Algorithms for Pupil Detection

Semester Thesis

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Abstract

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2.1.1 Description

The data set "Labelled Pupils in the Wild" [1] or short LPW was created by the Max Plank Institution and and contains 66 high-quality, high-speed eye region videos for the development and evaluation of pupil detection algorithms. All videos are labeled with the center of the pupil. 22 participant's eye region with five different ethnicities, five different eye colors were recorded.

The goal of the data set was to record samples of participants under conditions that are present in the reality. By having strong reflections, wearing glasses, wearing make up and so on the data set becomes a difficult challenge for pupil detection algorithms and a good evaluation of the algorithms is possible.

Figure 2.1: three example frames from the LPW data set.

2.1.2 Procedure

The participants were asked to look at a moving red ball as it moved around. The recording location was randomly picked and was in and around several buildings. Each location was chosen once, creating a diverse range of real life situations.

For the recording a high-speed Pupil Pro head-mounted eye tracker was used that took 95 frames per second with a resolution of 640x480 pixels. With this frame rate even fast eye movements last through several frames making it more robust to detect the pupil.

Location	Number of Videos
Outside	34.3%
Inside	65.7%

Table 2.1: Location of recordings

Light source	Percentage of recordings
Natural light	84.7%
Artificial light	33.6%

Table 2.2: Light Source of recordings

2.1.3 Ground truth annotation

In many cases the pupil area has a clear boundary the center can be annotated easily. One or two points inside the pupil were manualy selected and used as seed points. From these point area with the same intensity value are extracted and used for annotation. But in some difficult scenarios this method was not possibl because of strong noise over the pupil. In this case the additional information gained from the participants following a red fall with their eyes was used to create the labels. The frames then were manually annotated and this data was then used as calibration data to cross reference the center of the pupil with the position of the red ball.

The complete data set has labels for the center of the pupil for every frame. Given the ground truth annotation, it is possible to evaluate algorithms and compare them to each other.

2.1.4 Folder structure

The folder stands for the part, file number stands the participants and the file extension for the file type. The text file contains the ground truth annotation of the pupil center in x and y coordinates for each frame.

The labels.ods file contains the information about the location, light source, vision aid used, prescription, nationality, eye color and gender. The README.txt file contains the information about the data set and the folder structure.

The folder structure is shown in the following tree diagram.

```
LPW/
├── labels.ods
└── README.txt
```

2.2 Eye characteristics

2.2.1 Anatomy of the eye

The eye is surrounded by the **eye lid**. Its purpose is to shield the eye from debris and lubricate the eye by spreading tears over its surface with each blink. Then there are four different parts of the eye that matters for pupil detection. The white part of the eye is the **sclera** and is separated from the **iris** by the **limbus**.

Name	Radius	Additional Information
Iris	12 mm	in average
Pupil	2-9 mm	varies with light intensity

Table 2.3: Oversight iris and pupil radius

The iris regulates the radius size of the **pupil** is and thereby regulates how much light comes through the pupil. The pupil is in the center of the iris and is the black part of the eye. The Darker the environment is, the larger the pupil radius. The Iris is colored and can be blue, green/hazle, brown, gray, amber and black. There are also other colors but they are in connection with health issues or genetic defects and play no role in this thesis.

The most important fact for pupil detection is that the iris, pupil and sclera have different brightness values. The pupil is the darkest area in the eye, the sclera is the brightest and the iris is in between. Depending on the color of the iris, the brightness value can vary. Those characteristics are often used in pupil detection algorithms and will be discussed in the next chapter. There can also be reflections of the environment be seen in the eye. This reflection

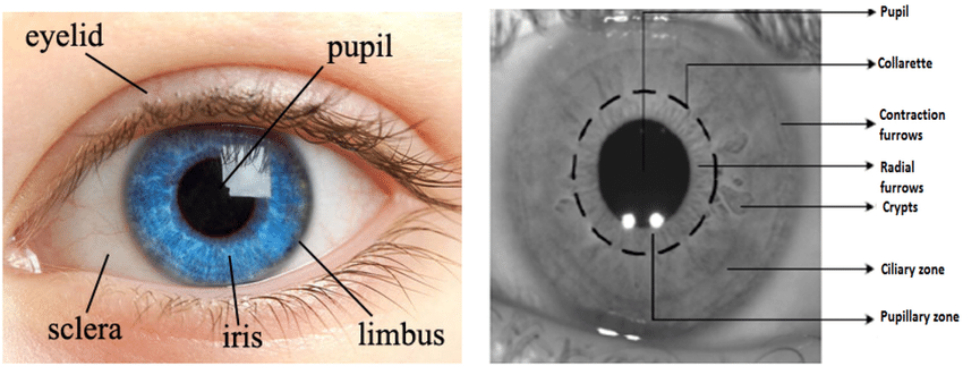


Figure 2.2: Overview of the different parts of the eye.

is called **purkinje reflection** and can temper with pupil detection algorithms accuracy and therefore is a challenge for pupil detection algorithms.

2.2.2 Image characteristics

Histogram

When inspecting the histogram of a randomly chosen frame from the LPW data set, it can be seen that the histogram shows different peaks in different intensities. In this particular example it can be seen that the peak between 0 and 50 corresponds to the pupil it self. This can be be seen in figure 2.3 because when comparing both histograms the lowest peak stays unchanged. The intensity of the iris is therefore between 90 and 150.

The sclera is the brightest part of the eye but in this frame the skin intensity will be around the same magnitude as the sclera intensity. Subtracting the ROI from the original picture will result in a histogram almost only containing the skin and sclera. The histogram is therefore a strong tool for eye segmentation.

If there is more reflection on the pupil, the histogram will also change in its shape. The peak created by the pupil will shrink and flatten out. The mean of the histogram will increase as dark points are substituted by brighter points. This ultimately makes it hard for using a fixed threshold for segmentation. Using Histogram equalization increases the contrast of the image but stretches the peak into a wider intensity range.

Also interesting to inspect is one pixel row of the image with their intensity values. Here it can be observed that the pupil creates a valley in the intensity values. The reflection on the pupil generates a peak right after the valley. This can be seen in figure 2.4.

The x coordinate represents the pixel at this coordination and the y coordinate

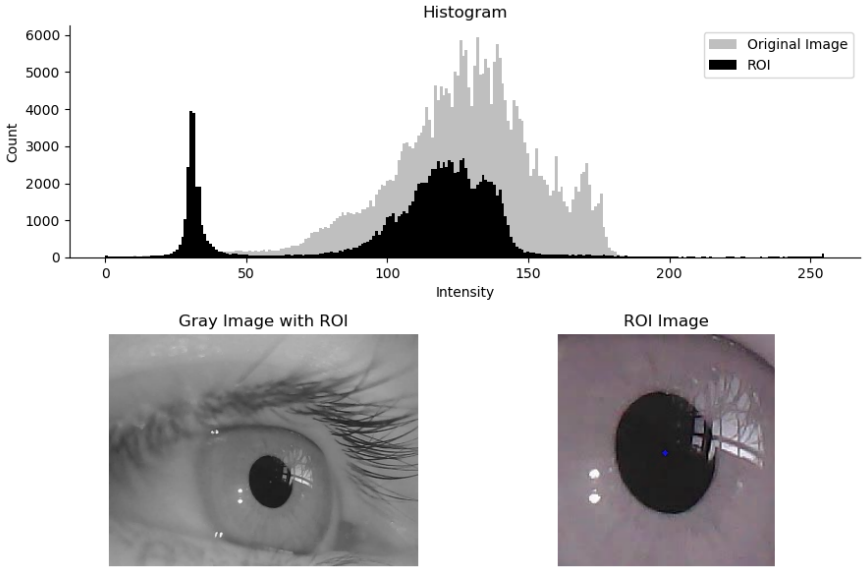


Figure 2.3: Comparison histogram of the whole image and the region of interest.

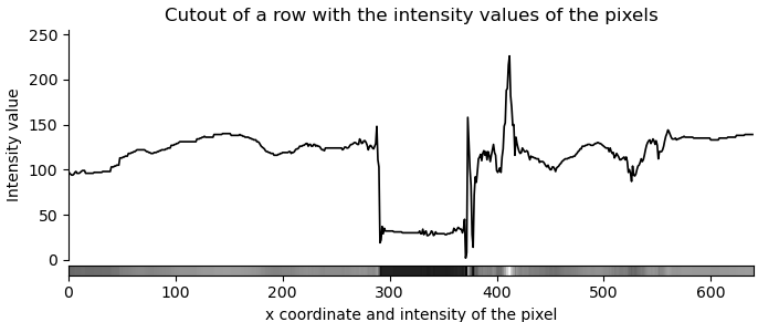


Figure 2.4: Plot of the intensity values of one pixel row.

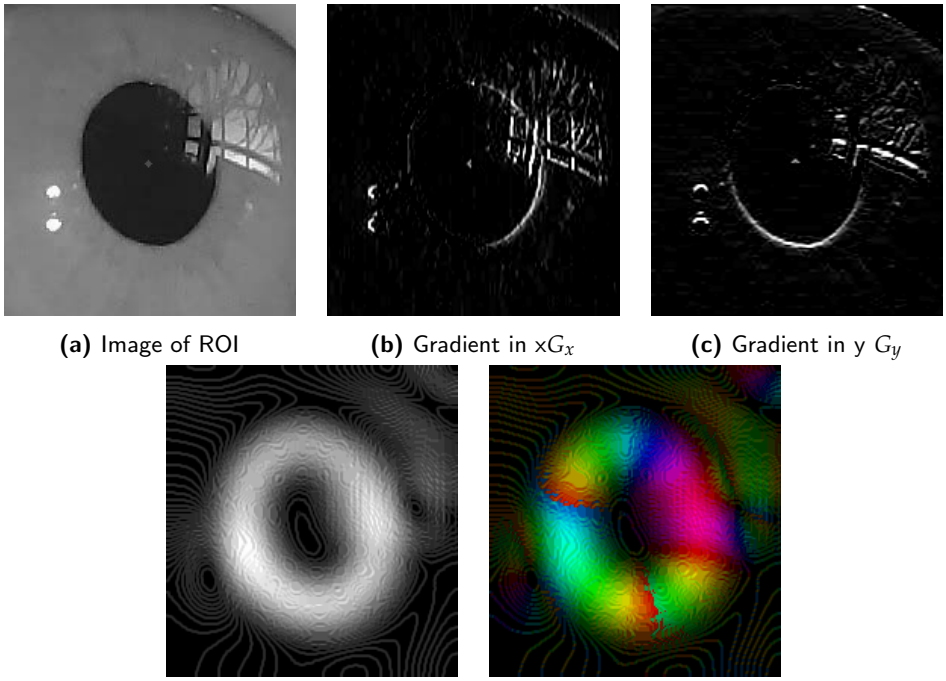
represents the intensity value of the pixel.

The conclusion is, that the histogram is a strong tool to gain insight information of the frame but can vary a lot depending on the environment, therefore adaptive algorithms have to be used.

Gradients

As already shown in the previous subsection, the pupil creates a valley in the intensity values. This is also reflected in the gradient of the image. The gradient responds strongly to the change of intensity in the transition from the pupil to the iris. The orientation of the gradient points from the pupil towards the iris and can be of great help for edge detection. The gradient is calculated by using the Sobel operator. The Sobel operator is a discrete

differentiation operator. It convolves the image with a differential kernel and therefore emphasizes regions with high spatial frequency. The Sobel operator will be discussed in a later chapter in more detail. But for know it's important to get an overview of the different characteristics of the eye.



(a) Image of ROI **(b)** Gradient in xG_x **(c)** Gradient in y G_y

(d) Magnitude G **(e)** Orientation θ

Figure 2.5: Plot of gradient characteristics of the ROI.

The gradient in x direction is shown in figure 2.5b and the gradient in y direction is shown in figure 2.5c. The magnitude of the gradient is shown in figure 2.5d and the orientation of the gradient is shown in figure 2.5e. Depending on the environment, the gradient itself could already sufficient for pupil detection. In image processing the gradient is often used for edge detection and all sorts of different filters. Used correctly it can be very powerful. The magnitude is calulated by the following formula:

$$G = \sqrt{G_x^2 + G_y^2} \tag{2.1}$$

Because we have the gradient in x and y direction, we can calculate the orientation of the magnitude by using the following formula:

$$\theta = \arctan \frac{G_y}{G_x} \tag{2.2}$$

The orientation is calculated in radians and can be converted to degrees by multiplying it with $\frac{180}{\pi}$. The orientation is perpenticular to the edge and points in the direction of the gradient. Because the pupil is darker than the iris, the gradient points from the pupil to the iris.

Noise

In the data set there are different kind of noise. The most common noise is the noise created by the camera. This noise is called **Gaussian noise**. Gaussian noise is a statistical noise having a probability density function equal to that of the normal distribution, which is also known as the Gaussian distribution. The noise is created by the camera and is therefore not dependent on the environment. The noise is therefore not a problem for the pupil detection algorithm.

Considering that the goal is to detect the pupil there can also be other noise that can be problematic. **Reflection** of the environment can have an impact on the accuracy of the algorithm and destroy information about the pupil. Also the **eyelashes** can be problematic. They are very dark and therefore temper with the edges of the pupil and iris. Especially when the eyelashes are in front of the pupil. Also if make up is used.

Another Noise term can be that the eye is not constantly open. The **eyelid** can cover parts of the pupil or even the whole pupil. Then also glasses or lenses can add noise to the image.

Chapter 3

Theory

3.1 Algorithms

This chapter will take a look at commonly used algorithms in image processing for edge detection, identifying areas of interest and applying them onto pupil detection. At first the algorithms will be discussed and analyzed on possible use cases, individual strengths and weaknesses. To show the nature of the algorithm, the same preprocessed images from the LPW data set are used and therefore it is possible to showcase the results and compare them. The general approach for pupil detection, even tough different algorithms are use, can be summarized by finding the region of interest (ROI), then find pupil edges and finally extract the pupil as ellipse. Depending on the algorithm, the steps can sometimes be extended or even combined. This chapter will therefore also discuss the possible combinations of algorithms.

3.1.1 Fundamental notation

Thru out this thesis the following notation is used to descripe the algorithms. The image with intensity level I is a function

$f(x,y) : \mathbb{N}^2 \to \mathbb{N}$, where $f(x,y)$ is the intensity $I \in [0,255]$ at position (x,y)

In image processing there the coordinate system is defined different than in mathematics. The origin is in the upper left corner and the x-axis is pointing to down vertically. The y axis is pointing to the right horizontally. this is shown in figure 3.1.

Also important to note it that the image is a discrete function, therefore each intensity value I comes with an quantization error. This is also the case when using an algorithm on the intensity values of the image. So it is not possible to have an exact result, it is always an approximation of the real result.

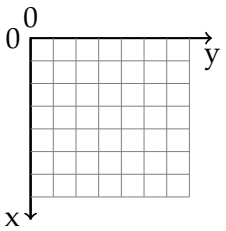


Figure 3.1: Coordinate system used in image processing.

Relationship between pixels

One also important theory in this thesis will be based on relationship between pixels. In this subsection the terms **neighborhood**, **adjacency**, **connectivity**, **region** and **boundaries** will be introduced and visualized, so that they can be used in the following chapters.

Neighborhood A pixel P at location (x, y) has two vertical neighbor pixels and two horizontal neighbor pixels in a 2D image. These neighbors are defined as $N_4(P)$ with coordinates:

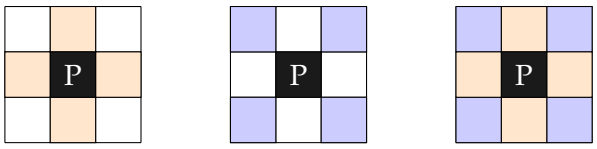
$$N_4(P) = \{(x, y+1), (x, y-1), (x+1, y), (x-1, y)\} \tag{3.1}$$

A pixel P at location (x, y) has four diagonal neighbor pixels in a 2D image. These neighbors are defined as $N_D(P)$ with coordinates:

$$N_D(P) = \{(x+1, y+1), (x+1, y-1), (x-1, y+1), (x-1, y-1)\} \tag{3.2}$$

Adding the neighbors from $N_4(P)$ and $N_D(P)$ results in the 8-neighborhood $N_8(P)$ of pixel P with coordinates:

$$N_8(P) = N_4(P) \cup N_D(P) \tag{3.3}$$



(a) $N_4(P)$ **(b)** $N_D(P)$ **(c)** $N_8(P)$

Figure 3.2: 3 different neighborhoods of pixel P at location (x, y).

Adjacency Mainly there are three types of adjacent pixels in a 2D image. Let V be the set of intensity values used to define adjacency. Depending on the intensity range of the image, it's possible to define different subsets

of V, containing the intensity values that are considered as adjacent in the neighborhood. In a binary image V is often defined as $V = \{1\}$, where 0 stands for background and 1 for foreground (This can also be considered a binary mask). In a grayscale image V can be defined as any subset of the intensity range. In this thesis the intensity range is $V = \{0, 1, 2, ..., 255\}$.

To keep it simple, the following explanations will be based on a binary image with $V = \{1\}$. Let define P as a pixel at location (x, y) and Q as a pixel at location (x', y'). P and Q are considered adjacent if Q is in the neighborhood of P and $f(Q)$ is in V. These are the three adjacency types:

- 4-adjacency, if $Q \in N_4(P) and f(Q) \in V$: The pixels that are directly above, below, left and right of the pixel.
- 8-adjacency, if $Q \in N_8(P) and f(Q) \in V$: The pixels that are directly above, below, left, right and the pixels that are diagonally adjacent to the pixel.
- m-adjacency $\begin{cases} \text{if } Q \in N_4(P) \text{ and } f(Q) \in V \\ Q \in N_D(P) \text{ and } N_D(P) \cap N_4(Q) \text{ has no intensities} \in V \end{cases}$

(a) 4-adjacency **(b)** $8 - adjacency$

Figure 3.3: 3 different neighborhoods of pixel P at location (x, y).

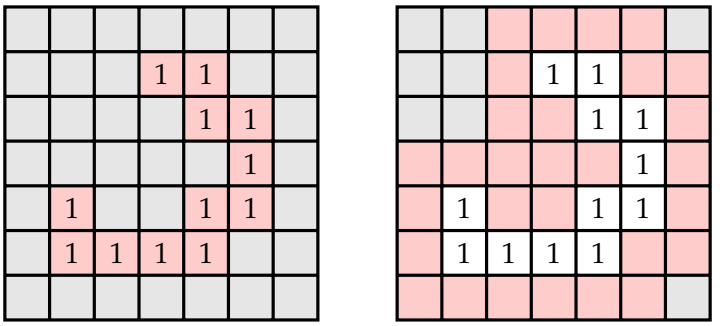
Connectivity The connectivity of point P is a set of points that can be reached in n steps with a given adjacency type and intensity set V. If point P is connected with Q then there exist a path (or curve) from P to Q that consists of a sequence of distinct pixels with coordinates

$$(x_0, y_0), (x_1, y_1), ..., (x_n, y_n), \text{ where } n \text{ is the lenght of the path.}$$

If the path is closed then $(x_0, y_0) = (x_n, y_n)$

Region Let's define R as an subset of pixels in an image. R is a region if all points $\in R$ are a connected set, meaning that all points in R are connected with each other and therefore form a region. This does not mean that the path connecting all points is closed. Two regions can be adjacent to each other, if their union again forms a connected set.

Boundary The outer boundary of a region R is the set of pixels not in R that are adjacent to pixels in R. In the definition of a boundary, the adjacency type is important. As a rule of thumb to define the boundary, the 8-adjacency is used. One important property of the outer boundary is, that it is a closed path. The inner boundary is the set of pixels that are in R but are adjacent to at least one pixel that is not in R and again the 8-adjacency is used. The inner boundary is not a closed path.



(a) region R **(b)** outer boundary (8-adjacent)

Figure 3.4: Region and outer border.

3.1.2 Preprocessing

To be able to compare the different approaches, it is important to define the used algorithms that lead to the wanted result. An image taken has to be preprocessed first. The idea behind this step is to create a common ground for narrowing the deviation of the images down, so that the algorithms are able to recreate the same result over span of different images.

Converting to grayscale

As already presented in the previous chapter, the only colorful part of the eye is the iris. But the color itself is of no interest for the detection. Therefore the frames are first converted to grayscale. This is done by converting the colors into a gray intensity value. During this chapter the grayscale images have certain properties:

It is important to note that these resolutions are resolutions are congruent with the resolutions used in the LPW paper [1]. This is important for the comparison of the results. When scaling an image there is always an image interpolation done to find the best approximation for the new intensity value I. The interpolation used in this thesis is the bilinear interpolation.[2] This is a linear interpolation in the x and y axis of the intensity values and solves equation 3.4. let $Q_{11} = (x_1, y_1)$, $Q_{12} = (x_1, y_2)$, $Q_{21} = (x_2, y_1)$ and

Scaling	Shape	numpy array type
100%	640x480	unit8
50%	320x240	unit8
25%	160x120	unit8
12.5%	80x60	unit8
6.25%	40x30	unit8

Table 3.1: Scaling of the frames used in this thesis.

$Q_{22}(x_2, y_2)$ be the four surrounding points. The intensity value I at (x, y) is then calculated by equation 3.4. The point P at (x, y) is the point of interest.

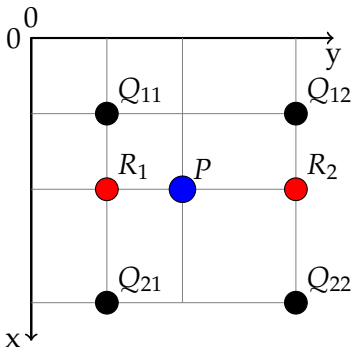


Figure 3.5: Bilinear interpolation.

$$v(x, y) = ax + by + cxy + d \tag{3.4}$$

$$f(R_1) \approx \frac{x_2 - x}{x_2 - x_1} f(Q_{11}) + \frac{x - x_1}{x_2 - x_1} f(Q_{21}) = R_1(x, y_1) \tag{3.5}$$

$$f(R_2) \approx \frac{x_2 - x}{x_2 - x_1} f(Q_{12}) + \frac{x - x_1}{x_2 - x_1} f(Q_{22}) = R_2(x, y_2) \tag{3.6}$$

$$f(P) \approx \frac{y_2 - y}{y_2 - y_1} f(R_1) + \frac{y - y_1}{y_2 - y_1} f(R_2) = v(x, y) \tag{3.7}$$

Histogram equalisation [3]

Another important aspect of preprocessing the frames is using Histogram equalization. This has the effect of increasing the contrast of the image. For this task Contras Limited Adaptive Histogram Equalization (CLAHE) is used. CLAHE is an Histogram Equalization method that has the benefit that it is adaptive to the local contrast of the image. This is very useful if the contrast of the image is not uniform. By using Histogram Equalization on the frames, noise is added to the image. This can be dealt with by using a low pass filter,

like an Gaussian filter for example.

Using a normal Histogram Equalization would lead to a loss of information in the region around the pupil and the iris. This is due to the fact that the contrast in this region is already very high. The CLAHE method splits the image into smaller blocks called ”tiles” and calculates the histogram on each block individualy and therefore does not lead to the same lose of important information at the pupil region. The CLAHE method is applied to the frames after they are converted to grayscale. The result of this step is shown in figure 3.6.

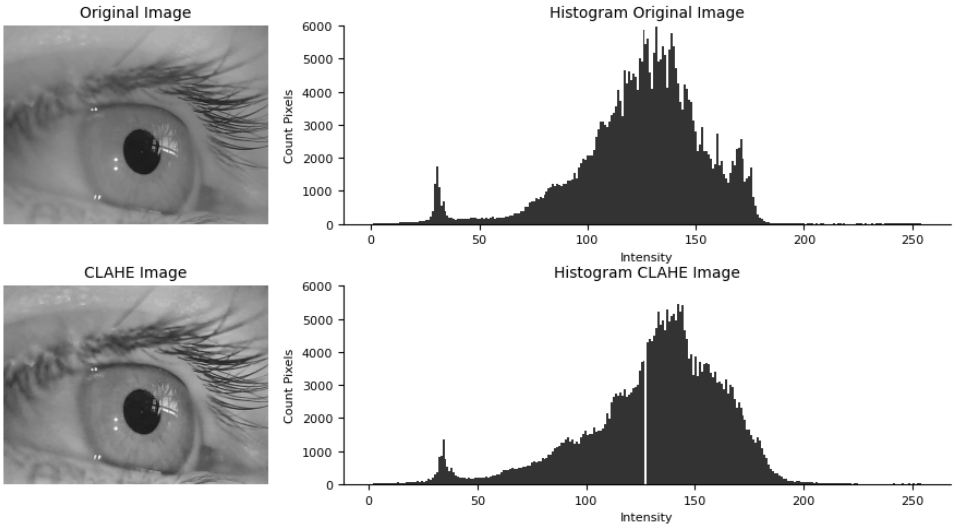


Figure 3.6: Example of CLAHE an its effect on the histogram.

These are the parameters used for the CLAHE plots

```
clahe = cv2.createCLAHE(clipLimit=1.0, tileGridSize=(11,11))
```

3.1.3 Edge Detection

The main goal behind edge detection is to find edges in the image. An edge is defined as a region that has a high contrast to its surrounding pixels, in other words a rapidly changing intensity in a small area. The edge detection is useful to filter the image for possible pupil contours. The edge detection analysis the image based on the change of intensity. Therefore a gradient calculation is used. There are different methods to calculate the gradient of an image. One of the most popular methods is the Sobel operator that makes use of the first differential of the image. The laplacian operator is another method that uses the second differential of the image. In this thesis the Sobel operator is used to calculate the gradient of the image. After calculating

the gradient Canny edge detection is used to refine the edges. Canny edge detection is a multi step algorithm that uses a hysteresis thresholding to filter the edges. The result of the edge detection is a one pixel thick binary edge map. All edges are now possible candidates for the pupil contour.

Sobel Operators [4]

The Sobel Operators are used is a common algorithm for edge detection. It is a gradient calculation that uses a 3x3 differential kernel to calculate the gradient of the image. The Sobel gradient is calculated in x and y direction. The gradient in x and y direction are calculated by convolving the image with two different kernels. The image with gray values is defined as $f(x,y)$ and the kernels are defined as k_x and k_y. Therefore the gradient is calculated as follows:

$$\nabla f = \begin{bmatrix} G_x \\ G_y \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix} \tag{3.8}$$

$$k_x = \begin{bmatrix} -1 & 0 & +1 \\ -2 & 0 & +2 \\ -1 & 0 & +1 \end{bmatrix} \tag{3.9}$$

$$k_y = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ +1 & +2 & +1 \end{bmatrix} \tag{3.10}$$

$$G_x(x,y) = f(x,y) * k_x = \sum_{s=-a}^{a} \sum_{t=-b}^{b} k_x(s,t) f(x+s, y+t) \tag{3.11}$$

$$G_y(x,y) = f(x,y) * k_y = \sum_{s=-a}^{a} \sum_{t=-b}^{b} k_y(s,t) f(x+s, y+t) \tag{3.12}$$

$G_x(x,y)$ and $G_y(x,y)$ are the gradients in x and y direction. $f(x,y)$ is the image and k_x and k_y are the kernels. The kernels convolved with the image $f(x,y)$ and the result is the gradient magnitude in x and y direction. In other words the convolution of an image with k_x or k_y gives as result the change from pixel to pixel in x or y direction. In python the Sobel in x and y are calculated with the OpenCV Library for example:

```
G_x = cv2.Sobel(img,cv2.CV_64F,1,0,ksize=3)
G_y = cv2.Sobel(img,cv2.CV_64F,0,1,ksize=3)
```

The total gradient magnitude G is calculated with this equation:

$$G = \sqrt{G_x^2 + G_y^2} \tag{3.13}$$

and the direction θ of the gradient is calculated with this equation:

$$\theta = \arctan \frac{G_y}{G_x} \tag{3.14}$$

The result of the convolution can be seen in figure 2.5 in chapter two.

Canny Edge Detection [5]

The Canny Edge Detection is used to recieve single edge points form the gradient magnitude image. This algorithm can summarized in four steps:

1. Noise reduction, smoothing the image with a Gaussian filter
2. Compute the gradient magnitude and direction
3. Non-maximum suppression to the gradient magnitude image
4. Use double thresholding and connectivity analysis to detect and link edges

Step 1: Noise reduction
The first step is to reduce noise from the input image. This is done by convolving the image with a low pass filter. For this task a Gaussian filter is used. The Gaussian filter is defined as:

$$f_{filter}(x,y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}} \tag{3.15}$$

The Gaussian filter is then convolved with the image so smooth the image.

$$f_{smoothed}(x,y) = f(x,y) * f_{filter}(x,y) \tag{3.16}$$

Step 2: Compute the gradient magnitude and direction
The gradient magnitude is calculated with the equation 3.13 and the gradient direction is calculated with the equation 3.14.

Step 3: Non-maximum suppression
The non-maximum suppression is used to thin the edges out, so that the edges are only one pixel wide. This can be achieved with an loop that goes over all edges and checks if the current pixel, belonging to the edge, is the local maximum in the direction of the $\pm$gradient vector. If that is the case the pixel is kept, otherwise it is set to zero. Because as already described in 3.1.1 the coordinate system is defined different. The gradient direction is in reference to the x axis.

Because an image is quantized, this also means that θ needs to be quantized to four directions to evaluate their neighbors.

This leads following quantizations:

$$\theta_q = \begin{cases} 90, & \text{if } 67.5^\circ < \theta \leq 112.5^\circ \vee -112.5^\circ < \theta \leq -67.5^\circ \\ -45^\circ, & \text{if } 22.5^\circ < \theta \leq 67.5^\circ \vee -157.5^\circ < \theta \leq -112.5^\circ \\ +45^\circ, & \text{if } 112.5^\circ < \theta \leq 157.5^\circ \vee -67.5^\circ < \theta \leq -22.5^\circ \\ 0^\circ, & \text{if } -22.5^\circ < \theta \leq 22.5^\circ \vee -157.5^\circ < \theta \leq 157.5^\circ \end{cases} \tag{3.17}$$

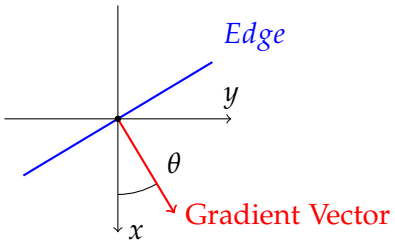


Figure 3.7: Definition of the gradient direction

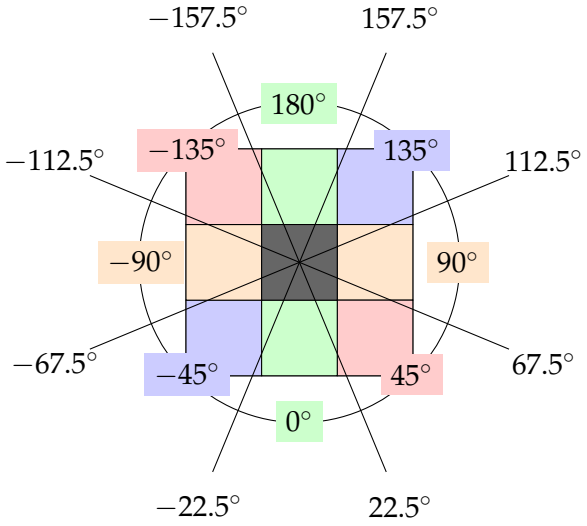
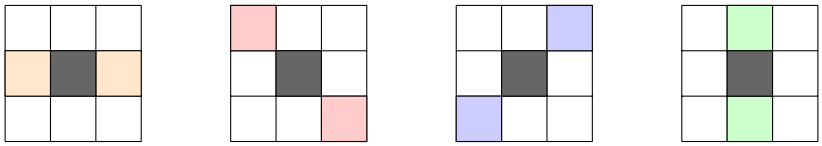


Figure 3.8: Quantization of the gradient direction

It is important to note that when following the gradient direction, the two neighboring pixels are used to evaulate the gradient magnitute maximum. This is shown in figure 3.8 and 3.9. If the gradient is maximal at the current pixel at (x, y), meaning it is a local maximum in the previous defined neighborhood in respect to the gradient direction, the value of the pixel is written into $g_n(x, y)$, otherwise it is set to zero $g_n(x, y) = 0$. This is called non-maximum suppression. Therefore $g_n(x, y)$ contaians only the thinned edges.

Step 4: Double thresholding and connectivity analysis
After Step 3, $g_n(x, y)$ still shows edges that can be thicker than one pixel. $g_n(x, y)$ is then thresholded with a high an low threshold (hysteresis thresh-



(a) Neighbors: 90° **(b)** Neighbors: 45° **(c)** Neighbors: -45° **(d)** Neighbors: 0°

Figure 3.9: The gradient magnitude is evaluated in the direction of the gradient.

olding) creating two images:

$$g_{low}(x,y) = \begin{cases} g_n(x,y), & \text{if } g_n(x,y) \geq T_{low} \\ 0, & \text{otherwise} \end{cases} \tag{3.18}$$

$$g_{high}(x,y) = \begin{cases} g_n(x,y), & \text{if } g_n(x,y) \geq T_{high} \\ 0, & \text{otherwise} \end{cases} \tag{3.19}$$

Because two different thresholds were used, there is still overlap between g_{low} and g_{high}. All non zero pixels in g_{high} are considered strong edge pixels. To recieve all weak edge pixels, the strong edge pixels are substracted from g_{low}. The remaining pixels are considered weak edge pixels.

$$g_{weak}(x,y) = g_{low}(x,y) - g_{high}(x,y) \tag{3.20}$$

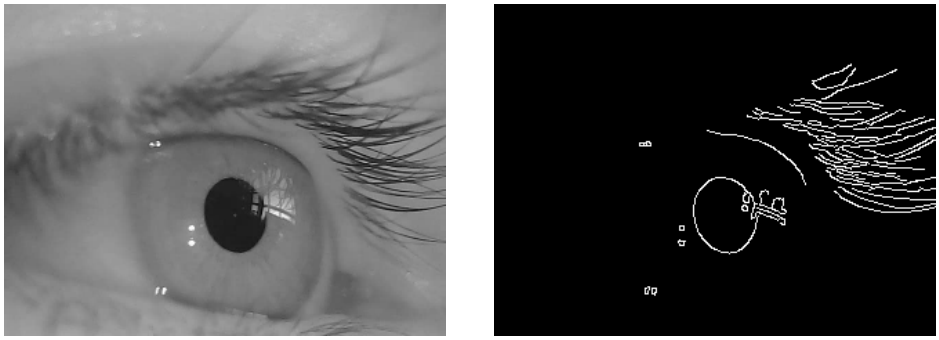
Next step is to connect the weak edge pixels to the strong edge pixels. This is done by checking the 8-neighborhood of each strong edge pixel. If there is a weak edge pixel in the neighborhood, it is considered a strong edge pixel. This is done until no more weak edge pixels are found. The result is a binary image $g_{final}(x,y)$ containing all edges. But this still does not return a one pixel thick edge.

To solve this the edges are passed on an edge-thinning algorithmus. Let's define the edges as set A and B as a structuring element. The equation for thining then becomes:

$$A \otimes B = A - (A \circledast B) \tag{3.21}$$

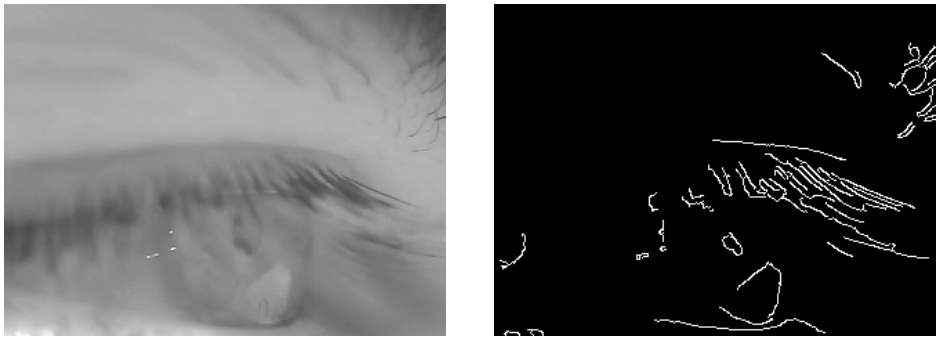
Where $\otimes$ is the thinning operator, $\circledast$ is the dilation operator.

Results In a frame where the pupil region clearly can be distinceted from the rest, the Canny edge detector can be used to find the pupil region. but as soon as more noise to the pupil is added, the hysteresis thresholding becomes more tricky and the detection accuracy decreases immensely. Also is it not possible to differentiate between the eye leashes, eye brows and the pupil. Therefore by using only the Canny edge detector. The pupil edges can not be found reliable and the algorithm itself is not adaptable to a great varriety of environments.



(a) Original image, scaling 0.5 **(b)** Clear pupil region, Canny edge detection

Figure 3.10: Canny edge detection on a clear pupil region



(a) Original image, scaling 0.5 **(b)** Weak pupil region, Canny edge detection

Figure 3.11: Canny Edge detection on a frame with weak pupil region

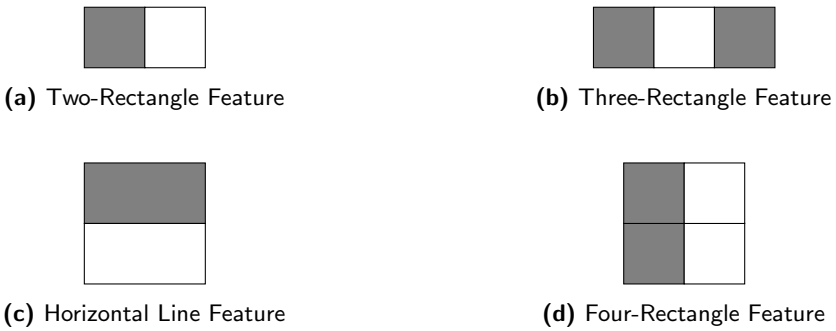
3.1.4 Haar like feature detection [6]

Concept

Haar like features is based on the observation that object have a certain local intensity variation in an image and this fact can be useful to identify objects. Haar like features makes use of this and divides the image into rectangular regions and then calculates the difference between the sum of the pixels in the white and gray regions, show in figure 3.12. This is done for all possible rectangular positions in the image. The result is a feature vector that is used to classify the image and give insight about the position of the object. In theory the feature is convolved with the image to calculate the feature vector but to make the detection faster, the integral image is calculated once and then used to calculate the feature vector at each possible position in the image. This leads to an easy calculation of the feature vector than now only needs four values to be calculated. The properties of the different features bring different abilities to detect specific local intensity patterns that are an indicator for a certain object or image region property.

Features characteristics

To get a better understanding of the different features and their characteristics of the resulting feature vector the four features are explained a little more in detail.



(a) Two-Rectangle Feature

(b) Three-Rectangle Feature

(c) Horizontal Line Feature

(d) Four-Rectangle Feature

Figure 3.12: Haar-like Features

The key concept to keep in mind is, that the features generate a response based on the difference of the sum of the pixels in the white and gray regions and therefore are able to detect local intensity patterns. The response of each feature also depends on the size of the feature and stands in relation to the size of the object that is to be detected. The result of the Haar-like features provides information about the presence or absence of a certain pattern that is reflected in the structure of the feature used. Because an object can have more than one fitting feature, the features are often used in a cascade of classifiers. The first classifier is a very simple classifier that is able to detect a lot of features. The following classifiers are more complex and are only used if the previous classifier was able to detect a certain feature. This leads to a very fast detection of the object. This safes computational resources and time. Often Machine learning is involved to classify if an object is present or not, based on the feature vectors.

Haar-Like Feature for pupil detection [7]

One very useful feature for finding a point in the pupil is given by a feature described in the paper . This feature is constructed differently but used the same way as the other features to calculate an feature vector or response map.

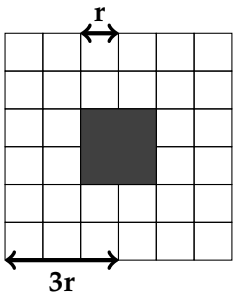


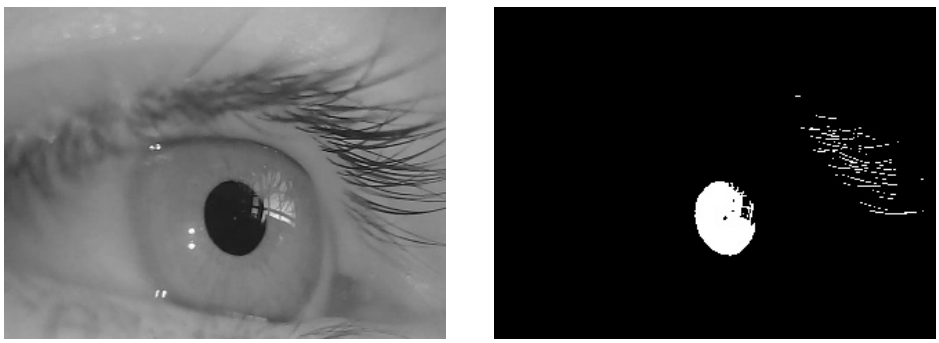
Figure 3.13: Haar-like feature for pupil detection

3.1.5 Thresholding and Ellipse fitting

Thresholding

In Thresholding the characteristic of the pupil is used, that it is the darkest part of the eye as seen in 2.3 . Here it is important to find a certain thresholding value to only extract the pupil. Considering the best scenario that the pupil is not affected by too much noise: no eyelash, no reflection. This method can be reliable to find the pupil region. This process shows to be less computation expensive and finds the pupil fast.

The most difficult part is to find a fitting threshold value. This could be done by using the histogram of the frame. When inspecting the histogram, one could consider the lowest peak in the histogram as the given center value for thresholding. But this is not always the case. There is also the possibility for adaptive thresholding for example otsu thresholding. but throughout the work with thresholding otsu was also tested but was found to be less reliable than the histogram approach itself.



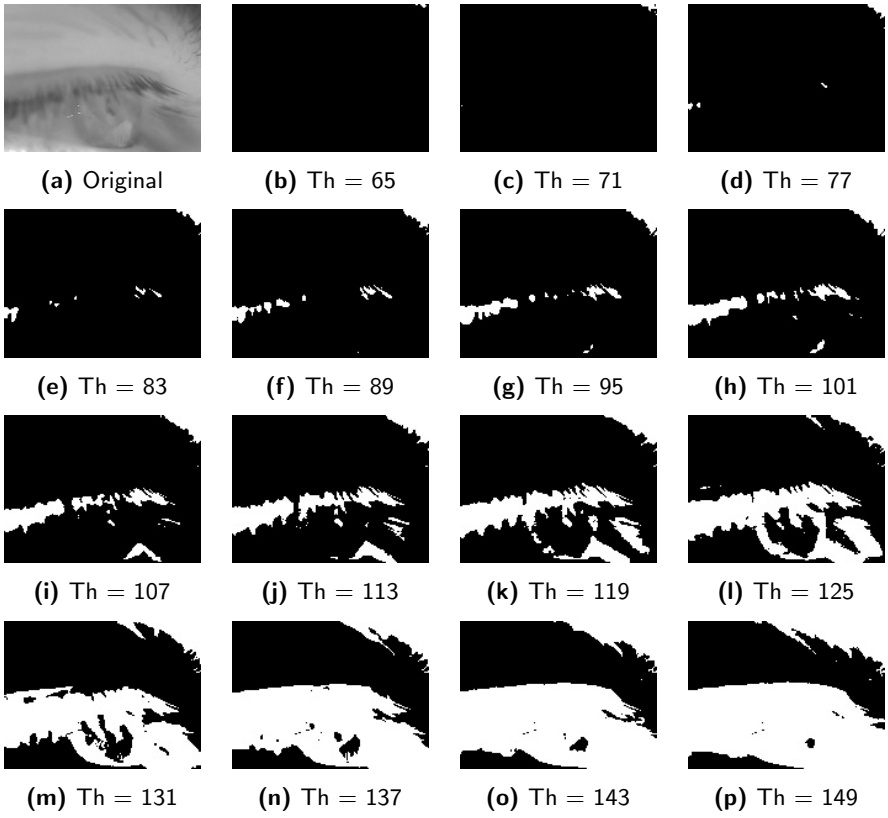
(a) Original frame **(b)** thresholded frame

Figure 3.14: Original and thresholded frame

in 3.14 it can be seen that the thresholding is extremely volatile to reflec-

tions, eyelashes and other noise. This is why thresholding only has a good performance in a strictly defined environment with almost no noise.

To showcase the volatility and the dependency on the threshold value, in figure 3.15 a series of the same frame is shown with different threshold values (Th). It can be seen that the threshold value has to be chosen very carefully. If the value is chosen to low, the pupil region is not found. If the value is chosen to high, too much information is extracted and the pupil region can not be differentiated from the rest.



(a) Original **(b)** Th = 65 **(c)** Th = 71 **(d)** Th = 77

(e) Th = 83 **(f)** Th = 89 **(g)** Th = 95 **(h)** Th = 101

(i) Th = 107 **(j)** Th = 113 **(k)** Th = 119 **(l)** Th = 125

(m) Th = 131 **(n)** Th = 137 **(o)** Th = 143 **(p)** Th = 149

Figure 3.15: Thresholded images

Ellipse fitting

Ellipse fitting is based on finding a contour and then fit an ellipse to it. The accuracy of the ellipse fit is strongly dependent on the quality of the contour extracted. The goal is to find the pupil as a connected region and than use ellipse fit to reconstruct the pupil as an ellipse. This involves solving equations with an approximation algorithm. For example least squares fitting. The least squares fitting is a method to find the best possible fit based on

a set of data points. In this case points on the contour. The least squares fitting calculates the distance from the possible ellipse curve to all points on the contour and minimizes the distance to all points by finding changing the ellipse parameters. This is done by solving the following equations:

The general equation for an ellipse with center (x_c, y_c), major axis a and minor axis b is:

$$\frac{(x - x_c)^2}{a^2} + \frac{(y - y_c)^2}{b^2} = 1 \tag{3.22}$$

To find all the values there is a need for at least 5 points on the contour of an possible ellipse.

$$J(a, b, x_c, y_c) = \sum_{i=1}^{n} \left(\frac{(x_i - x_c)^2}{a^2} + \frac{(y_i - y_c)^2}{b^2} - 1 \right)^2 \tag{3.23}$$

where n is the number of data points. The goal is to find the values of a, b, x_c, and y_c that minimize this cost function.

We can use calculus to find the values of a, b, x_c, and y_c that minimize J. The partial derivatives of J with respect to a, b, x_c, and y_c are:

$$\frac{\partial J}{\partial a} = 4 \sum_{i=1}^{n} \frac{(x_i - x_c)^2}{a^3} \left(\frac{(x_i - x_c)^2}{a^2} + \frac{(y_i - y_c)^2}{b^2} - 1 \right) \tag{3.24}$$

$$\frac{\partial J}{\partial b} = 4 \sum_{i=1}^{n} \frac{(y_i - y_c)^2}{b^3} \left(\frac{(x_i - x_c)^2}{a^2} + \frac{(y_i - y_c)^2}{b^2} - 1 \right) \tag{3.25}$$

$$\frac{\partial J}{\partial x_c} = -8 \sum_{i=1}^{n} \frac{x_i - x_c}{a^2} \left(\frac{(x_i - x_c)^2}{a^2} + \frac{(y_i - y_c)^2}{b^2} - 1 \right) \tag{3.26}$$

$$\frac{\partial J}{\partial y_c} = -8 \sum_{i=1}^{n} \frac{y_i - y_c}{b^2} \left(\frac{(x_i - x_c)^2}{a^2} + \frac{(y_i - y_c)^2}{b^2} - 1 \right) \tag{3.27}$$

We can set these partial derivatives to zero and solve for a, b, x_c, and y_c to find the values that minimize J. This can be done using numerical methods such as Newton's method or gradient descent.

Once we have the values of a, b, x_c, and y_c, we can use the equation for an ellipse to draw the fitted ellipse on the data points.

Least square ellipse fitting is commonly used in image processing, computer vision, and pattern recognition to extract features from objects that can be approximated by ellipses, such as eyes, faces, or particles. When solving for the five different parameters, using the conic way described as:

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0 \tag{3.28}$$

defining an ellipse if this condition is satisfied:

$$B^2 - 4AC < 0 \tag{3.29}$$

or if using the parametric was described as:

$$(x-h)^2/a^2 + (y-k)^2/b^2 = 1 \tag{3.30}$$

the solution the the least square problem becomes a non linear problem and needs some sort of approximation. Depending on the kind of approximation different ellipses will be calculated. Also important to keep in mind, the ellipses are multiplied with an rotation matrix R to fit the ellipse also to rotated ellipses.

$$R = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \tag{3.31}$$

Contours

When working with thresholds the contour is of interests. The contour is defined as the outer boundary of the threshold binary matrix and is the foundation for the ellipse fit method. Here it is not given to extract one single contour and therefore the contours need to be filtered based on which contour represents the pupil the best. Mainly three criteria are used.

Circularity / Compactness

$$\frac{4\pi \mathrm{A}}{() \oint_{\partial S} dS)^2} \tag{3.32}$$

Where A is the Area of the contour, and it is devided by the integral over the outer boundary of the contour also known as the permitter of the contour.

Similarity The for the similarity the OpenCV library is used. This methods compares a given shape with another. As base for the comparison a simple ellipse is taken and then compared.

Area The Area of the contour should be maximal to find the greatest pupil area and filter out smaller area contours.

Each algorithm that returns a list of contours needs them to be checked with these four conditions and in the best case scenario only the contour of the pupil survives. This is then the contour on which a ellipse will be fitted.

3.1.6 Random Sample Consensus (RANSAC) [8]

The Random Sample Consensus (RANSAC) algorithm is an iterative method that is used to estimate the parameters for a known problem. For ellipse fitting a contour or a set of possible points inside or on the contour C is the input. The algorithm then randomly selects a subset S_c of 5 random selected points in C. These 5 points are then used to fit an ellipse as described in the section Ellipse Fitting.

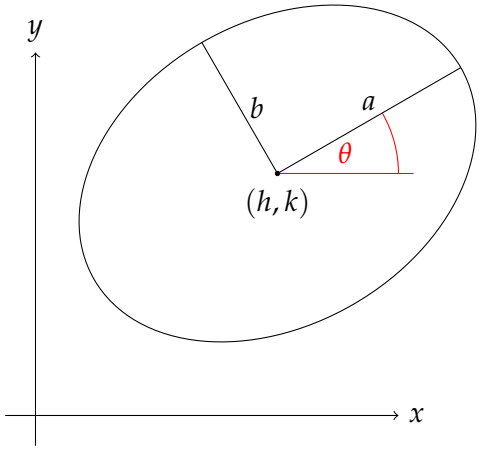


Figure 3.16: Ellipse with center (h,k), semi-axes a and b, and rotation θ

$$\frac{((x-h)\cos(\theta)+(y-k)\sin(\theta))^2}{a^2}+\frac{((x-h)\sin(\theta)-(y-k)\cos(\theta))^2}{b^2}=1 \tag{3.33}$$

The equation 3.33 represents an ellipse with major axis a, minor axis b and the center of the ellipse at (h,k). The angle θ is the rotation of the ellipse in the coordinate system. When comparing it with the general ellipse equation and compare coefficients the covariance matrix can be derived:

$$\mathbf{A}x^2+\mathbf{B}xy+\mathbf{C}y^2=1 \tag{3.34}$$

$$\mathbf{A}=a^2*\sin(\theta)^2+b^2*\cos(\theta)^2 \tag{3.35}$$
$$\mathbf{B}=2(b^2-a^2)*\sin(\theta)*\cos(\theta) \tag{3.36}$$
$$\mathbf{C}=a^2*\cos(\theta)^2+b^2*\sin(\theta)^2 \tag{3.37}$$

The covariance matrix is then defined as:

$$\mathbf{V}=\begin{bmatrix}\mathbf{A} & \mathbf{B}/2\\ \mathbf{B}/2 & \mathbf{C}\end{bmatrix} \tag{3.38}$$

The benefit of using the covariance matrix is, that the eigenvalues and eigenvectors have a direct relation to the major and minor axis and the rotation of the ellipse. The eigenvalues represent the major and minor of the axis and the eigenvectors represent the rotation matrix R to rotate the ellipse in the coordinate system. This has the benefit that all parameters are known to transform all points into a new coordinate system where the calculated ellipse equation is represented in an unit circle.

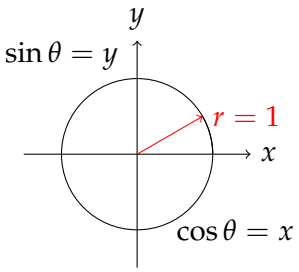


Figure 3.17: Unit circle with trigonometric identities

This allows for a simple distance calculation. Because $\lambda_{1,2}$ are known, the new coordsystem can be transformed to the unit circle by dividing the x and y values by the eigenvalues. The points form C are then multiplied by the transposed eigenvectors to rotate the points into the new system and then divide by the eigenvalues. Let p be a point in C and p' the transformed point in the new coordinate system.

$$p' = R^T \begin{bmatrix} p_x - h \\ p_y - k \end{bmatrix} \odot \begin{bmatrix} \frac{1}{\lambda_1} \\ \frac{1}{\lambda_2} \end{bmatrix} \tag{3.39}$$

$$r = \sqrt{p_x'^2 + p_y'^2} \tag{3.40}$$

So the distance calculation of a point p to the ellipse curve simplifies to the distance of the transformed point p' to the unit circle.

$$d = \sqrt{p_x'^2 + p_y'^2} - 1 \tag{3.41}$$

By using this knowledge, the RANSAC algorithm iterates n times to find the best ellipse with the most inliers. The inliers are calculated like following:

$$p = \begin{cases} \text{inlier} = \{p \in C | d < 0\} \\ \text{border} = \{p \in C | d = 0\} \\ \text{outlier} = \{p \in C | d > 0\} \end{cases} \tag{3.42}$$

The RANSAC algorithm calculates the ellipse parameters with five random points S_c for n iterations and evaluates each iteration the number of inliers.

The ellipse with the most inliers is then used to calculate the final ellipse parameters with the best fit. The differentiation between on border an inlier is important to weight the focus on enclosing ellipses or on the border of the ellipse. Because the calculation of the distance d is not an integer, it is possible that the distance is not exactly zero but still on the border. So it is important to keep in mind that comparing floats with each other, to alway use a small epsilon value that d can differ from 0 to set the threshold. In python functions like *isclose* from the math module can be used to compare floats with each other. The number of iterations to find a 99% chance of finding the best ellipse with the most inliers can be calculated with the following formula:

$$n = \frac{\log(1-p)}{\log(1-w^s)} \tag{3.43}$$

Where p = is the probability of finding the best ellipse and w is the ratio of inliers to outliers (Probability that a point is an inlier). This formula can then be used to estimate the number of iterations n for the RANSAC algorithm to be 99% sure to find the best ellipse with size s as sample. In our case we use $s = 5$, $p = 0.99$ and for w an approximation needs to be made. Because the number of inliers is unknown the ratio of inliers to outliers can not be calculated and needs to be estimated. In this case we assume that the ratio of inliers to outliers is $w = 0.90$ which can be adapted later on.

$$n = \frac{\log(1-0.99)}{\log(1-0.90^5)} = 5.158 \tag{3.44}$$

So the number of iterations to find the best ellipse with the most inliers is 6. But it is important to keep in mind that this is only true if inliers are the sum of border and inliers. Otherwise the number of iterations needs to increase because the ratio inliers to outliers changes drastically.

3.1.7 Active Contouring [9]

Active contour segmentation is a method that is used to find the boundary curve of an object given that the object exists. There a many different approaches in active contouring but in this thesis two variants will be discussed. The first one being the classic snakes approach (Kass et al., 1988) [10] with a simple energy function and the other variant being the active contouring without edges (ACWE) based on level sets. In both cases a initial contour is set and through iterative changes tries to find the boundary curve of an object. Each iteration the contour is changed by a small amount and then evaluated with an energy function. The energy function is a measurement of how well the contour is fitting to the edge of the object and in both cases the energy function is minimized.

Let $C(q) : [0,1] \rightarrow \mathbb{R}^2$ be a parametrized planar curve and let $I : [0,a] \times [0,b] \rightarrow \mathbb{R}^+$ be a given image used to detect the object boundaries with active contouring. The classical snakes approach (Kass et al., 1988) [10] associates the curve C with an energy given in 3.46 But the most basic description for the energy minimization can be summarized in E_{int} and E_{ext}

$$E = E_{int} + E_{ext} \tag{3.45}$$

$$E(C) = \underbrace{\alpha \int_0^1 |C'(q)|^2 \, dq + \beta \int_0^1 |C''(q)|^2 \, dq}_{\mathrm{E}_{int}} \underbrace{- \lambda \int_0^1 |\nabla I(C(q))| \, dq}_{\mathrm{E}_{\mathrm{ext}}} \tag{3.46}$$

The goal is to minimize the energy function and by doing so the contour C will find local minimals and depending on the sign of *lambda* be attracted to light or dark edges. The first two terms of the energy describe the intern energy of the contour and the last term describes the external energy. In other words, the intern energy describes the curve smoothness and the external energy describes the relation of the curve to the edge at the curves boundary. In the following the three terms will be discussed in more detail.

To use these Equations, they have to be applied on a discrete grid. The curve C is discretized into N points $c_i(x_i, y_i)$ with $i \in [0, N-1]$. The energy function is then calculated for each point C_i and summed up to get the total energy of the curve. The energy function is then minimized by changing the position of the points $c_i(x_i, y_i)$ and the process is repeated until the energy function converges. The energy function is minimized by using the gradient descent method. The gradient of the energy function is calculated and the points $c_i(x_i, y_i)$ are changed by a small amount in the direction of the gradient. The gradient descent method is repeated until the energy function converge to a local minimal.

The formula can be discretized by sampling the initial curve position evenly and then the formula can be expressed as following:

$$E = \sum_{i=0}^{N-1} \alpha(c_{i+1} - c_i)^2 + \beta(c_{i+2} - 2c_{i+1} + c_i)^2 - \lambda|\nabla I(c_i)| \tag{3.47}$$

Here it is important to note, that the points are circular, when i is equal to $N-1$ the next point is c_0. Therefore the indices can also be interpreted as $c_{(i+1 \bmod N)}$ and $c_{(i+2 \bmod N)}$

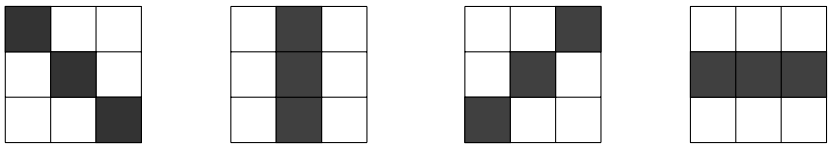
$$\text{substitute: } \begin{cases} \sum_{i=0}^{N-1}(c_{i+1} - c_i) = C'(q) \\ \sum_{i=0}^{N-1}(c_{i+2} - 2c_{i+1} + c_i) = C''(q) \end{cases}$$

Where $C'(q)$ calculate the length of the curve and $C''(q)$ calculates the curvature of the curve.

Because the classic snakes implementation searches for the best boundary curve that localy minimizes the energy function, the initial contour need to be initialized close to the best solution. Otherwise the algorithm will converge to a local minimum and not the global minimum. The classic snakes approach searches for the minimal with PDEs that is solved using the steepest descent method which brings performance issues and is prone to terminate early if a local minimum is found.

Active Contouring without Edges (ACWE) [9]

In ACWE the same principle is used to find the best object boundary. But in this case the energy function is minimized using level sets and morphological operations to solve the PDEs. **??** The morphological operation use four different structuring elements.



(a) Element 1 **(b)** Element 2 **(c)** Element 3 **(d)** Element 4

Figure 3.18: The four morphological structuring elements used in ACWE.

ACWE is based on Geodesic Active Contours (GAC). GAC has the benefit that it already solves key problems from the classical active contouring. Based on thresholds the contour is evaluated and decided if it flows expands the contour to a new pixel or not. GAC is based on classic active contours and geodesic curves in a Riemannian space. The level sets have the benefit that they can easily be implemented in a discrete grid and open the door to solve the PDEs using morphological operations. The level set approach brings more stability and is less sensitive to the initial contour compared

to the classical active contouring method. It is also possible to detect the interior and exterior boundary. In the classical approach the interior force is based on first and second derivatives of the curve. This leads to stability, computational complexity and performance issues. This is solved in the GAC by using the evolution of an implicitly defined curve. The GAC algorithm based on a level set function u is defined as following:

$$\frac{\partial u}{\partial t} = \underbrace{g(I) \cdot |\nabla u| \cdot \operatorname{div}\left(\frac{\nabla u}{|\nabla u|}\right)}_{\text{Smoothing force}} + \underbrace{g(I) \cdot \nu \cdot |\nabla u|}_{\text{Balloon force}} + \underbrace{\nabla g(I) \cdot \nabla u}_{\text{External force}}. \tag{3.48}$$

Where u is the level set function, a binary mask that is 1 inside the object and 0 outside. I is the image, $g(I)$ is the edge indicator function, ν is a variable that decides in which direction and how strong the curve should move. For chosing $g(I)$ there are different variants but two of the most common are the following:

$$g(I) = \begin{cases} \frac{1}{1+|\nabla I|*\alpha} & (1) \\ \\ -|\nabla I| & (2) \end{cases} \tag{3.49}$$

In (1) $g(1)$ is low when the gradient is high and vice versa. Meaning that $g(I)$ regulates the influence of the forces depending on the gradient. It is around 0 when the gradient is high, in other words it stops the curve from moving when the the pixel has reached an edge. α is a variable that regulates how strong the influence of the external energy is. In (2) $g(I)$ is negative when the gradient is high. So both definition of $g(I)$ will stop the curve from moving when it reaches an edge.

Smoothing force The smoothing force is the same as the curvature term in the classical snakes method. The difference is that in GAC the smoothing force is calculated using the gradient of the level set function u instead of the curve.

$$g(I) \cdot |\nabla u| \cdot \operatorname{div}\left(\frac{\nabla u}{|\nabla u|}\right) \tag{3.50}$$

Now the smoothing force is calculated with using an recursive call of SI and IS of operator F that approximates the curvature flow that is based on PDE.

$$\mathrm{F}(u) = SI(u) \circ IS(u) \tag{3.51}$$

Where SI means supremum of infimum and IS means infimum of supremum. In other words, SI stands for erode and IS stands for dilate. One step of

smoothing looks like following:

$$1) u_{SI \circ IS} = \text{erode}(u) \circ \text{dilate}(u)$$
$$2) u_{IS \circ SI} = \text{dilate}(u) \circ \text{erode}(u)$$

by repeating step 1 to 2 the smoothing force can be stronger or weaker. But it is important so always use step 1 and step 2 in the same order.

Balloon force The balloon force is important in regions where the gradient is low and hinders the curve flow to get stuck at a local minimum. Expecially when looking at the external force, where the velocity of the curve is determined by the gradient of $g(I)$ and the gradient of u. Ifh $g(I)$ goes to zero the curve will have no external velocity, therefore the balloon force is important to make the algorithm more robust.

$$g(I) \cdot \nu \cdot |\nabla u| \tag{3.52}$$

To make GAC more computational efficient the ballon force and also the smoothing force are computed differently. The Balloon force is $g(I) * |\nabla u| * v$ with parameter $v \in -1, 1$ can be substituted with dilation over the four different neighborhood structuring elements. Because dilation is a morphological operation it is computed very efficient and also gets rid of the PDEs that need to be solved in the classical snakes method. By using dialate the balloon force leads to the wanted result by making the curve grow.

$$\text{dialate}(u) = u \bigoplus Element_i \quad \text{for} \quad i \in \{1, 2, 3, 4\} \tag{3.53}$$

External force The external force is the same as in the classical snakes method but in GAC it is weighted by $g(I)$ to stop the curve from moving when it reaches an edge.

$$\nabla g(I) \cdot \nabla u \tag{3.54}$$

The change in the level set function u is only applied to the points where $g(I) > T$ where T is a defined threshold to control the velocity and the characteristic that the curve moves faster in areas where the gradient is low. This leads to an algorithm that moves faster in a region with the same intensity and slower in regions with a high gradient.

The external force or the attracting force is calculated for each point p in the binary mask from the before modified level set function u and evaluated with the following equation:

$$p = \begin{cases} 1 & iff \nabla u * \nabla g(I) > 0 \\ 0 & iff \nabla u * \nabla g(I) < 0 \end{cases} \tag{3.55}$$

Summary To conclude, the equation 3.55 describes the deformation of u, the binary level set function. and is the main equation for Geodesic Active Contours (GAC).

Convert GAC to ACWE

The GAC only inspects places where the curve could move iteratively and this is the main difference to ACWE. ACWE also known as Chan-Vese algorithm takes the complete image into consideration to evaluate the boundary or the curvature flow of the level set. Let c_i the the mean intensity of an subset *Oomega* of the image I and let c_1 be the mean intensity inside the curve or level set u defined as Ω_1 and c_2 the mean intensity outside level set u defined as Ω_2.

$$c_i = \frac{\int_{\Omega_i} I(x)\,dx}{\int_{\Omega_i} dx} \tag{3.56}$$

Then the external force function $F(c_1, c_2, C)$ is defined as

$$\begin{aligned} F(c_1, c_2, C) =& \mu \cdot \text{length}(u) + \nu \cdot |\text{inside}(u)| \\ &+ \lambda_1 \cdot \int_{\Omega_1} (I(x) - c_1)^2\,dx \\ &+ \lambda_2 \cdot \int_{\Omega_2} (I(x) - c_2)^2\,dx \end{aligned} \tag{3.57}$$

where the first two terms $\mu \cdot \text{length}(u) + \nu \cdot |\text{inside}(u)|$ are the same as in the GAC The third and fourth terms sort of calculate the unscaled intensity variance in the different subsets Ω_1 and Ω_2.

The only difference now between GAC and the ACWE is that the external forceis is calculated differently. In GAC the external force is calculated with $\nabla g(I) \cdot \nabla u$ and based on a threshold decides if a point is inside or outside the curve, as seen in equation 3.55.

In ACWE the external force is calculated with the mean intensity of the subsets Ω_1 and Ω_2 and the intensity of the image $I(x)$ as seen in equation 3.57. Converting the equation onto an discrete grid it can be written as:

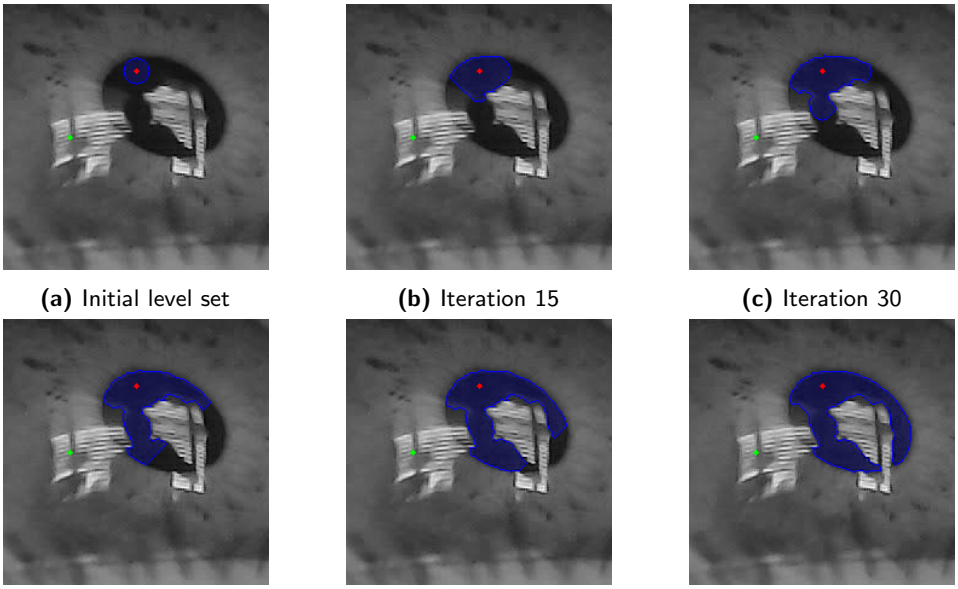
$$\xi = \underbrace{\lambda_1 \cdot |\nabla u_{mask}| \cdot (I(x) - c_1)^2}_{Inside} - \underbrace{\lambda_2 \cdot |\nabla u_{mask}| \cdot (I(x) - c_2)^2}_{outside} \tag{3.58}$$

ξ is then used to calculate the new level set function u with the following equation:

$$u(x, y) = \begin{cases} 1 \text{ if } \xi < 0 \\ 0 \text{ if } \xi > 0 \\ \text{do nothing otherwise} \end{cases} \tag{3.59}$$

The Gradient of the mask will only be nonzero at the contour of the mask. Now $I(x)$ is subtracted by the mean and each point on the contour will have two values, their inside and outside weighted intensities. By subtracting the mean intensity from the pixel intensities on the boundary of u, multiplying it with λ_i and subtracting the outside term from the inside term an relationship between the inside and outside intensities is created and can be tuned with the sensitivity parameters λ_i .

This relationship is then used to calculate the new level set function u and the process is repeated until the level set function converges. In a case where the function reaches the object boundary the inside and outside intensities will be the same and the curve will not move anymore and the equation goes to zero. Here is an example of the evolution the the level set u when the ACWE algorithm is used with a given starting point inside the pupil.



(a) Initial level set **(b)** Iteration 15 **(c)** Iteration 30

(d) Iteration 60 **(e)** Iteration 90 **(f)** Iteration 115

Figure 3.19: ACWE iterations

Active contouring with ellipse parameters

A different approach is to let an ellipse grow and modify its parameters based on the gradient information. By sampling an amount of equally spaced points on the ellipse and evaluate an energy function based on the image intensity gradient and the normal vector of the ellipse at those points. The energy function can be described as the scalar product of the gradient and the normal vector of the ellipse curve.

$$E = -\sum_{i=0}^{N-1} \nabla I(p_i) \cdot n(\vec{p}_i) \tag{3.60}$$

Where p_i is a point on the ellipse and $n(\vec{p}_i)$ is the normal vector of the ellipse at that point. $\nabla I(p_i)$ is the gradient of the image at that point. The energy function is minimized by changing the ellipse parameters This converts the

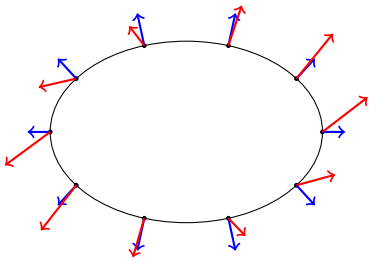


Figure 3.20: Ellipse with normal vector (blue) and gradient vectors (red)

problem into a minimization problem. The energy function still needs a term to make sure that the curve grows and does not get stuck at a local minimum. Here it is not possible to use the morphological operators because the shape of the ellipse is not given as a level set. Instead the ellipse is based on only the five ellipse parameter: center(x, y), axis: (a, b) , angle θ.

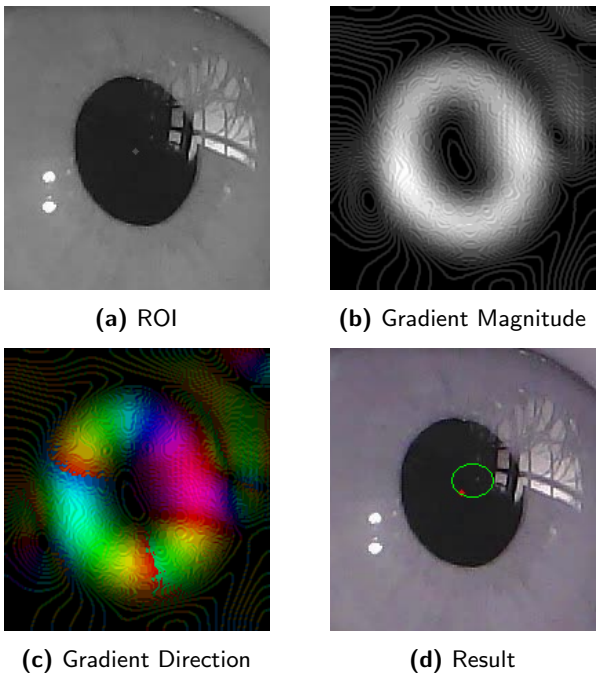
The energy function is the lowest when at each point the gradient vector and normal vector point in the same direction. The gradient vector consists of the magnitude of the gradient and the orientation. Whereas the normal vector is normalized and points in the direction of the normal of the ellipse at that point.

The problem with this approach is overlapping with the classical active contouring. The initial contour needs to be close to the optimal solution. Otherwise the algorithm stops at an local minimum and convergence to the optimal solution is not guaranteed. Because it is an minimization with five parameters it is a very complex problem and the algorithm needs to be run multiple times to find the best local solution.

For this approach, the image needs preprocessing and the algorithm is very

sensitive to noise. Therefore it is a must to first apply a low pass filter onto the image. But because of blurring the image gradients and directions aren't as accurate as the could be and struggles with reflections and other noise.

Here is an example with the convergence to an local minimum. The initial ellipse is nowhere near to the optimal solution and the algorithm converges to an local minimum.



(a) ROI **(b)** Gradient Magnitude

(c) Gradient Direction **(d)** Result

Figure 3.21: Convergence to local minimum

The problem with expanding the energy function with more terms is, that the algorithm gets more complex. For example when the size is used as a reward for the energy function (the bigger the better), it is not given anymore to stop at the boundary of the object. The parameters have to be chosen smart and carefully to make sure that the algorithm still converges to the optimal solution.

Chapter 4

Algorithm Implementation

In this chapter some of the algorithm in the theory chapter are combined, tested and evaluated. The goal is to find the most robust combination of algorithms to detect the pupil even in a very demanding data set like the LPW. The LPW data set is already discussed in its chapter and will not be discussed here again. In general the evaluation can be split up into two part. The first part is localization of the pupil and second part is finding the pupil ellipse. The importance of the localization is that with the information of the location of the pupil it is possible to create a region of interest (ROI). This has the benefit that in the second part the image size is already decreased and additional noise can be even more limited. Also the computational effort is reduced because the image size is smaller. Important to note is, that all algorithms make the basic assumption, that the pupil is always visible. So blinking is not counted as a failure and excluded of the evaluation if possible. For detecting blinking, other algorithms need to be used to preprocess every frame and detect if the eye is closed or not.

4.1 Localization

For the localization mainly thresholding, edge detection and haar-like features were implemented and can now be evaluated. To make thresholding more flexible a semi adaptive algorithm was created to choose the best fitting threshold value. The histogram is used to determine the highest peak in the low intensity range. This value is then used to threshold the image.

$$t = \text{argmax}\{h(i)|i \in [0, 255]\} \tag{4.1}$$

With $h(i)$ being the histogram of the image. The threshold value is then used to calculate a range for using double thresholding to extract the pupil. The image is modified by setting all values below and above the threshold value

to 0.

$$f(x,y) = \begin{cases} 0 & iff \quad I(x,y) < t-35 \quad \text{or} \quad I(x,y) > t+25 \\ I(x,y) & otherwise \end{cases} \tag{4.2}$$

4.1.1 Thresholding

Whereas the under limit of the threshold is set lower than the higher limit. This derives from testing and can be concluded, that the probability that the pixels with lower intensity values belongs to the pupil is higher than the probability that the pixels with higher intensity values belong to the pupil. In a environment with almost no noise and most important no reflections this approach works very well. But as also already mentioned in the theory section, reflections lead to a less higher peak in the histogram and the possibility exist that the threshold value has no peak in the lower values range and therefore leads to a faulty result that can not be used to create an ROI or use it with edge detection to find the boundary of the pupil. Also it is important to note that with this thresholding approach the mask created is not a binary mask but a mask with values between $[t-35, t+25]$. But the possibility to use the binary mask still exist and the all contours found in this threshold range are evaluated by their circularity and similarity to an ellipse. The best fitting contour is then used to create the ROI or ellipse fit directly to find the ellipse parameters. [RESULTS] This method works in a environment with almost no noise flawless, it is has benefit to reach the goal in almost realtime but in with the LPW data set it is keen to strugle with most of the conditions and is therefore not usefull for the LPW data set.

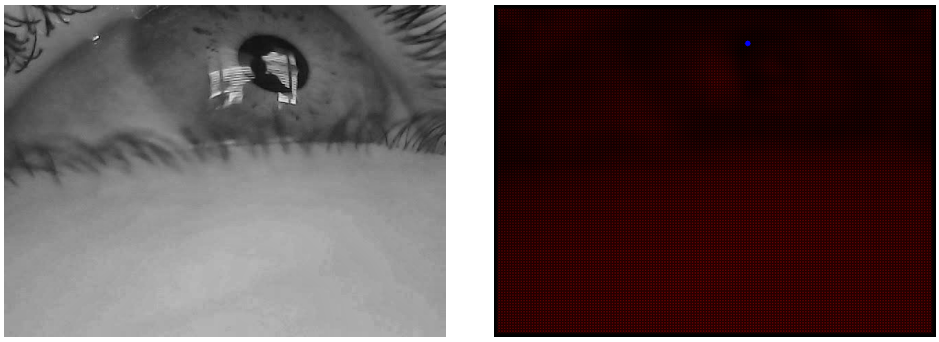
4.1.2 Edge detection

Edge detection is strongly effected by noise and therefore the preprocessing is key for useful results. Every frame undergoes the same preprocessing as in the other algorithms but a gaussian blur is used additionally to smooth fast changing intensity regions out. Even though the gaussian blur is used, there is still information missing that is covered by noise. The edges are detected but the same problem as with thresholding arises here. By using Sobel and than the Canny edge detection the results are useful in a environment with almost no noise but as already said, LPW is known for its noise and reflections. Canny edge detection needs two threshold parameters to work properly and also here arises the problem that these parameters need to be adaptive to the environment which can be tricky. [RESULTS]

4.1.3 Haar-like features

The Haar-like feature is the approach that is proposed by this thesis to find the region of interest. It has the best detection rate of all approaches tested

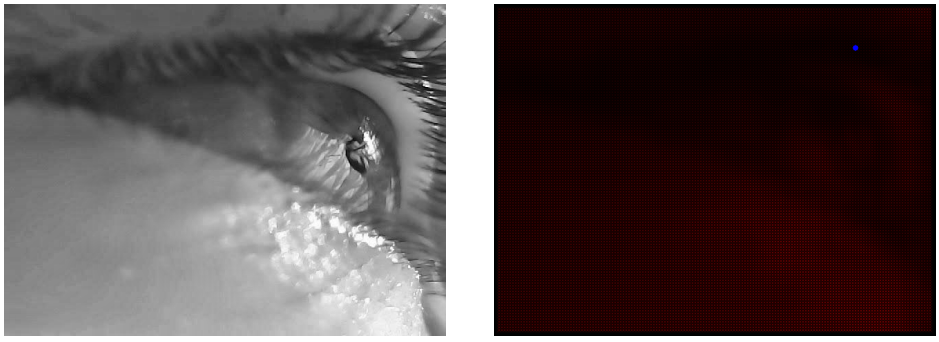
on the LPW data set and is therefore the best approach to find the ROI. The Haar-like feature is constructed as described in the theory part and is shown in **??**. The calculation of the feature vector is done by using the integral image and is done 3 times with variating Radius r all feature vectors are then compared and the location of the highest response is returned as a point that lies on the pupil.



(a) Original Frame **(b)** feature vector, blue is the best response

Figure 4.1: Feature vector for pupil detection

The stronges response lies then within the pupil and the location is then used to create a ROI. In this case a ROI of $110x110$ was the norm but depending on the rescaling of the frames this can be adapted. Important to note is, that the returned point in the pupil is not given to be in the center. This in an important fact when choosing the size of the ROI that is created. But this approach is still not perfect and even though it can handle noise really good, there are limits to the amount of noise until the algorithm fails. Also the algorithm is not able to detect the pupil when the eye is closed.



(a) Extreme noise example **(b)** feature vector result with best response

Figure 4.2: Limits to the Haar-like feature approach

The problem with the LPW data set is, that it is so versatile and the conditions

change during the recording. This makes it hard to find an approach that can adapt to all the conditions and still perform well in a given time interval. The implementation can still be improved and sped up, in the Haar-like feature the following libraries were used to speed up the algorithm:

```
from concurrent.futures import ThreadPoolExecutor
from numba import njit
```

ThreadPool Executor makes it possible to use multithreading and njit compiles the sliding window over the integral image of the Haar-like feature to machine code for more efficient calculation.

4.2 Ellipse parameter estimation

In the second part of an pupil detection algorithm it is necessary to make use of the informations from the first part and build on this foundation and find the five ellipse parameter: center: (x, y), axis: (major, minor) and angle. The LPW has labels for the center only of the pupil and therefore only the center can be used to evaluate the performance of the algorithms. A second evaluation has to be done manually by inspecting the fit of the ellipse to the pupil. This is hard to evaluate with numerical methods and the result needs therefore to be taken with a grain of salt. In this section 4 different algorithms will be discussed and evaluated: The OpenCV ellipse fit based on contours (binary thresholding), ACWE with OpenCV Ellipse fit, ACWE combined with RANSAC, and Canny edge detection with OpenCV Ellipse fit.

4.2.1 Thresholding and OpenCV ellipse fit

The benefit of this method is that it can almost run in realtime and still perform well in certain frames where the noise is low and the pupil is clearly visible. But as already mentioned in the localization part, this method is not robust enough to handle the LPW data set. The thresholding is done with the same approach as in the localization part and the binary mask is then used to find the contours. The contours are then evaluated by their circularity and similarity to an ellipse. The best fitting contour is then used to fit an ellipse with the OpenCV ellipse fit function. The OpenCV ellipse fit function is based on the least square method and therefore very robust and fast. The ellipse fit is then evaluated by the center and the distance to the ground truth center. The problem with least square method is, lies in the definition itself. Because the method tries to find an ellipse that minimizes the distance from every point on the boundary of the binary mask to the ellipse curve. When the pupil is partially covered by noise, the contour does not match the original shape of the pupil and does not represent the total pupil boundary.

Using least square method in this case leads to a faulty ellipse fit where the result is not usable. The amount of outliers is to high and the ellipse fit is not accurate enough because there is information missing.

(a) ROI **(b)** Binary thresholding **(c)** OpenCV Ellipse fit

Figure 4.3: Binary thresholding with OpenCV ellipse fit

4.2.2 Canny edge detection with OpenCV ellipse fit

4.2.3 ACWE with OpenCV ellipse fit

4.2.4 ACWE combined with RANSAC

Chapter 5

Proposal

5.1 Proposed Algorithm

After intensive research and analysis, the algorithm proposed for pupil detection consist of the following steps:

1. **Preprocessing:** The image is converted to grayscale and then histogram equalization method CLAHE is used to improve the contrast of the image.
2. **Haar-like features:** From the image the feature vector is calculated using the Haar-like feature for pupil detection proposed by [**?**]. The feature vector is then used to find the strongest repsonse in the image. This point is considered to be inside the pupil area.
3. **ACWE** The active contour without edges algorithm is applied to the image with the point returned by the Haar-like features as center of the initial contour. Returns the contour of the pupil.
4. **RANSAC** The RANSAC algorithm is applied to the mask returned by the ACWE algorithm. Iterates over the mask contour and fits a circle to a random subset of the contour points. Returns the circle with the highest number of inliers.

Chapter 6

Results

In this chapter the proposed algorithm is evaluated on the LPW data set and numerical results are presented, followed by a discussion of the results.

6.1 Evaluation

6.2 Discussion

6.3 Possible Improvements

Chapter 7

Conclusion

7.1 Summary

Appendix A

Appendix

A.1 Choosing the right model

Depending on the level of noise different algorithms can be considered. Here only algorithms that are implemented in the thesis are considered. There are even more algorithms that could be used for object sedimentation for example MSER or Machine Learning.

Low noise If there is almost no noise in the image, meaning the pupil is all the time clearly visible and no reflections in the pupil area, it is possible to use the Haar-like feature to localize the pupil. Create a ROI and then use the intensity Value of the pixel with the best response from the Haar-Like Feature to get a threshold value. Then create a binary mask and inspect all contours in the ROI and choose the contour with the best circularity and similarity to an ellipse. Use OpenCV ellipse fit to retrieve the ellipse parameters. This approach is very fast and accurate in low noise conditions and can almost run in realtime.

Medium noise / High noise Here it becomes more tricky to choose the right mode. In general the more noise is introduced the more robust the algorithm has to be. This comes with the cost of speed. Here the proposed algorithm should be used to extract the pupil parameters.

A.2 Code

All the code can be found on github under: https://github.com/parisj/Algorithms_Eye_Detection. The code is implemented in python and for the packages used, take a look a the requirement.txt file

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