CPSC 8420 Advanced Machine Learning
Week 8: Spectral Clustering and Manifold Learning

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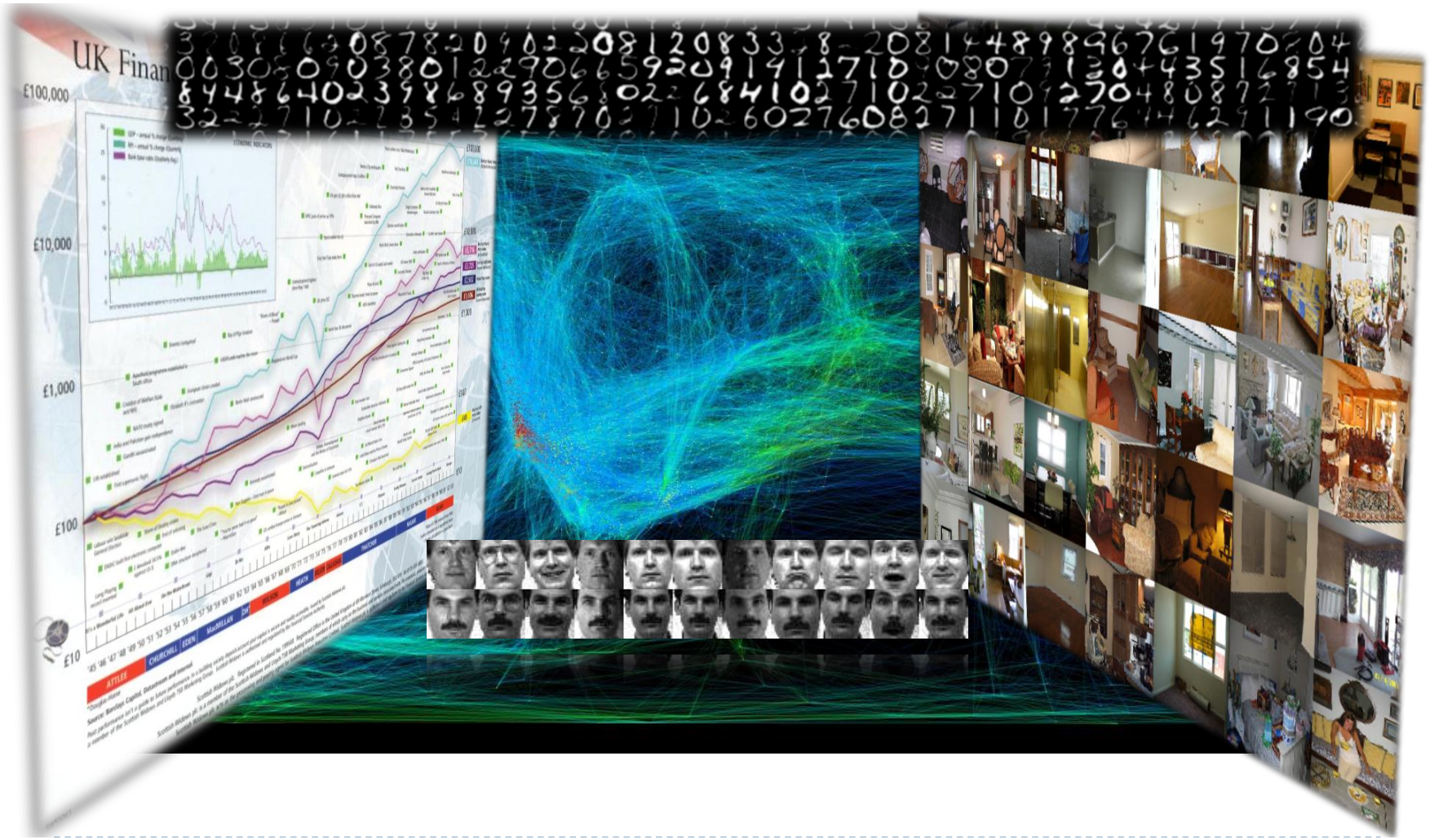
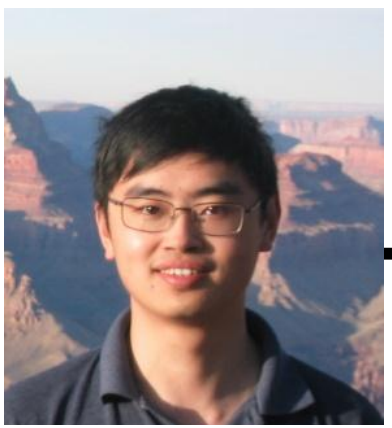
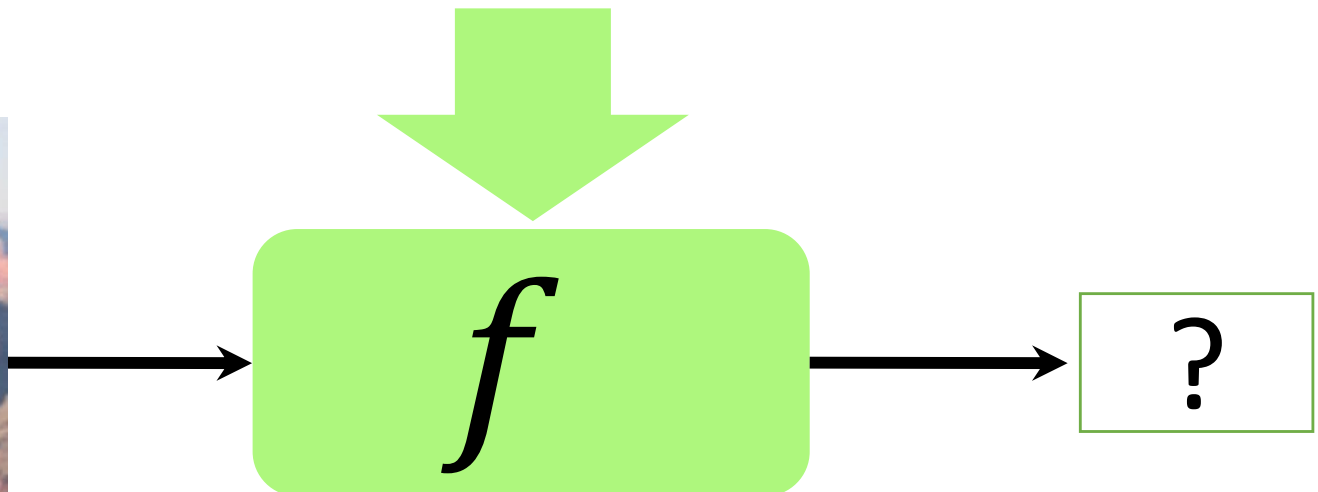
October 8, 2020

Machine Learning



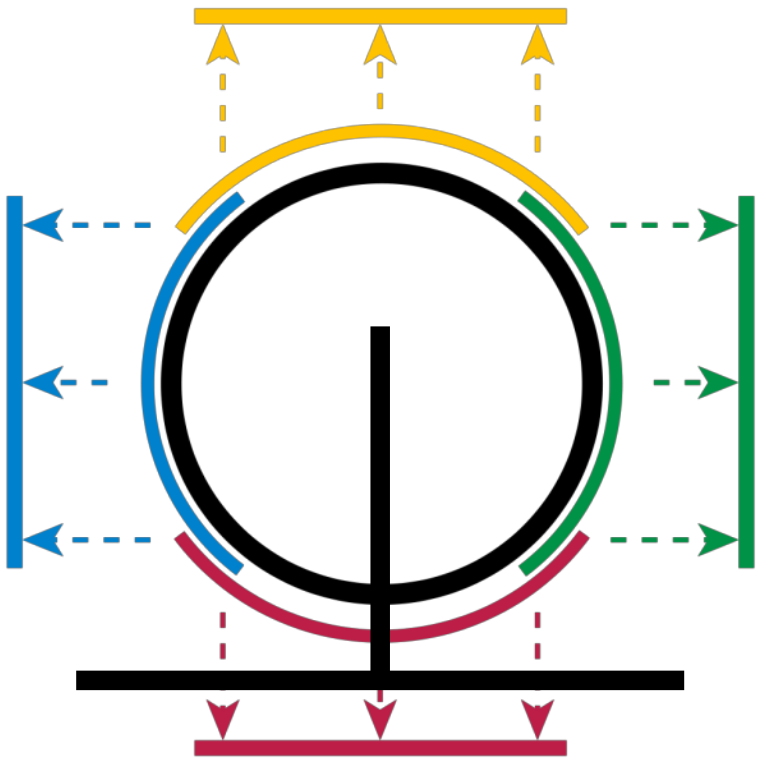
Machine Learning





Manifold

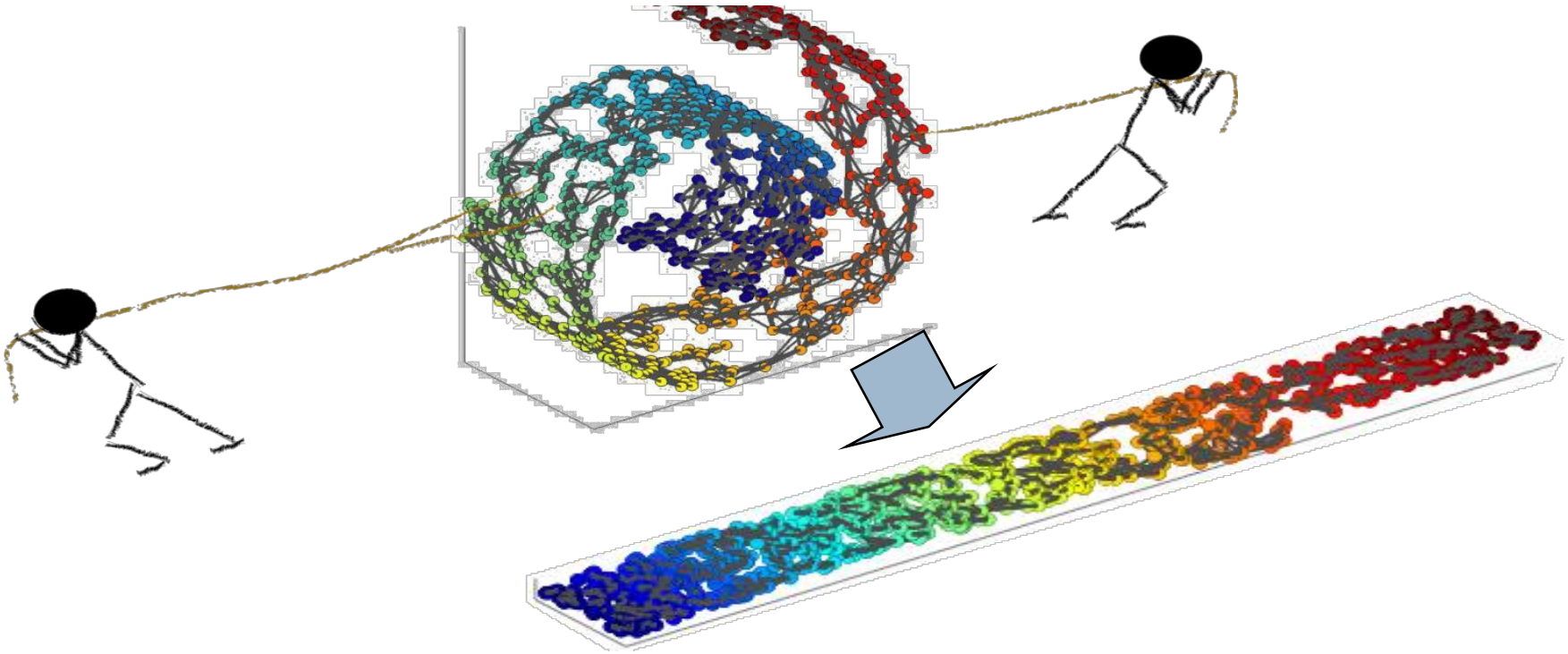
Manifold = Many + Fold



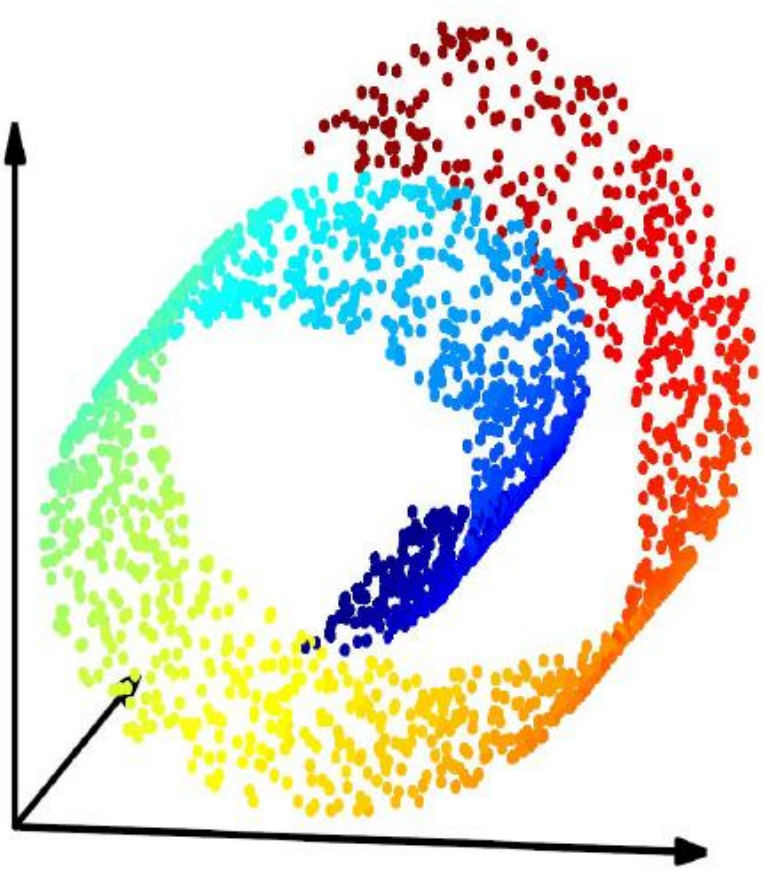
Manifold

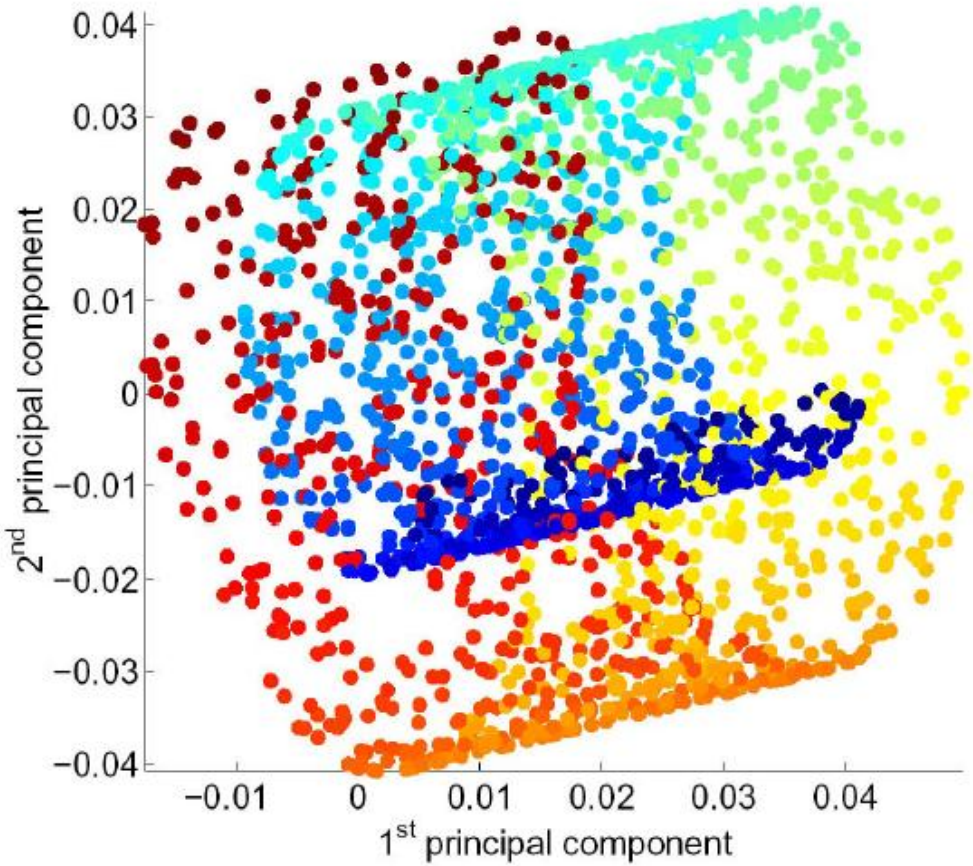
Manifold

Manifold

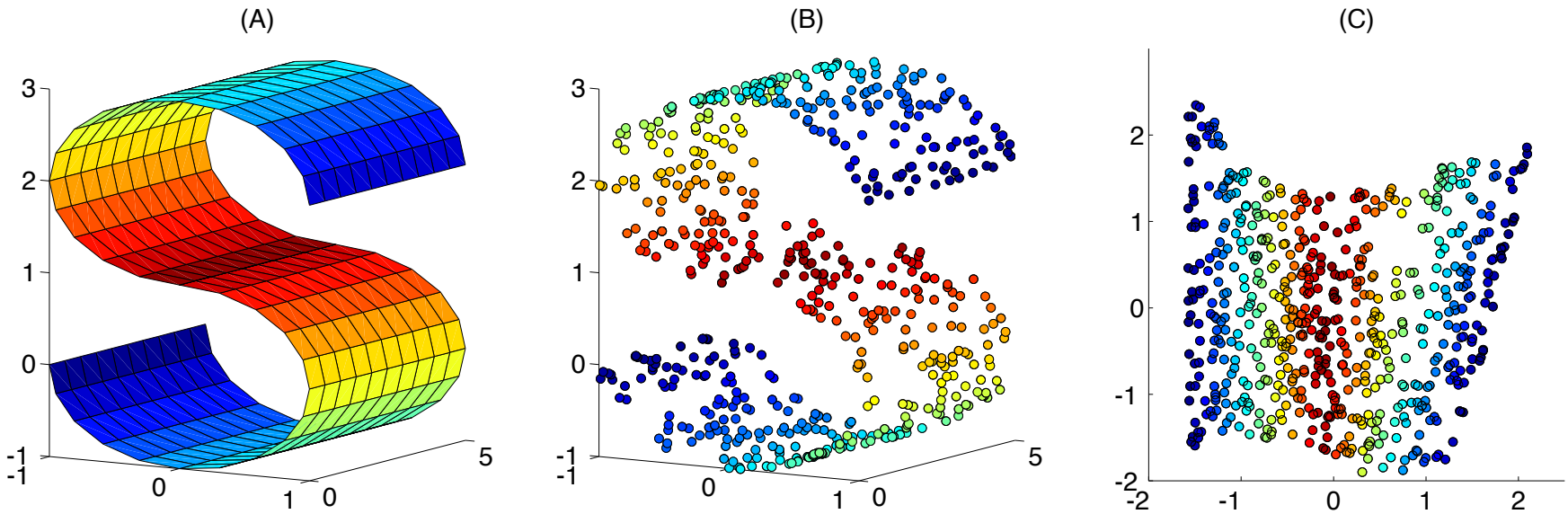


Manifold

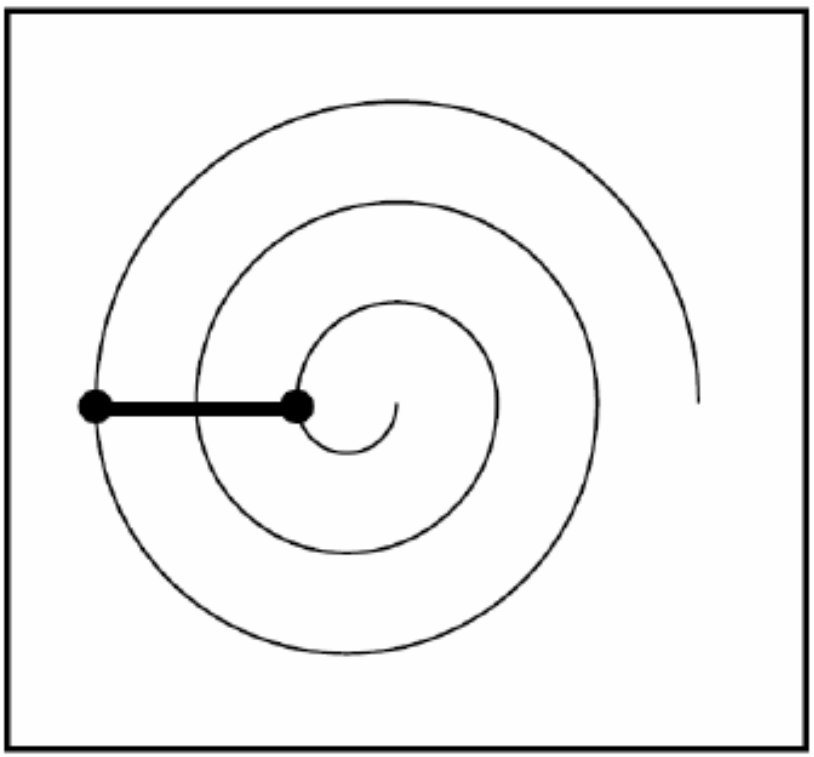


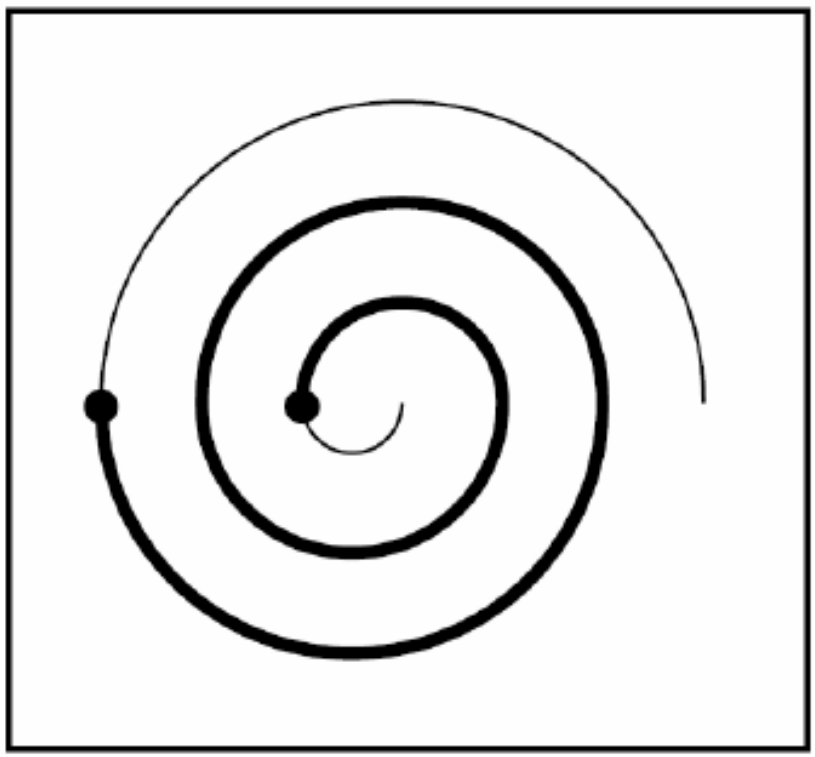

0.04
0.03
0.02
0.01
0
−0.01
−0.02
−0.03
−0.04
2nd principal component
−0.01
0
0.01
0.02
0.03
0.04
1st principal component

Manifold

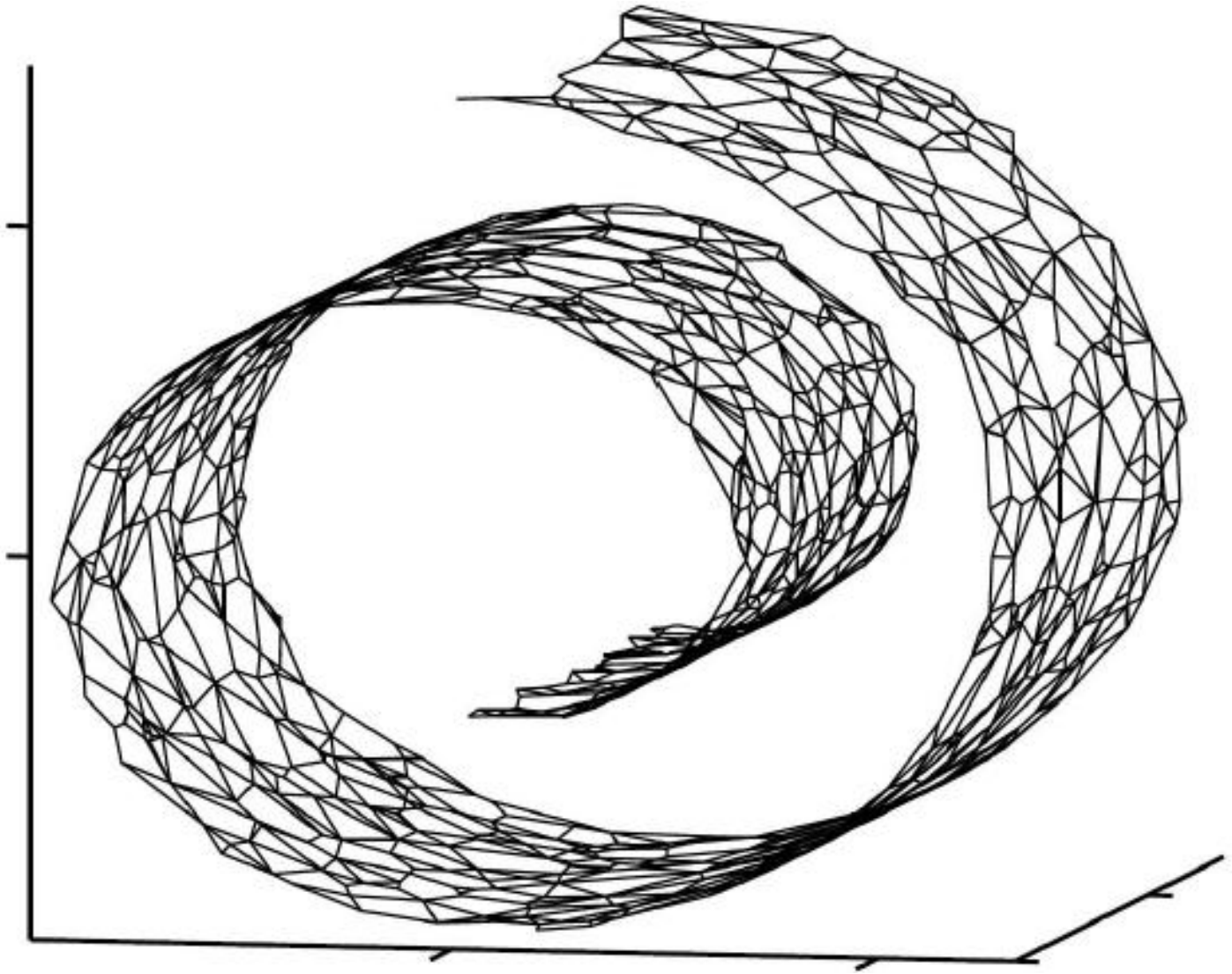


ISOMAP



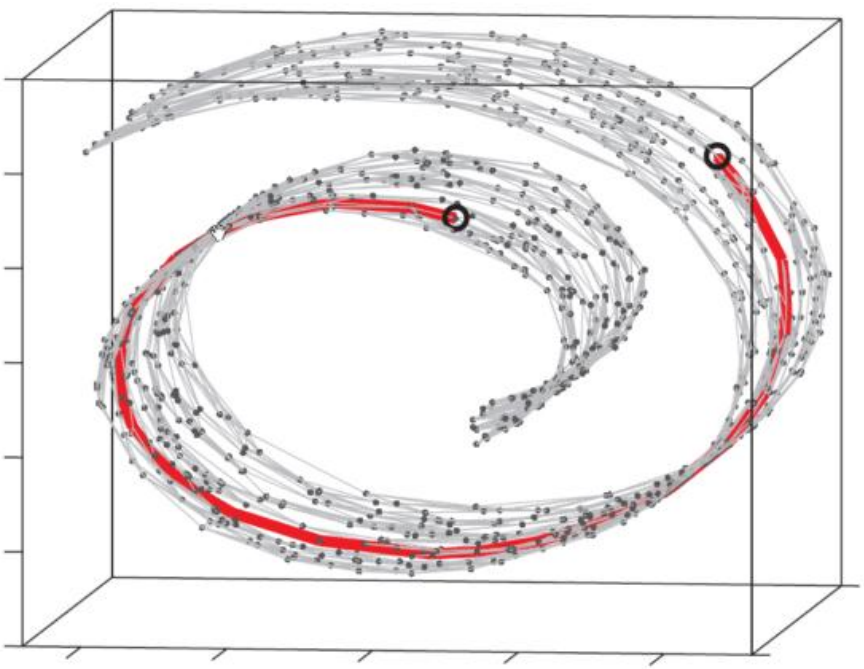


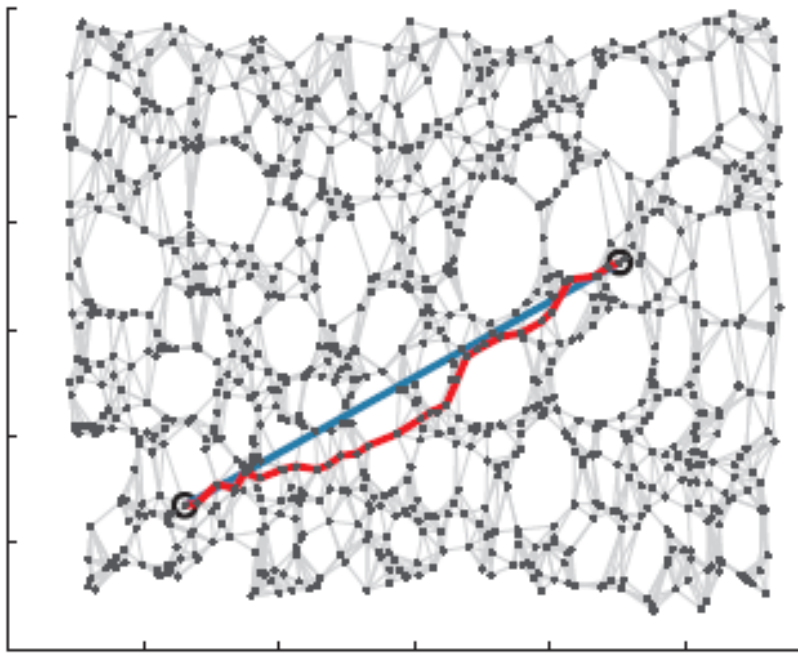
ISOMAP



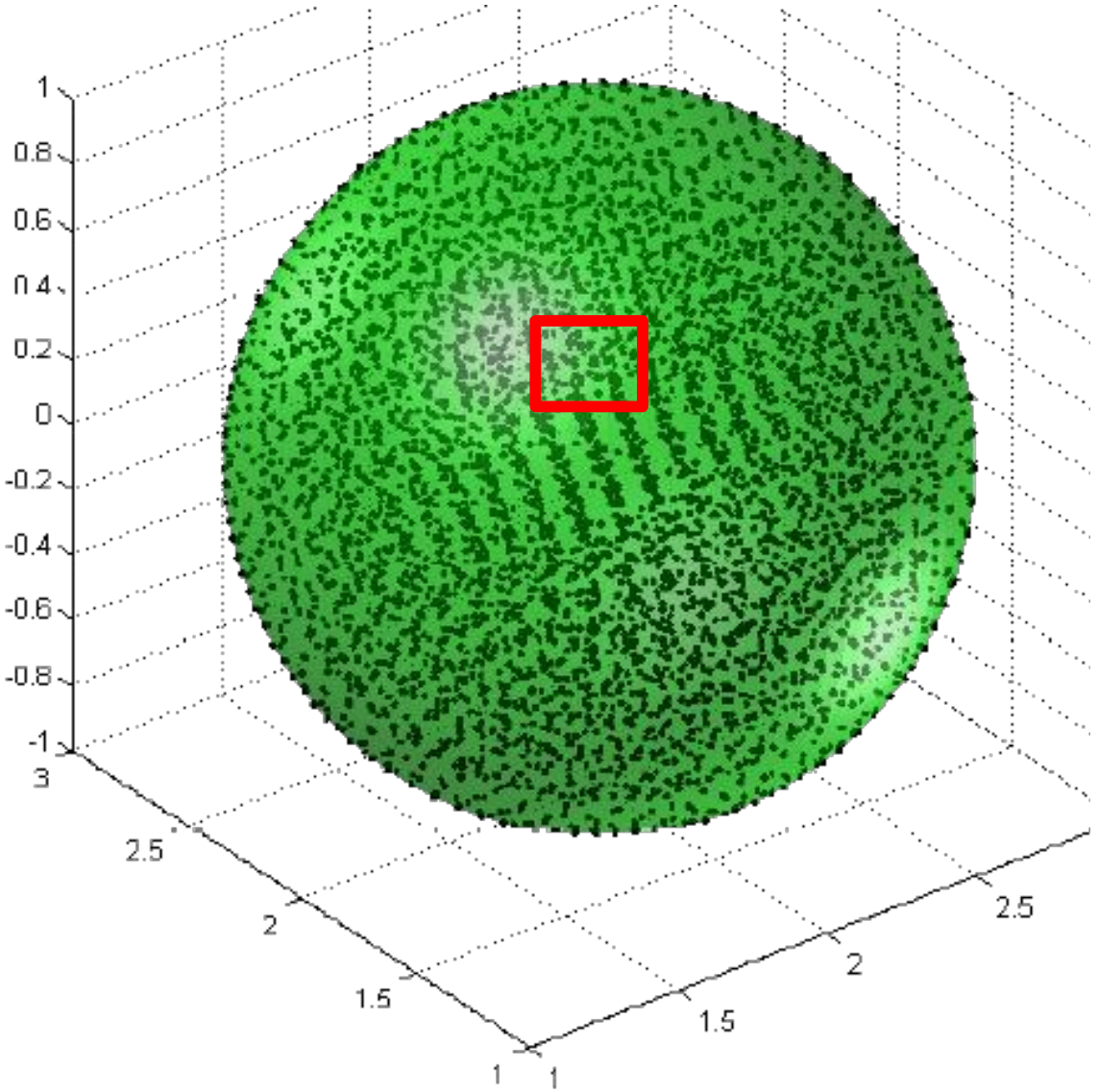
ISOMAP

$$\min_{y} \sum_{i,j} \left(d_{\mathcal{M}}(x_i, x_j) - \|y_i - y_j\| \right)^2$$

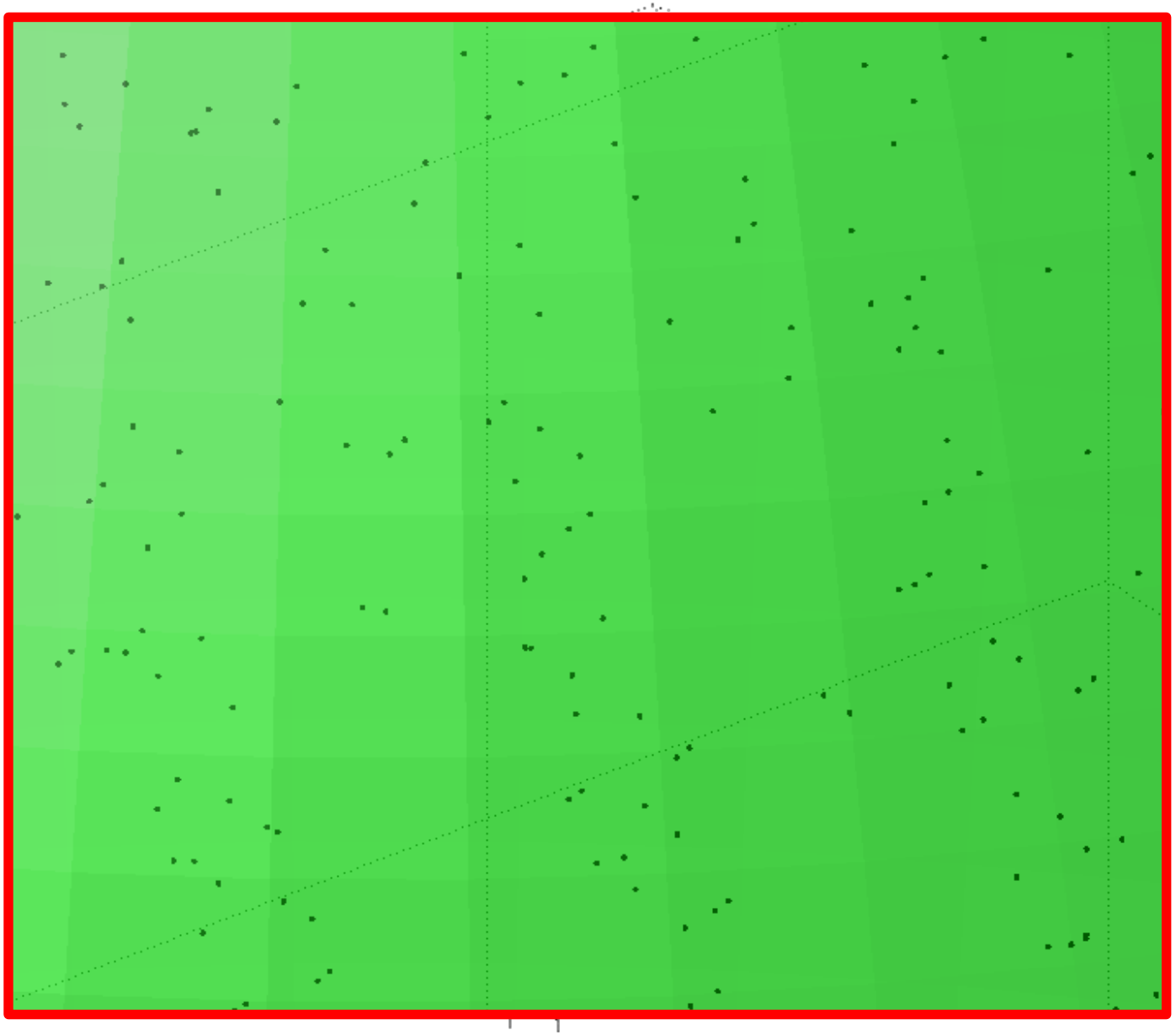




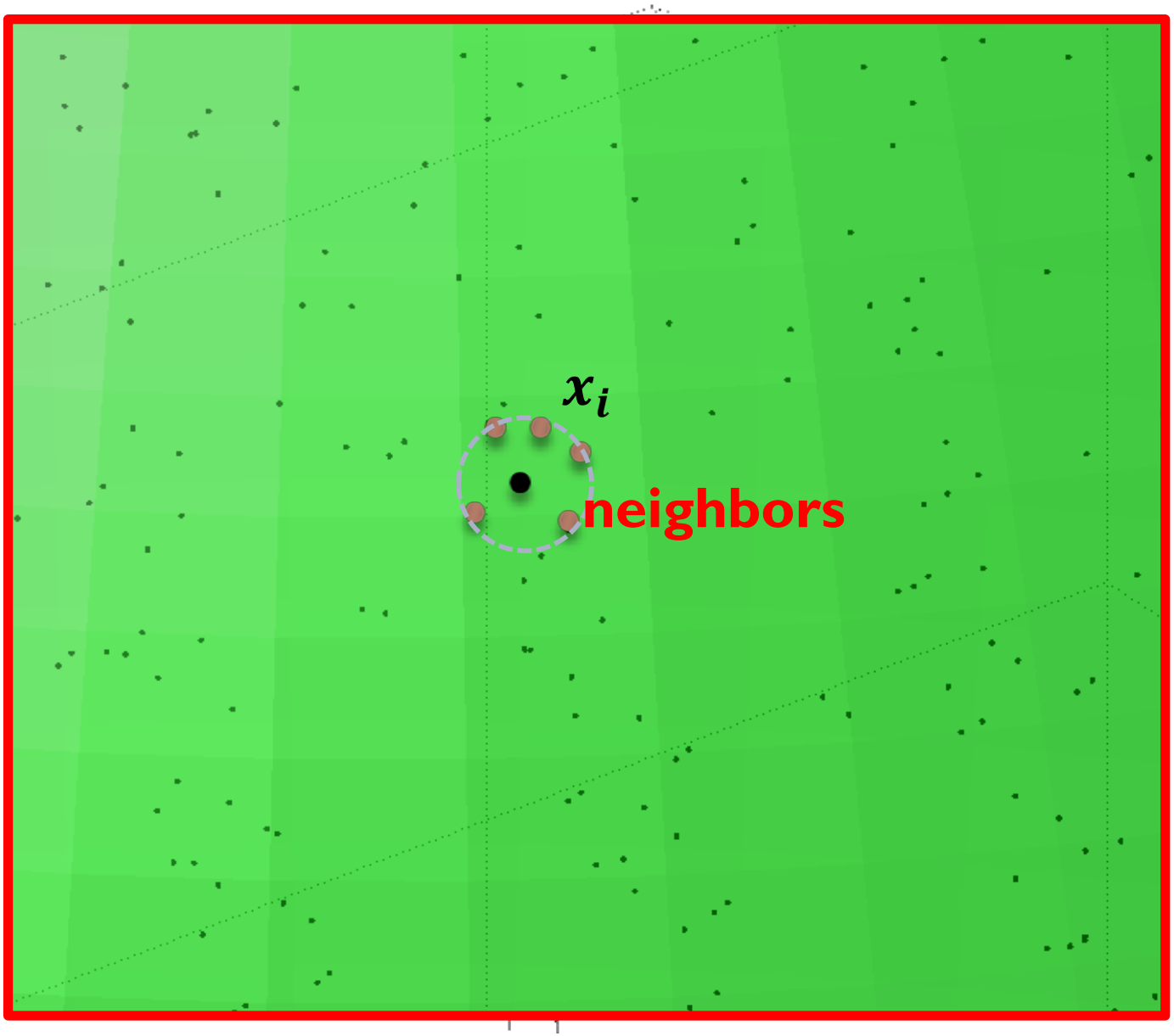
LLE



LLE

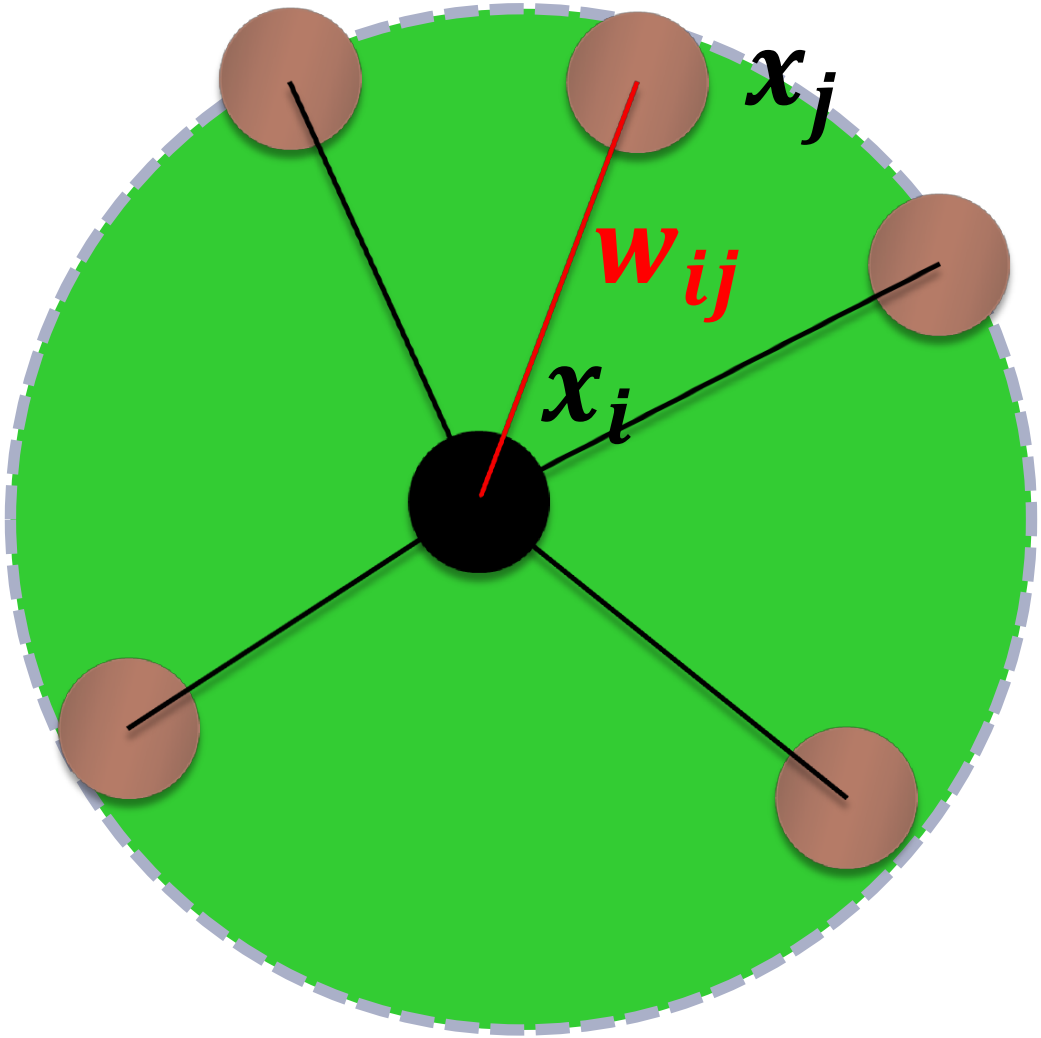


LLE



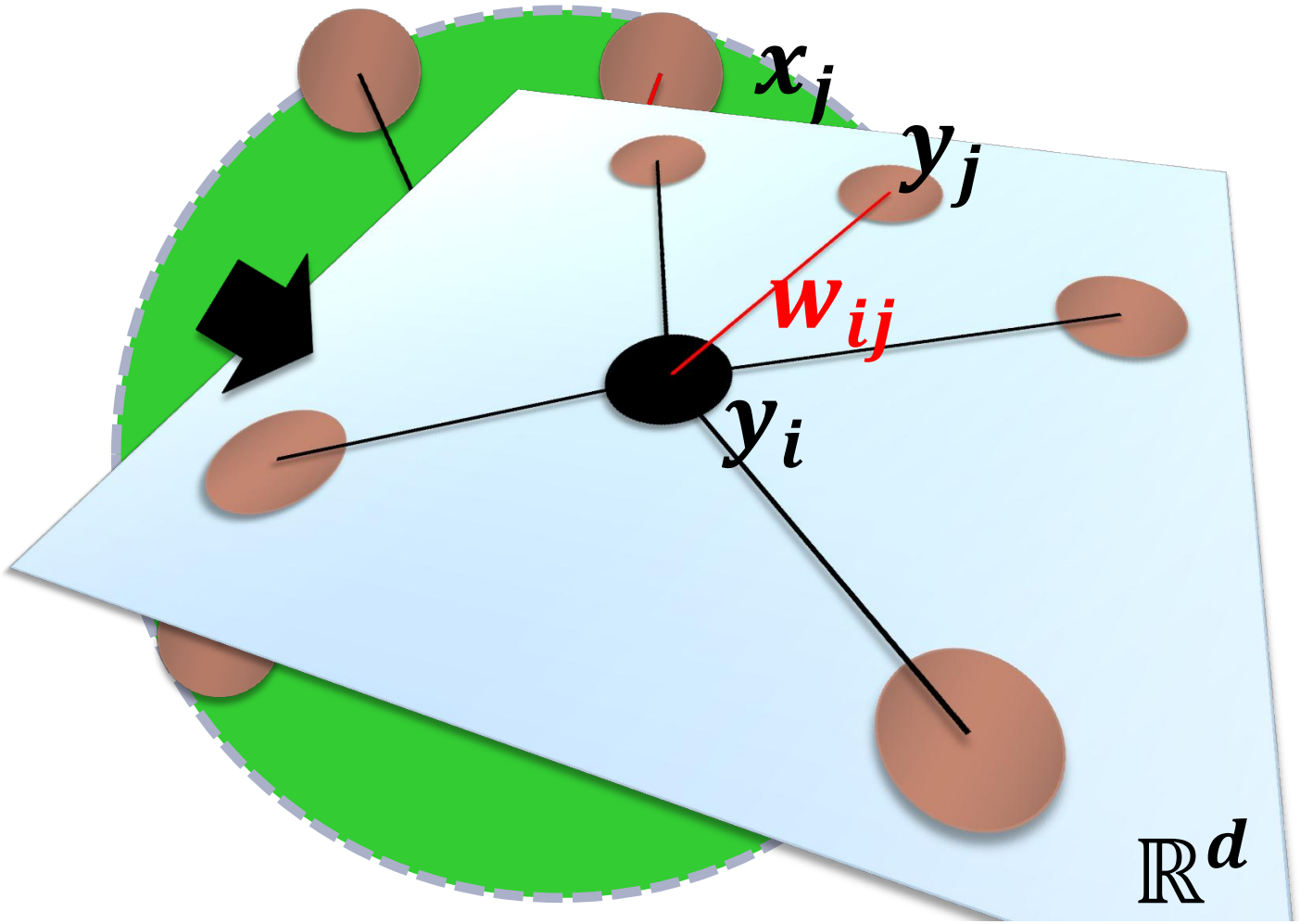
LLE

$$x_1 = w_{12}x_2 + w_{13}x_3 + w_{14}x_4$$



LLE

$$x_1' \approx w_{12}x_2' + w_{13}x_3' + w_{14}x_4'$$



Weight Learning

$$J(w) = \sum_{i=1}^{m} ||x_i - \sum_{j \in Q(i)} w_{ij} x_j||_2^2, \quad s.t. \quad \sum_{j \in Q(i)} w_{ij} = 1$$

$$\begin{aligned} J(W) &= \sum_{i=1}^{m} ||x_i - \sum_{j \in Q(i)} w_{ij} x_j||_2^2 \\ &= \sum_{i=1}^{m} || \sum_{j \in Q(i)} w_{ij} x_i - \sum_{j \in Q(i)} w_{ij} x_j||_2^2 \\ &= \sum_{i=1}^{m} || \sum_{j \in Q(i)} w_{ij} (x_i - x_j)||_2^2 \\ &= \sum_{i=1}^{m} W_i^T (x_i - x_j)(x_i - x_j)^T W_i, \quad s.t. \quad \sum_{j \in Q(i)} w_{ij} = W_i^T 1_k = 1 \end{aligned} \tag{1}$$

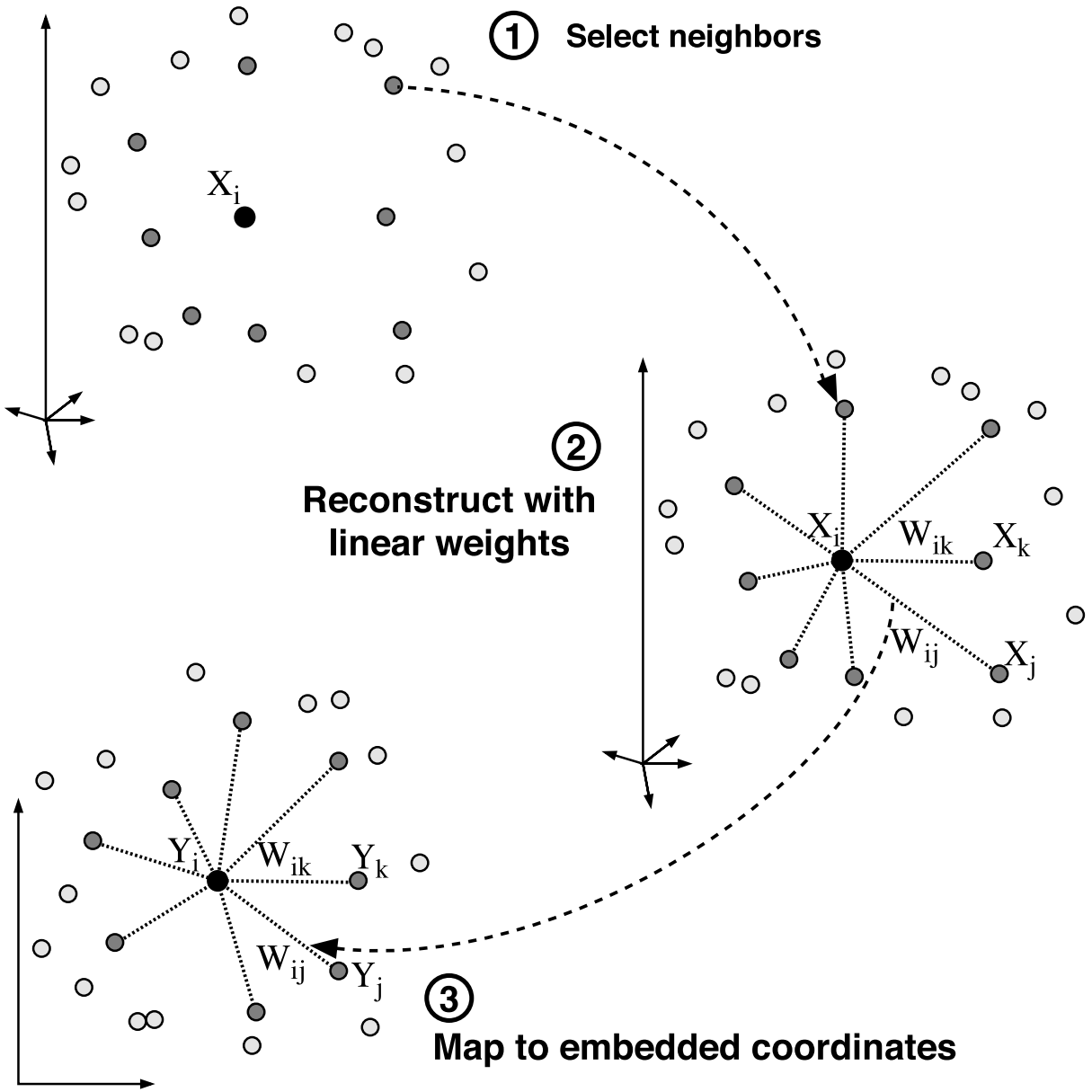
Projection

$$J(y) = \sum_{i=1}^{m} ||y_i - \sum_{j=1}^{m} w_{ij} y_j||_2^2, \quad s.t. \ \sum_{i=1}^{m} y_i = 0; \quad \frac{1}{m} \sum_{i=1}^{m} y_i y_i^T = I$$

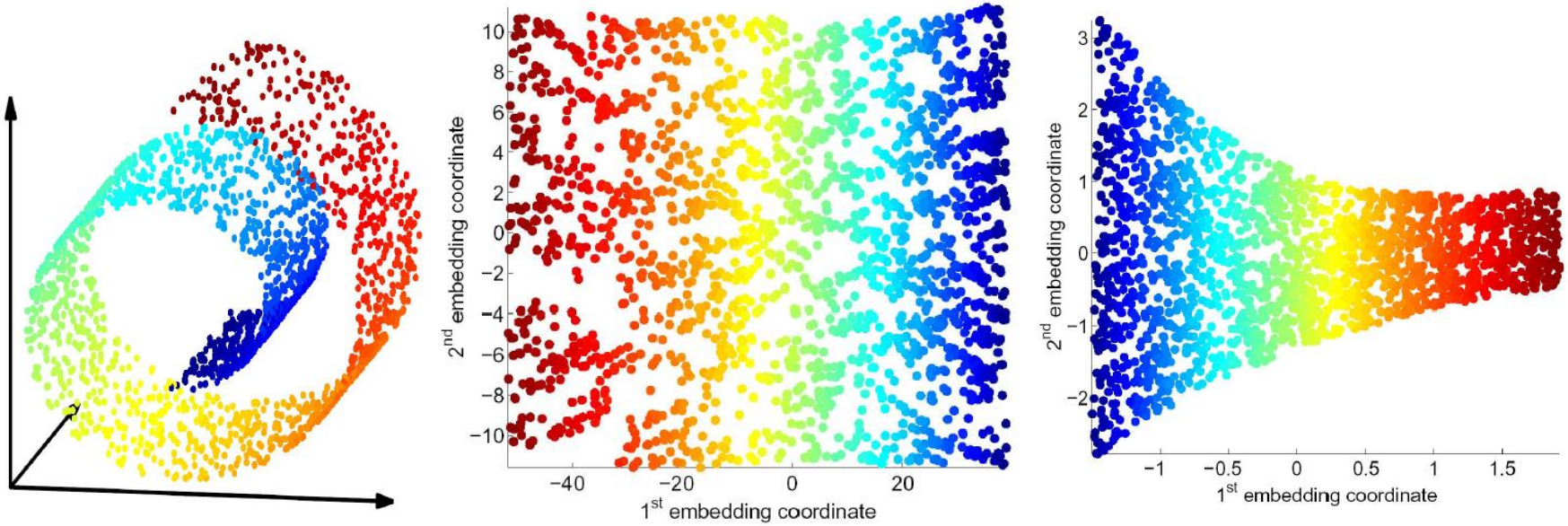
$$\begin{aligned} J(Y) &= \sum_{i=1}^{m} ||y_i - \sum_{j=1}^{m} w_{ij} y_j||_2^2 \\ &= \sum_{i=1}^{m} ||YI_i - YW_i||_2^2 \\ &= tr(Y(I-W)(I-W)^T Y^T) \end{aligned} \tag{2}$$

$$L(Y) = tr(YMY^T + \lambda(YY^T - mI))$$

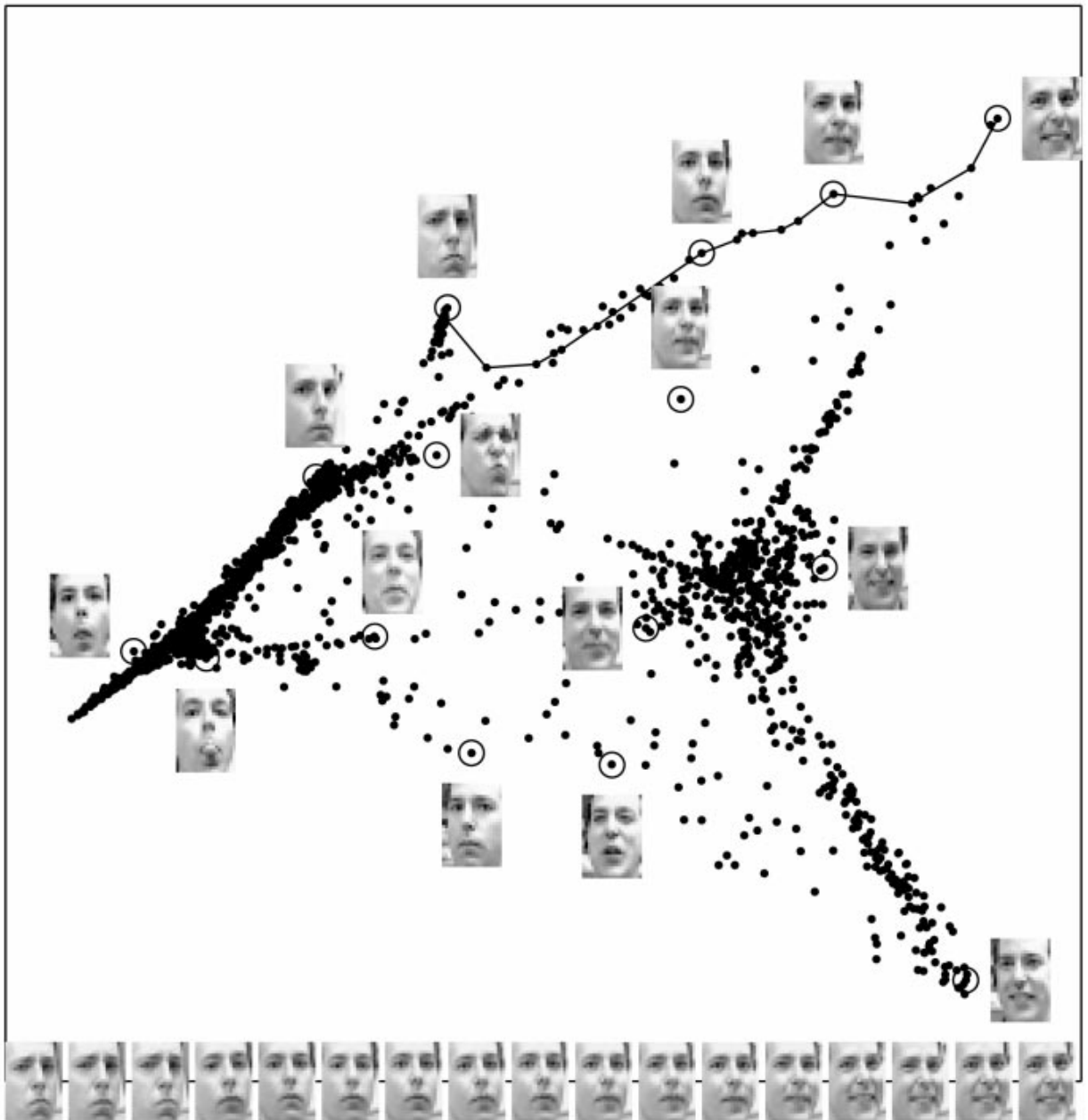
LLE Algorithm



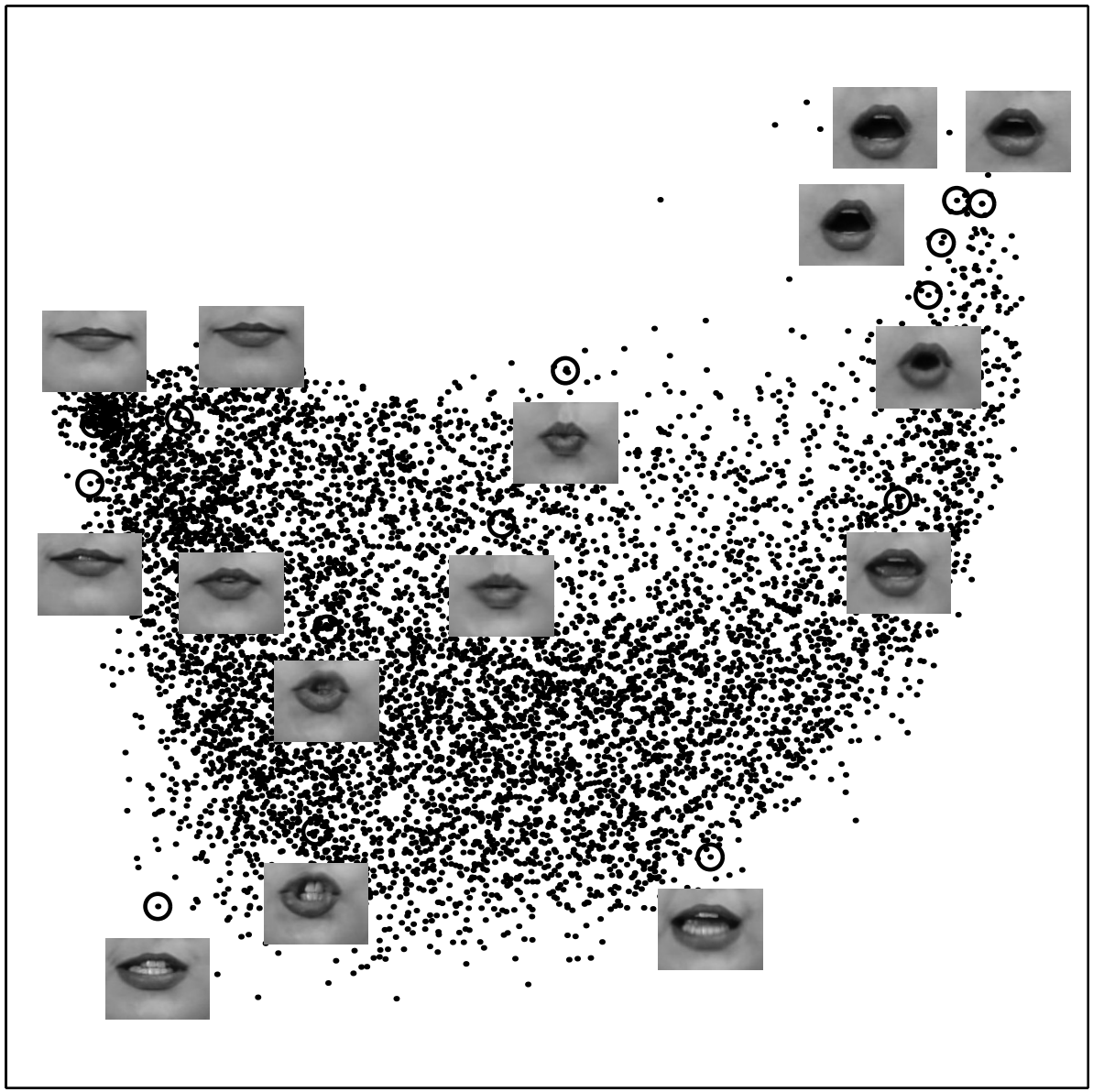
ISOMAP vs. LLE



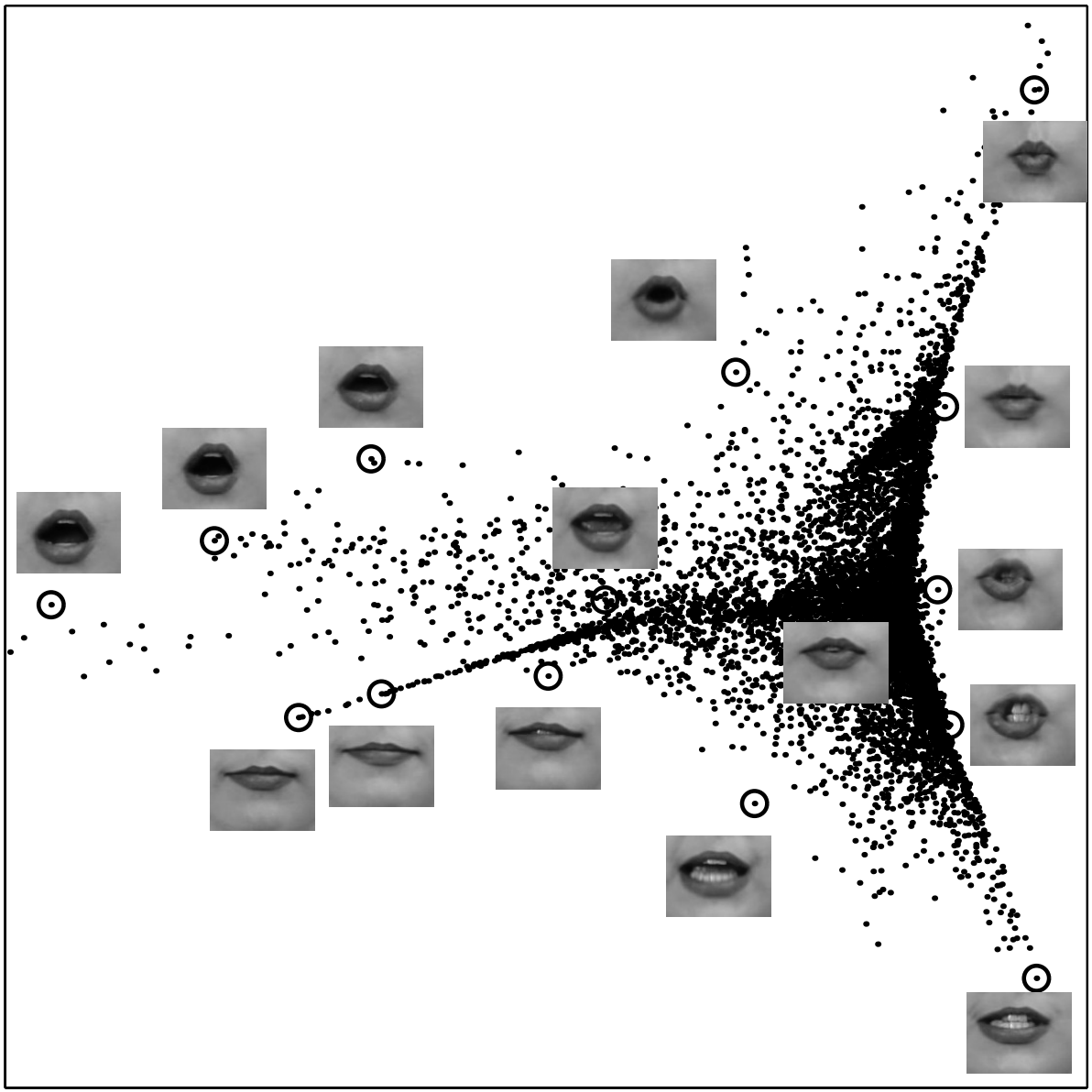
Facial Expression



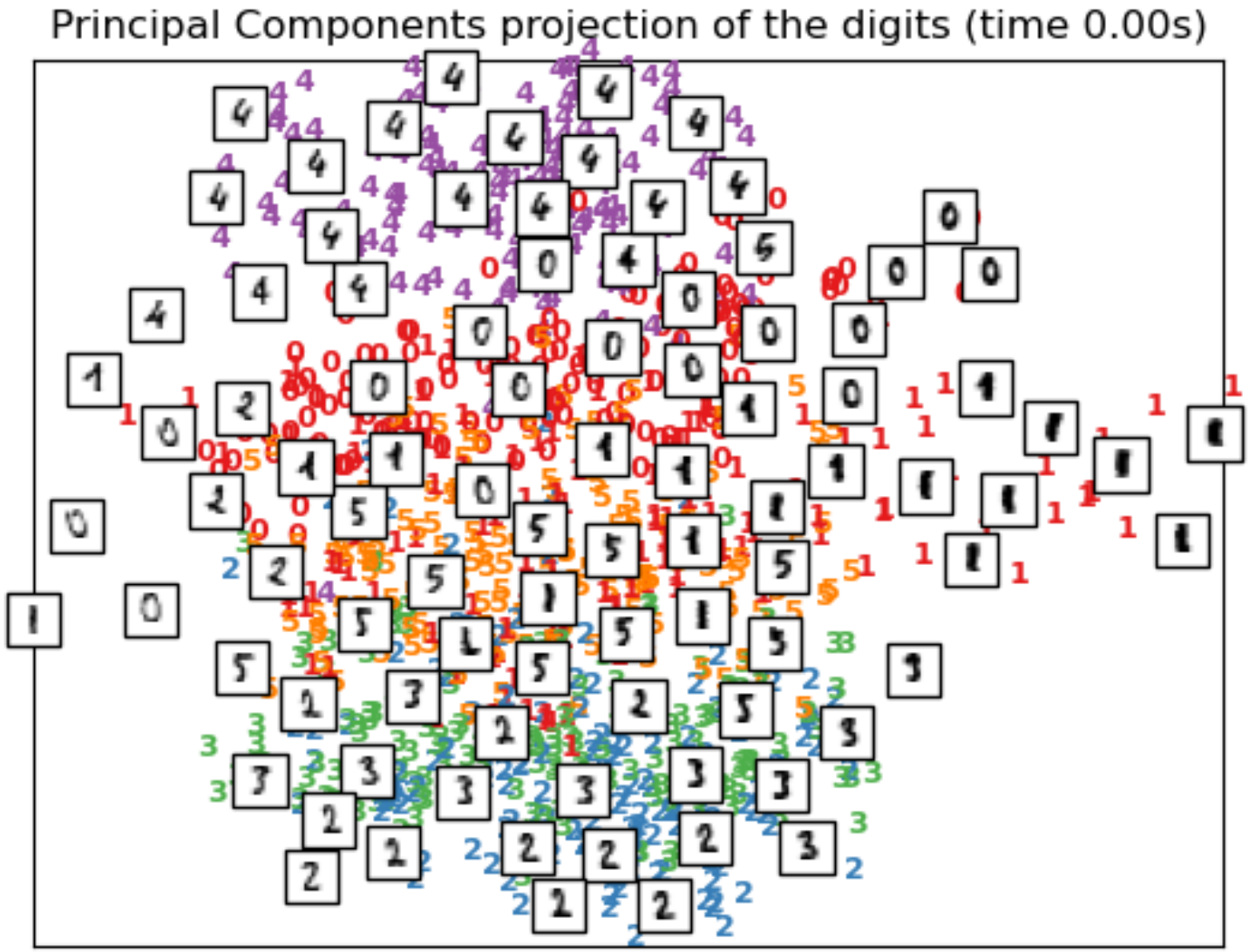
Lips



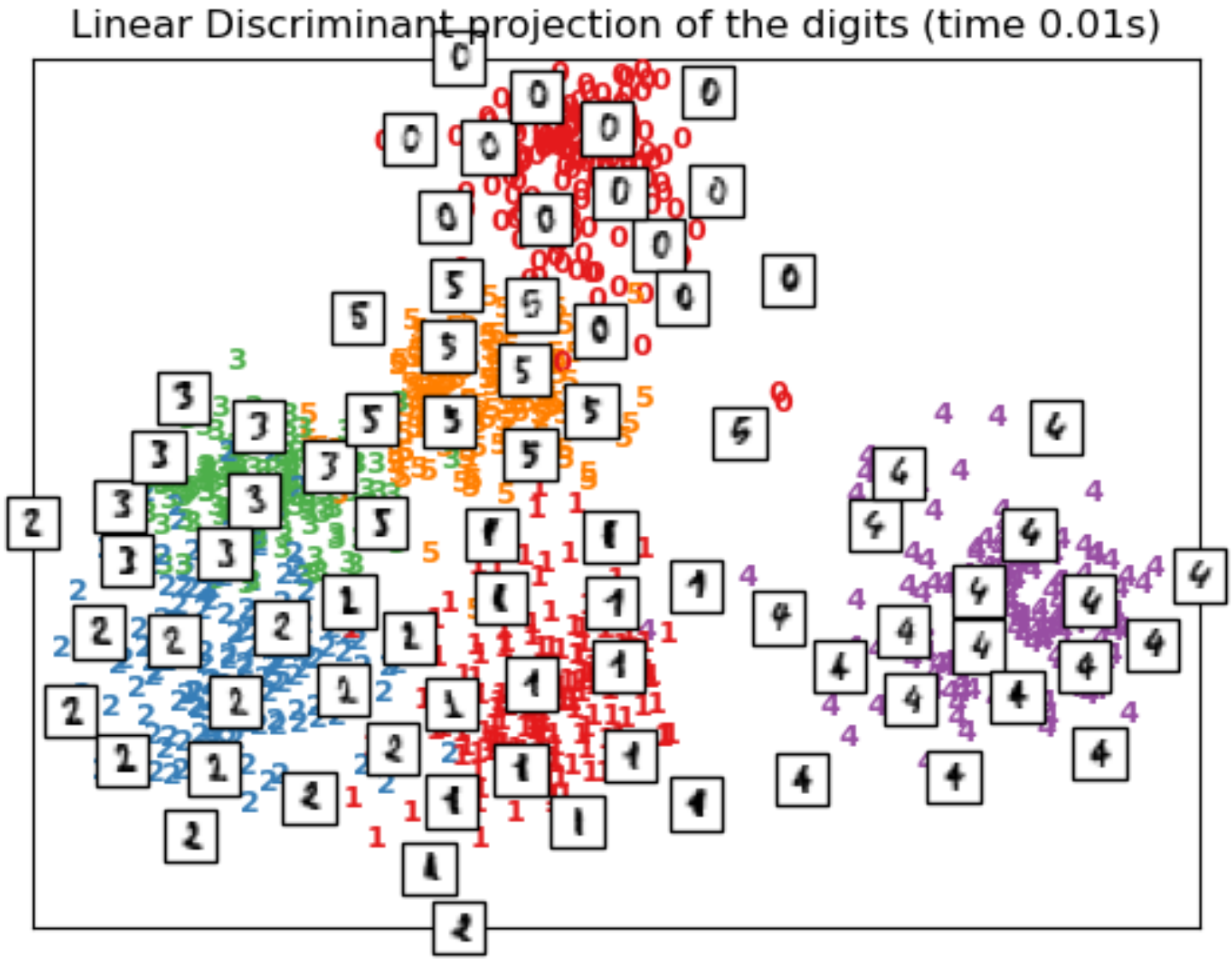
Lips



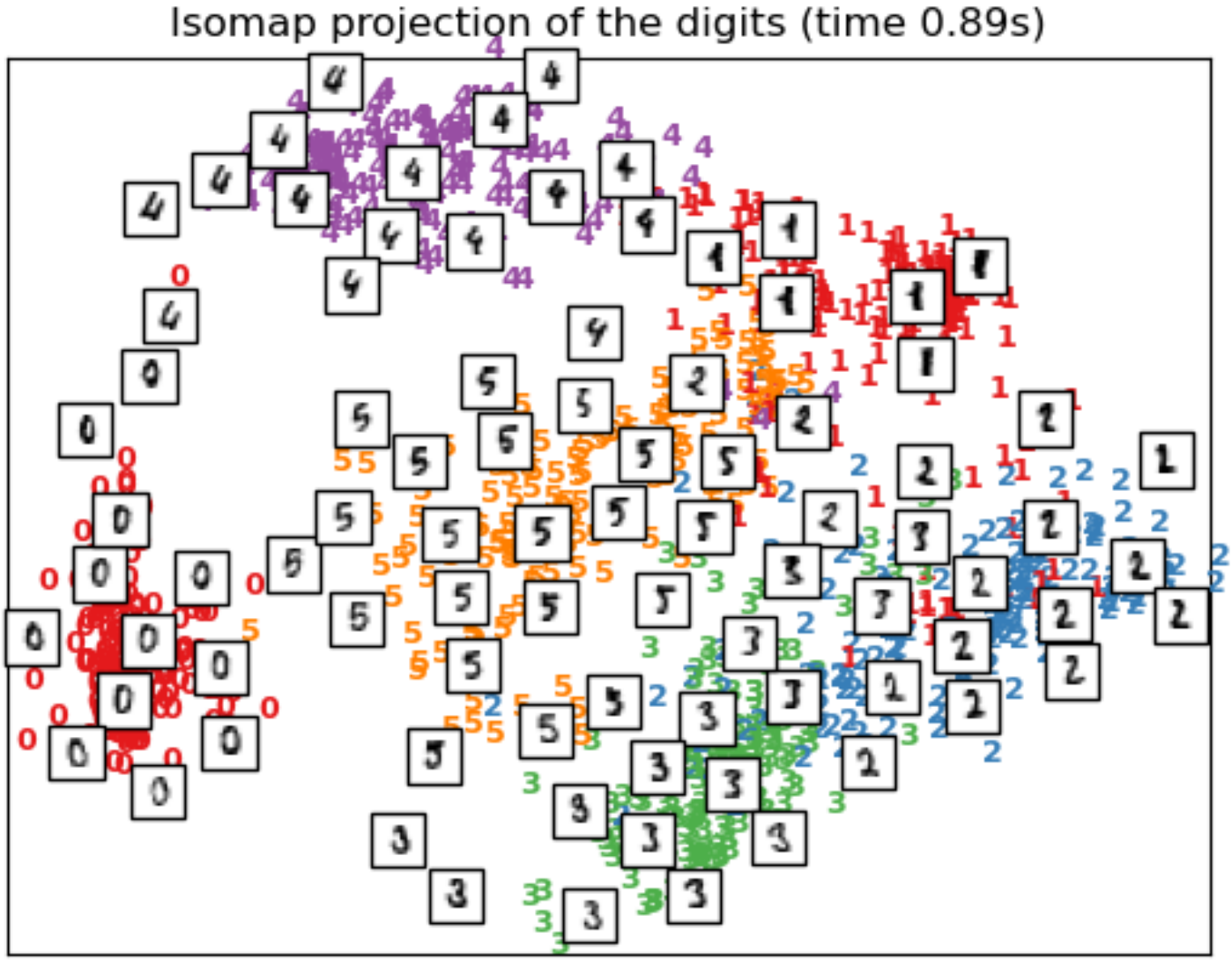
Digits



Digits



Digits



Digits