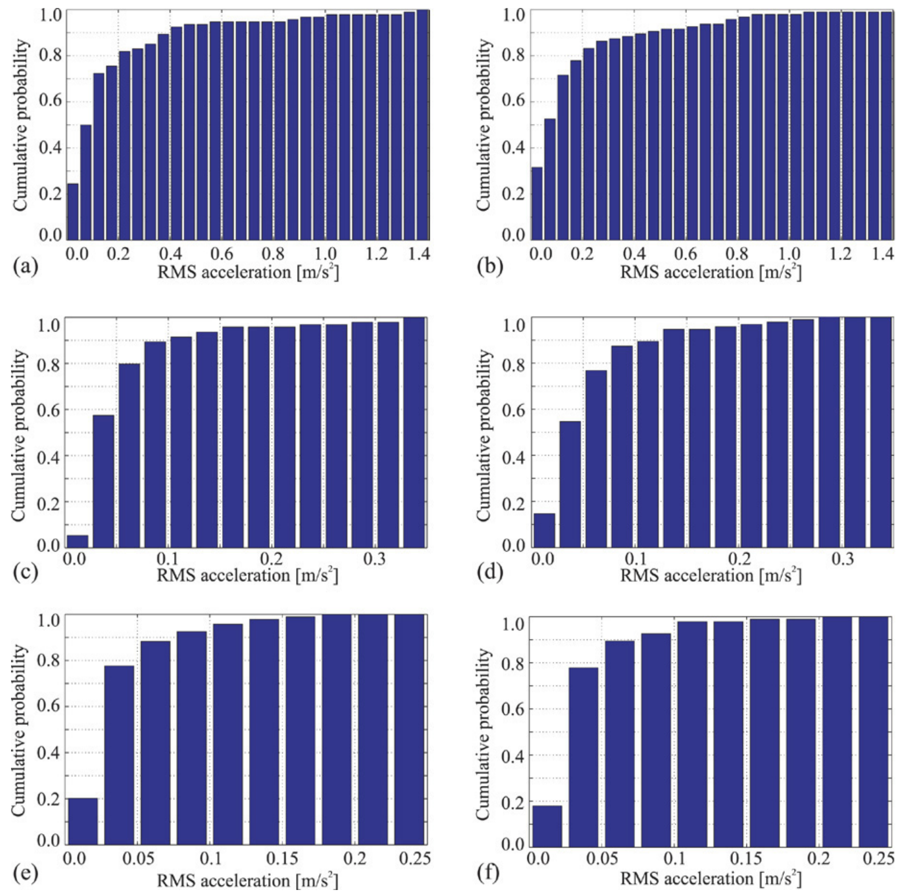

Fig. 8. Cumulative probability that the RMS acceleration level for the whole response signal is smaller than or equal to the RMS acceleration level shown on the horizontal axis for the response in (a) Mode 1 due to measured forces, (b) Mode 1 due to simulated forces, (c) Mode 2 due to measured forces, (d) Mode 2 due to simulated forces, (e) Mode 3 due to measured forces and (f) Mode 3 due to simulated forces.

The structural response was calculated in two ways. Firstly, 95 responses to 95 measured forces, lasting 60 s each, were calculated. Then, 95 forces were generated using the force model described in Section 5.2. The walking frequencies and DLFs for all harmonics and subharmonics for these 95 forces were manually adjusted to be the same as those for measured forces. The responses obtained in this way were then compared with those obtained from measured force time histories. Before presenting the results, it should be said that all simulations were conducted under an assumption of the force being stationary, i.e. acting at the point of maximum modal response in each mode. This does not reduce the value of this comparison.

Obtained results can be summarised as follows:

- The cumulative distribution of the RMS value for the response in each vibration mode individually under the measured forces was almost the same as that under the simulated forces (Fig. 8).
- The cumulative distribution of RMS values for the sum of responses in any two modes was almost the same for cases of measured and simulated forces. For comparison, only the sum of responses in the second and third mode is presented in Fig. 9(a) and (b).
- The cumulative distribution of RMS values for the total response of all three modes to measured forces was almost the same as the one to simulated forces (Fig. 9(c) and (d)).

Table 4
Modal properties of the catenary footbridge

Mode	1	2	3	4	5
Natural frequency [Hz]	2.44	3.66	4.86	6.66	9.50
Modal damping [%]	0.53	0.65	0.96	0.73	0.77
Modal mass [kg]	10 520	5880	8690	10 767	10 319

This example shows that taking phases as random variables that are uniformly distributed, when defining the force model, is a reasonable approximation of the real situation.

6.2. *As-built footbridge*

The as-built footbridge analysed is a 30 000 kg catenary structure near Sheffield spanning 34 m (Fig. 10(a)). The footbridge is quite short and of simple structure. It has five well separated vertical modes of vibration in the frequency range up to 10 Hz. Their mode shapes are presented in Fig. 10(b), while their natural frequencies, modal dampings and modal masses, as identified by Pavic and Reynolds [18], are listed in Table 4. Note that the first mode is anti-symmetric, which is a consequence of the curvature of the underformed shape of the bridge deck.

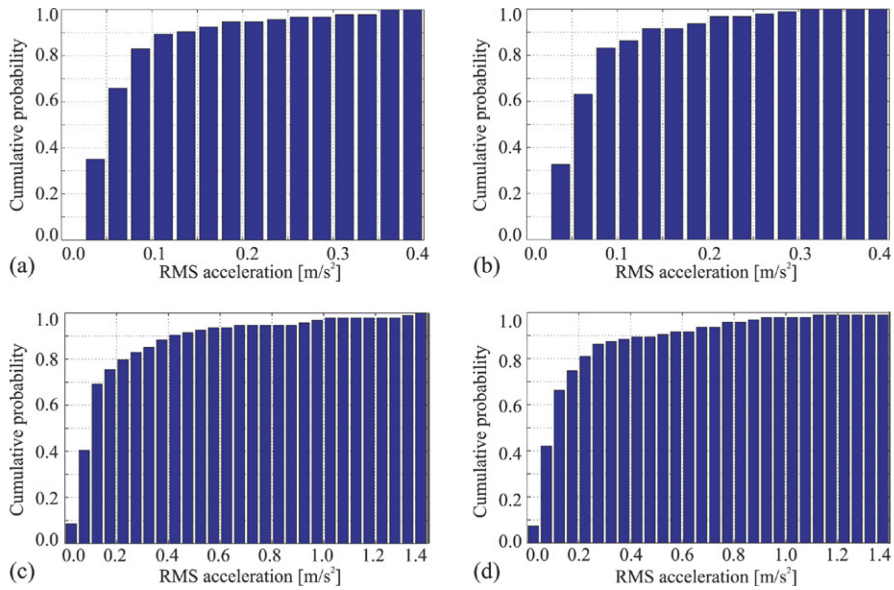

Fig. 9. Cumulative probability of RMS acceleration response in Modes 2 and 3 due to (a) measured forces, (b) simulated forces. Cumulative probability of (c) total RMS response to measured forces, (d) total RMS response to simulated forces.

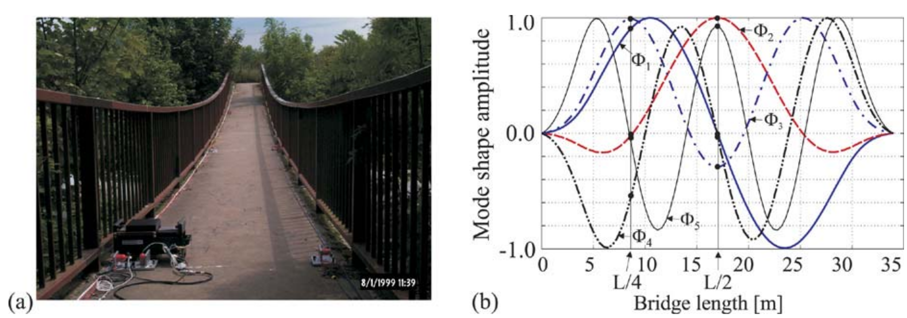

Fig. 10. (a) Catenary footbridge. (b) Mode shapes for five vertical vibration modes.

6.2.1. Response measurements

The acceleration response of this footbridge to a single person excitation was measured at a quarter-span (hereafter referred to as test point 1) and at the midspan point (test point 2) [19]. Seven test subjects were asked to cross the bridge with a pacing rate they personally considered to be normal. The exercise was repeated twice for each test subject so that in total 14 responses were recorded. By using a video camera, the crossing time was also recorded for every crossing. Typical acceleration time histories measured at test point 1 (TP1) and test point 2 (TP2) are shown in Fig. 11(a) and (b), respectively. Their Fourier spectra are presented in Fig. 11(c) and (d). It can be seen that at the quarter-span point (TP1) several modes respond significantly to walking excitation. These are Φ_1, Φ_3 and Φ_4, i.e. all modes that have nonzero amplitude at TP1 (Fig. 10(b)). Also, at the midspan point (TP2), the response is mainly a combination of the second and the fifth modes (Fig. 10(b)). Based on these measurements it is evident that the contribution of several vibration modes should be taken into account when estimating the vibration response to human-induced force. The measured accelerations are low-pass filtered (up to 10 Hz) in order to contain only the frequency content related to the first five vertical modes analysed. The results are summarised in Figs. 12 and 13 in the form of peak acceleration and RMS acceleration of the whole signal. They are shown as solid lines on the probability histograms presented in Fig. 12(a) and (b) for TP1, and Fig. 12(c) and (d) for TP2. Cumulative probabilities are also presented as solid lines in Fig. 13(a) and (b) for TP1, and Fig. 13(c) and (d) for TP2.

6.2.2. Multi-mode response simulations

The response of the bridge was calculated for 2000 different force time histories generated according to the procedure described in Section 5.3. The modal responses at TP1 and TP2 were obtained for individual vibration modes and then summed after multiplication of each modal response by the mode shape amplitude at the point considered. In this way the total physical responses were obtained at both test points.

The probability and the corresponding cumulative distribution for peak and RMS values of total acceleration responses in these two points are shown as grey in Figs. 12 and 13, respectively. Comparing them with the probability distribution of measured accelerations (solid lines in Figs. 12 and 13), it can be seen that probability of having a high level of peak responses generally exists for the calculated responses only (Fig. 12(a) and (c)). This is an estimate that is on the safe side and is a

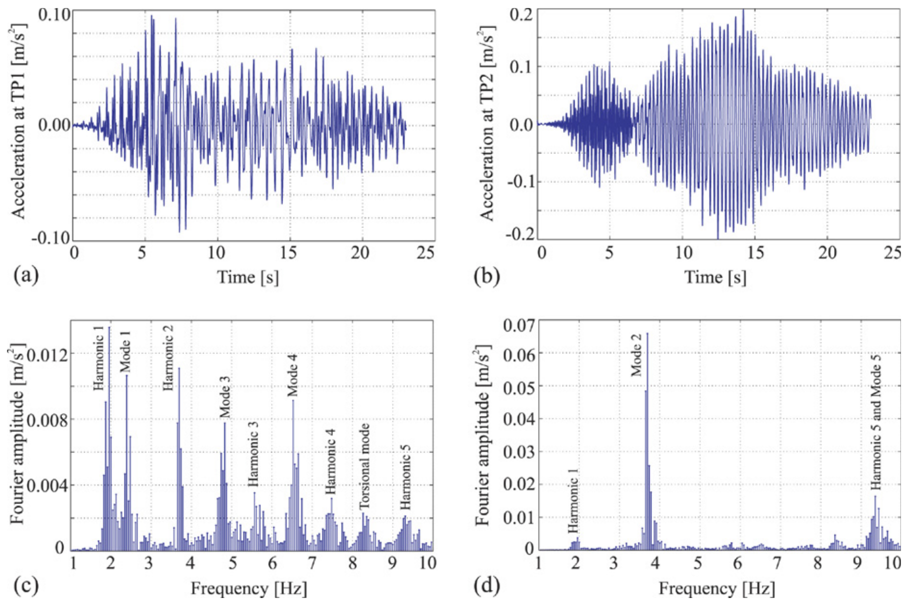

Fig. 11. Measured time history at (a) TP1 and (b) TP2. Fourier spectrum of signal at (c) TP1 and (d) TP2.

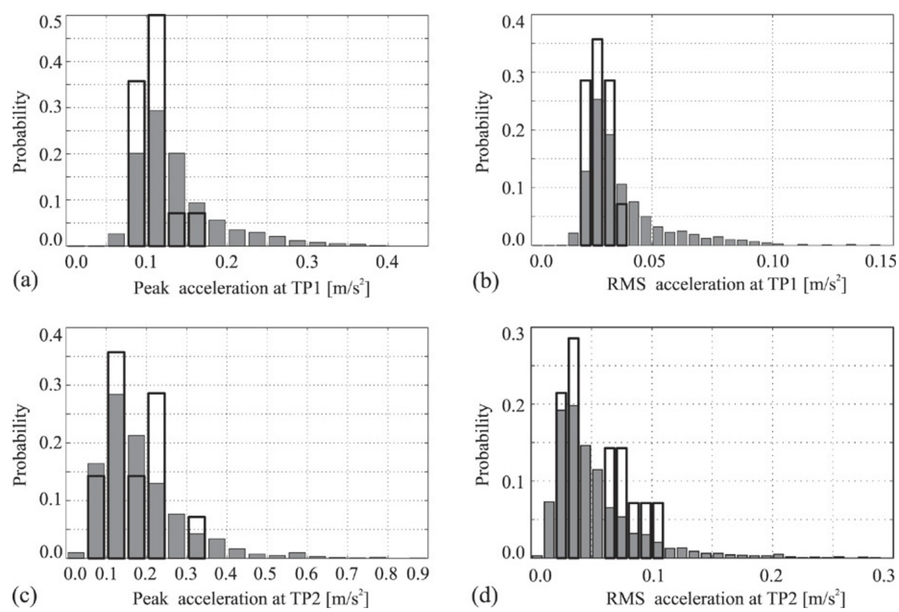

Fig. 12. Probability of certain acceleration level for (a) peak acceleration at TP1, (b) RMS acceleration at TP1, (c) peak acceleration at TP2, and (d) RMS acceleration at TP2. Measured data are presented as solid lines while calculated responses are shown as grey.

consequence of the fact that the probability of having forces with greater amplitudes and with frequencies of their harmonics closer to some of the natural frequencies is much greater in simulations than in the measured sample of only seven pedestrians making 14 crossings in total. An additional reason not to have high level responses in the measured data is that some pedestrians lost their steady step due to human–structure interaction when perceiving a vibration level that they personally considered as disturbing [20].

It should be emphasised here that, under the condition that probability distributions of walking frequency and step length for crossings of the bridge analysed can be described by functions in Fig. 2, the result from 2000 simulations presented is actually more reliable in a statistical sense than the measurement set limited to seven people only, that cannot represent inter-subject variability.

Finally, the model is much more accurate in the prediction of vibration response of footbridges to single person excitation than BS 5400 [1] currently used in the UK. The same applies to a more recent document BD 37/01 [5] that makes use of the same calculation procedure as BS 5400. To illustrate this, a deterministic 'single-harmonic–single-mode' response defined in BS was applied on the first two modes individually. In both cases, the amplitude of the dynamic force is taken to be 180 N, i.e. 25.7% of the test subject weight, being 700 N. It was assumed, based on the recommendations in BS, that the pedestrian walking at a step frequency that matches the natural frequency of the first mode at 2.44 Hz crosses the bridge with a

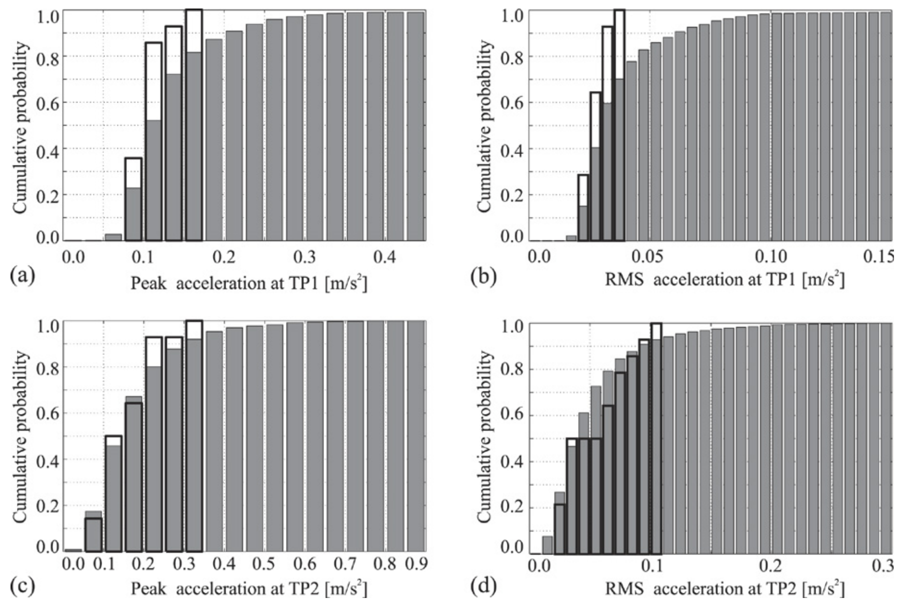

Fig. 13. Cumulative probability for measured and simulated (a) peak acceleration at TP1, (b) RMS acceleration at TP1, (c) peak acceleration at TP2, and (d) RMS acceleration at TP2. Measured data are presented as solid lines while calculated responses are shown as grey.

quite fast, and also quite improbable, walking speed of 2.2 m/s, while the one walking at the step frequency that matches the second mode at 3.66 Hz moves with an even less probable speed of 3.3 m/s. The peak acceleration responses calculated in this way were 0.42 m/s^2 and 0.72 m/s^2 for the first two modes respectively. Comparing these results with those presented in Fig. 13(a) and (c), it can be concluded that the values estimated by BS 5400 are highly unlikely to occur on the bridge analysed. This is despite the fact that the BS takes into consideration only a single vibration mode.

7. Conclusions

This paper describes the modelling of the human-induced walking force for a single pedestrian. A probability-based model is proposed that takes into account inter- and intra-subject variability in the walking force. The model takes into account all of the frequency content of the walking force up to the fifth harmonic. In this way, a general multi-harmonic force model is formulated that allows for calculation of multi-mode response of a structure.

This model is an extension of a probability-based 'single-harmonic–single-mode' response calculation model developed in a previous stage of the research [12]. The inter-subject variability is modelled via probability distributions of walking frequencies, force amplitudes and step lengths. The intra-subject variability is modelled via a frequency domain representation of both amplitudes and phases in the spectrum of the walking force. As such, this modelling of intra-subject variability is an extension of the model formulated by Brownjohn et al. [10].

Based on case studies of one imaginary 3DOF footbridge simulation model and one real-life as-built footbridge, the proposed model was successfully verified. It was shown that it predicts the multi-mode response of footbridges with sufficient accuracy. The new challenge in the next stage of research is the verification of the model on slender floor structures, where multi-mode response occurs more frequently, as well as an implementation of the model for multi-person traffic.

It should be noted that the model defined is easily programmable and as such could present a powerful tool for estimating efficiently the probability of footbridge vibration response due to single person walking. The novel methodology has the potential to revolutionise the current codes of practice dealing with vibration serviceability assessment due to dynamic excitation caused by human walking.

Acknowledgments

The authors would like to thank to Professor J.M.W. Brownjohn of the University of Sheffield and the National Institute of Education in Singapore for their permission to use the data related to measured walking forces. We also acknowledge the financial support which came from the UK Engineering and Physical Sciences Research Council (EPSRC) for grant reference GR/S14924/01 ("Investigation of the As-Built Vibration Performance of System Built Floors") as well as for grant reference GR/T03000/01 ("Stochastic Approach to Human-Structure Dynamic Interaction") provided to Professor A. Pavic, for his Advanced Research Fellowship. Finally, the authors are grateful for the financial support from the Universities UK Overseas Research Student Award Scheme (reference ORS/2002036023).

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