HW2_psp2133

February 27, 2018

1 Deep Learning for Computer Vision: Assignment 2

1.1 Computer Science: COMS W 4995 004

1.2 Due: February 27, 2018

Problem You are given the noisy XOR data generated for you below. Your task is to implement a multi-layer perceptron binary classifier with one hidden layer. For the activiation function of the hidden units use ReLu or leaky ReLu. For the loss function use a softplus on a linear output layer as we did in class.

a) Implement each layer of the network as a separate function with both forward propagation and backpropagation.

b) Train the network using stochastic gradient descent with mini-batches.

c) Show the decision regions of the trained classifier by densely generating points in the plane and color coding these points with the binary labels.

d) Repeat (b) and (c) varying the number of hidden units: 3, 16, 512. Discuss how the number of hidden units affects the solution.

e) Try choosing your own loss function (without asking me or the TAs what you should choose), repeating (d).

f) Now try with three inputs, generating your own training and validation data. (For this XOR the output should be a 1 if and only if exactly one of the inputs is 1. But make the training data noisey as before.) Use softplus loss. Do not try to show the decision regions, instead generate a validation set in the same manner as the training set, classify the samples, and compute the classification accuracy.

If you are struggling to get the network to converge, experiment with different learning rates.
Grading: a-d = 85%, e=10%, f=5%.

NOTE: Do not to use keras, tensorflow, pytorch, sklearn, etc. to do this. You must build the machine learning components from scratch. Let's start by importing some libraries.

```
In [476]: import numpy as np
          import random
```

```
import pandas as pd
import matplotlib.pyplot as plt
from tqdm import tqdm
```

Let's make up some noisy XOR data.

```
In [501]: data = pd.DataFrame(np.zeros((5000, 3)), columns=['x1', 'x2', 'y'])

          # Let's make up some noisy XOR data to use to build our binary classifier
          for i in range(len(data.index)):
              x1 = 1.0 * random.randint(0,1)
              x2 = 1.0 * random.randint(0,1)
              y = 1.0 * np.logical_xor(x1 == 1 , x2 == 1)
              x1 = x1 + 0.18 * np.random.normal()
              x2 = x2 + 0.18 * np.random.normal()
              data.iloc[i,0] = x1
              data.iloc[i,1] = x2
              data.iloc[i,2] = y

          data.head()
```

```
Out[501]:          x1        x2    y
          0 -0.225245  0.115329  0.0
          1  0.184413  1.030207  1.0
          2  0.914806 -0.179590  1.0
          3  0.081391  0.877957  1.0
          4  0.860010  0.811776  0.0
```

Let's message this data into a numpy format.

```
In [502]: # set X (training data) and y (target variable)
          cols = data.shape[1]
          X = data.iloc[:,0:cols-1]
          y = data.iloc[:,cols-1:cols]

          # The cost function is expecting numpy matrices so we need to convert X and y before
          X = np.matrix(X.values)
          y = np.matrix(y.values)
```

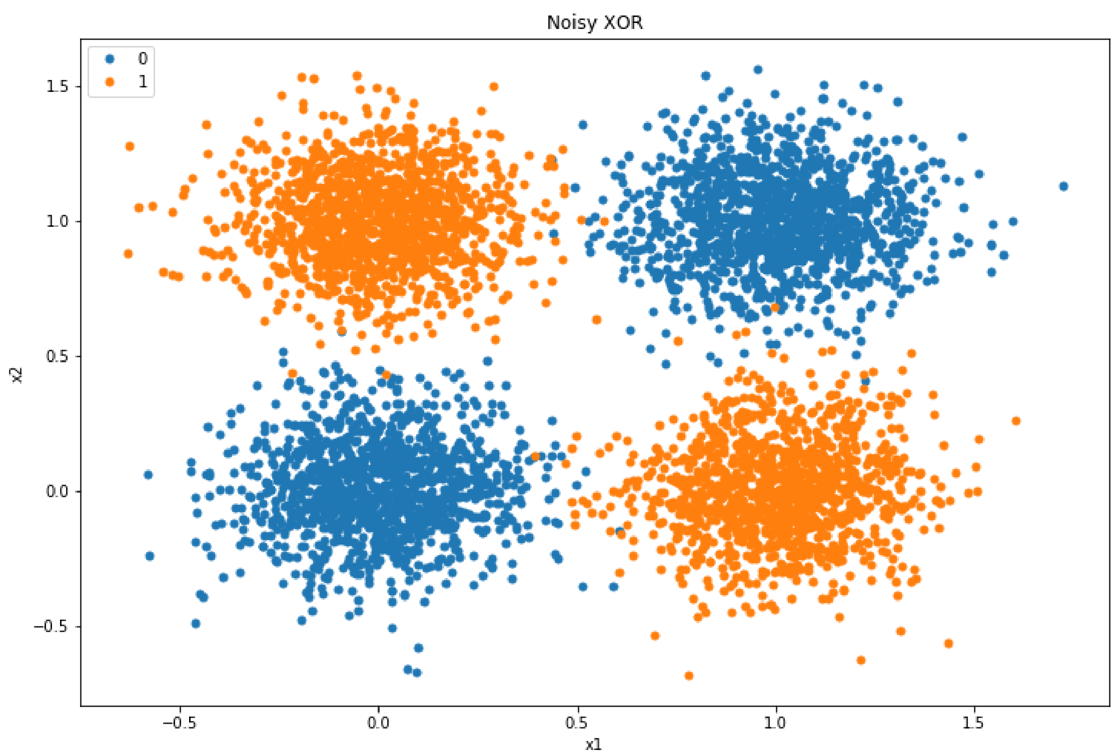
Let's make a sloppy plotting function for our binary data.

```
In [503]: # Sloppy function for plotting our data
          def plot_data(X, y_prob):

              fig, ax = plt.subplots(figsize=(12,8))
              ax.margins(0.05) # Optional, just adds 5% padding to the autoscaling

              y_predict = y_prob > 0.5
              indices_0 = [k for k in range(0, X.shape[0])
```

```
                if not y_predict[k]]
indices_1 = [k for k in range(0, X.shape[0])
                if y_predict[k]]

ax.plot(X[indices_0, 0], X[indices_0,1], marker='o',
        linestyle='', ms=5, label='0')
ax.plot(X[indices_1, 0], X[indices_1,1], marker='o',
        linestyle='', ms=5, label='1')

ax.legend()
ax.legend(loc=2)
ax.set_xlabel('x1')
ax.set_ylabel('x2')
ax.set_title('Noisy XOR')
plt.show()
```

Now let's plot it.

```
In [504]: plot_data(X, y)
```



Now let's create functions for forward and backward prop through the layers and we are off...

```
In [5]: # Add a dimension for the bias
        # X = np.c_[X, np.ones(X.shape[0])]
```

```
In [505]: def relu(a):
              a[a < 0] = 0
              return a

          def leaky_relu(a, eps):
              a[np.where(a < 0)] *= eps
              return a

In [506]: def sigmoid(z):
              return 1.0/(1.0 + np.exp(-z))

In [507]: def softplus(data, labels, a):
              loss = np.multiply(1 - 2*labels, a)
              loss = np.log(1 + np.exp(loss))
              return loss

In [577]: class Network():
              def __init__(self, in_units = 2, hidden_units = 3,
                           out_units = 1, lr = 0.1,
                           batch_size = 100, debug = False):

                  # Network specifications
                  self.in_units = in_units
                  self.hidden_units = hidden_units
                  self.batch_size = batch_size
                  self.out_units = out_units
                  self.lr = lr
                  self.debug = debug

                  # Initialize network parameters
                  w1 = np.random.rand(self.hidden_units, self.in_units)
                  w2 = np.random.rand(self.out_units, self.hidden_units)
                  b1 = np.zeros((hidden_units, 1))
                  b2 = np.zeros((self.out_units, 1))

                  self.weights = [w1, w2]
                  self.bias = [b1, b2]

                  # Initialize batchwise layer outputs as empty
                  self.linear1 = []
                  self.relu1 = []
                  self.eps = 0.001
                  self.linear2 = []

                  # Initialize batchwise gradients as empty
                  self.allgrad_weight1 = []
                  self.allgrad_weight2 = []
                  self.allgrad_bias1 = []
```

```
        self.allgrad_bias2 = []

    # Feedforward for entire batch
    def forward(self, data, save = True):
        self.linear1 = []
        self.relu1 = []
        self.linear2 = []
        for idx in range(data.shape[0]):
            x = data[idx, ::]
            x = x.reshape(self.in_units, 1)
            self._forward(x, idx, save)
        return np.array(self.linear2)

    # Feedforward for a single data point
    def _forward(self, x, idx, save = True):

        linear1 = np.dot(self.weights[0], x) + self.bias[0]
        relu1 = leaky_relu(linear1, self.eps)
        linear2 = np.dot(self.weights[1], relu1) + self.bias[1]

        self.linear2.append(linear2)
        self.linear1.append(linear1)
        self.relu1.append(relu1)

        if self.debug and idx == 0:
            print("\n---- PRINT FORWARD DATA ----\n")
            print("x = ", x, x.shape)
            print("weights1 = ", self.weights[0], self.weights[0].shape)
            print("linear1 = ", linear1, linear1.shape)
            print("relu1 = ", relu1, relu1.shape)
            print("weights2 = ", self.weights[1], self.weights[1].shape)
            print("linear2 = ", linear2, linear2.shape)

        return linear2

    # Backprop for entire batch
    def backward(self, L, data, labels):
        if (data.shape[0] != labels.shape[0]):
            print("Data and labels shapes mismatch")
            return None

        self.allgrad_weight1 = []
        self.allgrad_weight2 = []
        self.allgrad_bias1 = []
        self.allgrad_bias2 = []
        for idx in range(data.shape[0]):
            x = data[idx, ::]
            x = x.reshape(self.in_units, 1)
```

```
            y = labels[idx]
            self._backward(L, x, y, idx)

        self.grad_weights = [np.mean(self.allgrad_weight1),
                             np.mean(self.allgrad_weight2)]
        self.grad_bias = [np.mean(self.allgrad_bias1),
                          np.mean(self.allgrad_bias2)]

    # Backprop for a single data point
    def _backward(self, L, x, y, idx):
        r = 1.0 - 2*y

        grad_bias2 = sigmoid(r * self.linear2[idx]) * r
        grad_weight2 = np.multiply(grad_bias2, self.relu1[idx])
        grad_relu1 = grad_bias2 * self.weights[1]
        grad_linear1 = grad_relu1.T
        grad_linear1[np.where(self.relu1[idx] <= 0)] *= self.eps
        grad_weight1 = np.dot(grad_linear1, x.T)
        grad_bias1 = grad_linear1

        self.allgrad_weight1.append(grad_weight1)
        self.allgrad_weight2.append(grad_weight2)
        self.allgrad_bias1.append(grad_bias1)
        self.allgrad_bias2.append(grad_bias2)

        if (self.debug and idx == 0):
            print("\n---- PRINT BACKWARD DATA ----\n")
            print("linear2 = ", self.linear2[idx])
            print("r:", r, "y:", y)
            print("weight2:", self.weights[1])
            print("grad_bias2:", grad_bias2)
            print("grad_weight2:", grad_weight2)
            print("weight1:", self.weights[0])
            print("grad_relu1:", grad_relu1)
            print("grad_linear1:", grad_linear1)
            print("grad_weight1:", grad_weight1)
            print("grad_bias1:", grad_bias1)

    # Update mini batch weights
    def update_weights(self, lr = None):
        if (lr):
            self.lr = lr
        self.weights[0] -= (self.lr * self.grad_weights[0])
        self.weights[1] -= (self.lr * self.grad_weights[1])
        self.bias[0] -= (self.lr * self.grad_bias[0])
        self.bias[1] -= (self.lr * self.grad_bias[1])

    # Predict labels
```

```python
    def predict(self, data):
        out = self.forward(data, save = False)
        out[out >= 0] = 1
        out[out < 0] = 0
        return out

    # Compute accuracy
    def accuracy(self, data, labels):
        predicted_labels = self.predict(data).reshape(-1, 1)
        matches = np.where(predicted_labels == labels)
        return np.sum(labels == predicted_labels) / labels.shape[0] * 100.0
```

In [509]:
```python
# Split indexes for training and testing
def split_data(data_X, data_y, test_split = 5):
    train_n = data_X.shape[0]
    test_n = batch_size * test_split
    train_n = train_n - test_n
    train_idx = np.arange(train_n)
    test_idx = np.arange(train_n, train_n + test_n)
    print("Split (train/test): (%d/%d)" %(train_n, test_n))
    data, labels = data_X[train_idx, ::], data_y[train_idx, ::]
    test_data, test_labels = data_X[test_idx, ::], data_y[test_idx, ::]
    return data, labels, test_data, test_labels
```

In [584]:
```python
# Train an epoch
def train(net, epoch, data, labels, batch_size,
          adaptive_lr = False, debug = False):
    train_n = data.shape[0]
    train_idx = np.arange(train_n)
    np.random.shuffle(train_idx)

    train_loss = []
    train_acc = []
    for i in range(0, train_n, batch_size):
        batch_idx = train_idx[i:i + batch_size]
        batch_X = data[batch_idx, ::]
        batch_y = labels[batch_idx, ::]

        # Feedforward
        a = net.forward(batch_X)
        loss = softplus(batch_X, batch_y, a.reshape(batch_size, 1))
        train_loss.append(loss)
        acc = net.accuracy(batch_X, batch_y)
        train_acc.append(acc)
        net.backward(loss, batch_X, batch_y)
        if adaptive_lr:
            lr = 1/(1 + epoch) * 0.1
        else:
```

```
                lr = None
            net.update_weights(lr)

        if debug:
            print("Epoch %d, Train loss: %.4f" %(epoch, np.mean(train_loss)))
            print("Epoch %d, Train Accuracy: %.4f" %(epoch, np.mean(train_acc)))
```

```
In [590]: # Test validation data
          def test(net, test_data, test_labels, batch_size, debug = True):
              test_n = test_data.shape[0]
              test_idx = np.arange(test_n)
              test_loss = []
              test_acc = []
              for i in range(0, test_n, batch_size):
                  batch_idx = test_idx[i:i + batch_size]
                  batch_X = test_data[batch_idx, :]
                  batch_y = test_labels[batch_idx, ::]
                  a = net.forward(batch_X)
                  loss = softplus(batch_X, batch_y, a.reshape(batch_size, 1))
                  test_loss.append(loss)
                  acc = net.accuracy(batch_X, batch_y)
                  test_acc.append(acc)
                  if debug:
                      print("Batch %d: %d/%d Accuracy: %f" %(i/batch_size, i, test_n, acc))

              if debug:
                  print("Test loss:", np.mean(test_loss))
                  print("Accuracy:", np.mean(test_acc))
```

```
In [591]: # Define decision boundary
          def plot_boundary(X, y_prob):

              fig, ax = plt.subplots(figsize=(12,8))
              ax.margins(0.05) # Optional, just adds 5% padding to the autoscaling

          #     indices_0 = np.where(y_prob <= 0.5)
          #     indices_1 = np.where(y_prob > 0.5)
          #     print(y_prob.shape)
              indices_0 = [k for k in range(0, X.shape[0])
                           if y_prob[k, ::] <= 0.5]
              indices_1 = [k for k in range(0, X.shape[0])
                           if y_prob[k, ::] > 0.5]

              ax.plot(X[indices_0, 0], X[indices_0,1],
                      'b', marker='o', linestyle='', ms=5, label='0')
              ax.plot(X[indices_1, 0], X[indices_1,1],
                      'y', marker='o', linestyle='', ms=5, label='1')
```

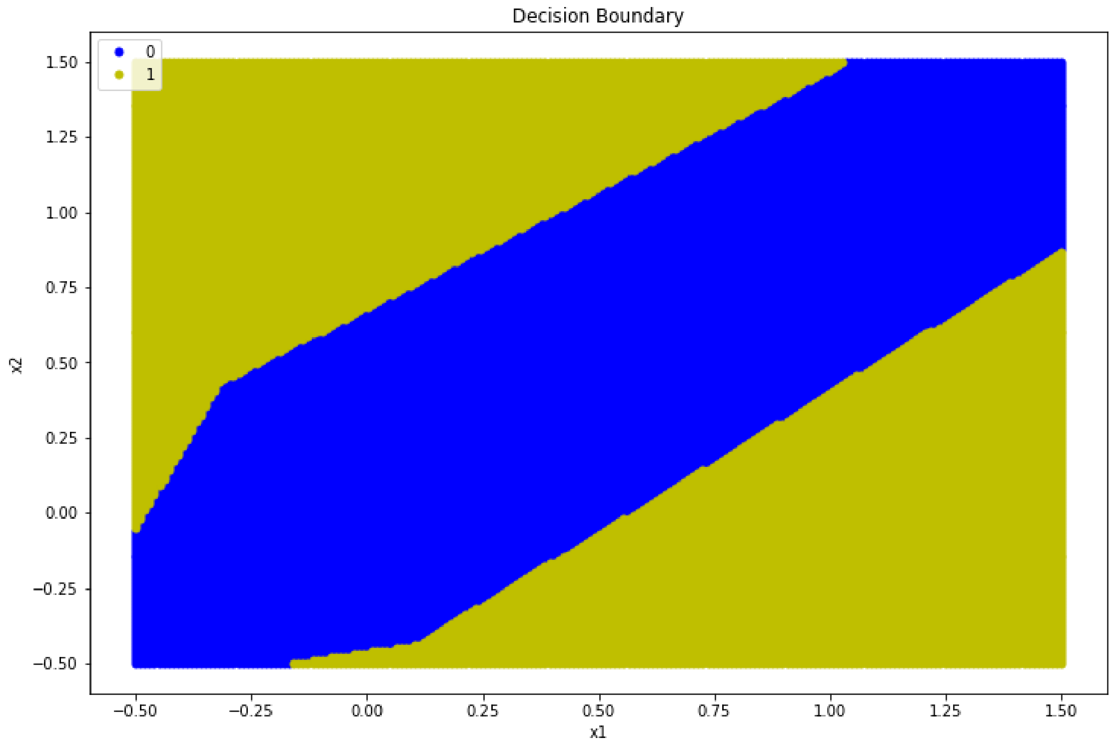
```
    ax.legend()
    ax.legend(loc=2)
    ax.set_xlabel('x1')
    ax.set_ylabel('x2')
    ax.set_title('Decision Boundary')
    plt.show()
```

In [592]:
```
# Random initialization that works for network with 3 hidden units
seed = int(np.random.rand(1) * 10000)
seed = 6524
print("Random seed:", seed)
np.random.seed(seed)

# Network parameters
batch_size = 50
num_epochs = 10
hidden_units = 3

# Initialize network
net3 = Network(in_units = 2, hidden_units = hidden_units, lr = 0.1,
              batch_size = batch_size, debug = False)

# Split Data into training and test sets
data, labels, test_data, test_labels = split_data(X, y)

# Train the network for 'num_epochs'
for epoch in range(num_epochs):
    train(net3, epoch, data, labels, batch_size,
          adaptive_lr = False, debug = False)

test(net3, test_data, test_labels, batch_size, debug = True)
```

```
Random seed: 6524
Split (train/test): (4750/250)
Batch 0: 0/250 Accuracy: 94.000000
Batch 1: 50/250 Accuracy: 90.000000
Batch 2: 100/250 Accuracy: 92.000000
Batch 3: 150/250 Accuracy: 98.000000
Batch 4: 200/250 Accuracy: 94.000000
Test loss: 0.5732782175474475
Accuracy: 93.6
```

In [594]:
```
n = 200
idx = np.linspace(-0.5, 1.5, n)
x1, x2 = np.meshgrid(idx, idx)
new_X = np.zeros((n*n, 2))
new_X[:, 0] = x1.flatten()
```

```
new_X[:, 1] = x2.flatten()

new_y = net3.predict(new_X)

plot_boundary(new_X, new_y)
```

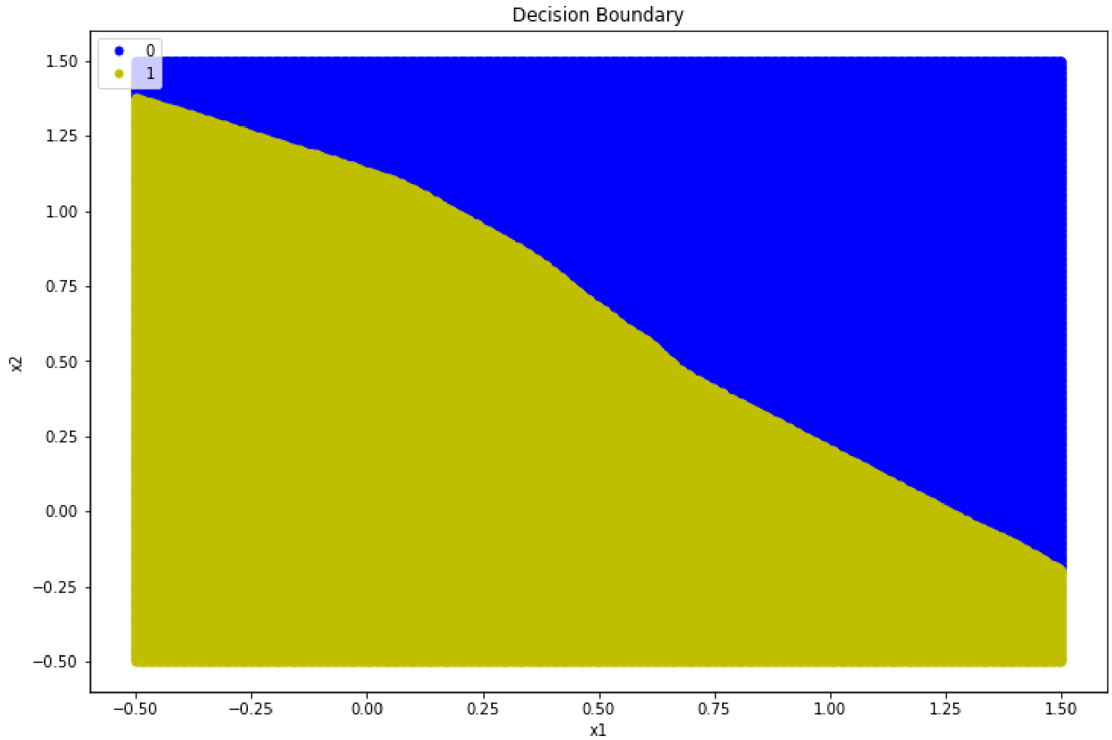


In [597]:
```
# Random initialization that works for network with 3 hidden units
seed = int(np.random.rand(1) * 10000)
seed = 3581
seed = 6524
print("Random seed:", seed)
np.random.seed(seed)

# Network parameters
batch_size = 50
num_epochs = 10
hidden_units = 16

# Initialize network
net16 = Network(hidden_units = hidden_units, lr = 0.1,
                batch_size = batch_size, debug = False)

# Split Data into training and test sets
```

```
data, labels, test_data, test_labels = split_data(X, y)

# Train the network for 'num_epochs'
for epoch in range(num_epochs):
    train(net16, epoch, data, labels, batch_size, debug = False)

test(net16, test_data, test_labels, batch_size, debug = True)
```

```
Random seed: 6524
Split (train/test): (4750/250)
Batch 0: 0/250 Accuracy: 62.000000
Batch 1: 50/250 Accuracy: 48.000000
Batch 2: 100/250 Accuracy: 70.000000
Batch 3: 150/250 Accuracy: 70.000000
Batch 4: 200/250 Accuracy: 60.000000
Test loss: 0.6531193933053229
Accuracy: 62.0
```

```
In [598]: n = 500
          idx = np.linspace(-0.5, 1.5, n)
          x1, x2 = np.meshgrid(idx, idx)
          new_X = np.zeros((n*n, 2))
          new_X[:, 0] = x1.flatten()
          new_X[:, 1] = x2.flatten()

          new_y = net16.predict(new_X)

          new_labels = net16.predict(test_data)
          plot_boundary(new_X, new_y)
```

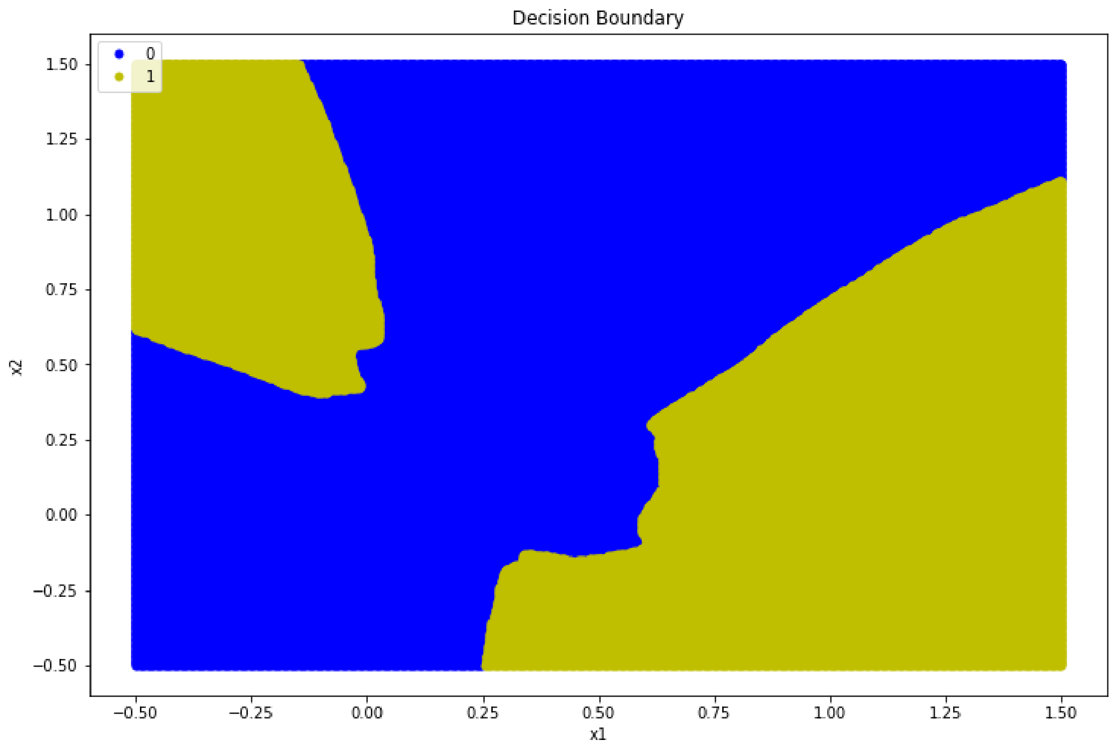


In [607]:
```
# Random initialization that works for network with 3 hidden units
seed = int(np.random.rand(1) * 10000)
seed = 3997
print("Random seed:", seed)
np.random.seed(seed)

# Network parameters
batch_size = 50
num_epochs = 4
hidden_units = 512

# Initialize network
net512 = Network(hidden_units = hidden_units, lr = 0.10,
              batch_size = batch_size, debug = False)

# Split Data into training and test sets
data, labels, test_data, test_labels = split_data(X, y)

# Train the network for 'num_epochs'
for epoch in range(num_epochs):
    train(net512, epoch, data, labels, batch_size,
          adaptive_lr = True, debug = True)
    test(net512, test_data, test_labels, batch_size, debug = True)
```

```
test(net512, test_data, test_labels, batch_size, debug = True)
```

```
Random seed: 3997
Split (train/test): (4750/250)
Epoch 0, Train loss: 7.4746
Epoch 0, Train Accuracy: 60.2947
Batch 0: 0/250 Accuracy: 66.000000
Batch 1: 50/250 Accuracy: 54.000000
Batch 2: 100/250 Accuracy: 56.000000
Batch 3: 150/250 Accuracy: 68.000000
Batch 4: 200/250 Accuracy: 60.000000
Test loss: 0.6487984018715431
Accuracy: 60.8
Epoch 1, Train loss: 0.6384
Epoch 1, Train Accuracy: 67.6421
Batch 0: 0/250 Accuracy: 82.000000
Batch 1: 50/250 Accuracy: 62.000000
Batch 2: 100/250 Accuracy: 68.000000
Batch 3: 150/250 Accuracy: 76.000000
Batch 4: 200/250 Accuracy: 74.000000
Test loss: 0.6372125502330925
Accuracy: 72.4
Epoch 2, Train loss: 0.6284
Epoch 2, Train Accuracy: 75.4526
Batch 0: 0/250 Accuracy: 82.000000
Batch 1: 50/250 Accuracy: 58.000000
Batch 2: 100/250 Accuracy: 62.000000
Batch 3: 150/250 Accuracy: 80.000000
Batch 4: 200/250 Accuracy: 72.000000
Test loss: 0.6405757812452638
Accuracy: 70.8
Epoch 3, Train loss: 0.6263
Epoch 3, Train Accuracy: 78.3158
Batch 0: 0/250 Accuracy: 86.000000
Batch 1: 50/250 Accuracy: 76.000000
Batch 2: 100/250 Accuracy: 82.000000
Batch 3: 150/250 Accuracy: 84.000000
Batch 4: 200/250 Accuracy: 80.000000
Test loss: 0.6305097199985602
Accuracy: 81.6
Batch 0: 0/250 Accuracy: 86.000000
Batch 1: 50/250 Accuracy: 76.000000
Batch 2: 100/250 Accuracy: 82.000000
Batch 3: 150/250 Accuracy: 84.000000
Batch 4: 200/250 Accuracy: 80.000000
Test loss: 0.6305097199985602
Accuracy: 81.6
```

In [608]:
```
n = 500
idx = np.linspace(-0.5, 1.5, n)
x1, x2 = np.meshgrid(idx, idx)
new_X = np.zeros((n*n, 2))
new_X[:, 0] = x1.flatten()
new_X[:, 1] = x2.flatten()

new_y = net512.predict(new_X)

plot_boundary(new_X, new_y)
```



In [609]:
```
class Network_cce():
    def __init__(self, in_units = 2, out_units = 1,
                 hidden_units = 3, lr = 0.1,
                 batch_size = 100, debug = False):

        # Network specifications
        self.in_units = in_units
        self.hidden_units = hidden_units
        self.out_units = out_units
        self.batch_size = batch_size
```

```
        self.lr = lr
        self.debug = debug

        # Initialize network parameters
        w1 = np.random.rand(hidden_units, self.in_units) - 0.5
        w2 = np.random.rand(self.out_units, hidden_units) - 0.5
        b1 = np.zeros((hidden_units, 1))
        b2 = np.zeros((self.out_units, 1))

        self.weights = [w1, w2]
        self.bias = [b1, b2]

        # Initialize batchwise layer outputs as empty
        self.linear1 = []
        self.relu1 = []
        self.eps = 0.00
        self.linear2 = []

        # Initialize batchwise gradients as empty
        self.allgrad_weight1 = []
        self.allgrad_weight2 = []
        self.allgrad_bias1 = []
        self.allgrad_bias2 = []

    # Feedforward for entire batch
    def forward(self, data, save = True):
        self.linear1 = []
        self.relu1 = []
        self.linear2 = []
        for idx in range(data.shape[0]):
            x = data[idx, ::]
            x = x.reshape(self.in_units, 1)
            self._forward(x, idx, save)
        return np.array(self.linear2)

    # Feedforward for a single data point
    def _forward(self, x, idx, save = True):

        linear1 = np.dot(self.weights[0], x) + self.bias[0]
        relu1 = leaky_relu(linear1, self.eps)
        linear2 = np.dot(self.weights[1], relu1) + self.bias[1]

        self.linear2.append(linear2)
        self.linear1.append(linear1)
        self.relu1.append(relu1)

        if self.debug and idx == 0:
            print("\n---- PRINT FORWARD DATA ----\n")
```

```
            print("x = ", x, x.shape)
            print("weights1 = ", self.weights[0], self.weights[0].shape)
            print("linear1 = ", linear1, linear1.shape)
            print("relu1 = ", relu1, relu1.shape)
            print("weights2 = ", self.weights[1], self.weights[1].shape)
            print("linear2 = ", linear2, linear2.shape)

        return linear2

    # Backprop for entire batch
    def backward(self, L, data, labels):
        if (data.shape[0] != labels.shape[0]):
            print("Data and labels shapes mismatch")
            return None

        self.allgrad_weight1 = []
        self.allgrad_weight2 = []
        self.allgrad_bias1 = []
        self.allgrad_bias2 = []
        for idx in range(data.shape[0]):
            x = data[idx, ::]
            x = x.reshape(self.in_units, 1)
            y = labels[idx]
            self._backward(L, x, y, idx)

        self.grad_weights = [np.mean(self.allgrad_weight1),
                             np.mean(self.allgrad_weight2)]
        self.grad_bias = [np.mean(self.allgrad_bias1),
                          np.mean(self.allgrad_bias2)]

    # Backprop for a single data point
    def _backward(self, L, x, y, idx):
        eps = 0.01
        grad_bias2 = sigmoid(self.linear2[idx]) - y
#         grad_bias2 = 2*(y - self.linear2[idx])
        grad_weight2 = np.multiply(grad_bias2, self.relu1[idx])
        grad_relu1 = grad_bias2 * self.weights[1]
        grad_linear1 = grad_relu1.T
        grad_linear1[np.where(self.relu1[idx] <= 0)] *= self.eps
        grad_weight1 = np.dot(grad_linear1, x.T)
        grad_bias1 = grad_linear1

        self.allgrad_weight1.append(grad_weight1)
        self.allgrad_weight2.append(grad_weight2)
        self.allgrad_bias1.append(grad_bias1)
        self.allgrad_bias2.append(grad_bias2)

        if (self.debug and idx == 0):
```

```
                print("\n---- PRINT BACKWARD DATA ----\n")
                print("linear2 = ", self.linear2[idx])
                print("r:", r, "y:", y)
                print("weight2:", self.weights[1])
                print("grad_bias2:", grad_bias2)
                print("grad_weight2:", grad_weight2)
                print("weight1:", self.weights[0])
                print("grad_relu1:", grad_relu1)
                print("grad_linear1:", grad_linear1)
                print("grad_weight1:", grad_weight1)
                print("grad_bias1:", grad_bias1)

        # Update mini batch weights
        def update_weights(self, lr = None):
            if lr is not None:
                self.lr = lr
            self.weights[0] -= (self.lr * self.grad_weights[0])
            self.weights[1] -= (self.lr * self.grad_weights[1])
            self.bias[0] -= (self.lr * self.grad_bias[0])
            self.bias[1] -= (self.lr * self.grad_bias[1])

        # Predict labels
        def predict(self, data):
            out = self.forward(data, save = False)
            out[out >= 0] = 1
            out[out < 0] = 0
            return out

        # Compute accuracy
        def accuracy(self, data, labels):
            predicted_labels = self.predict(data).reshape(-1, 1)
            matches = np.where(predicted_labels == labels)
            return np.sum(labels == predicted_labels) / labels.shape[0] * 100.0
```

In [610]:
```
def categorical_cross_entropy(data, labels, out):
    n = data.shape[0]
    eps = 0.01
    L = - np.multiply(labels, np.log(sigmoid(out)))
    L = L - np.multiply(1.0 - labels, np.log(1.0 - sigmoid(out)))
    L = np.sum(L)/n
    return L
```

In [614]:
```
# Train an epoch
def train_cce(net, epoch, data, labels, batch_size,
              adaptive_lr = False, debug = False):
    train_n = data.shape[0]
    train_idx = np.arange(train_n)
    np.random.shuffle(train_idx)
```

```
    train_loss = []
    train_acc = []
    for i in range(0, train_n, batch_size):
        batch_idx = train_idx[i:i + batch_size]
        batch_X = data[batch_idx, ::]
        batch_y = labels[batch_idx, ::]

        # Feedforward
        a = net.forward(batch_X)
        loss = categorical_cross_entropy(batch_X, batch_y, a.reshape(batch_size, 1))
#          loss = quadratic(batch_X, batch_y, a.reshape(batch_size, 1))
        train_loss.append(loss)
        acc = net.accuracy(batch_X, batch_y)
        train_acc.append(acc)
        net.backward(loss, batch_X, batch_y)
        if adaptive_lr:
            lr = np.exp(-epoch * 0.1)
        else:
            lr = None
        net.update_weights(lr)

    if debug:
        print("Epoch %d, Train loss: %.4f" %(epoch, np.mean(train_loss)))
        print("Epoch %d, Train Accuracy: %.4f" %(epoch, np.mean(train_acc)))
```

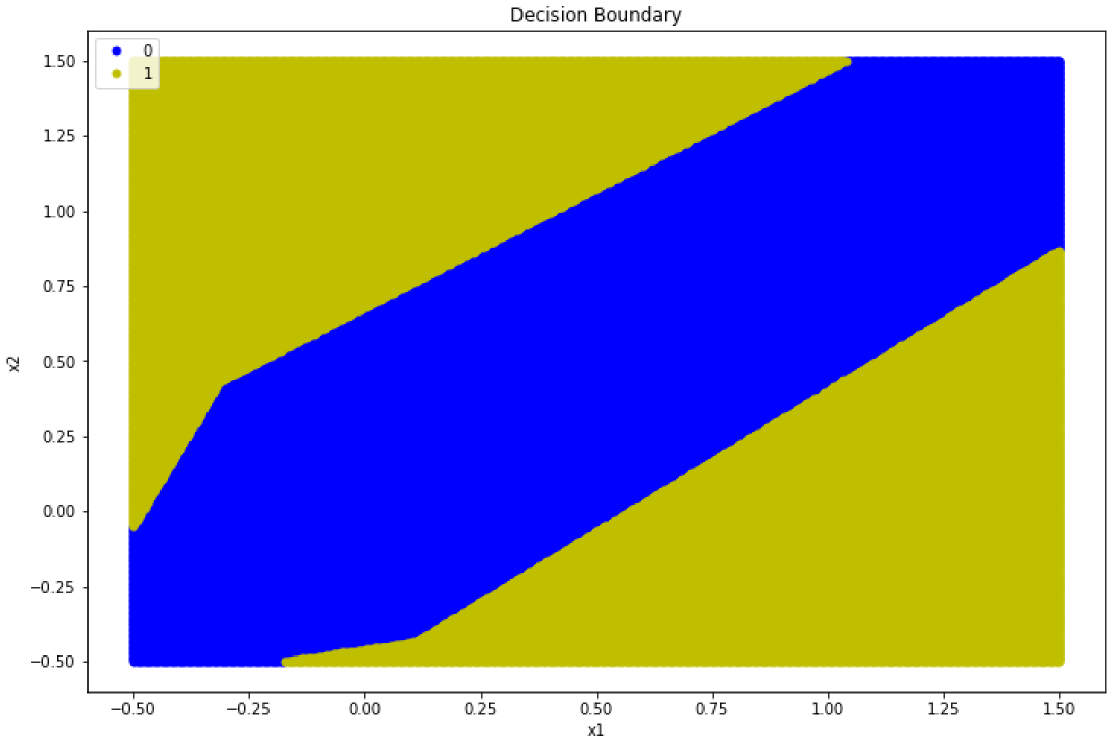
In [613]:
```
# Test validation data
def test_cce(net, test_data, test_labels, batch_size, debug = True):
    test_n = test_data.shape[0]
    test_idx = np.arange(test_n)
    test_loss = []
    test_acc = []
    for i in range(0, test_n, batch_size):
        batch_idx = test_idx[i:i + batch_size]
        batch_X = test_data[batch_idx, :]
        batch_y = test_labels[batch_idx, ::]
        a = net.forward(batch_X)
        loss = categorical_cross_entropy(batch_X, batch_y, a.reshape(batch_size, 1))
        test_loss.append(loss)
        acc = net.accuracy(batch_X, batch_y)
        test_acc.append(acc)
        if debug:
            print("Batch %d: %d/%d Accuracy: %f" %(i/batch_size, i, test_n, acc))

    if debug:
        print("Test loss:", np.mean(test_loss))
        print("Accuracy:", np.mean(test_acc))
```

In [615]:
```
# Random initialization that works for network with 3 hidden units
```

```
seed = int(np.random.rand(1) * 10000)
seed = 6524
print("Random seed:", seed)
np.random.seed(seed)

# Network parameters
batch_size = 50
num_epochs = 10
hidden_units = 3

# Initialize network
net3_cce = Network_cce(hidden_units = hidden_units, lr = 0.1,
                       out_units = 1, batch_size = batch_size,
                       debug = False)

# Split Data into training and test sets
data, labels, test_data, test_labels = split_data(X, y)

# Train the network for 'num_epochs'
for epoch in range(num_epochs):
    train_cce(net3_cce, epoch, data, labels, batch_size, debug = False)

test_cce(net3_cce, test_data, test_labels, batch_size, debug = True)
```

```
Random seed: 6524
Split (train/test): (4750/250)
Batch 0: 0/250 Accuracy: 94.000000
Batch 1: 50/250 Accuracy: 90.000000
Batch 2: 100/250 Accuracy: 94.000000
Batch 3: 150/250 Accuracy: 98.000000
Batch 4: 200/250 Accuracy: 94.000000
Test loss: 0.5813633907518609
Accuracy: 94.0
```

```
In [616]: n = 500
          idx = np.linspace(-0.5, 1.5, n)
          x1, x2 = np.meshgrid(idx, idx)
          new_X = np.zeros((n*n, 2))
          new_X[:, 0] = x1.flatten()
          new_X[:, 1] = x2.flatten()

          new_y = net3_cce.predict(new_X)

          plot_boundary(new_X, new_y)
```

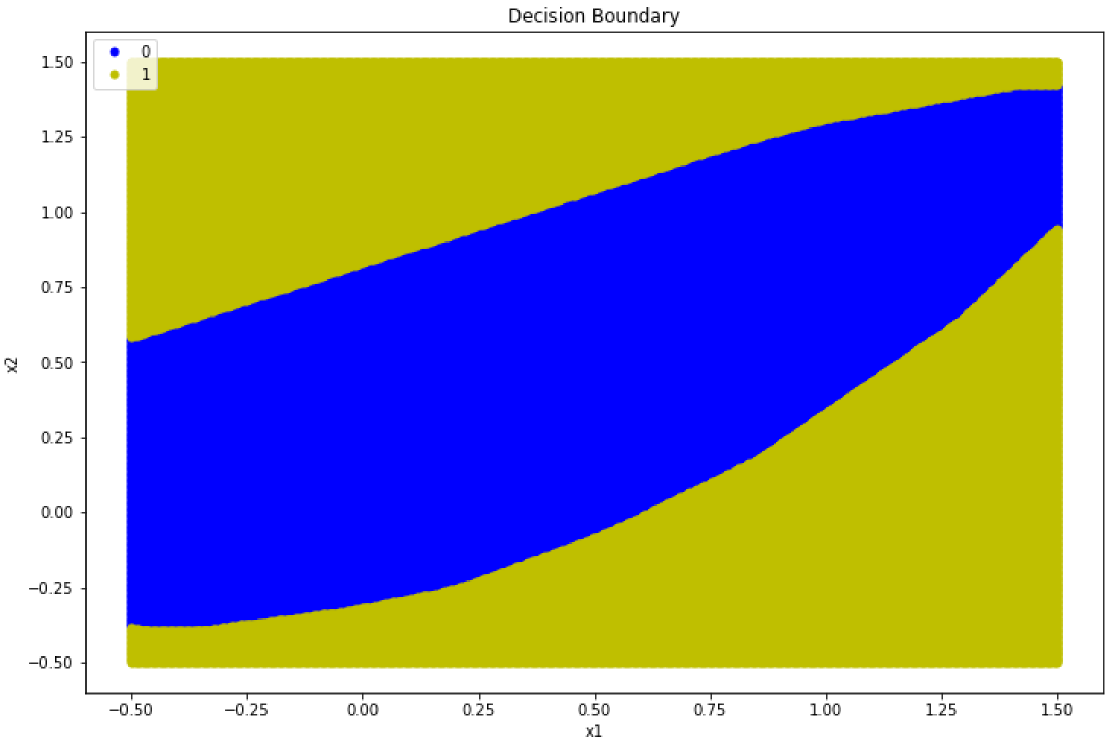


In [617]:
```
# Random initialization that works for network with 3 hidden units
seed = int(np.random.rand(1) * 10000)
# seed = 6524
# seed = 5792
# seed = 6257
# seed = 2013
# seed = 9928
seed = 3581
print("Random seed:", seed)
np.random.seed(seed)

# Network parameters
batch_size = 100
num_epochs = 10
hidden_units = 16

# Initialize network
net16_cce = Network_cce(hidden_units = hidden_units, lr = 0.2,
                        out_units = 1, batch_size = batch_size, debug = False)

# Split Data into training and test sets
data, labels, test_data, test_labels = split_data(X, y)
```

```
# Train the network for 'num_epochs'
for epoch in range(num_epochs):
    train(net16_cce, epoch, data, labels, batch_size, debug = False)

test(net16_cce, test_data, test_labels, batch_size, debug = True)
```

```
Random seed: 3581
Split (train/test): (4500/500)
Batch 0: 0/500 Accuracy: 87.000000
Batch 1: 100/500 Accuracy: 93.000000
Batch 2: 200/500 Accuracy: 89.000000
Batch 3: 300/500 Accuracy: 84.000000
Batch 4: 400/500 Accuracy: 90.000000
Test loss: 0.5537342012391172
Accuracy: 88.6
```

In [618]:
```
n = 500
idx = np.linspace(-0.5, 1.5, n)
x1, x2 = np.meshgrid(idx, idx)
new_X = np.zeros((n*n, 2))
new_X[:, 0] = x1.flatten()
new_X[:, 1] = x2.flatten()

new_y = net16_cce.predict(new_X)

plot_boundary(new_X, new_y)
```

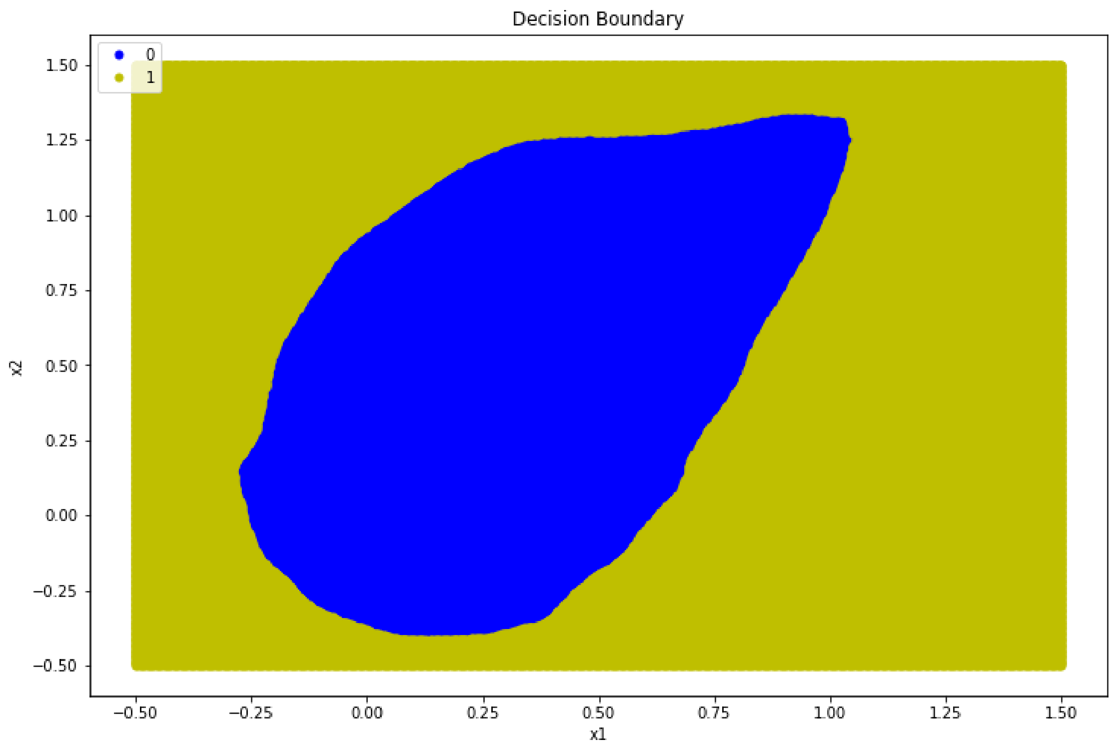


In [625]:
```
# Random initialization that works for network with 3 hidden units
seed = int(np.random.rand(1) * 10000)
# seed = 8128
# seed = 7900
# seed = 2247
# seed = 9928
# seed = 3581
# seed = 1386
seed = 6118
print("Random seed:", seed)
np.random.seed(seed)

# Network parameters
batch_size = 50
num_epochs = 10
hidden_units = 512

# Initialize network
net512_cce = Network_cce(hidden_units = hidden_units, lr = 0.1,
              out_units = 1, batch_size = batch_size, debug = False)

# Split Data into training and test sets
data, labels, test_data, test_labels = split_data(X, y)

# Train the network for 'num_epochs'
for epoch in range(num_epochs):
    train(net512_cce, epoch, data, labels, batch_size,
          adaptive_lr = True, debug = False)
#     test(net512_cce, test_data, test_labels, batch_size, debug = True)

test(net512_cce, test_data, test_labels, batch_size, debug = True)
```

```
Random seed: 6118
Split (train/test): (4750/250)
Batch 0: 0/250 Accuracy: 74.000000
Batch 1: 50/250 Accuracy: 62.000000
Batch 2: 100/250 Accuracy: 70.000000
Batch 3: 150/250 Accuracy: 78.000000
Batch 4: 200/250 Accuracy: 74.000000
Test loss: 0.5836245096929604
Accuracy: 71.6
```

In [627]:
```
n = 500
idx = np.linspace(-0.5, 1.5, n)
```

```
x1, x2 = np.meshgrid(idx, idx)
new_X = np.zeros((n*n, 2))
new_X[:, 0] = x1.flatten()
new_X[:, 1] = x2.flatten()

new_y = net512_cce.predict(new_X)

plot_boundary(new_X, new_y)
```



In [628]:
```
data = pd.DataFrame(np.zeros((6000, 4)), columns=['x1', 'x2', 'x3', 'y'])

# Let's make up some noisy XOR data to use to build our binary classifier
for i in range(len(data.index)):
    x1 = 1.0 * random.randint(0, 1)
    x2 = 1.0 * random.randint(0, 1)
    x3 = 1.0 * random.randint(0, 1)
    xor = 1.0 * np.logical_xor(np.logical_xor(x1 == 1, x2 == 1), x3 == 1)
    x1 = x1 + 0.18 * np.random.normal()
    x2 = x2 + 0.18 * np.random.normal()
    x3 = x3 + 0.18 * np.random.normal()
    data.iloc[i,0] = x1
    data.iloc[i,1] = x2
    data.iloc[i,2] = x2
```

```
        data.iloc[i,3] = xor

data.head()

# set X (training data) and y (target variable)
cols = data.shape[1]
Xf = data.iloc[:, 0:cols - 1]
yf = data.iloc[:, cols - 1:cols]

# The cost function is expecting numpy matrices so we need to convert X and y before
Xf = np.matrix(Xf.values)
yf = np.matrix(yf.values)
```

In [632]:
```
# Random initialization that works for network with 3 hidden units
seed = int(np.random.rand(1) * 1000)
# seed = 6524
print("Random seed:", seed)
np.random.seed(seed)

# Network parameters
batch_size = 20
num_epochs = 10
hidden_units = 3

# Initialize network
net3_f = Network(in_units = Xf.shape[1], hidden_units = hidden_units,
                    out_units = yf.shape[1], lr = 0.1,
                    batch_size = batch_size, debug = False)

# Split Data into training and test sets
data, labels, test_data, test_labels = split_data(Xf, yf, test_split = 20)
print(data.shape, test_data.shape)

# Train the network for 'num_epochs'
for epoch in range(num_epochs):
    train(net3_f, epoch, data, labels, batch_size,
          adaptive_lr = False, debug = False)

test(net3_f, test_data, test_labels, batch_size, debug = True)
```

```
Random seed: 306
Split (train/test): (5600/400)
(5600, 3) (400, 3)
Batch 0: 0/400 Accuracy: 45.000000
Batch 1: 20/400 Accuracy: 55.000000
Batch 2: 40/400 Accuracy: 45.000000
Batch 3: 60/400 Accuracy: 35.000000
Batch 4: 80/400 Accuracy: 50.000000
```

```
Batch 5: 100/400 Accuracy: 45.000000
Batch 6: 120/400 Accuracy: 40.000000
Batch 7: 140/400 Accuracy: 60.000000
Batch 8: 160/400 Accuracy: 45.000000
Batch 9: 180/400 Accuracy: 40.000000
Batch 10: 200/400 Accuracy: 35.000000
Batch 11: 220/400 Accuracy: 55.000000
Batch 12: 240/400 Accuracy: 70.000000
Batch 13: 260/400 Accuracy: 45.000000
Batch 14: 280/400 Accuracy: 50.000000
Batch 15: 300/400 Accuracy: 60.000000
Batch 16: 320/400 Accuracy: 45.000000
Batch 17: 340/400 Accuracy: 55.000000
Batch 18: 360/400 Accuracy: 15.000000
Batch 19: 380/400 Accuracy: 50.000000
Test loss: 0.6982227977229182
Accuracy: 47.0
```

In [630]:
```python
# Random initialization that works for network with 3 hidden units
seed = int(np.random.rand(1) * 1000)
# seed = 6524
print("Random seed:", seed)
np.random.seed(seed)

# Network parameters
batch_size = 50
num_epochs = 10
hidden_units = 16

# Initialize network
net16_f = Network(in_units = Xf.shape[1], hidden_units = hidden_units,
                     out_units = yf.shape[1], lr = 0.5,
                     batch_size = batch_size, debug = False)

# Split Data into training and test sets
data, labels, test_data, test_labels = split_data(Xf, yf, test_split = 20)
print(data.shape, test_data.shape)

# Train the network for 'num_epochs'
for epoch in range(num_epochs):
    train(net16_f, epoch, data, labels, batch_size,
          adaptive_lr = True, debug = False)
#     test(net16_f, test_data, test_labels, batch_size, debug = True)

test(net16_f, test_data, test_labels, batch_size, debug = True)
```

```
Random seed: 42
Split (train/test): (5000/1000)
```

```
(5000, 3) (1000, 3)
Batch 0: 0/1000 Accuracy: 54.000000
Batch 1: 50/1000 Accuracy: 54.000000
Batch 2: 100/1000 Accuracy: 52.000000
Batch 3: 150/1000 Accuracy: 56.000000
Batch 4: 200/1000 Accuracy: 58.000000
Batch 5: 250/1000 Accuracy: 58.000000
Batch 6: 300/1000 Accuracy: 48.000000
Batch 7: 350/1000 Accuracy: 58.000000
Batch 8: 400/1000 Accuracy: 48.000000
Batch 9: 450/1000 Accuracy: 40.000000
Batch 10: 500/1000 Accuracy: 58.000000
Batch 11: 550/1000 Accuracy: 40.000000
Batch 12: 600/1000 Accuracy: 44.000000
Batch 13: 650/1000 Accuracy: 50.000000
Batch 14: 700/1000 Accuracy: 42.000000
Batch 15: 750/1000 Accuracy: 46.000000
Batch 16: 800/1000 Accuracy: 56.000000
Batch 17: 850/1000 Accuracy: 54.000000
Batch 18: 900/1000 Accuracy: 52.000000
Batch 19: 950/1000 Accuracy: 54.000000
Test loss: 0.7010135698934545
Accuracy: 51.1
```

In [631]:
```python
# Random initialization that works for network with 3 hidden units
seed = int(np.random.rand(1) * 10000)
# seed = 6524
print("Random seed:", seed)
np.random.seed(seed)

# Network parameters
batch_size = 50
num_epochs = 10
hidden_units = 512

# Initialize network
net512_f = Network(in_units = Xf.shape[1], hidden_units = hidden_units,
                     out_units = yf.shape[1], lr = 0.1,
                     batch_size = batch_size, debug = False)

# Split Data into training and test sets
data, labels, test_data, test_labels = split_data(Xf, yf, test_split = 20)
print(data.shape, test_data.shape)

# Train the network for 'num_epochs'
for epoch in range(num_epochs):
    train(net512_f, epoch, data, labels, batch_size,
```

```python
                adaptive_lr = False, debug = False)
#     print(net512_f.weights)
#     test(net512_f, test_data, test_labels, batch_size, debug = True)

test(net512_f, test_data, test_labels, batch_size, debug = True)
```

```
Random seed: 5298
Split (train/test): (5000/1000)
(5000, 3) (1000, 3)
Batch 0: 0/1000 Accuracy: 56.000000
Batch 1: 50/1000 Accuracy: 54.000000
Batch 2: 100/1000 Accuracy: 58.000000
Batch 3: 150/1000 Accuracy: 44.000000
Batch 4: 200/1000 Accuracy: 38.000000
Batch 5: 250/1000 Accuracy: 36.000000
Batch 6: 300/1000 Accuracy: 52.000000
Batch 7: 350/1000 Accuracy: 48.000000
Batch 8: 400/1000 Accuracy: 46.000000
Batch 9: 450/1000 Accuracy: 50.000000
Batch 10: 500/1000 Accuracy: 56.000000
Batch 11: 550/1000 Accuracy: 64.000000
Batch 12: 600/1000 Accuracy: 50.000000
Batch 13: 650/1000 Accuracy: 54.000000
Batch 14: 700/1000 Accuracy: 52.000000
Batch 15: 750/1000 Accuracy: 50.000000
Batch 16: 800/1000 Accuracy: 54.000000
Batch 17: 850/1000 Accuracy: 52.000000
Batch 18: 900/1000 Accuracy: 46.000000
Batch 19: 950/1000 Accuracy: 56.000000
Test loss: 0.7332692589848414
Accuracy: 50.8
```